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**IN5230**
**Electronic noise –**
**Estimates and countermeasures**

## Lecture X (Razavi 7)
## Noise – Razavi – Chapter 7

1

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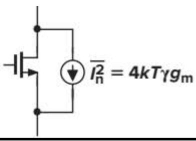
## MOSFET Thermal Noise (29)

- MOS transistors exhibit thermal noise with the most significant source being the noise generated in the channel
- For long-channel MOS devices operating in saturation, the channel noise can be modeled by a current source connected between the drain and source terminals with a spectral density

$$\overline{I_n^2} = 4kT\gamma g_m$$

**Red: Equation**

- The coefficient ‘γ’ (not body effect coefficient) is derived to be 2/3 for long-channel transistors and is higher for submicron MOSFETs
- As a rule of thumb, Razavi assume γ=1




**Red: Figure**

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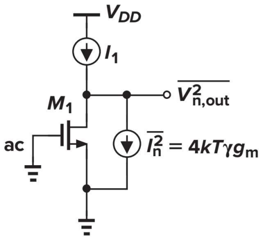
2

## MOSFET Thermal: Example




- The maximum output noise occurs if the transistor sees only its own output impedance as the load, i.e., if the external load is an ideal current source
- Output noise voltage spectrum is given by (7.29,7.30)

$$S_{out}(f) = S_{in}(f)|H(f)|^2$$

$$\overline{V_n^2} = \overline{I_n^2} r_O^2 = (4kT\gamma g_m) r_O^2$$





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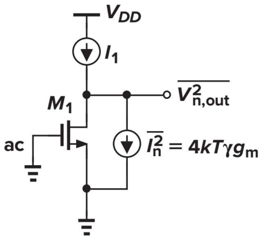
## MOSFET Thermal: Example




- Noise current of a MOS transistor decreases if the transconductance drops
- Noise measured at the output of the circuit does not depend on where the input terminal is because input is set to zero for noise calculation. (Vgs is not a part of the expression for In).
- The output resistance $r_o$ does not produce noise because it is not a physical resistor

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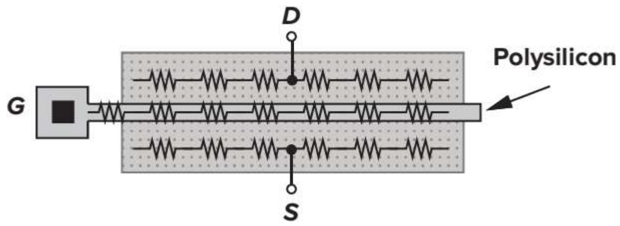
# MOSFET Thermal Noise




- Ohmic sections of a MOSFET have a finite resistivity and exhibit thermal noise
- For a wide transistor, source and drain resistance is negligible whereas the gate distributed resistance may become noticeable

5

5

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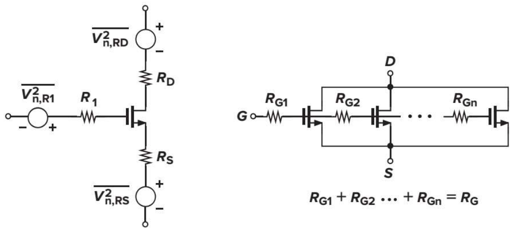
# MOSFET Thermal Noise




- In the noise model (left fig), the lumped resistance $R_1$ represents the distributed gate resistance
- In the distributed structure of the right figure, unit transistors near the left end see the noise of only a fraction of $R_G$ whereas those near the right end see the noise of most of $R_G$

Red: Equation

- It can be proven that $R_1 = R_G/3$ and hence the noise generated by gate resistance is $\overline{V^2_{nRG}} = 4kT\,R_G/3$

6

6

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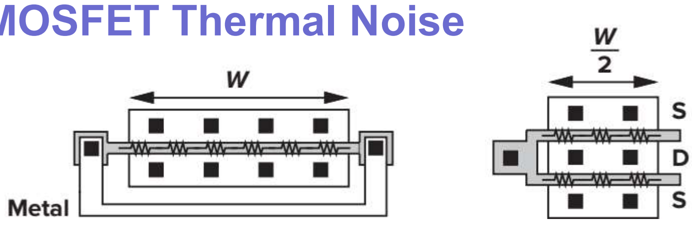
# MOSFET Thermal Noise




- Effect of $R_G$ can be reduced by proper layout
- In left fig, the two ends of the gate are shorted by a metal line, reducing the distributed resistance from $R_G$ to $R_G/4$
- Alternatively, the transistor can be folded as in the right figure so that each gate “finger” exhibits a resistance of $R_G/4$ for composite transistor

7

7

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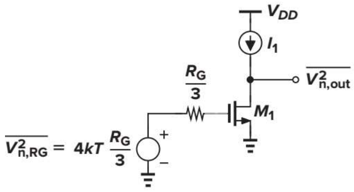
# MOSFET Thermal Noise: Example




- If the total distributed gate resistance is $R_G$, the output noise voltage due to $R_G$ is given by (7.32)

$$\overline{V^2_{n,out}} = 4kT\frac{R_G}{3}(g_m r_O)^2$$

- For the gate resistance noise to be negligible, we must ensure (7.33)

$$\frac{R_G}{3} \ll \frac{\gamma}{g_m}$$

Yellow: Equation

8

8

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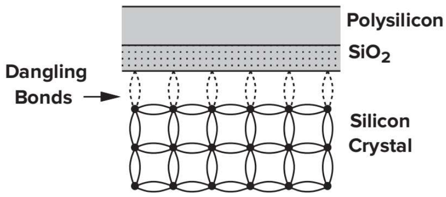
## Flicker Noise (36)




Yellow: Figure

- At the interface between the gate oxide and silicon substrate, many “dangling” bonds appear, giving rise to extra energy states
- Charge carriers moving at the interface are randomly trapped and later released by such energy states, introducing “flicker” noise in the drain current
- Other mechanisms in addition are believed to generate flicker noise

