The phase diagram of QCD

Thermodynamics, order parameters & dynamics

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The Functional Renormalization Group

Lecture notes Saalburg summer school

JMP, Jacqueline Bonnet, Stefan Rechenberger

Material

Lecture notes

hand-written

Non-perturbative methods in gauge theories

Critical phenomena

Topical reviews

Collection of reviews & lecture notes on the FRG & DSE

Structure of the FRG: Aspects of the FRG JMP ’05, Annals Phys.322:2831-2915,2007

talks

The FRG approach to gauge theories & applications to QCD JMP, ERG 2012 Aussois

Aspects of the QCD phase diagram and the EoS B.-J. Schaefer, CompStar 2012 School Zadar

Schladming 2011: Physics at all scales: The Renormalization Group

Outline

- (I) Introduction to the phase diagram of QCD & functional methods
- (II) Phase structure of QCD at finite temperature
- (III) Phase diagram of QCD
- (IV) Dynamics

(I) Introduction to the phase diagram of QCD & funMethods

- **Phase diagram of QCD**
  - Perturbative QCD & asymptotic freedom
  - Confinement
  - Chiral symmetry breaking

- **Functional methods for QCD**
  - FRG, DSE, 2PI
  - Phase structure with the FRG & optimisation
  - FRG for QCD & dynamical hadronisation

(II) Phase structure of QCD at finite temperature

Yang-Mills theory & QCD at T=0

- **Yang-Mills theory at finite temperature**
  - Confinement
  - Thermodynamics

- **Phase structure of QCD at finite temperature**
  - Order parameter
  - Comparison with other methods

(III) Phase diagram of QCD

- **Phase structure at imaginary chemical potential**
  - Imaginary chemical potential & Roberge-Weiss symmetry
  - Dual order parameters
  - Chiral versus confinement-deconfinement temperatures

- **Phase structure at finite density**
  - Chiral versus confinement-deconfinement temperatures
  - Phase structure with QCD-improved effective models
  - High density phases: To be or not to be

(IV) Dynamics

- **Turbulence in gauge theories**
  - **Abelian Higgs model & beyond**
- **Transport in YM & QCD**
  - **Spectral functions**
  - **transport coefficients**

(I) Introduction to the phase diagram of QCD & funMethods

- **Phase diagram of QCD**
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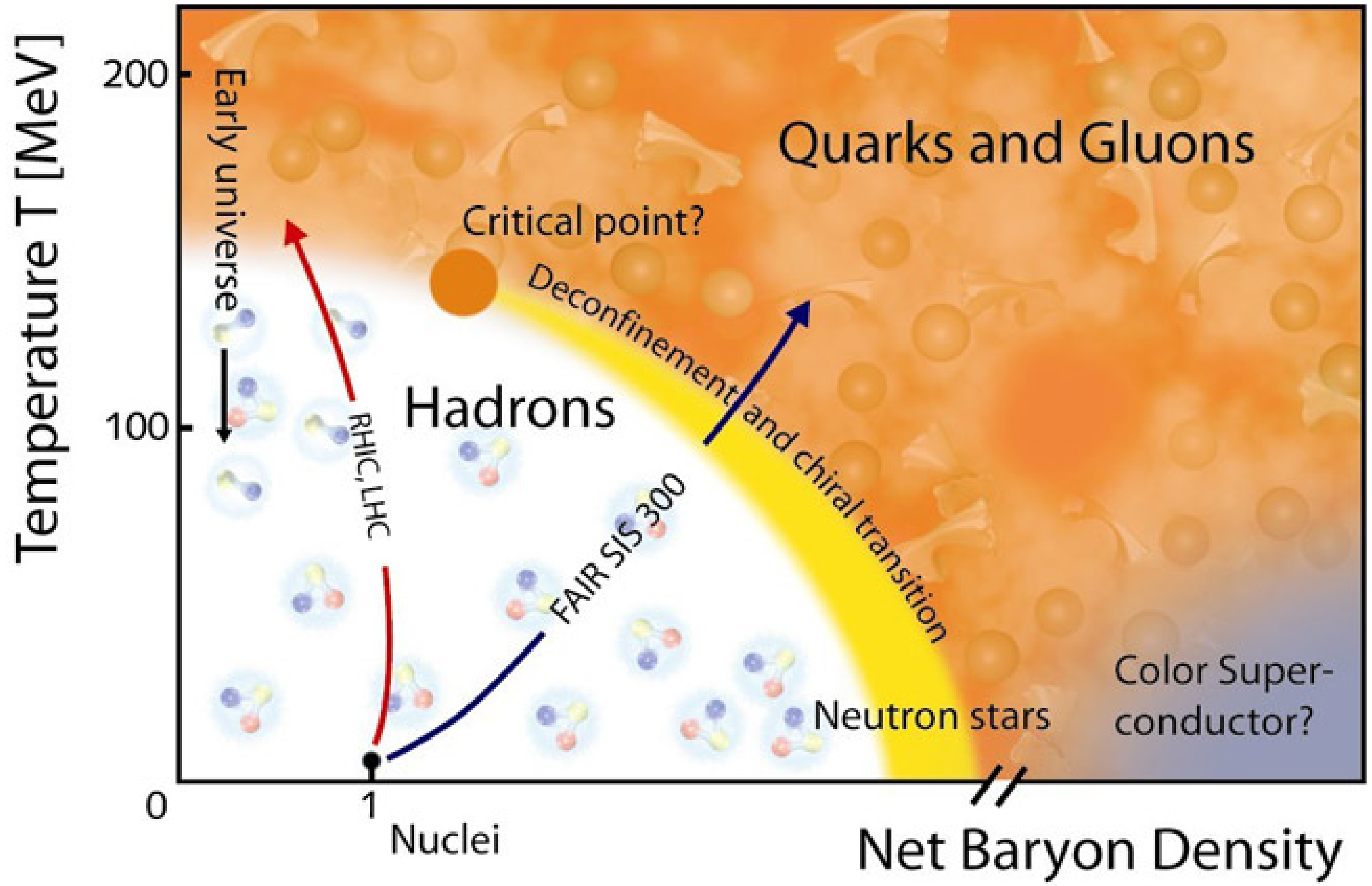
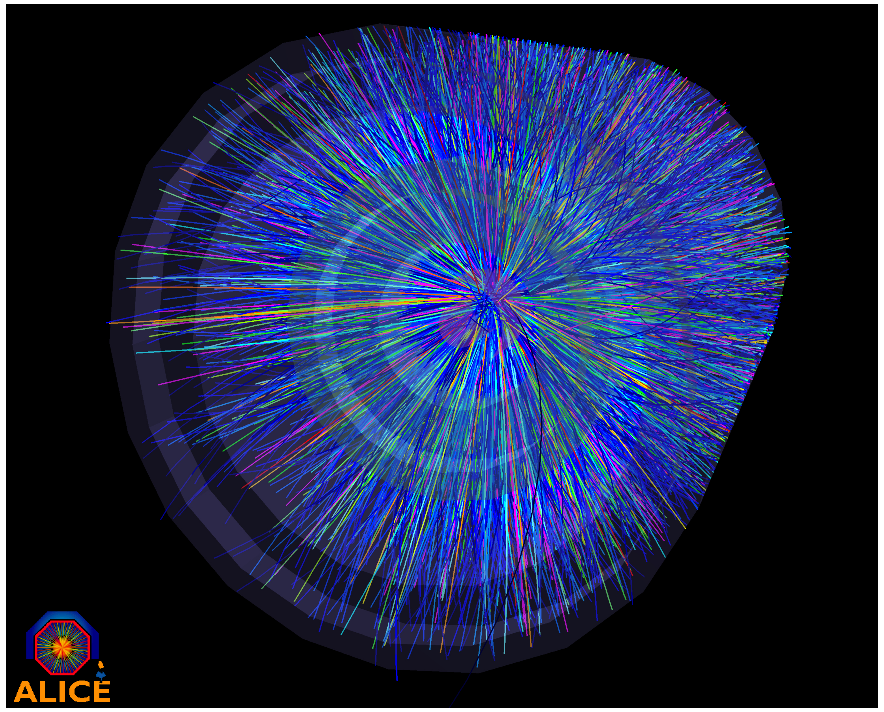
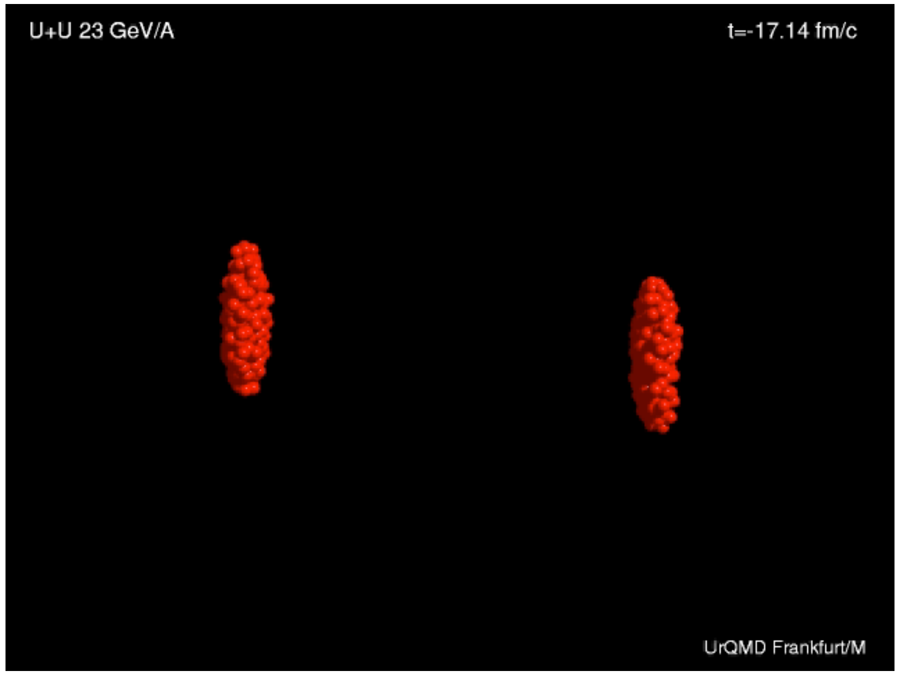
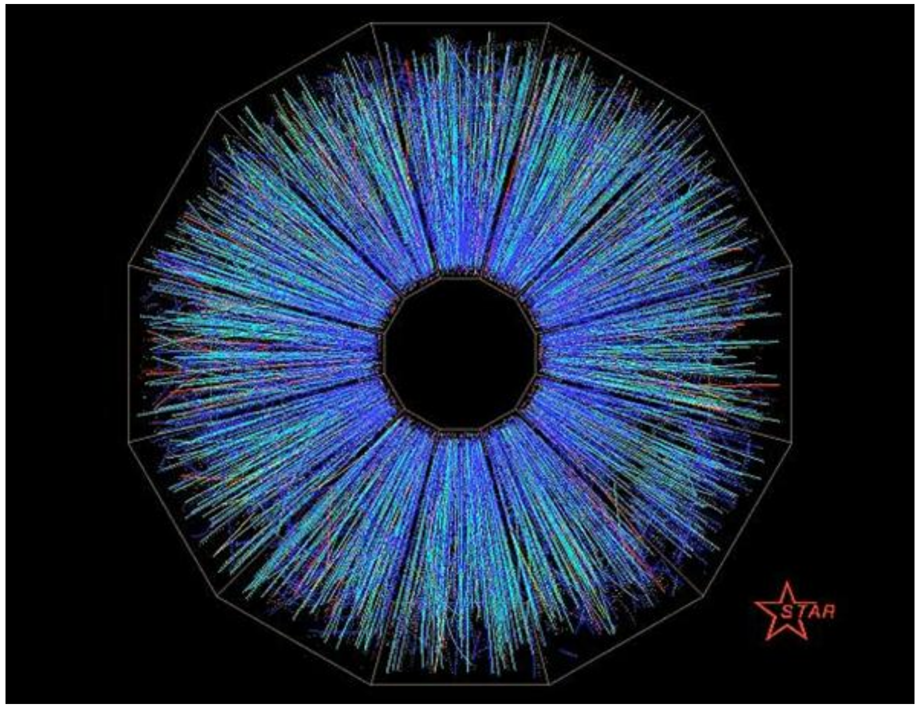
Heavy ion collisions







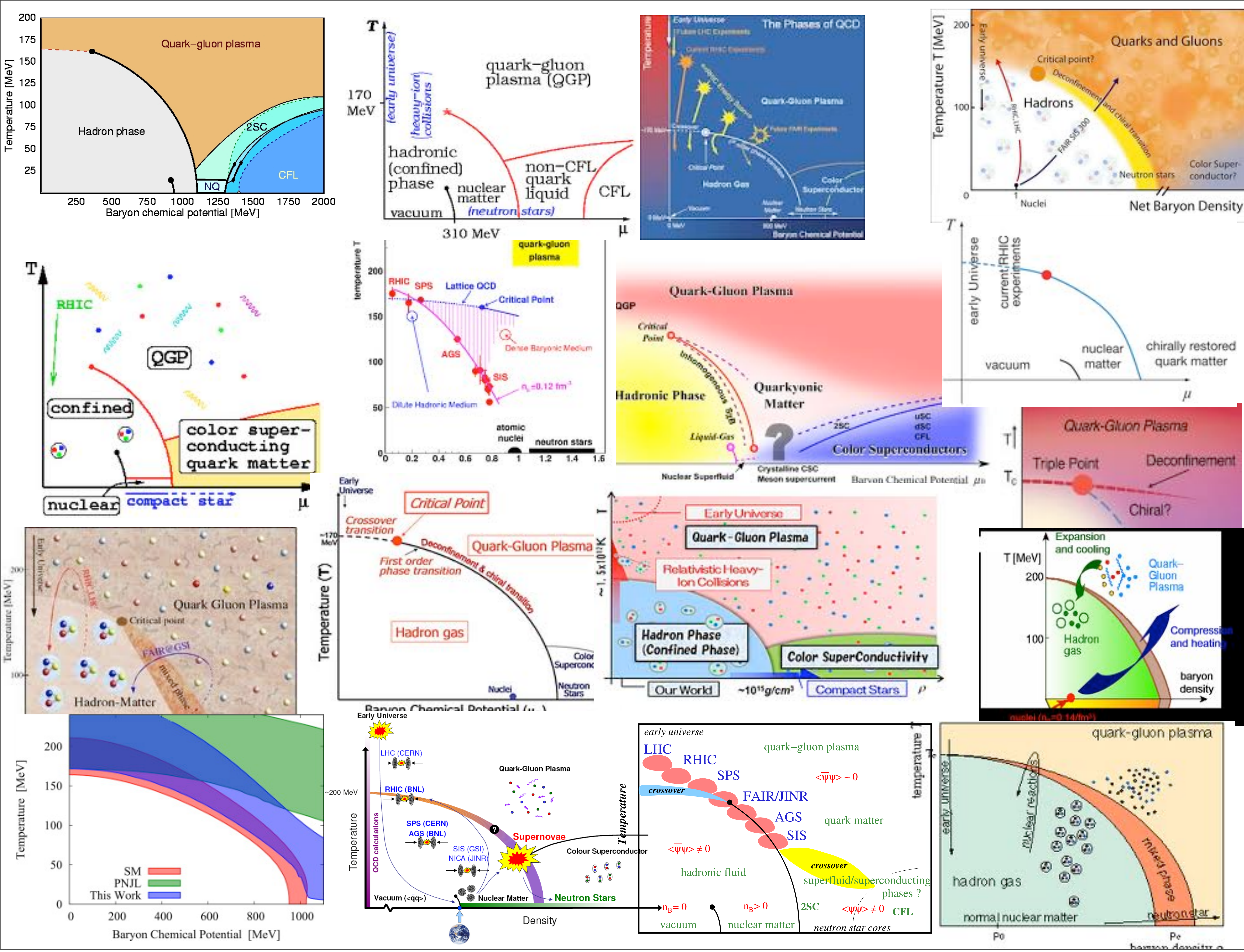


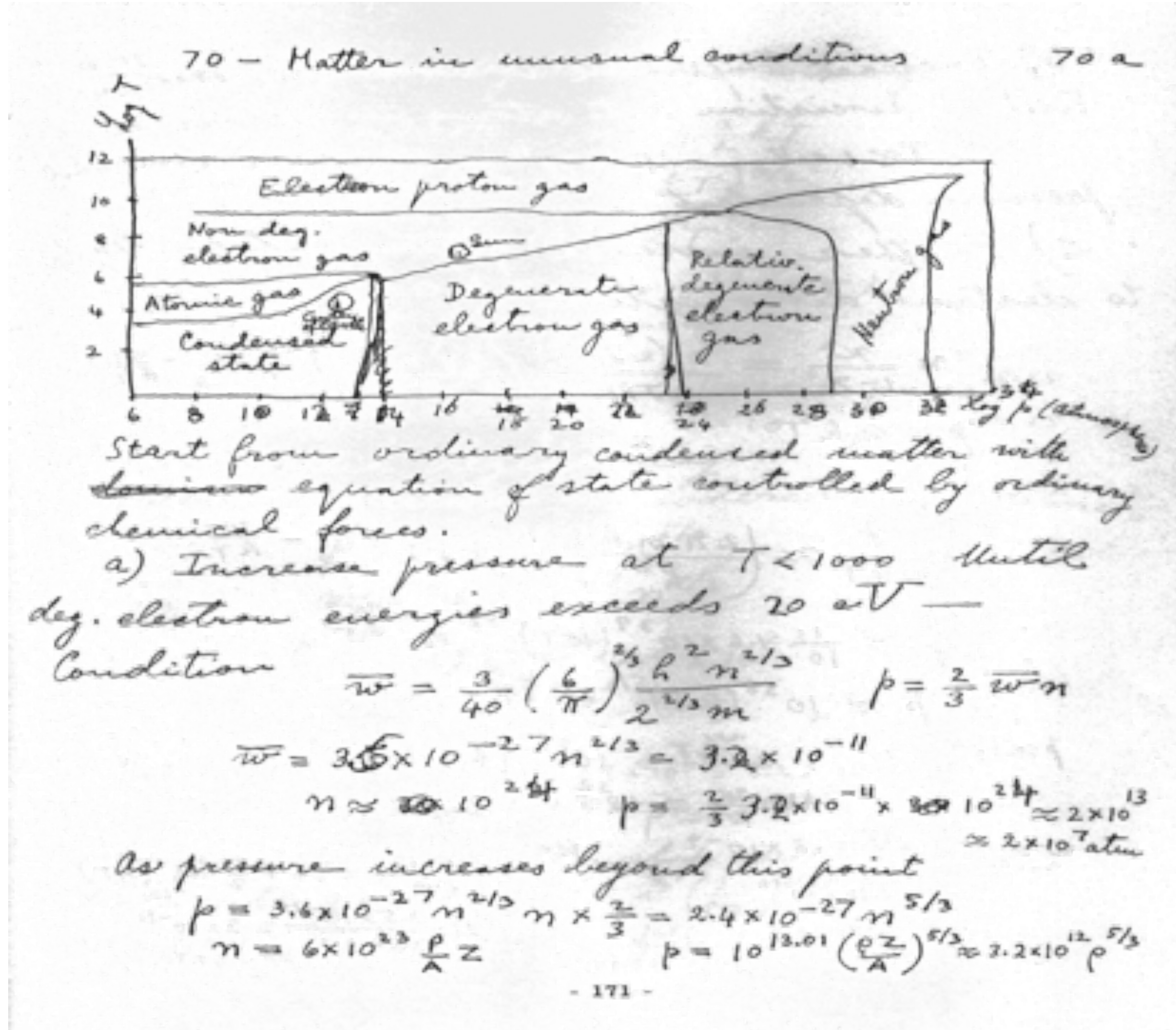
UrQMD Frankfurt/M

ALICE, LHC

Simulation of a heavy ion collision

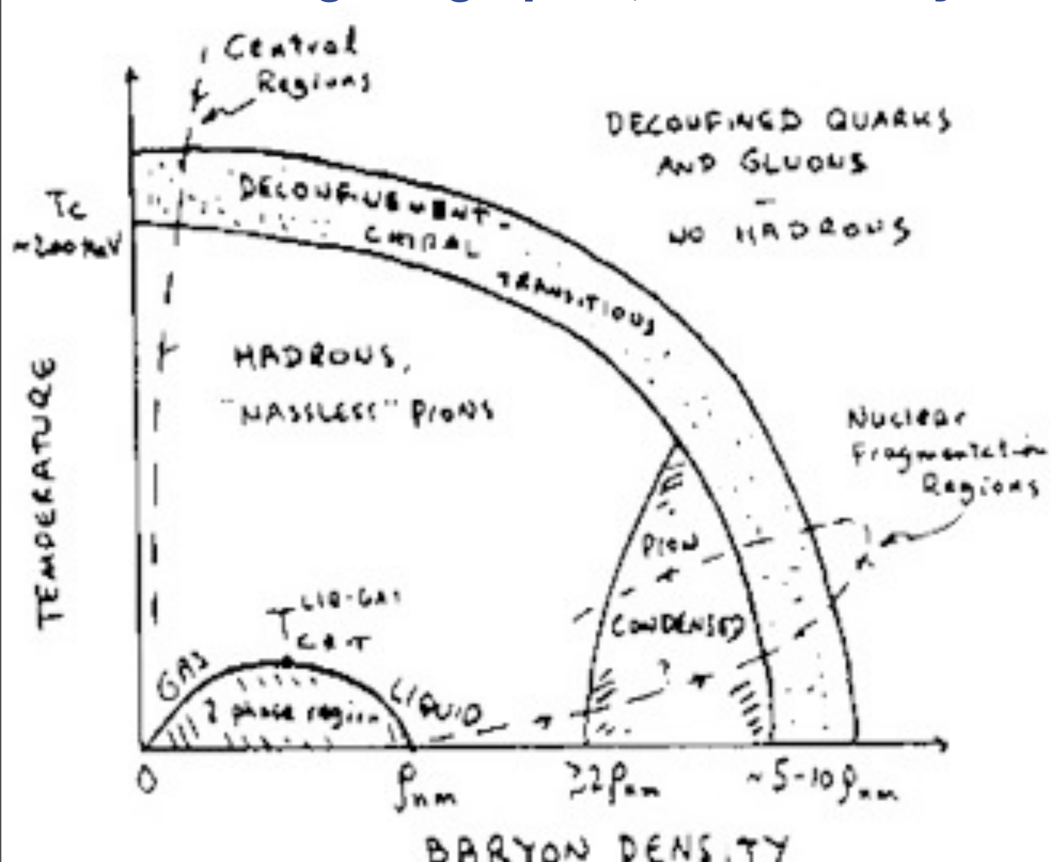
STAR, RHIC


Quark–gluon plasma
Hadron phase
2SC
CFL
NQ
Temperature [MeV]
Baryon chemical potential [MeV]
quark–gluon plasma (QGP)
170 MeV
(early universe)
(heavy-ion collisions)
hadronic (confined) phase
nuclear matter
non-CFL quark liquid
CFL
vacuum
(neutron stars)
310 MeV
The Phases of QCD
Quark-Gluon Plasma
Hadron Gas
Color Superconductor
Baryon Chemical Potential
Quarks and Gluons
Critical point?
Hadrons
Deconfinement and chiral transition
Neutron stars
Color Super-conductor?
Nuclei
Net Baryon Density
Temperature T [MeV]
RHIC
QGP
confined
color super-conducting quark matter
nuclear
compact star
quark-gluon plasma
RHIC SPS Lattice QCD
Critical Point
AGS
SIS
Dense Baryonic Medium
Dilute Hadronic Medium
atomic nuclei
neutron stars
Quark-Gluon Plasma
Critical Point
Hadronic Phase
Quarkyonic Matter
Liquid-Gas
Color Superconductors
Nuclear Superfluid
Crystalline CSC
Meson supercurrent
Baryon Chemical Potential
early Universe
current RHIC experiments
nuclear matter
chirally restored quark matter
vacuum
Quark-Gluon Plasma
Triple Point
Deconfinement
Chiral?
Early Universe
Critical Point
Crossover transition
Deconfinement & chiral transition
First order phase transition
Quark-Gluon Plasma
Hadron gas
Color Superconductor
Nuclei
Neutron Stars
Temperature (T)
Baryon Chemical Potential
Early Universe
Quark-Gluon Plasma
Relativistic Heavy-Ion Collisions
Hadron Phase (Confined Phase)
Color SuperConductivity
Our World
Compact Stars
Expansion and cooling
T [MeV]
Quark-Gluon Plasma
Hadron gas
Compression and heating
baryon density
Early Universe
Quark Gluon Plasma
Critical point
FAIR@GSI
mixed phase
Hadron-Matter
Temperature [MeV]
SM
PNJL
This Work
Baryon Chemical Potential [MeV]
Early Universe
LHC (CERN)
RHIC (BNL)
SPS (CERN)
AGS (BNL)
SIS (GSI)
NICA (JINR)
QCD calculations
Quark-Gluon Plasma
Supernovae
Colour Superconductor
Vacuum
Nuclear Matter
Neutron Stars
Temperature
Density
early universe
LHC
RHIC
SPS
FAIR/JINR
AGS
SIS
crossover
quark–gluon plasma
quark matter
hadronic fluid
superfluid/superconducting phases ?
2SC
CFL
vacuum
nuclear matter
neutron star cores
temperature
quark-gluon plasma
early universe
nuclear reactions
hadron gas
mixed phase
normal nuclear matter
neutron star

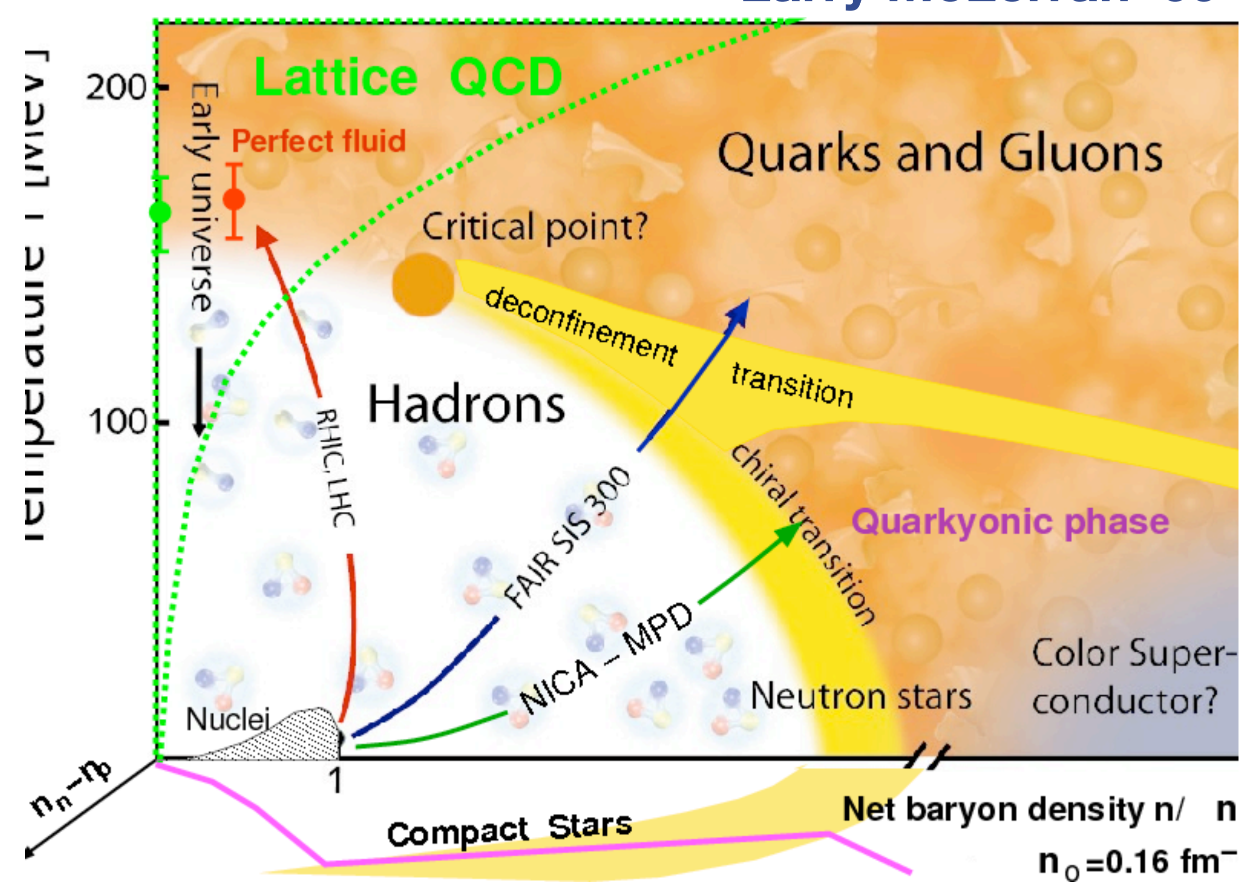


1953 Enrico Fermi

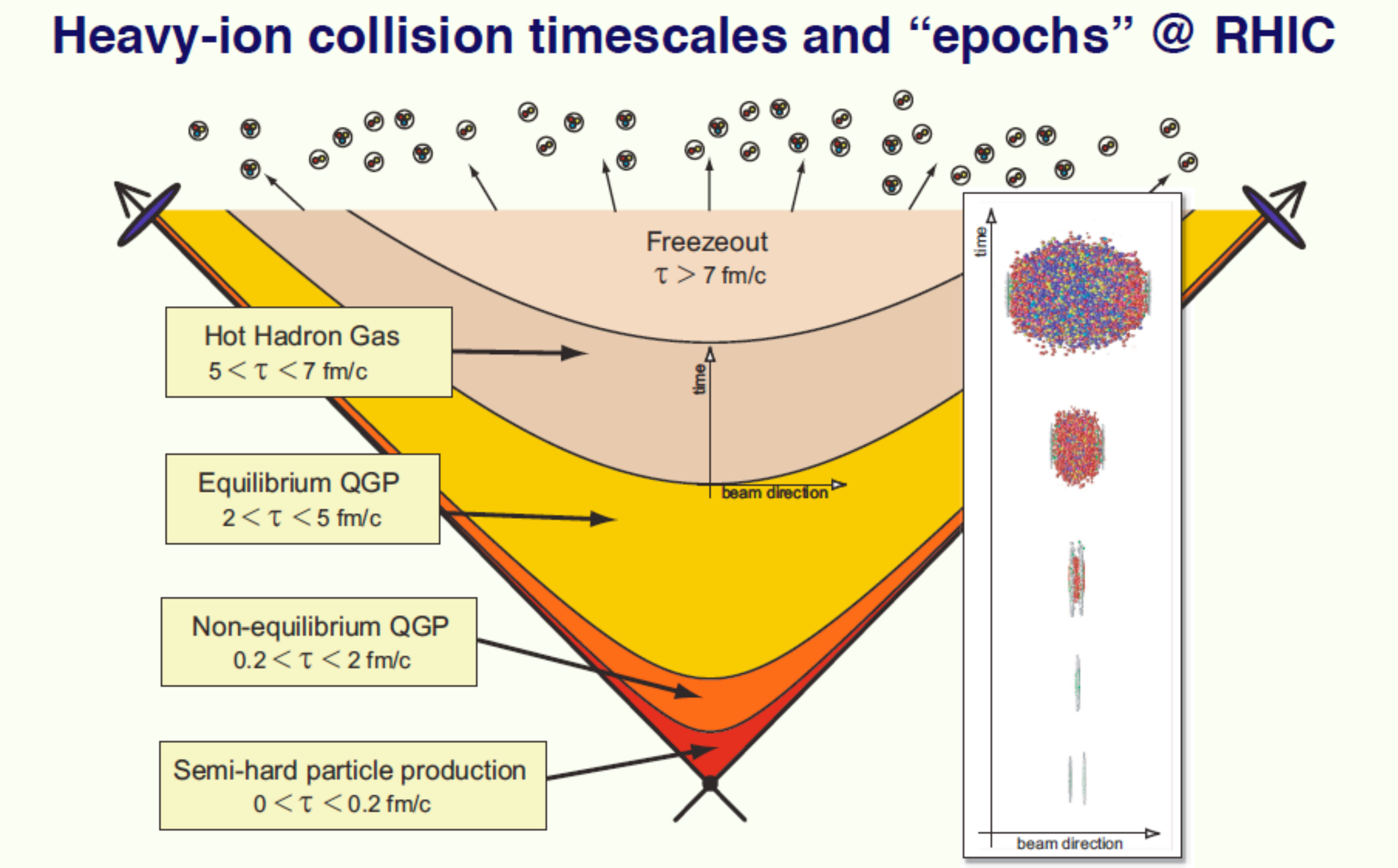
1983 US long range plan, Gordon Baym



Larry McLerran '09



Heavy ion collisions

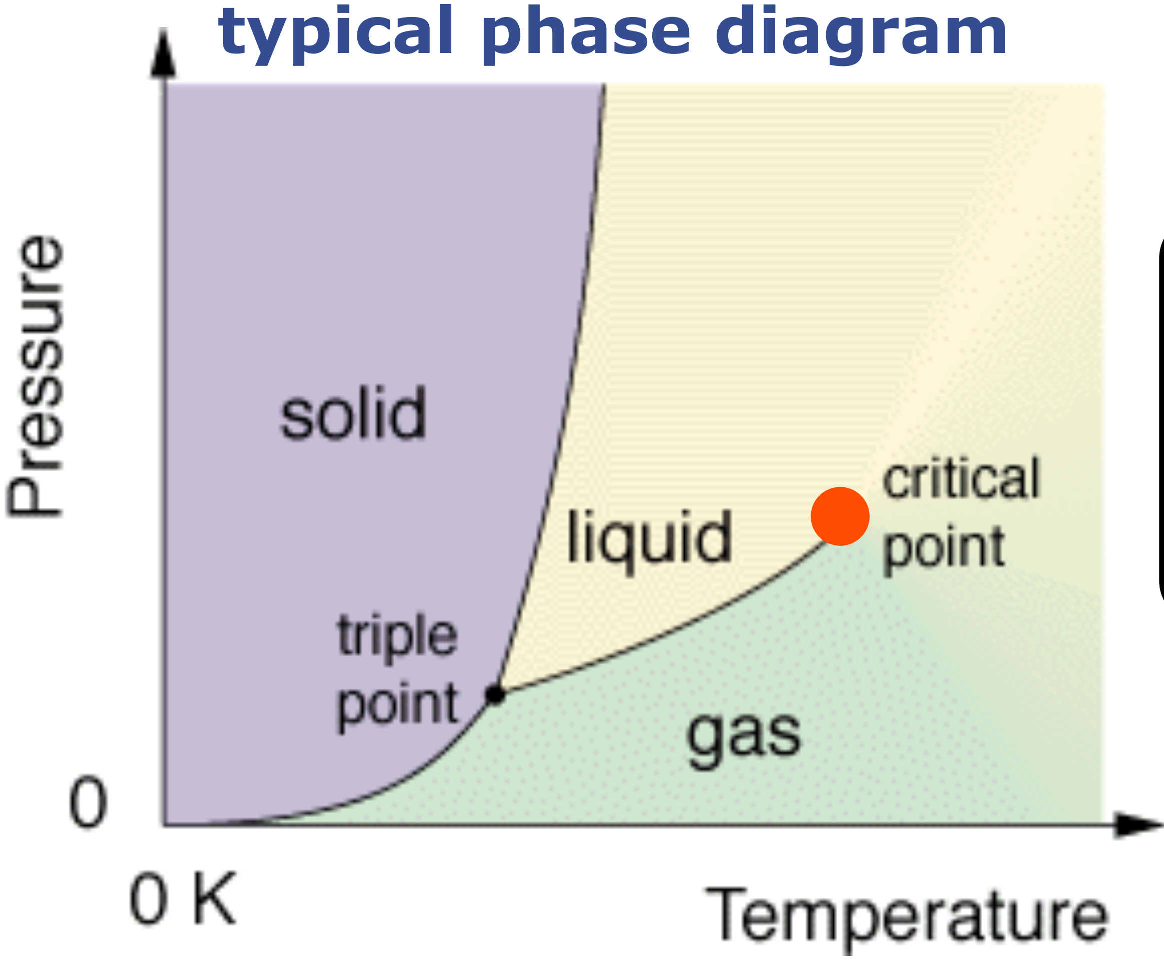


*1 fm/c $\simeq 3 \times 10^{-24}$ seconds

Strickland

Phase diagrams & order parameters

typical phase diagram



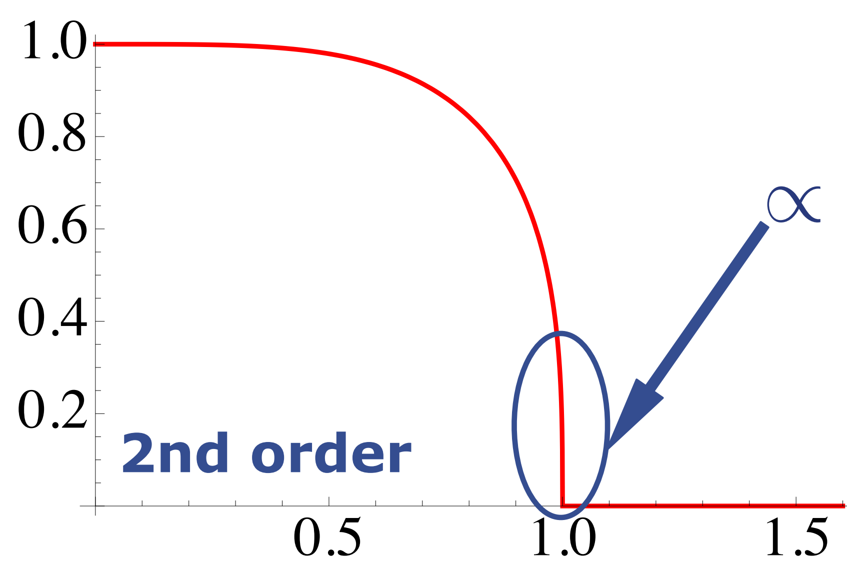
http://ltl.tkk.fi/research/theory/TypicalPD.gif

Order parameter: density n

density jumps	1st order phase transition
derivative of density jumps	2nd order phase transition
density smooth	cross-over

Ising model in 3d: $(\downarrow \uparrow)$-spin system

Order parameter: $\langle \uparrow \rangle$



$$\frac{\langle \uparrow \rangle}{\langle \uparrow \rangle_0}$$

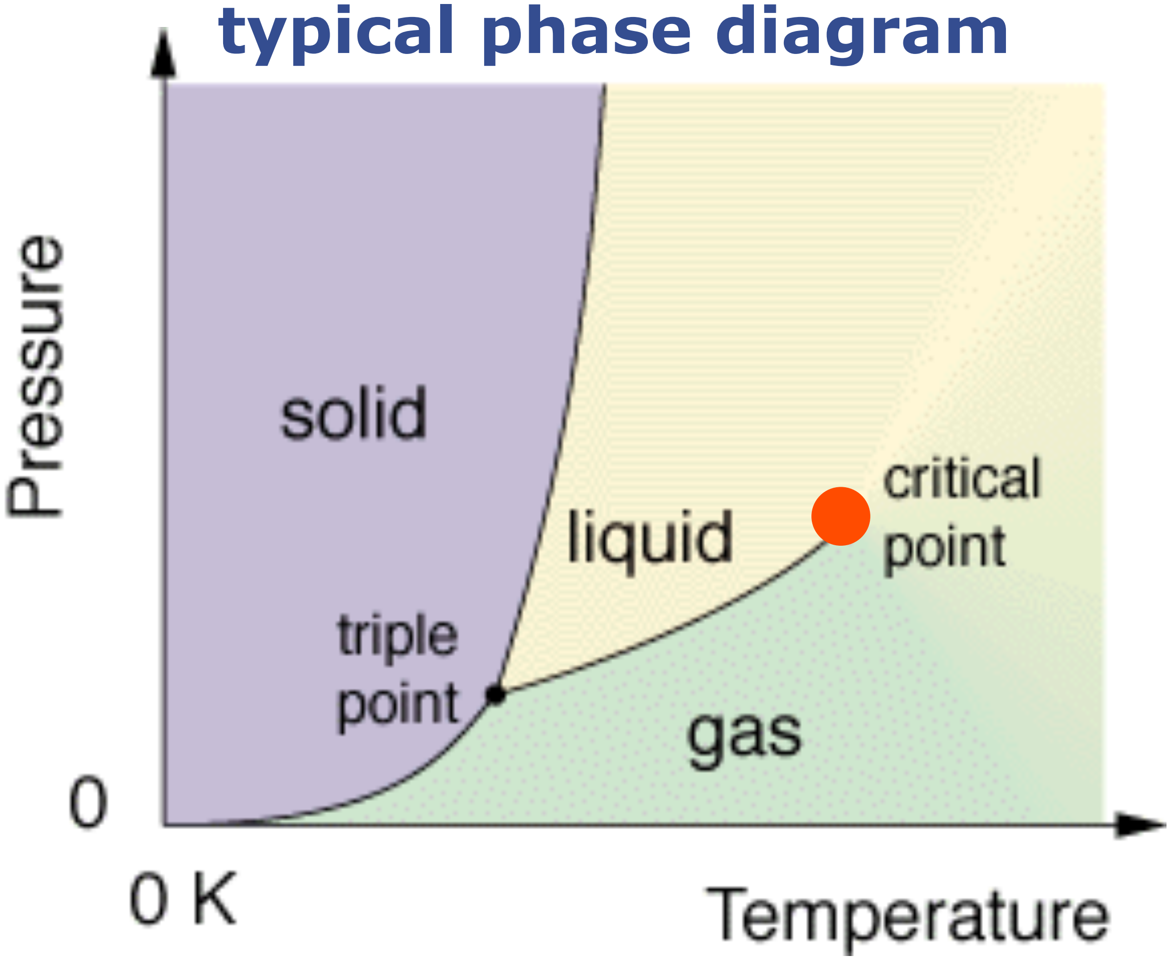
$$\frac{T}{T_c}$$

universal for $\mathbf{Z_2}$

$$\propto (1 - T/T_c)^{0.325\ldots}$$

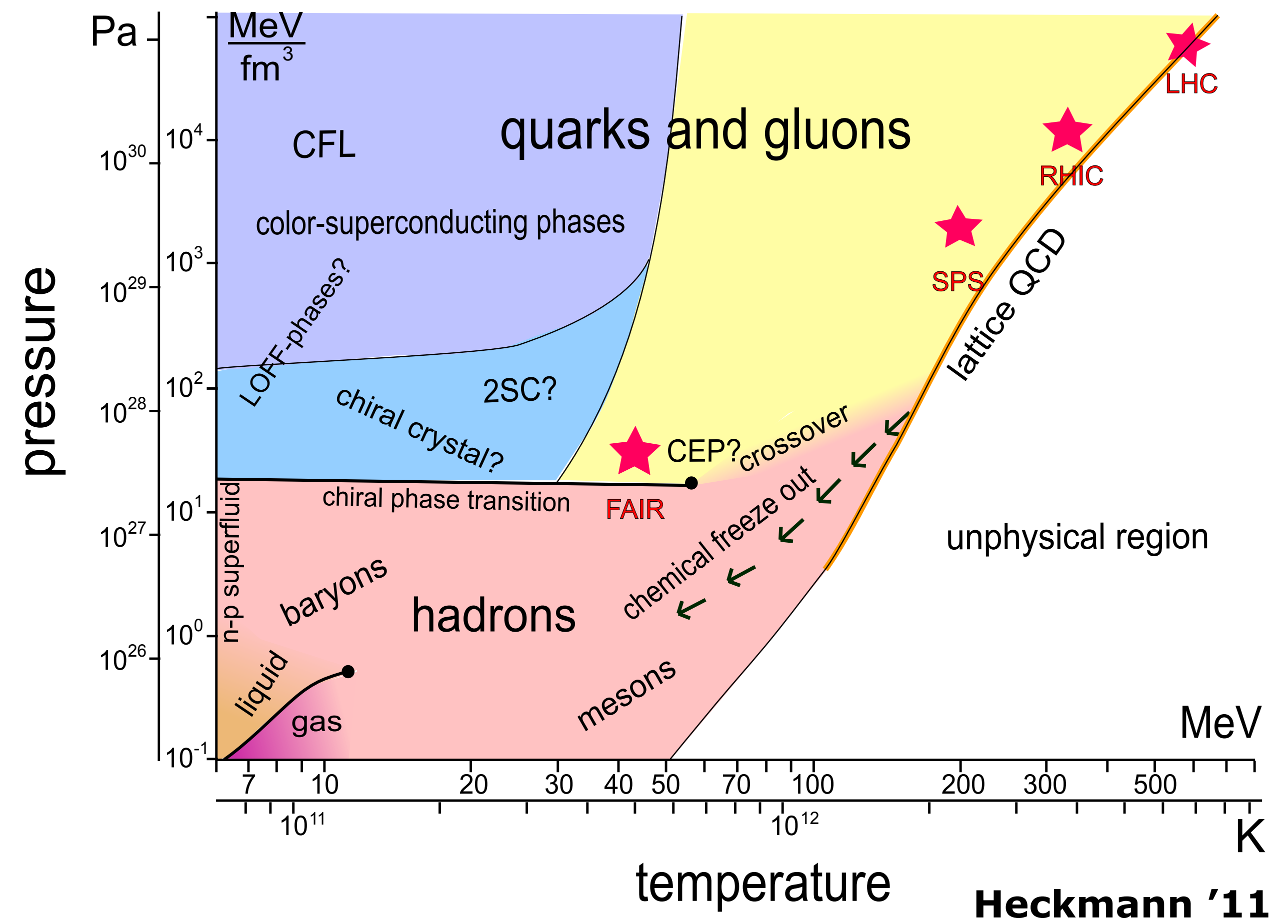
Phase diagrams & order parameters

typical phase diagram



http://ltl.tkk.fi/research/theory/TypicalPD.gif

phase diagram of QCD



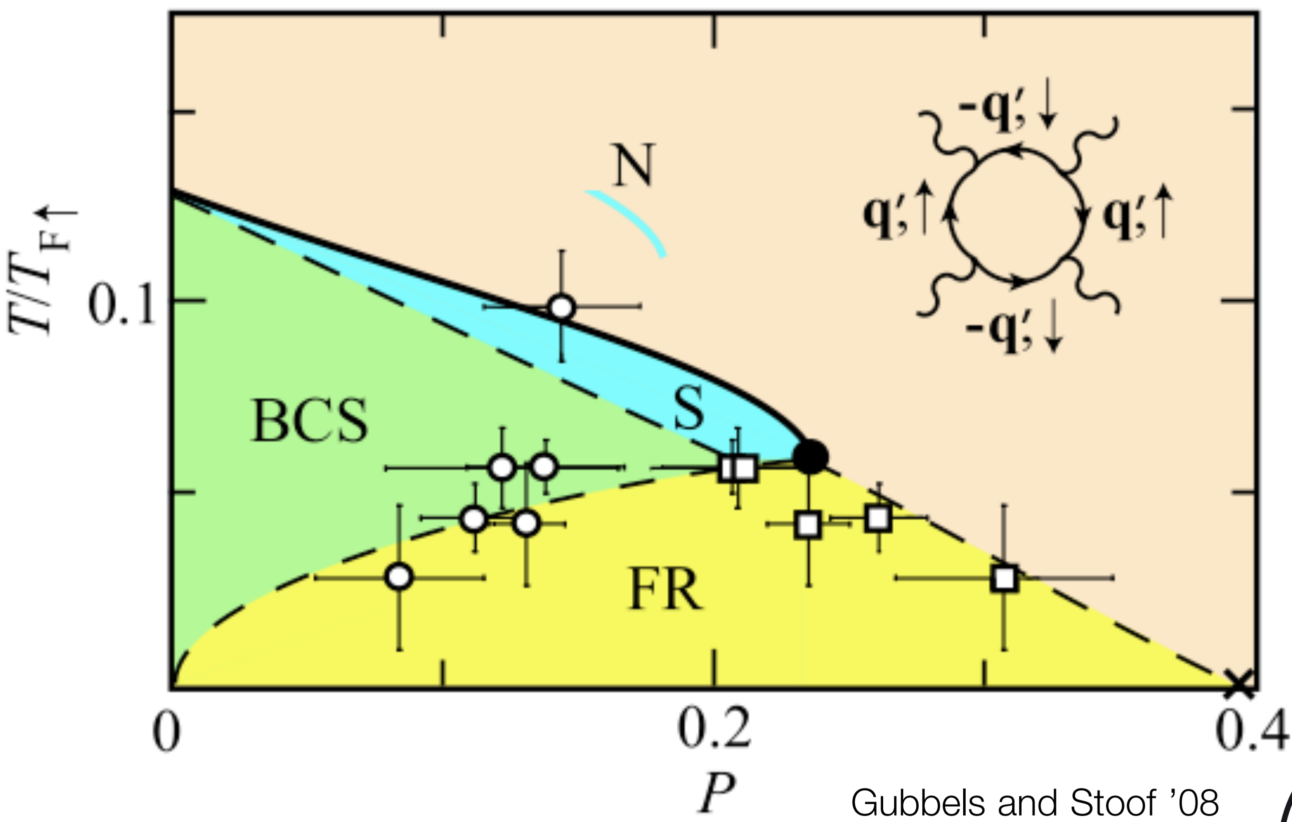
Phases in QCD

quarks massless - massive

quarks confined - deconfined

Phase diagrams & order parameters

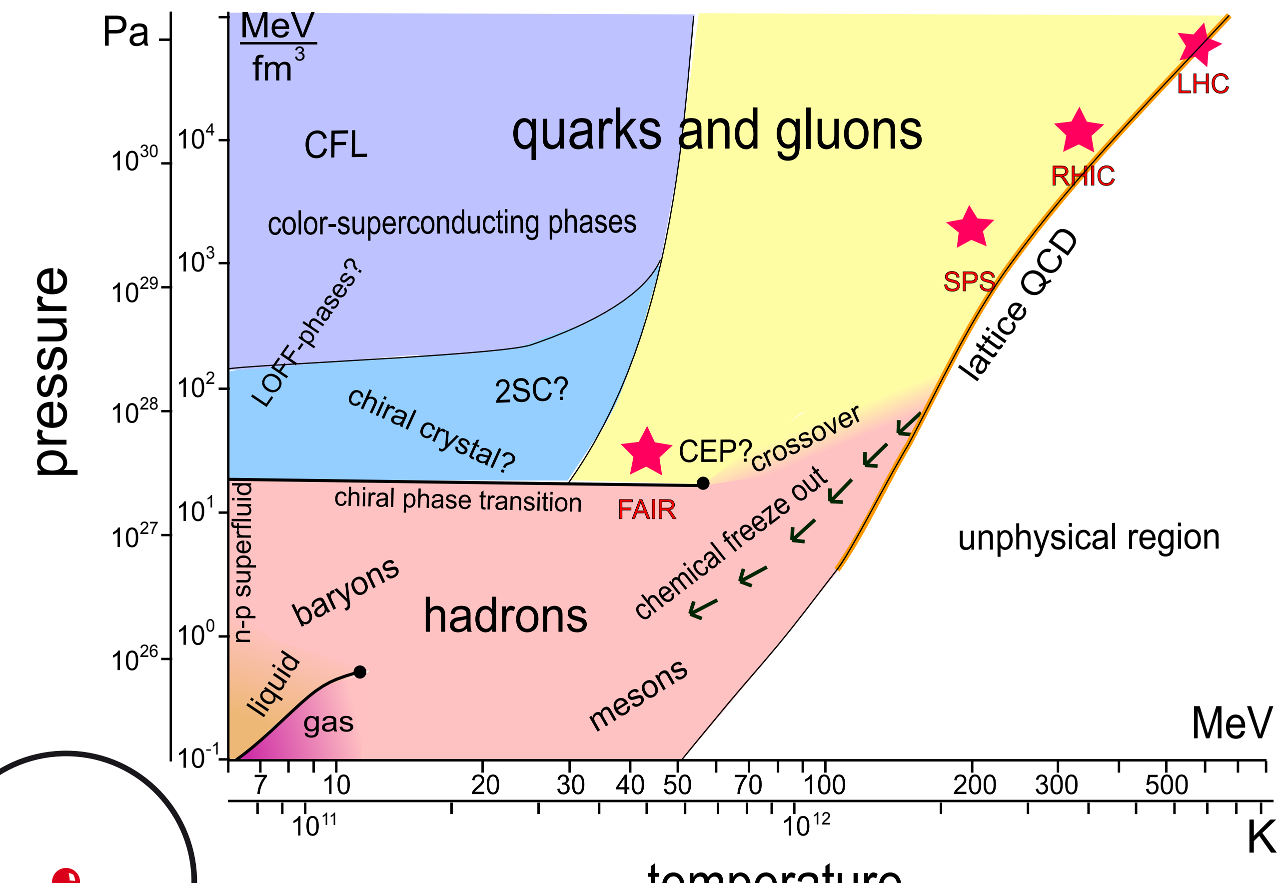
Phase diagram of cold atoms



Gubbels and Stoof '08

see talk of I. Boettcher

phase diagram of QCD



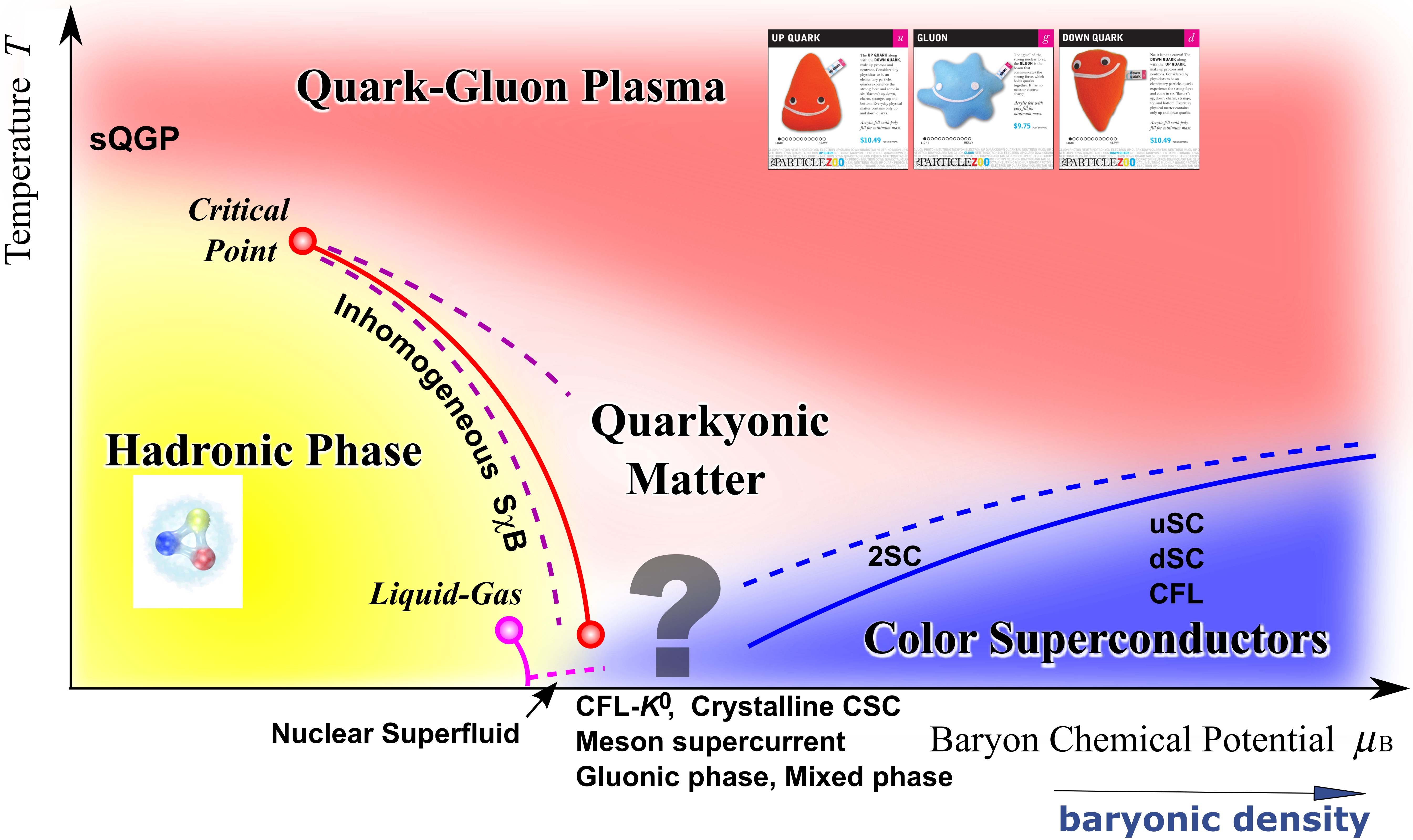
Heckmann '11

Phases in QCD

quarks massless - massive

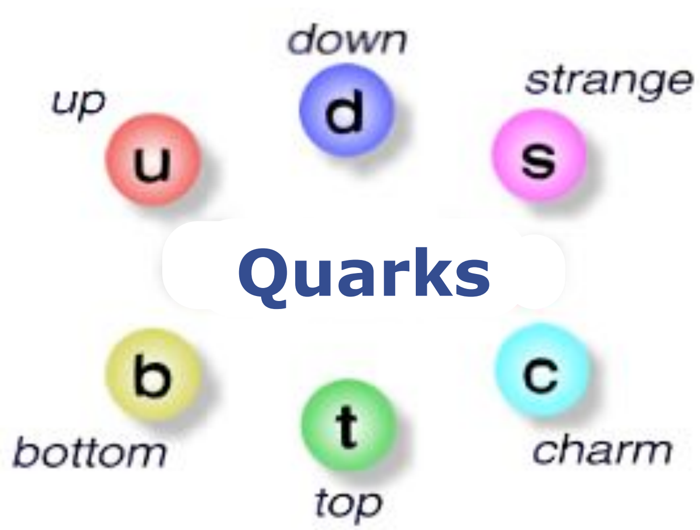
quarks confined - deconfined

Phase diagram of QCD



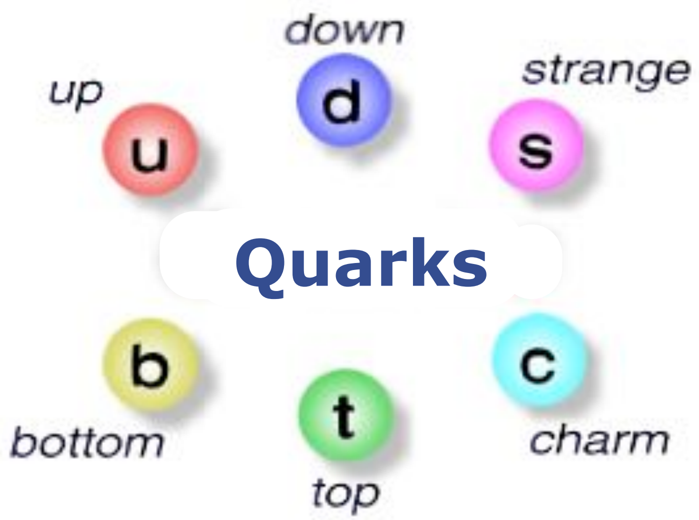
Fukushima

QCD



Gluons

Perturbative QCD & asymptotic freedom



QCD, asymptotic freedom and all that

Action and interactions

QCD action $S_{\rm QCD}$

Yang-Mills **gauge fixing**

$$\frac{1}{4}\int_x F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}\int_x (\partial_\mu A^a_\mu)^2 + \int_x \bar c^a \partial_\mu D^{ab}_\mu c^b + \int_x \bar q \cdot (i\not{D} + i\, m_\psi + i\mu\gamma_0)\cdot q$$

gluon **ghost** **quarks**

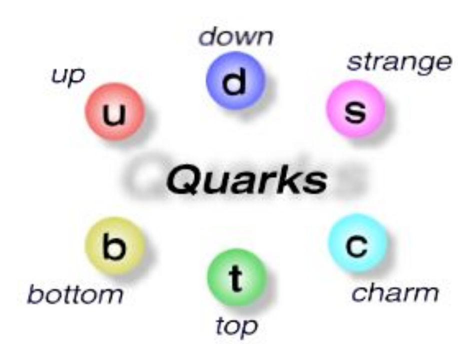
Pure gauge theory **matter sector**

$$\mathbf{F}^{\mathbf{a}}_{\mu\nu} = \partial_\mu \mathbf{A}^{\mathbf{a}}_\nu - \partial_\nu \mathbf{A}^{\mathbf{a}}_\mu + \mathbf{i}\mathbf{g}\,\mathbf{f}^{\mathbf{abc}}\mathbf{A}^{\mathbf{b}}_\mu\,\mathbf{A}^{\mathbf{c}}_\nu$$

$$\not{\mathbf{D}} = \gamma_\mu \mathbf{D}_\mu$$

$$\mathbf{a},\mathbf{b},\mathbf{c} = \mathbf{1},\ldots,\mathbf{N}^{\mathbf{2}}_{\mathbf{c}} - \mathbf{1}$$

$$\mathbf{N}_{\mathbf{f}} = \mathbf{6}$$



$$\mathbf{D}^{\mathbf{ab}}_\mu(\mathbf{A}) = \partial_\mu \delta^{\mathbf{ab}} - \mathbf{i}\,\mathbf{g}\,\mathbf{f}^{\mathbf{abc}}\mathbf{A}^{\mathbf{c}}_\mu$$