9

9

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## Flicker Noise

- Average power of flicker noise cannot be predicted easily
- It varies depending on cleanness of oxide-silicon interface and from one CMOS technology to another
- Flicker noise is more easily modelled as a voltage source in series with the gate and in saturation region, is roughly given by

$$\overline{V_n^2} = \frac{K}{C_{ox} W L} \cdot \frac{1}{f}$$

Red: Equation

- K is a process-dependent constant on the order of 10E-25V²F

10

10

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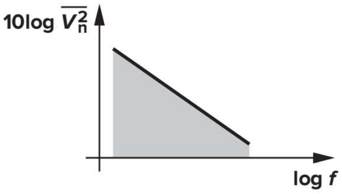
## Flicker Noise




- The noise spectral density is inversely proportional to frequency
  - Trap and release phenomenon occurs at low frequencies more often
- Flicker noise is also called “1/f” noise
- To reduce 1/f noise, device area must be increased
- Generally, PMOS devices exhibit less 1/f noise than NMOS transistors
  - Holes are carried in a “buried” channel, at some distance from the oxide-silicon interface

11

11

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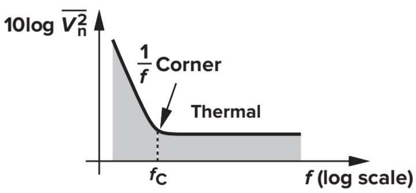
## Flicker Noise Corner Frequency

- At low frequencies, the flicker noise power approaches infinity
- At very slow rates, flicker noise becomes indistinguishable from thermal drift or aging of devices
  - Noise component below the lowest frequency in the signal of interest does not corrupt it significantly
- Intersection point of thermal noise and flicker noise spectral densities is called “corner frequency” fc




Yellow: Figure

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12

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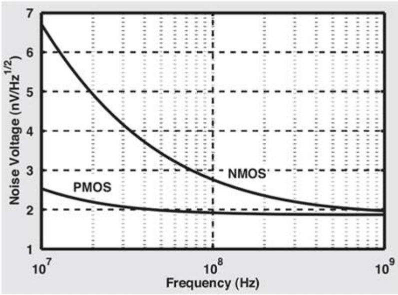
## Nanometer Design Notes

- W/L = 5µm/40nm,
- ID = 250µA

Yellow: Figure




- Low frequencies: Flicker noise
- High frequencies: Thermal noise

- ⇒ PMOS exhibit less noise than NMOS (and PNP less than NPN)
- ⇒NMOS noise corner at several hundred MHz

13

13

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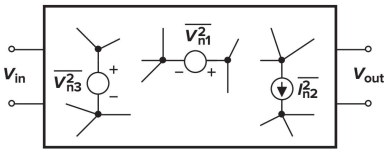
## Representation of Noise in Circuits



- To find the output noise, the input is set to zero and total noise is calculated at the output due to all the noise sources in the circuit
- This is how noise is measured in laboratories and in simulations

14

14

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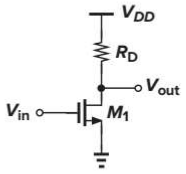
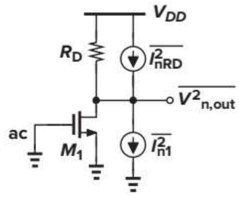
## Representation of Noise in Circuits: Example




- To find: Total output noise voltage of the common-source stage (left fig)
- Follow noise analysis procedure described earlier
- Thermal and flicker noise of $M_1$ and thermal noise of $R_D$ are modelled as current sources (right fig)

$$\overline{I^2_{n,th}} = 4kT\gamma g_m \quad \overline{I^2_{n,1/f}} = Kg_m^2/(C_{ox}WLf) \quad \overline{I^2_{n,RD}} = 4kT/R_D$$

- Output noise voltage per unit bandwidth, added as power quantities is

$$\overline{V^2_{n,out}} = \left(4kT\gamma g_m + \frac{K}{C_{ox}WL} \cdot \frac{1}{f} \cdot g_m^2 + \frac{4kT}{R_D}\right) R_D^2$$

Yellow: Equation

15



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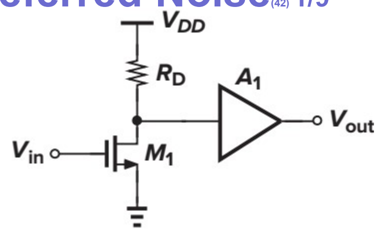
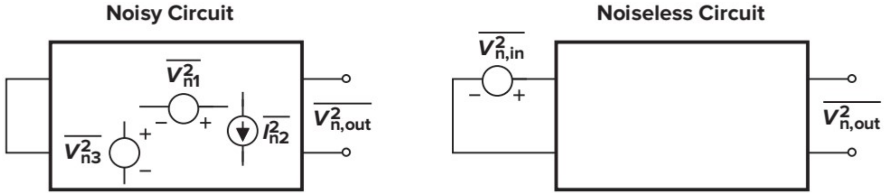
## Input-Referred Noise(42) 1/9




- Output-referred noise does not allow a fair comparison of noise in different circuits since it depends on the gain
- In above figure, a CS (Common-Source) stage is succeeded by a noiseless amplifier with voltage gain $A_1$, then the net output noise is now multiplied by $A_1^2$
- This may indicate that circuit becomes noisier as $A_1$ increases, which is incorrect since the signal level also increases proportionally, and net SNR at the output does not depend on $A_1$

16

## Input-Referred Noise 2/9




- Input-referred noise represents the effect of all noise sources in the circuit by a single source $\overline{V^2_{n,in}}$, at the input such that the output noise in right fig is equal to that in left fig.
- If the voltage gain is Av, then we must have

$$\overline{V^2_{n,out}} = A_v^2 \overline{V^2_{n,in}}$$

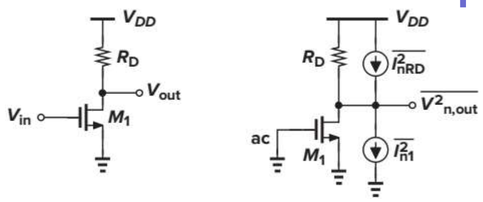
- The input-referred noise voltage in this simple case is simply the output noise divided by the gain squared.

17

17



## Input-Referred Noise: Example 3/9




- For the simple CS stage, the input-referred noise voltage is given by (7.45,7.46,7.47) $\overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{A_v^2}$

$$= \left(4kT\gamma g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right) R_D^2 \frac{1}{g_m^2 R_D^2}$$

$$= 4kT\frac{\gamma}{g_m} + \frac{K}{C_{ox}WL}\cdot\frac{1}{f} + \frac{4kT}{g_m^2 R_D}$$

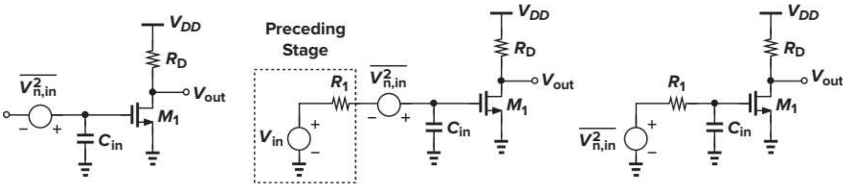
- First and third terms combined can be viewed as thermal noise of an equivalent resistance $R_T$, so that the equivalent input-referred thermal noise is $4kTR_T$

18

18

## Input-Referred Noise 4/9

- Single voltage source in series with the input is an incomplete representation of the input-referred noise for a circuit with a finite input impedance and driven by a finite source impedance
- For the CS stage, the input-referred noise voltage is independent of the preceding stage




- If the preceding stage is modelled by a Thevenin equivalent with an output impedance of $R_1$, the output noise due to voltage division is (7.48,7.49) $\overline{V^2_{n,out}} = \overline{V^2_{n,in}} \left|\frac{1}{R_1 C_{in} j\omega + 1}\right|^2 (g_m R_D)^2 = \frac{4kT\gamma g_m R_D^2}{R_1^2 C_{in}^2 \omega^2 + 1}$ 19



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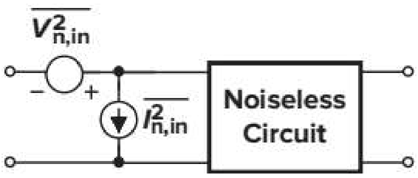
## Input-Referred Noise 5/9

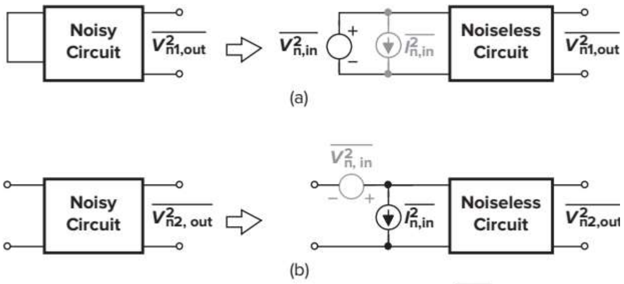
- The previous result is incorrect since the output noise due to $M_1$ does not diminish as $R_1$ increases
- To solve this issue, <u>we model the input-referred noise by both a series voltage source and a parallel current source</u>, so that if the output impedance of the previous stage assumes large values, thereby reducing the effect of $\overline{V^2_{n,in}}$, the noise current still flows through the finite impedance, producing noise at the input
- It can be proved that $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$ are necessary and sufficient to represent the noise of any linear two-port circuit




20

# Input-Referred Noise 6/9

- To calculate $\overline{V_{n,in}^2}$ and $\overline{I_{n,in}^2}$, two extreme cases are considered: zero and infinite source impedances




Yellow: Figure

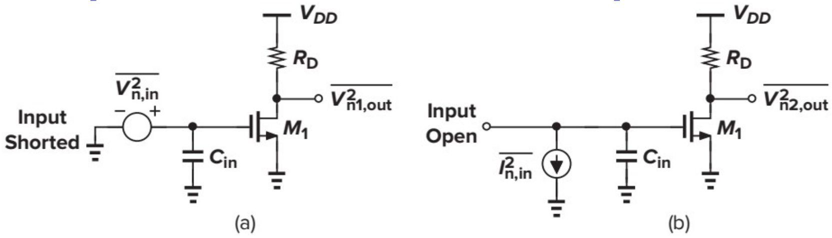
- If the source impedance is zero (fig a), $\overline{I_{n,in}^2}$ flows through $\overline{V_{n,in}^2}$ and has no effect at the output. i.e. the output noise measured arises solely from $\overline{V_{n,in}^2}$
- If the input is left open (fig b), then $\overline{V_{n,in}^2}$ has no effect and the output noise is only due to $\overline{I_{n,in}^2}$







# Input-Referred Noise: Example 7/9




- For the circuit in fig a), the input-referred noise voltage is simply (7.50)

$$\overline{V_{n,in}^2} = 4kT\frac{\gamma}{g_m} + \frac{4kT}{g_m^2 R_D}$$

- To obtain the input-referred noise current, the input is left open and we find the output noise in terms of $\overline{I_{n,in}^2}$
- The noise current flows through Cin, generating at the output (fig b) (7.51)

$$\overline{V_{n2,out}^2} = \overline{I_{n,in}^2}\left(\frac{1}{C_{in}\omega}\right)^2 g_m^2 R_D^2$$