QCD, asymptotic freedom and all that

Action and interactions

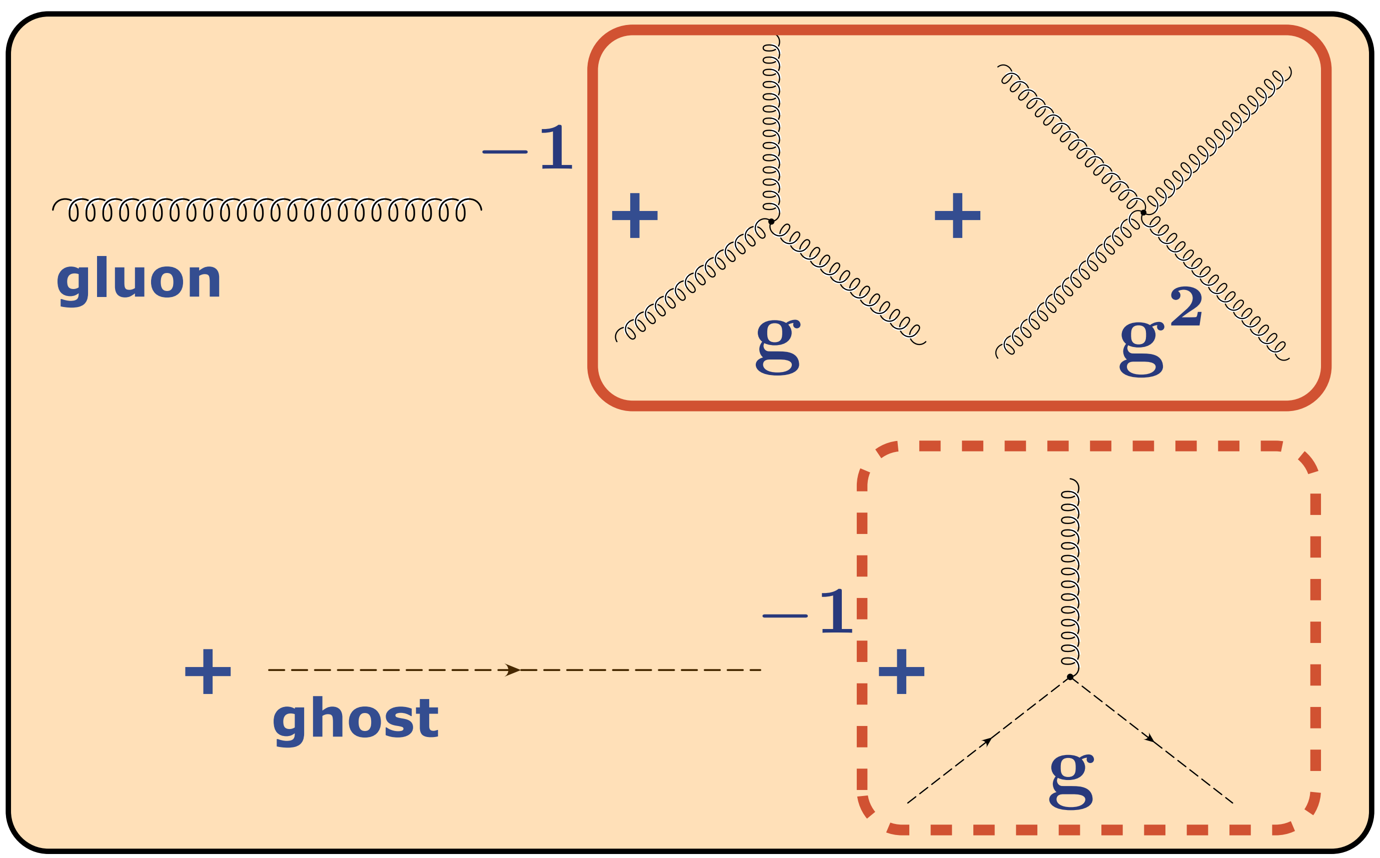
QCD action S_{QCD}

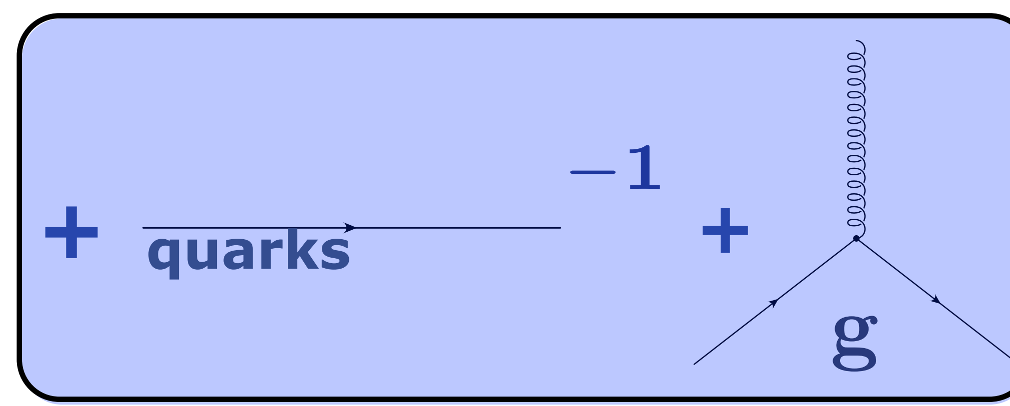
Yang-Mills **gauge fixing**

$$\frac{1}{4}\int_x F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}\int_x \left(\partial_\mu A^a_\mu\right)^2 + \int_x \bar{c}^a \partial_\mu D^{ab}_\mu c^b + \int_x \bar{q}\cdot\left(i\not{D} + i\,m_\psi + i\mu\gamma_0\right)\cdot q$$

gluon **ghost** **quarks**

Pure gauge theory **matter sector**



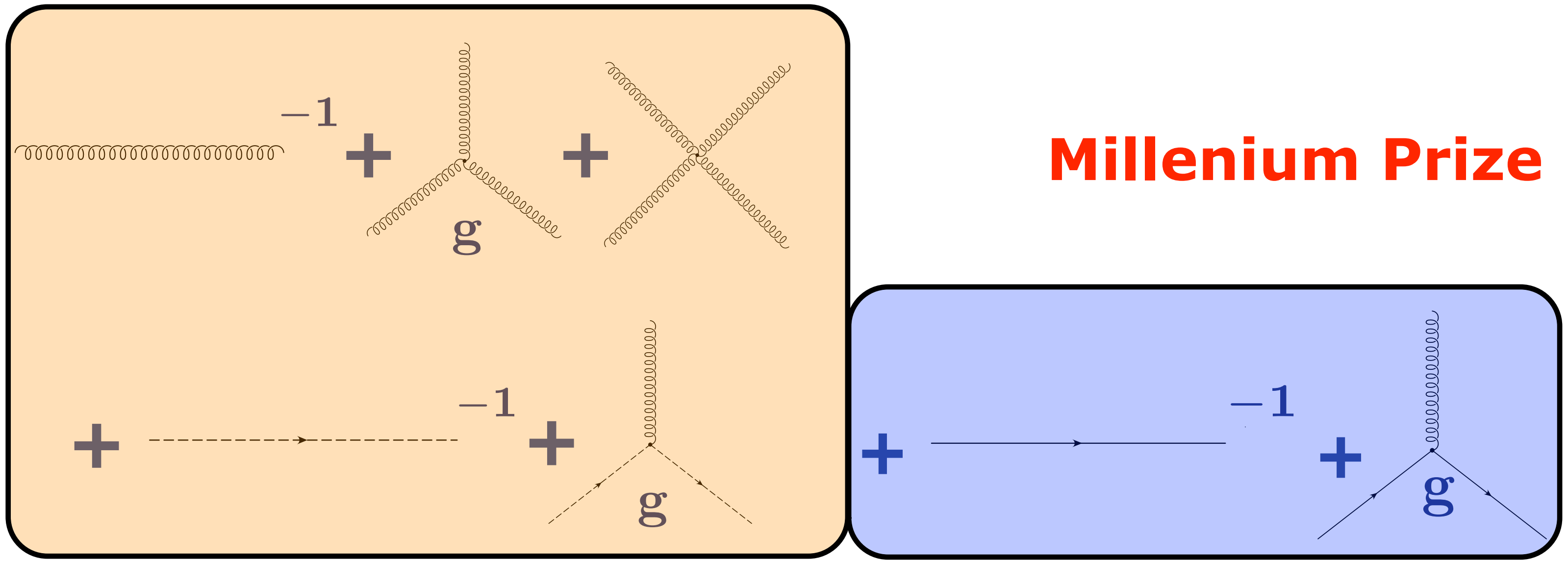


parameters

- **1 coupling** g
- **mass matrix** m_ψ $N_f \times N_f$

QCD, asymptotic freedom and all that

Running coupling at low and high energies

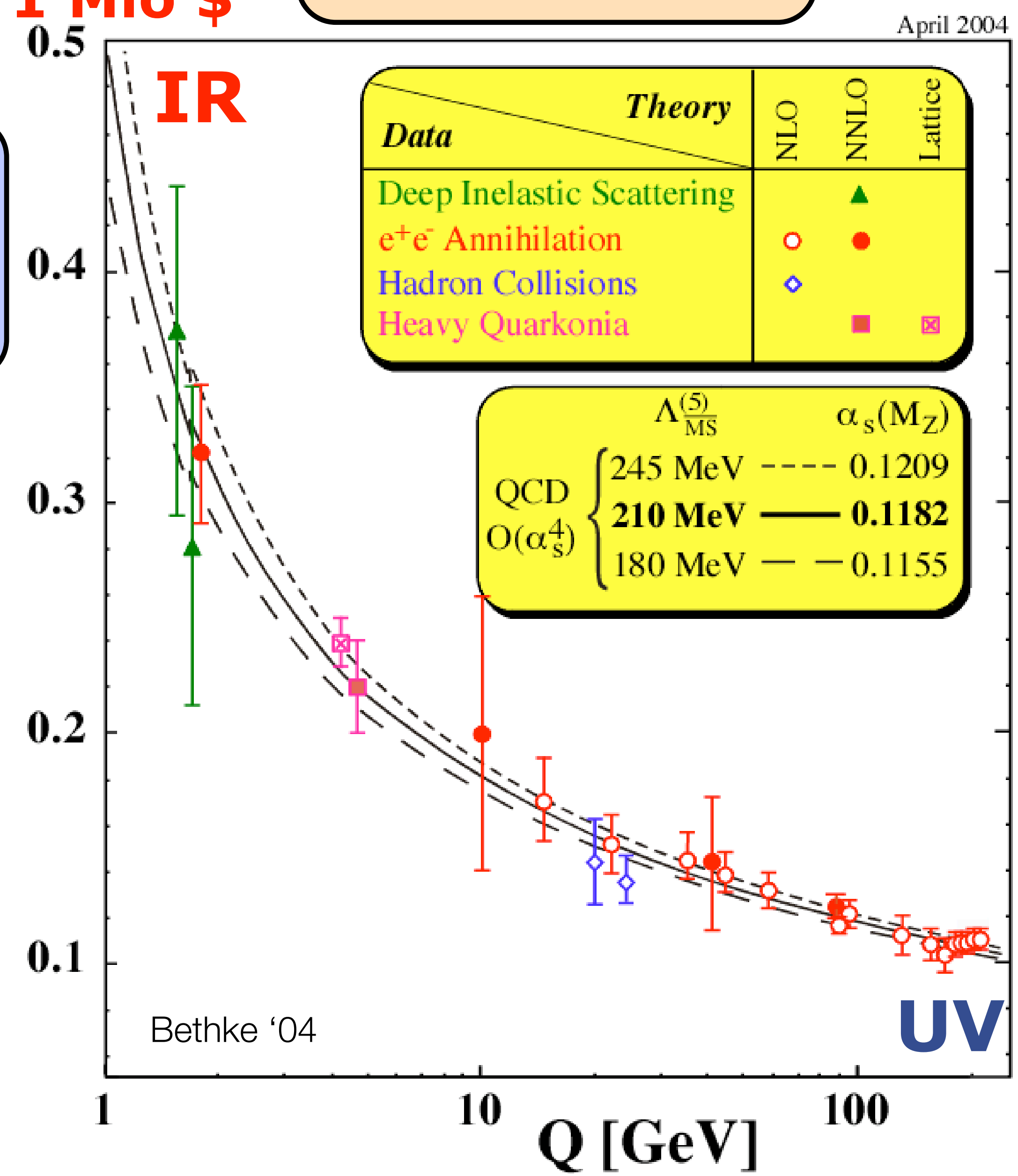


Pure gauge theory

matter sector

Millenium Prize 1 Mio $

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$

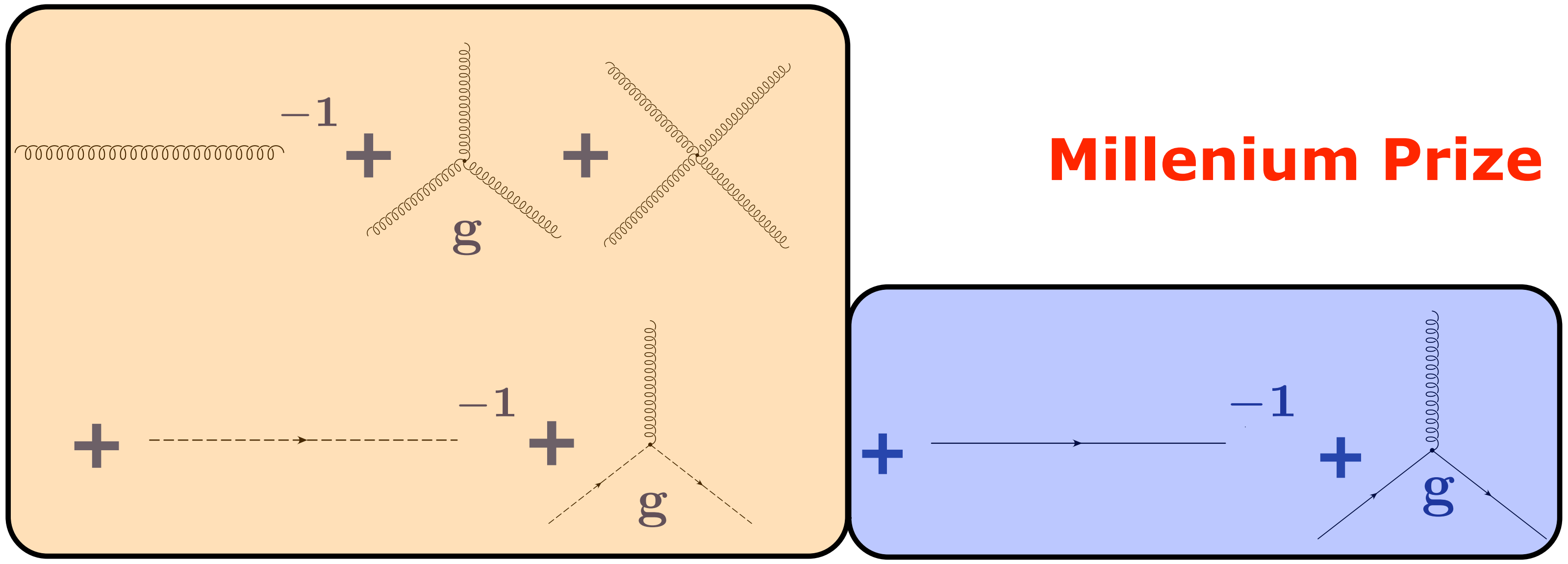


Nobel Prize '04

Gross, Politzer, Wilczek

QCD, asymptotic freedom and all that

Running coupling at low and high energies



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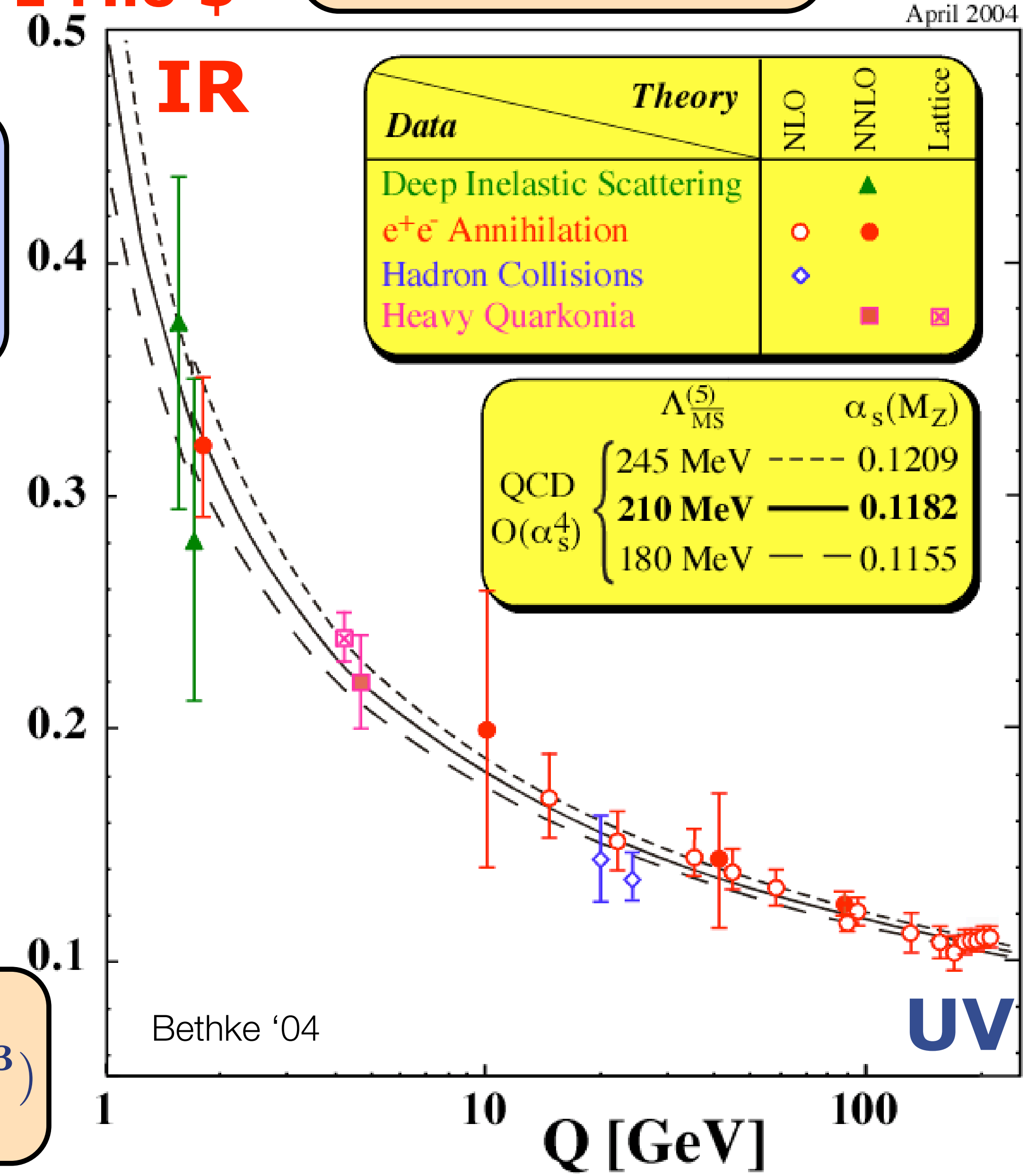


- running coupling (1-loop)

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 - \alpha_s(\mu)\beta_0 \log(Q^2/\mu^2)}$$

- beta function $\beta = Q^2 \frac{\partial \alpha_s(Q)}{\partial Q^2} = \beta_0 \alpha_s(\mu)^2 + O(\alpha_s(\mu)^3)$

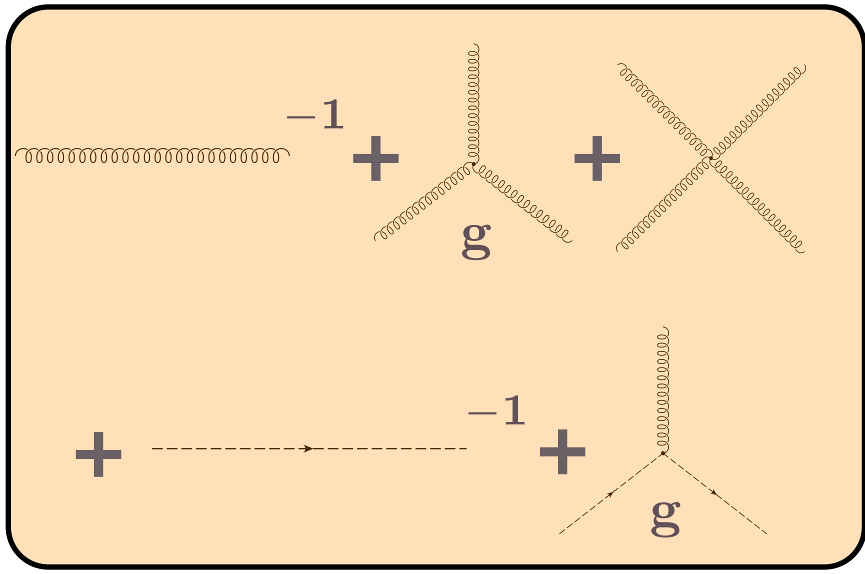
$$\beta = -\frac{1}{12\pi}(33 - 2N_f)\alpha_s^2 + O(\alpha_s^3)$$

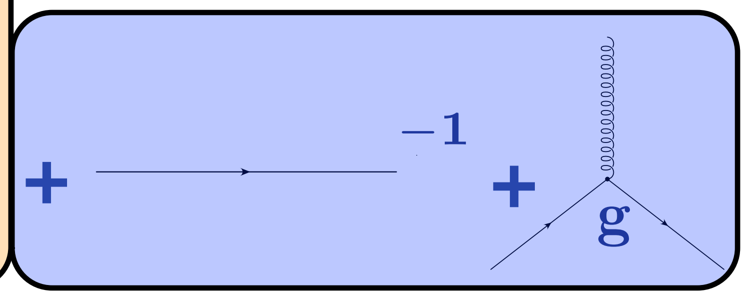
Nobel Prize '04

Gross, Politzer, Wilczek

QCD, asymptotic freedom and all that

Running coupling at low and high energies





Pure gauge theory

matter sector

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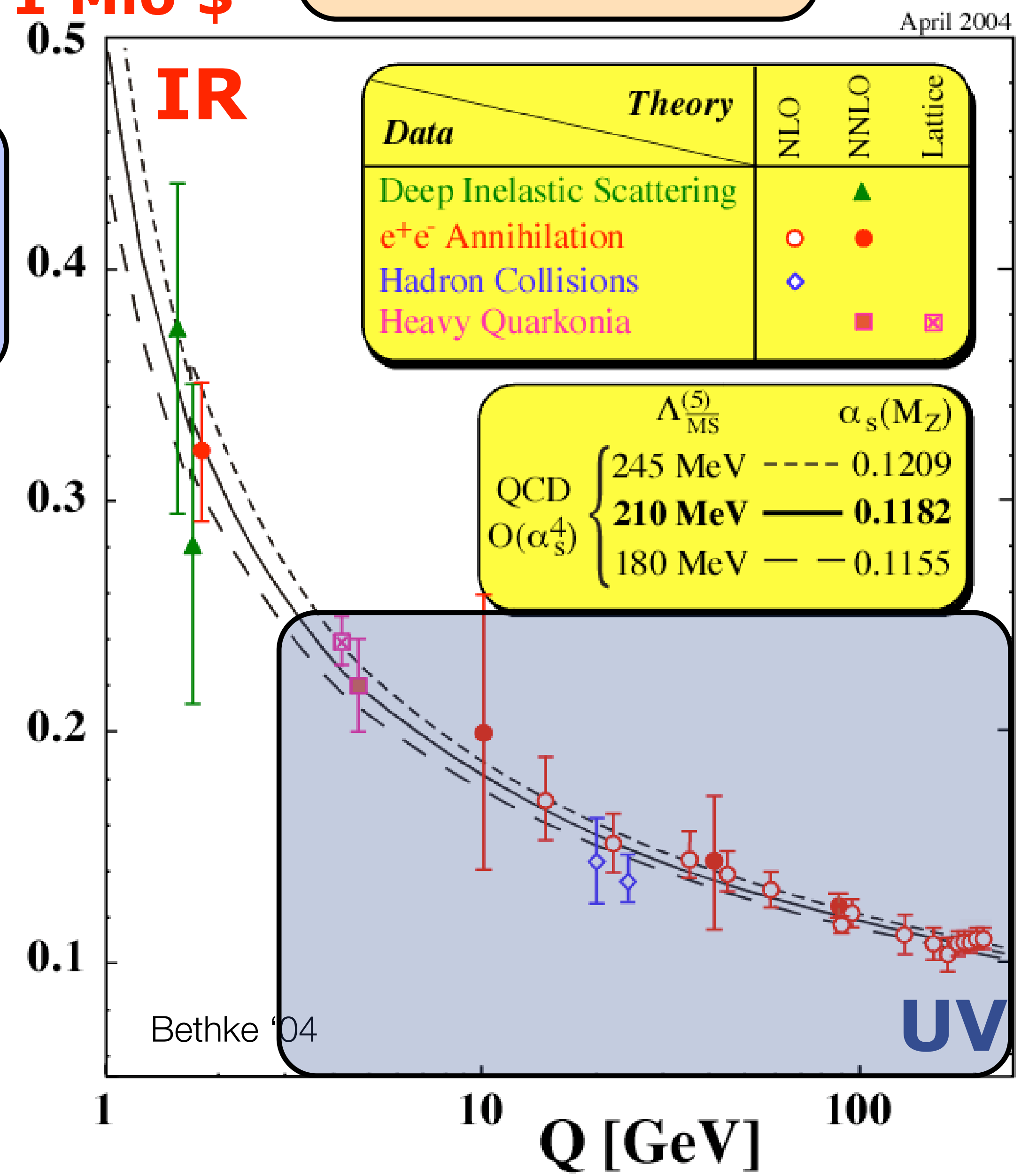


- running coupling (1-loop)

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 - \alpha_s(\mu)\beta_0 \log(Q^2/\mu^2)}$$

- UV: asymptotic freedom

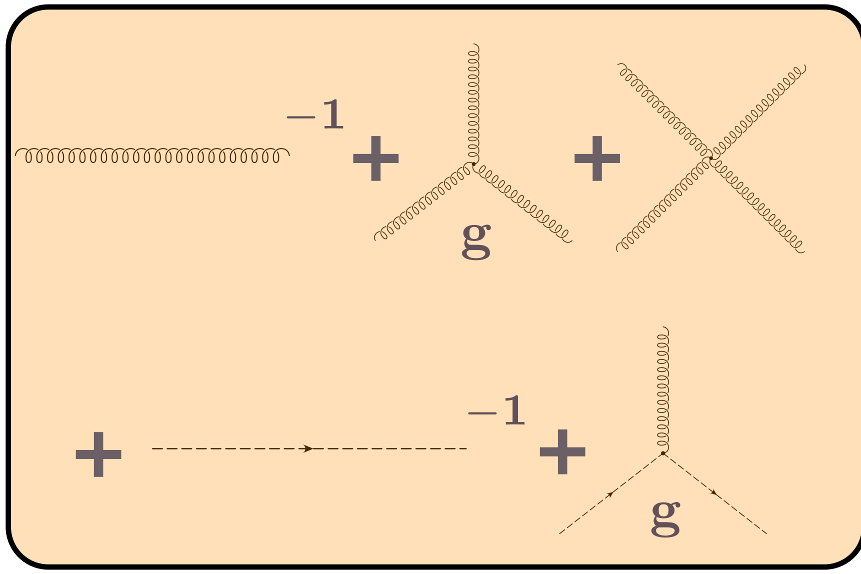
$$\alpha_s(Q \to \infty) = 0$$

Nobel Prize '04

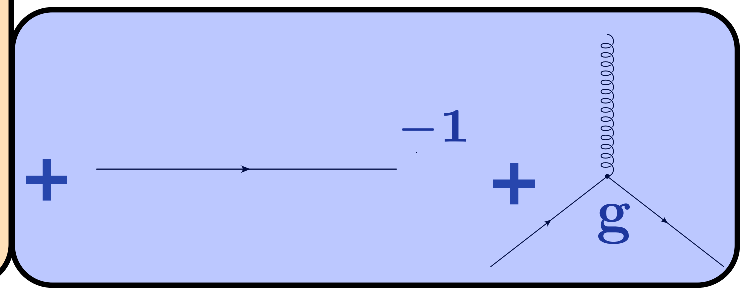
Gross, Politzer, Wilczek

QCD, asymptotic freedom and all that

Running coupling at low and high energies



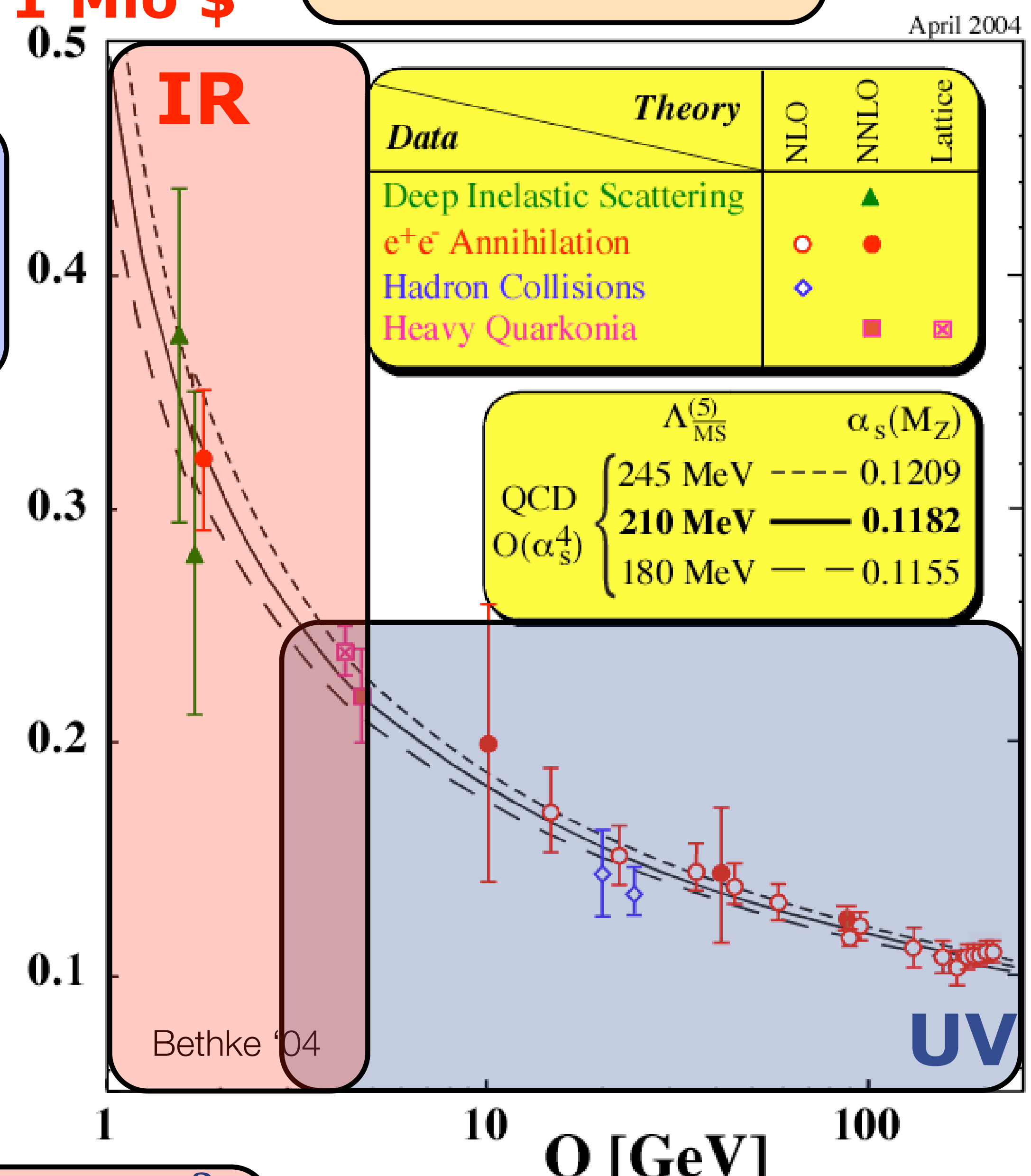
Pure gauge theory



matter sector

Millenium Prize 1 Mio $

$$\alpha_s(Q) = \frac{g^2(Q)}{4\pi}$$



- **running coupling (1-loop)**

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{1 - \alpha_s(\mu)\beta_0 \log(Q^2/\mu^2)}$$

- **IR: failure of perturbation theory**

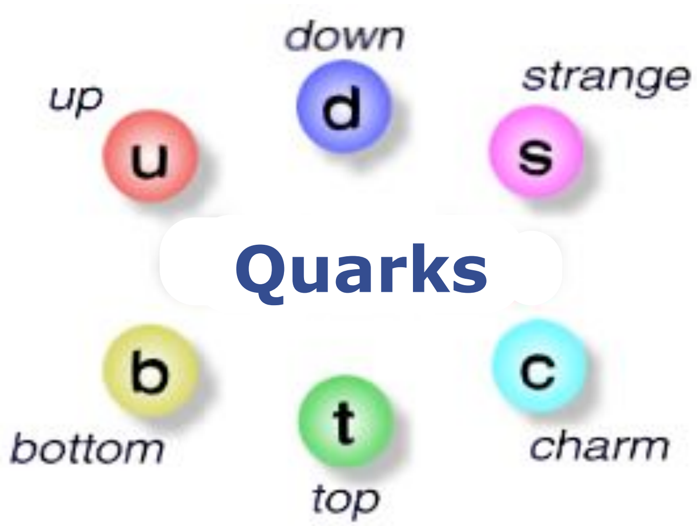
$$\alpha_s(\Lambda^2_{QCD}) = \infty \quad \text{at} \quad \Lambda^2_{QCD} = \mu^2 e^{-\beta_0/\alpha_s(\mu^2)}$$

$$\Lambda_{QCD} = 217^{+25}_{-23}\ \mathrm{MeV}$$

Nobel Prize '04

Gross, Politzer, Wilczek

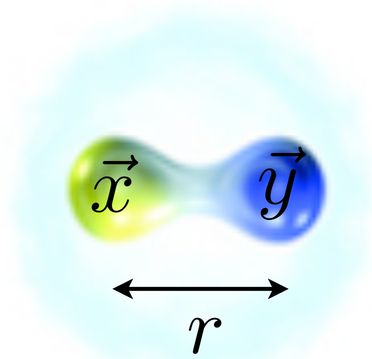
Confinement



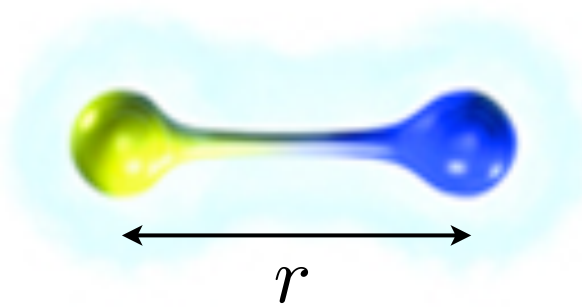
Gluons

Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

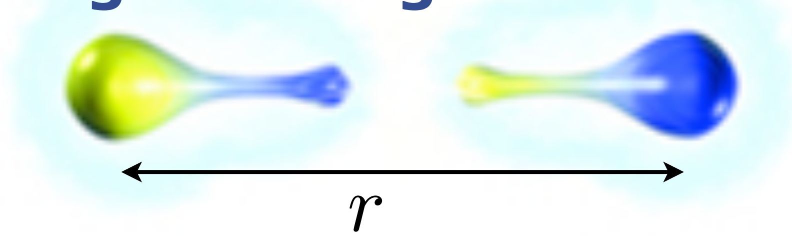


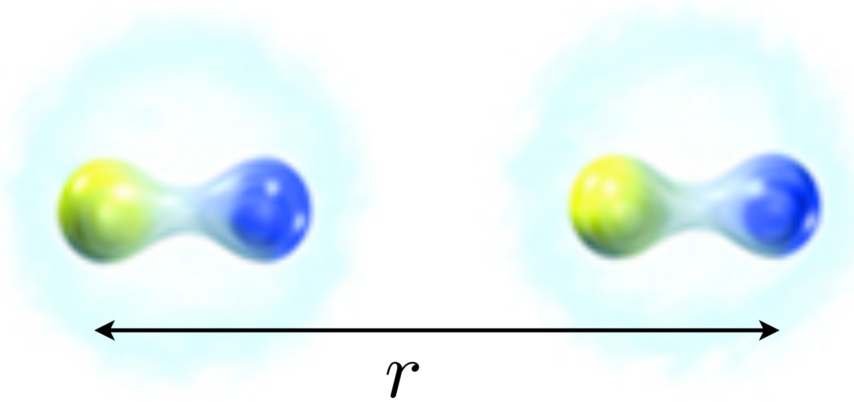
$$F_{q\bar{q}} \simeq -\frac{1}{r}$$



$$F_{q\bar{q}} \simeq \sigma r$$

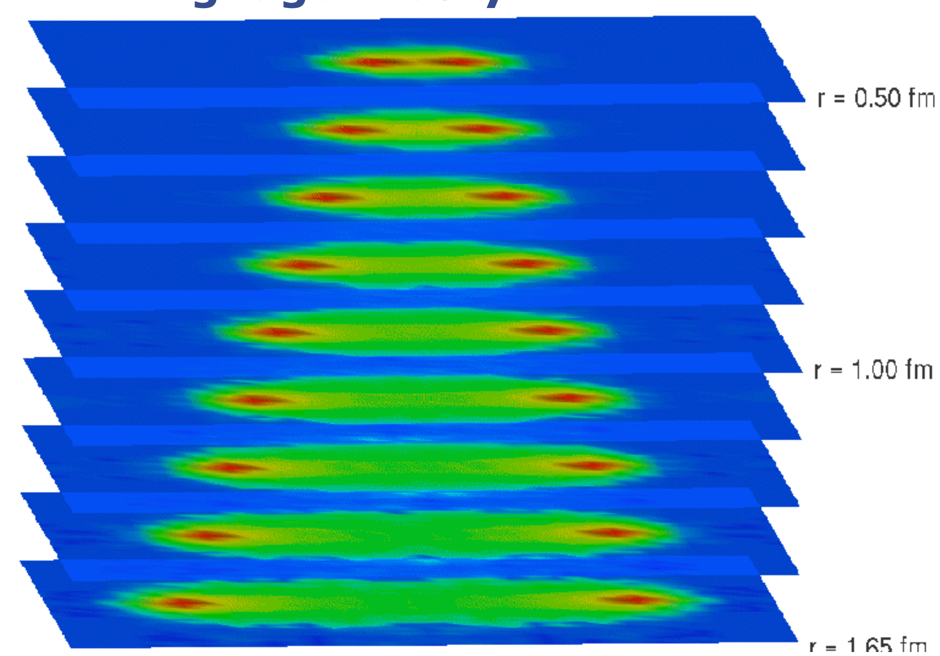
string breaking at $\mathrm{r} \approx 1\mathrm{fm}$



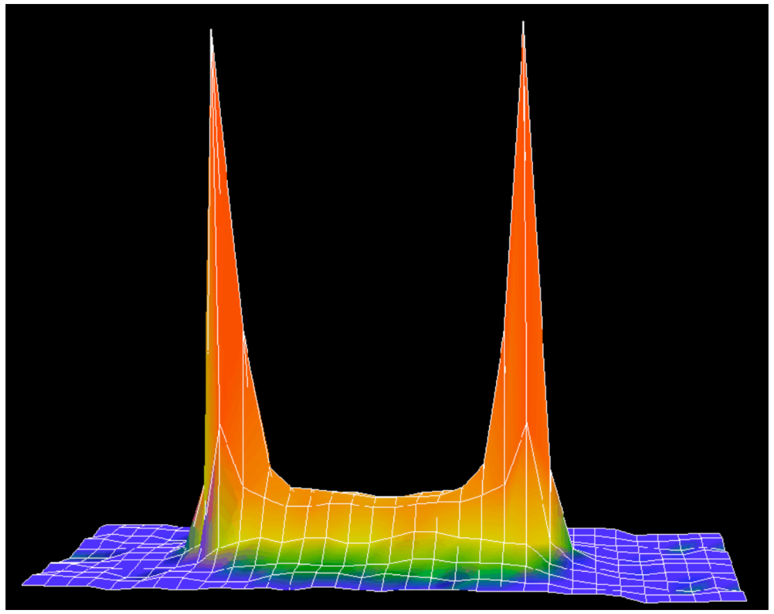


$$F_{q\bar{q}} \simeq \mathrm{const.}$$

gauge theory

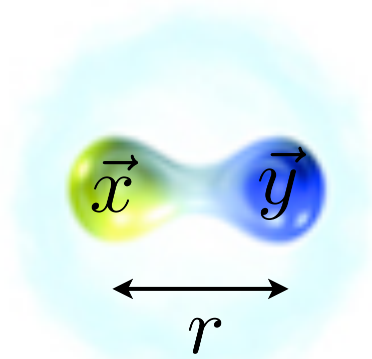


Energy density **Bali et al. '94**

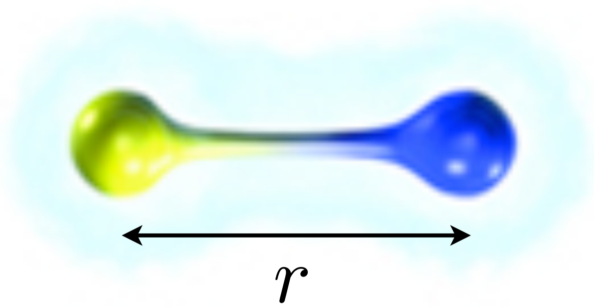


Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

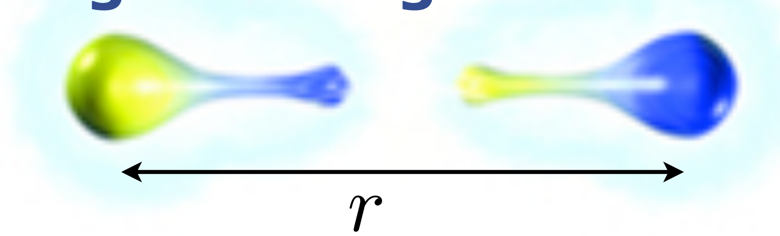


$$F_{q\bar{q}} \simeq -\frac{1}{r}$$

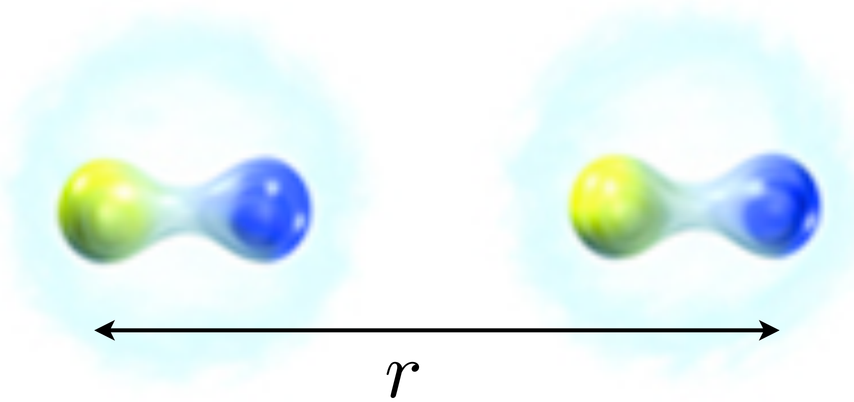


$$F_{q\bar{q}} \simeq \sigma r$$

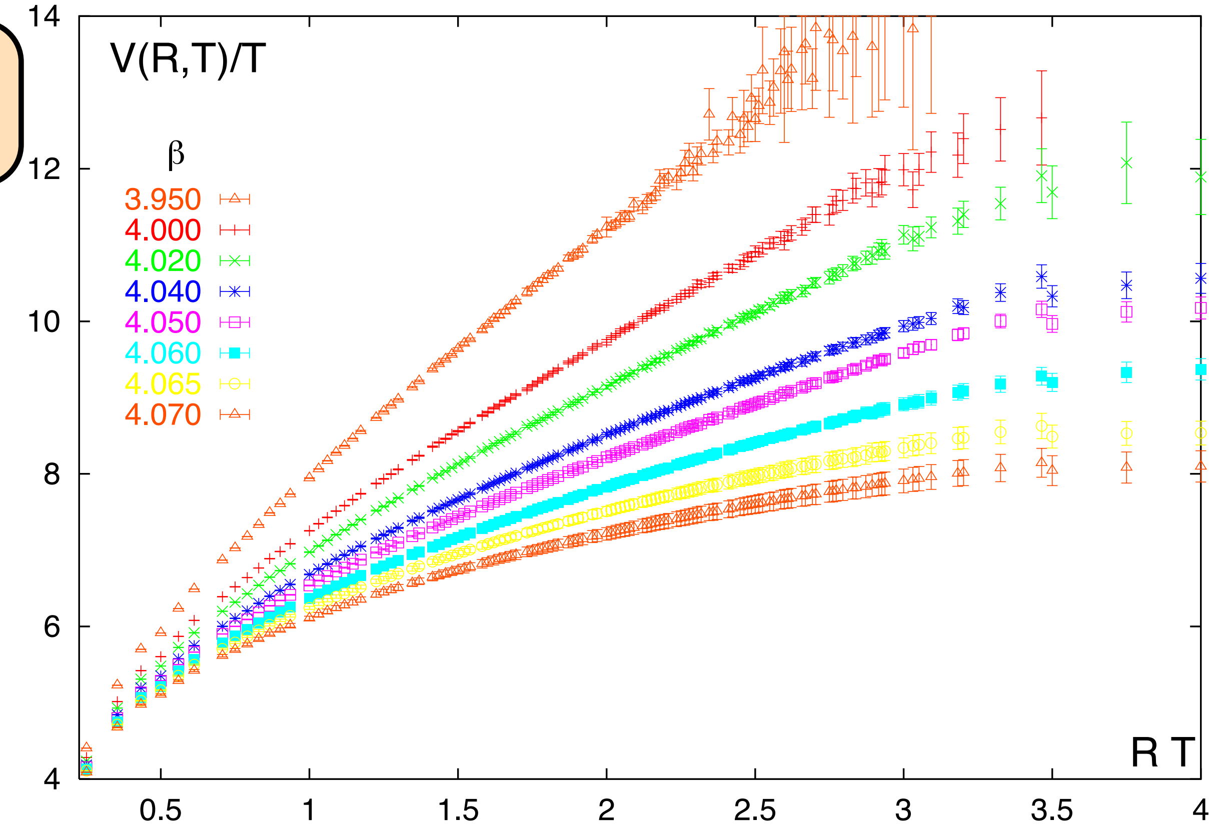
string breaking at $\mathrm{r} \approx 1\mathrm{fm}$



$$F_{q\bar{q}} \simeq \mathrm{const.}$$



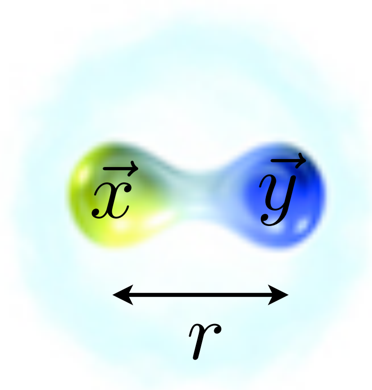
pure gauge theory



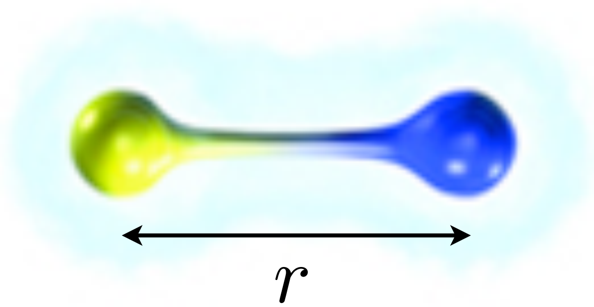
Kaczmarek et al '99

Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

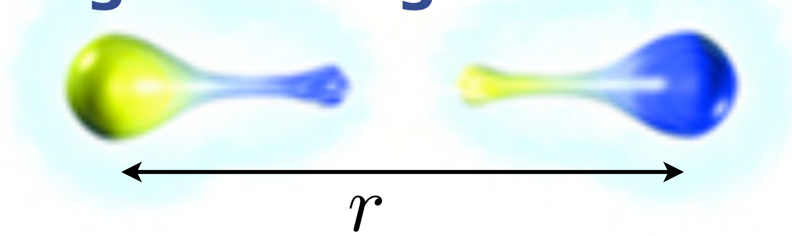


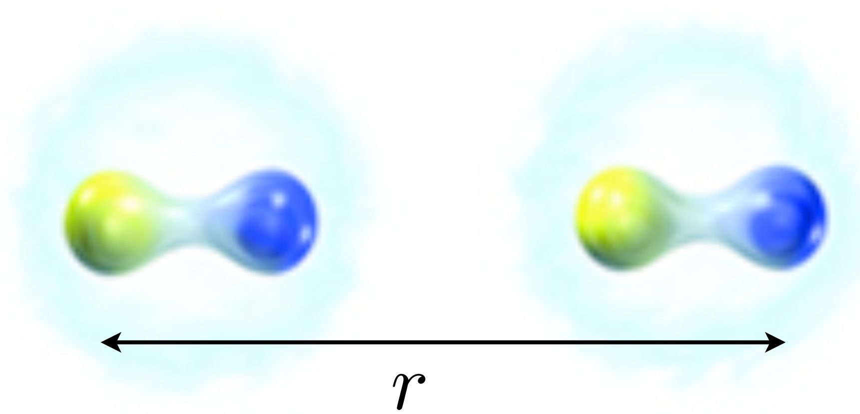
$$F_{q\bar{q}} \simeq -\frac{1}{r}$$



$$F_{q\bar{q}} \simeq \sigma r$$

string breaking at $\mathrm{r} \approx 1\mathrm{fm}$





$$F_{q\bar{q}} \simeq \mathrm{const.}$$

Order parameter $\sim{}'\langle q\rangle'$

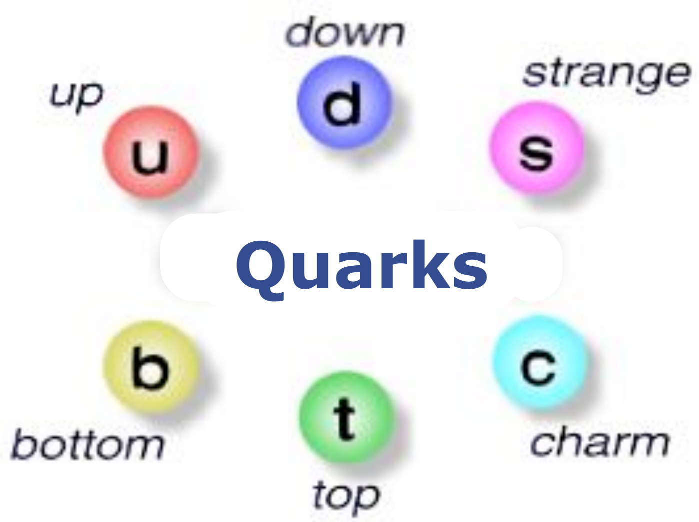
$$\Phi = e^{-\frac{1}{2T}F_{q\bar{q}}(\infty)}$$

- **Confinement** $\Phi = 0$
- **Deconfinement** $\Phi \neq 0$

Polyakov loop

$$\Phi = \frac{1}{3}\langle \mathrm{Tr}\,\mathcal{P}\exp\{ig\int_0^{1/T} dx_0 A_0\}\rangle$$

Chiral symmetry breaking



Chiral symmetry breaking

physical masses

strong chiral symmetry breaking $\Delta m_{\chi SB} \approx 400\,\mathrm{MeV}$

Generation	first	second	third	Charge
Mass [MeV]	1.5-4	1150-1350	170×10^3	
Quark	u	c	t	$\frac{2}{3}$
Quark	d	s	b	$-\frac{1}{3}$
Mass [MeV]	4-8	80-130	$(4.1\text{-}4.4)\times10^3$	

$$\Lambda_{\mathrm{QCD}} = 217^{+25}_{-23}\ \mathrm{MeV}$$

2 light flavours, one heavy flavour 2+1

Chiral symmetry breaking

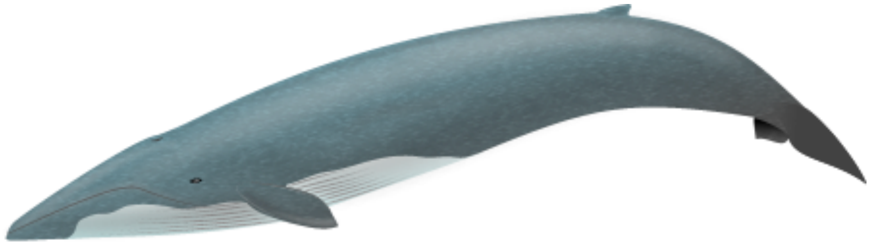
physical masses

strong chiral symmetry breaking $\Delta m_{\chi SB} \approx 400\,\mathrm{MeV}$

up

charm

top



down

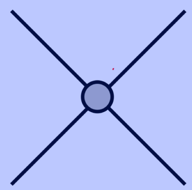
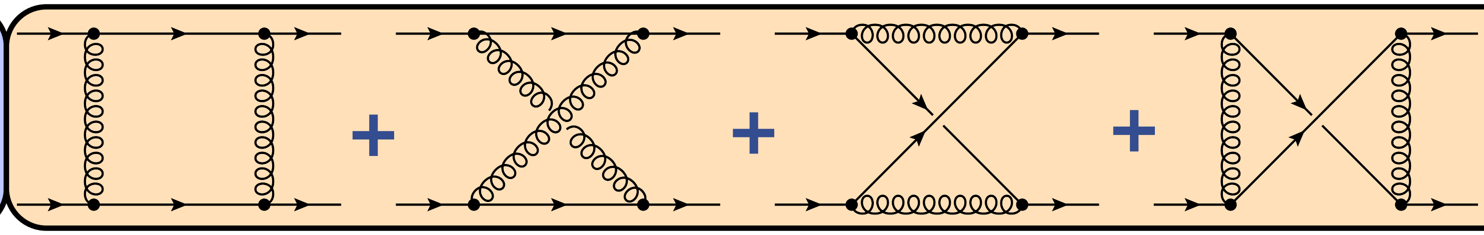
strange

bottom

2 light flavours, one heavy flavour 2+1

Chiral symmetry breaking

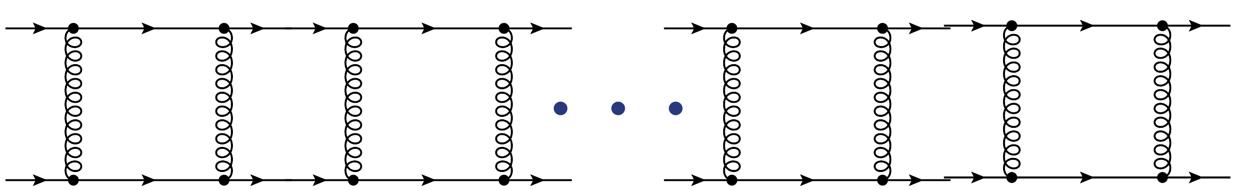
- **Perturbative four-fermi coupling** $\frac{\lambda_\psi}{2}\int\left[(\bar{q}q)^2+(i\bar{q}\gamma_5\vec{\tau}q)^2\right]$

$\lambda_\psi =$  $\propto$ 

$$\lambda_\psi \propto \alpha_s^2$$

$$\mathbf{N_f = 2}: \quad \vec{\tau} = (\sigma_1, \sigma_2, \sigma_3)$$

- **Fermionic mass term for** $\langle\bar{q}q\rangle \neq 0$

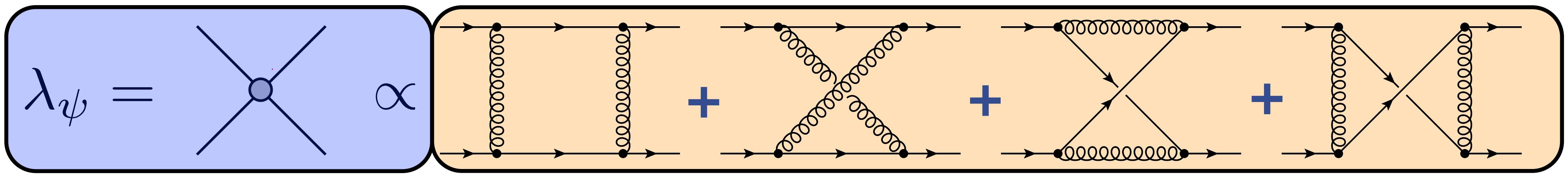


$$\frac{\lambda_\psi}{2}\int(\bar{q}q)^2 \longrightarrow \frac{\lambda_\psi}{2}\int\langle\bar{q}q\rangle\bar{q}q$$

mean field

Chiral symmetry breaking

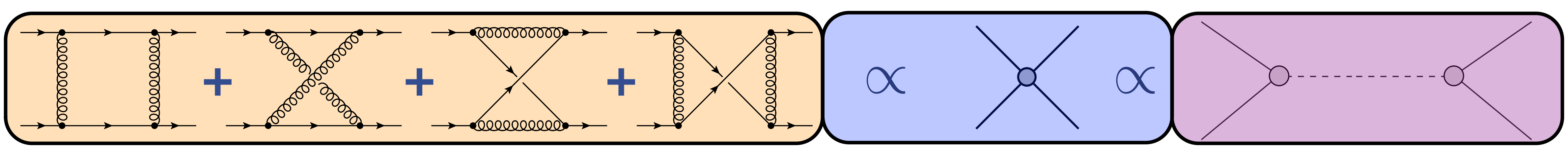
- **Perturbative four-fermi coupling** $\frac{\lambda_\psi}{2}\int \left[(\bar q q)^2 + (i\bar q \gamma_5 \vec\tau q)^2\right]$



$\lambda_\psi \propto \alpha_s^2$

$\mathbf{N_f = 2}: \quad \vec\tau = (\sigma_1, \sigma_2, \sigma_3)$

- **Bosonisation (Hubbard-Stratonovich)** $\langle\sigma\rangle \neq 0$



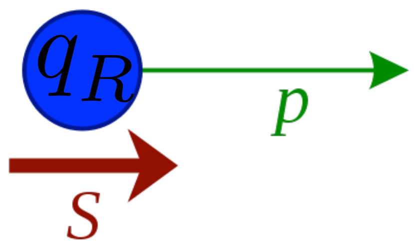
$$\frac{\lambda_\psi}{2}\int \left[(\bar\psi\psi)^2 + (i\bar\psi\gamma_5\vec\tau\psi)^2\right] = \frac{m_\sigma^2}{2}\int_x (\sigma^2 + \vec\pi^2) + i\,h\int_x \bar\psi(\sigma + i\gamma_5\vec\tau\vec\pi)\psi$$

$\mathbf{EOM}(\sigma)$

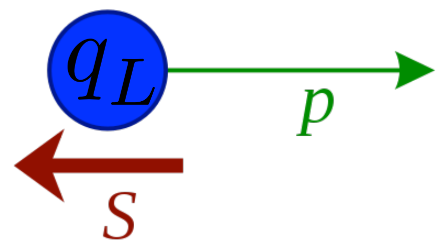
Chiral symmetry breaking

- **Chirality for massless particles**

Right-handed:



Left-handed:



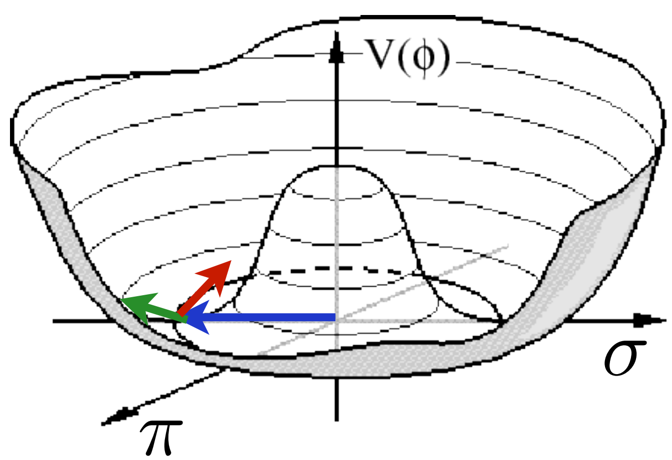
- **Order parameter**

$$\sigma \simeq \langle \bar{q} q \rangle$$ **chiral condensate**

$$\bar{q} q = q_R^\dagger \, q_L + q_L^\dagger \, q_R$$

  - **Chiral symmetry** $\sigma = 0$
  - **Symmetry broken** $\sigma \neq 0$

- **Meson potential**



Chiral symmetry breaking

- **Chirality for massless particles**

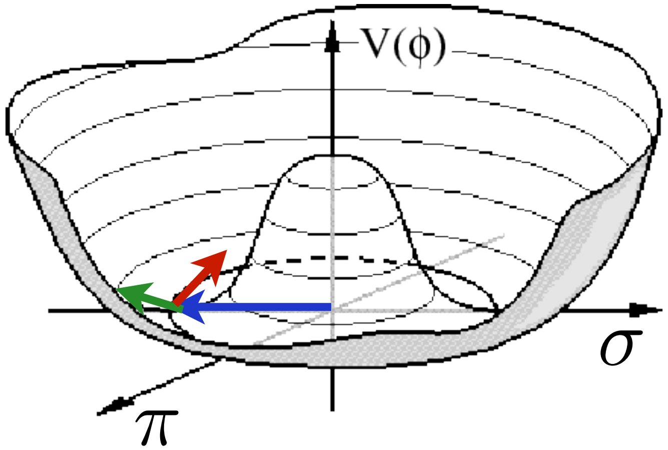
Right-handed: *Left-handed:*

- **Order parameter**

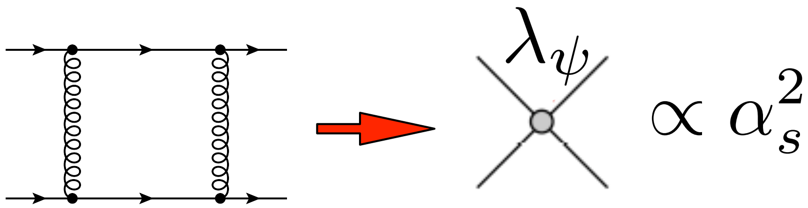
$$\sigma \simeq \langle \bar{q}q \rangle$$ **chiral condensate**

- **Chiral symmetry** $\sigma = 0$
- **Symmetry broken** $\sigma \neq 0$

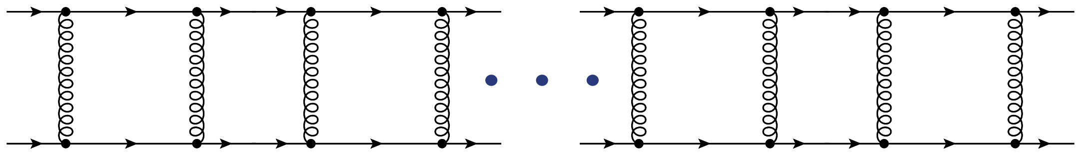
- **Meson potential**



chiral symmetry



$$\int d^4x\, \lambda_\psi \left[(\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2 \right]$$



$$\langle \bar{q}q \rangle \neq 0$$

mass term: $\langle \bar{q}q \rangle \, \bar{q}q$

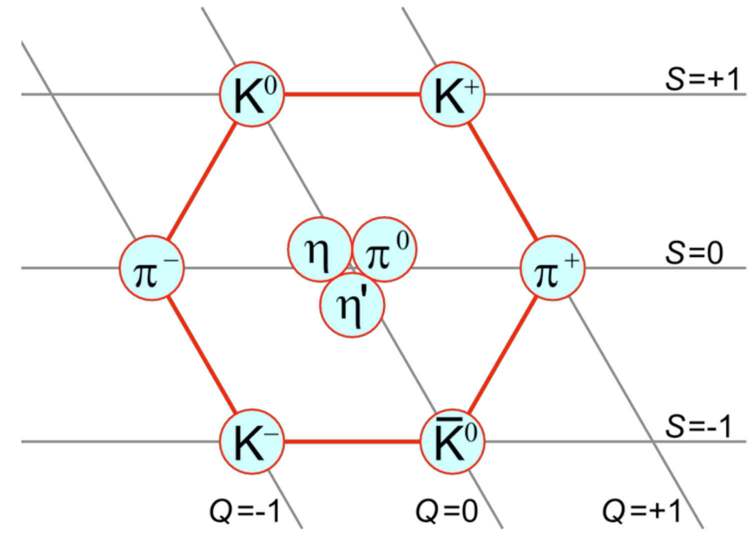
chiral symmetry broken

Chiral symmetry breaking

anomalous chiral symmetry breaking

- **Axial U(1)** $q \to e^{i\gamma_5 \alpha} q$ **with current** $J_{5,\mu} \propto \bar{q}\gamma_5\gamma_\mu q = q_R^\dagger q_L - q_L^\dagger q_R$

classically $\partial_\mu J_{5,\mu} = 0$



- **Anomalous breaking of the axial U(1)**

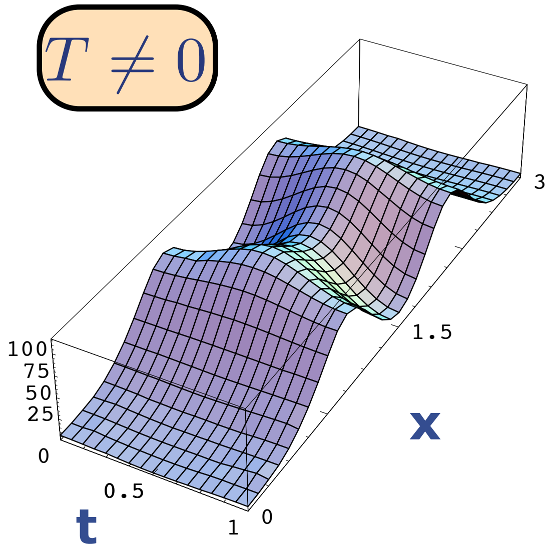
quantum $\partial_\mu \langle J_{5,\mu} \rangle = \frac{N_f}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} \langle F^a_{\mu\nu} F^a_{\rho\sigma} \rangle$

axial anomaly

Nonet of pseudoscalar mesons

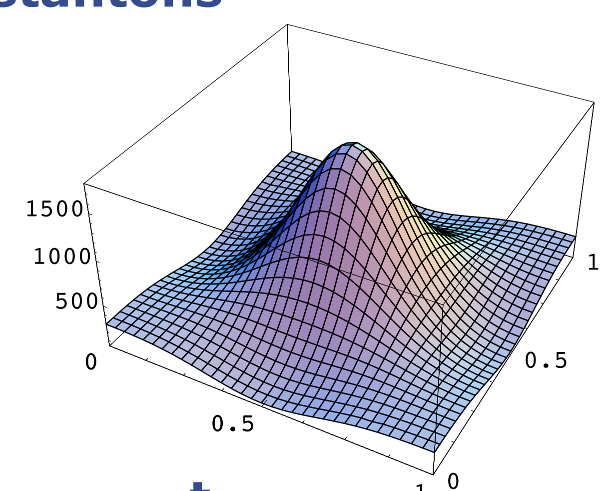
$\mathbf{m_{\eta'} \simeq 960\,MeV}$

$T \neq 0$



induced by instantons

$-\frac{1}{2}\mathrm{tr}F^2$

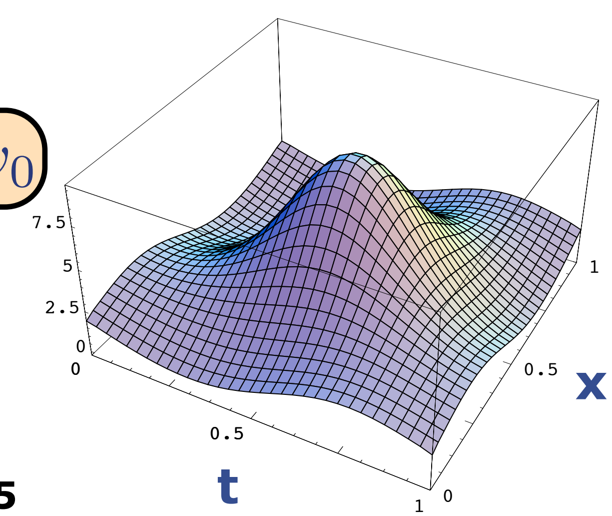


SU(2)

$T = 0$

fermionic zero modes

$\psi_0^\dagger \psi_0$



Plots from Ford, JMP '05

Chiral symmetry breaking

anomalous chiral symmetry breaking

- **Axial U(1)** $q \to e^{i\gamma_5 \alpha} q$ **with current** $J_{5,\mu} \propto \bar{q}\gamma_5\gamma_\mu q = q_R^\dagger q_L - q_L^\dagger q_R$

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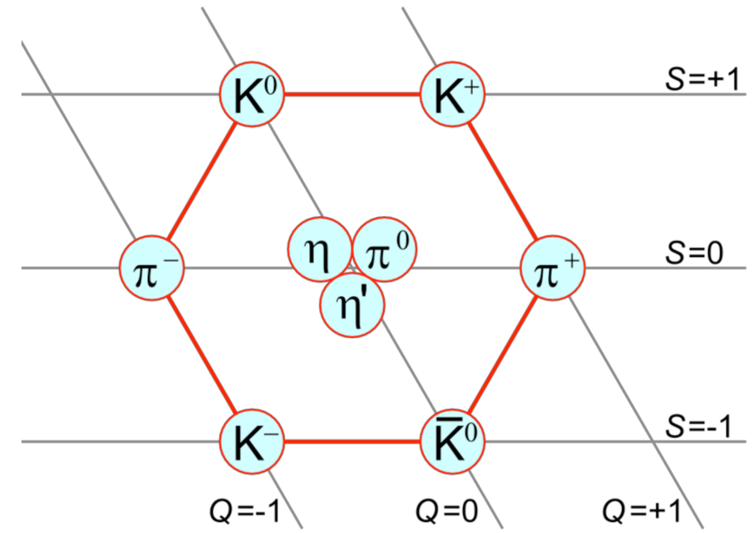


- **Anomalous breaking of the axial U(1)**

quantum

$$\partial_\mu \langle J_{5,\mu}\rangle = \frac{N_f}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\langle F^a_{\mu\nu}F^a_{\rho\sigma}\rangle$$

axial anomaly

Nonet of pseudoscalar mesons

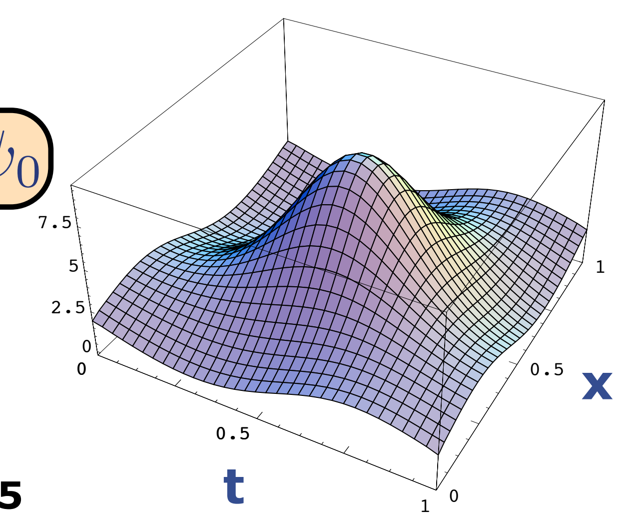
$$\mathrm{m}_{\eta'} \simeq 960\,\mathrm{MeV}$$

fermionic zero modes

't Hooft determinant

$$\Delta(k,\theta)\left(\det_{flav.}\bar{q}_L q_R + \det_{flav.}\bar{q}_R q_L\right)$$

$\psi_0^\dagger \psi_0$



UV-FRG
JMP '96

$$\Delta(k,\theta) \propto (k^2 + c_k\,\Delta m^2_{\chi sb})^{-\frac{3}{2}N_f+2} e^{-2\pi/\alpha_{s,k}}$$

Plots from Ford, JMP '05

Functional Methods for QCD

FunMethods: FRG-DSE-2PI-...