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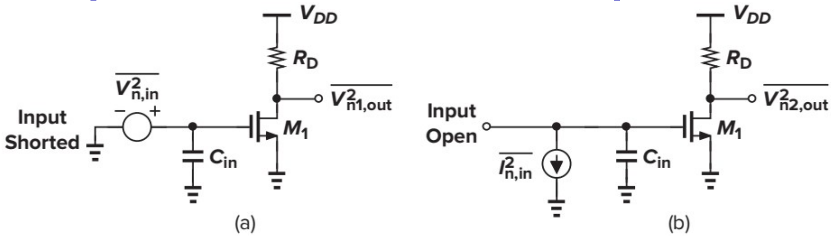
## Input-Referred Noise: Example 8/9




- This value must be equal to the output of the noisy circuit when the input is open. (7.52)

$$\overline{V^2_{n2,out}} = \left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2 \qquad \overline{V^2_{n2,out}} = \overline{I^2_{n,in}} \left(\frac{1}{C_{in}\omega}\right)^2 g_m^2 R_D^2$$

- Thus (7.53)

$$\overline{I^2_{n,in}} = (C_{in}\omega)^2 \frac{4kT}{g_m^2}\left(\gamma g_m + \frac{1}{R_D}\right)$$

23

23

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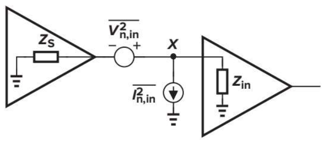
## Input-Referred Noise 9/9

- The input noise current $\overline{I^2_{n,in}}$, becomes significant for low enough values of the input impedance $Z_{in}$




- In above figure, $Z_S$ denotes the output impedance of the preceding circuit; total noise voltage sensed by the second stage at node $X$ is (7.54) $V_{n,X} = \frac{Z_{in}}{Z_{in}+Z_S} V_{n,in} + \frac{Z_{in} Z_S}{Z_{in}+Z_S} I_{n,in}$
- If $\overline{I^2_{n,in}} |Z_S|^2 \ll \overline{V^2_{n,in}}$, the effect of $I_{n,in}$ is negligible
- Thus, the input-referred noise current can be neglected if (7.55)

$$|Z_S|^2 \ll \frac{\overline{V^2_{n,in}}}{\overline{I^2_{n,in}}}$$

Yellow: Equation

24

24

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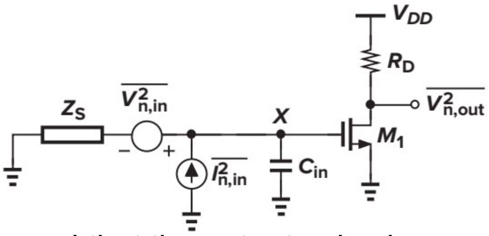
## Input-Referred Noise: Correlation 1/8

- Input-referred noise voltages and currents may be correlated
- Noise calculations must include correlations between the two
- Use of both a voltage source and a current source to represent the input-referred noise does not “count the noise twice”




- It can be proved that the output noise is correct for any source impedance $Z_S$, with both $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$ included

25

25

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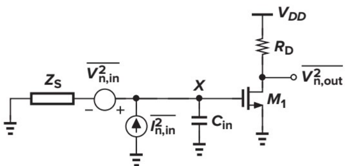
## Input-Referred Noise: Correlation 2/8




- Assuming $Z_S$ is noiseless for simplicity, we first calculate the total noise voltage at the gate of $M_1$ due to $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$.
- Cannot apply superposition of powers since they are correlated, but can be applied to voltages and currents since the circuit is linear and time-invariant
- Therefore, if $V_{n,M1}$ denotes the gate-referred noise voltage of $M_1$ and $V_{n,RD}$ the noise voltage of $R_D$ then (7.56,7.57)

$$V_{n,in} = V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD}$$

$$I_{n,in} = C_{in} s V_{n,M1} + \frac{C_{in} s}{g_m R_D} V_{n,RD}$$

26

26

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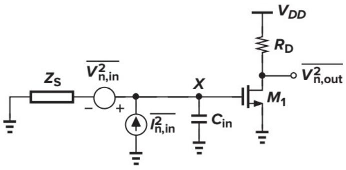
## Input-Referred Noise: Correlation 3/8




- $V_{n,M1}$ and $V_{n,RD}$ appear in both $V_{n,in}$ and $I_{n,in}$, creating a strong correlation between the two
- Adding the contributions of $V_{n,in}$ and $I_{n,in}$ at node $X$, as if they were deterministic quantities, we have (7.58,7.59)

$$V_{n,X} = V_{n,in}\frac{\frac{1}{C_{in}s}}{\frac{1}{C_{in}s}+Z_S} + I_{n,in}\frac{\frac{Z_S}{C_{in}s}}{\frac{1}{C_{in}s}+Z_S} = \frac{V_{n,in}+I_{n,in}Z_S}{Z_SC_{in}s+1}$$

27

27

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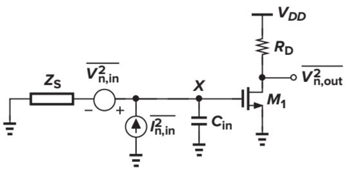
## Input-Referred Noise: Correlation 4/8




- Substituting for $V_{n,in}$ and $I_{n,in}$

$$V_{n,X} = \frac{1}{Z_SC_{in}s+1}\left[V_{n,M1}+\frac{1}{g_mR_D}V_{n,RD}+C_{in}sZ_S(V_{n,M1}+\frac{1}{g_mR_D}V_{n,RD})\right]$$
$$= V_{n,M1}+\frac{1}{g_mR_D}V_{n,RD}$$

- $V_{n,X}$ is independent
- It follows that

$$\overline{V^2_{n,out}} = g_m^2R_D^2\overline{V^2_{n,X}}$$
$$= 4kT\left(\gamma g_m+\frac{1}{R_D}\right)R_D^2$$

$\Rightarrow$ $V_{n,in}$ and $I_{n,in}$ do not double count the noise!

28

28

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## Input-Referred Noise: Correlation 5/8

- In some cases, it is simpler to consider the output short-circuit noise current-rather than the output noise voltage
- This current is then multiplied by the circuit's output resistance to yield the output noise voltage or simply divided by a proper gain to give input-referred quantities

29

29

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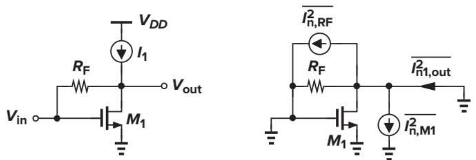
## (Example) (6/8)




- To find: Input-referred noise voltage and current. Assume $I_1$ is noiseless and $\lambda$=0
- To compute the input-referred noise voltage, we short the input port (right fig). Here, it is also possible to short the output port and hence (7.63)

$$\overline{I^2_{n1,out}} = \frac{4kT}{R_F} + 4kT\gamma g_m$$

- The output impedance of the circuit with the input shorted is simply $R_F$, therefore (7.64)

$$\overline{V^2_{n1,out}} = \left(\frac{4kT}{R_F} + 4kT\gamma g_m\right) R_F^2$$

30

30

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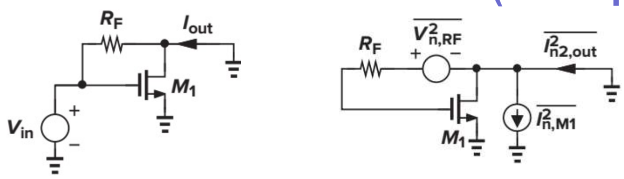
## Input-Referred Noise: Correlation (Example) 7/8




- Input-referred noise voltage can be found by dividing previous equation by voltage gain or $\overline{I^2_{n1,out}}$ by Gm²
- As shown in left fig, (7.65, 7.66)

$$G_m = \frac{I_{out}}{V_{in}} = g_m - \frac{1}{R_F}$$

- Therefore, (7.67)

$$\overline{V^2_{n,in}} = \frac{\frac{4kT}{R_F} + 4kT\gamma g_m}{(g_m - \frac{1}{R_F})^2}$$

31



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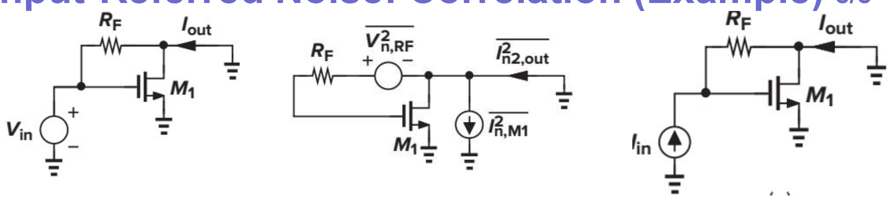
## Input-Referred Noise: Correlation (Example) 8/8




- To find the input-referred noise current (middle), we find the output noise current with the input left open (7.68)

$$\overline{I^2_{n2,out}} = 4kTR_F g_m^2 + 4kT\gamma g_m$$

- Next, we need to find the current gain of the circuit according to the arrangement in left figure
- Since, $V_{GS} = I_{in}R_F$ and $I_D = g_m I_{in} R_F$ (7.69,7.70)

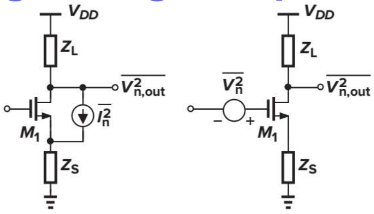
$$I_{out} = g_m R_F I_{in} - I_{in} = (g_m R_F - 1)I_{in}$$

- Thus, (7.71)

$$\overline{I^2_{n,in}} = \frac{4kTR_F g_m^2 + 4kT\gamma g_m}{(g_m R_F - 1)^2}$$

32

# Noise in Single-Stage Amplifiers: Lemma




- Lemma: The circuits in the figure are equivalent at low frequencies if $\overline{V_n^2} = \overline{I_n^2}/g_m^2$ and the circuits are driven by a finite impedance
- ⇒The noise source can be transformed from a drain-source current to a gate series voltage for arbitrary *Zs*

33



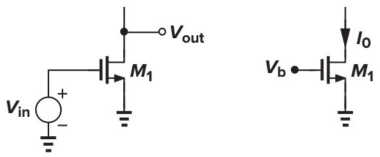


# Common-Source (CS) Stage 1/4

- From a previous example, the input-referred noise voltage of a simple CS stage was found to be (7.75)

$$\overline{V_{n,in}^2} = 4kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

- From above expression, $4kT\gamma/g_m$ is the thermal noise current expressed as a voltage in series with the gate




- To reduce input-referred noise voltage, transconductance must be maximized if the transistor is to amplify a voltage signal applied to its gate (left fig) whereas it must be minimized if operating as a current source (fig b).

34

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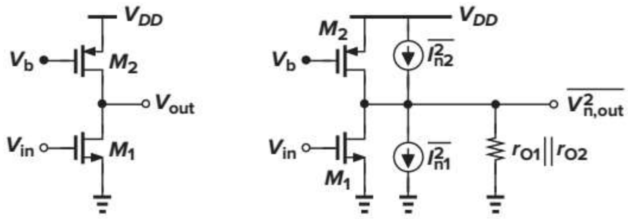
## Common-Source Stage: Example 2/4




- We want to find: a) Input-referred thermal noise, b) total output thermal noise with a load capacitance $C_L$
- Representing the thermal noise of $M_1$ and $M_2$ by current sources and noting that they are uncorrelated (7.76)

$$\overline{V_{n,out}^2} = 4kT(\gamma g_{m1} + \gamma g_{m2})(r_{O1}\|r_{O2})^2$$