Functional Renormalisation Group

Functional Renormalisation Group

Generating functional Z

$$Z[J] = \frac{1}{\mathcal{N}} \int d\varphi \, e^{-S[\varphi] + \int_x J\varphi}$$

$$\langle \varphi \rangle_J = \phi$$

partition function

$$S[\varphi] = \frac{1}{2} \int_x \left[\partial_\mu \varphi \partial_\mu \varphi + \mathrm{m}^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 \right]$$

classical action

zero-dimensional example: 'Functional' flows for integrals

Functional Renormalisation Group

Generating functional Z

$$Z[J] = \frac{1}{\mathcal{N}} \int d\varphi \, e^{-S[\varphi] + \int_x \, J\varphi}$$

partition function

$$\langle \varphi \rangle_J = \phi$$

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi} \, e^{-S[\hat{\varphi}+\phi] + \int_x \, \hat{\varphi} \, \frac{\delta \Gamma[\phi]}{\delta \phi}}$$

free energy

$$\varphi = \hat{\varphi} + \phi$$

$$\langle \hat{\varphi} \rangle_{\frac{\delta \Gamma}{\delta \phi}} = 0$$

$$J = \frac{\delta \Gamma}{\delta \phi}$$

$$\Gamma[\phi] = \sup_J \left(\int_x J \cdot \phi - \log Z[J] \right)$$

Legendre transform

Functional Renormalisation Group

Generating functional Z

$$Z[J] = \frac{1}{\mathcal{N}} \int d\varphi \, e^{-S[\varphi] + \int_x J\varphi}$$

partition function

$$\langle \varphi \rangle_J = \phi$$

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free energy

$$\varphi = \hat{\varphi} + \phi$$

$$\langle \hat{\varphi} \rangle_{\frac{\delta\Gamma}{\delta\phi}} = 0$$

$$J = \frac{\delta \Gamma}{\delta \phi}$$

Dyson-Schwinger equation

$$\frac{\delta \Gamma[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta \phi(x)} \right\rangle$$

quantum equation of motion

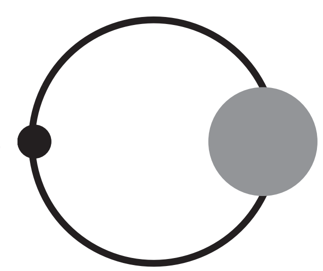
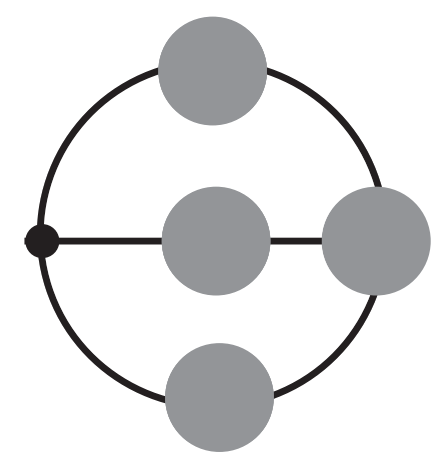
Functional Renormalisation Group

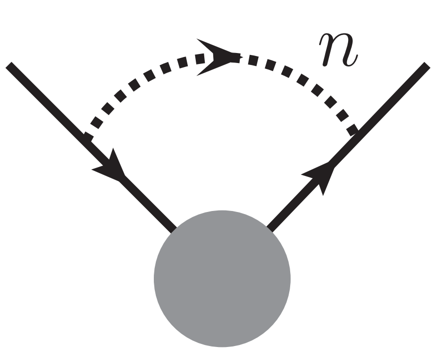
Dyson-Schwinger equation

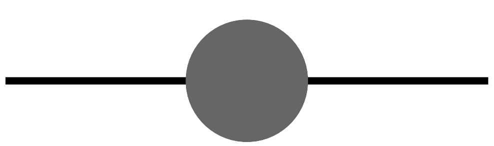
$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta\phi(x)} \right\rangle$$

Diagrammatics

$$S[\phi] = \frac{1}{2}\int_x \left[\partial_\mu\phi\partial_\mu\phi + \mathrm{m}^2\phi^2 + \frac{\lambda}{4}\phi^4\right]$$

$$\frac{\lambda}{2}\langle[\hat{\varphi}(x)+\phi(x)]^3\rangle = \frac{\lambda}{2}\phi^3(x) + \frac{3\lambda}{2}\phi(x)$$  $$- \quad \frac{\lambda}{2}$$ 

$$\Gamma^{(\mathbf{n})} =$$ 

G =  $$= \langle\hat{\varphi}(x)\hat{\varphi}(y)\rangle$$

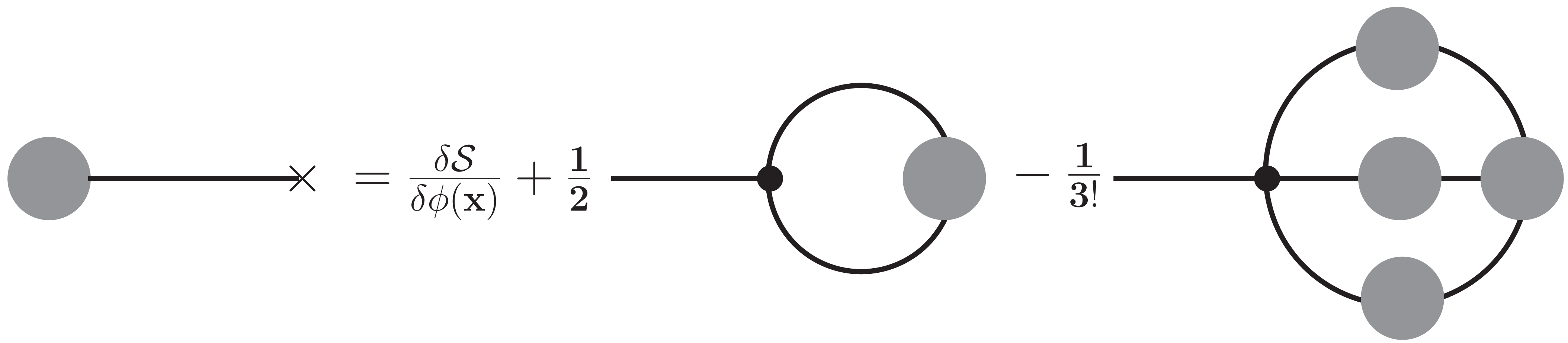
Functional Renormalisation Group

Dyson-Schwinger equation

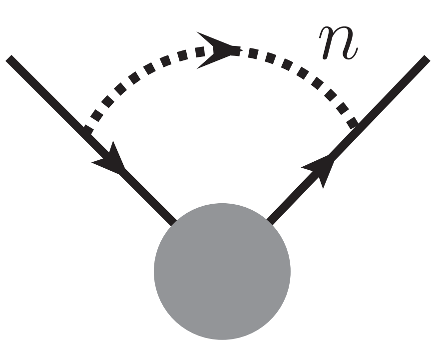
$$\frac{\delta\Gamma[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta\phi(x)} \right\rangle$$

Diagrammatics

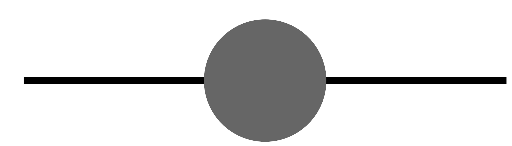
$$S[\phi] = \frac{1}{2}\int_x \left[\partial_\mu\phi\partial_\mu\phi + \mathrm{m}^2\phi^2 + \frac{\lambda}{4}\phi^4\right]$$



$\Gamma^{(\mathbf{n})} =$



G =



$= \langle\hat{\varphi}(x)\hat{\varphi}(y)\rangle$

Functional Renormalisation Group

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi]+\int_x \hat{\varphi}\,\frac{\delta\Gamma[\phi]}{\delta\phi}}$$

No quantum fluctuations

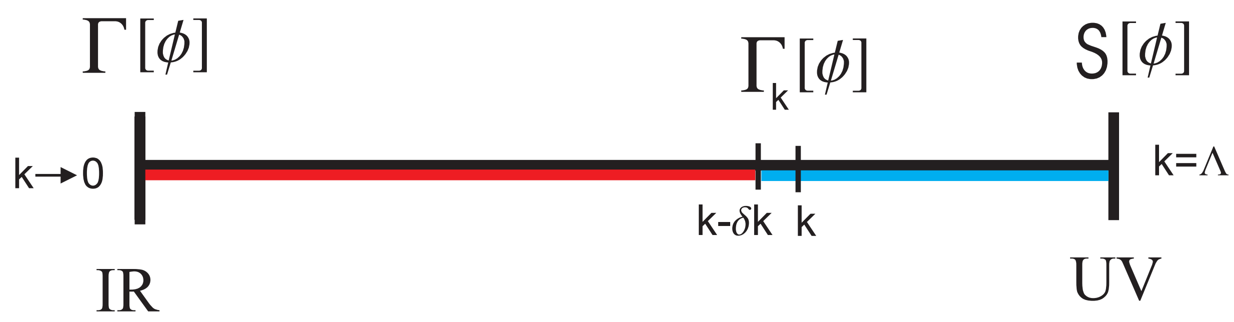
$$\Gamma[\phi] = -\log e^{-S[\phi]} = S[\phi]$$

Functional Renormalisation Group

Effective action Γ

$$\Gamma[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi]+\int_x \hat{\varphi}\, \frac{\delta\Gamma[\phi]}{\delta\phi}}$$

UV quantum fluctuations up to $p^2 = k^2$



Functional Renormalisation Group

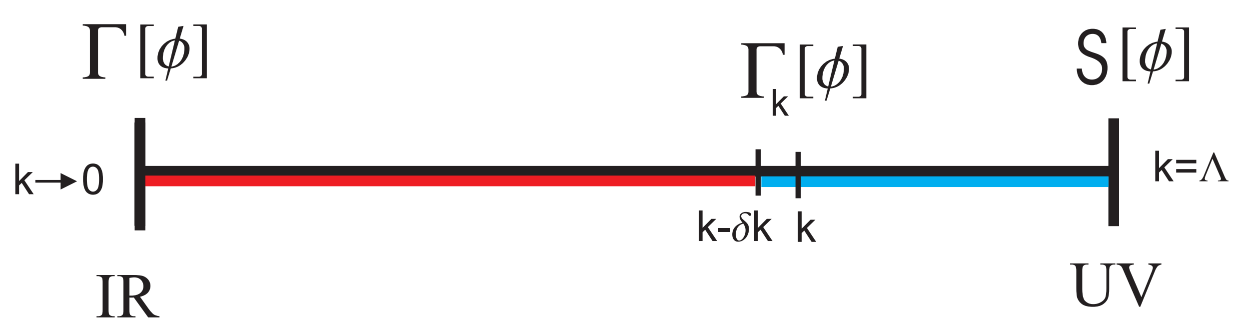
Effective action Γ_k

$$\Gamma_k[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi] + \frac{1}{2}\int_p \hat{\varphi}(p) R_k(p^2) \hat{\varphi}(-p) + \int_x \hat{\varphi}\, \frac{\delta \Gamma_k[\phi]}{\delta \phi}}$$

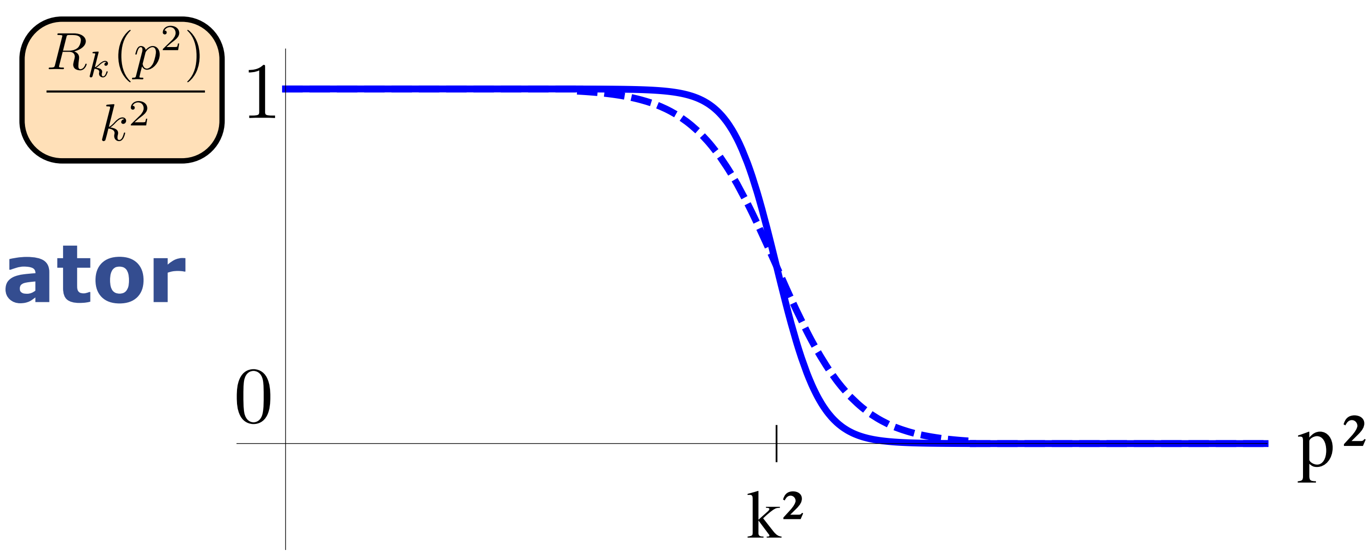
$$\frac{\delta \Gamma_k[\phi]}{\delta \phi(x)} = \left\langle \frac{\delta S[\hat{\varphi}+\phi]}{\delta \phi(x)} \right\rangle$$

DSE

UV quantum fluctuations up to $p^2 = k^2$



Regulator

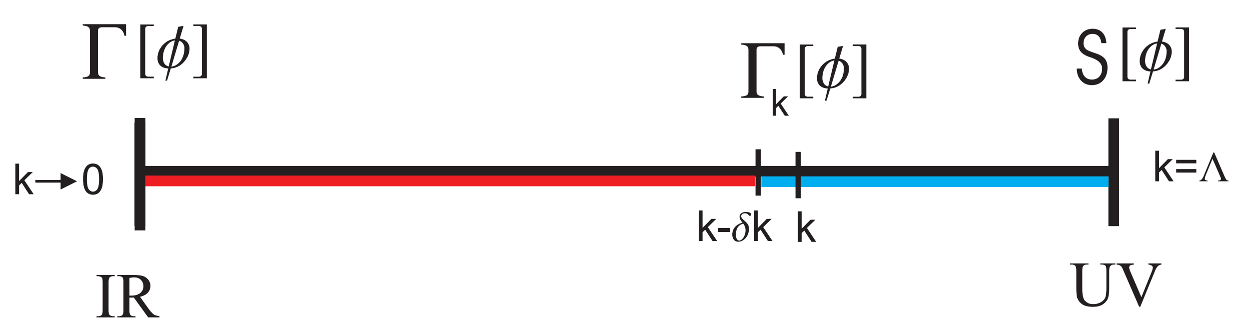


Functional Renormalisation Group

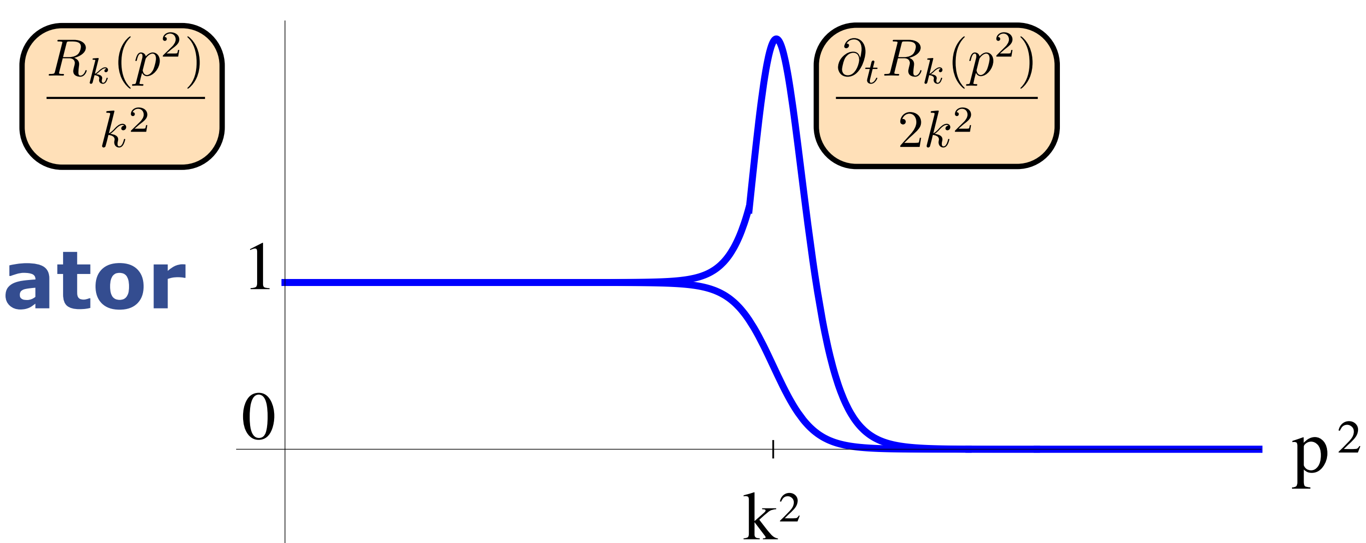
Effective action Γ_k

$$\Gamma_k[\phi] = -\log \int d\hat\varphi\, e^{-S[\hat\varphi+\phi] + \frac{1}{2}\int_p \hat\varphi(p) R_k(p^2)\hat\varphi(-p) + \int_x \hat\varphi \frac{\delta\Gamma_k[\phi]}{\delta\phi}}$$

UV quantum fluctuations up to $p^2 = k^2$



Regulator



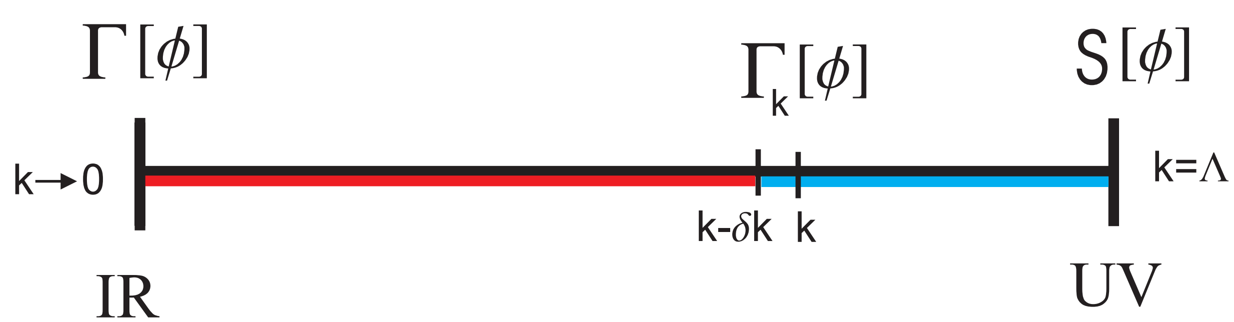
$$t = \log \frac{k}{\Lambda}$$

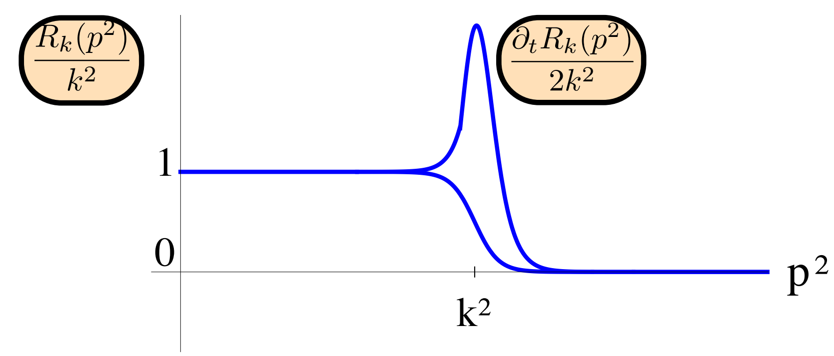
Functional Renormalisation Group

Effective action Γ_k

$$\Gamma_k[\phi] = -\log \int d\hat{\varphi}\, e^{-S[\hat{\varphi}+\phi] + \frac{1}{2}\int_p \hat{\varphi}(p) R_k(p^2)\hat{\varphi}(-p) + \int_x \hat{\varphi}\, \frac{\delta \Gamma_k[\phi]}{\delta \phi}}$$

UV quantum fluctuations up to $p^2 = k^2$





Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\int \frac{d^4 p}{(2\pi)^4}\, \langle \hat{\varphi}(p)\hat{\varphi}(-p)\rangle\, \partial_t R_k(p^2)$$

$$t = \log \frac{k}{\Lambda}$$

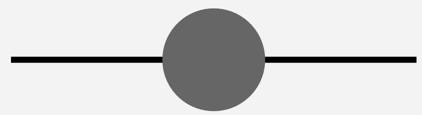
Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\, \langle \hat\varphi(p)\hat\varphi(-p)\rangle\, \partial_t R_k(p^2)$$

$$t = \log\frac{k}{\Lambda}$$

Propagator

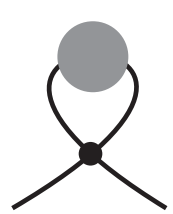
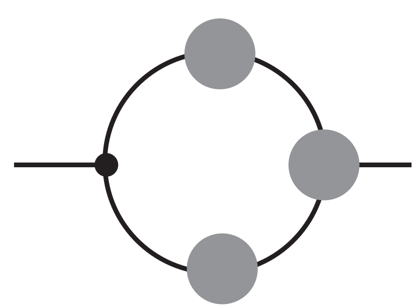
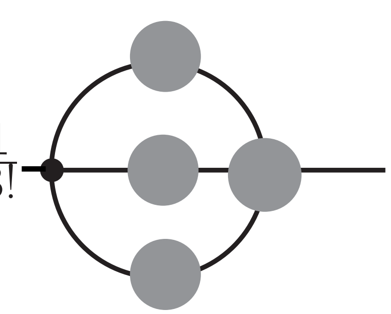
G =  $= \langle \hat\varphi(x)\hat\varphi(y)\rangle$

$$G^{-1}[\phi] = \Gamma_k^{(2)}[\phi] + R_k$$

$$\frac{\delta\Gamma_k[\phi]}{\delta\phi(x)} = \left\langle \frac{\delta S[\hat\varphi+\phi]}{\delta\phi(x)}\right\rangle$$

$$\Gamma_k[\phi] = -\log\int d\hat\varphi\, e^{-S[\hat\varphi+\phi]+\frac{1}{2}\int_p \hat\varphi(p)R_k(p^2)\hat\varphi(-p)+\int_x \hat\varphi\,\frac{\delta\Gamma_k[\phi]}{\delta\phi}}$$

DSE

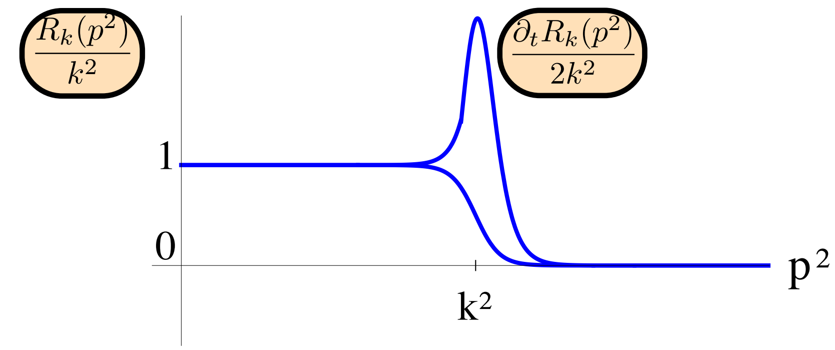
$\Gamma_k^{(2)}[\phi] - S^{(2)}[\phi] = \frac{1}{2}$  $+\frac{1}{2}$  $+\frac{1}{3!}$ 

Functional Renormalisation Group

Flow

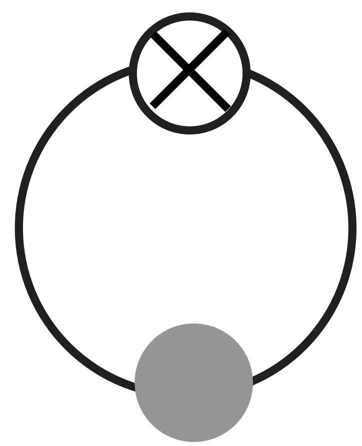
$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi] + R_k}\,\partial_t R_k$$

regulator



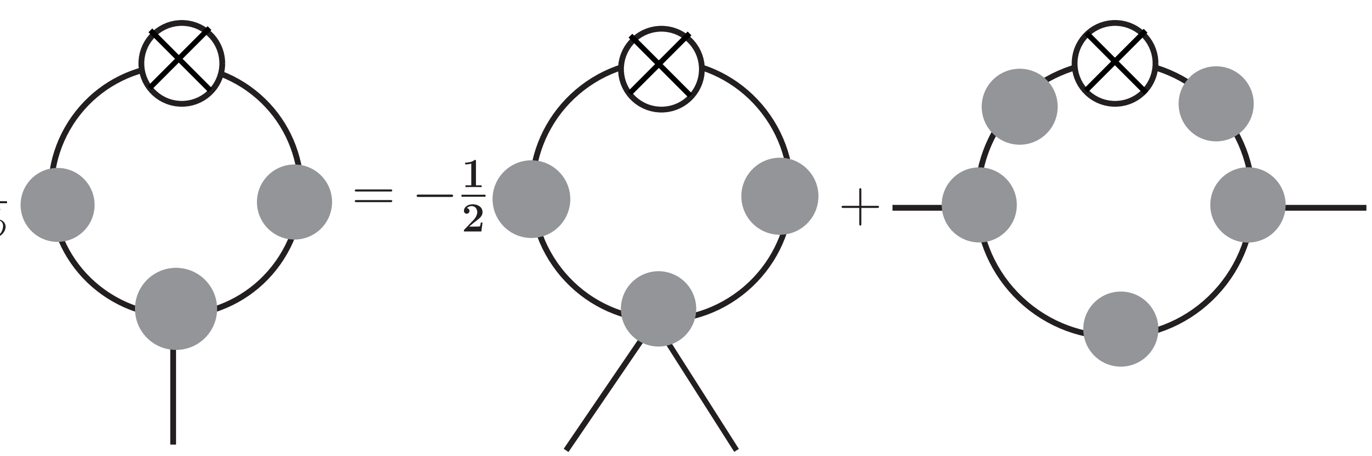
Diagrammatics

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



Propagator

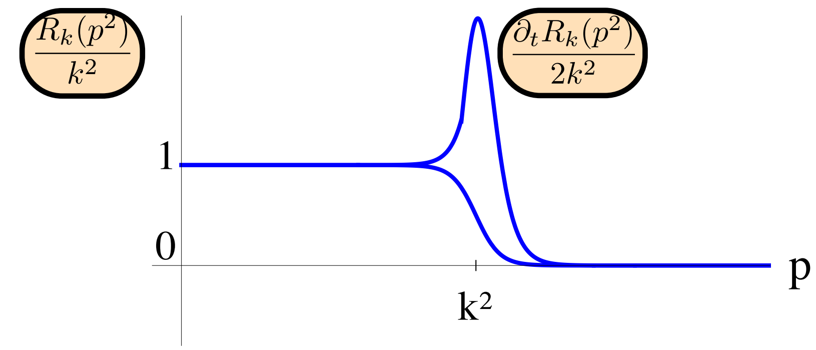
$$\partial_t \Gamma_k^{(2)}[\phi] = -\frac{1}{2}\frac{\delta}{\delta\phi} \quad = -\frac{1}{2} \quad +$$



Functional Renormalisation Group

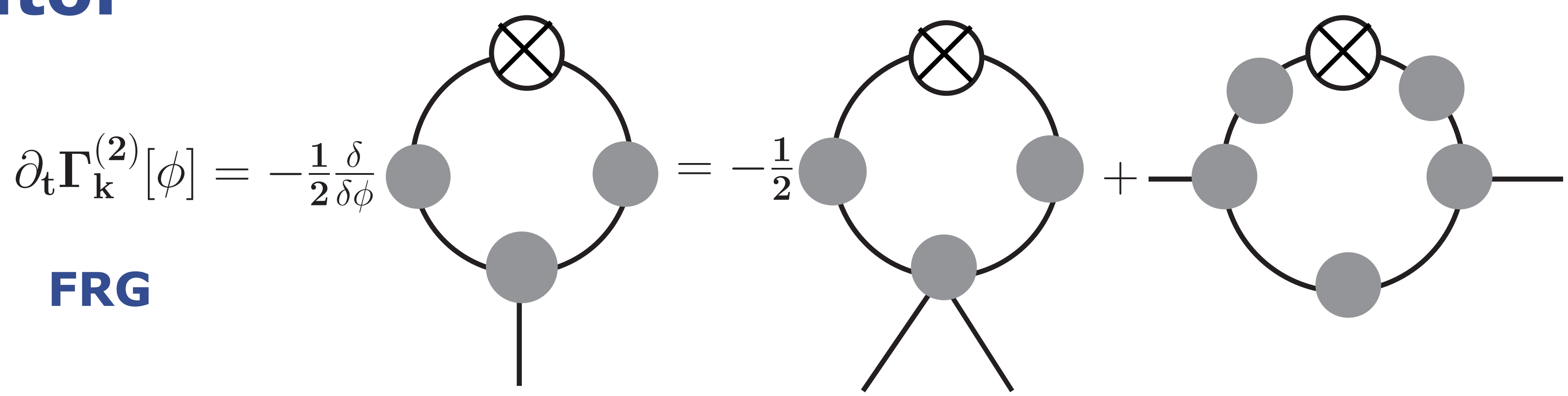
Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

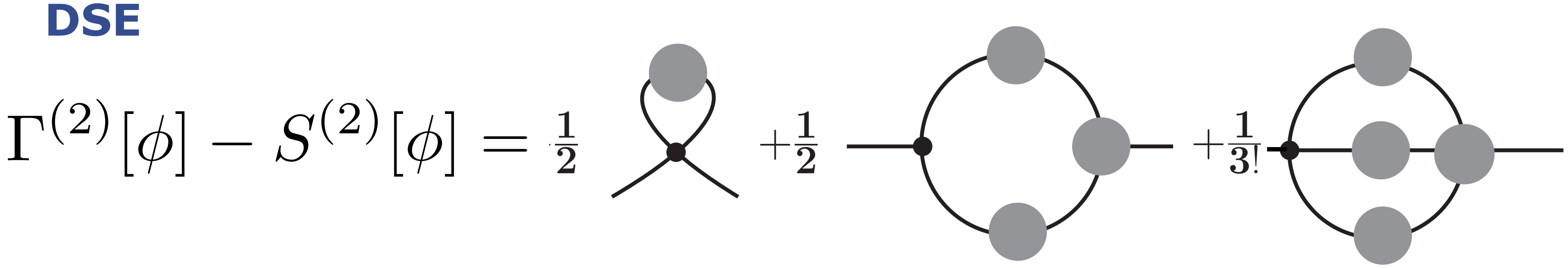


Propagator

FRG



DSE



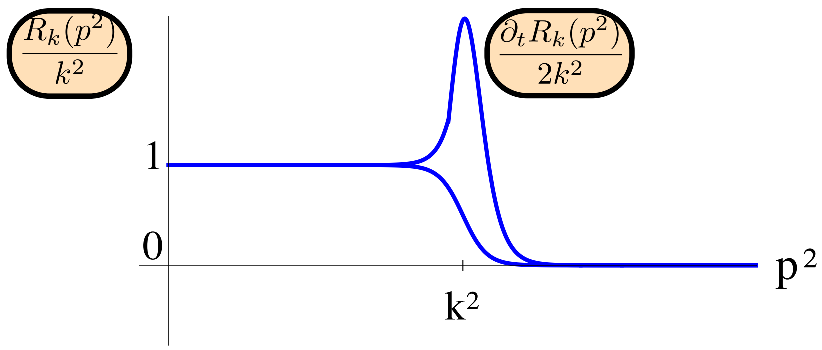
$\partial_t \Gamma^{(n)} = \mathrm{Flow}_n[\Gamma^{(m)}; m = 2, ..., n+2]$

$\Gamma^{(n)} = \mathrm{DSE}_n[S^{(m)}, \Gamma^{(m)}; m = 2, ..., n+2]$

Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



Properties

	FRG	DSE	2PI	3PI	4PI
• 1-loop exact	✓	–			
• closed	✓	✓			
• RG-scaling	✓	–	–	–	✓
• Energy/particle-number conserv.	–	–	✓	✓	✓

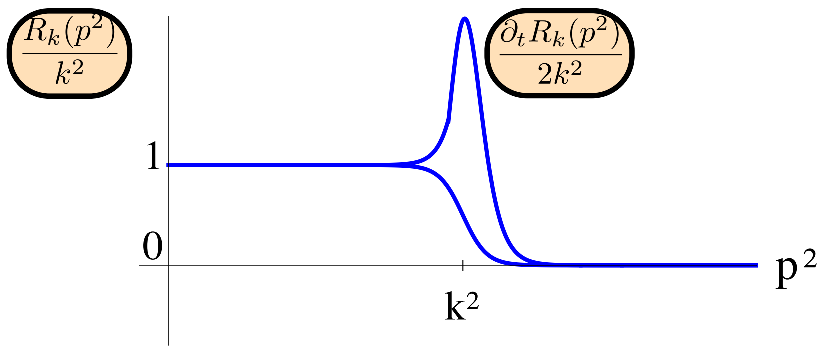
✓ automatic

– only in specific approximation schemes

Functional Renormalisation Group

Flow

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$



Properties

FunMethods

- **1-loop exact** ✓
- **closed** ✓
- **RG-scaling** ✓
- **Energy/particle-number conserv.** ✓

✓ **automatic**

– **only in specific approximation schemes**

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

$$\partial_t \Gamma^{(n)} = \mathrm{Flow}_n[\Gamma^{(m)}; m = 2, ..., n+2]$$

Derivative expansion

- **Expansion in powers of momenta**
- **controlled in the presence of a mass gap** m_{gap}
- **Expansion parameter** $\frac{p^2}{\max(k^2, m_{\mathrm{gap}}^2)}$

Vertex expansion

- **Expansion in number** n **of external fields**
- **controlled in perturbation theory/presence of symmetries**
- **Expansion parameter** n

Mixtures, exact resummation schemes, ...

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

- **Expansion in powers of momenta**
- **controlled in the presence of a mass gap** $m_{\rm gap}$
- **Expansion parameter** $\frac{p^2}{\max(k^2, m_{\rm gap}^2)}$

Lowest order: 0th order

$$\Gamma_k[\phi] = \frac{1}{2} \int_p \phi \, p^2 \phi + \int_x V_k(\phi) + O(p^2)$$

$$\Gamma_k^{(2)}[\phi](p,q) = \left(p^2 + V_k''(\phi)\right) (2\pi)^d \delta(p-q)$$

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

Lowest order: 0th order

$$\Gamma_k[\phi] = \frac{1}{2} \int_p \phi\, p^2 \phi + \int_x V_k(\phi) + O(p^2)$$

$$\Gamma_k^{(2)}[\phi](p,q) = \left(p^2 + V_k''(\phi)\right) (2\pi)^d \delta(p-q)$$

$$R_{k,\mathrm{opt}}(p^2) = (k^2 - p^2)\theta(k^2 - p^2)$$

$$\partial_t R_{k,\mathrm{opt}}(p^2) = 2k^2 \theta(k^2 - p^2)$$

Flow

$$\partial_t V_k[\phi] = \frac{1}{2d} \frac{\Omega_d}{(2\pi)^d} k^d \frac{k^2}{k^2 + V''(\phi)}$$

$$\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

Approximation schemes

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Derivative expansion

Lowest order: 0th order

$$\Gamma_k[\phi] = \frac{1}{2} \int_p \phi\, p^2 \phi + \int_x V_k(\phi) + O(p^2)$$

$$\Gamma_k^{(2)}[\phi](p,q) = \left(p^2 + V_k''(\phi)\right) (2\pi)^d \delta(p-q)$$

$$\Gamma^{(2)}[\phi](p) + R_{k,\mathrm{opt}}(p^2) = \left[k^2 + V''(\phi)\right] \theta(k^2 - p^2) + (p^2 + V''(\phi))\theta(p^2 - k^2)$$

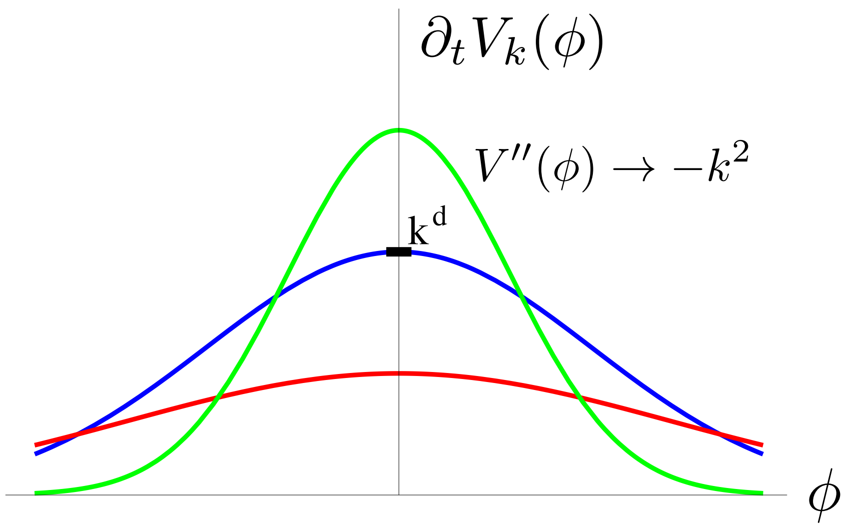
Flow

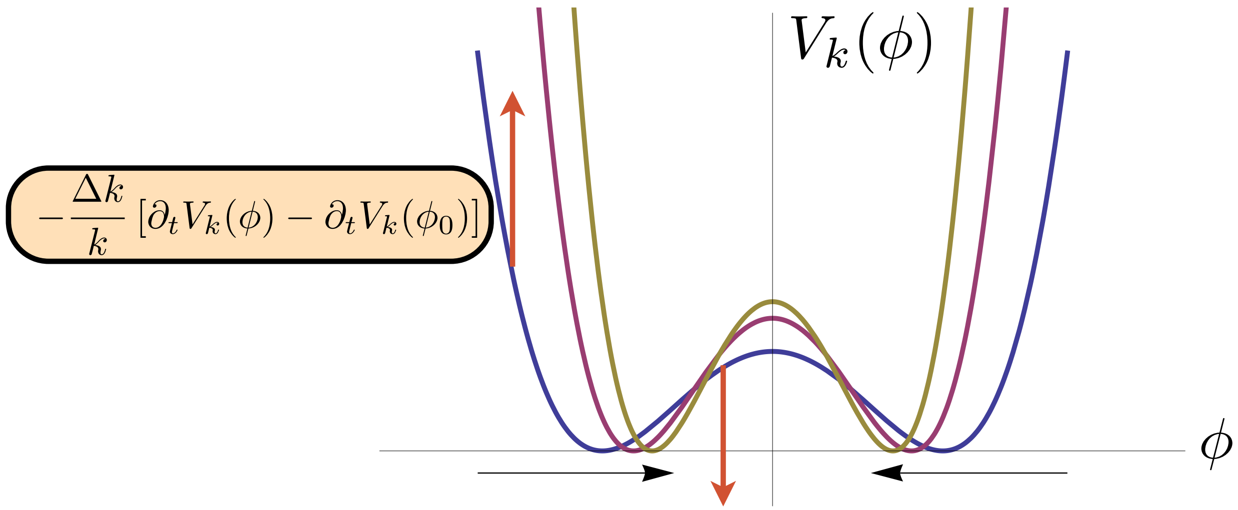
$$\partial_t V_k[\phi] = \frac{1}{2d} \frac{\Omega_d}{(2\pi)^d} k^d \frac{k^2}{k^2 + V''(\phi)}$$

$$\Omega_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

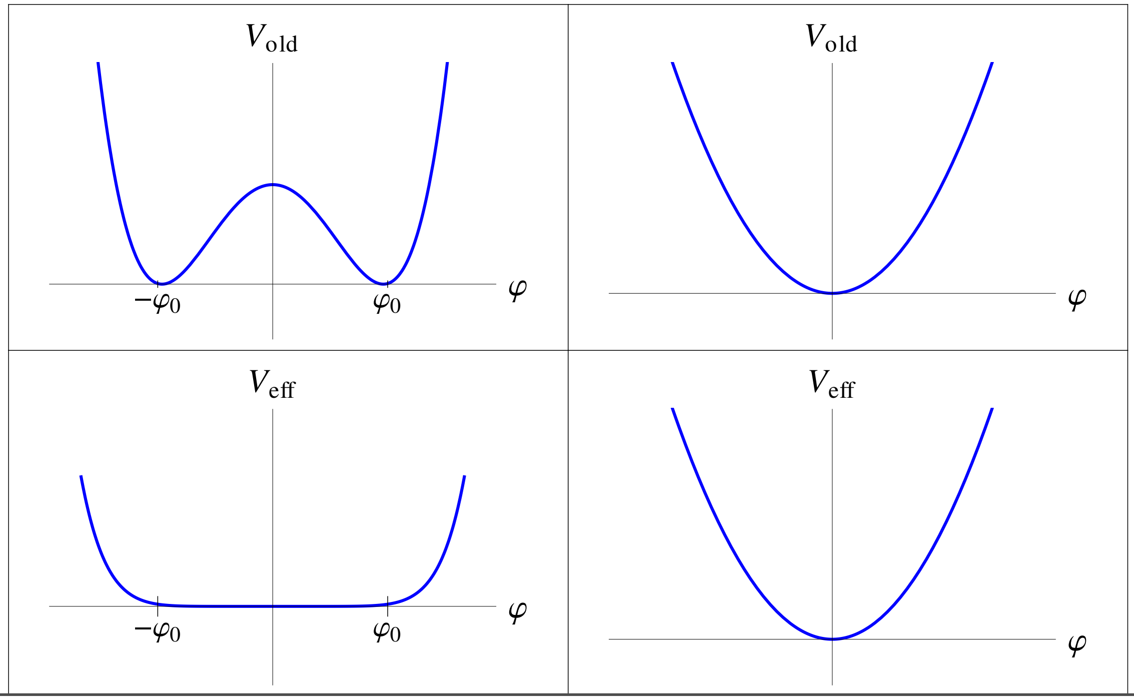
Approximation schemes & phase structure

$$\partial_t V_k[\phi] = \frac{1}{2d}\frac{\Omega_d}{(2\pi)^d}k^d\frac{k^2}{k^2+V''(\phi)}$$



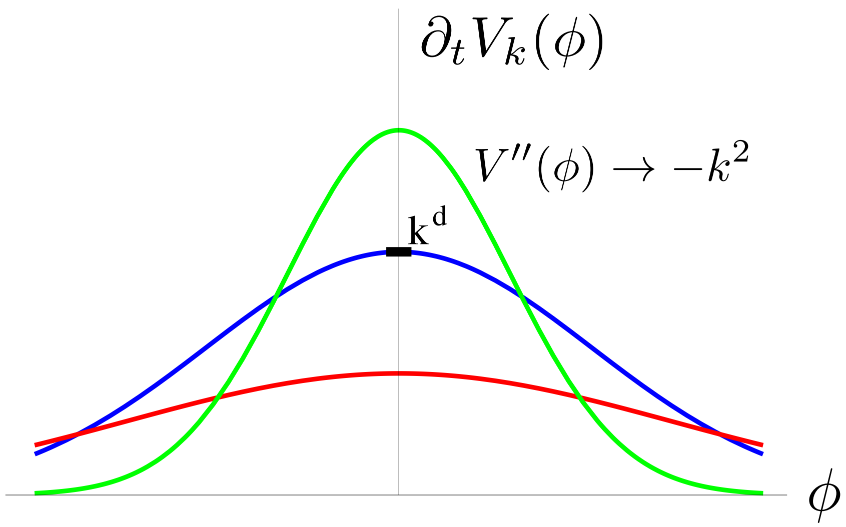


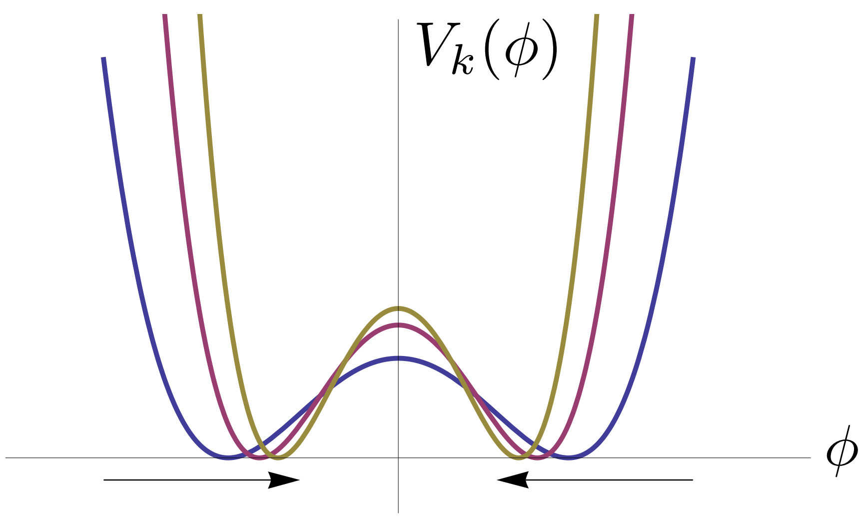
- bosonic flow is symmetry-restoring
- flow guarantees convexity



Approximation schemes & phase structure

$$\partial_t V_k[\phi] = \frac{1}{2d}\frac{\Omega_d}{(2\pi)^d}k^d \frac{k^2}{k^2+V''(\phi)}$$





- bosonic flow is symmetry-restoring
- flow guarantees convexity of effective action

Litim, JMP, Vergara '06

Example: 3d critical exponents with FRG

$$\Gamma_k[\phi] = \frac{1}{2}\int_p Z_k\,\phi\, p^2 \phi + \int_x V_k(\phi)$$

$$V_k(\phi) = \sum_{n=1}^{N_{\max}} \frac{\lambda_n}{n!}(\phi^2 - \phi_{0,k}^2)^n$$

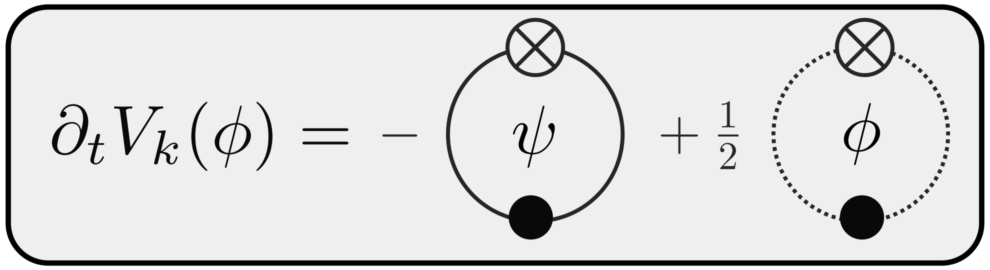
$N = 1:\ \nu_{\text{Ising}} = 0.630...$

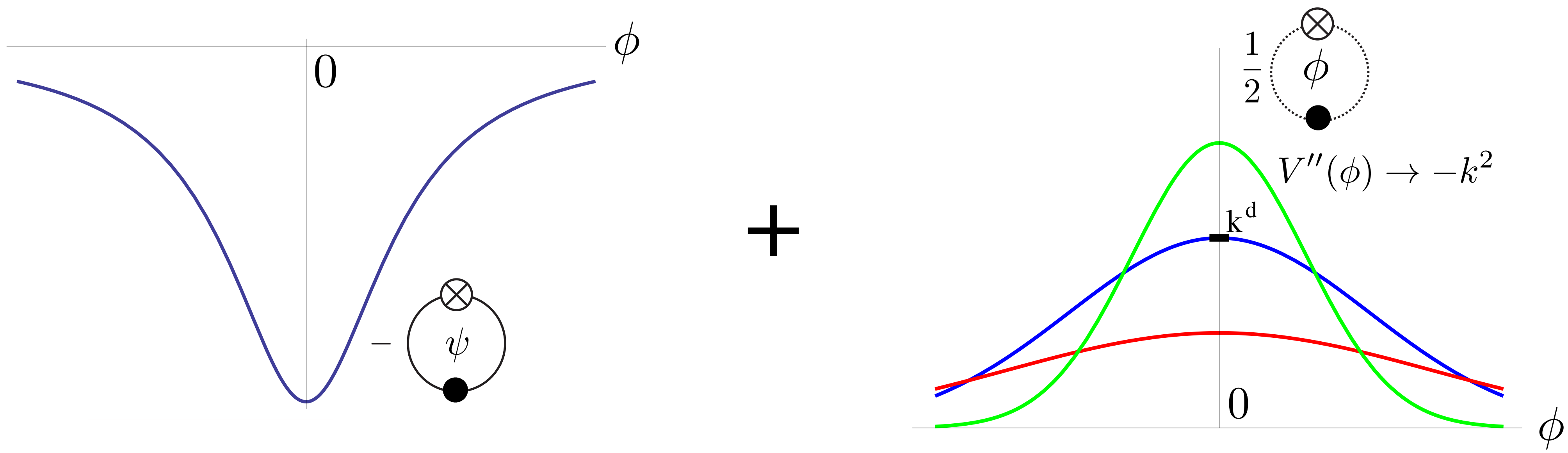
$N = 1:\ \nu_{\text{Ising}} = 0.637...$

A simple program to compute critical exponents in O(N)-models with the Wetterich equation

Michael Scherer

Approximation schemes & phase structure





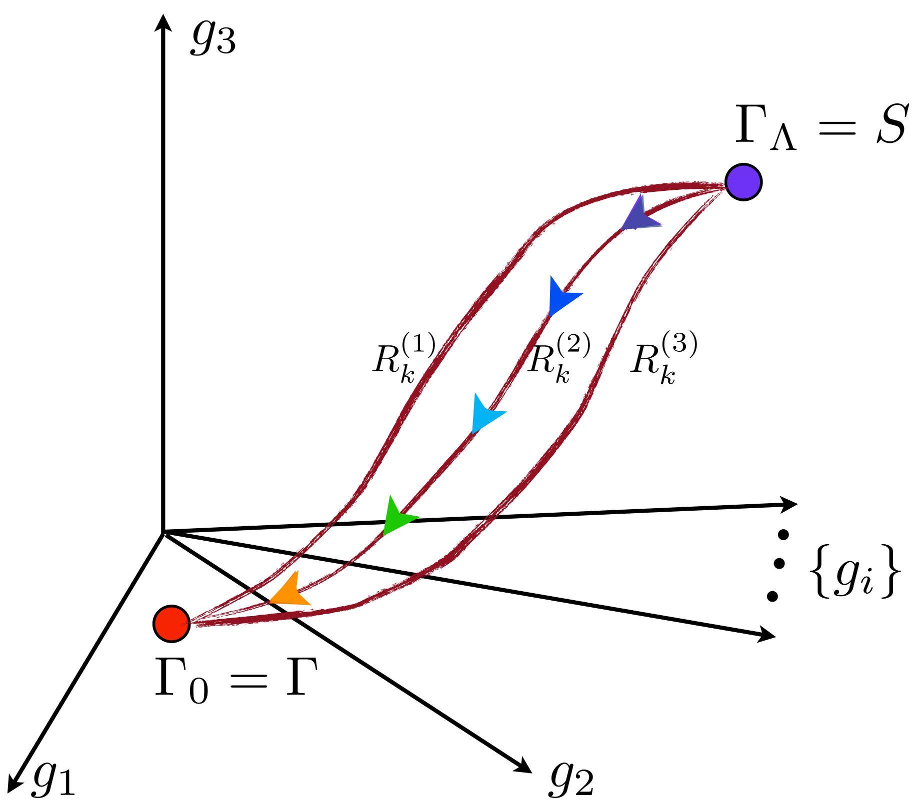
- **bosonic flow is symmetry-restoring**
- **fermionic flow is symmetry-breaking**
- **competing dynamics decides about fate of symmetries**
- **flow guarantees convexity**

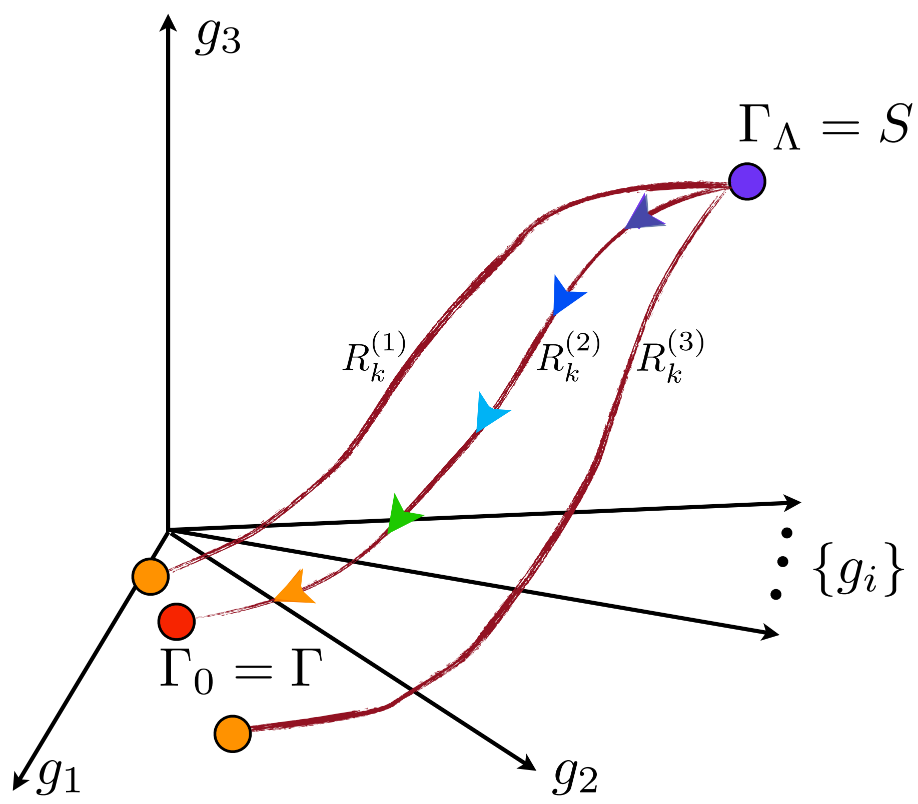
'governs general phase structures'

Approximation schemes & error control

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t R_k$$

Theory space





full flow

approximated flow

Optimisation: find $R_k^{(2)}$!

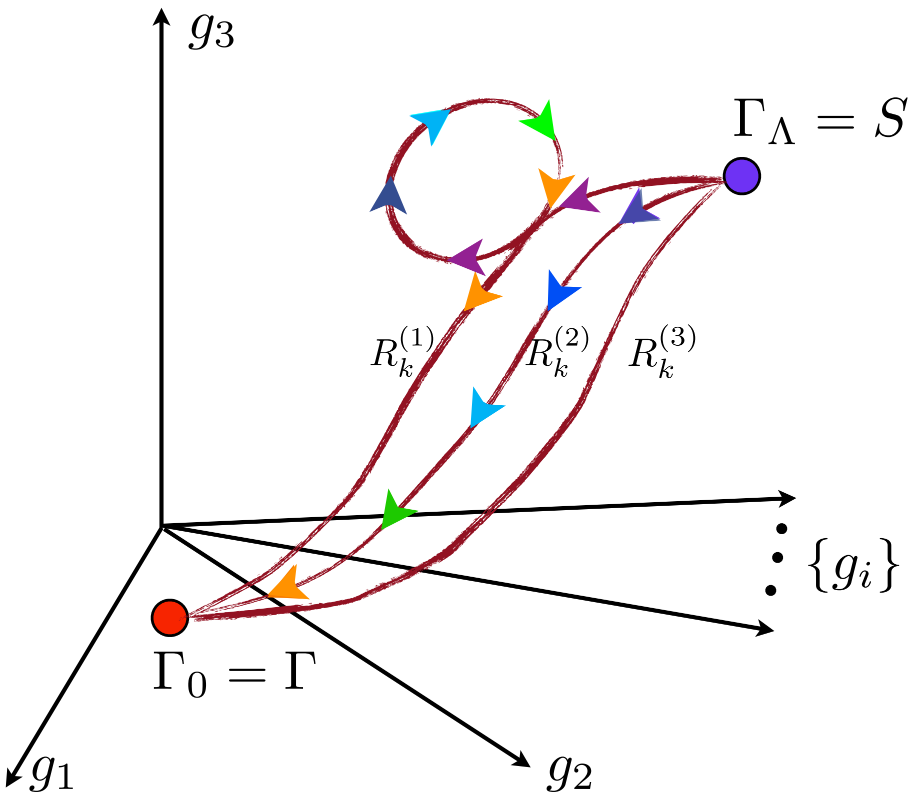
Litim '01: most rapid convergence

JMP '05: integrability

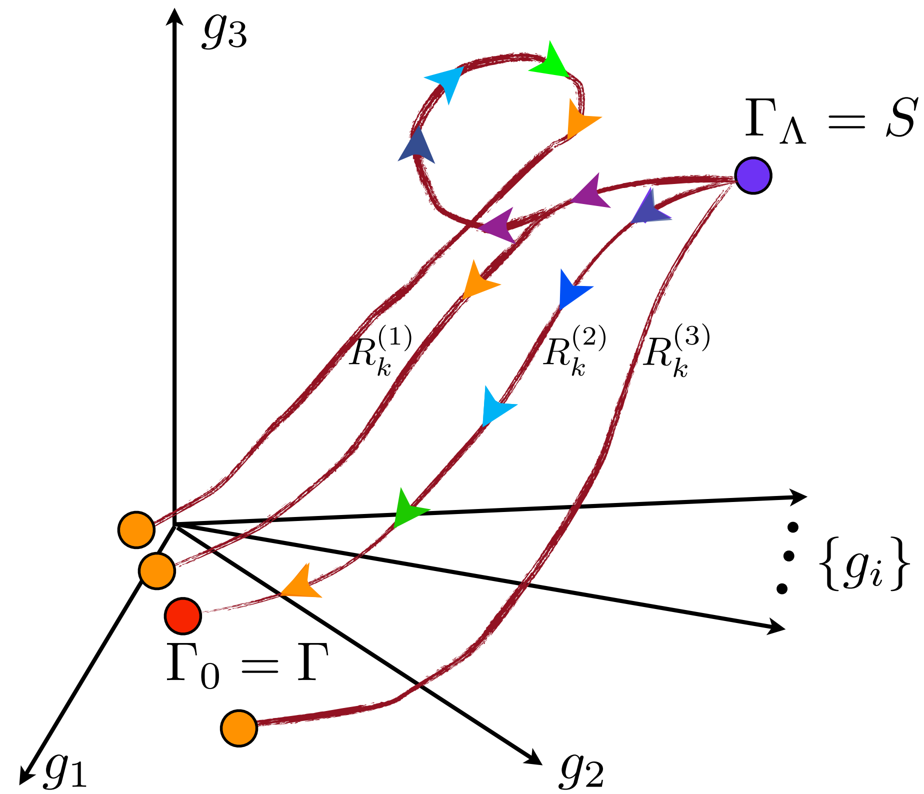
Approximation schemes & error control

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr} \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Theory space



full flow



approximated flow

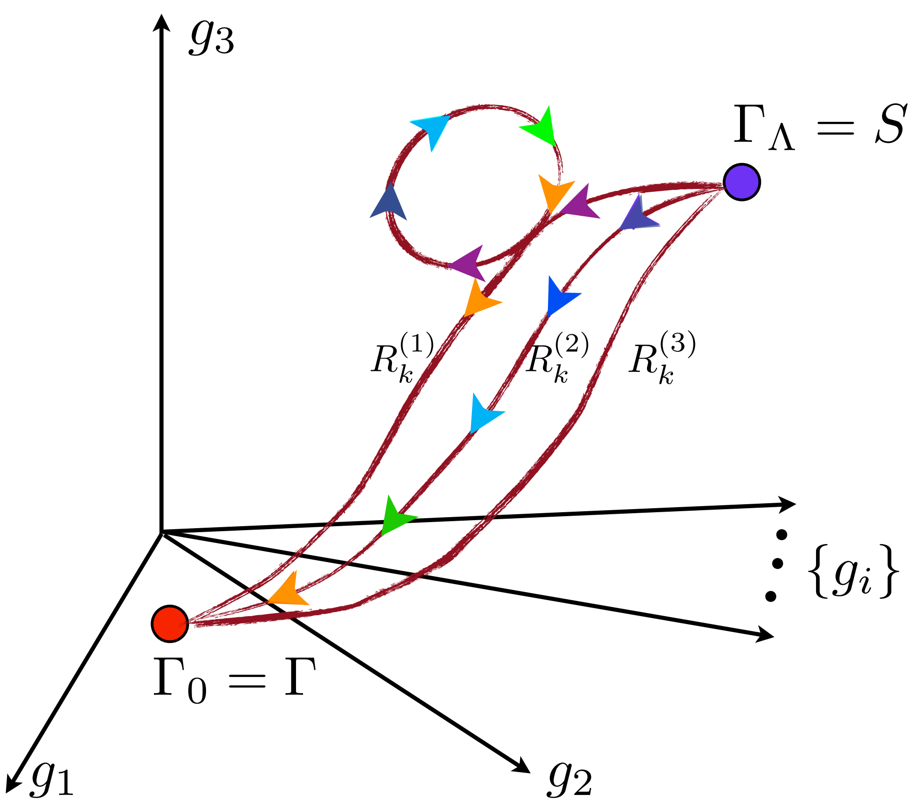
Optimisation: find $R_k^{(2)}$!

JMP '05: integrability

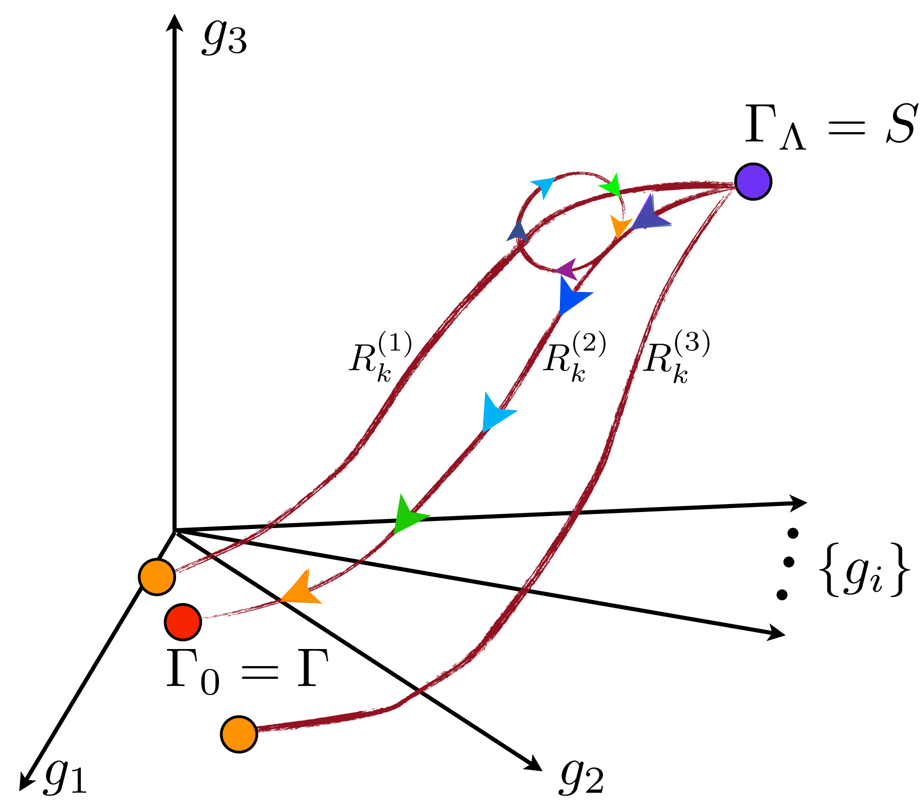
Approximation schemes & error control

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \frac{1}{\Gamma_k^{(2)}[\phi] + R_k} \partial_t R_k$$

Theory space

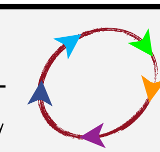


full flow



optimised flow

Optimisation: find $R_k^{(2)}$!

$\lim_{L \to 0} \frac{1}{L}$  $\to 0$

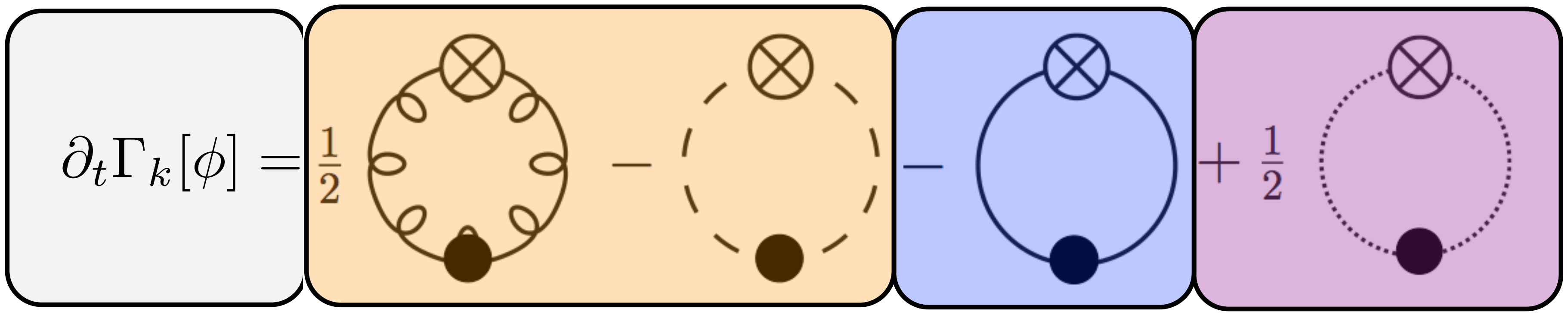
JMP '05: integrability

FRG for QCD

Functional Methods for QCD

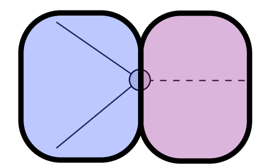
JMP, AIP Conf.Proc. 1343 (2011)

glue quantum fluctuations **hadronic quantum fluctuations**



free energy **quark quantum fluctuations**

RG-scale k: $t = \ln k$



dynamical

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

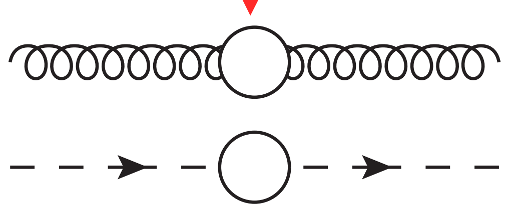
Yang-Mills:

$$\partial_t \Gamma_k[A, \bar{c}, c] = \frac{1}{2} \mathrm{Tr} \left\{ \frac{1}{\Gamma^{(2)}[A, \bar{c}, c] + R_k} \, \partial_t R_k \right\} - \partial_t C_k$$

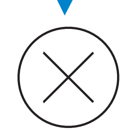
$\partial_t = k\ \partial_k$

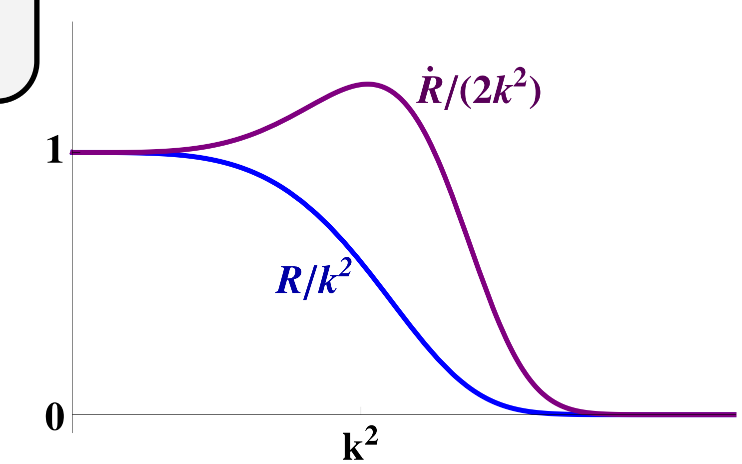
by L. Fister

full propagator



regulator

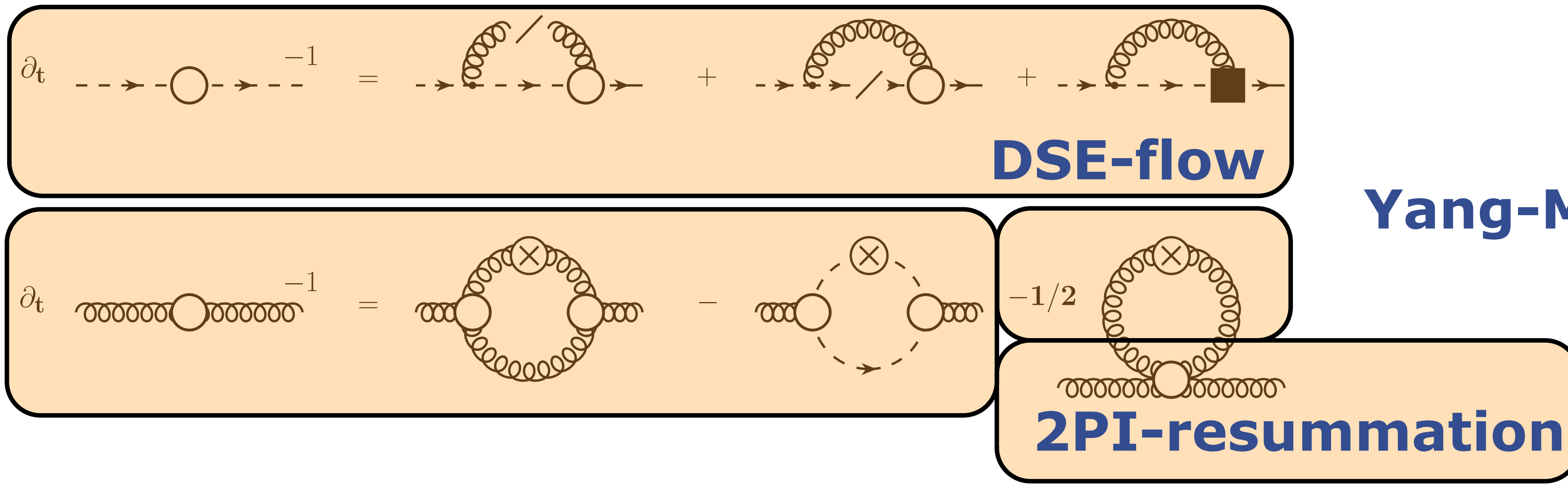


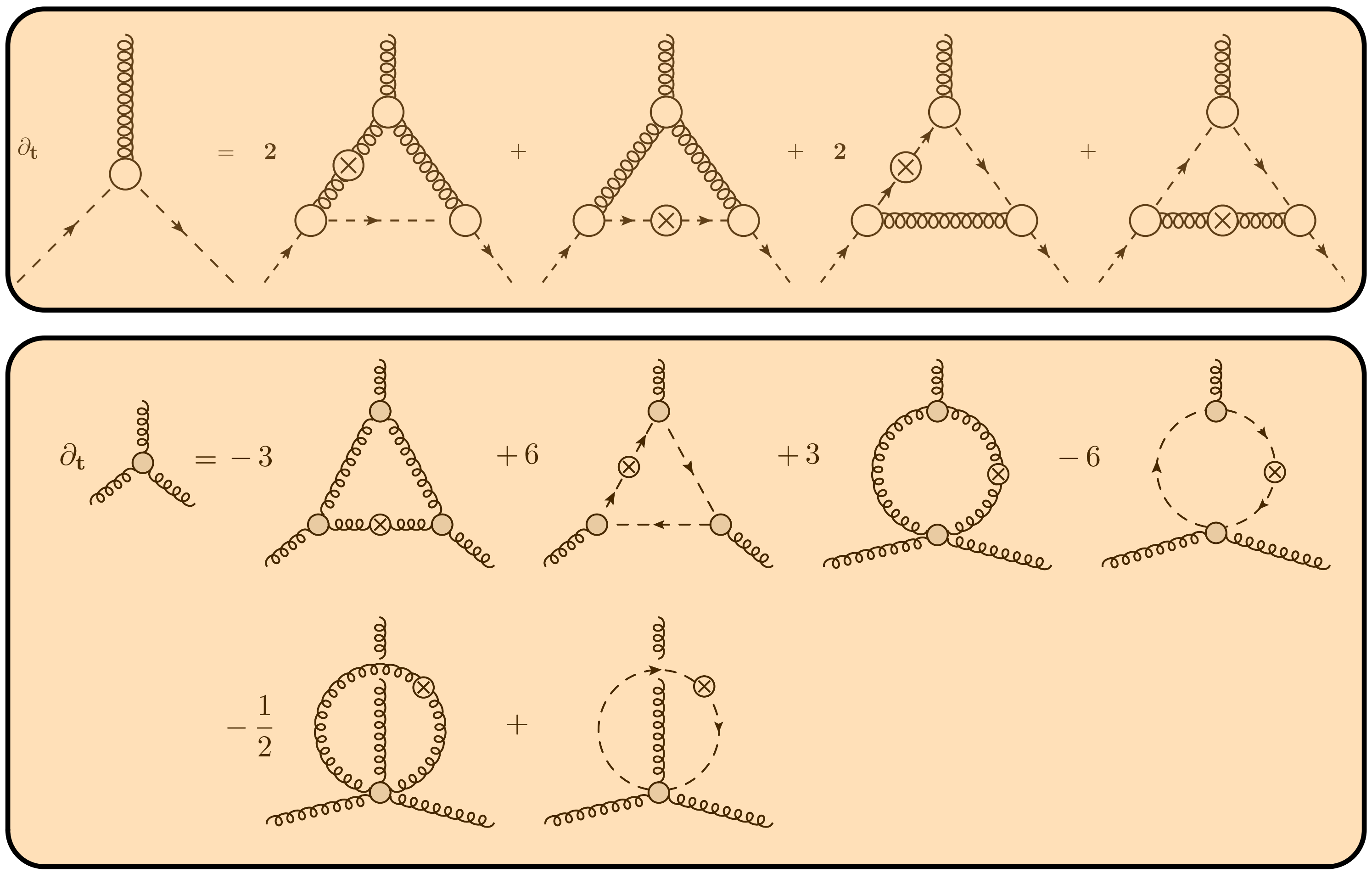


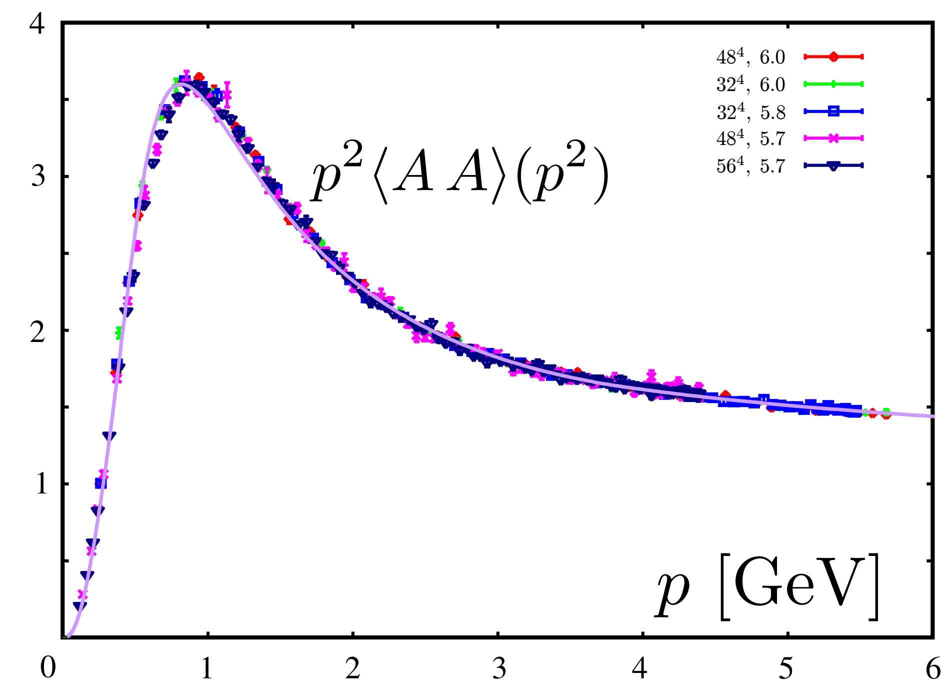
Functional Methods for QCD

Fister, JMP '11, 13

Yang-Mills propagators







FRG: Fischer, Maas, JMP '08

lattice: Sternbeck et al. '06

Functional Methods for QCD

...and now for something completely different .....

Nedelko, JMP unpublished '04
Fischer, Maas, JMP '08

Gauge invariance & Slavnov-Taylor identities

Landau gauge

STI

$$\Gamma_L^{(n)} = \mathrm{STI}_{\Gamma_L^{(n)}}[\,\{\Gamma_T^{(m)}\},\,\{\Gamma_L^{(m)}\}]$$

symmetries

FunEquations

$$\Gamma_T^{(n)} = \mathrm{F}_{\Gamma_T^{(n)}}[\{\Gamma_T^{(m)}\}]$$

dynamics

$$\Gamma_L^{(n)} = \mathrm{F}_{\Gamma_L^{(n)}}[\,\{\Gamma_T^{(m)}\},\,\{\Gamma_L^{(m)}\}]$$

symmetries

Functional Methods for QCD

...and now for something completely different .....

Nedelko, JMP unpublished '04
Fischer, Maas, JMP '08

Gauge invariance & Slavnov-Taylor identities

Landau gauge

STI

$$\Gamma_L^{(n)} = \mathrm{STI}_{\Gamma_L^{(n)}}[\,\{\Gamma_T^{(m)}\},\,\{\Gamma_L^{(m)}\}]$$

Uniformity/
Differentiability w.r.t. momentum

works in perturbation theory

not fully non-perturbatively

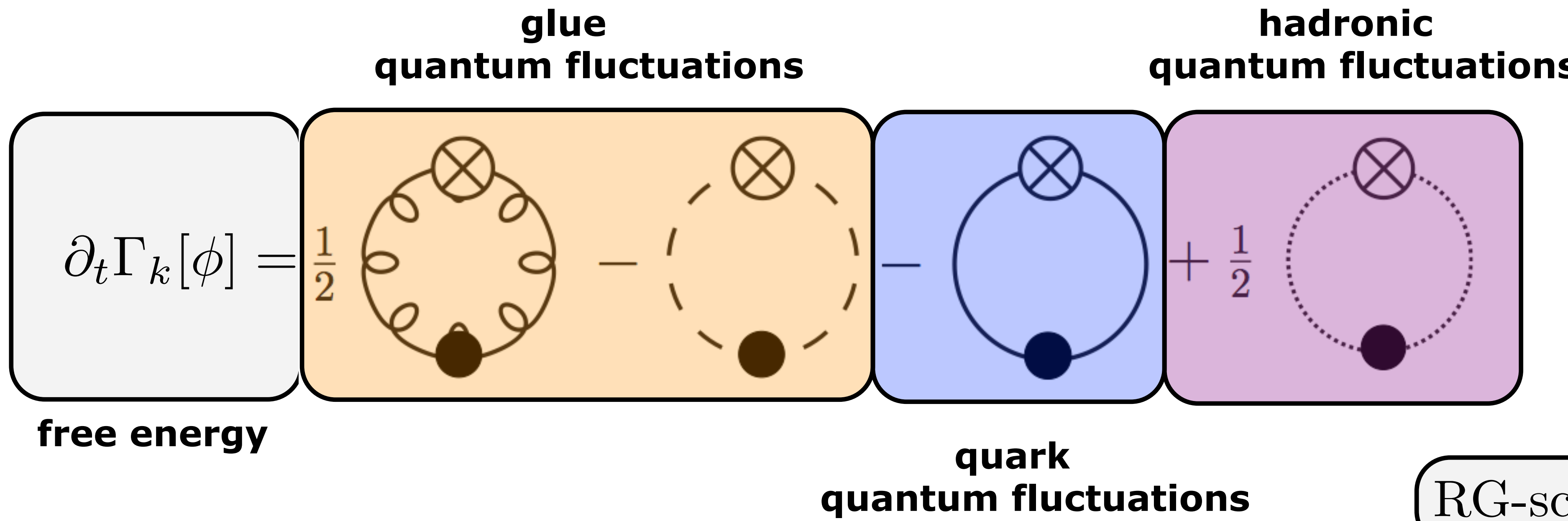
FunEquations

$$\Gamma_T^{(n)} = \mathrm{F}_{\Gamma_T^{(n)}}[\{\Gamma_T^{(m)}\}]$$

$$\Gamma_L^{(n)} = \mathrm{F}_{\Gamma_L^{(n)}}[\,\{\Gamma_T^{(m)}\},\,\{\Gamma_L^{(m)}\}]$$

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)



RG-scale k: $t = \ln k$

- **Gluons have cost us decades**

- **Fermions are straightforward** though 'physically' complicated
  - **no sign problem**
  - **chiral fermions**

- **bound states via dynamical hadronisation**

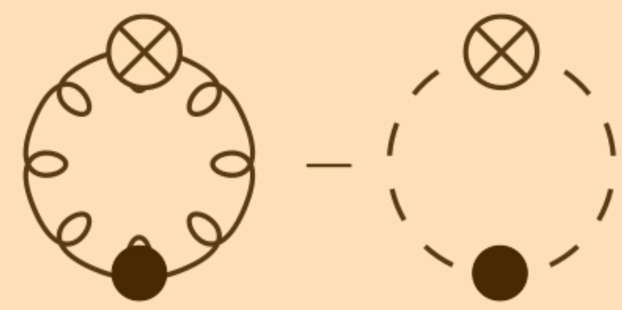
Complementary to lattice!

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

glue
quantum fluctuations

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$



free energy

free energy

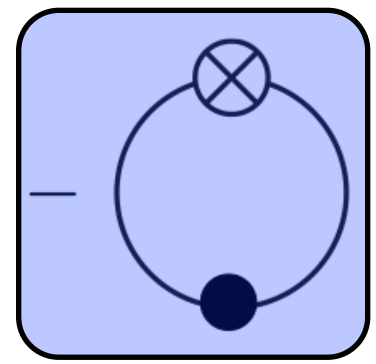
Yang-Mills theory

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

$$\partial_t \Gamma_k[\phi] =$$

free energy



quark
quantum fluctuations

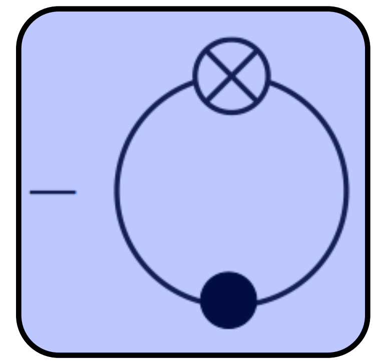
NJL-type models

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

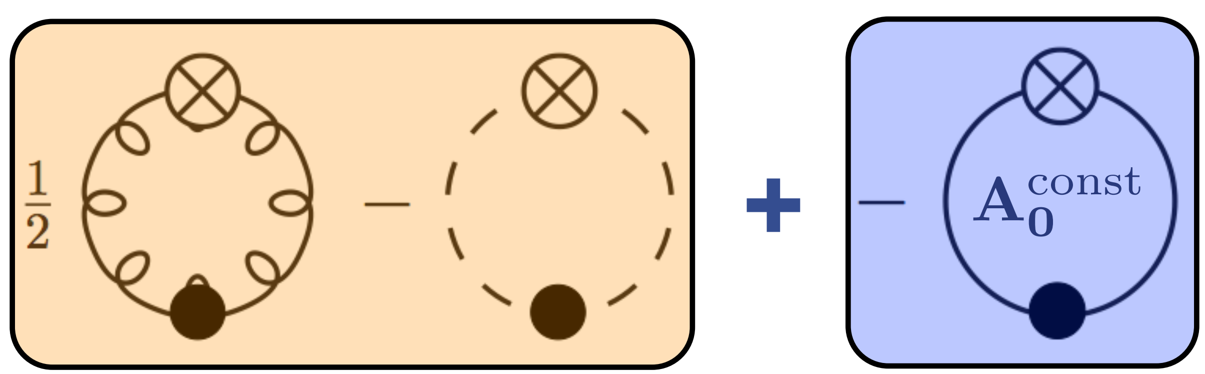
$$\partial_t \Gamma_k[\phi] =$$

free energy



quark
quantum fluctuations

NJL-type models



PNJL models

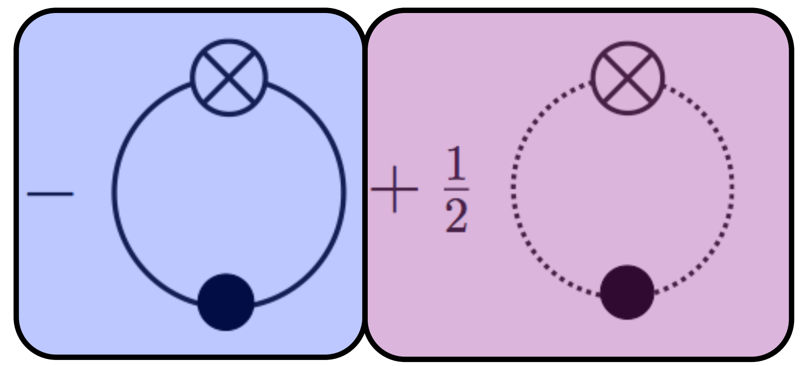
Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

benchmark in ultracold atoms

hadronic
quantum fluctuations

$$\partial_t \Gamma_k[\phi] =$$

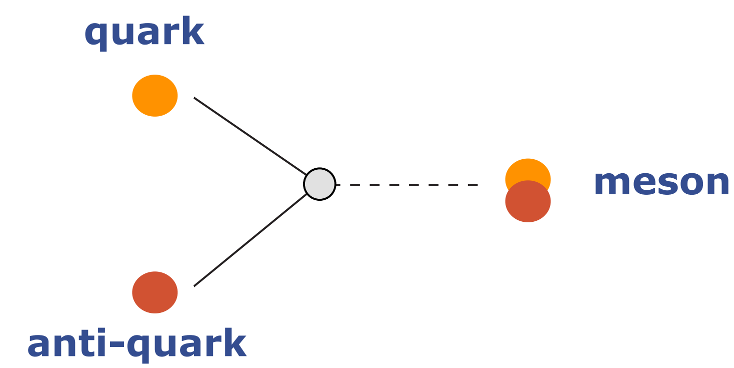


free energy

quark
quantum fluctuations

Quark-hadron models

'You name it, we do it'
John Thomas
QGP meets cold atoms-Episode III



bound states via dynamical hadronisation

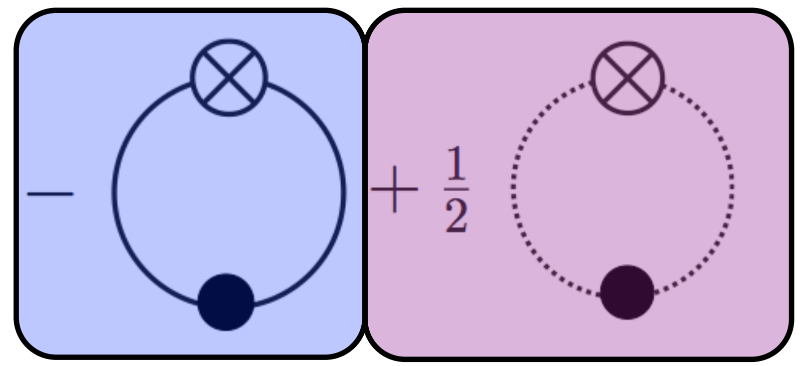
Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)

$$\partial_t \Gamma_k[\phi] =$$

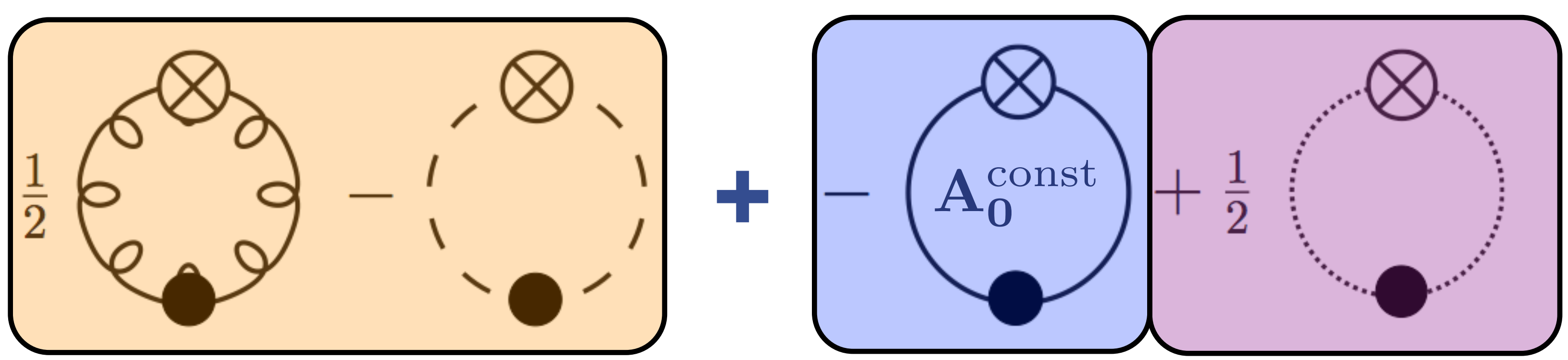
free energy

hadronic quantum fluctuations



quark quantum fluctuations

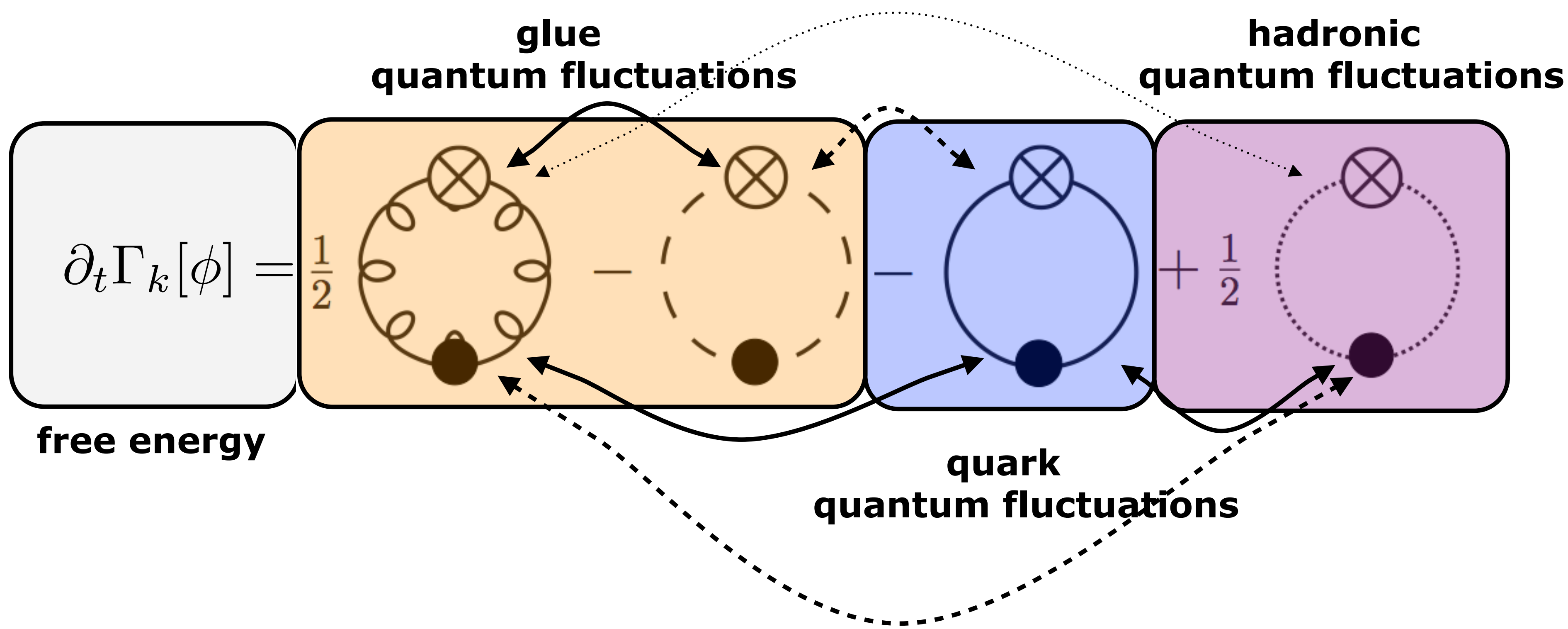
Quark-hadron models



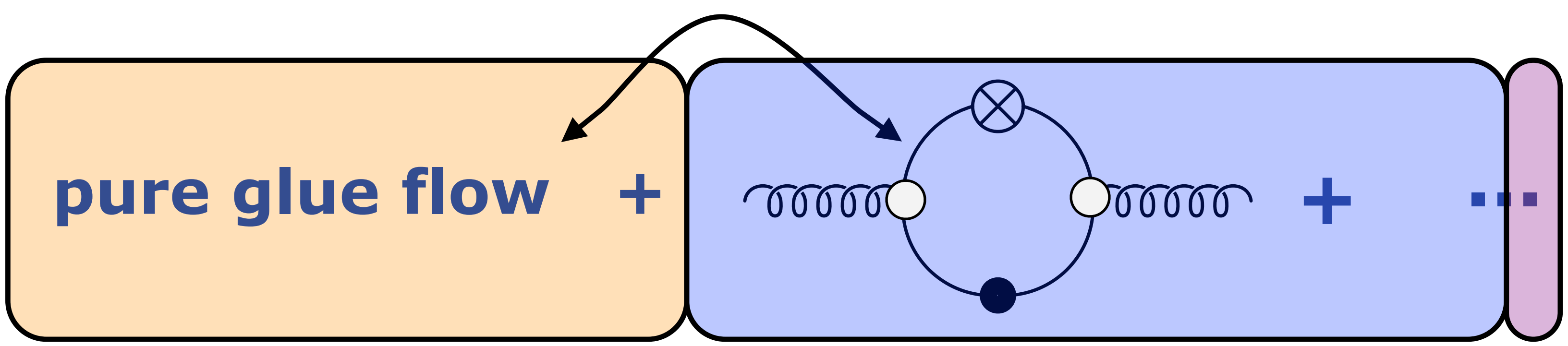
PQM models

Functional Methods for QCD

JMP, AIP Conf.Proc. 1343 (2011)



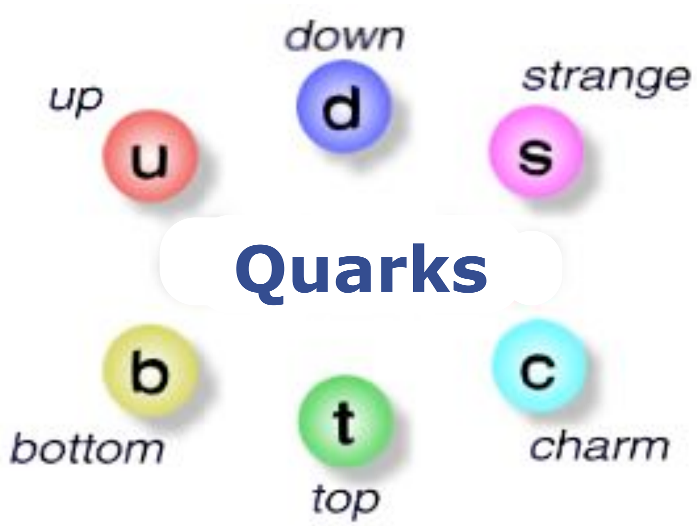
flow of gluon propagator



Naturally encorporates PQM/PNJL models as specific low order trunations

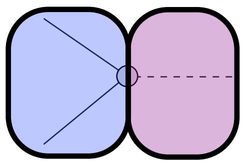
Dynamical hadronisation

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09



Quarks

Gluons



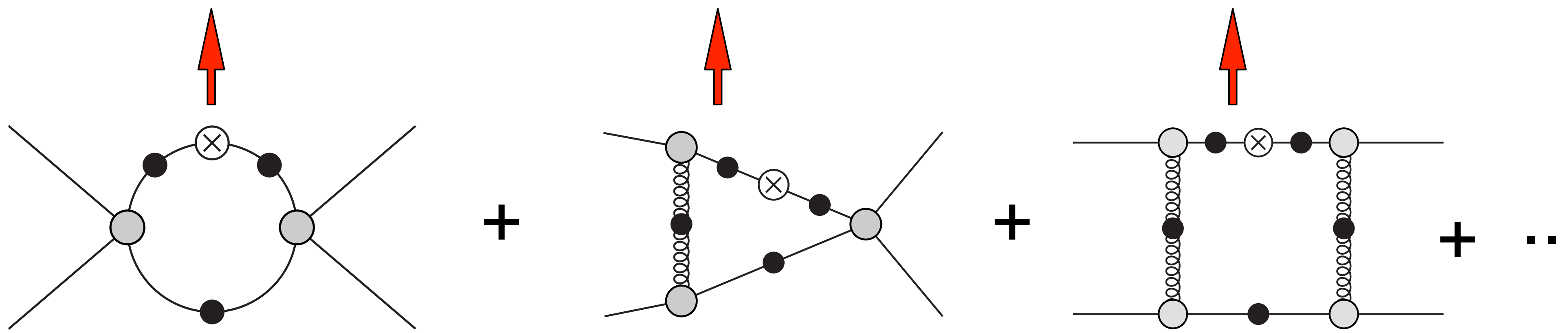
Hadrons

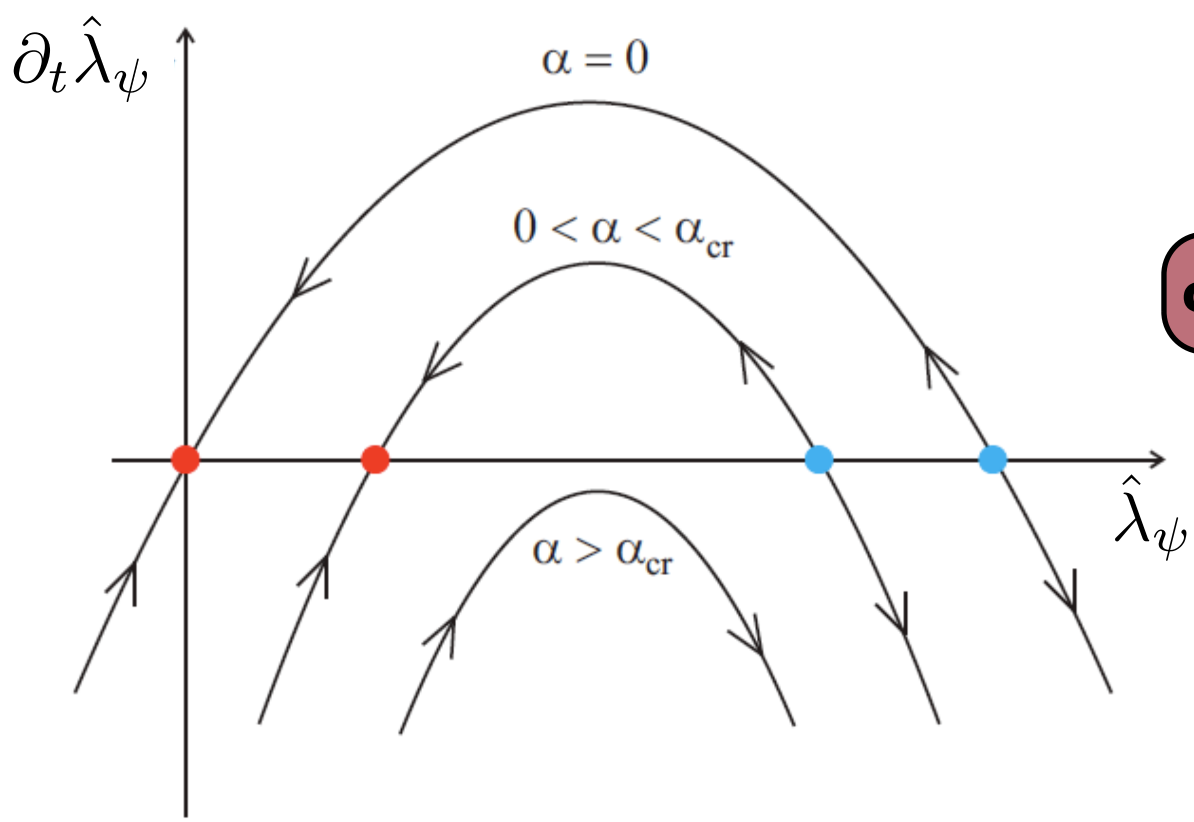
Chiral symmetry breaking

A glimpse at chiral symmetry breaking in QCD within the FRG

Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ **with infrared scale** k

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi + A\left(\frac{T}{k}\right)\hat{\lambda}_\psi^2 + B\left(\frac{T}{k}\right)\hat{\lambda}_\psi\,\alpha_s + C\left(\frac{T}{k}\right)\alpha_s^2 + \cdots$$





chiral symmetry breaking ⟷ $\alpha_s > \alpha_{s,cr}$

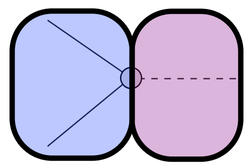
Dynamical hadronisation

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

$$\lambda_\psi=\frac{h^2}{m_\phi^2}$$

Hubbard-Stratonovich



$$\Phi=(\sigma,\vec\pi)$$

$$\tau\cdot\Phi=\sigma+i\gamma_5\vec\sigma\vec\pi$$

General dynamical hadronisation

hadronised Flow

$$\frac{\partial}{\partial t}\Big|_\phi\Gamma_k[\phi]=\frac{1}{2}G_{k,\phi}\dot R_{k,\phi}+R_kG_{k,\phi}\frac{\delta\dot\phi}{\delta\phi}-\frac{\delta\Gamma}{\delta\phi}\dot\phi$$

JMP '05

$$\phi=(A_\mu,C,\bar C,q,\bar q,\Phi,\ldots,n,\bar n,\ldots)$$

mesons baryons

$$-\frac{1}{2}\int_p\phi_k^*\cdot R_k\cdot\phi_k+J\cdot\phi_k$$

guarantees 1-loop flow

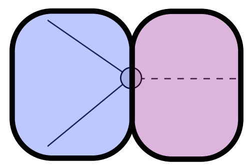
Dynamical hadronisation

Gies, Wetterich '01
JMP '05
Flörchinger, Wetterich '09

$$\frac{\lambda_\psi}{2}\left[(\bar\psi\psi)^2-(\bar\psi\gamma_5\vec\tau\psi)^2\right]=\left[i\,h\,\bar\psi(\tau\cdot\Phi)\psi+\frac{1}{2}m_\phi^2\Phi^2\right]_{\mathrm{EoM}(\Phi)}$$

$$\lambda_\psi=\frac{h^2}{m_\phi^2}$$

Hubbard-Stratonovich



$$\Phi=(\sigma,\vec\pi)$$

$$\tau\cdot\Phi=\sigma+i\gamma_5\vec\sigma\vec\pi$$

General dynamical hadronisation

hadronised Flow

$$\frac{\partial}{\partial t}\Big|_\phi\Gamma_k[\phi]=\frac{1}{2}G_{k,\phi}\dot R_{k,\phi}+R_kG_{k,\phi}\frac{\delta\dot\phi}{\delta\phi}-\frac{\delta\Gamma}{\delta\phi}\dot\phi$$

JMP '05

$$\phi=(A_\mu,C,\bar C,q,\bar q,\Phi,...,n,\bar n,...)$$

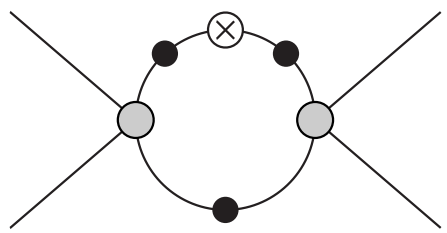
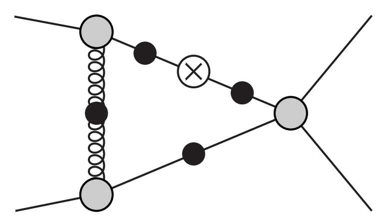
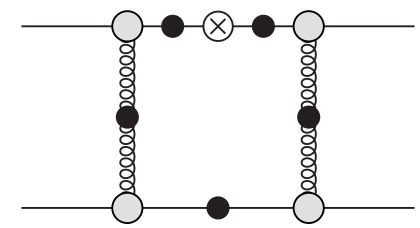
mesons **baryons**

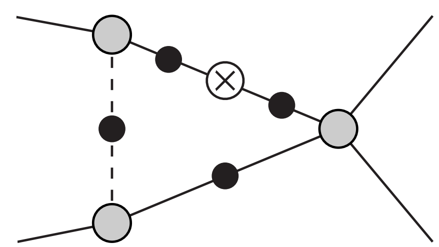
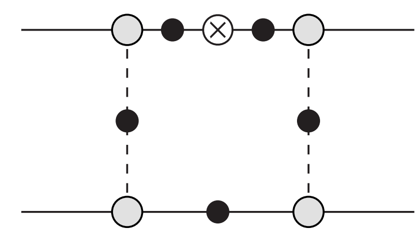
How to fix ϕ_k & $\dot\phi_k$?

$$\dot\Phi_k\simeq\dot A_k\bar\psi\tau\psi+\dot B_k\Phi_k+\dot C_k$$

Dynamical hadronisation

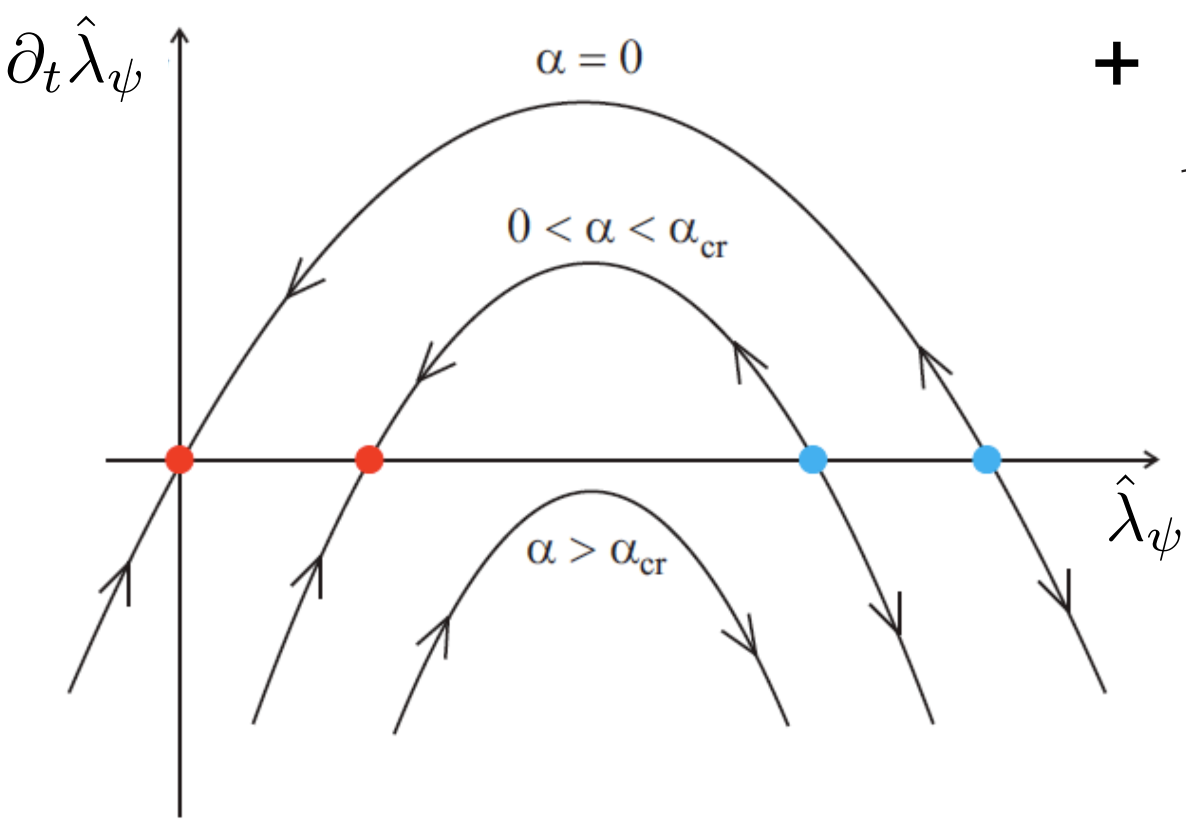
Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale k

$k\partial_k \hat{\lambda}_\psi$ = $2\hat{\lambda}_\psi$ +  +  + 

+  + 

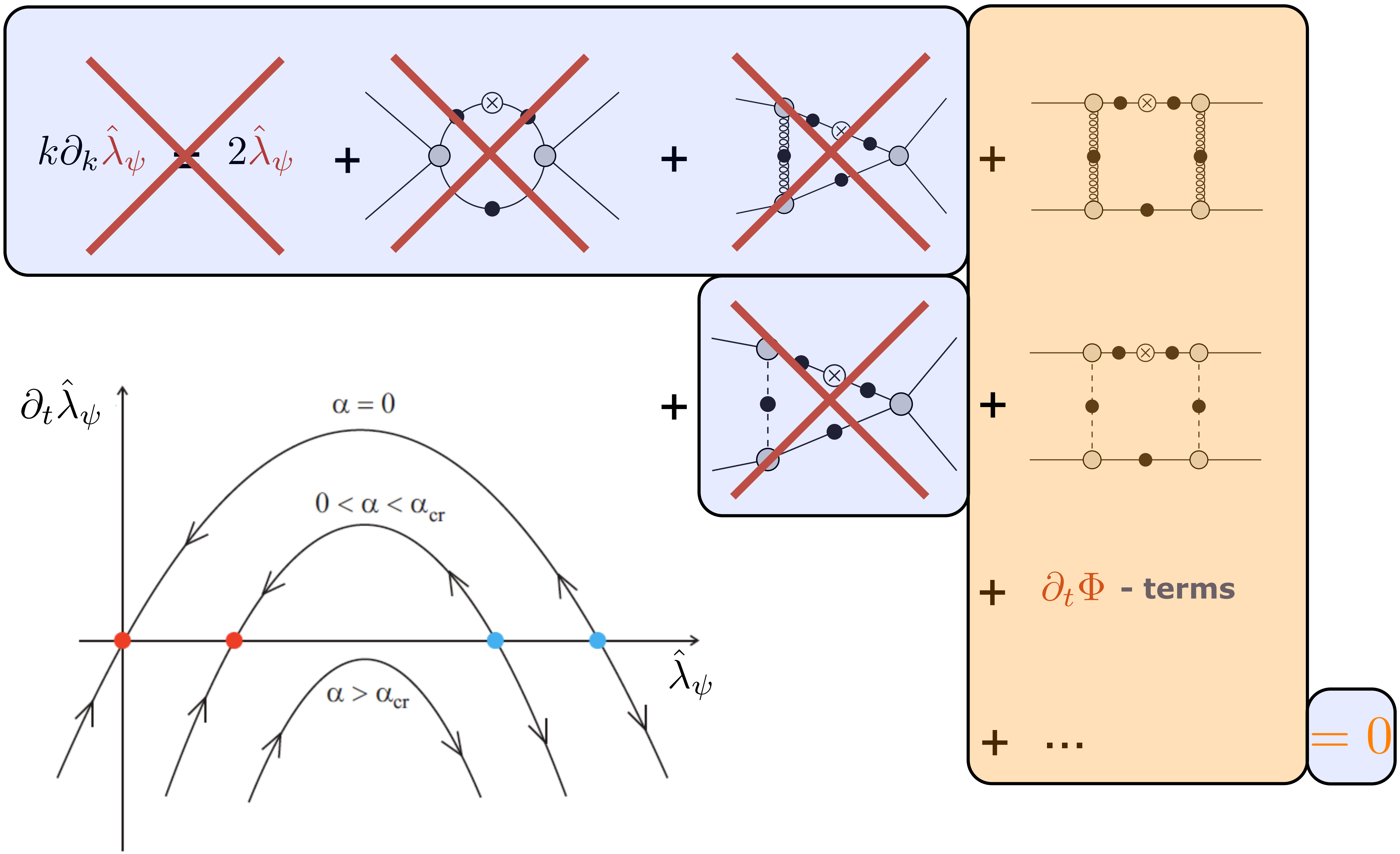
+ $\partial_t \Phi$ - terms

+ ...



Dynamical hadronisation

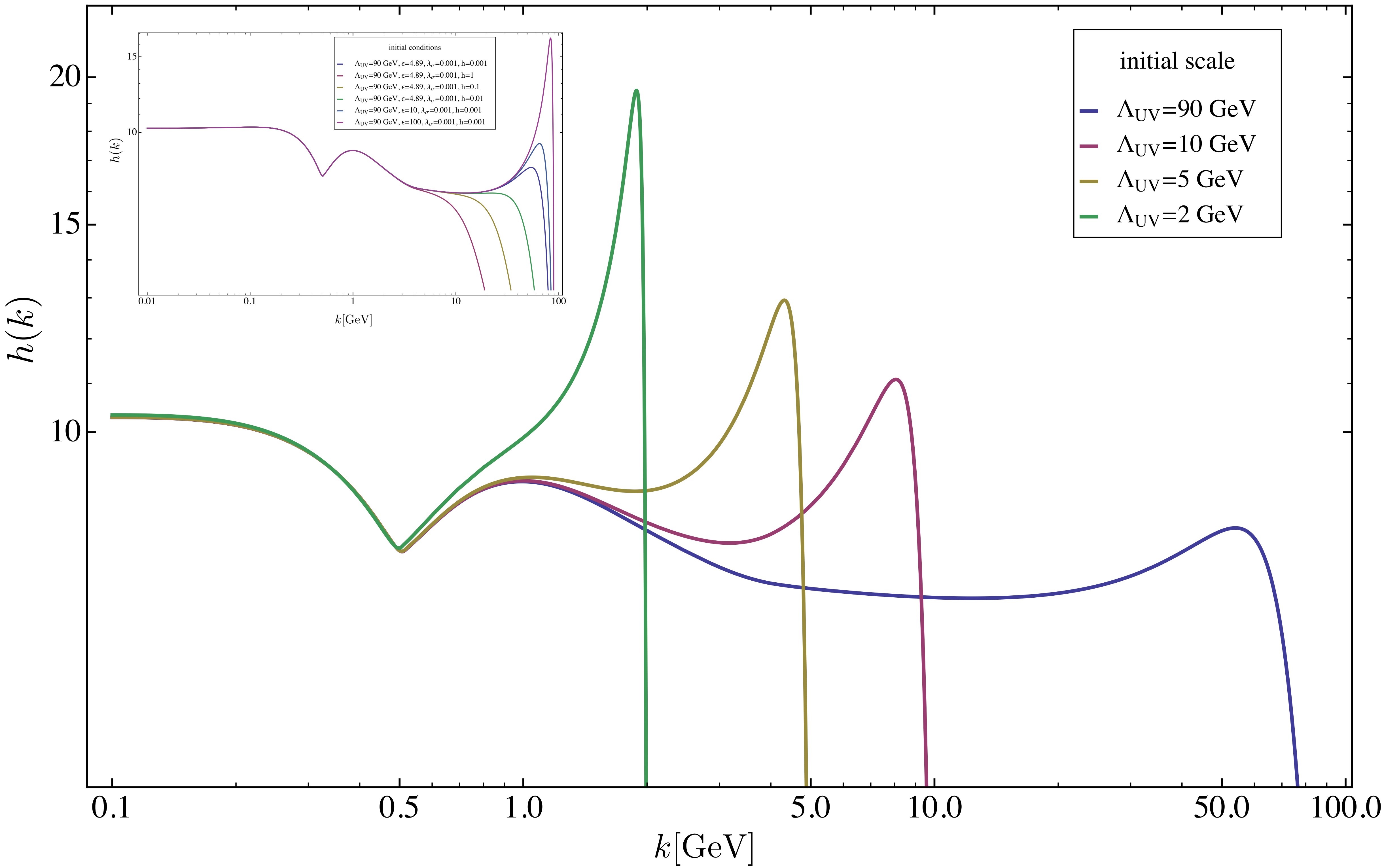
Full bosonisation $\hat{\lambda}_\psi = 0$



Dynamical hadronisation

Braun, Fister, Haas, JMP, Rennecke, in prep

Full bosonisation $\hat{\lambda}_\psi = 0$



Dynamical hadronisation

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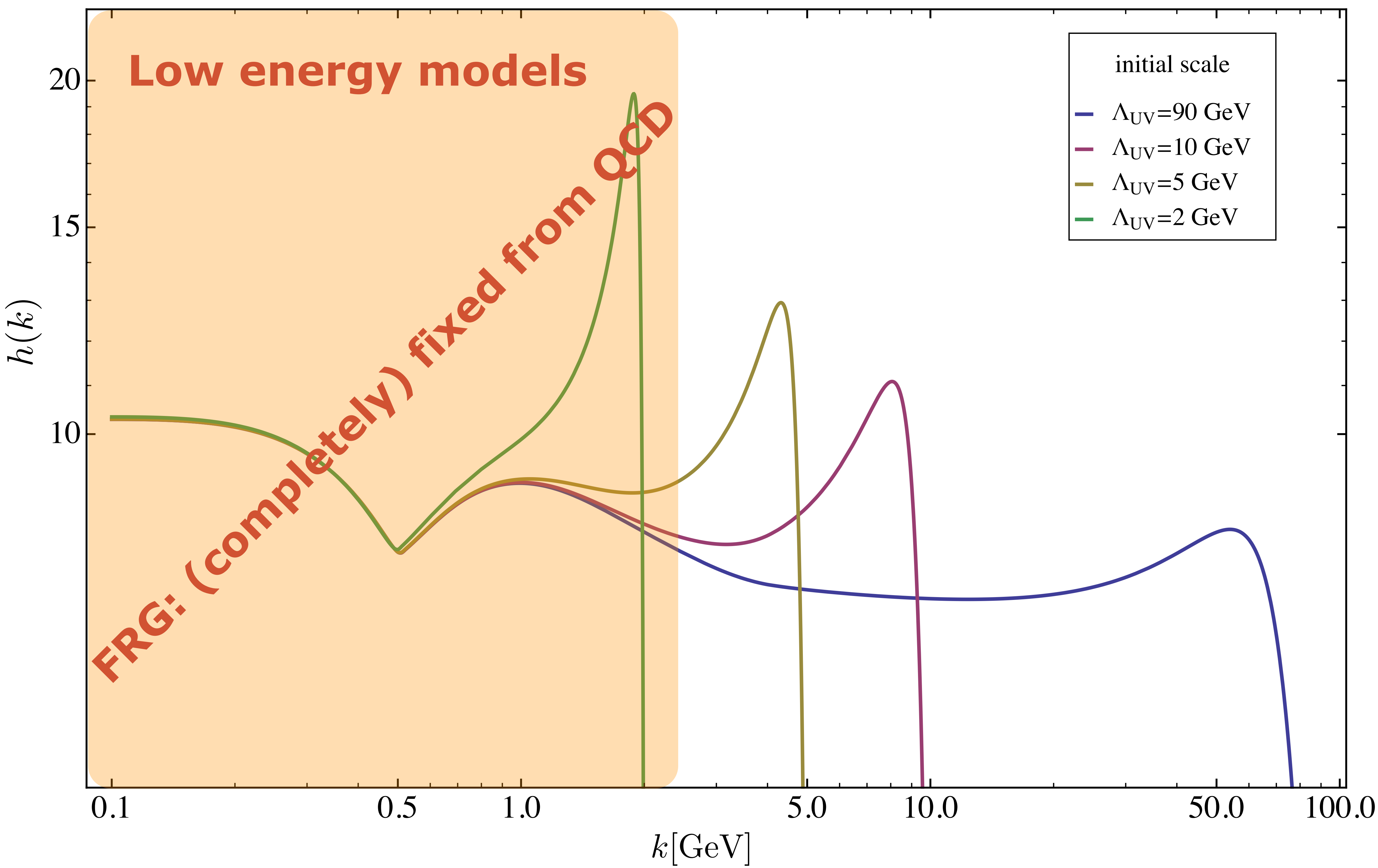
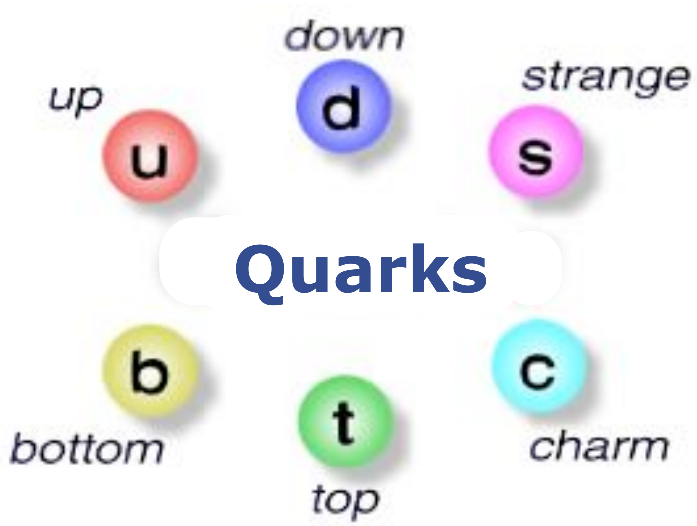


Approximation scheme



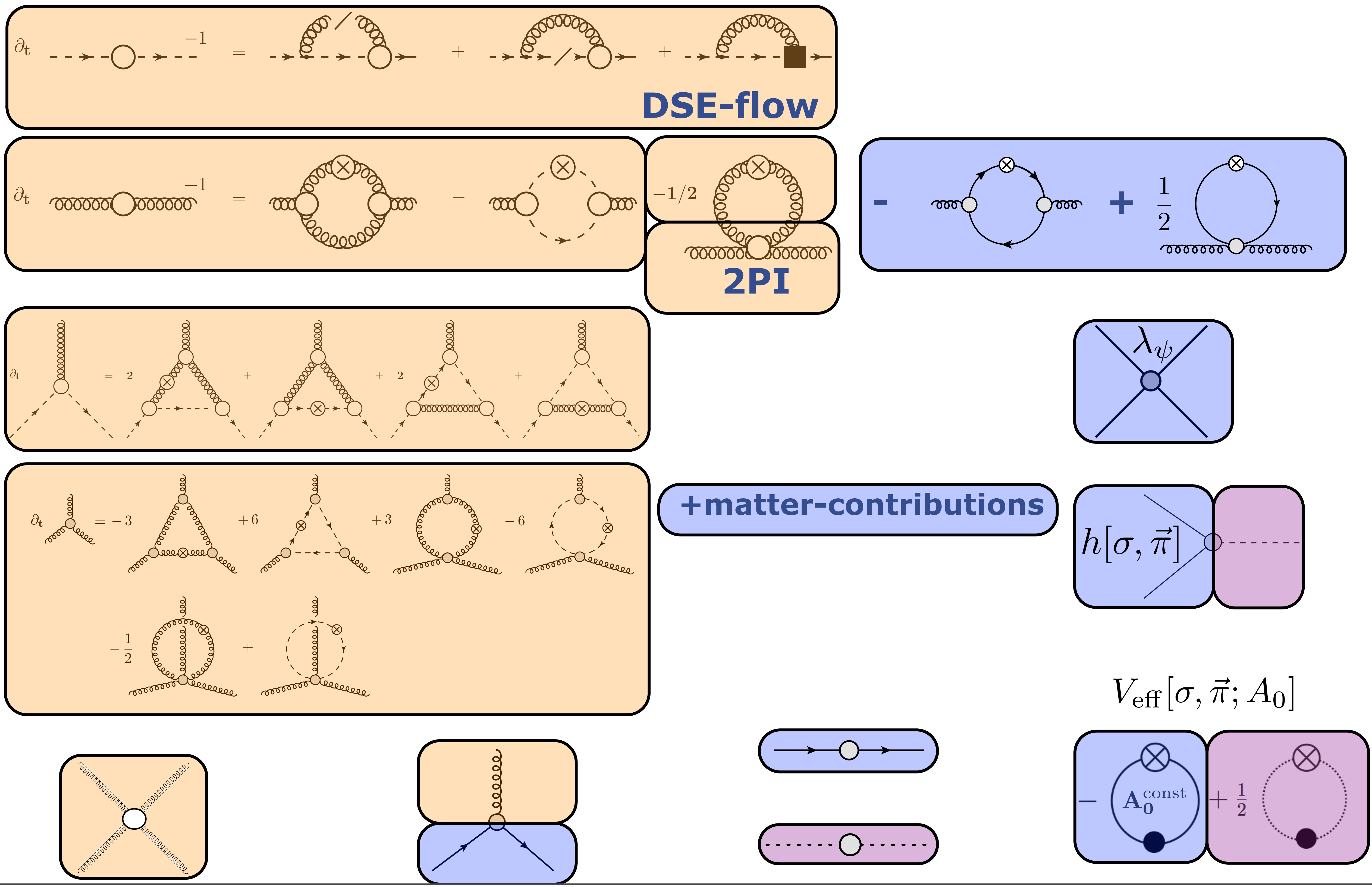
Gluons

Functional Methods for QCD

present approximation scheme

Yang-Mills

Matter



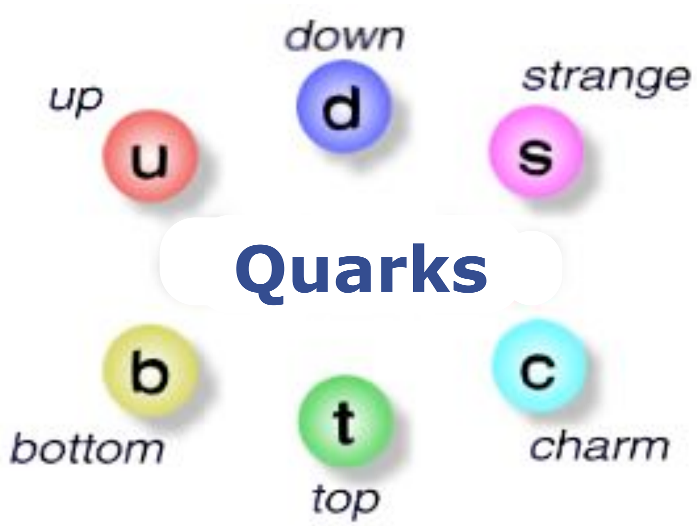
(II) Phase structure of QCD at finite temperature

Yang-Mills theory & QCD at T=0

- **Yang-Mills theory at finite temperature**
  - Confinement
  - Thermodynamics

- **Phase structure of QCD at finite temperature**
  - Order parameter
  - Comparison with other methods

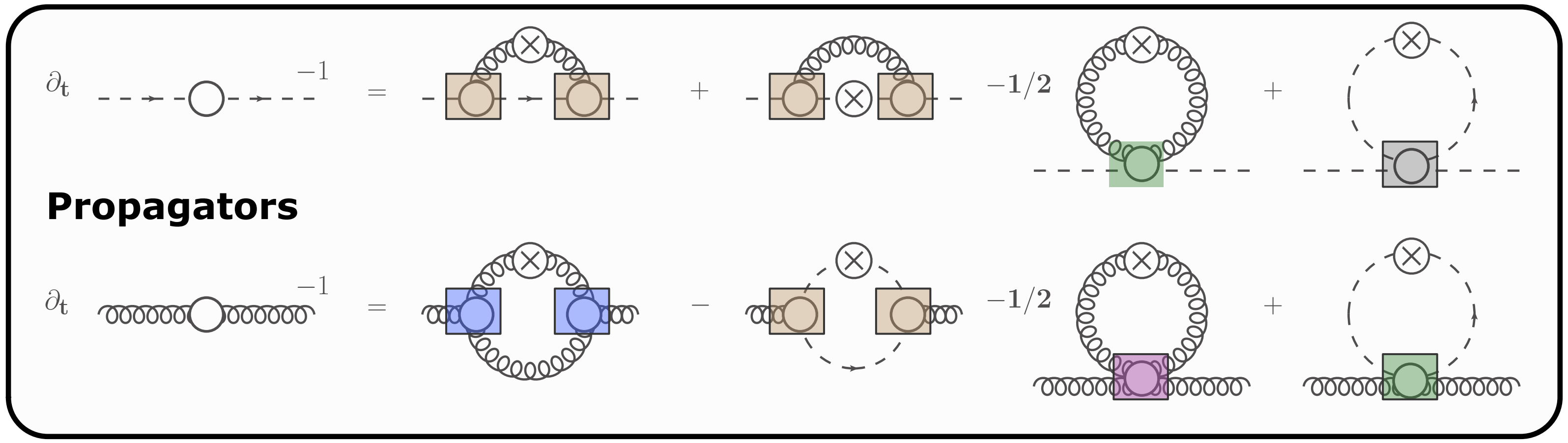
Yang-Mills theory & QCD at T=0

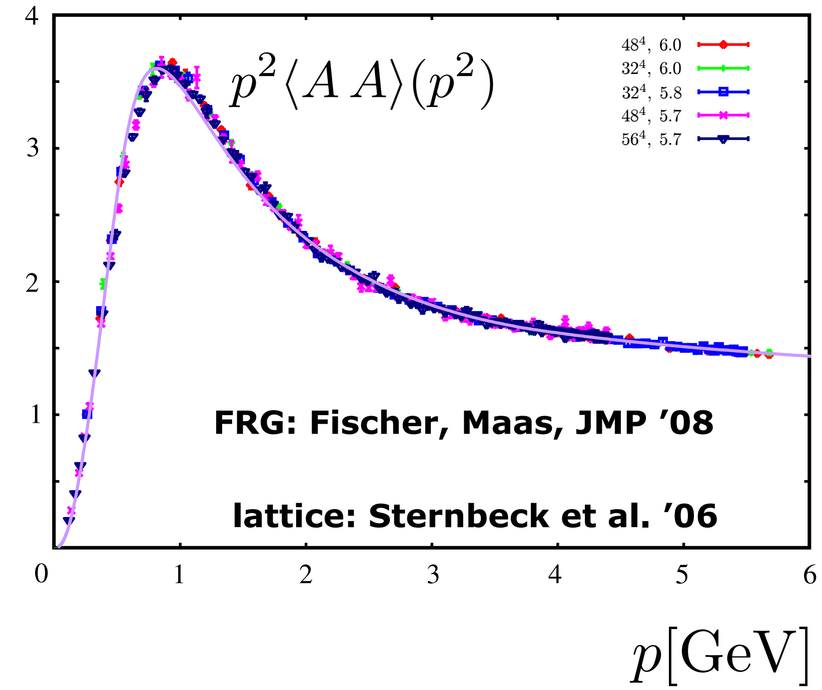


Gluons

Functional Methods for QCD

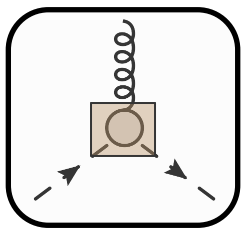
T=0 results for Yang-Mills correlation functions





see also talk of M. Huber

Vertices | **direct computations** | **resummations/ RG-dressing/STIs**



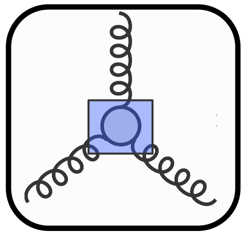
FRG: Fister, JMP '11

DSE: Huber, von Smekal '12

FRG: Ellwanger, Hirsch, Weber '96

DSE: von Smekal, Hauck, Alkofer '97

Landau gauge



FRG: Fister, PhD-thesis '12
Fister, JMP, in preparation

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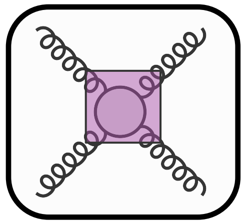

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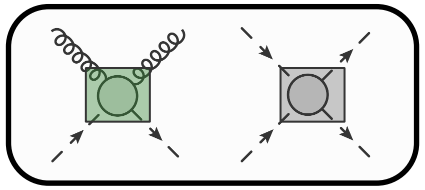
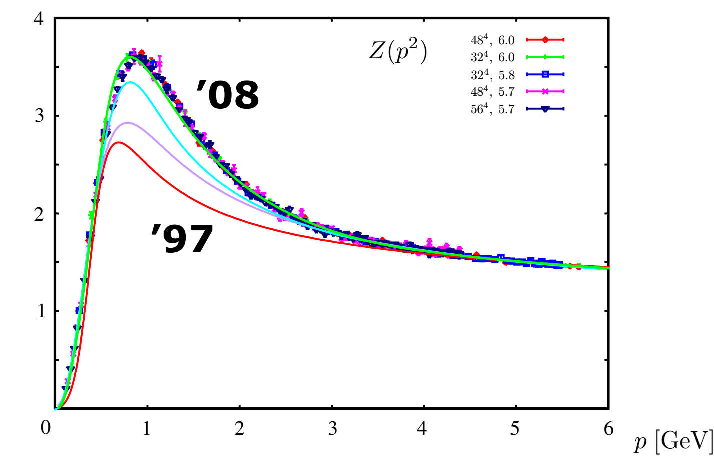
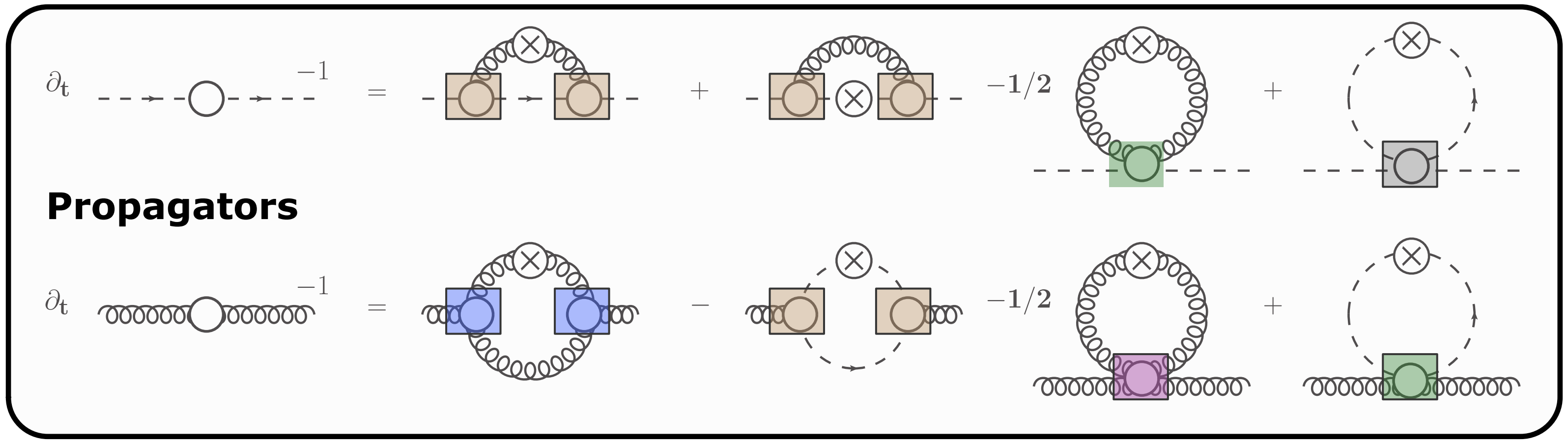


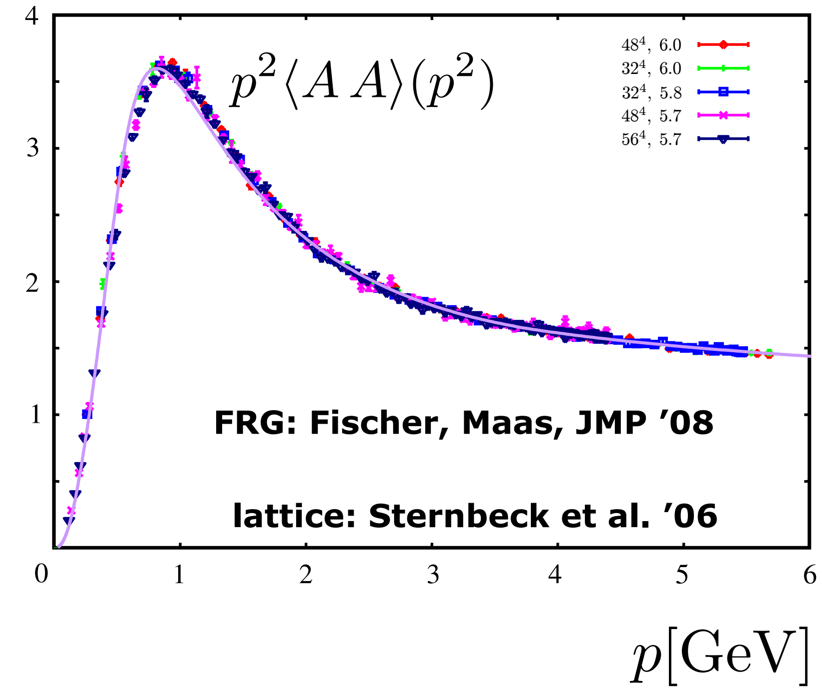
FRG: Fischer, JMP, Maas '08
Fister, JMP '11



Functional Methods for QCD

T=0 results for Yang-Mills correlation functions





Vertices | **direct computations** | **resummations/ RG-dressing/STIs**

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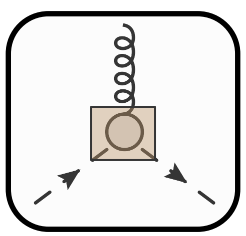

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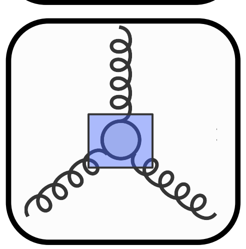

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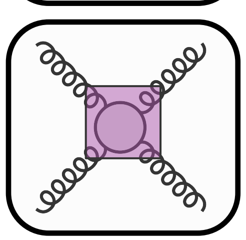

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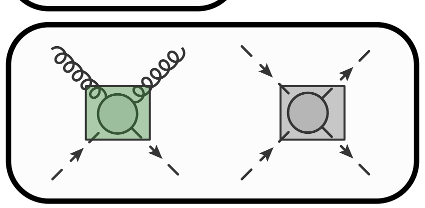

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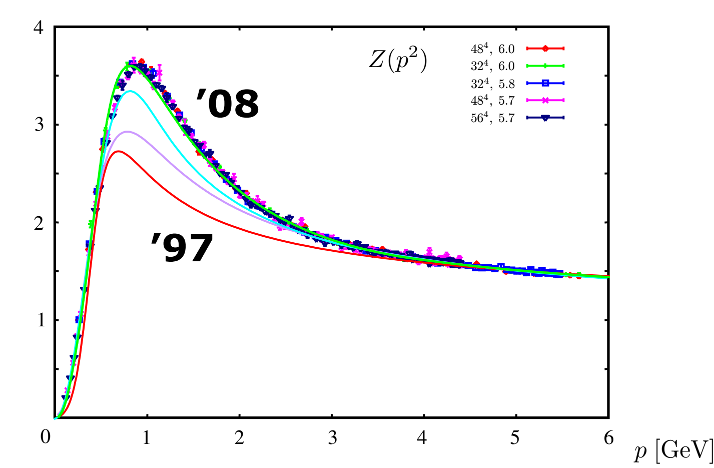
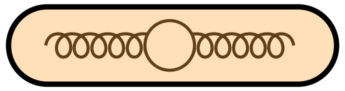
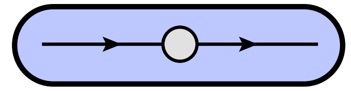


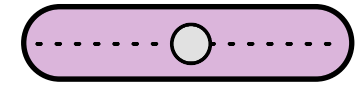
Functional Methods for QCD

T=0 results for QCD correlation functions

Propagators



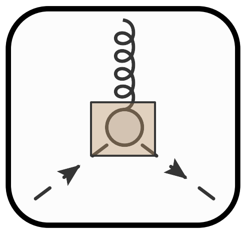




Vertices

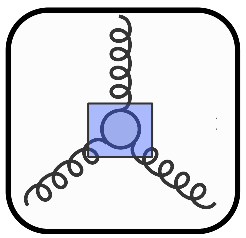
direct computations

resummations/ RG-dressing/STIs



FRG: Braun, Haas, Marhauser, JMP '09

DSE: Alkofer, Detmold, Fischer, Maris '04



stay tuned

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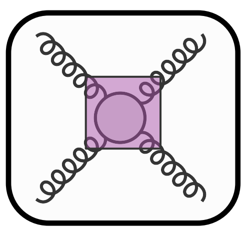

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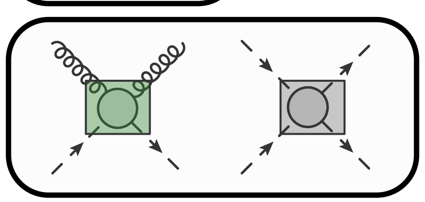

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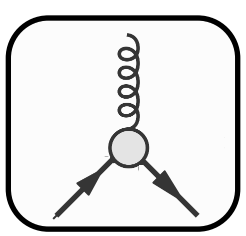

DSE: Hopfer, Windisch, Alkofer '12

FRG: Braun, Haas, Marhauser, JMP '09

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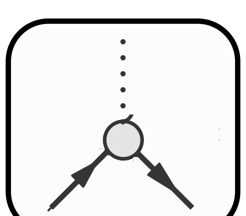


FRG: Braun, Haas, Marhauser, JMP '09

$m_{\psi;\text{constituent}}, \langle \bar{q} q \rangle, f_\pi \simeq \sigma, \cdots$

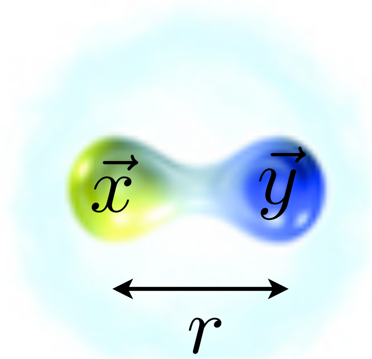
Yang-Mills theory at finite temperature

Confinement

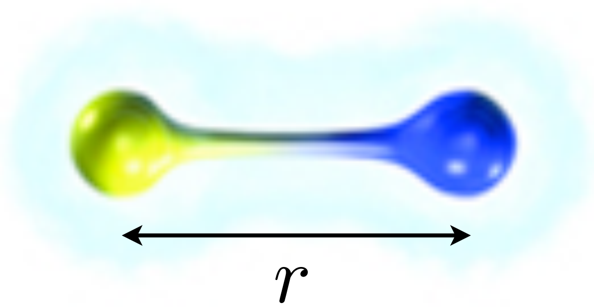
Confinement

Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

Reminder

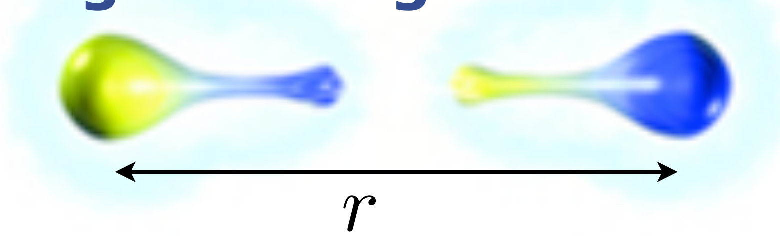


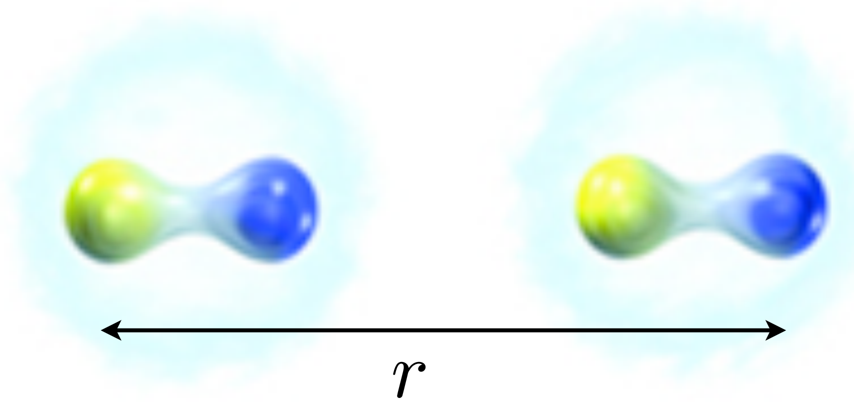
$$F_{q\bar{q}} \simeq -\frac{1}{r}$$



$$F_{q\bar{q}} \simeq \sigma r$$

string breaking at $\mathrm{r} \approx 1\mathrm{fm}$





$$F_{q\bar{q}} \simeq \mathrm{const.}$$

Order parameter $\sim {}'\langle q \rangle'$

$$\Phi = e^{-\frac{1}{2T} F_{q\bar{q}}(\infty)}$$

- **Confinement** $\Phi = 0$
- **Deconfinement** $\Phi \neq 0$

Polyakov loop

$$\Phi = \frac{1}{3} \langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\} \rangle$$