35

35

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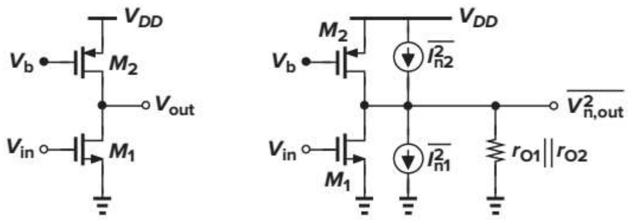
## Common-Source Stage: Example 3/4




- Since the voltage gain is equal to $g_{m1}(r_{O1}\|r_{O2})$, total noise voltage referred to the gate of $M_1$ is (7.77,7.78)

$$\overline{V_{n,in}^2} = 4kT(\gamma g_{m1} + \gamma g_{m2})\frac{1}{g_{m1}^2} = 4kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)$$

- Thus, $g_{m2}$ must be minimized because $M_2$ serves as a current source rather than a transconductor

36

36

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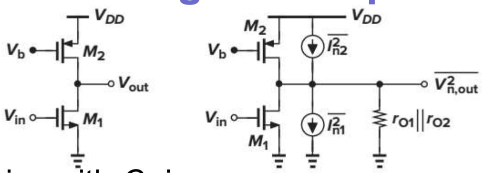
## Common-Source Stage: Example 4/4




- Total output noise with $C_L$ is (7.79,7.80)

$$\overline{V^2_{n,out,tot}} = \int_0^\infty 4kT\gamma(g_{m1}+g_{m2})(r_{O1}\|r_{O2})^2 \frac{df}{1+(r_{O1}\|r_{O2})^2 C_L^2 (2\pi f)^2}$$

$$\overline{V^2_{n,out,tot}} = \gamma(g_{m1}+g_{m2})(r_{O1}\|r_{O2})\frac{kT}{C_L}$$

- A low-frequency input sinusoid of amplitude Vm yields an output amplitude equal to $g_{m1}(r_{O1}\|r_{O2})V_m$ with an output SNR of (7.81,7.82)

Yellow: Equation

$$\mathrm{SNR}_{out} = \left[\frac{g_{m1}(r_{O1}\|r_{O2})V_m}{\sqrt{2}}\right]^2 \cdot \frac{1}{\gamma(g_{m1}+g_{m2})(r_{O1}\|r_{O2})(kT/C_L)}$$

$$= \frac{C_L}{2\gamma kT} \cdot \frac{g_{m1}^2(r_{O1}\|r_{O2})}{g_{m1}+g_{m2}} V_m^2$$

37

37

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## Common-Gate Stage: Thermal noise (64) 1/5

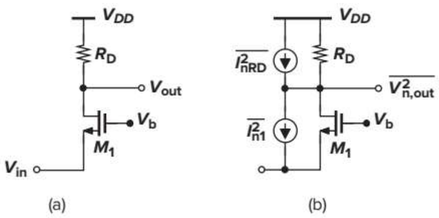


**Figure 7.46** (a) CG stage; (b) circuit including noise sources

- Neglecting channel-length modulation, we represent the thermal noise of $M_1$ and $R_D$ by two current sources
- Due to low input impedance of the circuit, the input-referred noise current is not negligible even at low frequencies

38

38

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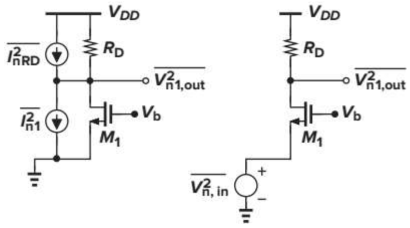
## Common-Gate Stage: Thermal noise 2/5




- To calculate the input-referred noise voltage, we short the input to ground and equate the output noises of the circuits. (7.93,7.94)

$$\left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2 = \overline{V^2_{n,in}}(g_m + g_{mb})^2 R_D^2$$

Yellow: Equation

$$\overline{V^2_{n,in}} = \frac{4kT(\gamma g_m + 1/R_D)}{(g_m + g_{mb})^2}$$

39



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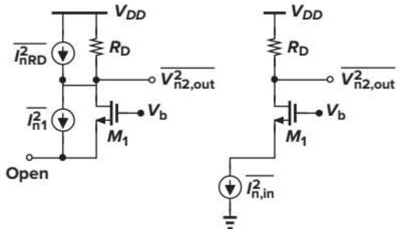
## Common-Gate Stage: Thermal noise 3/5




- To calculate the input-referred noise current, we leave the input open and equate the output noises of the circuits.
- $I_{n1}$ produces no noise at the output since the sum of the currents at the source of $M_1$ is zero.
- The output noise voltage is therefore $4kTR_D$ and hence:

(7.95) $$\overline{I^2_{n,in}} R_D^2 = 4kTR_D$$ $$\overline{I^2_{n,in}} = \frac{4kT}{R_D}$$

Yellow: Equation

40

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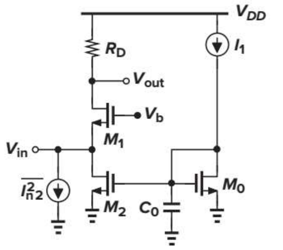
## Common-Gate Stage: Thermal noise 4/5




- Bias current source in the common-gate stage also contributes thermal noise
- Current mirror arrangement establishes bias current of M2 as a multiple of I1

First we look at M2:

- If input is <u>shorted</u> to ground, drain noise current of M2 does not contribute to input-referred noise <u>voltage</u>
- If input is <u>open</u>, all of $\overline{I_{n2}^2}$ flows from M1 and RD, producing an output noise of $\overline{I_{n2}^2}R_D^2$ and hence an input-referred noise <u>current</u> of $\overline{I_{n2}^2}$
- It is desirable to <u>minimize transconductance of M2</u>, at the cost of <u>reduced output swing</u>.