Confinement

Order parameters

Polyakov loop operator

$$L[A_0] = \frac{1}{N_c} \mathrm{tr}\mathcal{P} \quad e^{ig\int_0^{1/T} dt A_0}$$

$$\Phi = \langle L[A_0] \rangle$$

order parameter

$L[\langle A_0 \rangle]$ **order parameter**

$$L[\langle A_0 \rangle] = 0 \longleftrightarrow \langle L[A_0] \rangle = 0$$

$$L[\langle A_0 \rangle] \geq \langle L[A_0] \rangle$$

up to lattice renormalisation

Braun, Gies, JMP '07
Marhauser, JMP '08

$\langle A_0 \rangle$ **order parameter**

$$\left.\frac{\partial V[A_0]}{\partial A_0}\right|_{A_0 = \langle A_0 \rangle} = 0$$

$$V[A_0] = \frac{1}{\beta \,\mathrm{Vol}_3} \Gamma[A_0]$$

constant backgrounds

background Landau gauge

Confinement

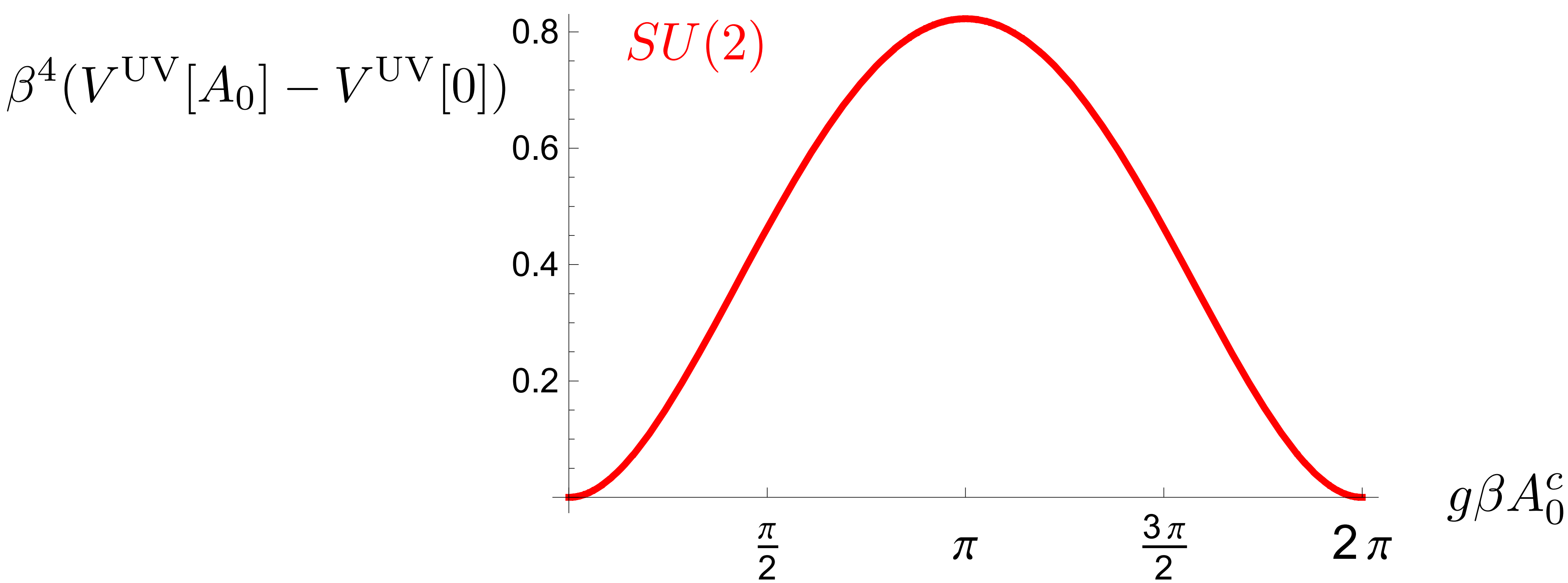
Effective Polyakov loop potential

One-loop

$$V^{\mathrm{UV}}[A_0] = \frac{1}{2\Omega}\,\mathrm{Tr}\log S^{(2)}_{AA}[A_0] - \frac{1}{\Omega}\mathrm{Tr}\log S^{(2)}_{C\bar{C}}[A_0]$$

free energy

Gross, Pisarski, Yaffe ’81
Weiss ’81



$$SU(2):\ \Phi[A_0] = \cos\frac{1}{2}\beta g A_0^c \qquad \text{with} \qquad A_0 = A_0^c\,\frac{\sigma_3}{2}$$

Non-perturbative effective potential

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

Confinement

Effective Polyakov loop potential

Non-perturbative effective potential

$$V[A_0] \simeq -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

flow $$\frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t R_k = \frac{1}{2}\mathrm{Tr}\,\partial_t\log(\Gamma_k^{(2)}[\phi]+R_k) - \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t\Gamma_k^{(2)}[\phi]$$

Propagators

sorry

$$\langle AA\rangle[A_0] \simeq \frac{1}{-D_\mu^2(A_0)}\;\frac{1}{Z[-D_\mu^2(A_0)]}$$

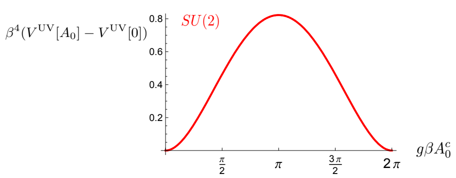
Integrals & sums

$$\mathrm{Tr}\,f[-D_\mu^2(A_0)] = \sum\!\!\!\!\!\!\int_{\vec{p},\pm} f[(2\pi T)^2(n\pm\varphi)^2 + \vec{p}^2] + \varphi - \text{indep.terms}$$

$$\beta^4 p_{\mathrm{SB}}$$

$$g\,A_0 = \frac{\varphi}{2\pi T}\tau^3_{\mathrm{ad}}$$

One-loop result



$$\beta^4 V^{UV}[A_0] = -2*3\left(\frac{\pi^2}{90} - \frac{2\pi^2}{3}\tilde{\varphi}^2(1-\tilde{\varphi})^2\right)$$

$$\tilde{\varphi} = \varphi \quad \mathrm{mod}\ 1$$

Confinement

Effective Polyakov loop potential

Non-perturbative effective potential

$$V[A_0] \simeq -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

free energy

flow

$$\frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t R_k = \frac{1}{2}\mathrm{Tr}\,\partial_t\log(\Gamma_k^{(2)}[\phi]+R_k) - \frac{1}{2}\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)}[\phi]+R_k}\partial_t\Gamma_k^{(2)}[\phi]$$

Propagators

sorry

$$\langle AA\rangle[A_0] \simeq \frac{1}{-D_\mu^2(A_0)}\,\frac{1}{Z[-D_\mu^2(A_0)]}$$

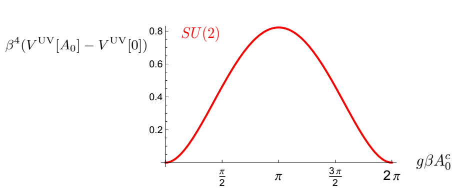
Integrals & sums

$$\mathrm{Tr}\,f[-D_\mu^2(A_0)] = \sum\!\!\!\!\!\!\int_{p,\pm} f[(2\pi T)^2(n\pm\varphi)^2 + \vec{p}^{\,2}] + \varphi - \text{indep.terms}$$

$$N_c^2 - 1$$

$$g\,A_0 = \frac{\varphi}{2\pi T}\tau^3_{\mathrm{ad}}$$

One-loop result



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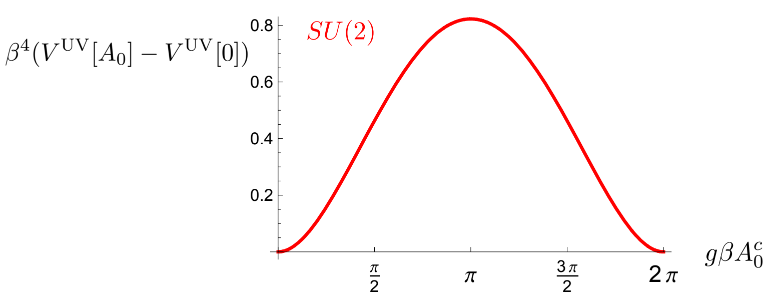
free energy

Braun, Gies, JMP '07

Fister, JMP '13

Confinement criterion



$$\beta^4 V^{UV}[A_0] = -2 * 3\left(\frac{\pi^2}{90} - \frac{2\pi^2}{3}\tilde{\varphi}^2(1-\tilde{\varphi})^2\right)$$

2 = 2 transversal physical polarisations+ 1 transversal (zero mode)+1 longitudinal - 2 ghosts

Gluon contribution deconfines

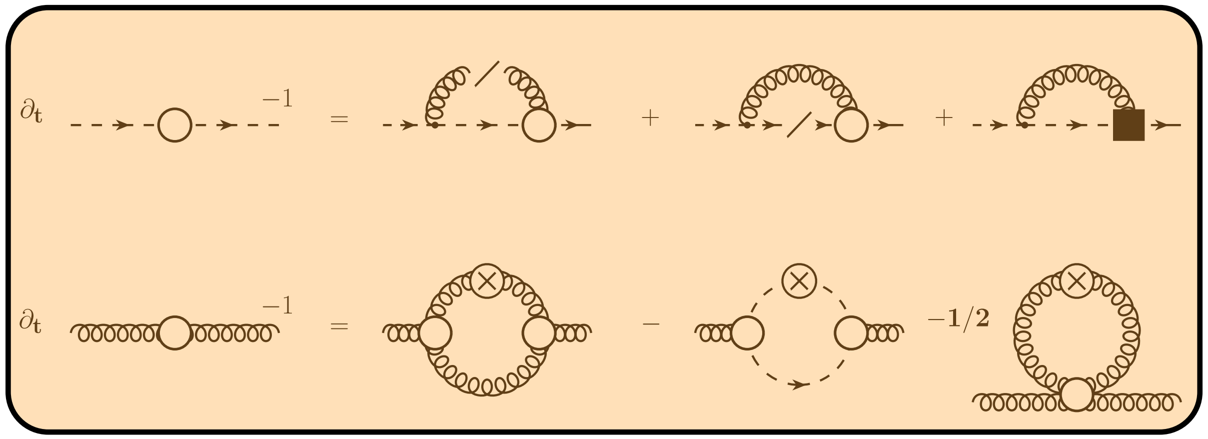
Ghost contribution confines

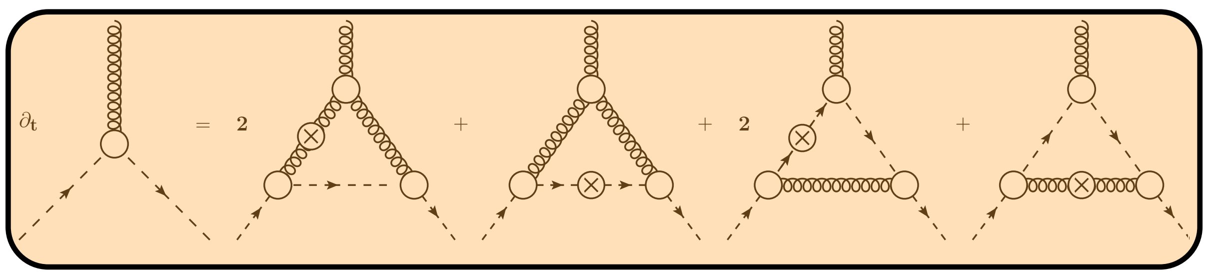
Confinement ⟷ suppression of the gluon relative to the ghost

Confinement

Thermal gluon propagators

Fister, JMP '11

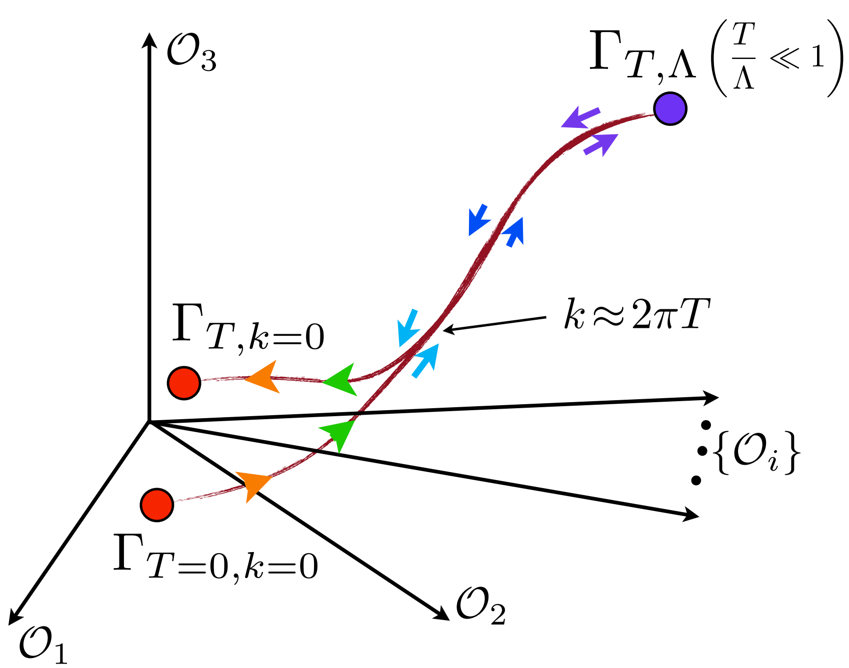


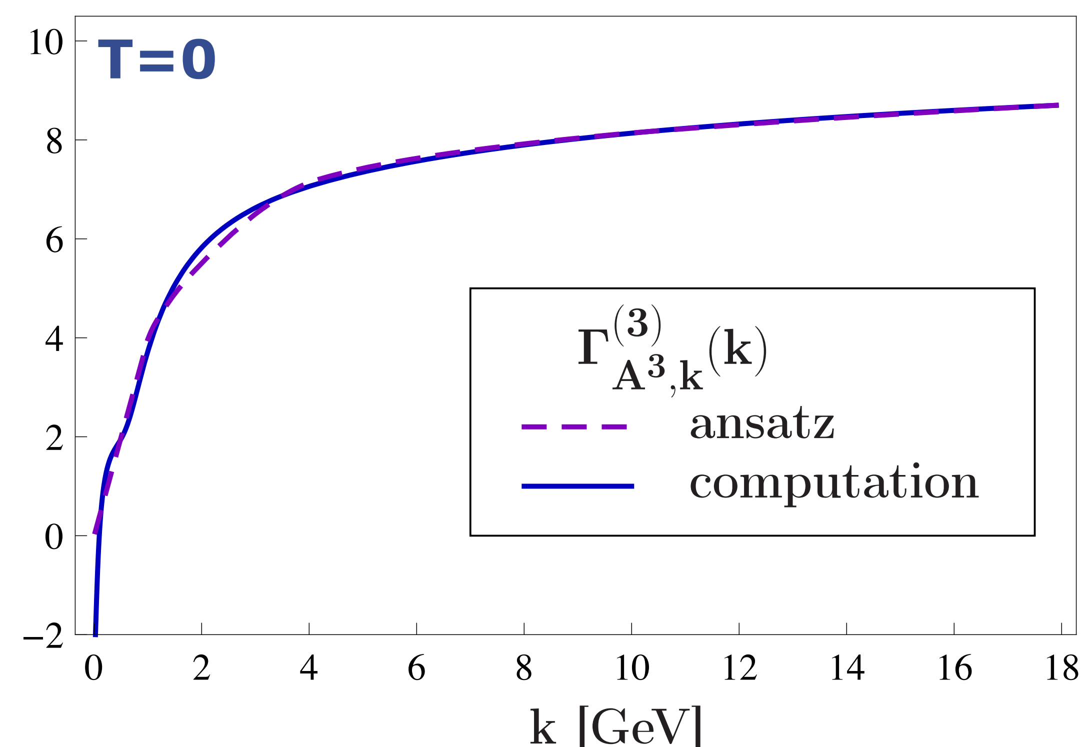


+ RG-dressed gluonic vertices

confirmed with the full system, JMP, Fister, in prep

Thermal flows





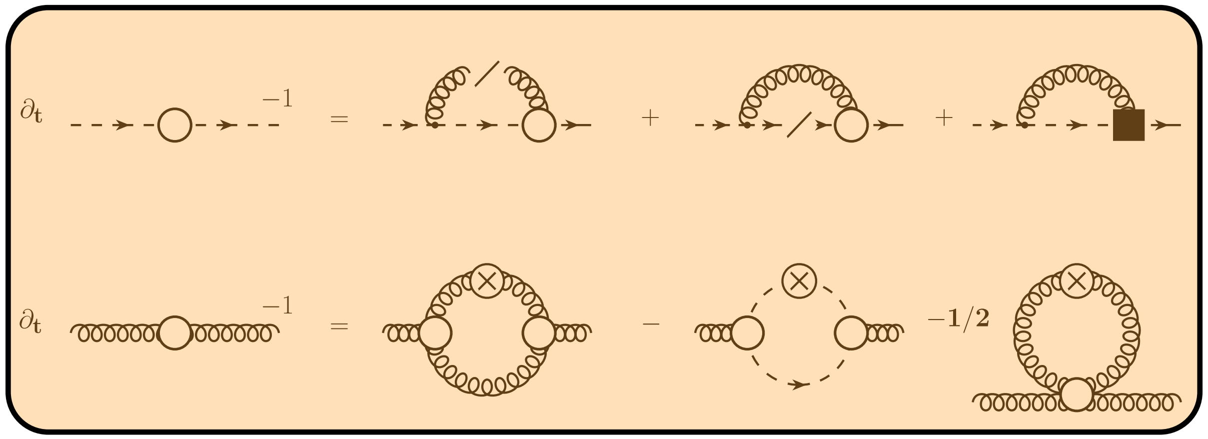
see also talk of M. Huber

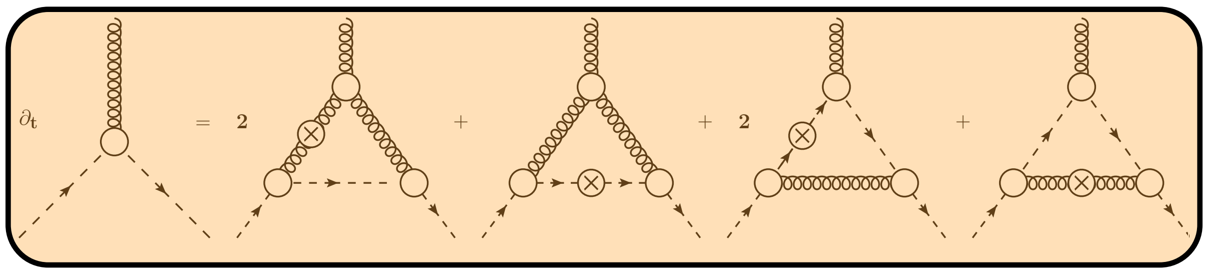
PhD thesis Fister '12

Confinement

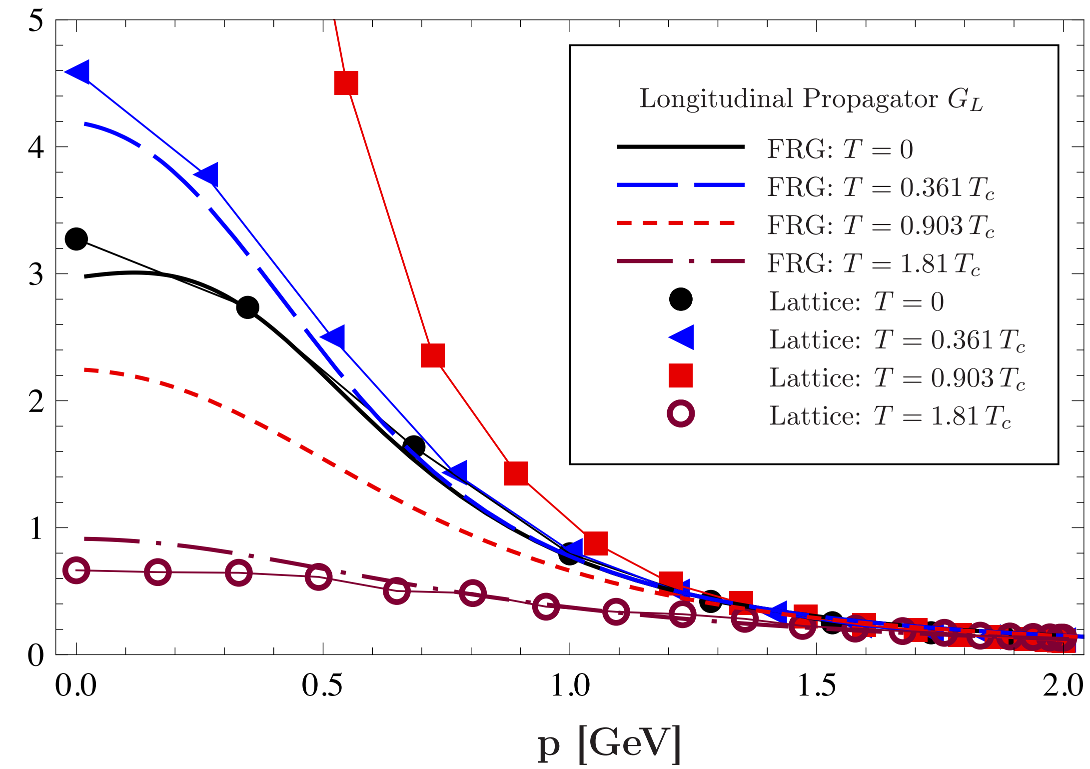
Thermal gluon propagators

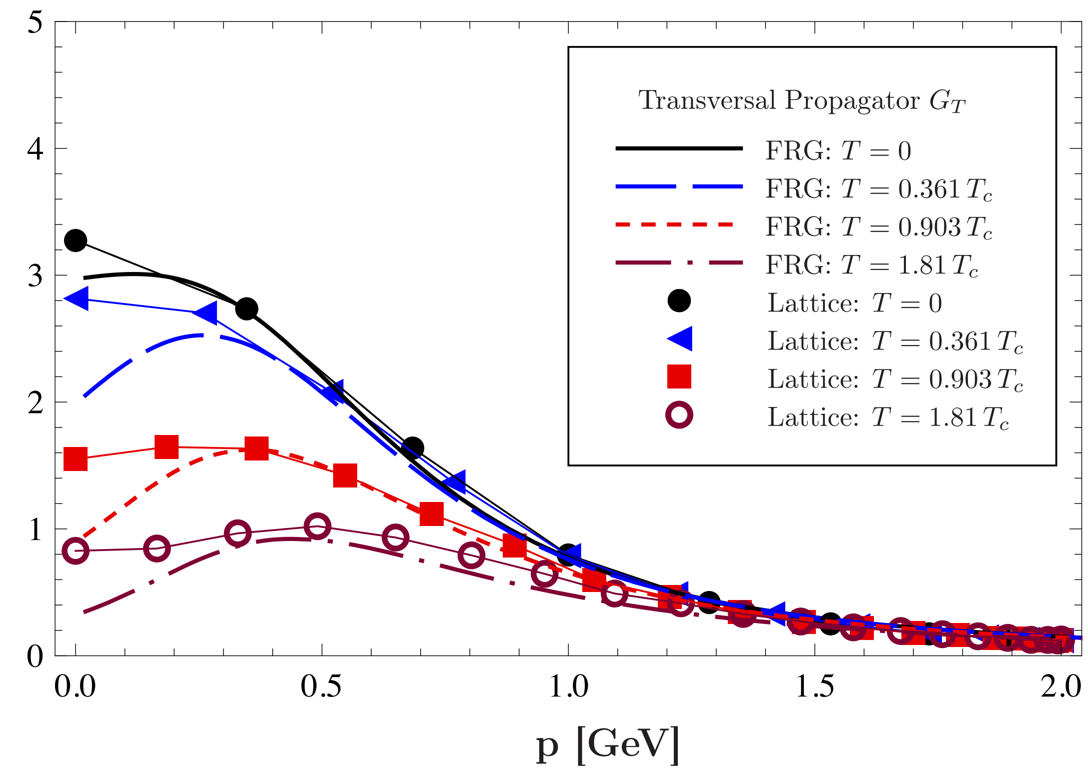
Fister, JMP '11





+ RG-dressed gluonic vertices



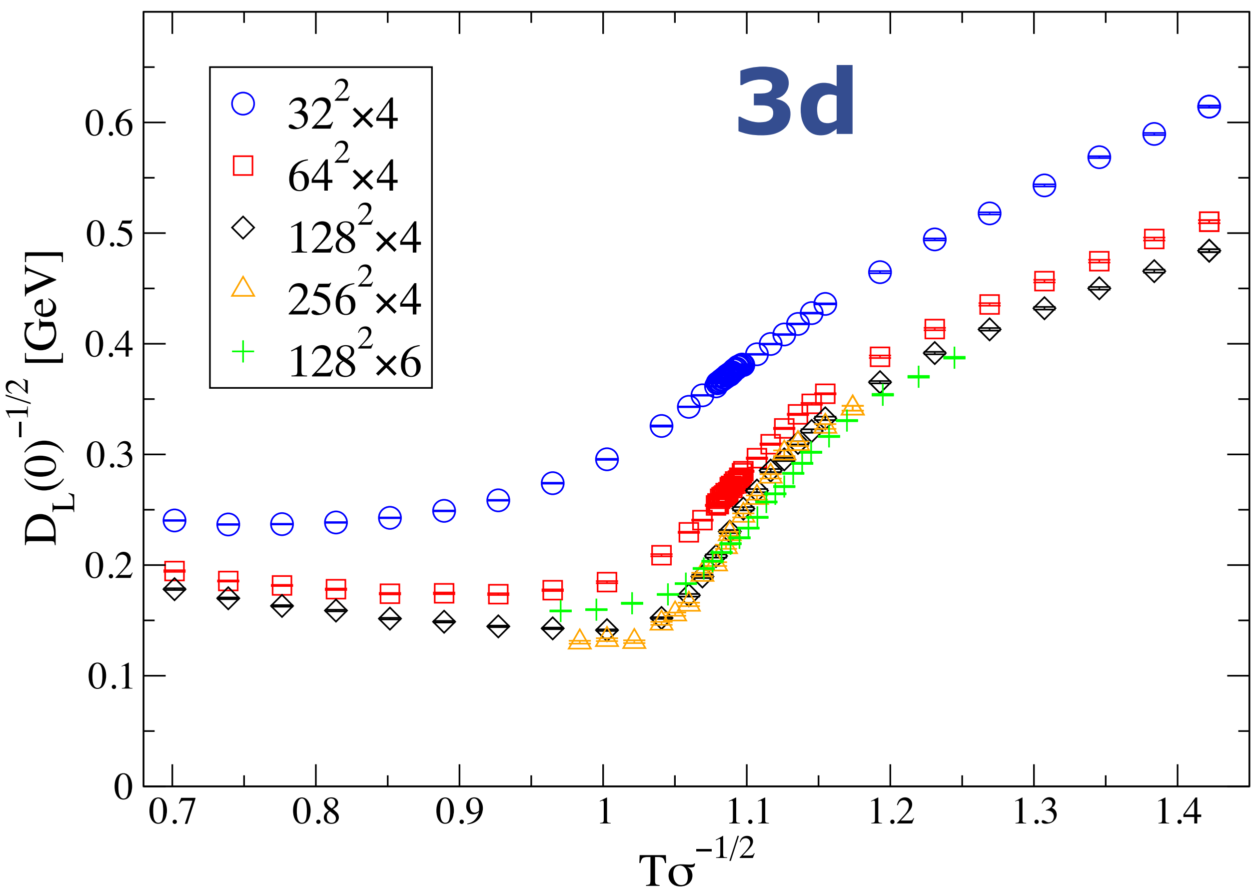


Lattice: Maas, JMP, Spielmann, von Smekal '11
Maas '11

Confinement

Chromo-electric propagator

Maas, JMP, Spielmann, von Smekal ’11

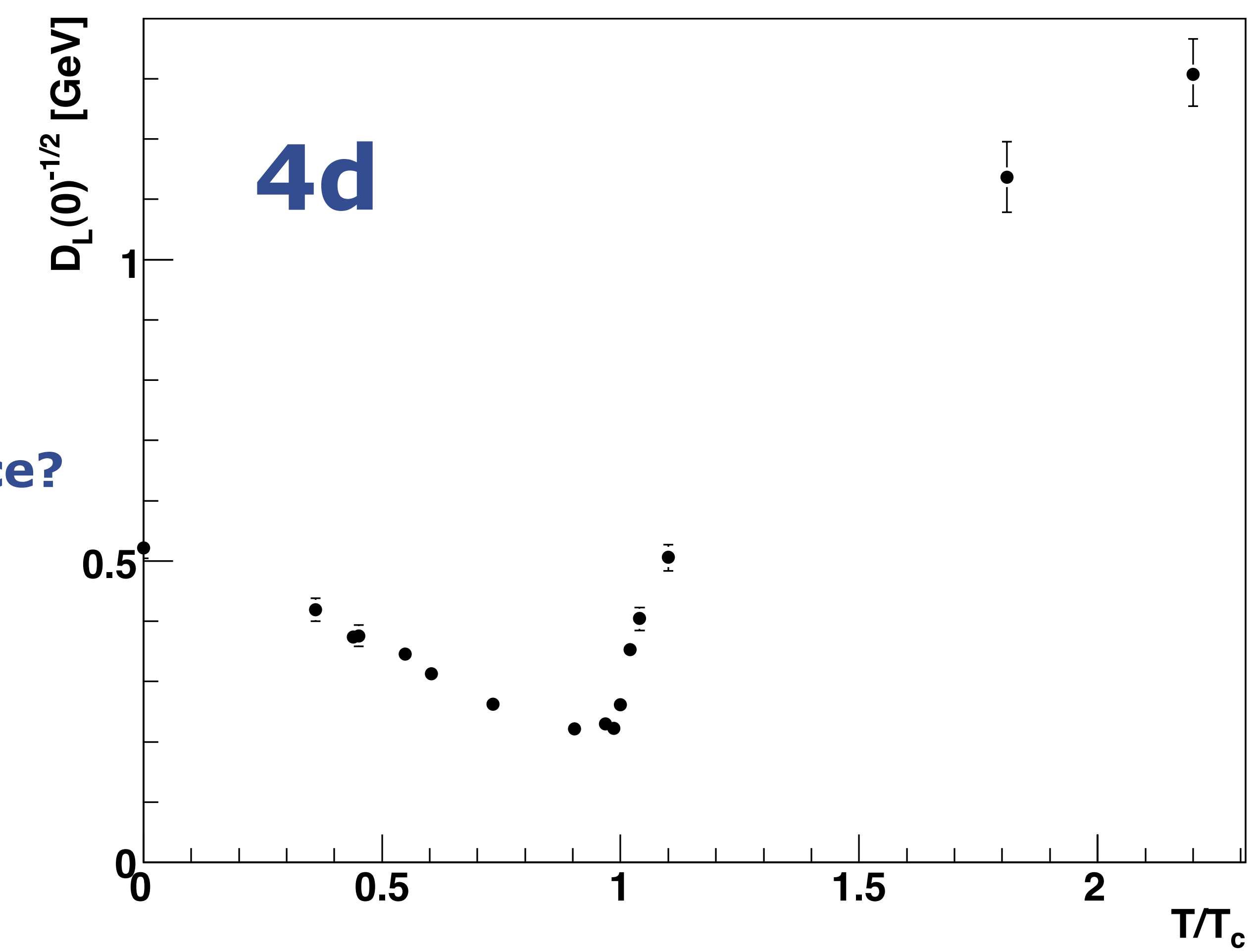


$\nu \approx 1$

$$\mathbf{D_L(0) = \langle A\ A \rangle_T(0)}$$

see also talk of P. Bicudo

Electric screening mass for SU(2)



$\nu \approx 0.68$

critical scaling in Landau gauge props on the lattice?

$$\mathbf{D_L(0)^{-1/2} \propto |T - T_c|^\nu + \cdots}$$

FRG

$$\mathbf{D_L(0) \propto V''[A_0] + \cdots}$$

global gauge fixing

Confinement

Order parameter

Braun, Gies, JMP '07

$$T_c = 275 \pm 27\,\mathrm{MeV}$$

$$T_c/\sqrt{\sigma} = 0.655 \pm 0.023$$

$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$

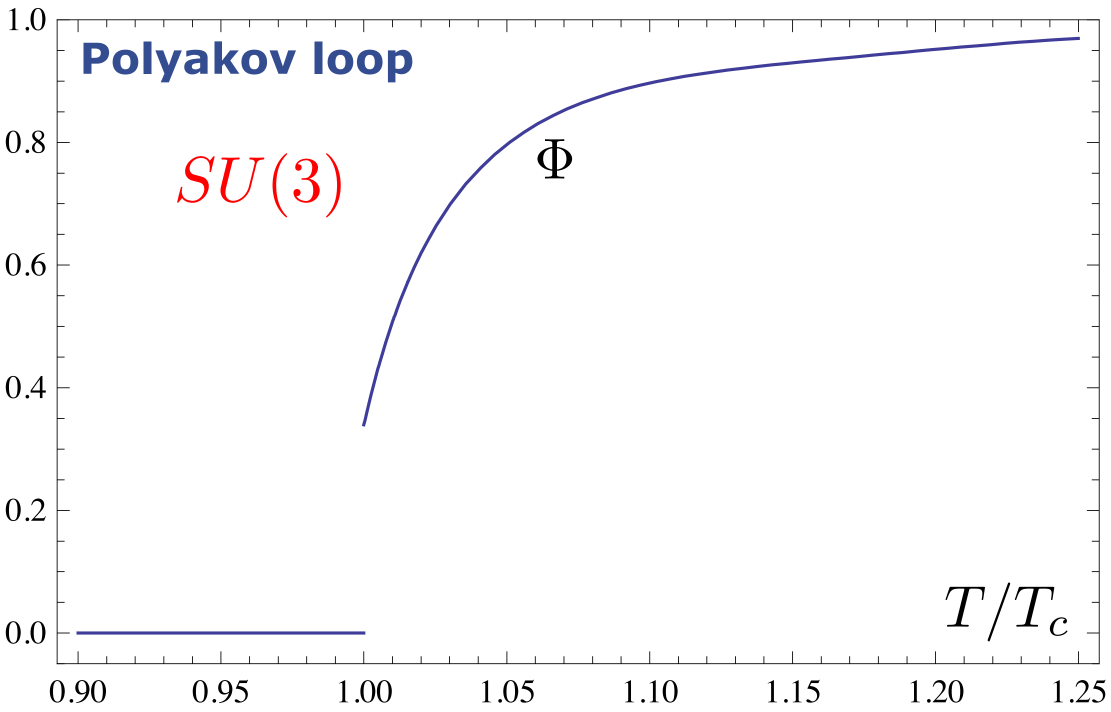
SU(2) & critical scaling: Marhauser, JMP '08

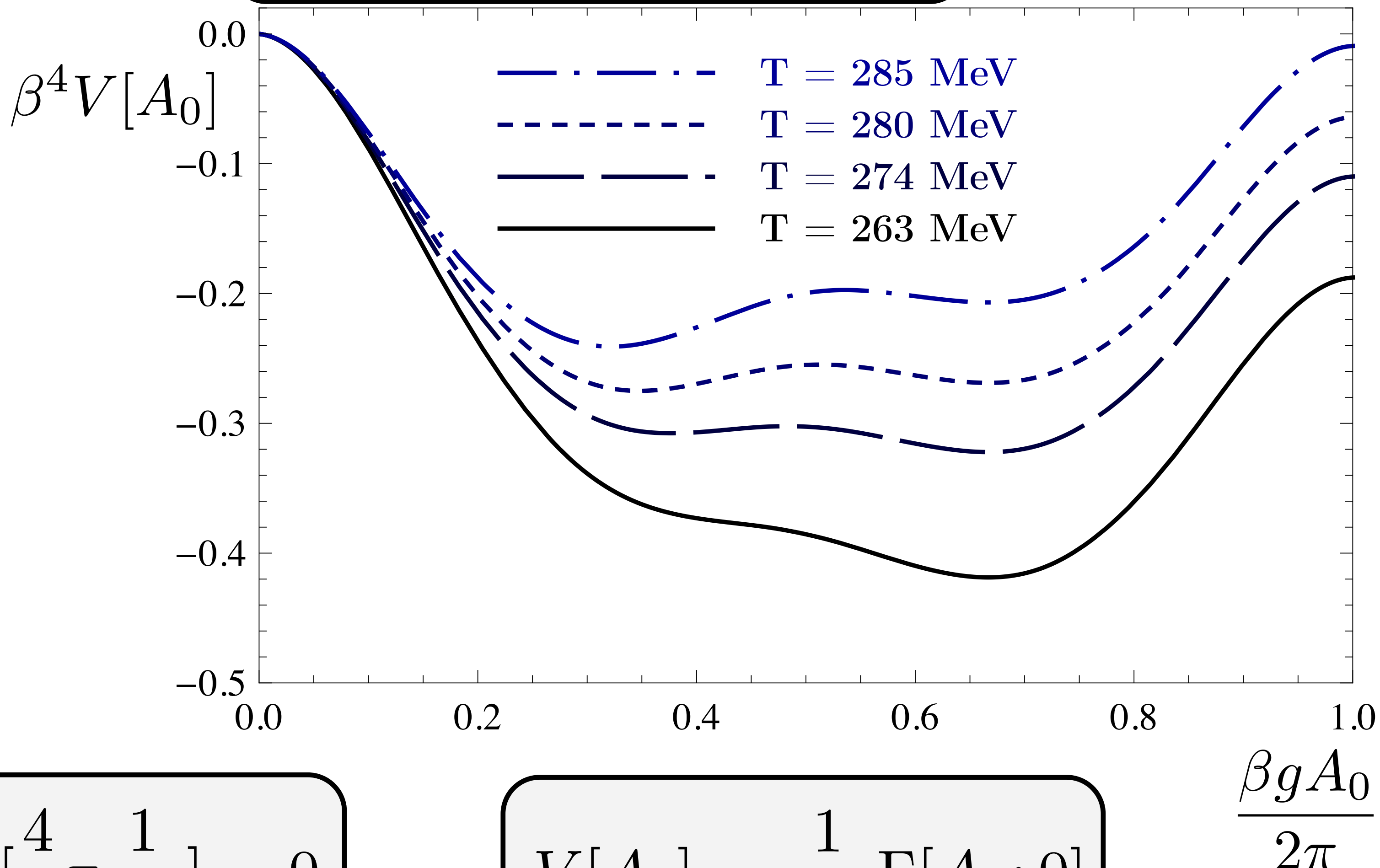
SU(N), Sp(2), E(7): Braun, Eichhorn, Gies, JMP '10

1st Lattice results

Diakonov, Gattringer, Schadler '12
Greensite '12
Greensite, Langfeld '13

FRG, DSE, 2PI agree quantitatively **Fister, JMP '13**





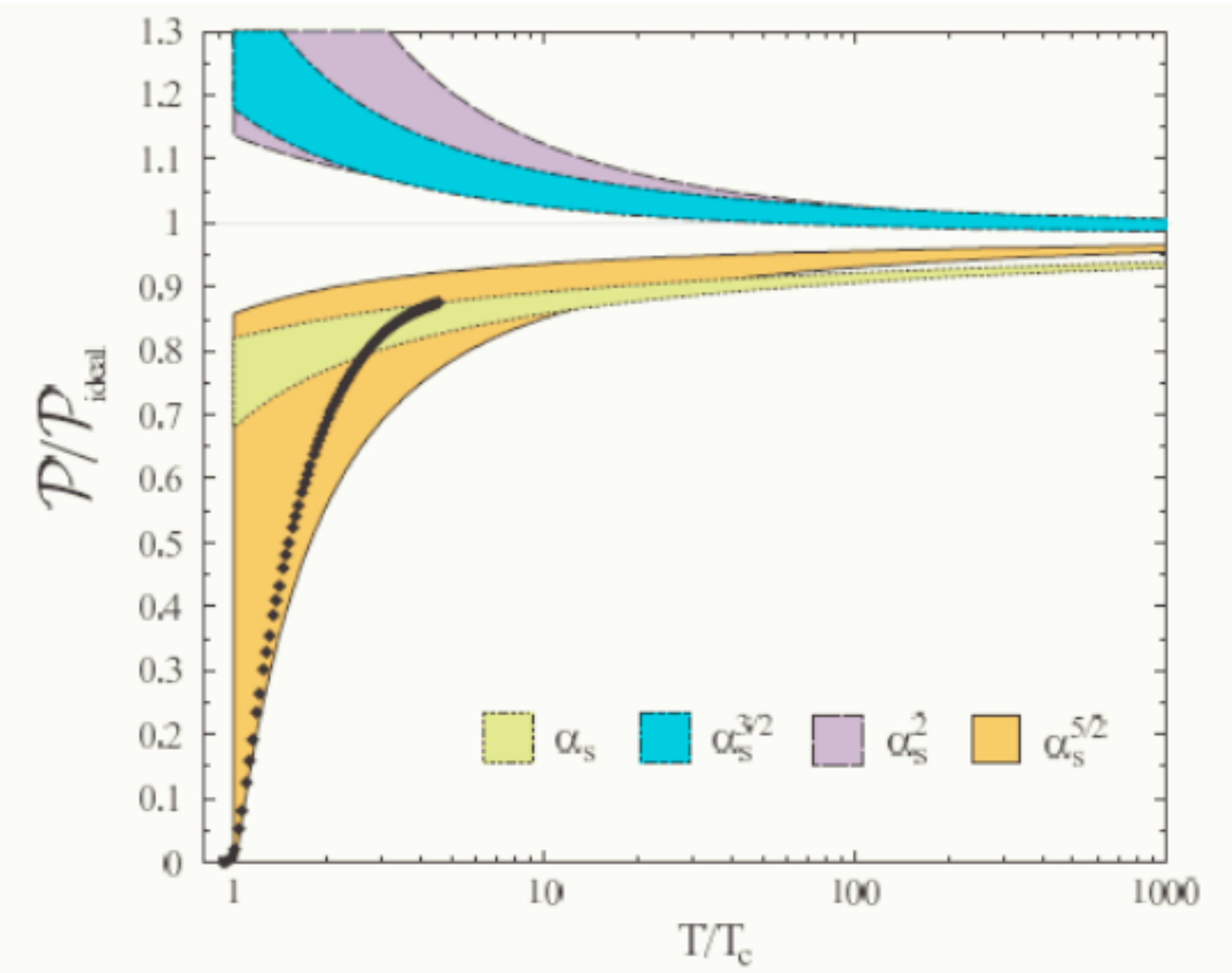
$$\Phi[A_0] = \frac{1}{3}(1 + 2\cos\frac{1}{2}\beta g A_0)$$

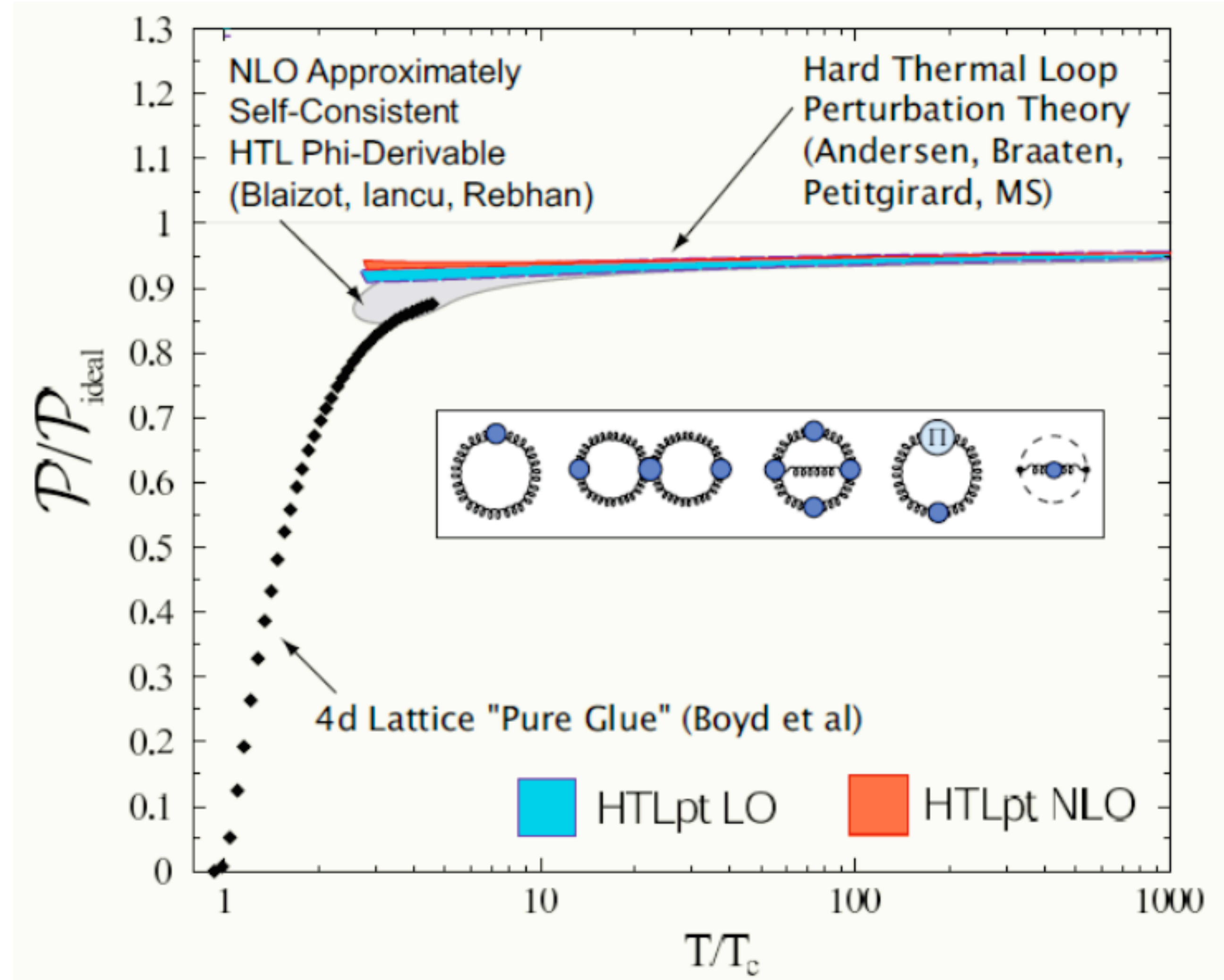
$$\Phi[\frac{4}{3}\pi\frac{1}{\beta g}] = 0$$

$$V[A_0] = \frac{1}{\beta\mathcal{V}}\Gamma[A_0;0]$$

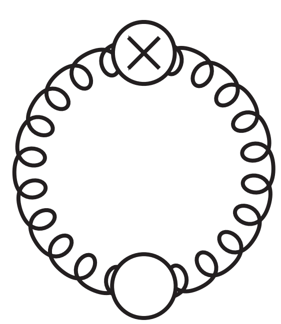
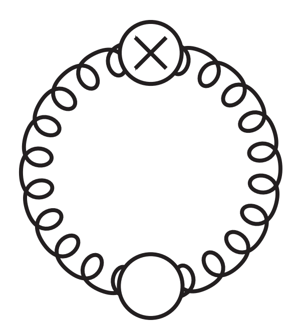
thermodynamics

Confinement & Thermodynamics





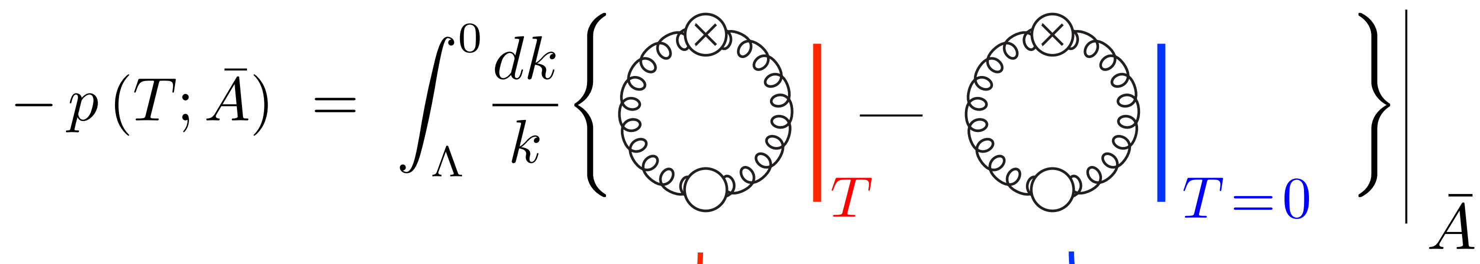
Strickland

$$-p\,(T;\bar{A}) \;=\; \int_\Lambda^0 \frac{dk}{k}\Bigg\{$$

$$\Bigg|_T \;-\;$$

$$\Bigg|_{T=0}\;\Bigg\}\Bigg|_{\bar{A}}$$

Fister, JMP

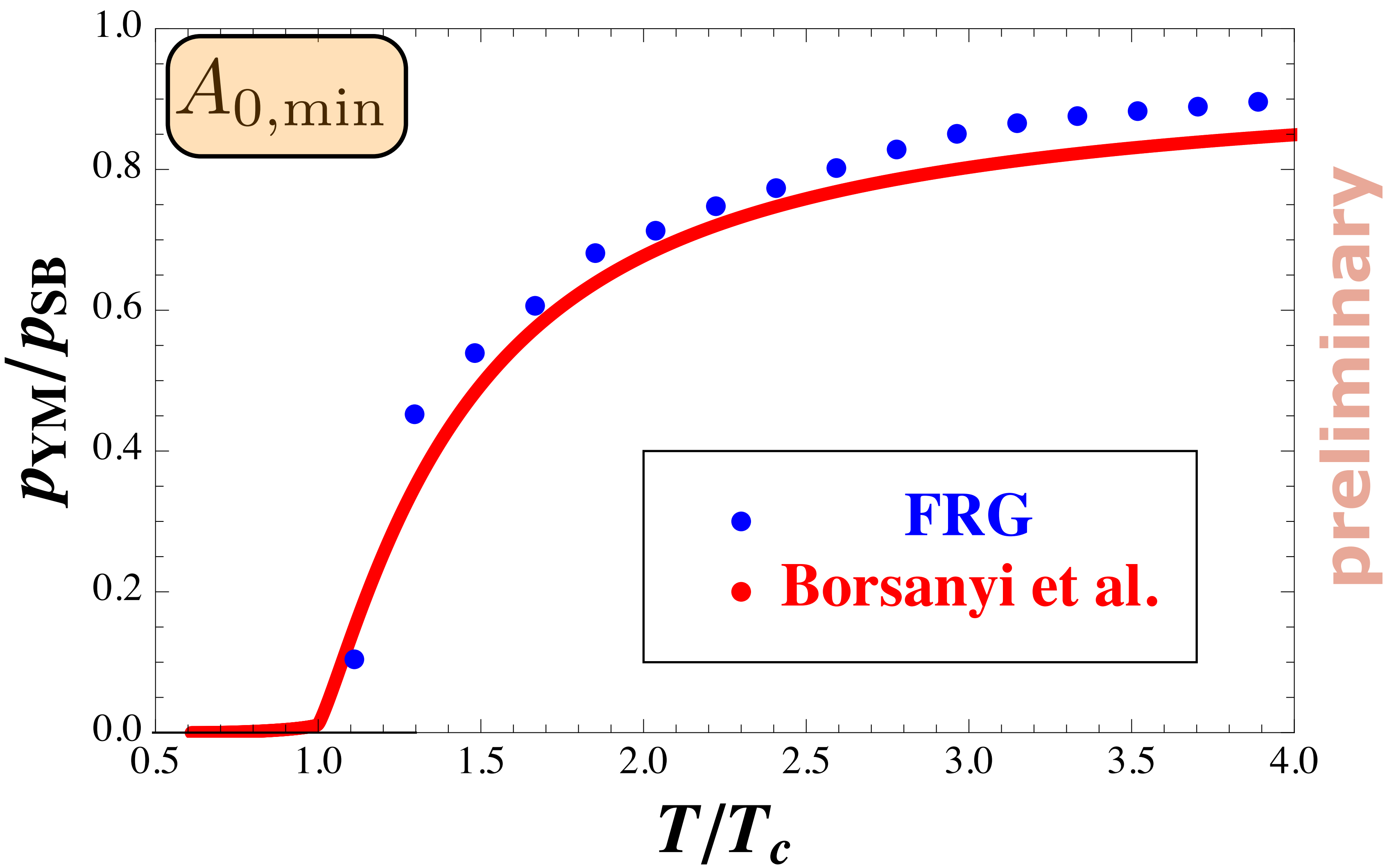
Confinement & Thermodynamics

Fister, JMP, in prep



1/2 * 2 polarisations

$$\sum\!\!\!\!\!\!\int_p G_{T,k}\;\partial_t R_k \qquad\qquad \int_p G_{T=0,k}\;\partial_t R_k$$

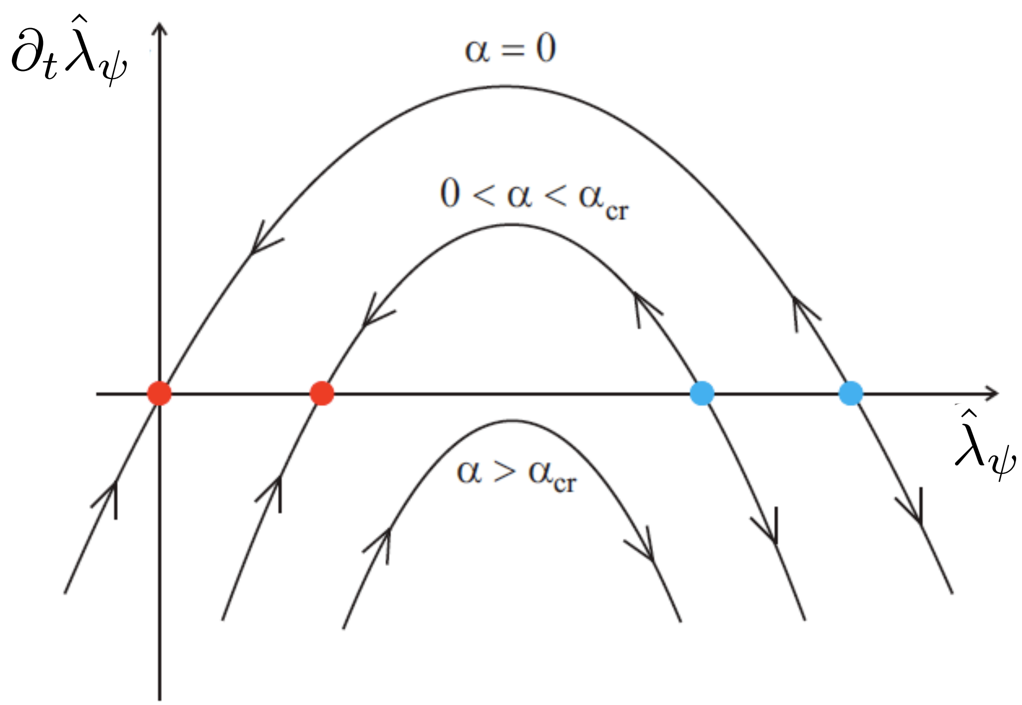


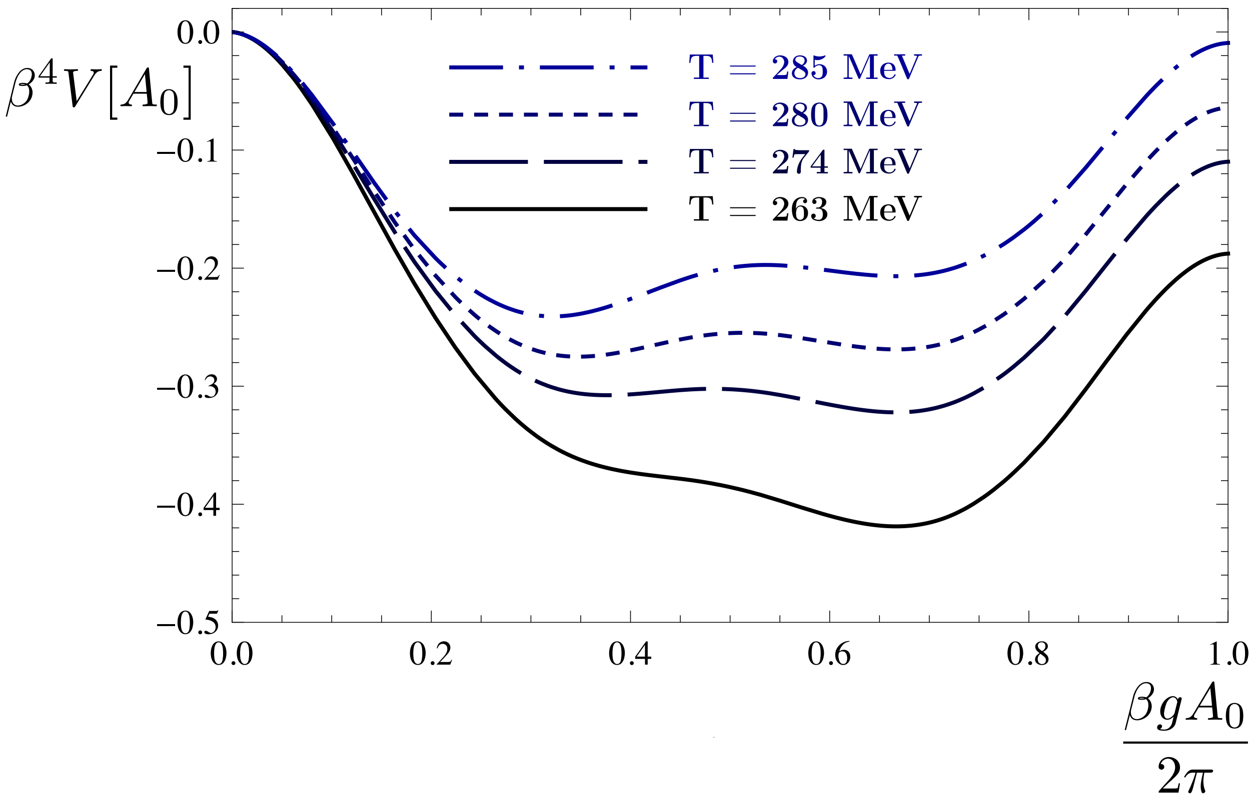
Phase structure of QCD at finite temperature

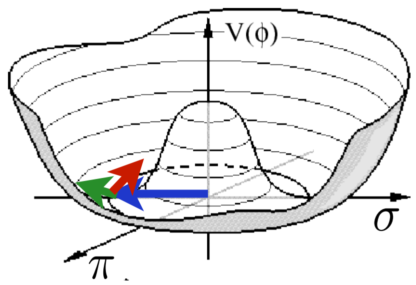
Full dynamical QCD: N_f = 2 & chiral limit

Phase structure

Reminder







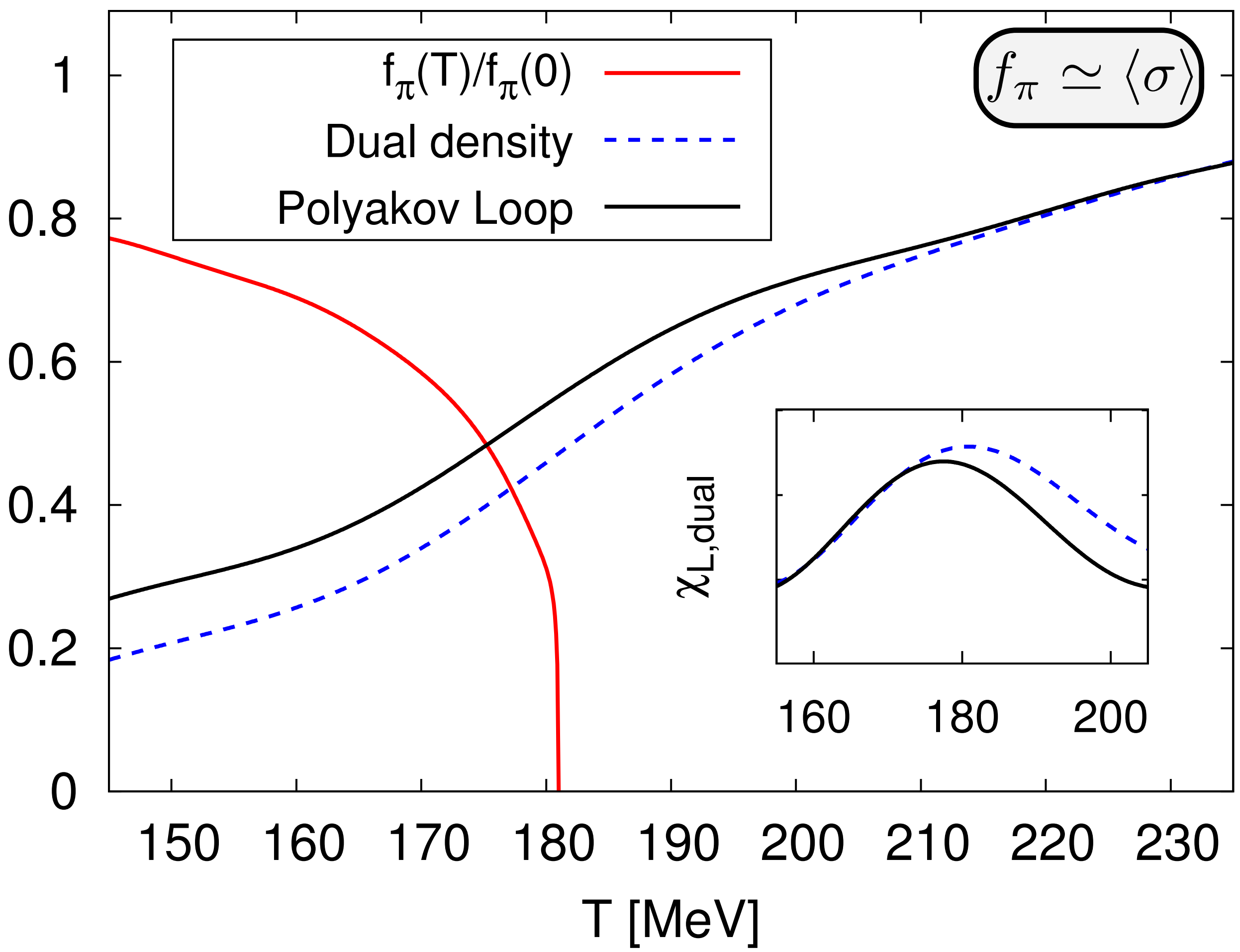
chiral symmetry breaking ⟷ $\alpha_s > \alpha_{s,cr}$

Confinement ⟷ **suppression of the gluon relative to the ghost**

Full dynamical QCD: $N_f = 2$ & chiral limit

Phase structure

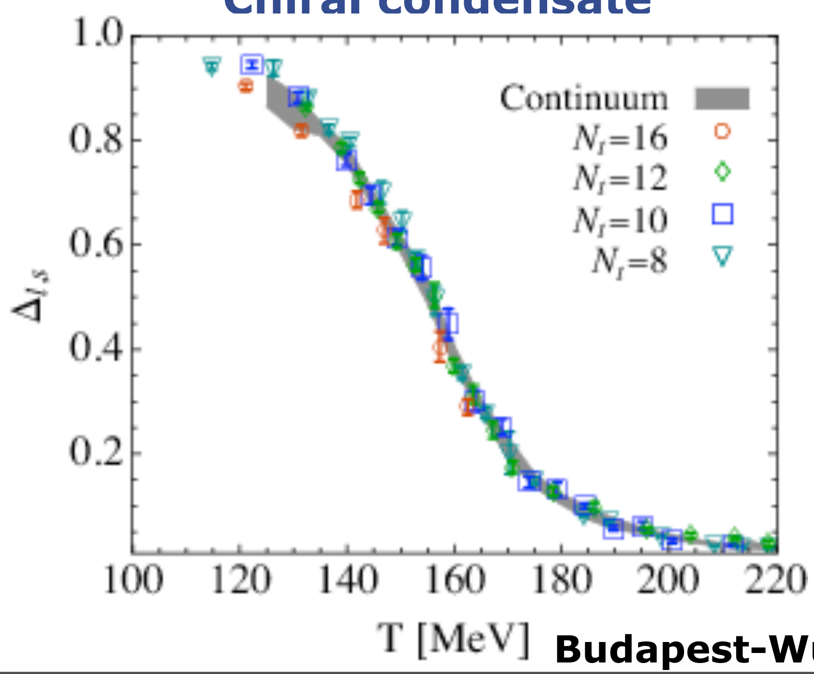
Braun, Haas, Marhauser, JMP '09



$T_\chi \simeq T_{\rm conf} \simeq 180{\rm MeV}$

Width $\Delta T_{\rm conf} \simeq \pm 20{\rm MeV}$

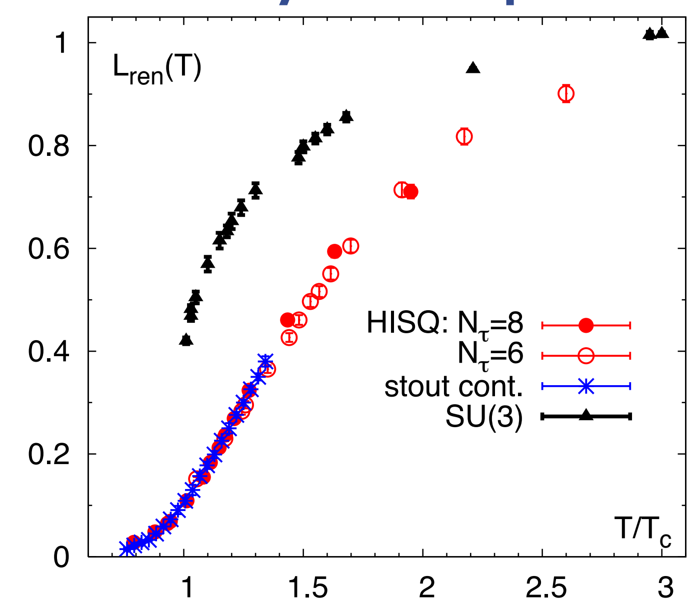
Chiral condensate



Budapest-Wuppertal '10

N_f **=2+1**

Polyakov loop



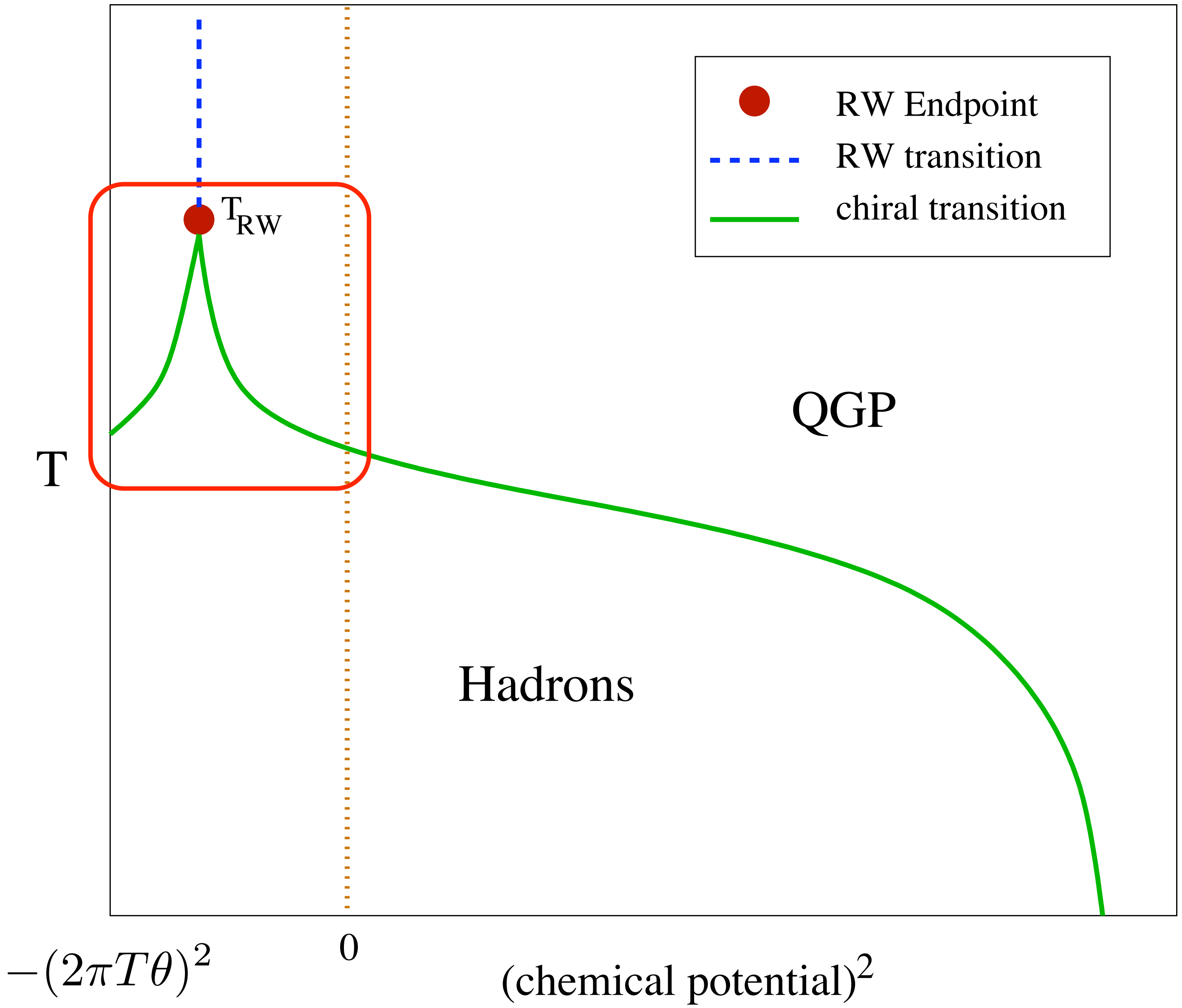
hotQCD '10

(III) Phase diagram of QCD

- **Phase structure at imaginary chemical potential**
  - **Imaginary chemical potential & Roberge-Weiss symmetry**
  - **Dual order parameters**
  - **Chiral versus confinement-deconfinement temperatures**

- **Phase structure at finite density**
  - **Chiral versus confinement-deconfinement temperatures**
  - **Phase structure with QCD-improved effective models**
  - **High density phases: To be or not to be**

Imaginary chemical potential



Dirac term

$$\int_x \bar{q}\cdot(i\not{D} + i\,m_\psi + i\mu\gamma_0)\cdot q$$

$$\mu = 2\pi T\theta\, i$$

$$\int_x \bar{q}_\theta \cdot (i\not{D} + i\,m_\psi)\cdot q_\theta$$

$$q_\theta(t,\vec{x}) = e^{2\pi T\theta\, i\, t} q(t,x)$$

Periodicity

$$q_\theta(t+\beta,\vec{x}) = -e^{2\pi\theta\, i} q_\theta(t,x)$$

Roberge-Weiss symmetry

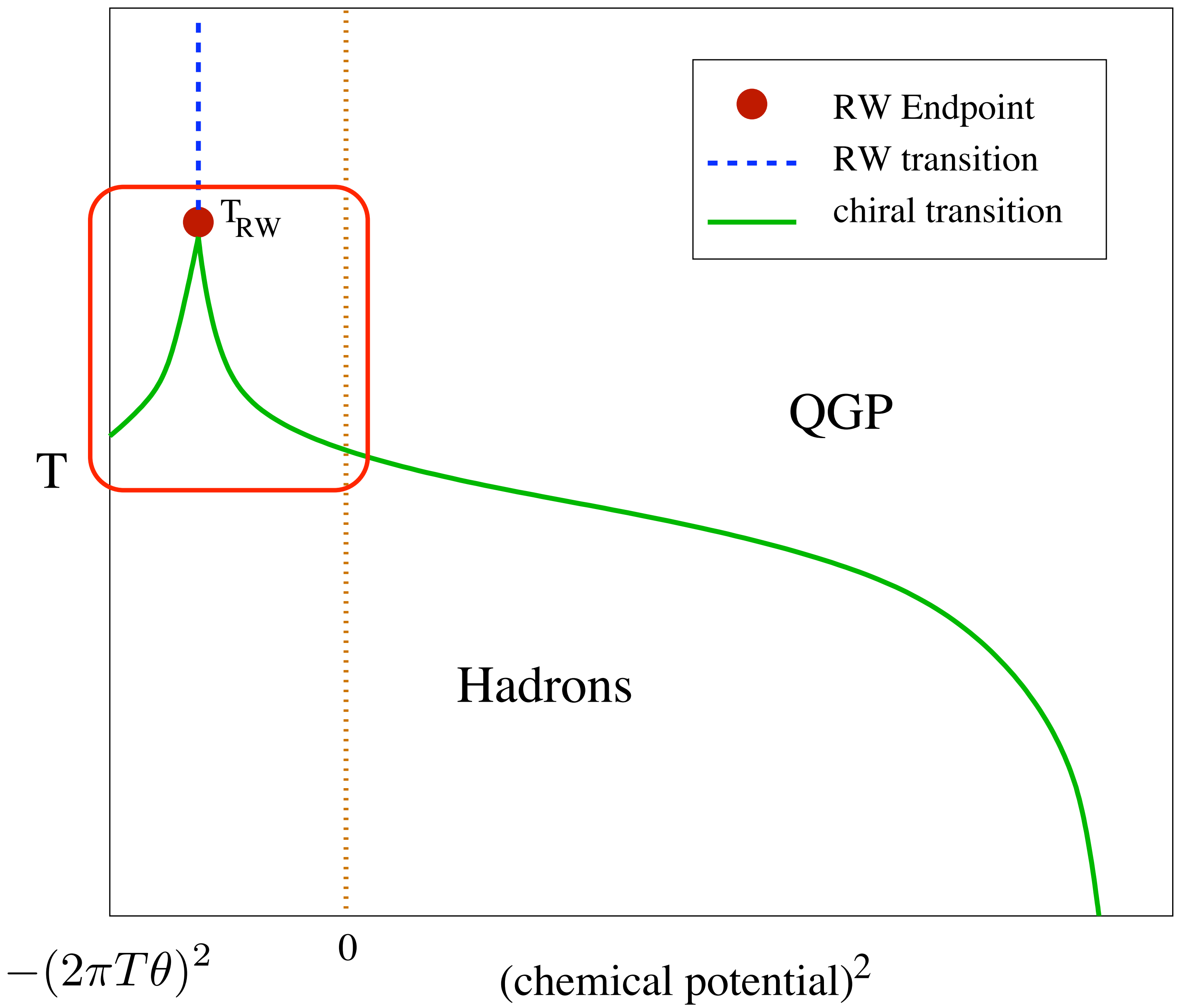
$$Z_\theta = Z_{\theta+1/3}$$

Partition function

via a center transformation $e^{\frac{2}{3}\pi\, i}\mathbb{1} \in \text{center}[SU(3)]$

gauge field insensitive to center transformations

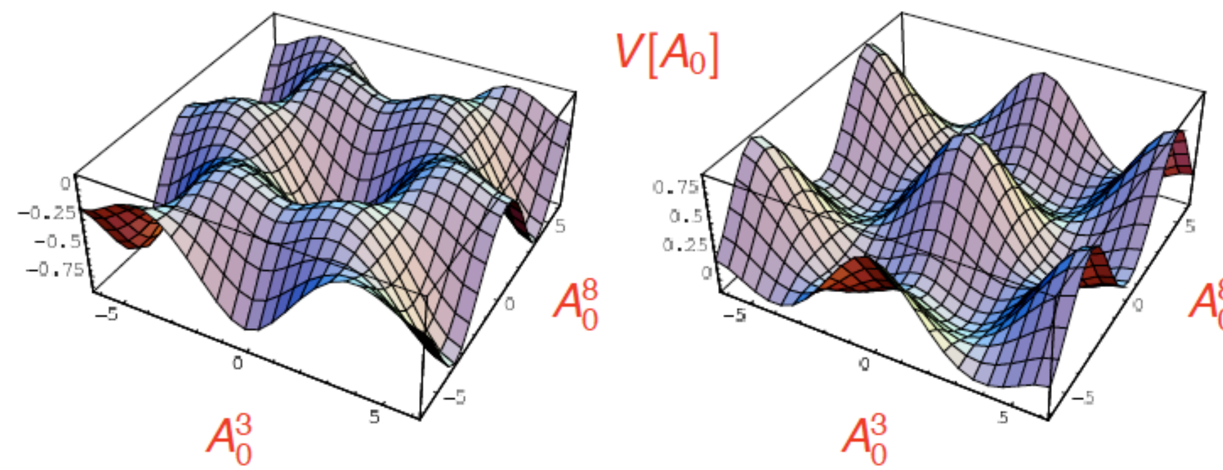
Imaginary chemical potential



Periodicity

$$q_\theta(t+\beta,\vec{x}) = -e^{2\pi\theta\, i} q_\theta(t,x)$$

Polyakov loop potential



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$$Z_\theta = Z_{\theta+1/3}$$

Partition function

via a center transformation $e^{\frac{2}{3}\pi\, i}\mathbb{1} \in \text{center}[SU(3)]$

gauge field insensitive to center transformations

Imaginary chemical potential

confinement order parameters

$$q_\theta(t+\beta,\vec{x}) = -e^{2\pi\theta\, i} q_\theta(t,x)$$

Center-sensitive observables

$$\mathcal{O}_\theta = \langle O[q_\theta]\rangle_\theta$$

at imaginary chemical potential

$$\mathcal{O}_\theta = \langle O[q_\theta]\rangle_{\theta=0}$$

at vanishing chemical potential

Dual order parameters

$$\tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

$$\tilde{\mathcal{O}} \xrightarrow{z} z\tilde{\mathcal{O}}$$

$$z = 1,\ e^{\frac{2}{3}\pi i},\ \ e^{\frac{4}{3}\pi i}$$

imaginary chemical potential

average over diff. theories

$$\tilde{\mathcal{O}}[\langle A_0\rangle_{\theta=0}]$$

breaking of RW-symmetry

Braun, Haas, Marhauser, JMP ’09

vanishing chemical potential

Lattice

Gattringer ‘06
Synatschke, Wipf, Wozar ’07
Bruckmann, Hagen, Bilgici, Gattringer ‘08

FunMethods

Fischer ’09
Fischer, Maas, Müller ’10

Imaginary chemical potential

confinement order parameters

$$\tilde{\mathcal{O}} = \int_0^1 d\theta\, \mathcal{O}_\theta e^{-2\pi i\theta}$$

at imaginary chemical potential			at vanishing chemical potential	
FRG	**DSE**		**FRG**	**DSE**
1-loop	1-loop	dual quark propagator	2-loop	3-loop
1-loop	–	dual pressure	–	–
1-loop	1-loop	dual density	2-loop	3-loop
1-loop	1-loop	dual susceptibilities	2-loop	3-loop

dual pressure=-T dual density

dual susceptibility= T dual density

Imaginary chemical potential

confinement order parameters

$$\tilde{\mathcal{O}} = \int_0^1 d\theta \, \mathcal{O}_\theta e^{-2\pi i\theta}$$

at imaginary chemical potential			at vanishing chemical potential	
FRG	**DSE**		**FRG**	**DSE**
1-loop	1-loop	dual quark propagator	2-loop	3-loop
1-loop	–	dual pressure	–	–
1-loop	1-loop	dual density	2-loop	3-loop
1-loop	1-loop	dual susceptibilities	2-loop	3-loop

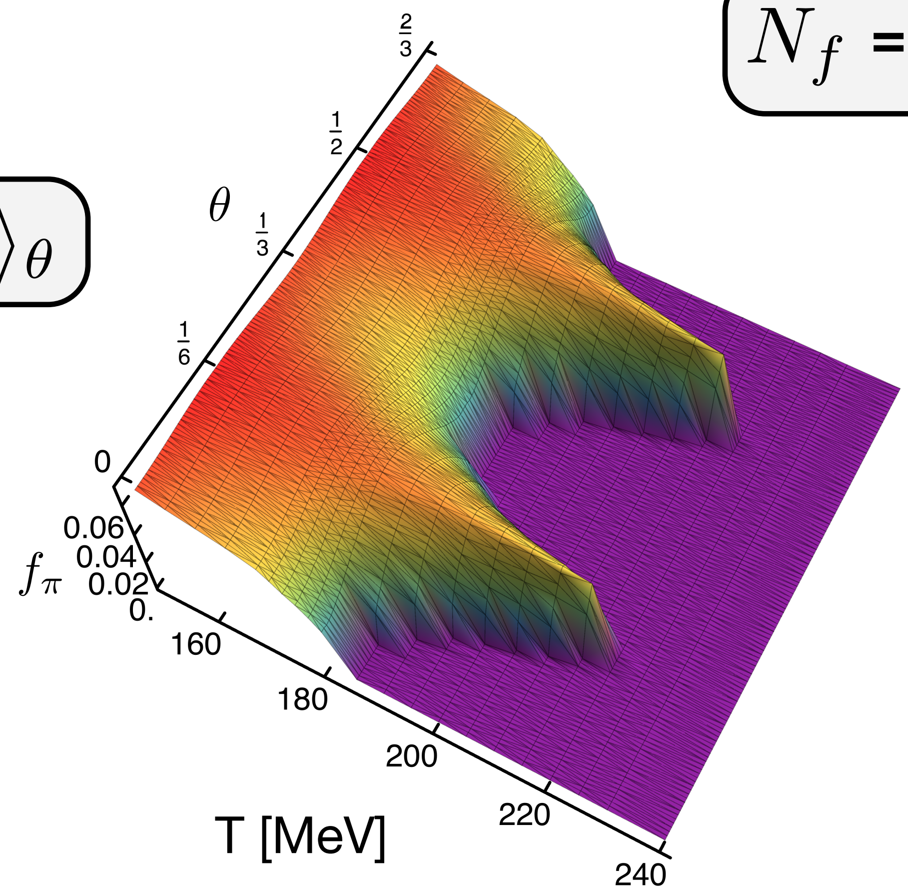
$$\frac{1}{2\pi T i} \int_0^1 d\theta \, (\partial_\theta \mathcal{O}_\theta) e^{-2\pi i\theta} = \frac{1}{T} \int_0^1 d\theta \, \mathcal{O}_\theta e^{-2\pi i\theta}$$

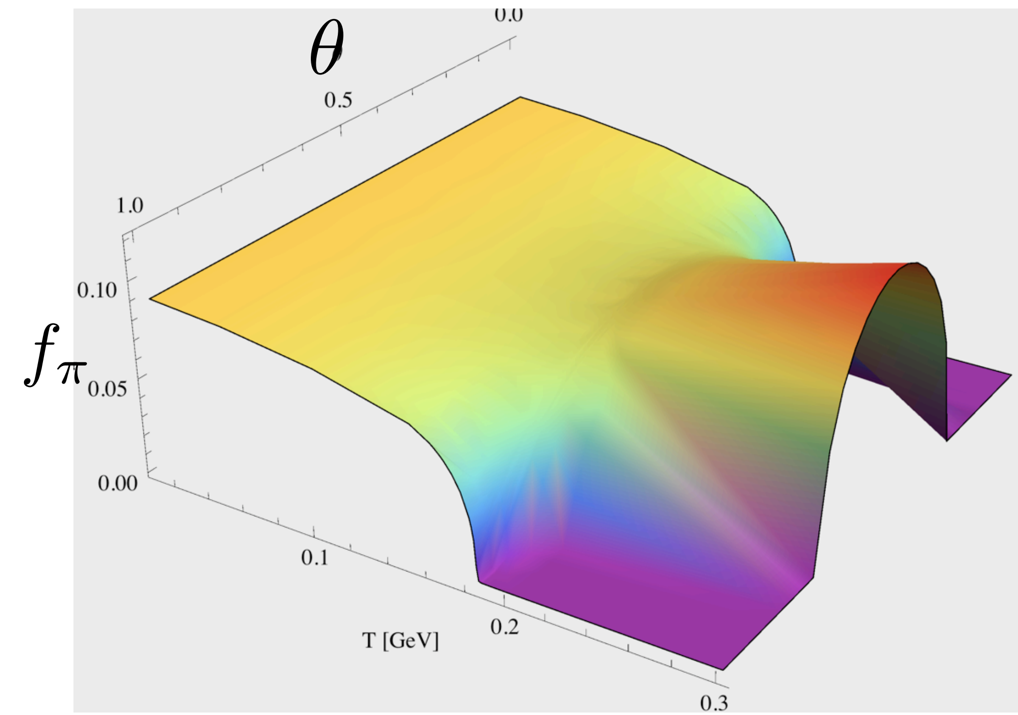
Imaginary chemical potential

N_f =2 & chiral limit

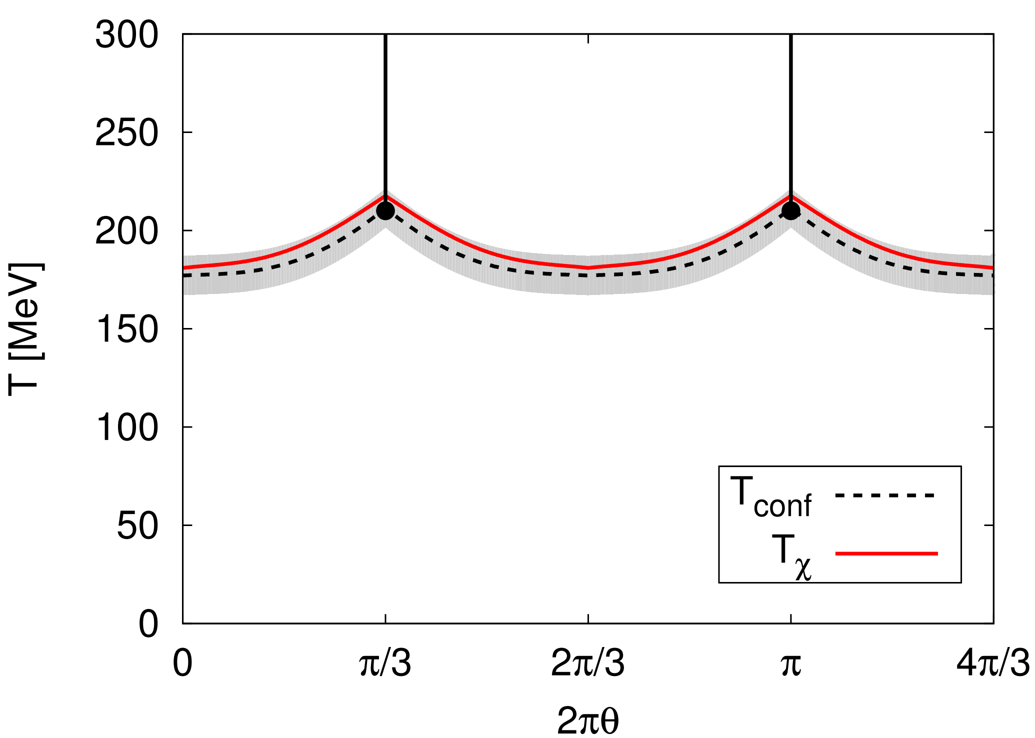
Braun, Haas, Marhauser, JMP '09

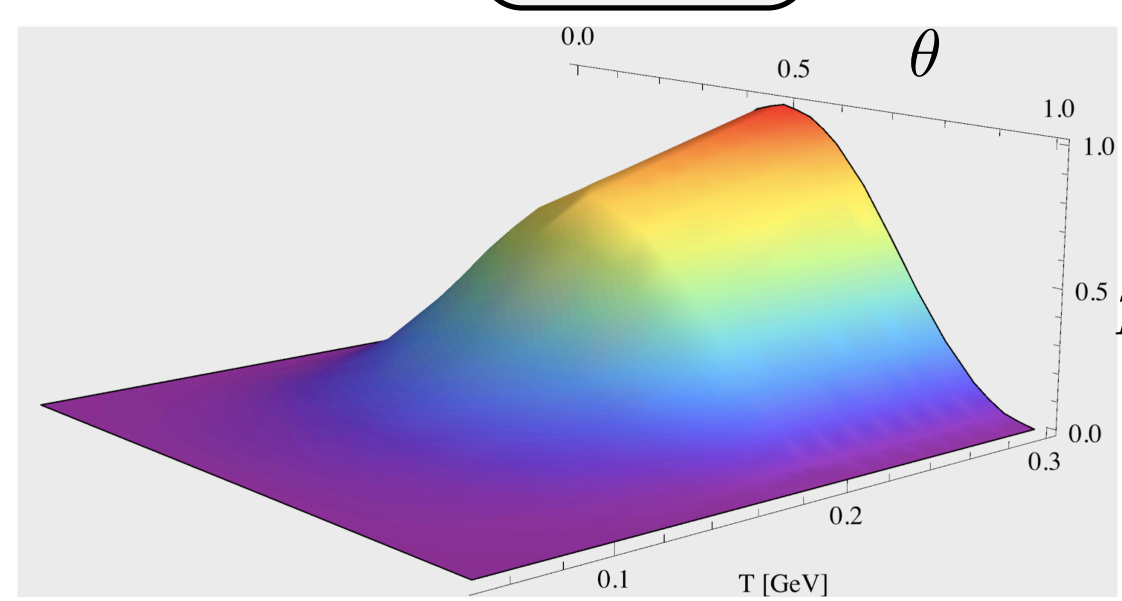
$\langle A_0 \rangle_\theta$





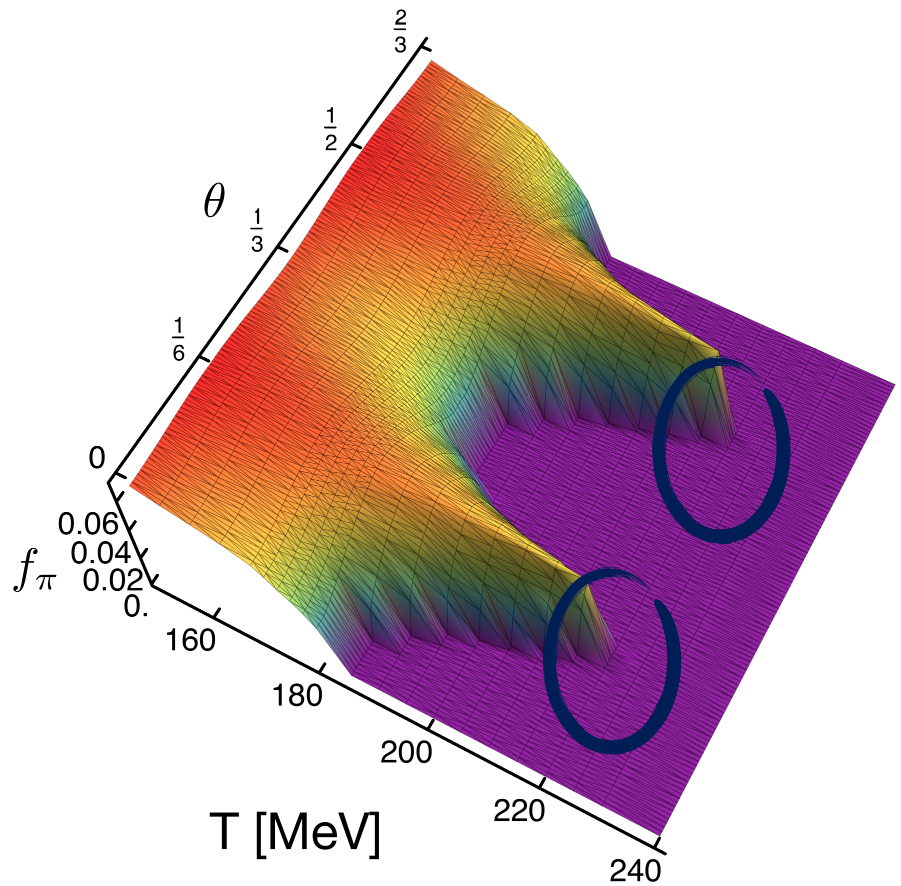
$\langle A_0 \rangle_{\theta=0}$

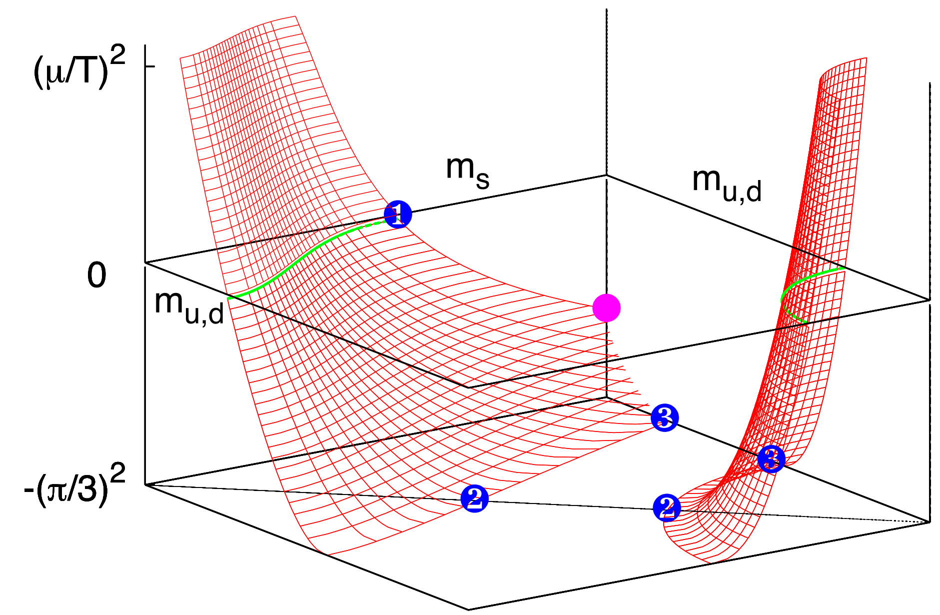




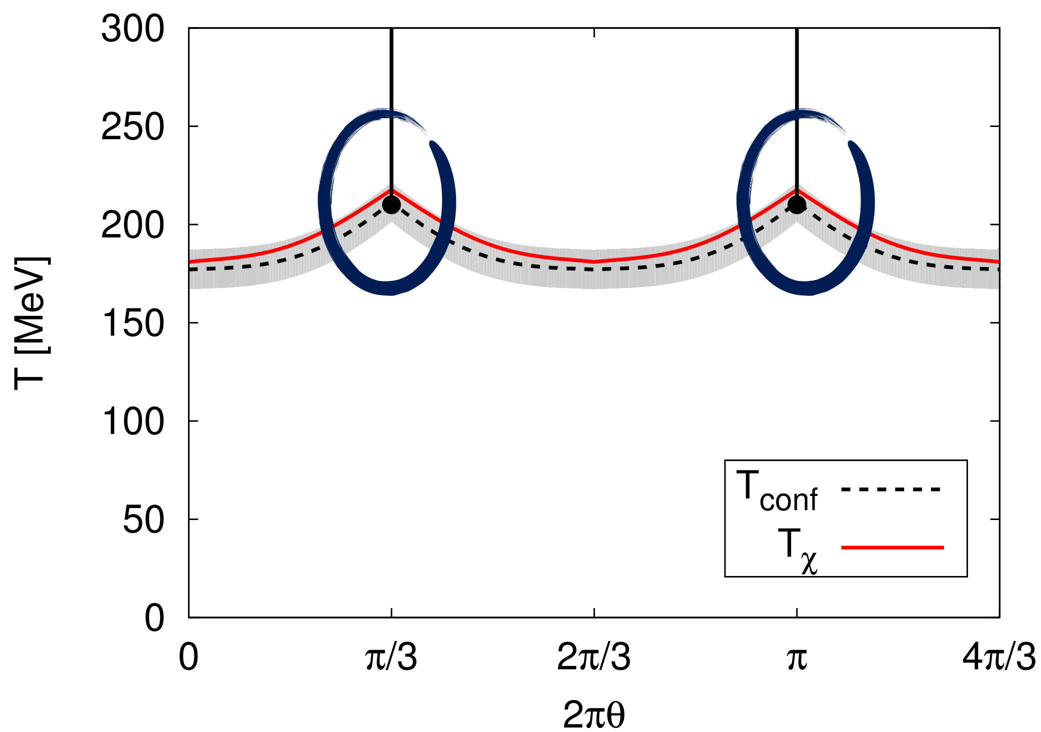
Imaginary chemical potential

Nature of the RW endpoint

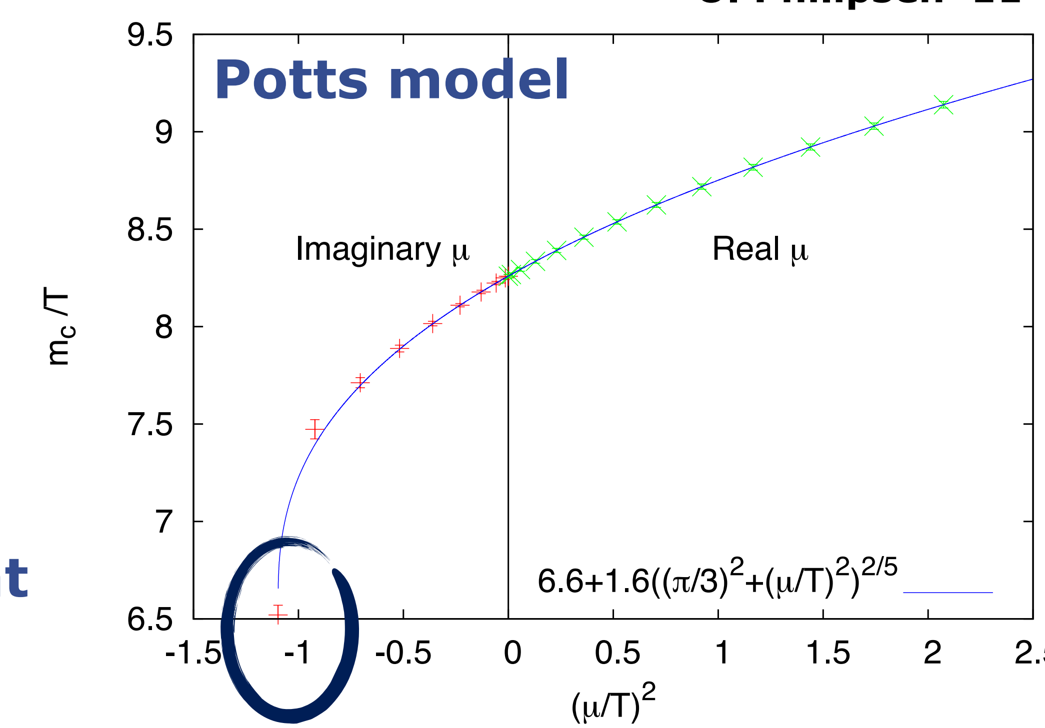




O. Philipsen '11



RW endpoint



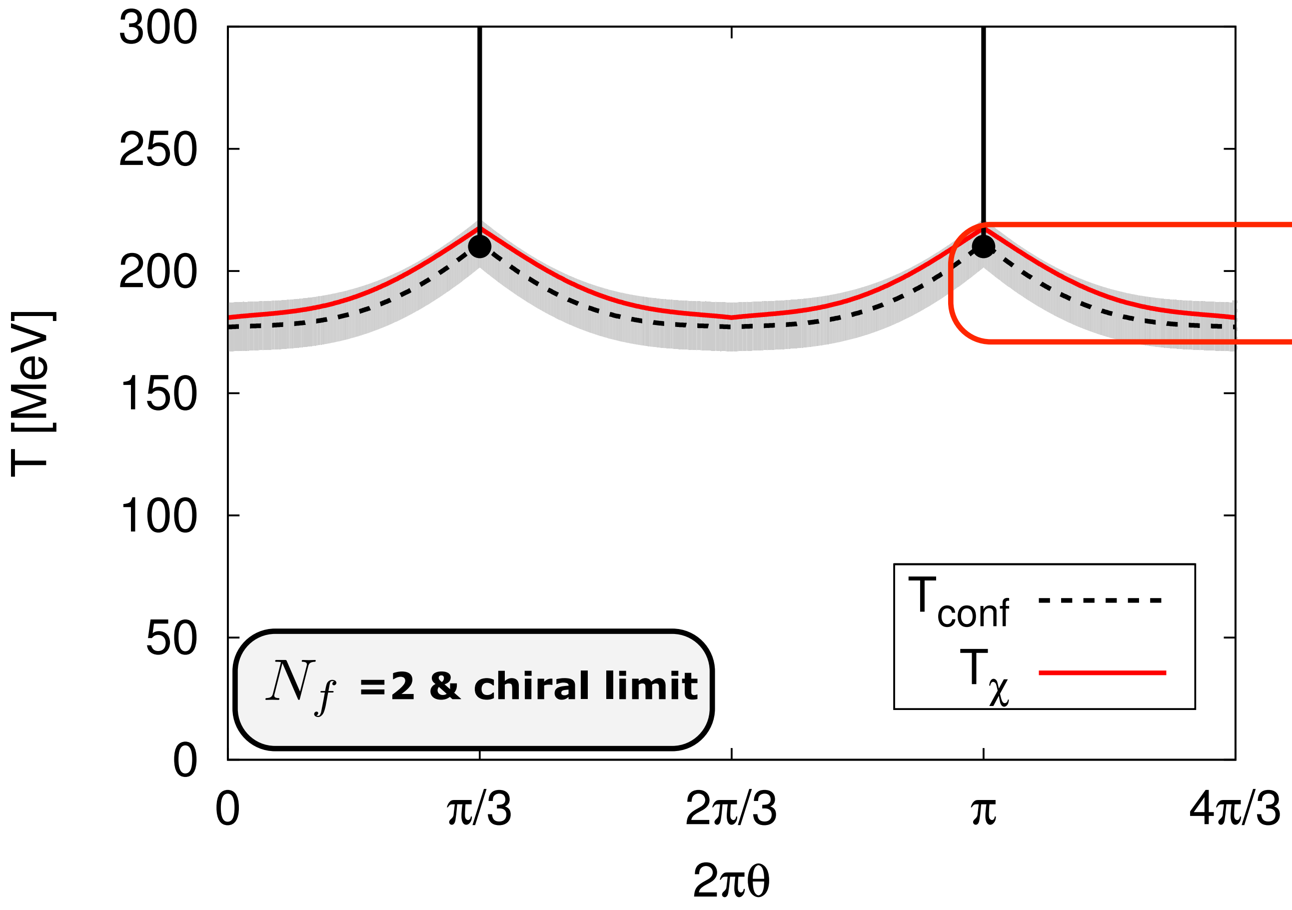
Nature of RW endpoint

lattice: D'Elia, Sanfilippo '09
de Forcrand, Philipsen '10
...

PNJL: Sakai et al '10,
Morita et al '11

Imaginary chemical potential

Braun, Haas, Marhauser, JMP '09

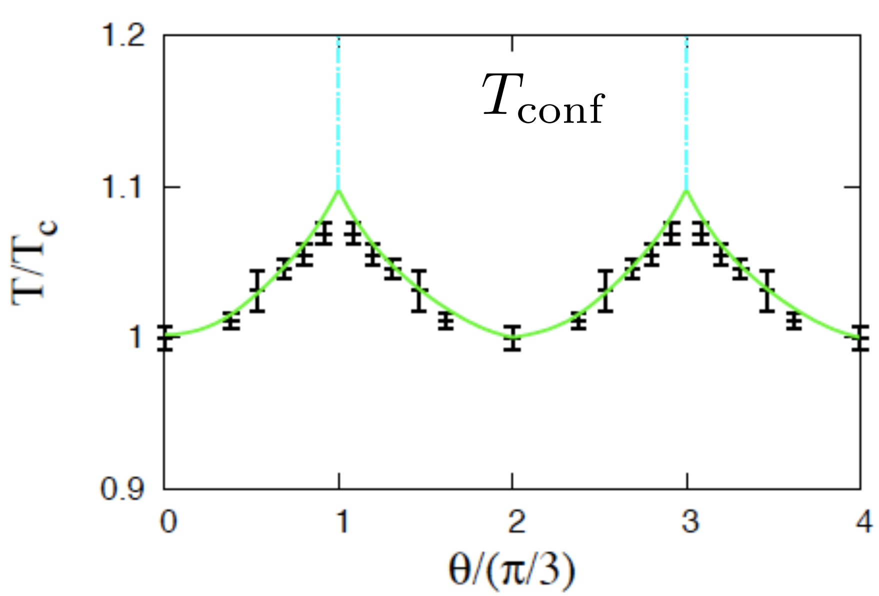


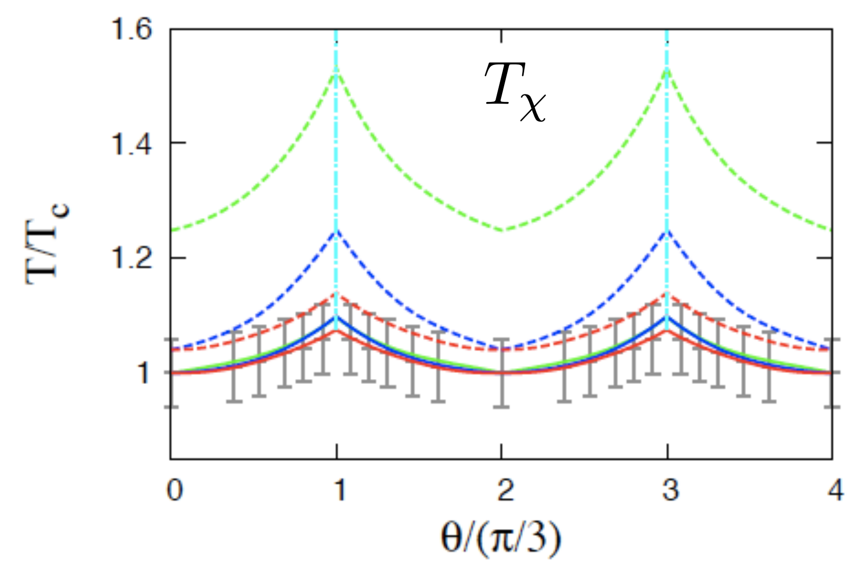
$\gtrsim |T_{\rm conf} - T_\chi|$

Dual order parameters

compatibility

lattice results, e.g.
Kratochvila et al '06, Wu et al '06
& D'Elia et al '07, Fromm et al '11



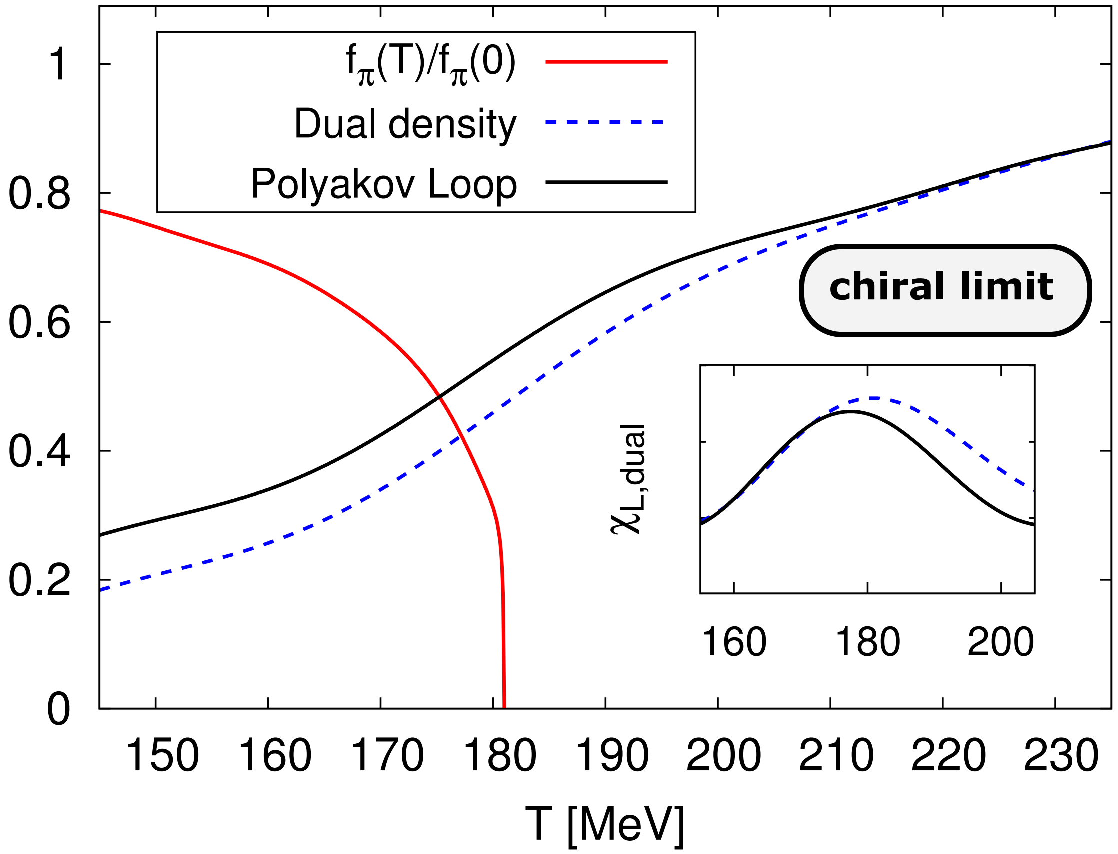


PNJL: Sakai et al '09

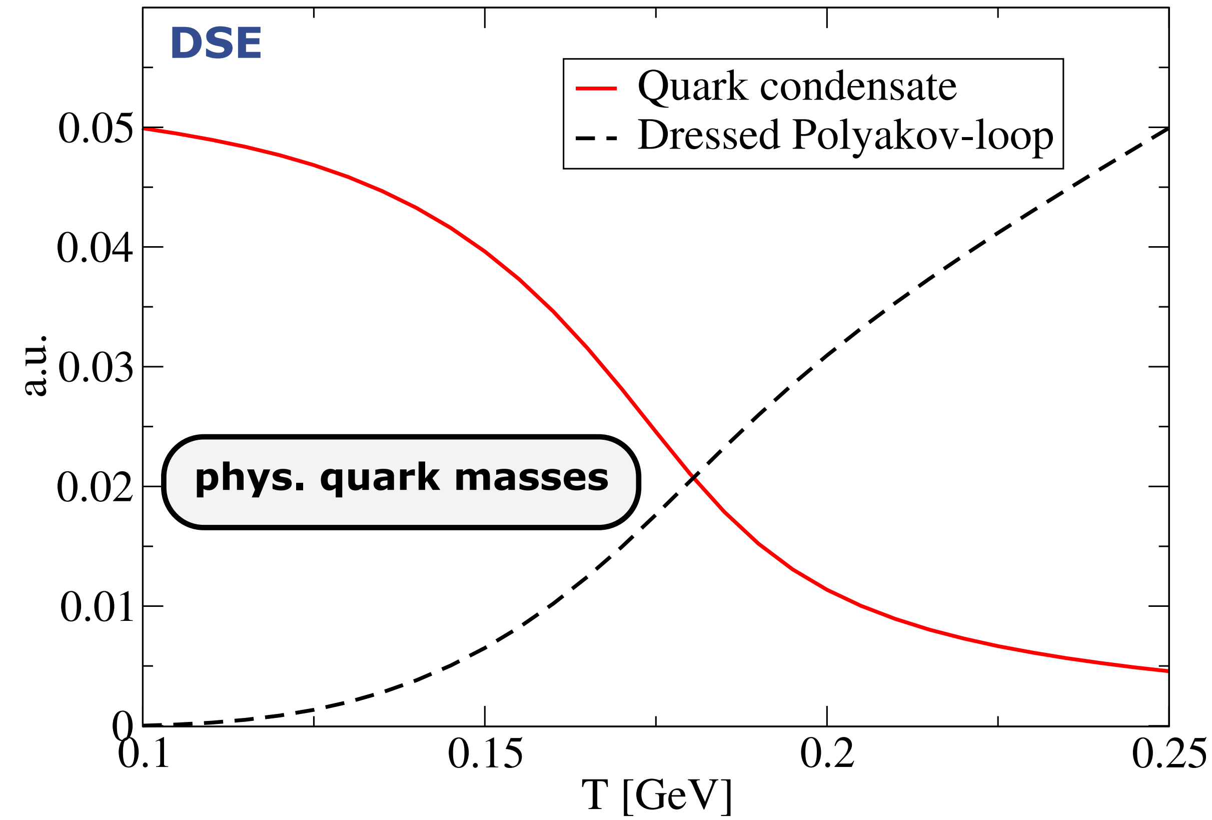
adjust 8-fermi interaction

Full dynamical QCD: $N_f = 2$ & chiral limit

Phase structure

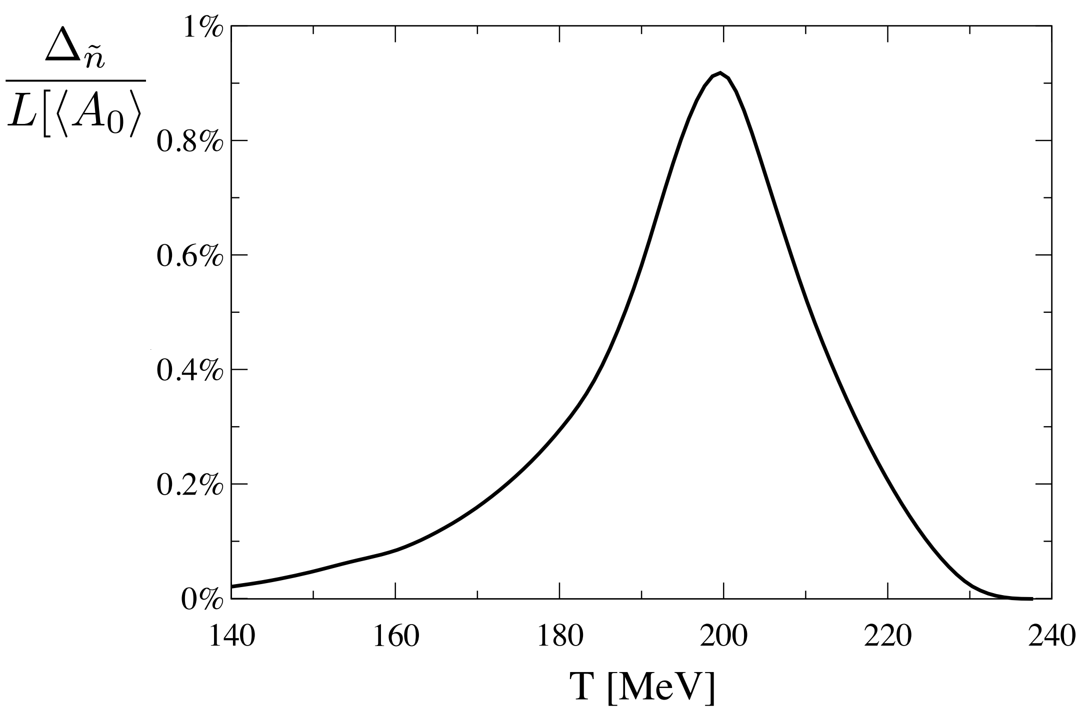


Braun, Haas, Marhauser, JMP '09



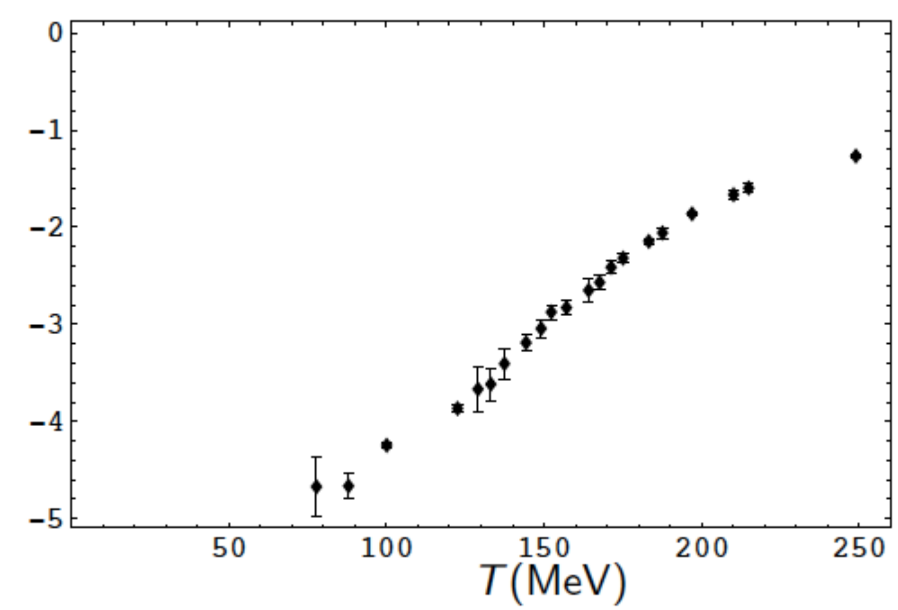
Fischer, Lücker, Müller '11

factorisation property of dual density



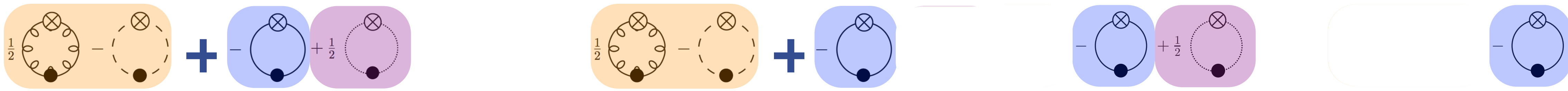
$$\Delta_{\tilde{n}} = \frac{\tilde{n}[\langle A_0 \rangle]}{\tilde{n}[0]} - L[\langle A_0 \rangle]$$

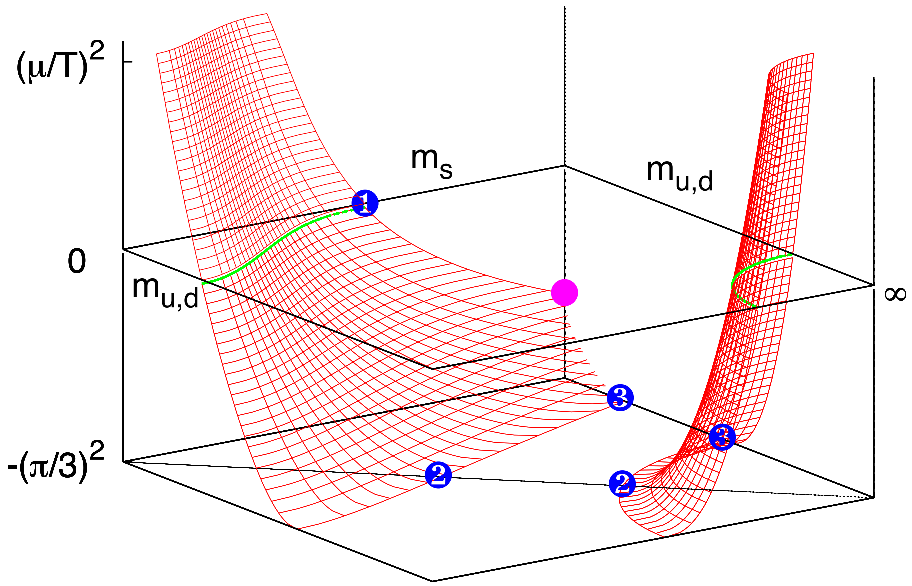
Log of dual condensate, m=60 MeV

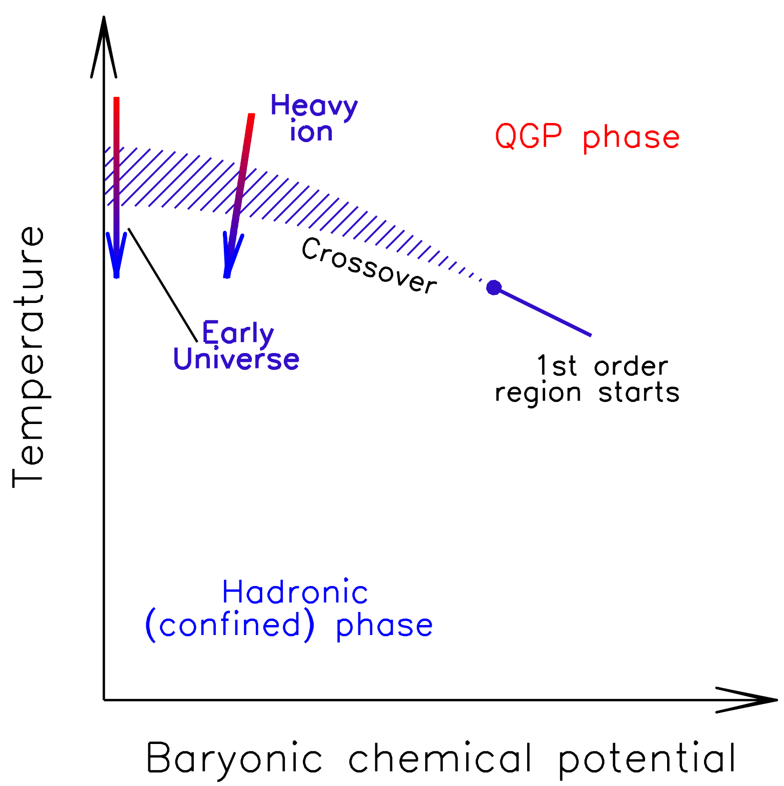


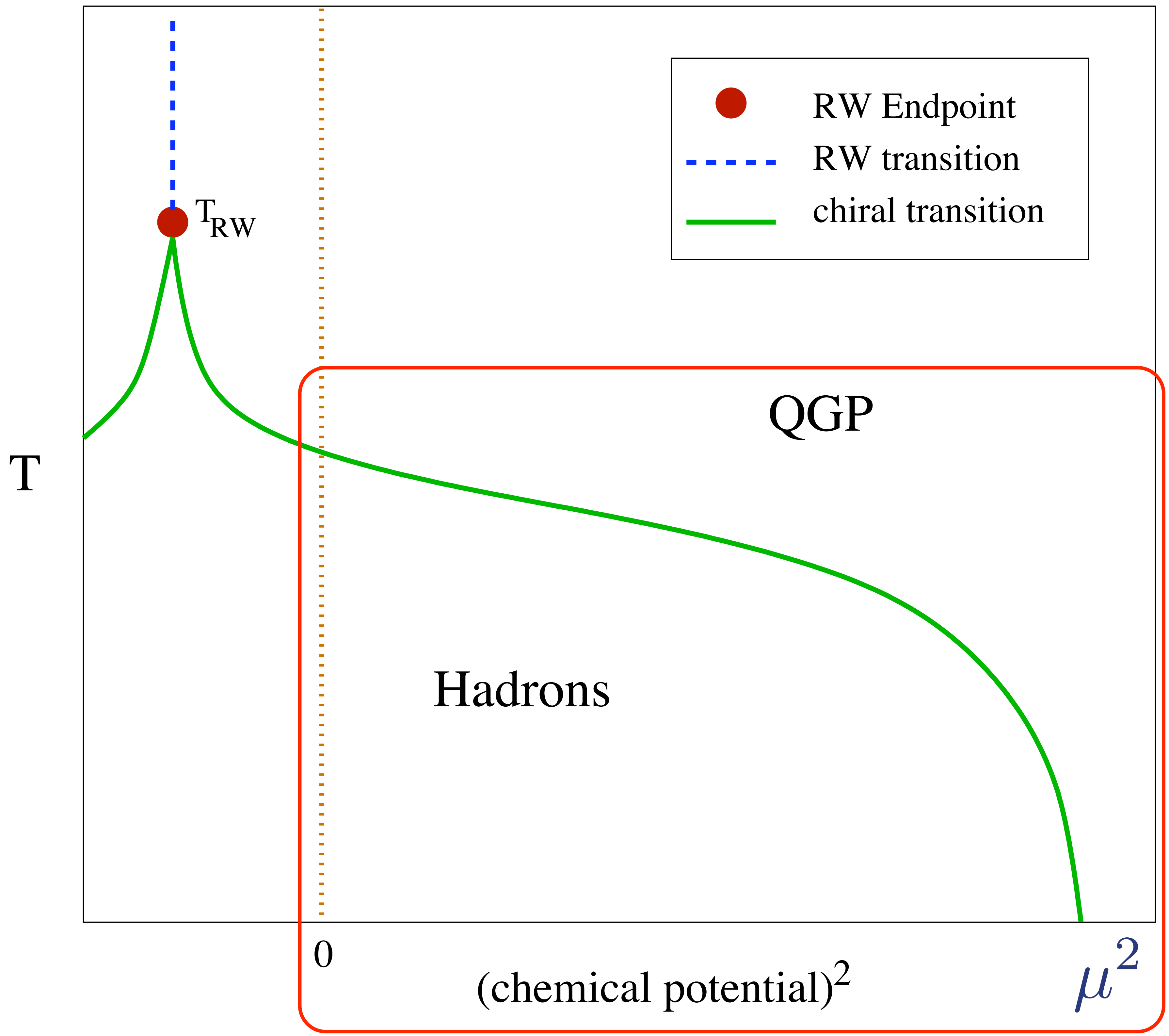
Zhang, Bruckmann, Gattringer, Fodor, Szabo '10

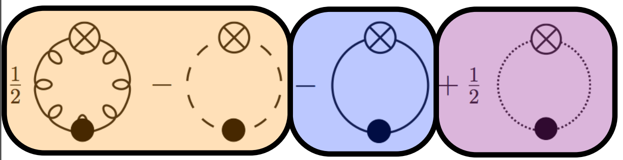
Real chemical potential







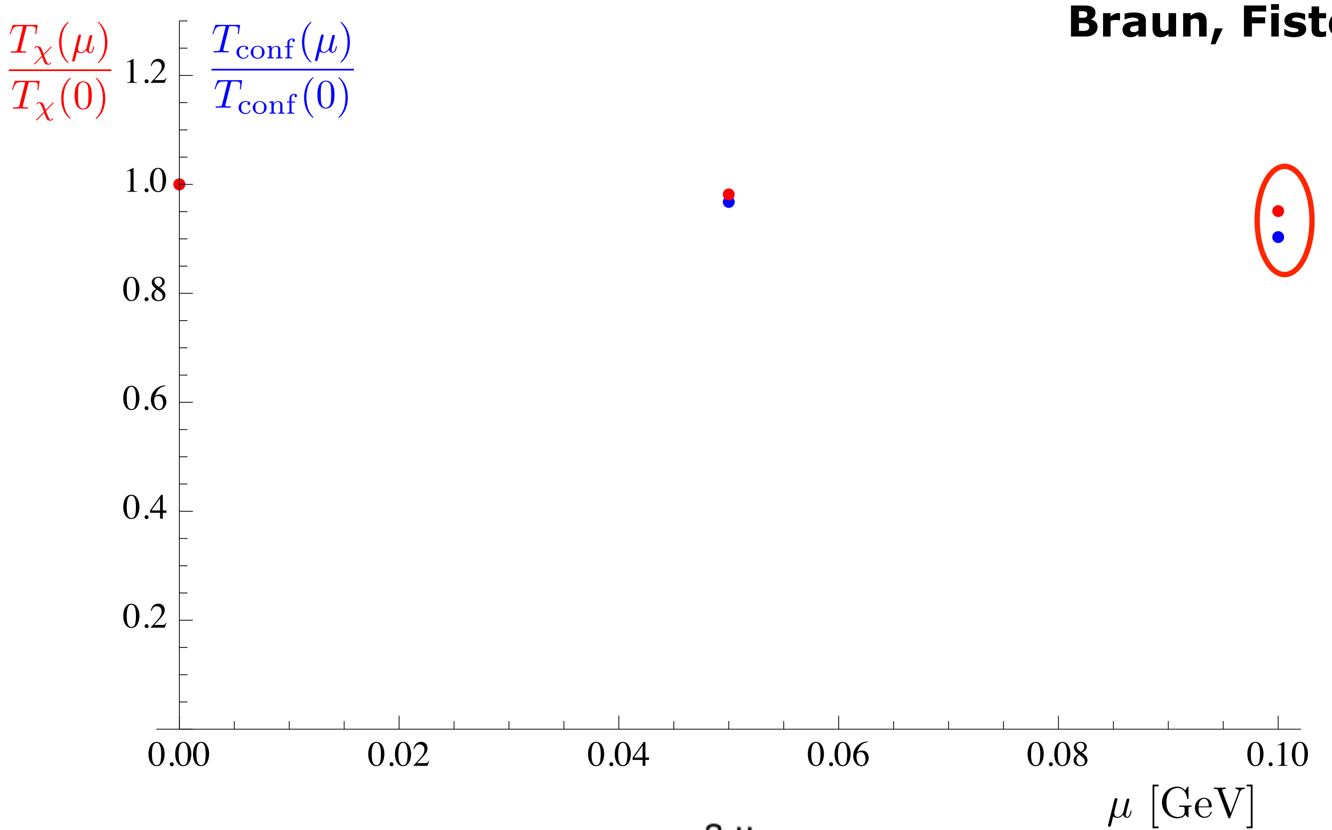


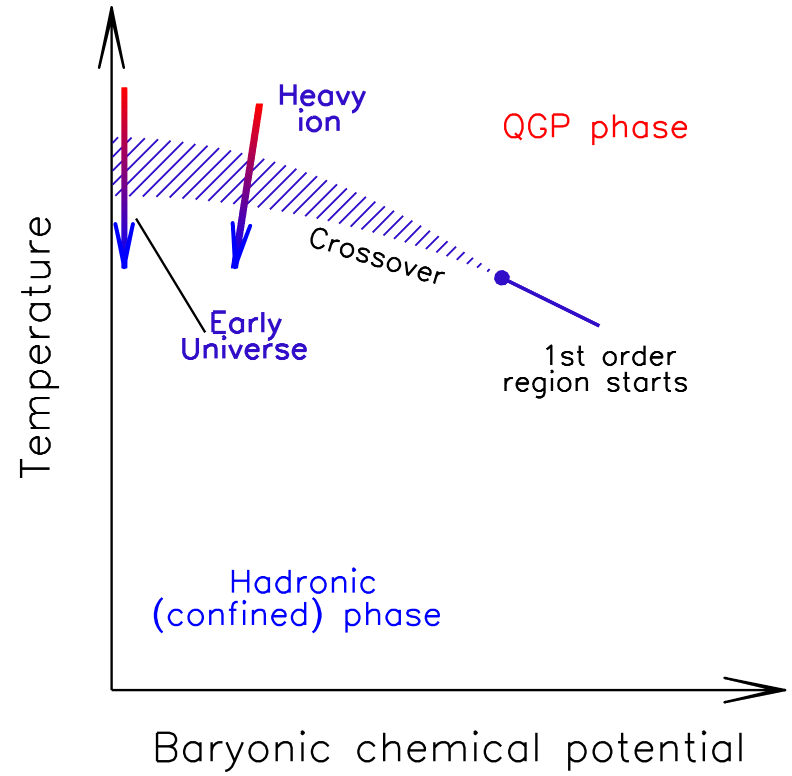


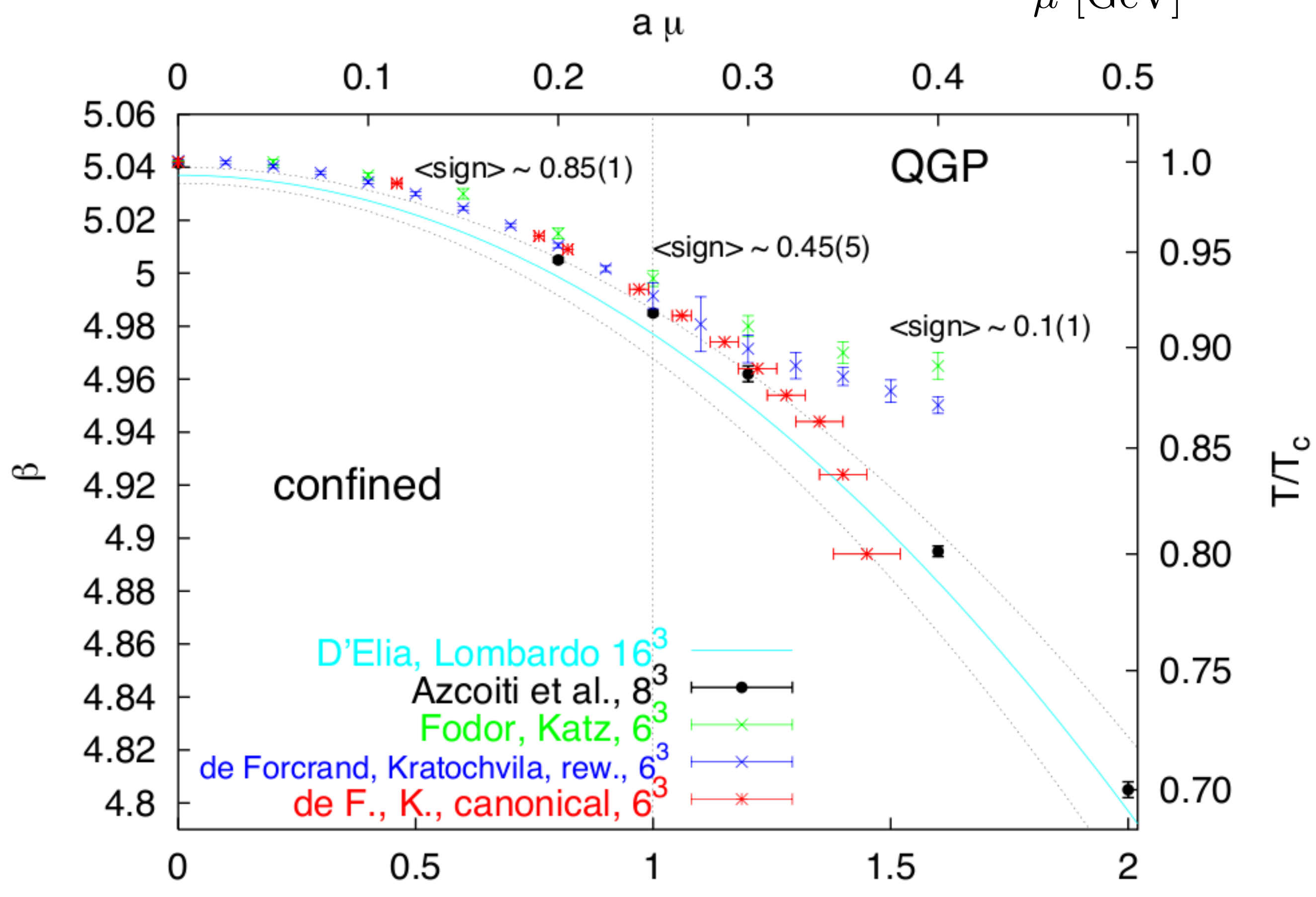
Chemical potential

Full dynamical QCD

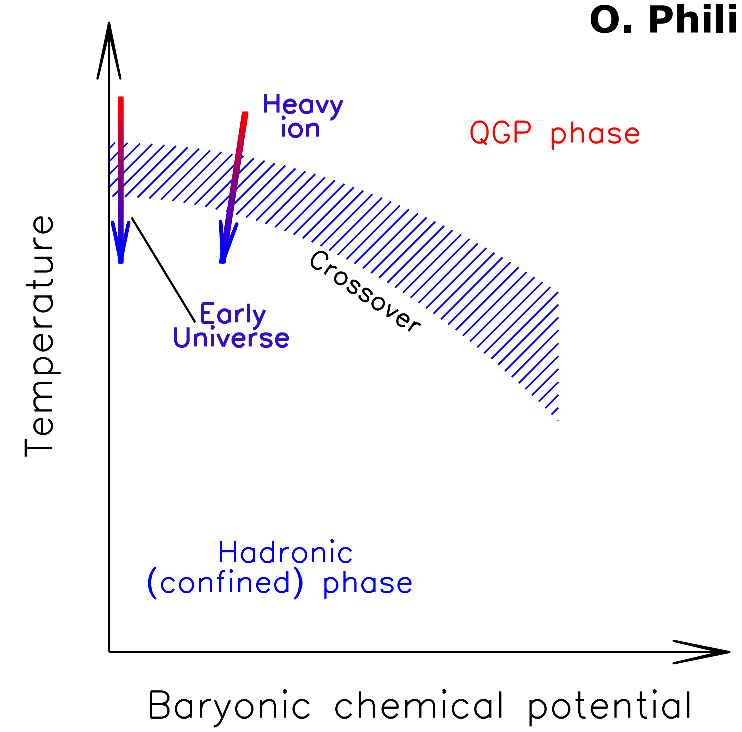
Braun, Fister, Haas, JMP



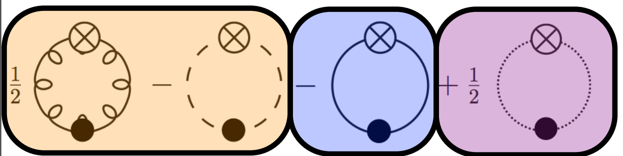


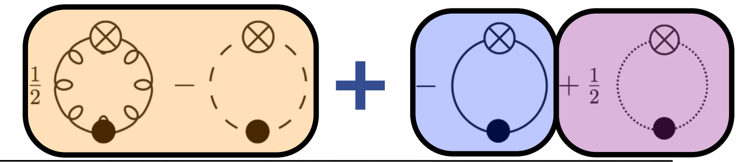


O. Philipsen '11



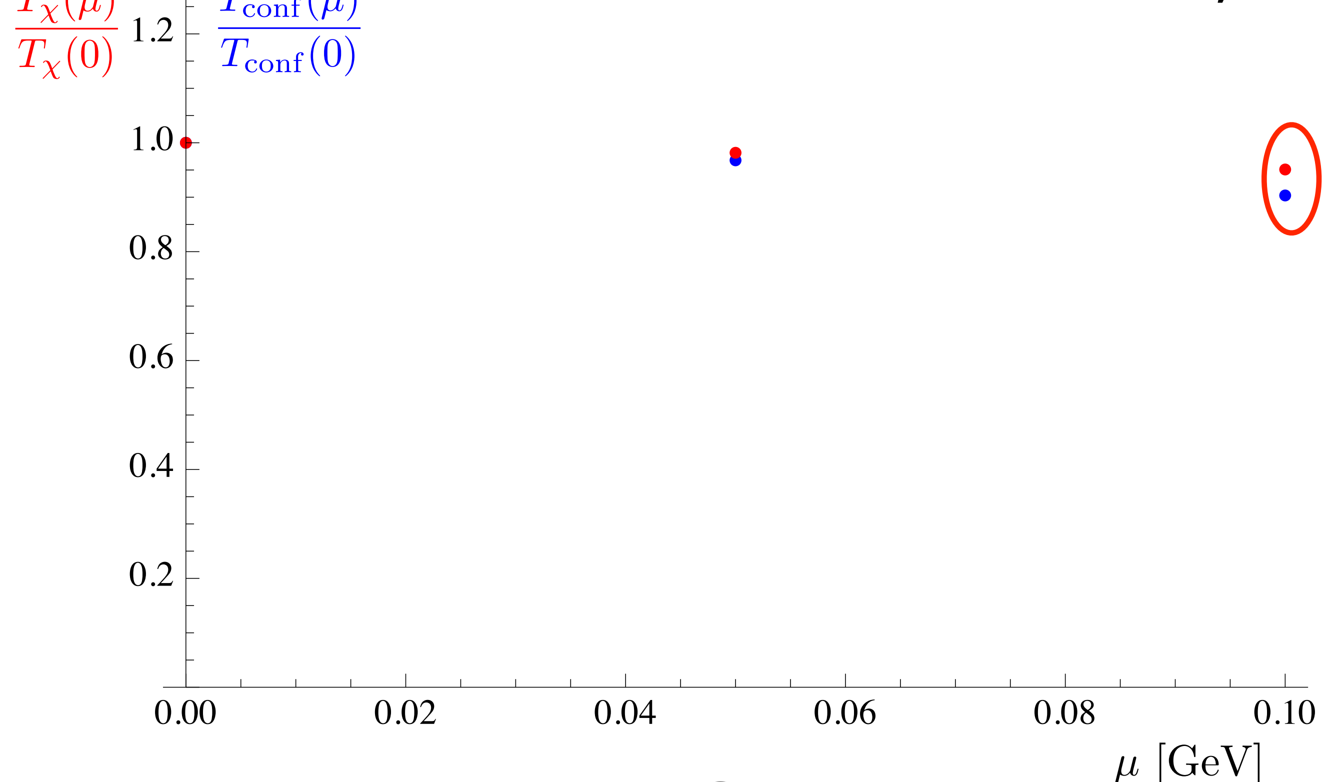
Chemical potential

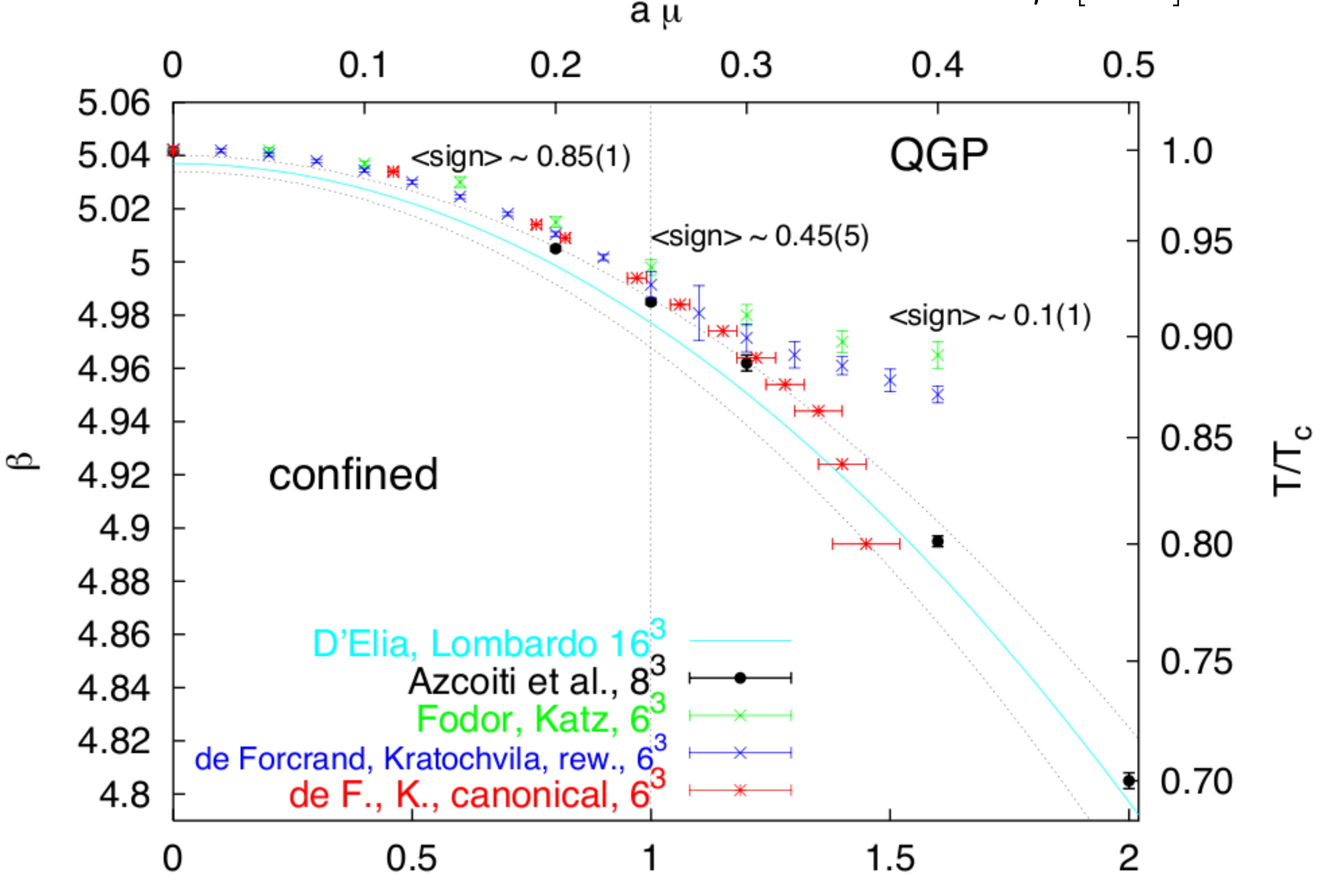




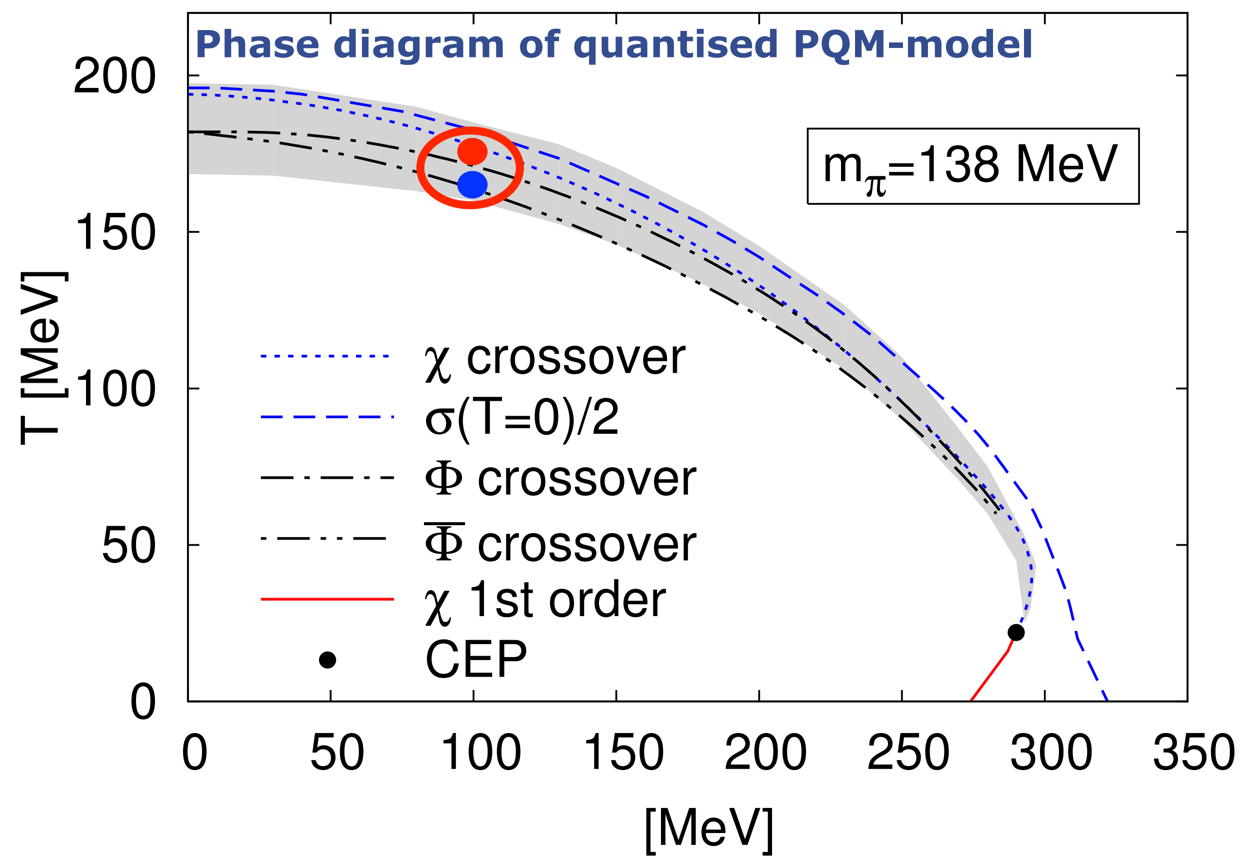
Full dynamical QCD

Braun, Fister, Haas, JMP





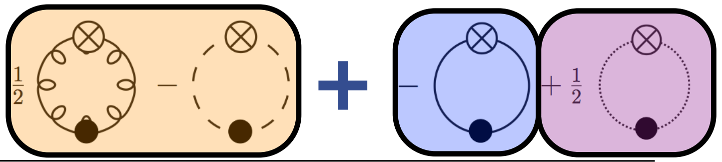
Herbst, JMP, Schaefer '10,'13



see talk of T. Herbst

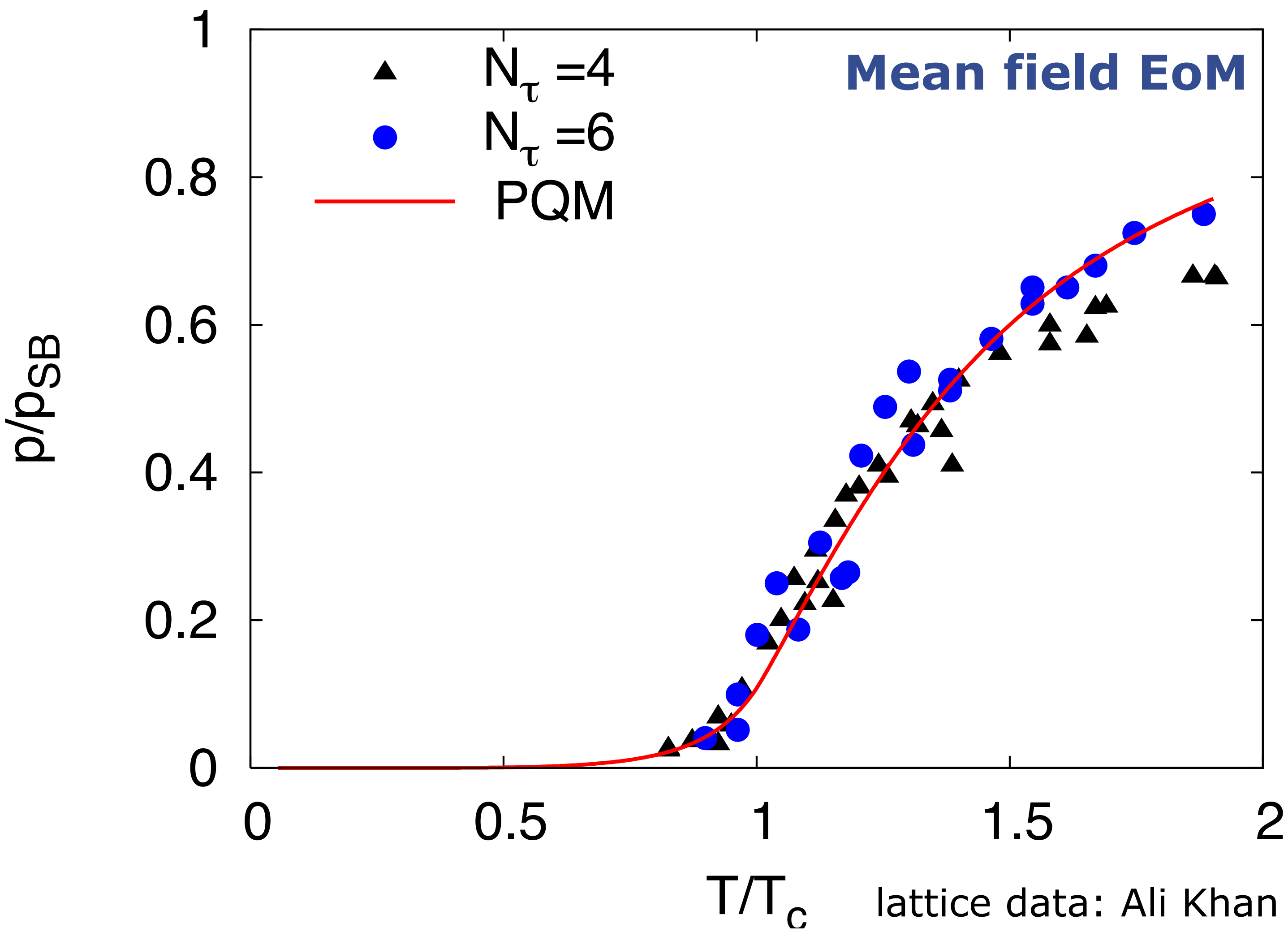
Chemical potential

Polyakov-extended models



Potential

Polyakov-loop Potential	Fermionic fluctuations	Mesonic potential
$U[\Phi, \bar{\Phi}]$	$+ \quad \Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$	$+ \quad V[\sigma, \vec{\pi}]$
Fit to YM-thermodynamics	fermionic fluctuations	mesonic fluctuations