41

41

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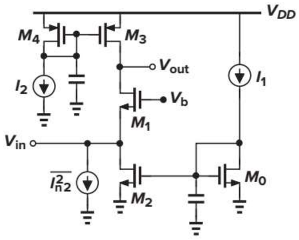
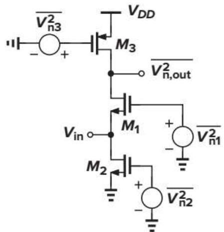
## Common-Gate Stage: Flicker noise 5/5




- Approximating the voltage gain as $(g_{m1} + g_{mb1})(r_{O1}||r_{O3})$ (7.105)

$$\overline{V_{n,in}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m1}^2 K_N}{(WL)_1} + \frac{g_{m3}^2 K_P}{(WL)_3}\right]\frac{1}{(g_{m1} + g_{mb1})^2}$$

- With the input open, the output noise is approximately (7.106)

$$\overline{V_{n2,out}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m2}^2 K_N}{(WL)_2} + \frac{g_{m3}^2 K_P}{(WL)_3}\right] R_{out}^2$$

- It follows that (7.107)

$$\overline{I_{n,in}^2} = \frac{1}{C_{ox} f}\left[\frac{g_{m2}^2 K_N}{(WL)_2} + \frac{g_{m3}^2 K_P}{(WL)_3}\right]$$

42

42

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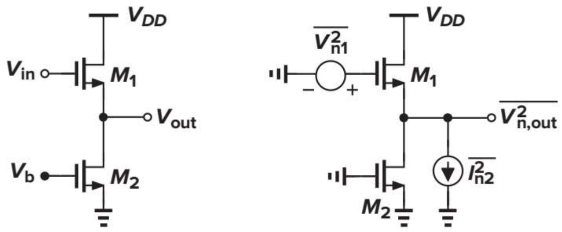
## Source Followers: Thermal Noise 1/2




- Since the input impedance of the source follower is quite high, the input-referred noise current can be neglected for moderate driving source impedances
- To compute the input-referred noise voltage, the output noise of M2 can be expressed as (7.108)

$$\overline{V^2_{n,out}}|_{M2} = \overline{I^2_{n2}}\left(\frac{1}{g_{m1}}\left\|\frac{1}{g_{mb1}}\right\|r_{O1}\|r_{O2}\right)^2$$

43

43

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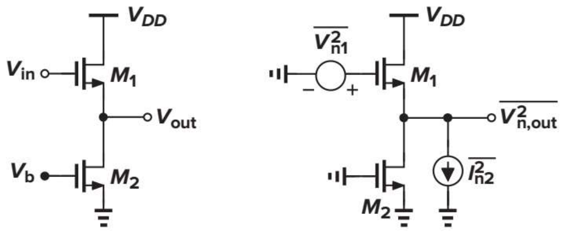
## Source Followers: Thermal Noise 2/2




- The voltage gain is

$$A_v = \frac{\frac{1}{g_{mb1}}\|r_{O1}\|r_{O2}}{\frac{1}{g_{mb1}}\|r_{O1}\|r_{O2} + \frac{1}{g_{m1}}}$$

- Total input-referred noise voltage is (7.110,7.111)

$$\overline{V^2_{n,in}} = \overline{V^2_{n1}} + \frac{\overline{V^2_{n,out}}|_{M2}}{A_v^2} = 4kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g^2_{m1}}\right)$$

Yellow: Equation

- Source followers add noise to the input signal and provide a voltage gain less than unity

44

44

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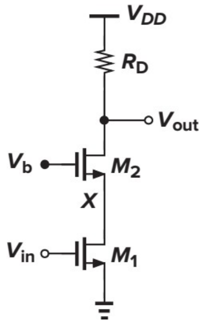
# Cascode Stage 1/3




- In the cascode stage, the noise currents of M1 and RD flow mostly through RD at low frequencies
- Noise contributed by M1 and RD is quantified in a common-source stage (7.112)

$$\overline{V_{n,in}^2}|_{M1,RD} = 4kT\left(\frac{\gamma}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)$$

45

45

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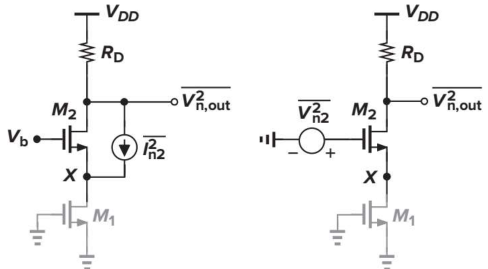
# Cascode Stage 2/3




- As shown in left figure, M2 contributes negligibly to noise at the output, especially at low frequencies
- If channel-length modulation of M1 is neglected, then $I_{n2} + I_{D2} = 0$ and hence M2 does not affect Vn,out
- From another perspective, in the equivalent circuit of the right figure, voltage gain from Vn2 to the output is small if impedance at node X is large (7.112)

46

46

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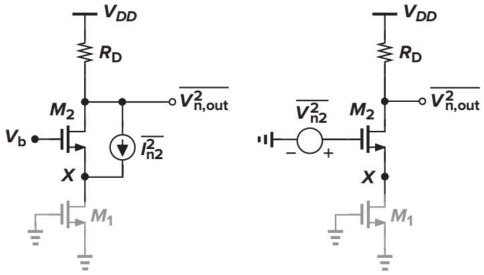
# Cascode Stage 3/3




- At high frequencies, the total capacitance at node X, $C_X$ gives rise to a gain, increasing the output noise (7.113)

$$\frac{V_{n,out}}{V_{n2}} \approx \frac{-R_D}{1/g_{m2} + 1/(C_X s)}$$

- This capacitance also reduces the gain from the main input to the output by shunting the signal current produces by $M_1$ to ground