lattice data: Ali Khan et al '01

Meisinger, Ogilvie '96

Pisarski '00

⋮

PNJL

Fukushima '04

Ratti, Thaler, Weise '06

Megias, Ruiz Arriola, Salcedo '06

Ghosh, Mukherjee, Mustafa, Ray '06

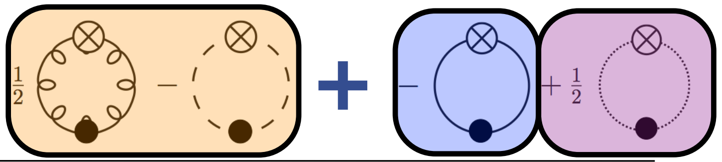
C. Sasaki, B. Friman and K. Redlich '06

PQM → Schaefer, JMP, Wambach '07

⋮

Chemical potential

Dynamical Polyakov-extended models



Potential

Polyakov-loop Potential	Fermionic fluctuations	Mesonic potential
$U[\Phi, \bar{\Phi}]$	$+ \quad \Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$	$+ \quad V[\sigma, \vec{\pi}]$
Fit to YM-thermodynamics	fermionic fluctuations	mesonic fluctuations

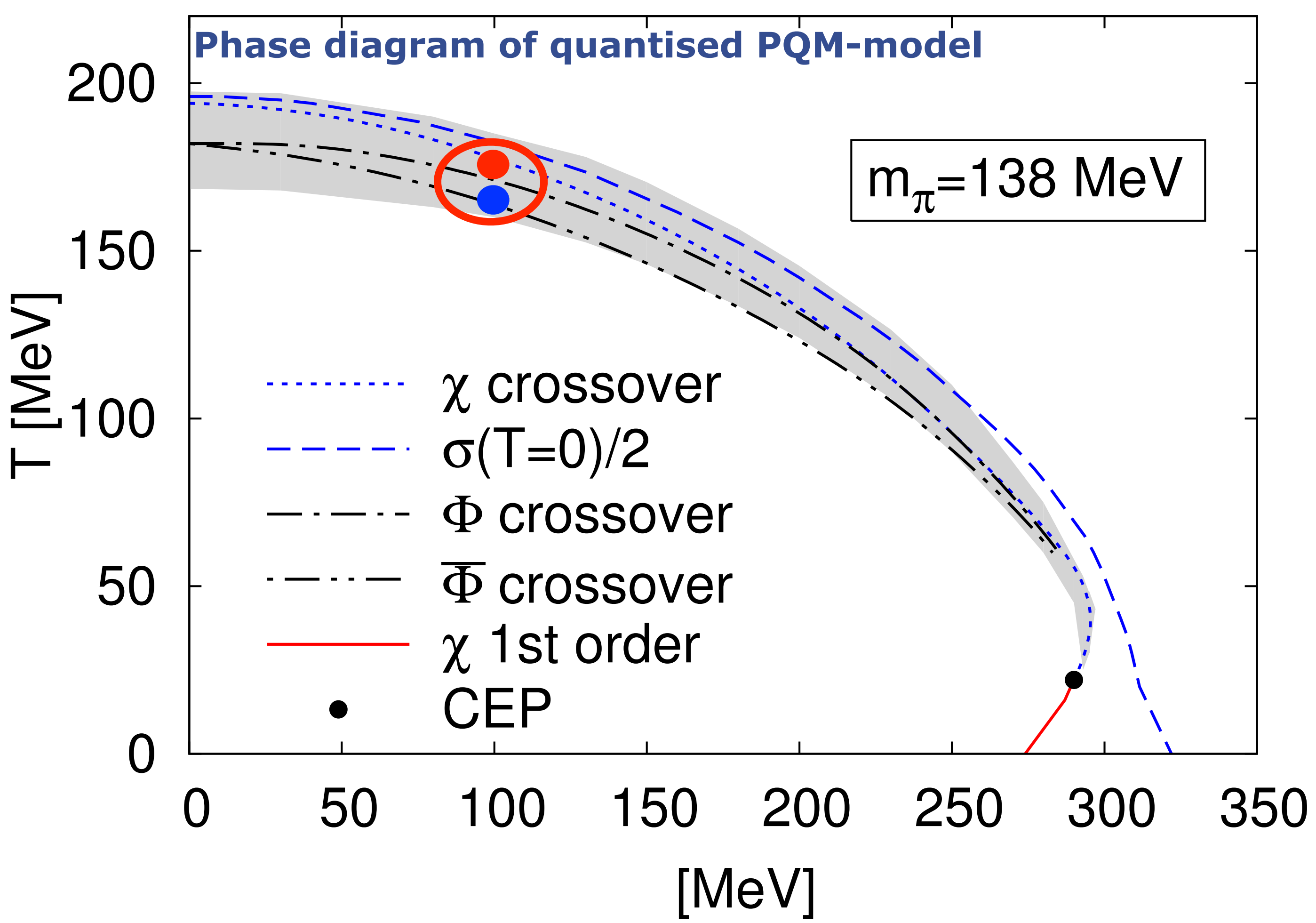
quark fluctuations change glue dynamics

$$T_{0\,\mathrm{YM}} \to T_0(N_f, \mu; m_q)$$

estimated via HTL/HDL computation

Schaefer, JMP, Wambach '07

Herbst, JMP, Schaefer '10,'13

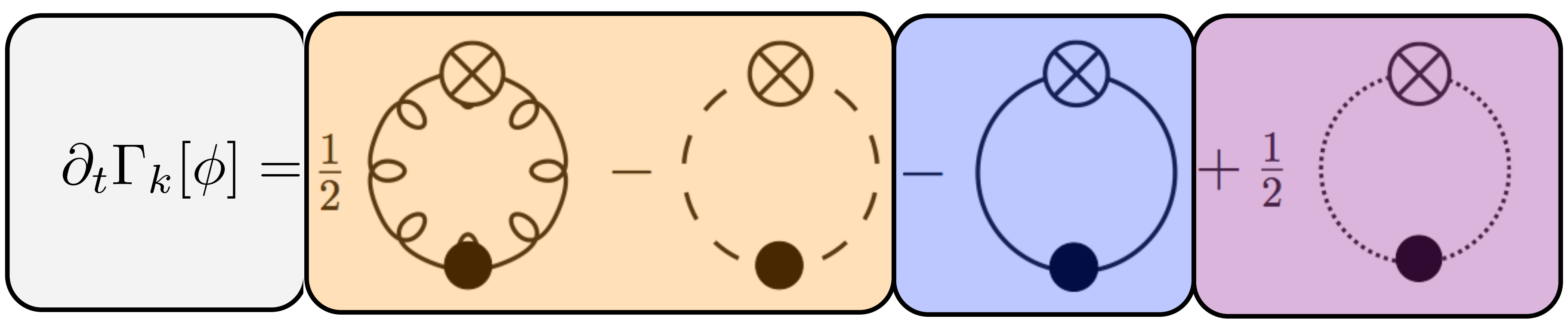


Chemical potential

Polyakov-extended models as reduced QCD

Effective potential

Polyakov-loop Potential	Fermionic fluctuations	Mesonic potential
$U[\Phi, \bar{\Phi}]$	$+ \quad \Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$	$+ \quad V[\sigma, \vec{\pi}]$
Fit to YM-thermodynamics	fermionic fluctuations	mesonic fluctuations



Reminder

Chemical potential

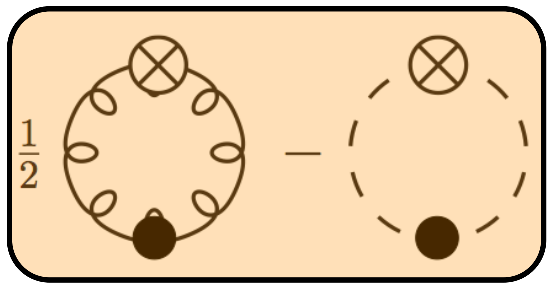
Polyakov-extended models as reduced QCD

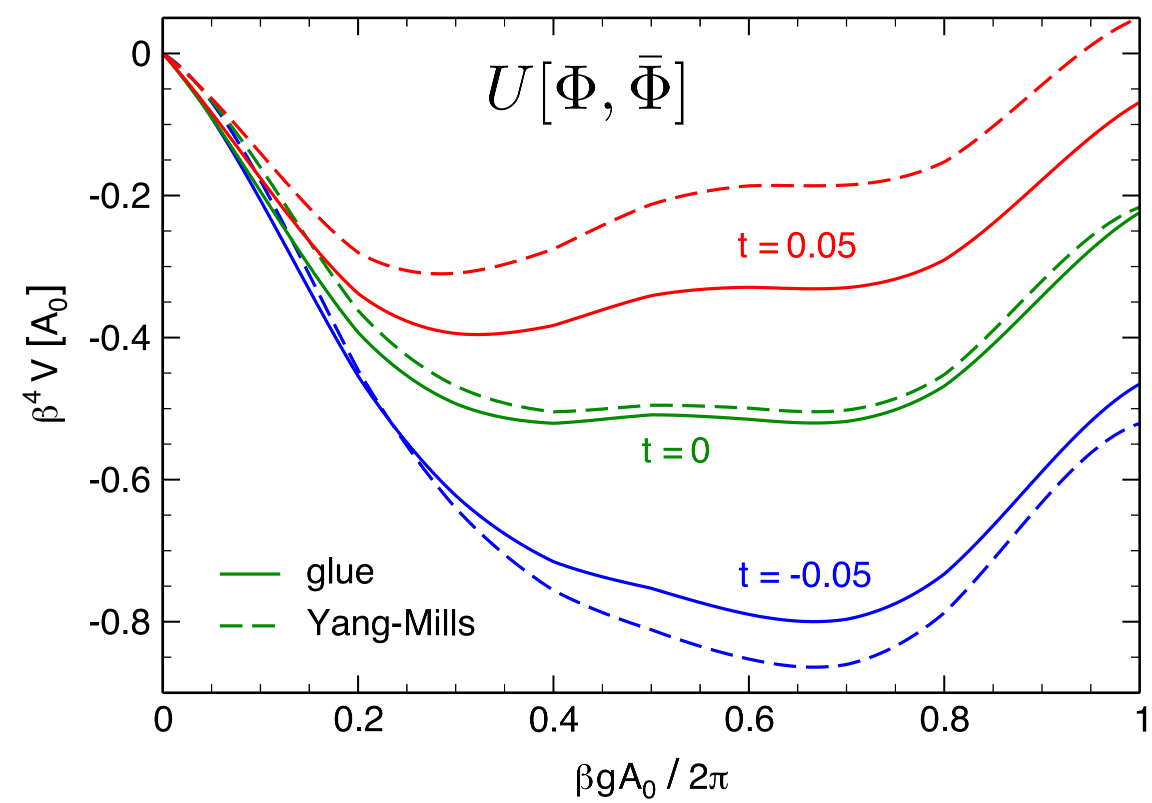
Towards QCD

Haas, Stiele, Braun, JMP, Schaffner-Bielich ’13

JMP ’10

Polyakov-loop Potential $U[\Phi, \bar{\Phi}]$ + **Fermionic fluctuations** $\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$ + **Mesonic potential** $V[\sigma, \vec{\pi}]$





QCD confirmation of

HTL/HDL quark estimate

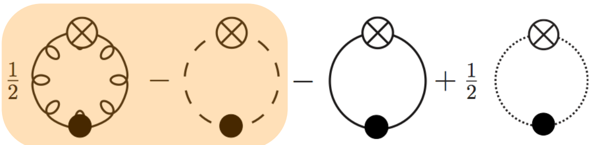
$$(\beta^4 V)_{\text{glue}}[t\,,\,A_0] \simeq (\beta^4 V)_{\text{YM}}[t_{\text{YM}}(t)\,,\,A_0]$$

Chemical potential

Improving models towards full QCD

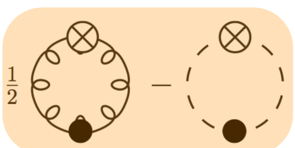
Haas, Stiele, Braun, JMP, Schaffner-Bielich '13

Glue Potential

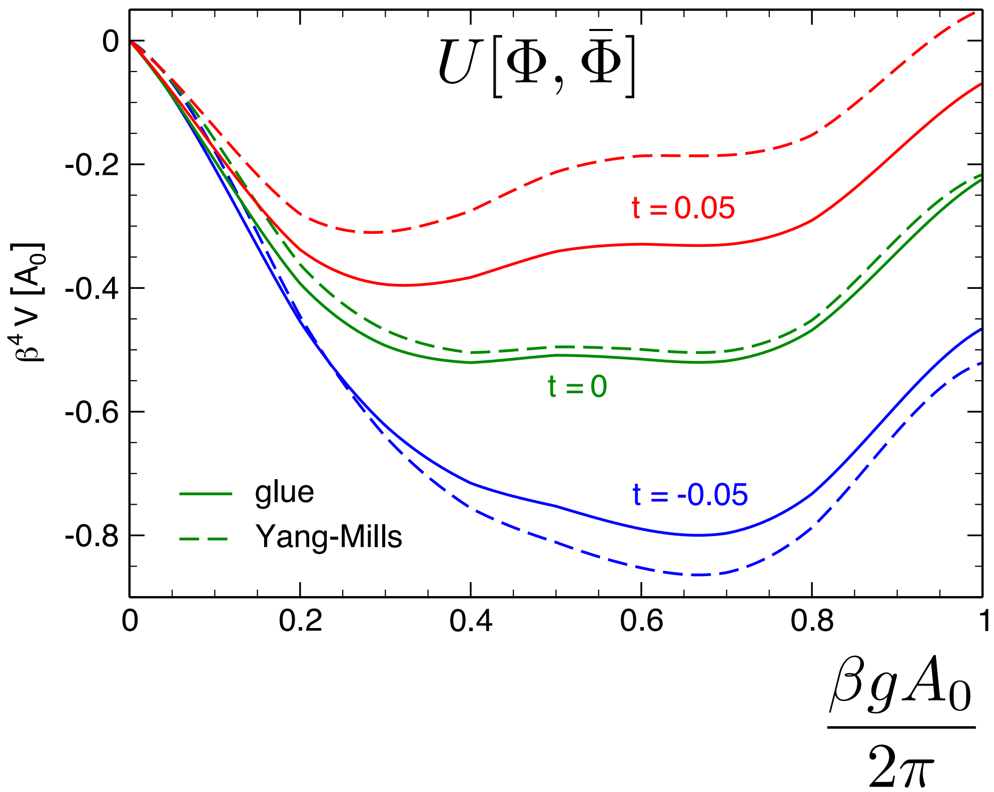


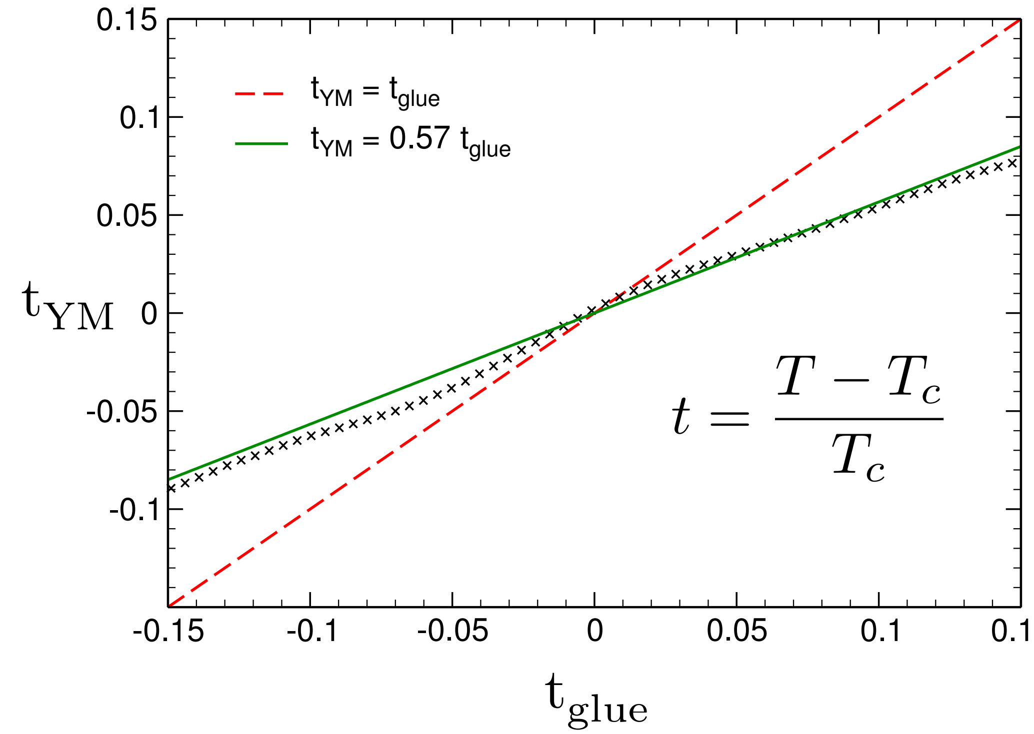
Braun, Haas, Marhauser, JMP '09

Yang-Mills Potential

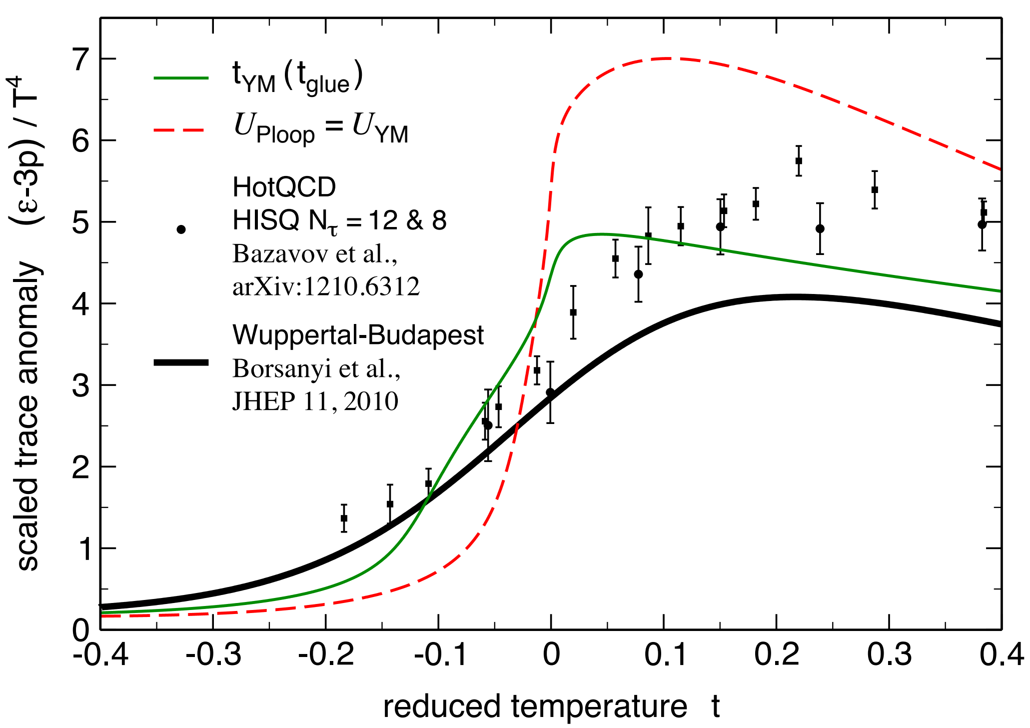


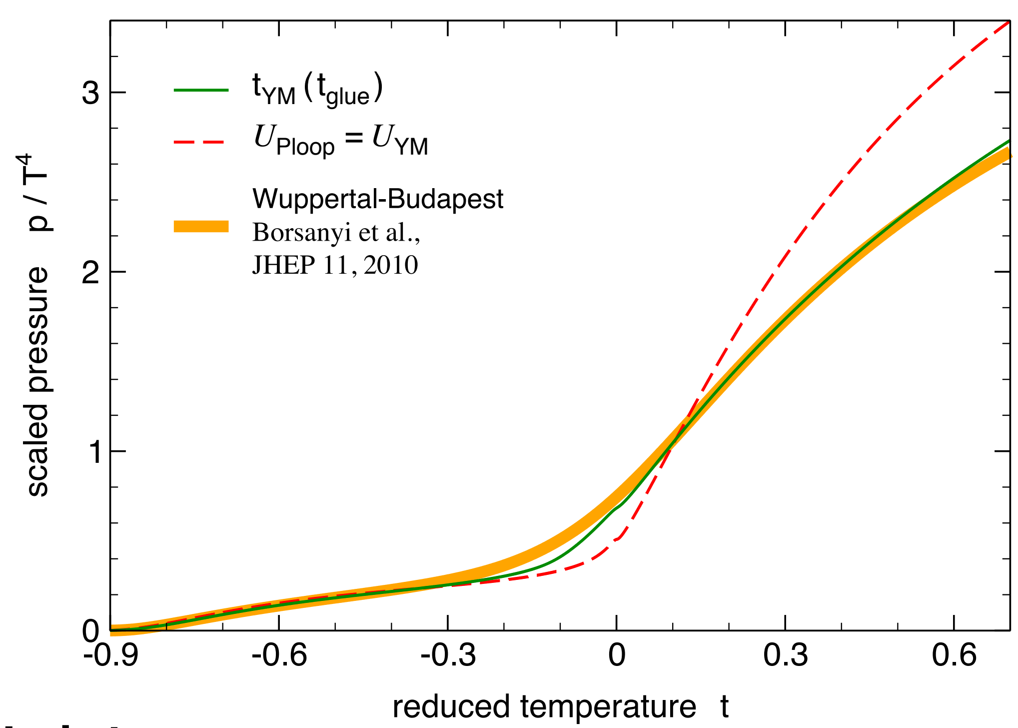
Braun, JMP, Gies '07





mean field

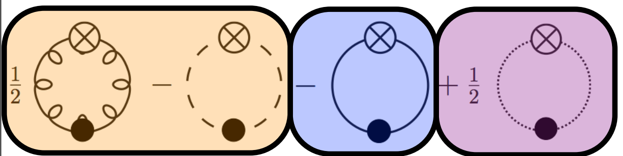




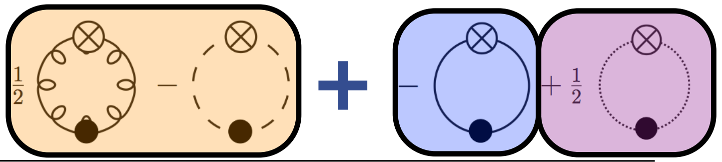
see also talk of T. Herbst

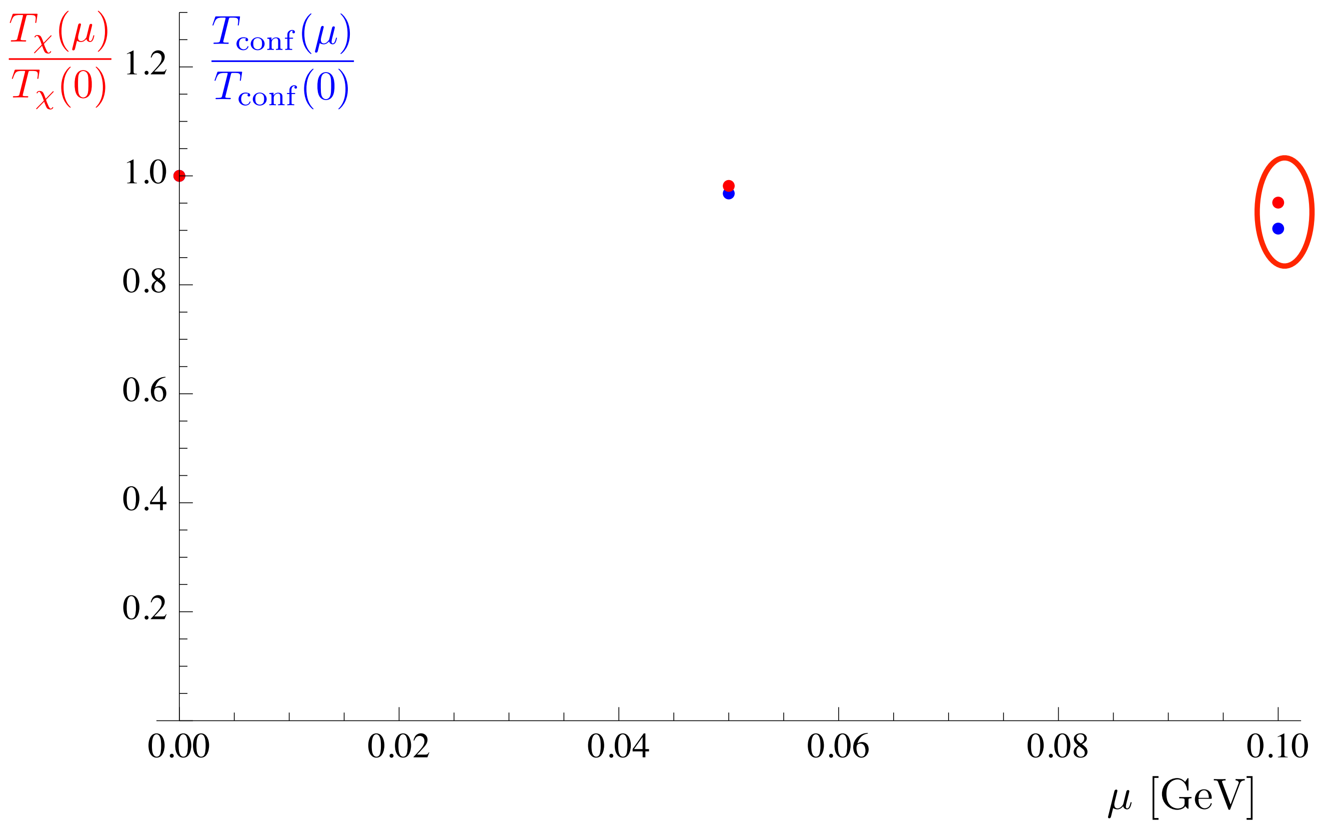
see also Fukushima, Kashiwa '12

Chemical potential

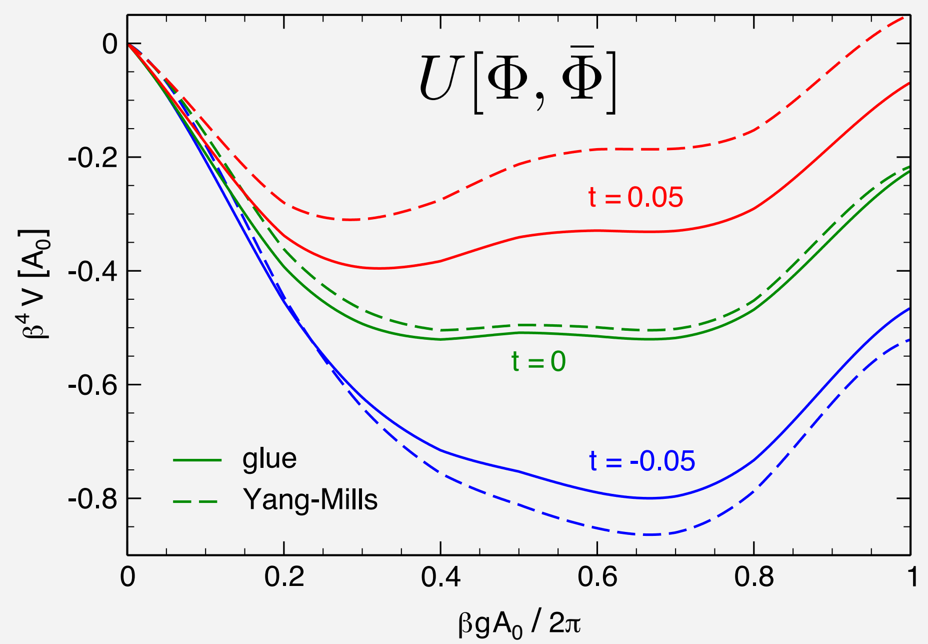


Full dynamical QCD

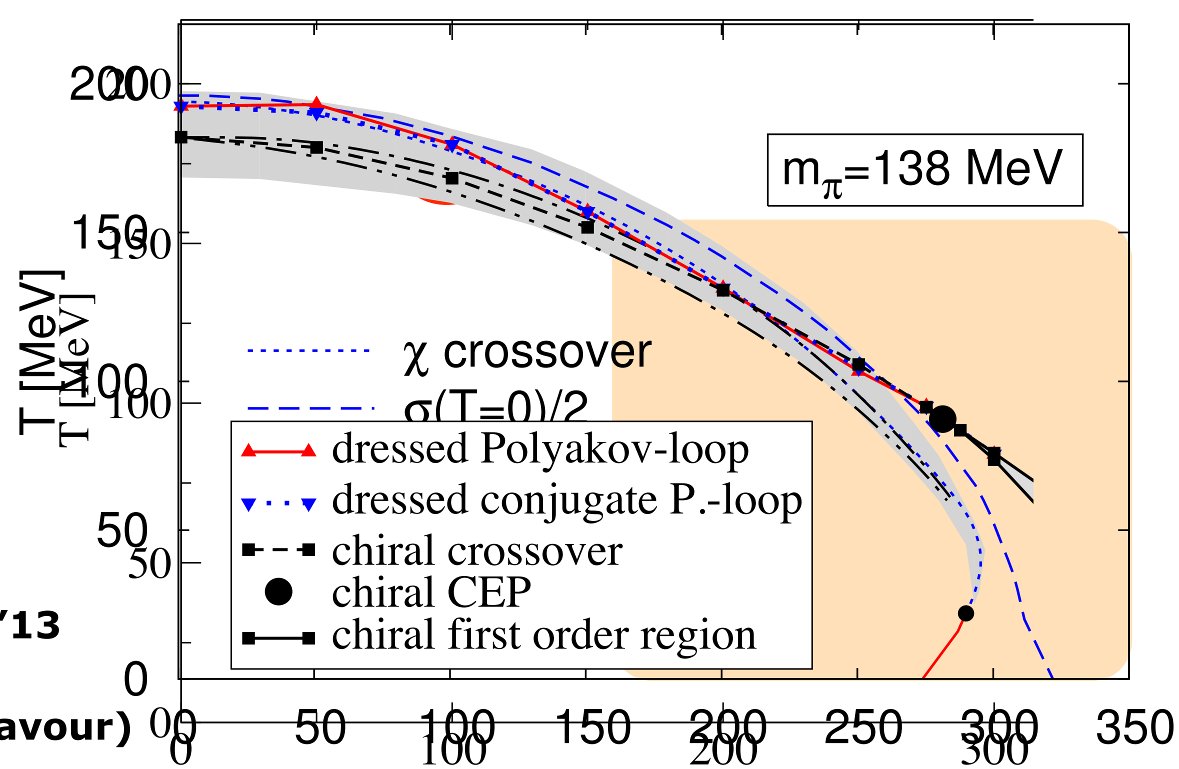




Polyakov loop potential in full QCD



Phase diagram of quantised PQM-model & DSE



Critical point unlikely for

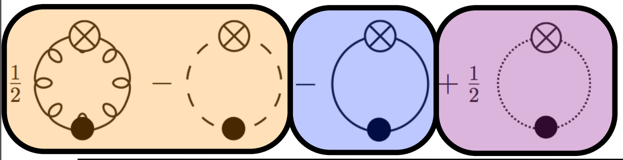
$$\frac{\mu_B}{T} < 2$$

PQM: Herbst, JMP, Schaefer '10, '13

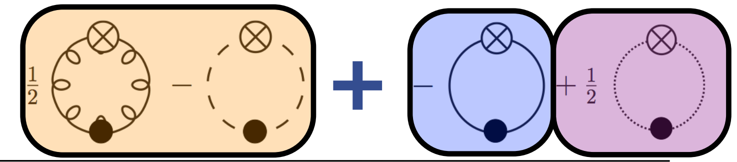
DSE: Fischer, Lücker, Mueller '11

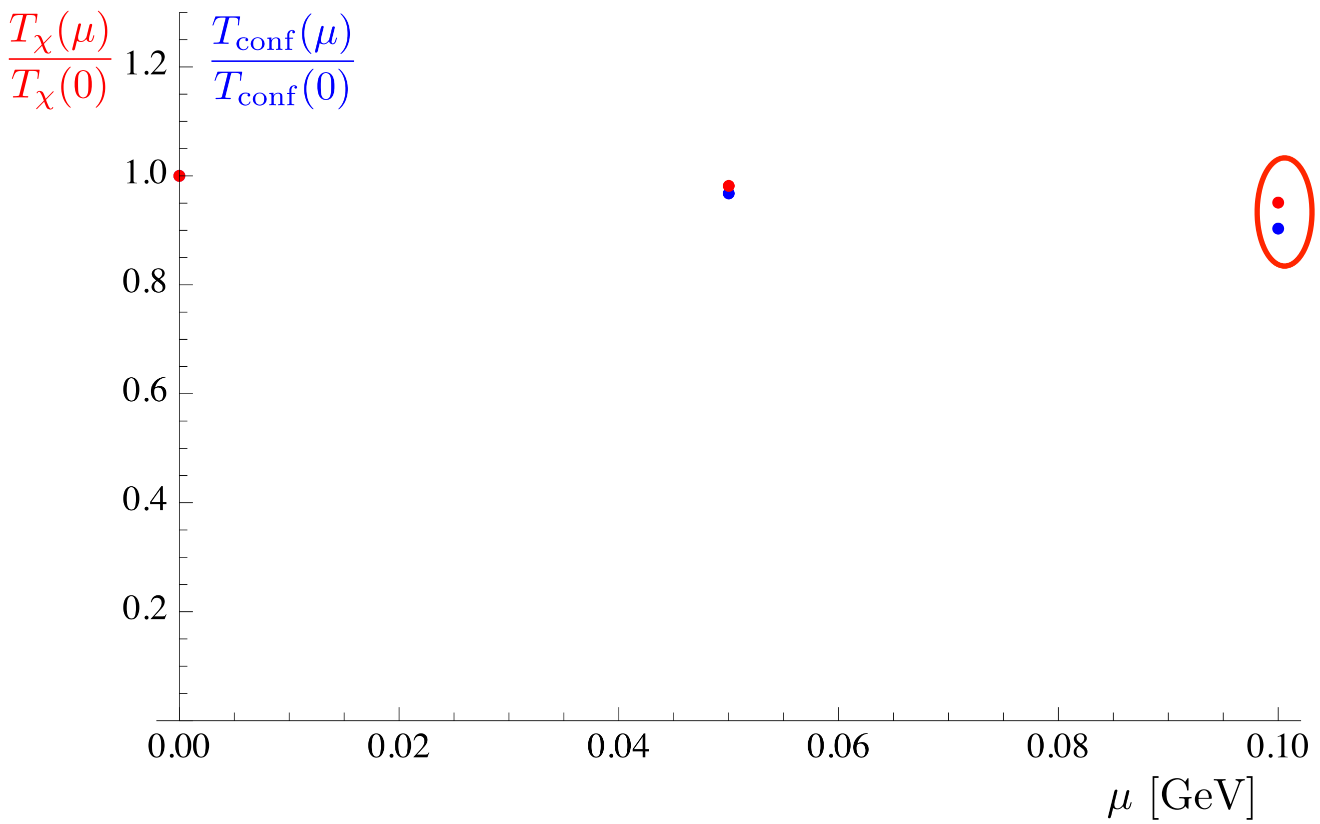
Fischer, Lücker '12 (2+1 flavour)

Chemical potential

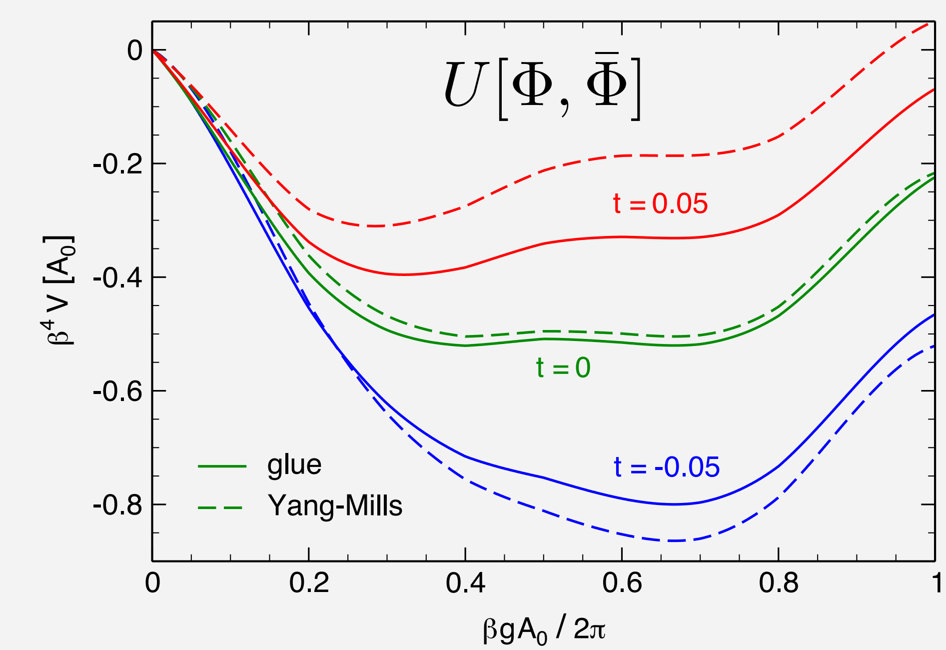


Full dynamical QCD

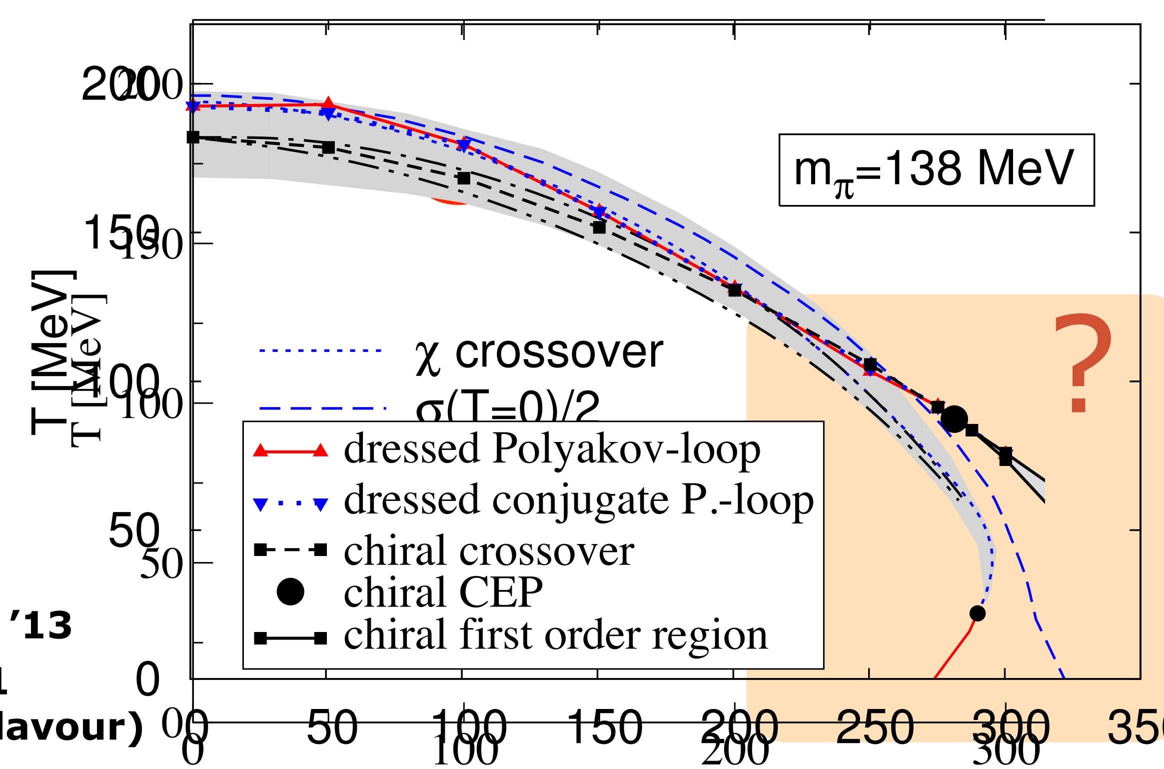




Polyakov loop potential in full QCD



Phase diagram of quantised PQM-model & DSE

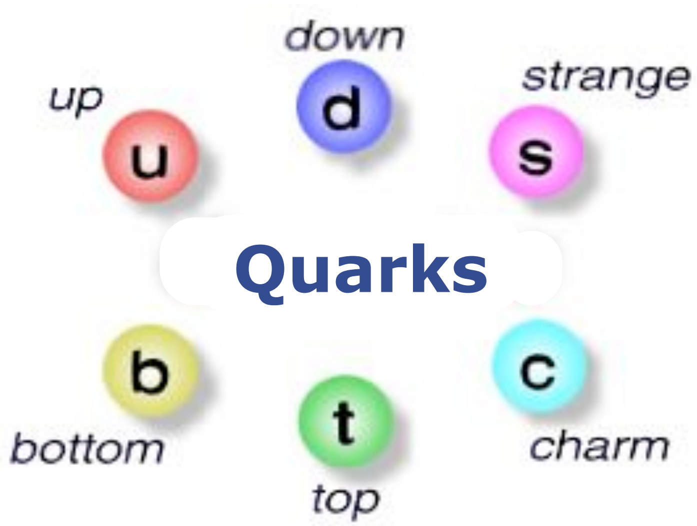


Critical point unlikely for

$$\frac{\mu_B}{T} < 4.5$$

PQM: Herbst, JMP, Schaefer '10, '13
DSE: Fischer, Lücker, Mueller '11
Fischer, Lücker '12 (2+1 flavour)

Technical report



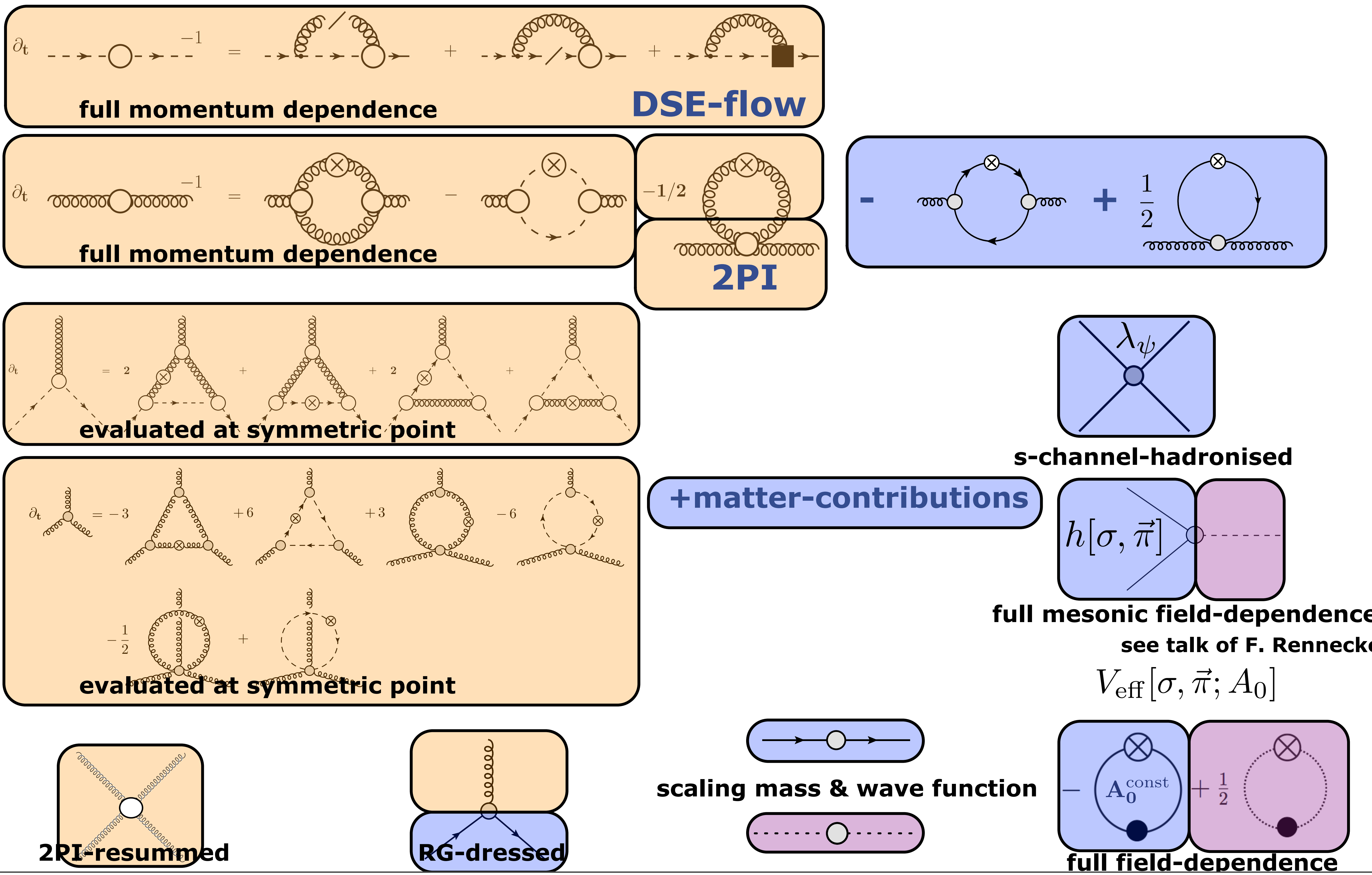
Gluons

[FRG QCD survey]()

Functional Methods for QCD

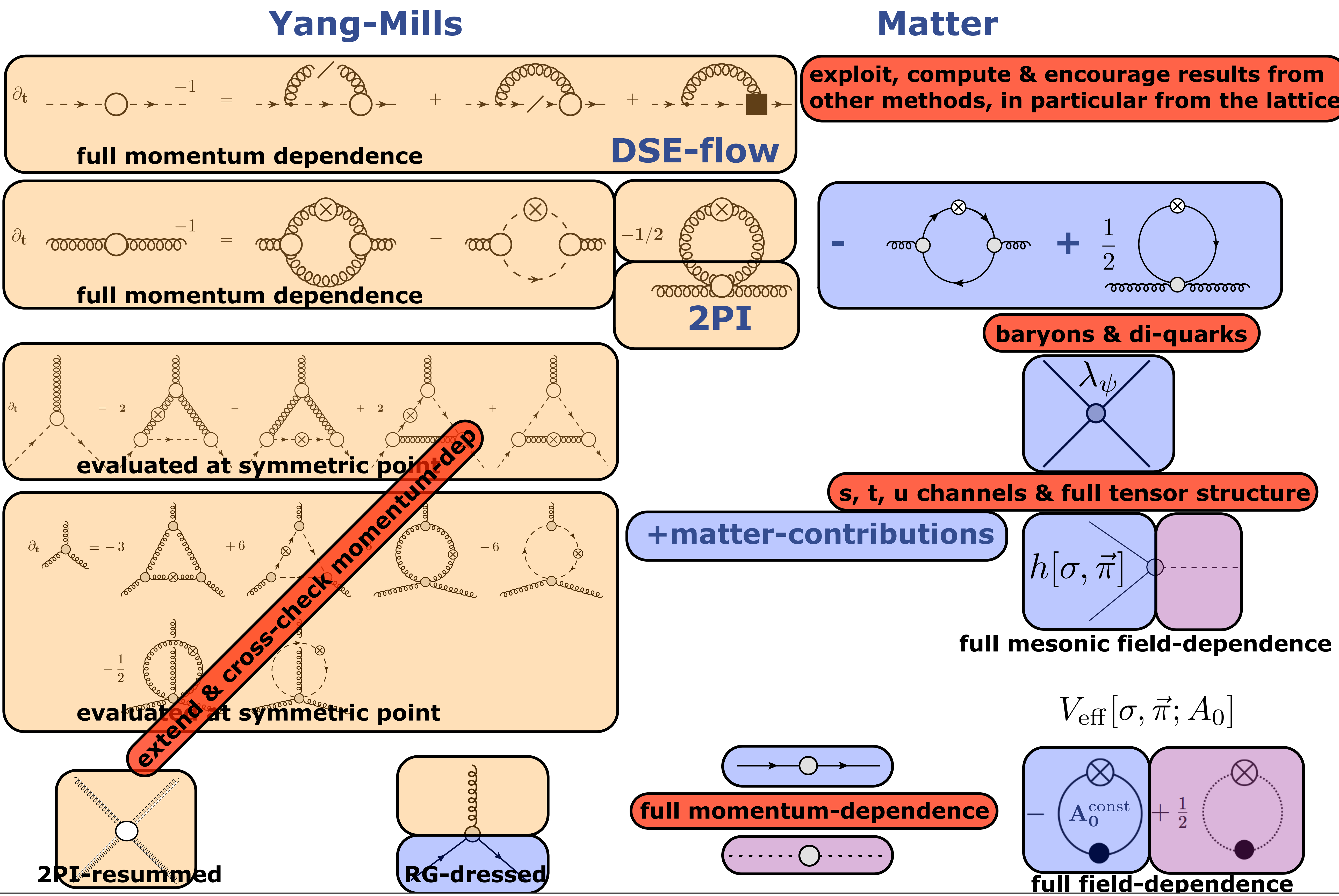
present approximation scheme

Yang-Mills   Matter



Functional Methods for QCD

present approximation scheme



(IV) Dynamics

- **Turbulence in gauge theories**
  - **Abelian Higgs model & beyond**

- **Transport in YM & QCD**
  - **Spectral functions**
  - **transport coefficients**

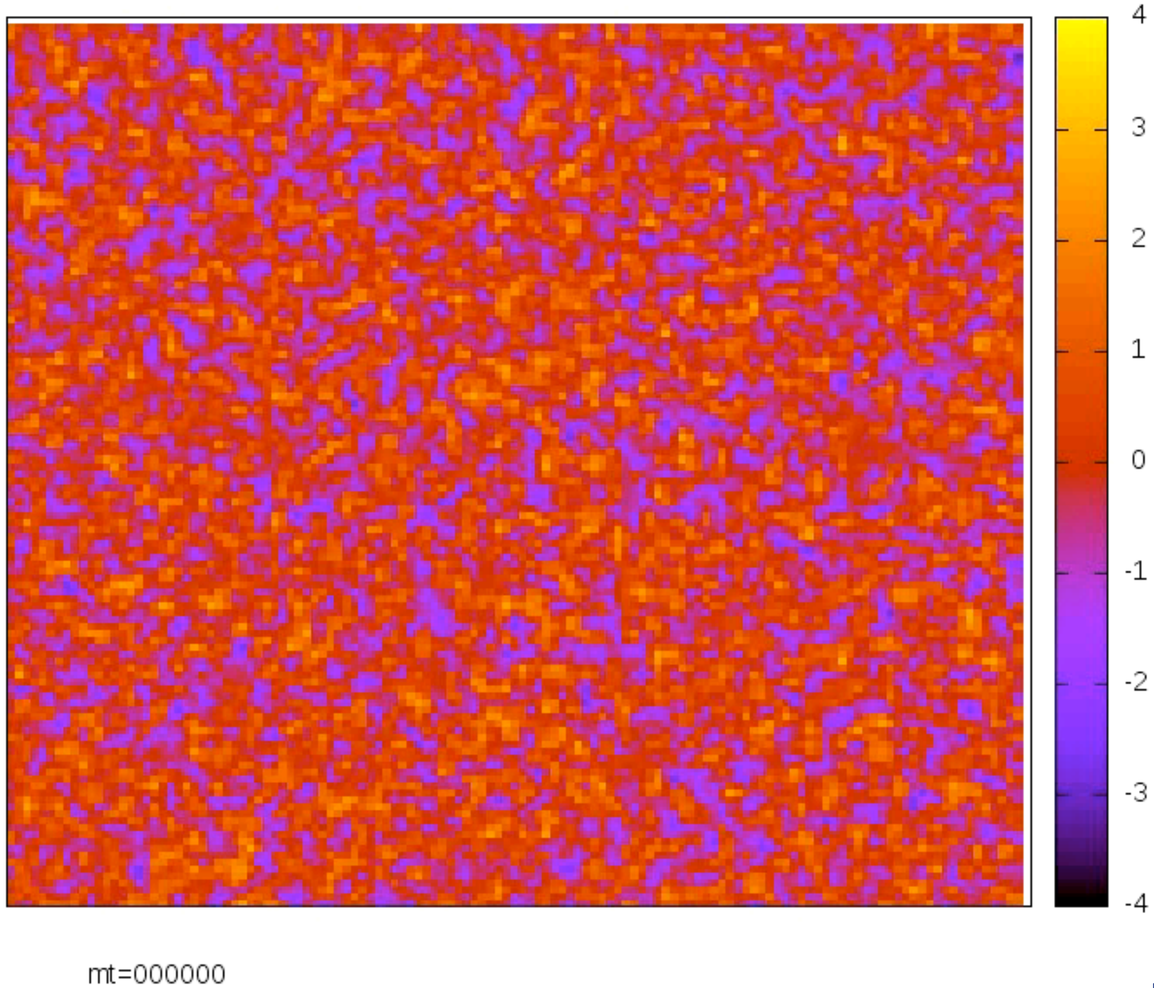
Non-equilibrium dynamics in QCD

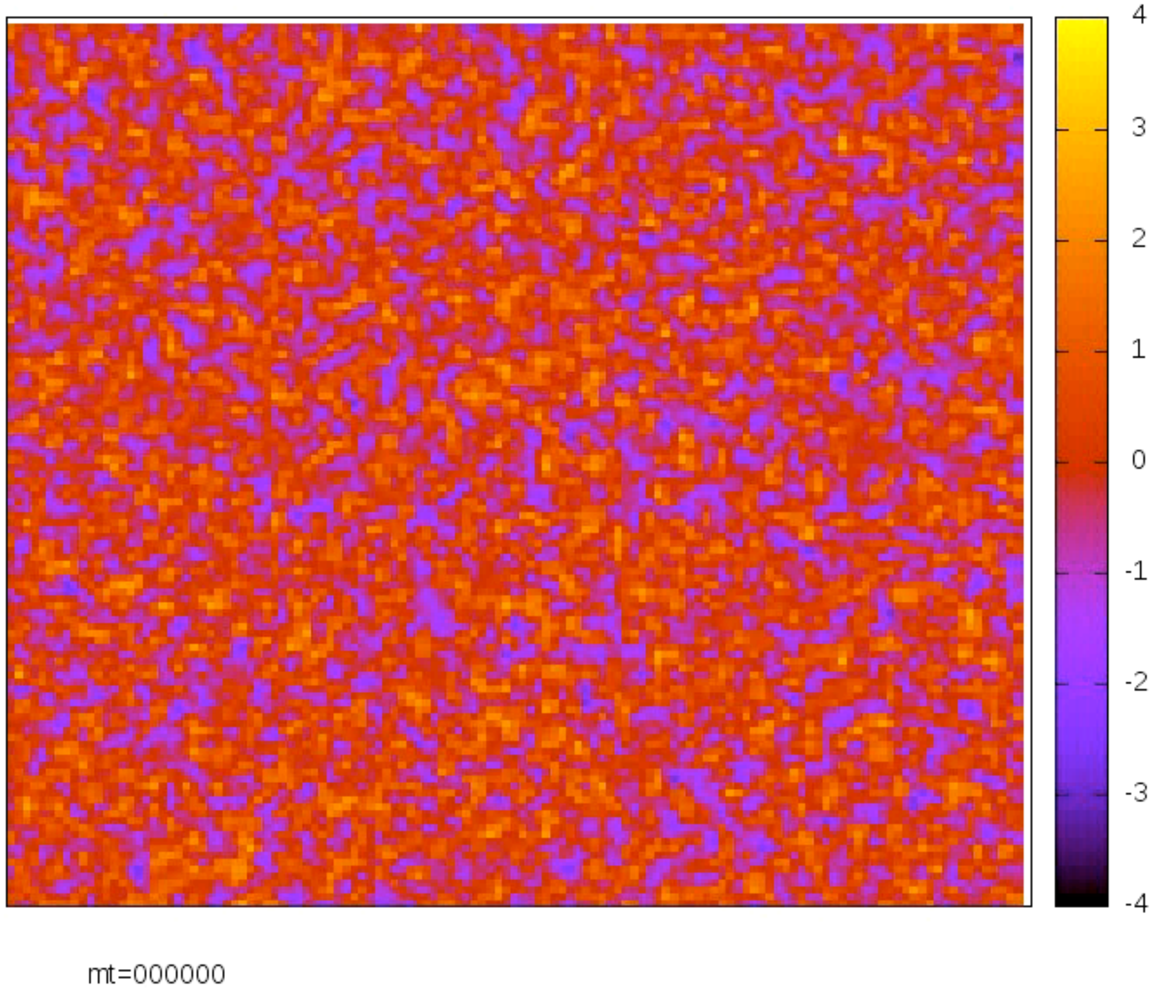
Gauge dynamics far from equilibrium

Quiz

Complex scalar vs Abelian Higgs

phase of scalar field





2+1 dim

Which is which?

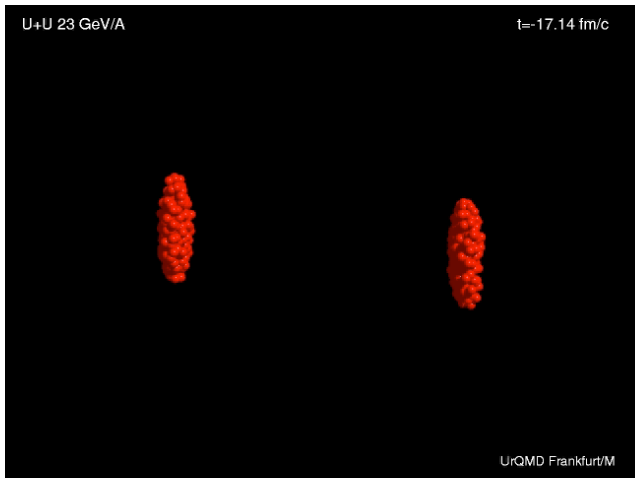
Gasenzer, McLerran, JMP, Sexty, in prep

Heavy ion collisions

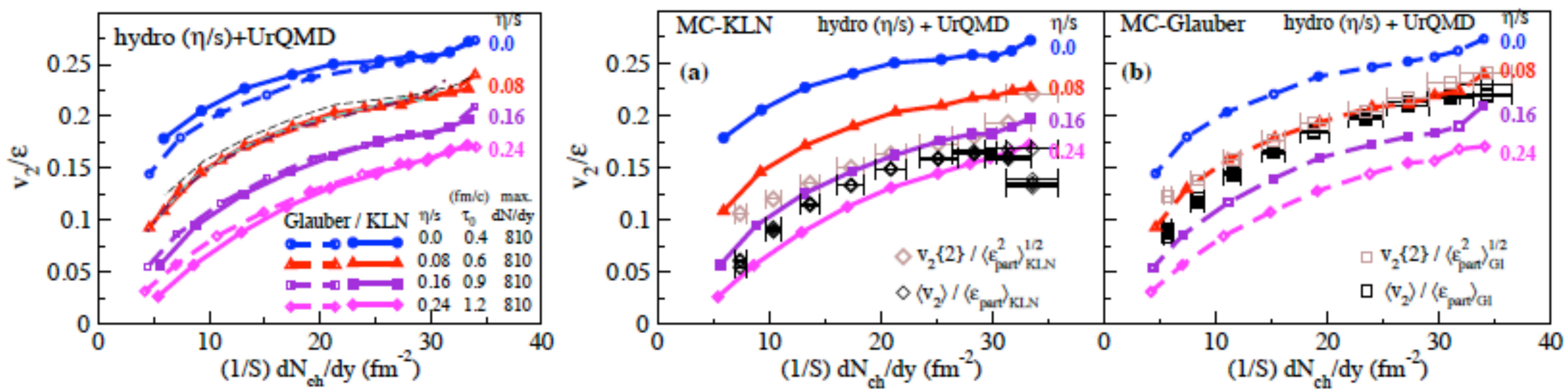
Far from equilibrium & hydrodynamics

Extraction of $(\eta/s)_{QGP}$ from AuAu@RHIC

H. Song, S.A. Bass, U. Heinz, T. Hirano, C. Shen, PRL106 (2011) 192301



UrQMD Frankfurt/M



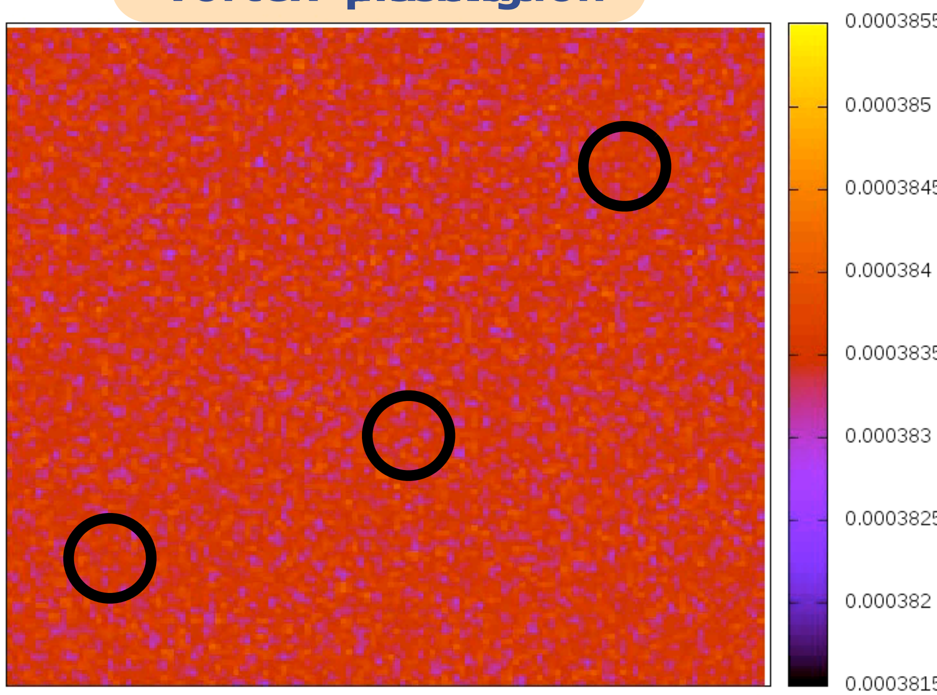
$1 < 4\pi(\eta/s)_{QGP} < 2.5$

U. Heinz, talk at RETUNE '12

Abelian Higgs

vortex pinning dissolution

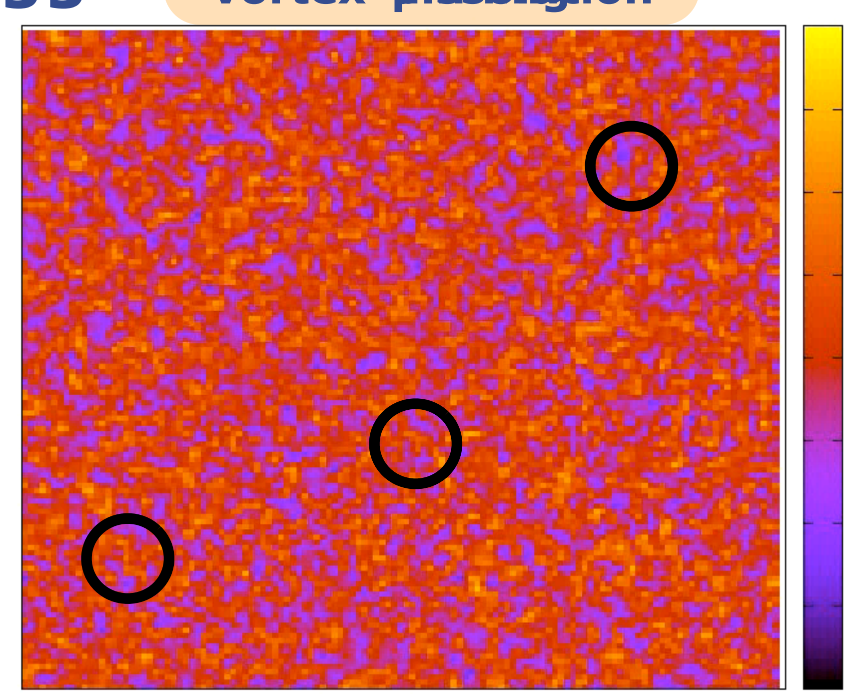
magnetic field



mt=000000

vortex pinning dissolution

phase of Higgs



mt=000000

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Heavy ion collisions

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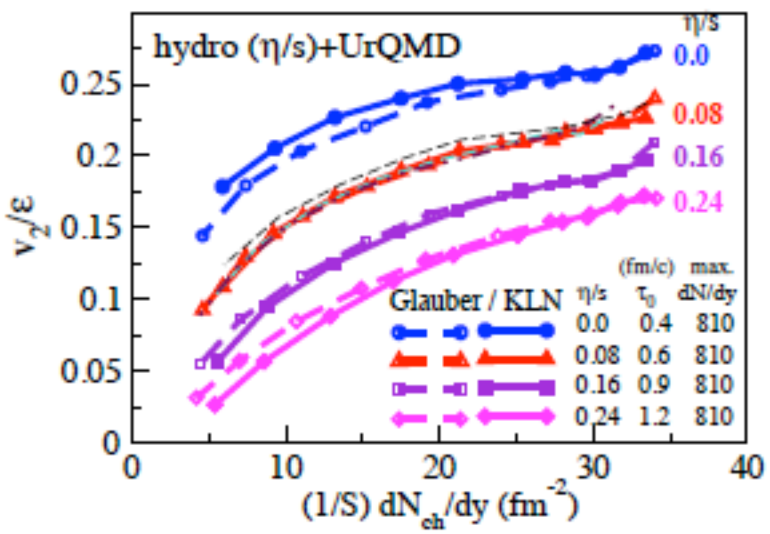
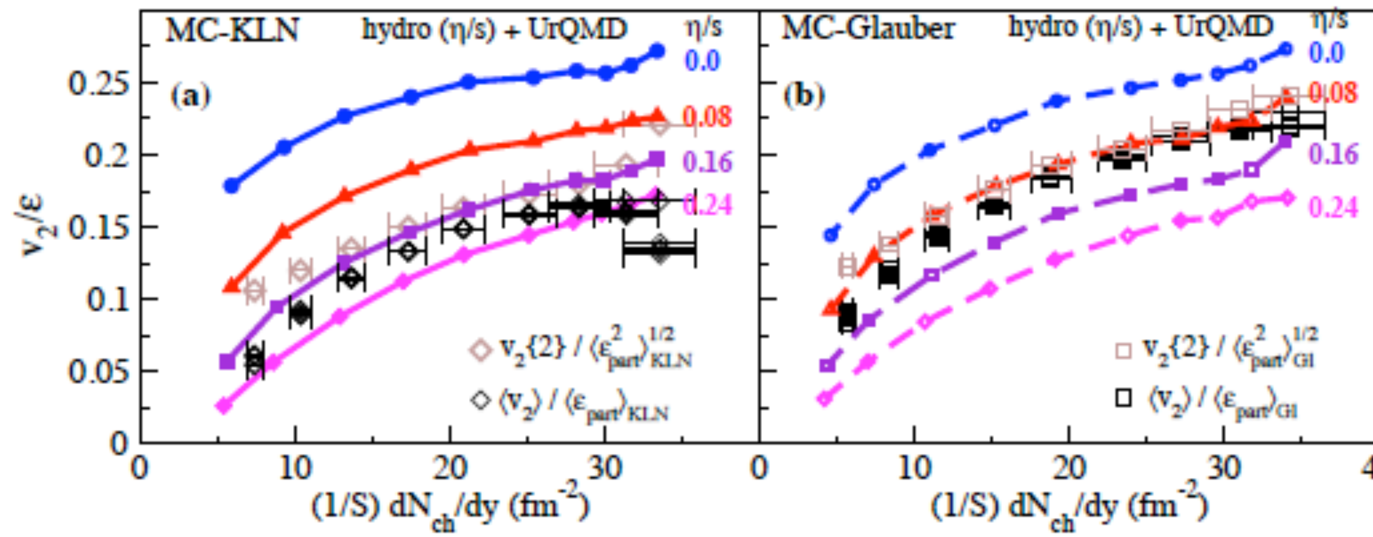
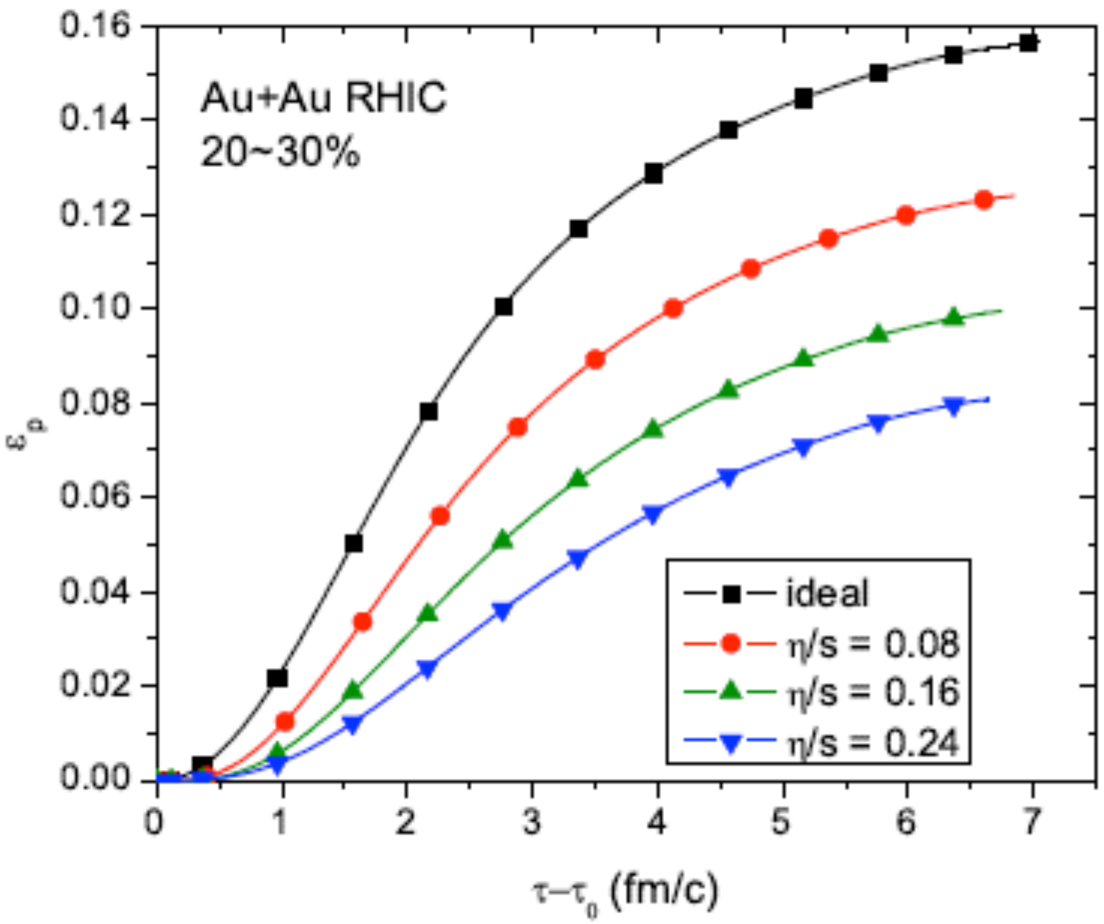




$$1 < 4\pi(\eta/s)_{\mathrm{QGP}} < 2.5$$



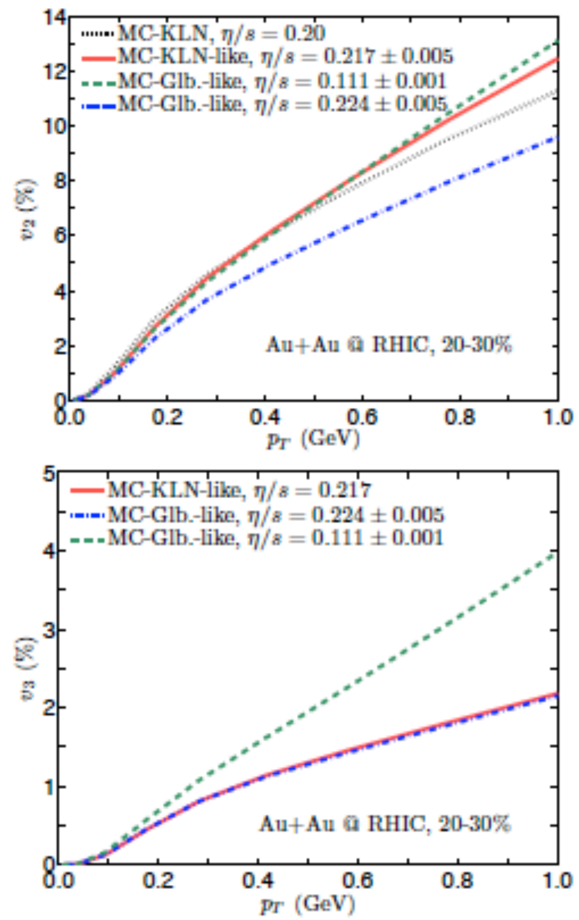
U. Heinz, talk at RETUNE '12

Heavy ion collisions

Shooting the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published



- Take ensemble of sum of deformed Gaussian profiles, $s(\boldsymbol{r}_\perp) = s_2(\boldsymbol{r}_\perp; \bar{\varepsilon}_2, \psi_2) + s_3(\boldsymbol{r}_\perp; \bar{\varepsilon}_3, \psi_3)$, with
  1. equal Gaussian radii $R_2^2 = R_3^2 = 8\,\mathrm{fm}^2$ to reproduce $\langle r_\perp^2 \rangle$ of MC-KLN source for 20-30% AuAu
  2. $\bar{\varepsilon}_2$ and $\bar{\varepsilon}_3$ adjusted such that
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{KLN}}^{20-30\%}$ ("MC-KLN-like")
     - $\bar{\varepsilon}_{2,3} = \langle \varepsilon_{2,3} \rangle_{\mathrm{Gl}}^{20-30\%}$ ("MC-Glauber-like")
  3. $\psi_2 = 0$, ψ_3 (direction of triangularity) distributed randomly
- Use $v_2^\pi(p_T)$ from VISH2+1 for $\eta/s = 0.20$ with MC-KLN initial conditions for 20-30% AuAu as "mock data"
- Fit mock $v_2^\pi(p_T)$ data with VISH2+1 for "MC-Glauber-like" or "MC-KLN-like" Gaussian initial conditions with both elliptic and triangular deformations by adjusting η/s
  $\Longrightarrow (\eta/s)_{\mathrm{KLN}} = 0.217 \pm 0.005$ for "MC-KLN-like",
  $(\eta/s)_{\mathrm{Gl}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\mathrm{Gl}}$=0.217 and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
  $\Longrightarrow (\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data $\Longrightarrow$ clearly visible (and measurable) difference!

This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields the same η/s (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and only one (more precisely: at most one!) of the models will fit the corresponding $v_2(p_T)$ data.

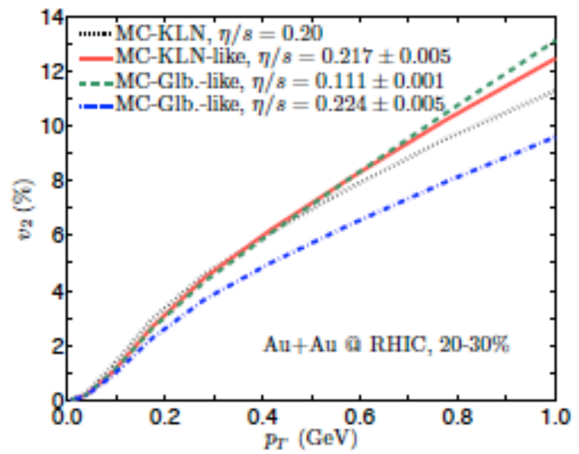
U. Heinz, talk at RETUNE '12

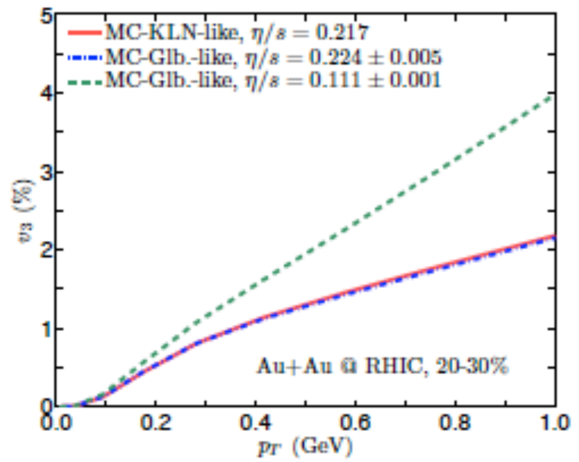
Heavy ion collisions

Computing the elephant

Proof of principle calculation:

Zhi Qiu and U. Heinz, to be published





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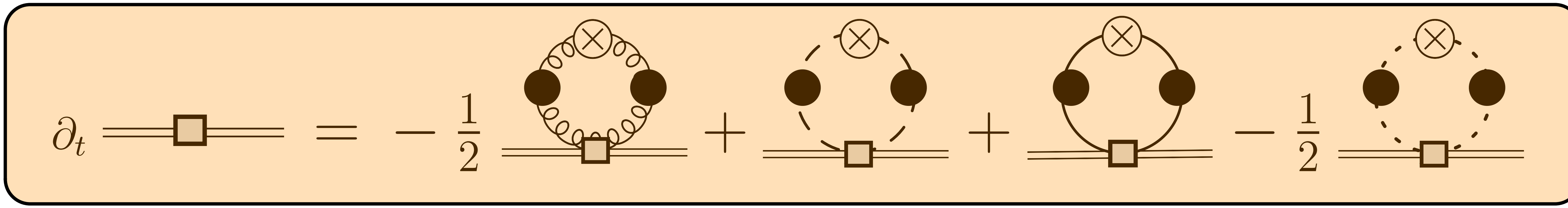
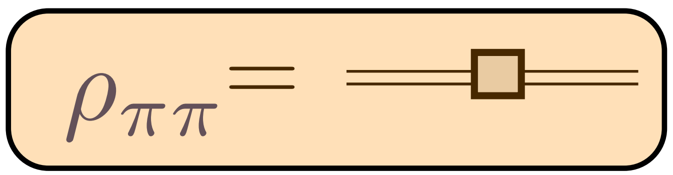
  $(\eta/s)_{\mathrm{Gl}} = 0.111 \pm 0.001$ for "MC-Glauber-like"
- Compute $v_3^\pi(p_T)$ for "MC-KLN-like" fit with $(\eta/s)_{\mathrm{Gl}}$=0.217 and reproduce it with "MC-Glauber-like" initial condition by readjusting η/s
  $\Longrightarrow$ $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224 \pm 0.005$ for "MC-Glauber-like"
- Compute $v_2^\pi(p_T)$ for "MC-Glauber-like" initial profiles with readjusted $(\eta/s)_{\mathrm{Gl}}^{v_3} = 0.224$ and compare with "MC-Glauber-like" fit to original mock data $\Longrightarrow$ clearly visible (and measurable) difference!

This exercise proves: (i) Fitting $v_3(p_T)$ data with MC-Glauber and MC-KLN initial conditions yields the same η/s (within narrow error band); (ii) The corresponding $v_2(p_T)$ fits are quite different, and only one (more precisely: at most one!) of the models will fit the corresponding $v_2(p_T)$ data.

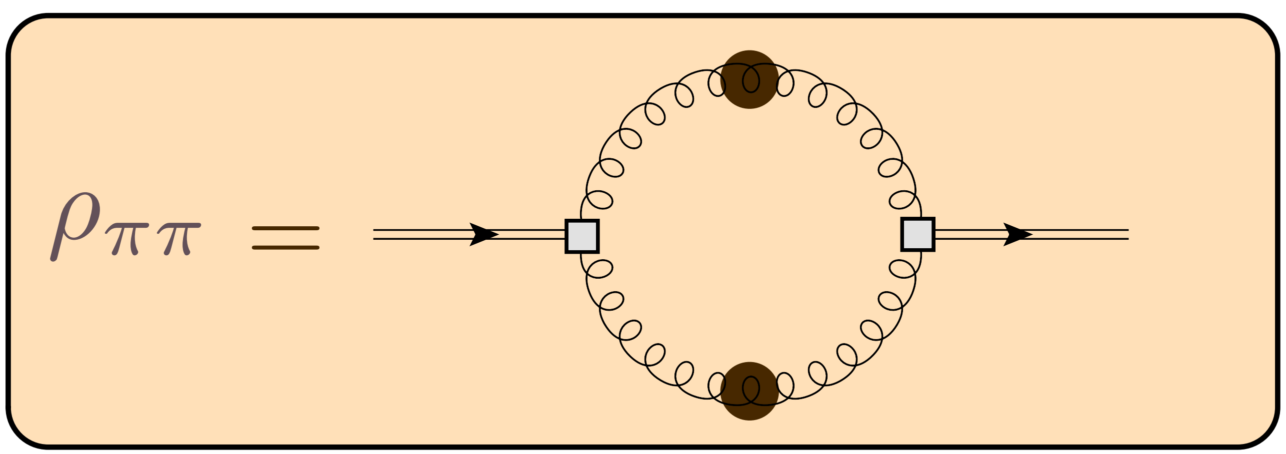
Transport in QCD

correlations of energy-momentum tensor





current approximation



'Those are my methods (principles), and if you don't like them...well, I have others'

direct computation **Groucho Marx**

$\rho_{T/L}$ **with MEM**

$$\rho_{\pi\pi}(p) = \frac{2d_A}{3}\int \frac{d^4k}{(2\pi)^4}\left[n(k^0) - n(k^0+p_0)\right]\left(V_{TT}\rho_T(k)\rho_T(k+p) + V_{TL}\rho_T(k)\rho_L(k+p) + V_{LL}\rho_L(k)\rho_L(k+p)\right)$$

Viscosity in QCD

Shear viscosity

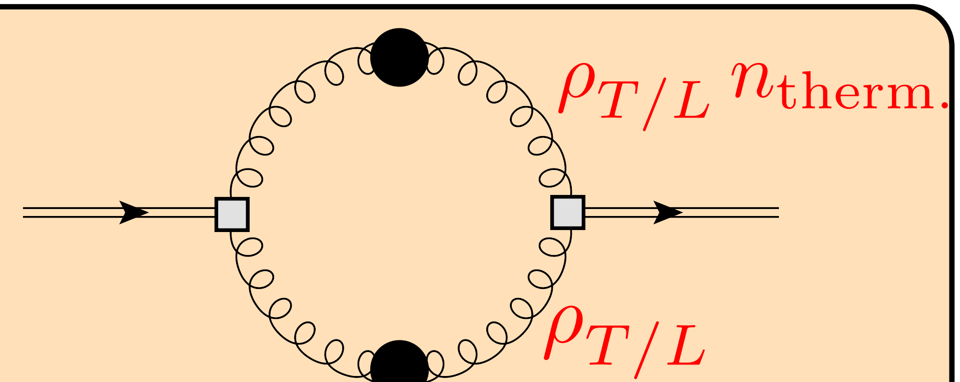
$$\eta = \frac{1}{20} \frac{d}{d\omega}\bigg|_{\omega=0} \rho_{\pi\pi}(\omega, 0)$$

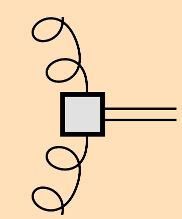
Kubo relation

see talk of M. Laine

current approximation

$\rho_{\pi\pi} =$





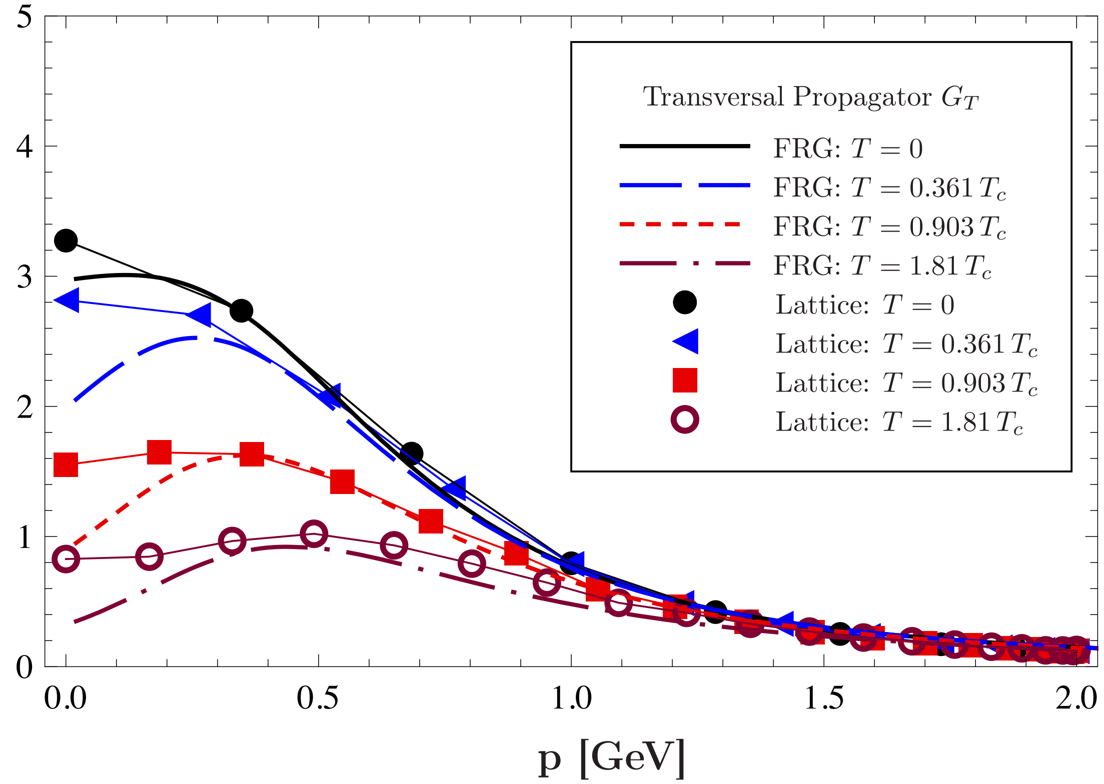
full vertex

$\rho_{T/L}$ **with MEM**

Viscosity in pure glue

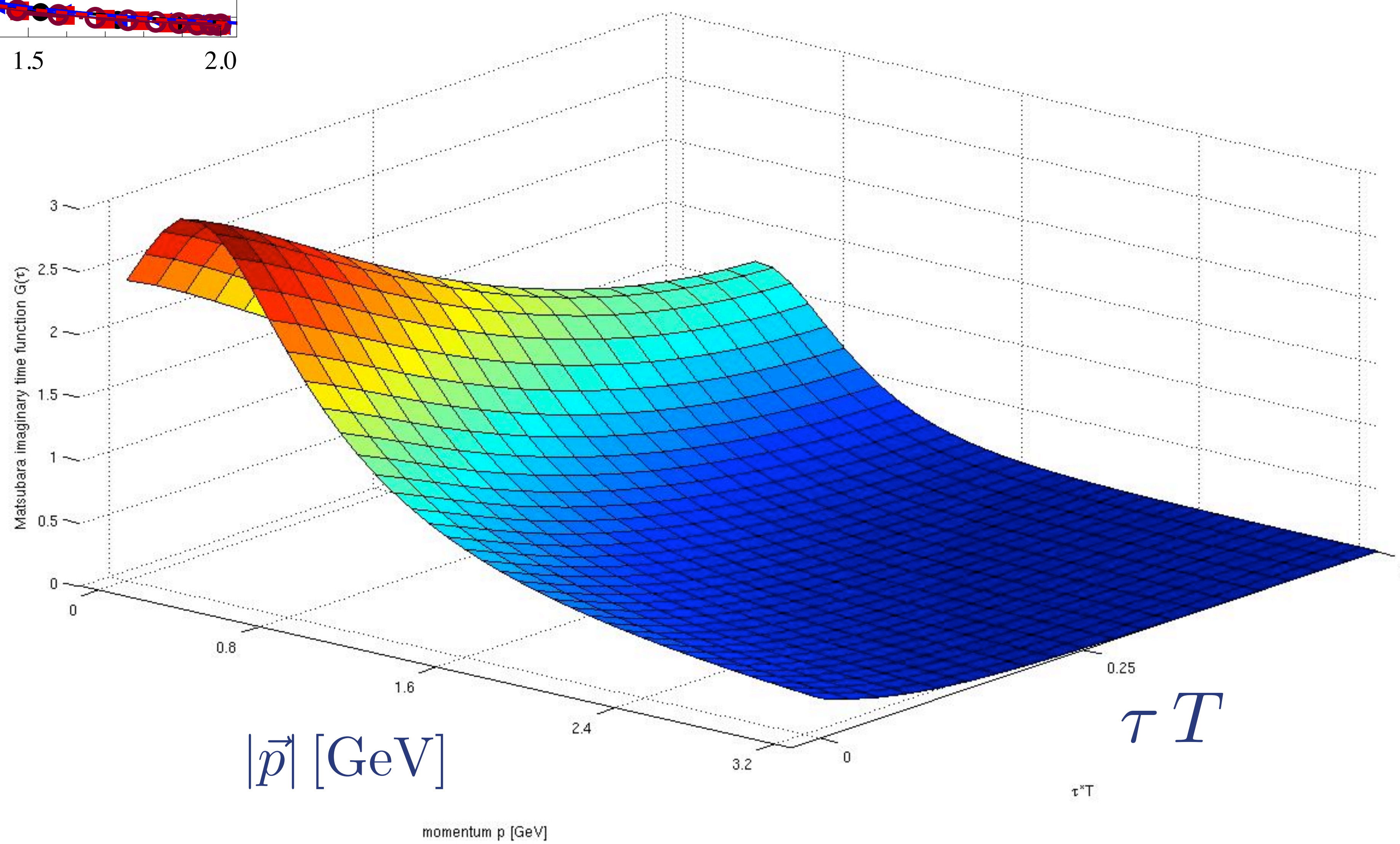
imaginary time correlations

Fister, M. Haas, JMP, in prep



transversal gluon propagator

$G_T(\tau, \vec{p})$



$|\vec{p}|$ [GeV]

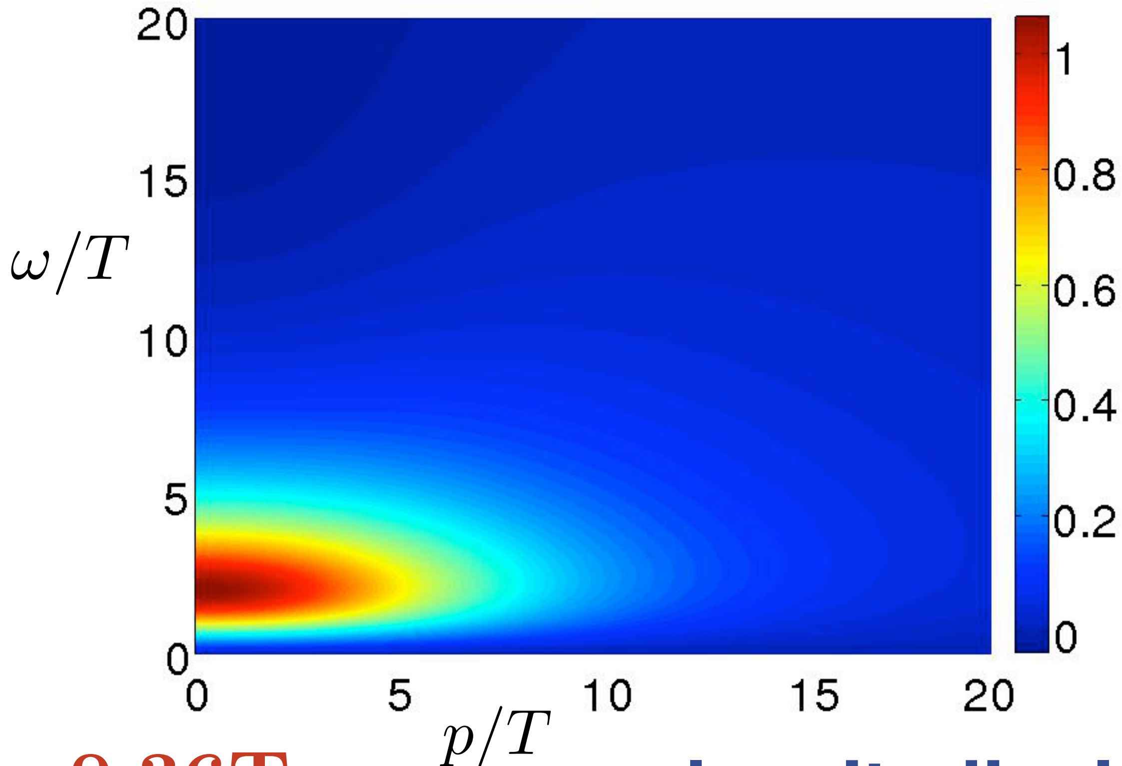
$\tau\, T$

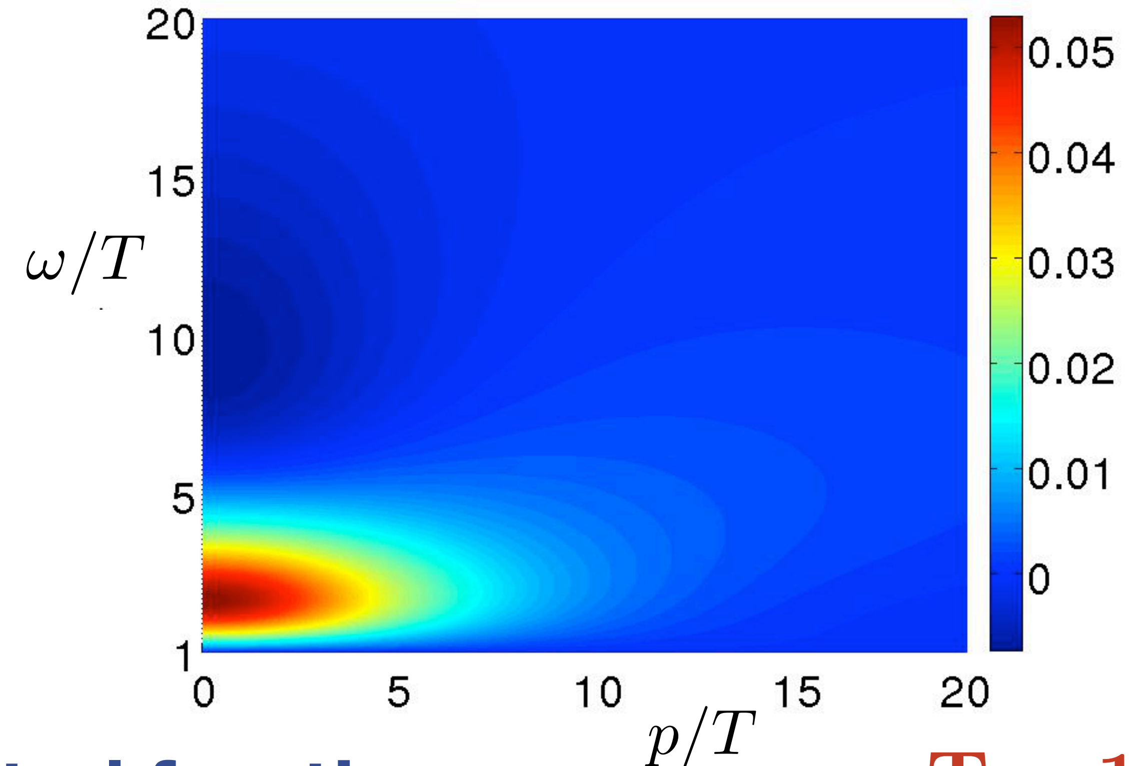
Viscosity in pure glue

spectral functions

transversal spectral functions

Fister, M. Haas, JMP, in prep

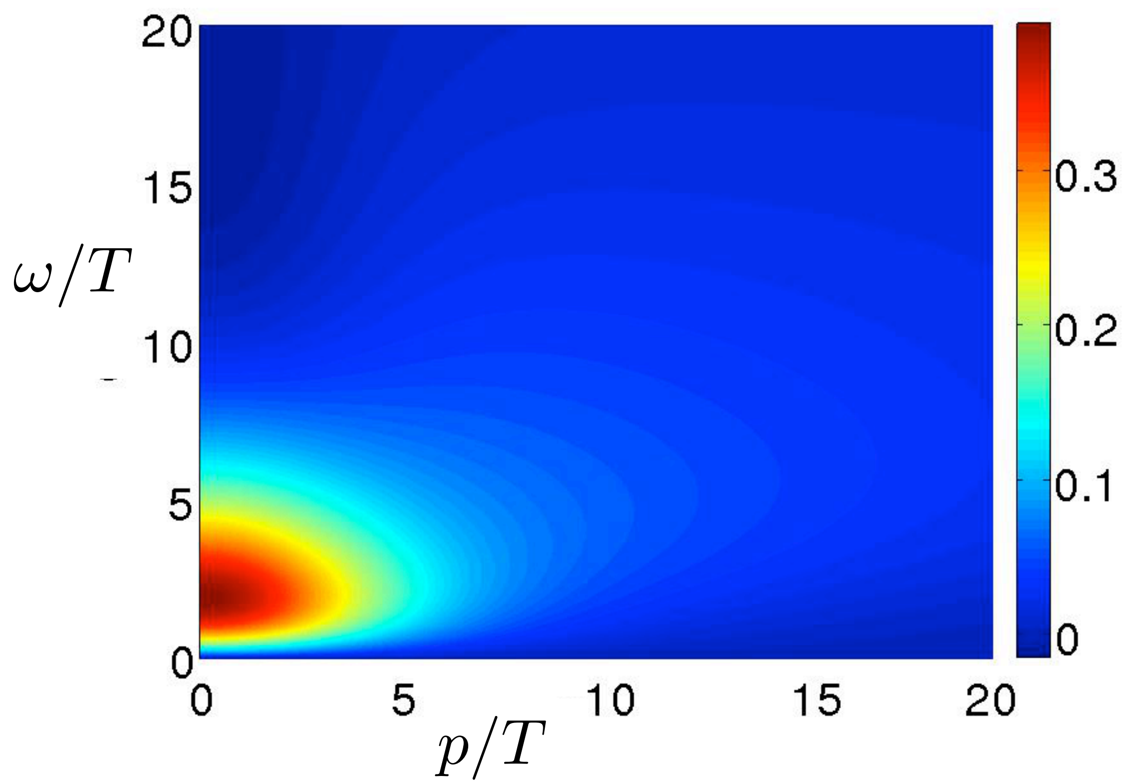


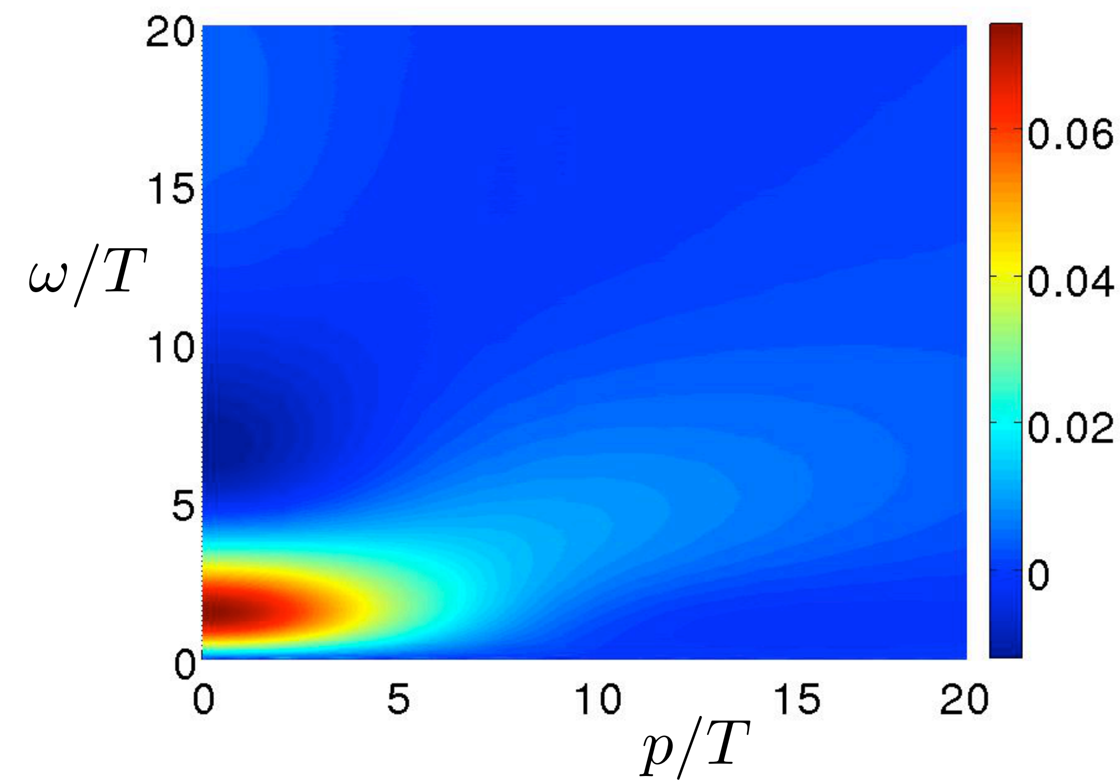


$\mathbf{T = 0.36 T_c}$

longitudinal spectral functions

$\mathbf{T = 1.8 T_c}$



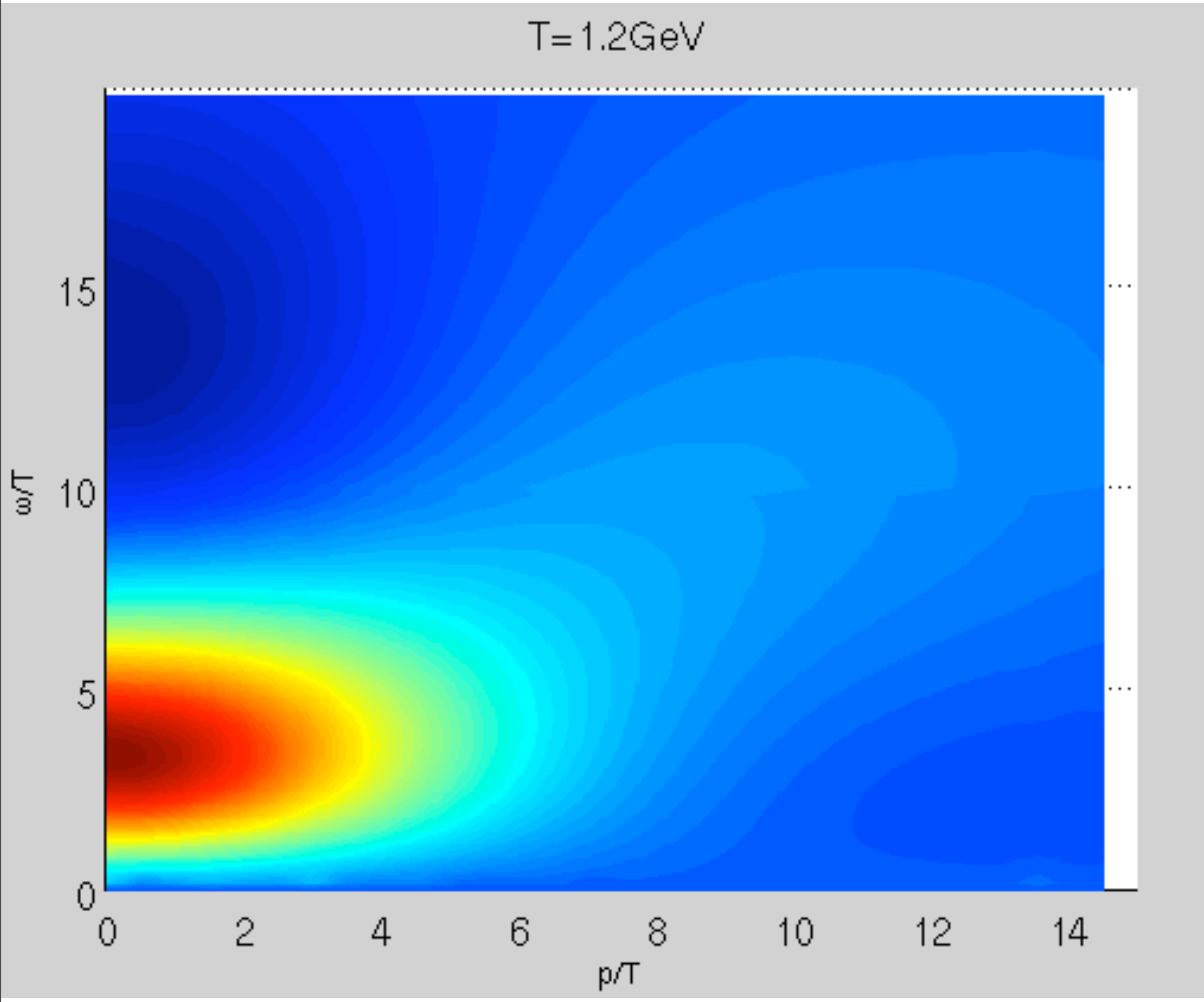


Viscosity in pure glue

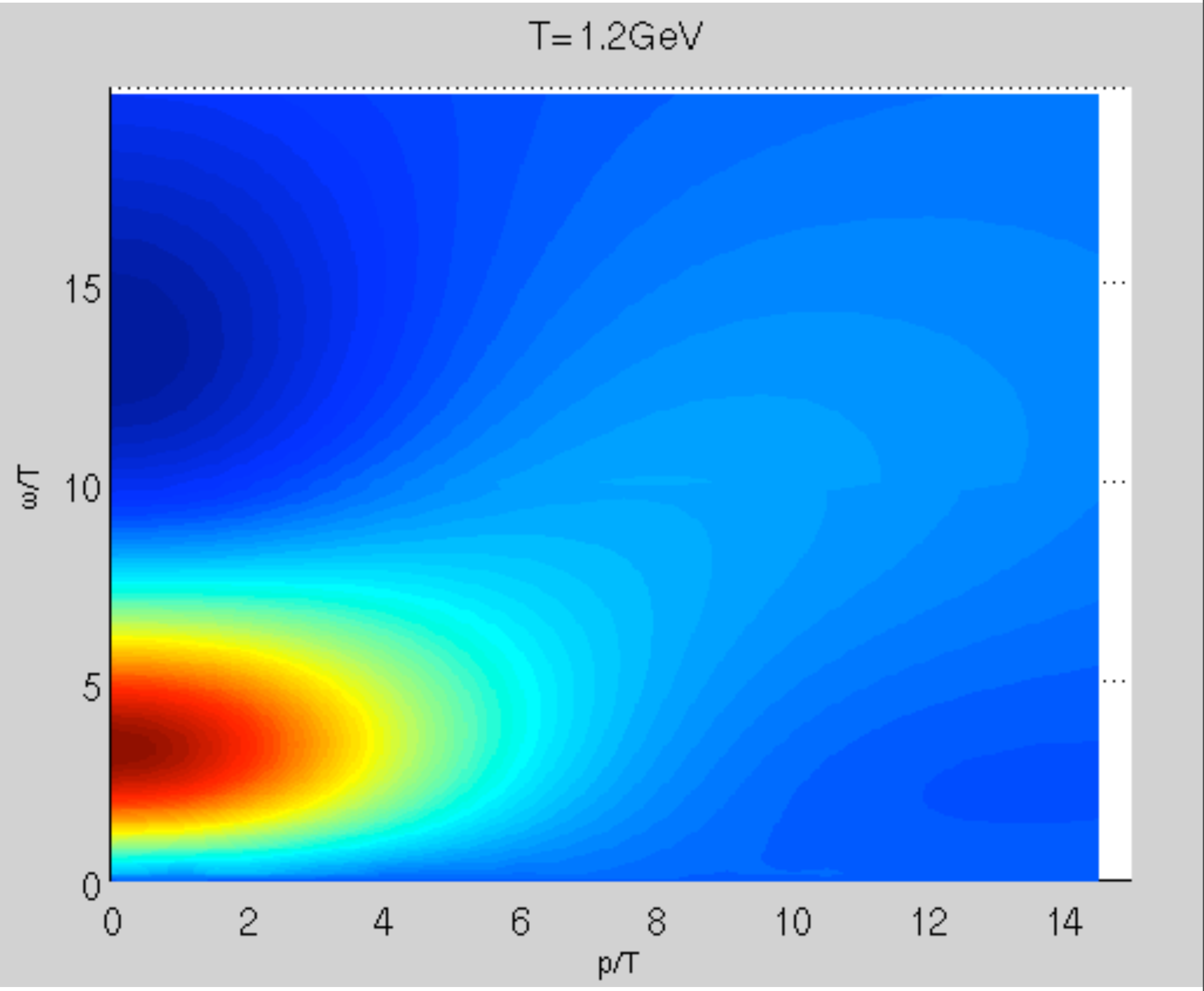
spectral functions

Fister, M. Haas, JMP, in prep

longitudinal spectral functions



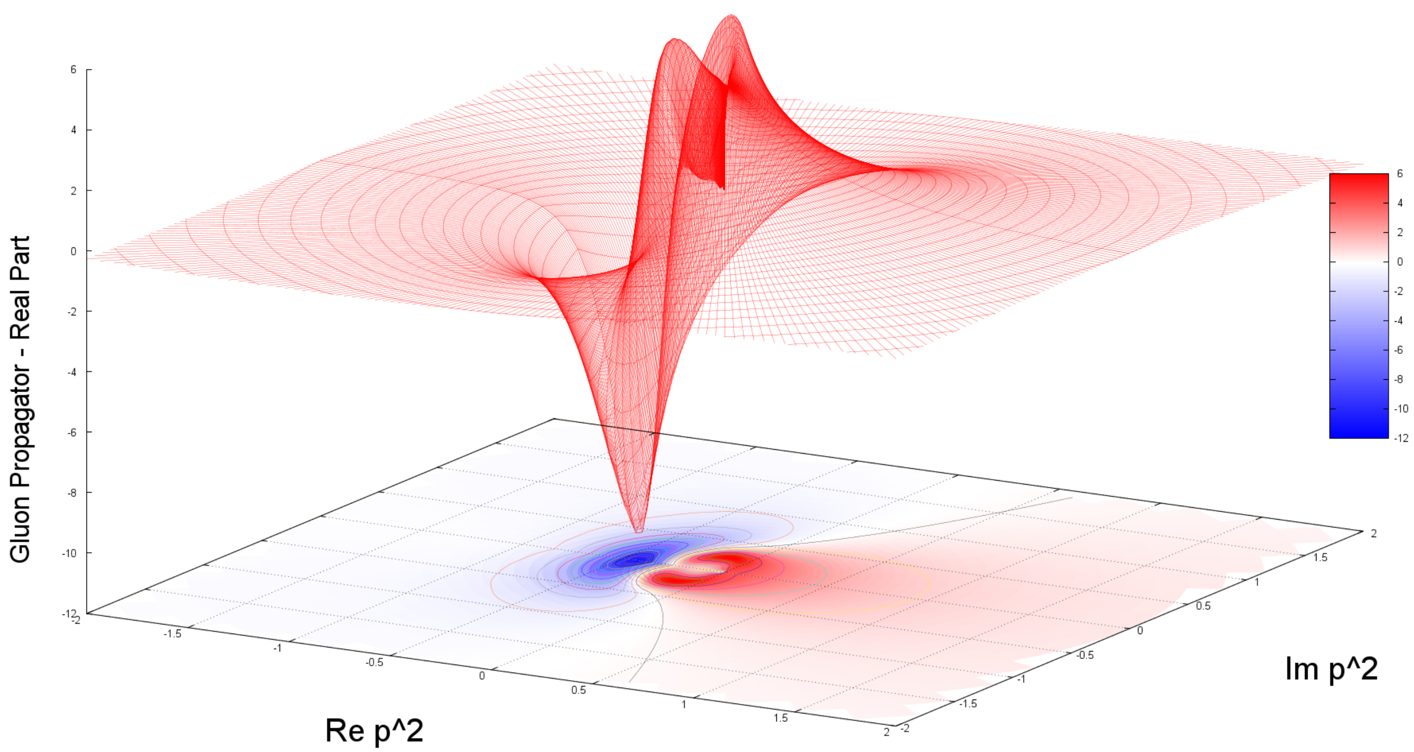
transversal spectral functions

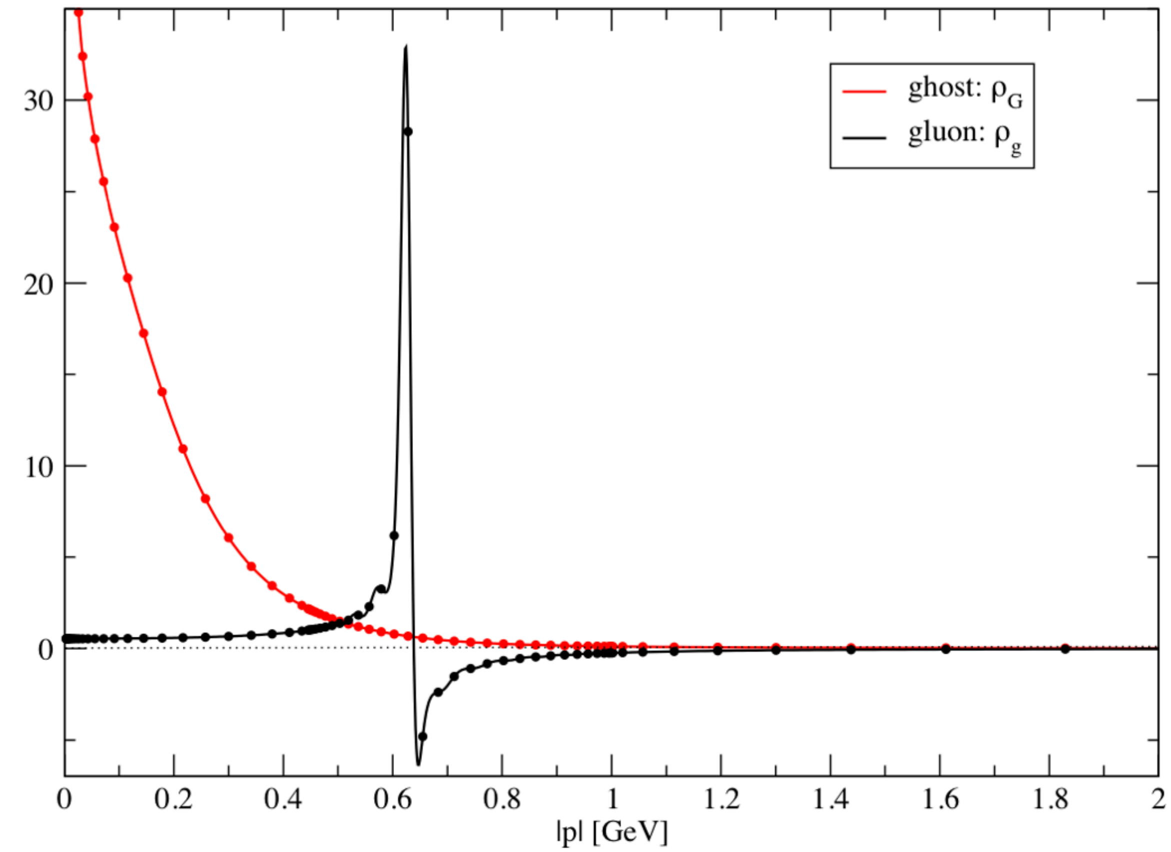


Viscosity in pure glue

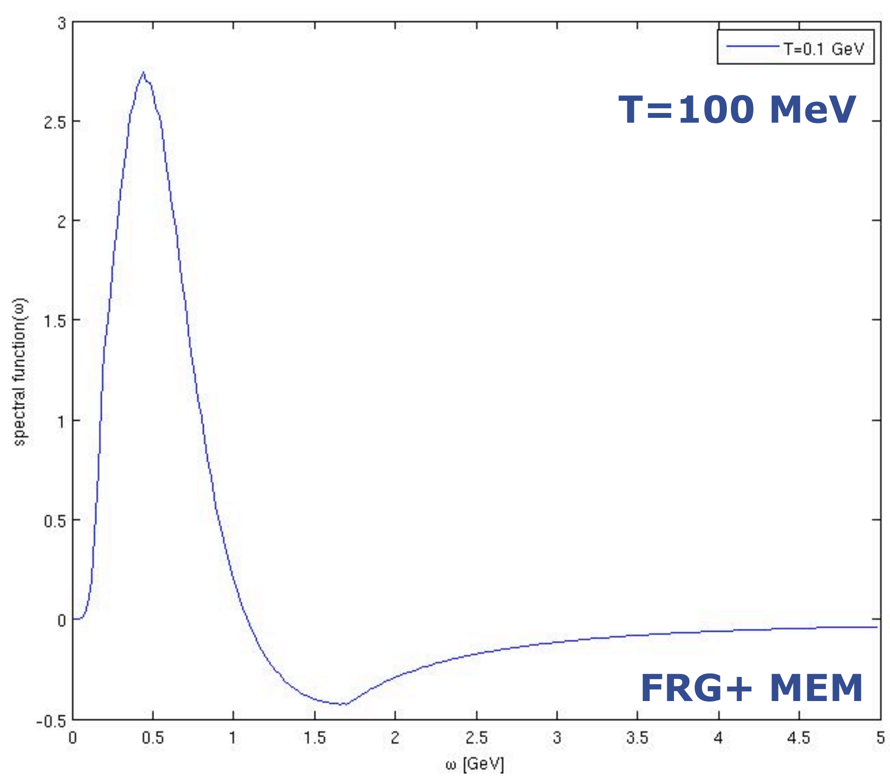
spectral functions

Complex DSEs





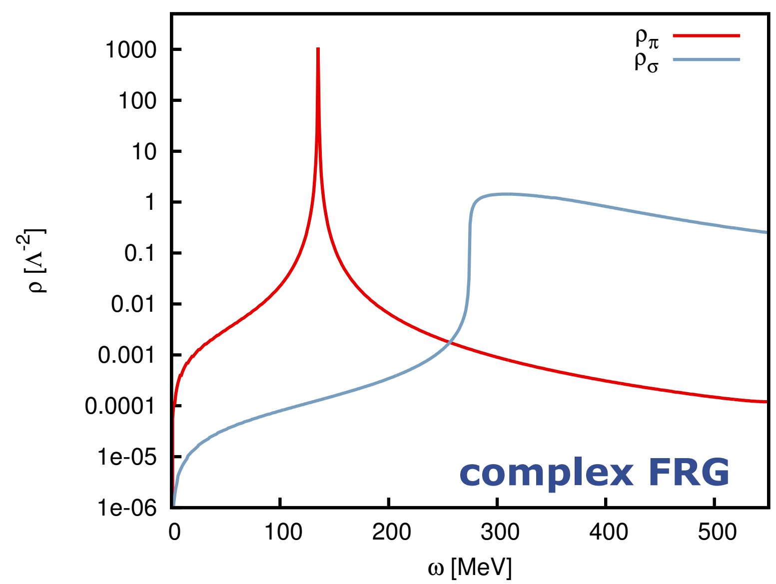
Strauss, Fischer, Kellermann '12



transversal spectral function

Fister, M. Haas, JMP, in prep

pion and sigma spectral functions

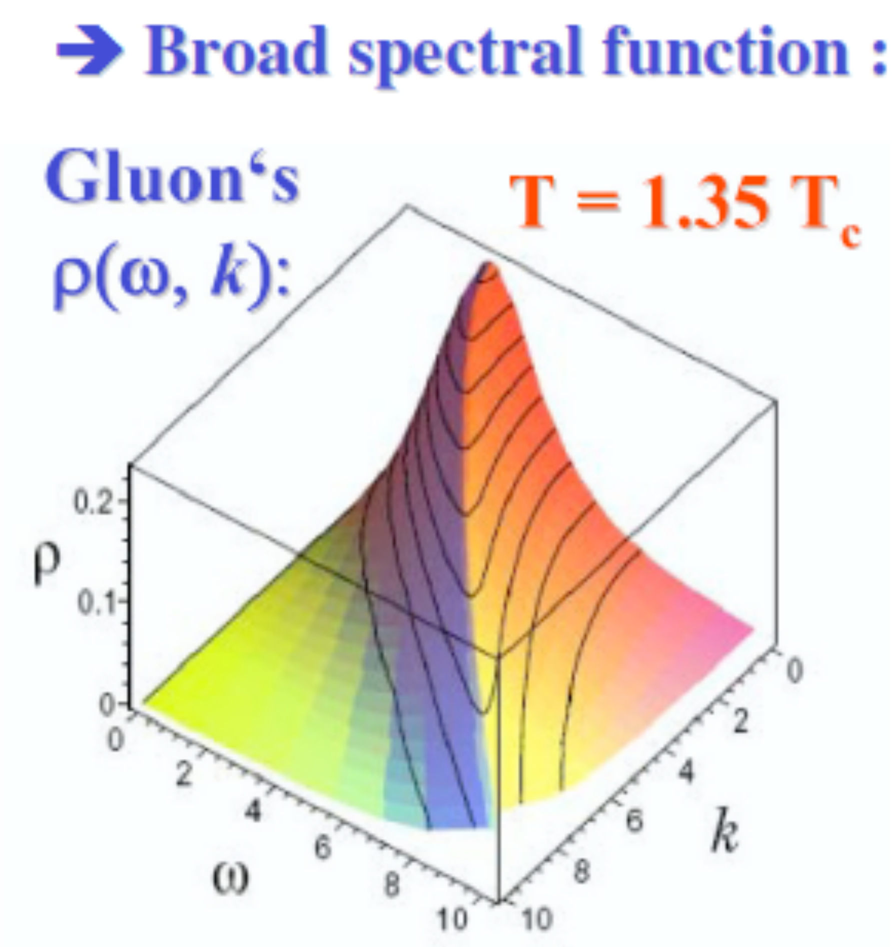


Kamikado, Strodthoff, von Smekal, Wambach '13

Viscosity in pure glue

spectral functions

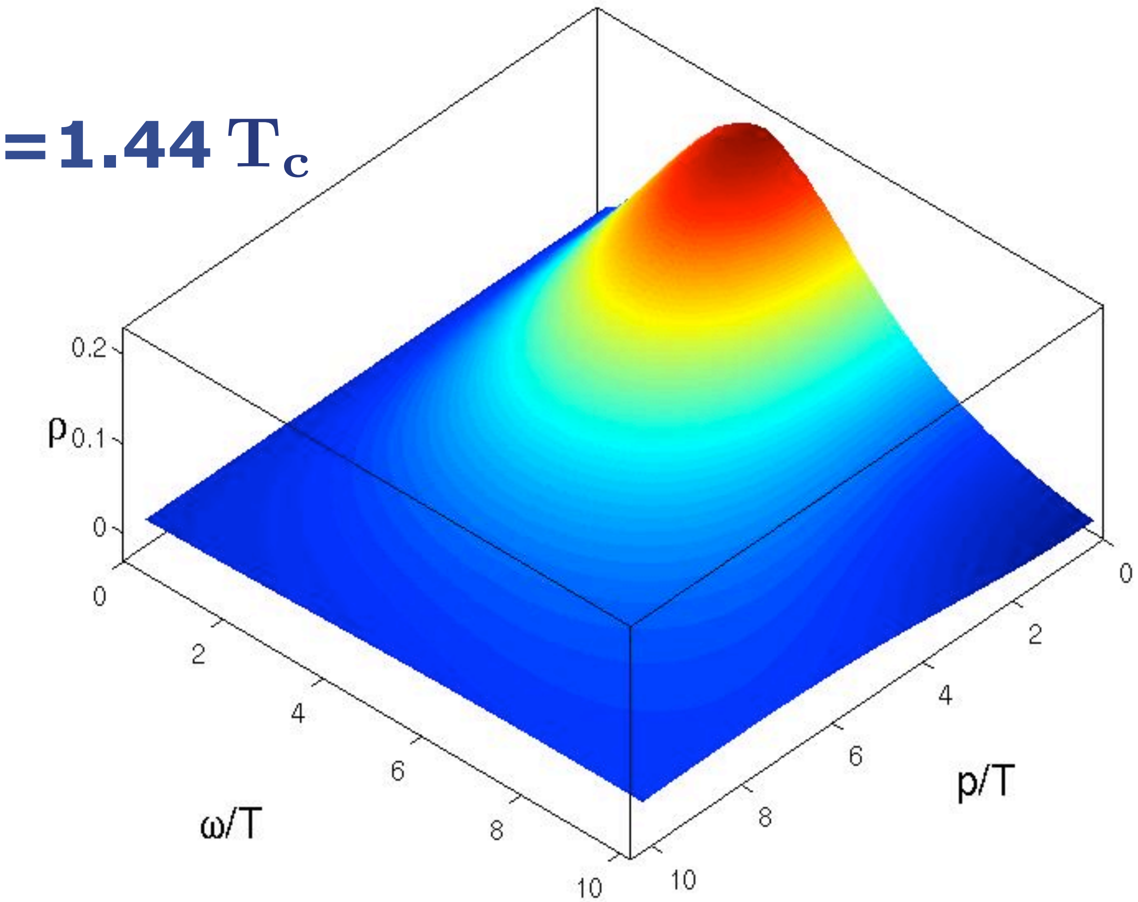
Fister, M. Haas, JMP, in prep



E. Bratkovskaya, talk at RETUNE '12

transversal spectral function

$T=1.44\,T_c$

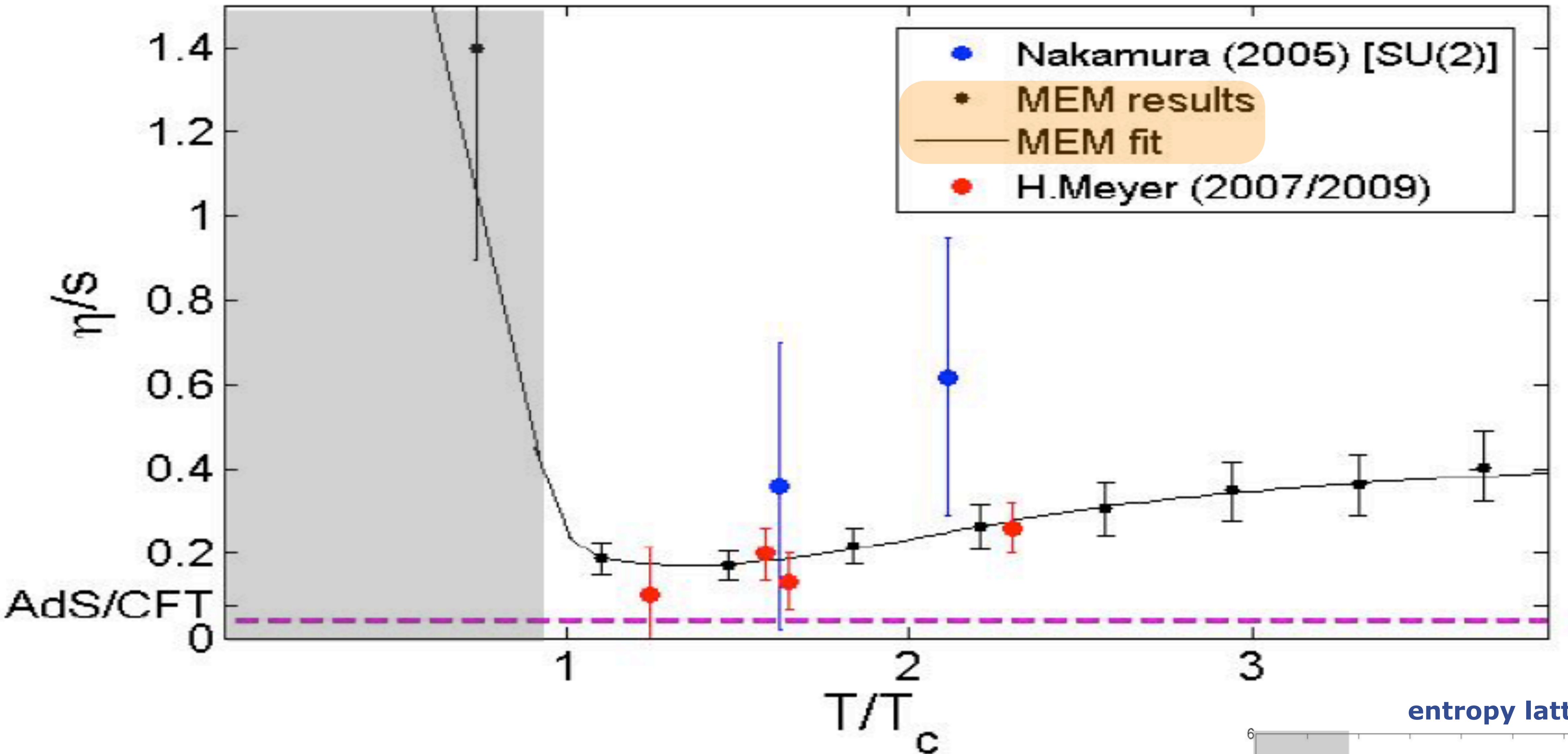


confirmed at T=0 with complex DSEs
Strauss, Fischer, Kellermann '12

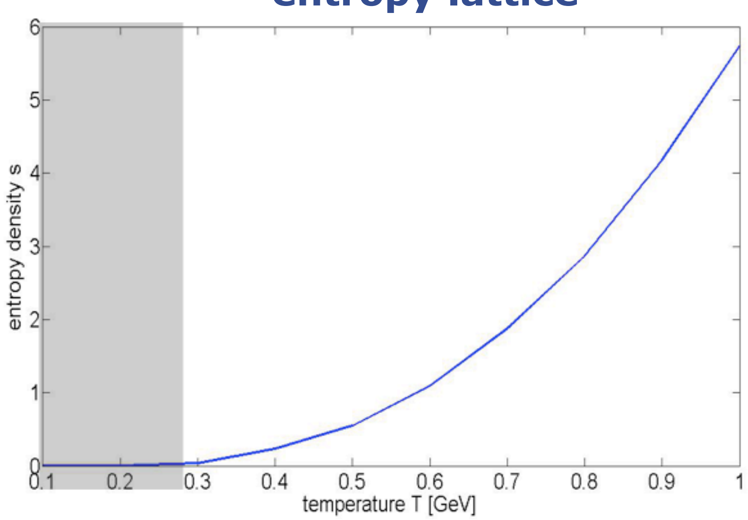
Viscosity in pure glue

shear viscosity

Fister, M. Haas, JMP, in prep



entropy lattice



H. Meyer '09

Boyd, Engels, Karsch '95


Thanx a lot
for the smooth organisation!!
of a very interesting winterschool
as always in Schladming!
Some participants at the lectures