47

47

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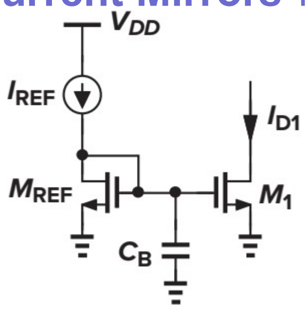
# Noise in Current Mirrors 1/4




- In the above current-mirror topology :$(W/L)_1 = N(W/L)_{REF}$
- The factor $N$ is in the range of 5 to 10 to minimize power consumed by the reference branch
- To determine the flicker noise in $I_{D1}$, we assume $\lambda$= 0 and $I_{REF}$ is noiseless

48

48

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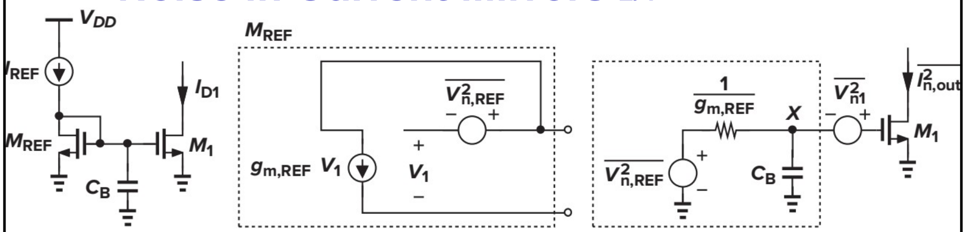
# Noise in Current Mirrors 2/4




- In the Thevenin equivalent for $M_{REF}$ and its flicker noise, the open-circuit voltage $V_{n,REF}$ (middle fig) and the Thevenin resistance is $1/g_{m,REF}$ (right fig)
- The noise voltage at node X and $V_{n1}$ add and drive the gate of $M_1$ producing (7.114)

$$\overline{I^2_{n,out}} = \left( \frac{g^2_{m,REF}}{C^2_B\omega^2 + g^2_{m,REF}} \overline{V^2_{n,REF}} + \overline{V^2_{n1}} \right) g^2_{m1}$$

- Since $(W/L)_1 = N(W/L)_{REF}$ and typically $L_1 = L_{REF}$ , we observe that $\overline{V^2_{n,REF}} = N\overline{V^2_{n1}}$

49

49

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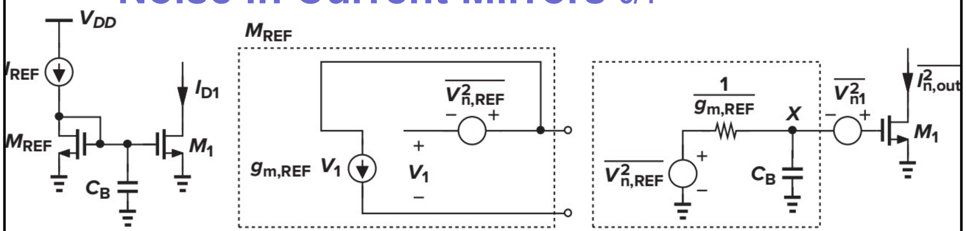
# Noise in Current Mirrors 3/4




- It follows that (7.115)

$$\overline{I^2_{n,out}} = \left( \frac{N g^2_{m,REF}}{C^2_B\omega^2 + g^2_{m,REF}} + 1 \right) g^2_{m1} \overline{V^2_{n1}}$$

- For the noise of the diode-connected device to be small, we need $(N-1)g^2_{m,REF} \ll C^2_B\omega^2$ and hence (7.117)

$$C^2_B \gg \frac{(N-1)g^2_{m,REF}}{\omega^2}$$

- This can be lead to $C_B$ being very high

50

50

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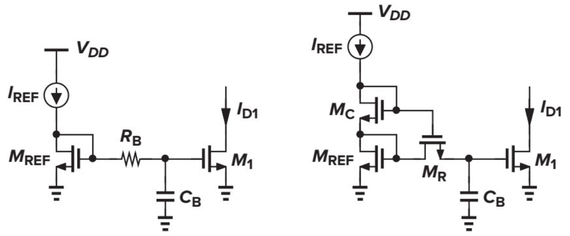
# Noise in Current Mirrors 4/4




- In order to reduce noise contributed by $M_{REF}$ and avoid a large capacitor, we can insert a resistance between its gate and $C_B$, so that It follows that (7.118)

$$\overline{I_{n,out}^2} = \left[ \frac{g_{m,REF}^2}{(1+g_{m,REF}R_B)^2 C_B^2 \omega^2 + g_{m,REF}^2} (\overline{V_{n,REF}^2} + \overline{V_{n,RB}^2}) + \overline{V_{n1}^2} \right] g_{m1}^2$$

- $R_B$ lowers the filter cutoff frequency but also contributes its own noise
- The MOS device $M_R$ with a small overdrive provides a high resistance and occupies a moderate area

51

51

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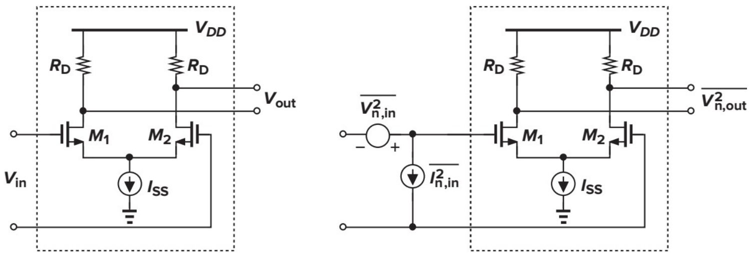
# Noise in Differential Pairs




- As shown in left figure, a differential pair can be viewed as a two-port circuit
- It is possible to model the overall noise as depicted in right figure
- For low-frequency operation, $\overline{I_{n,in}^2}$ is negligible

52

52

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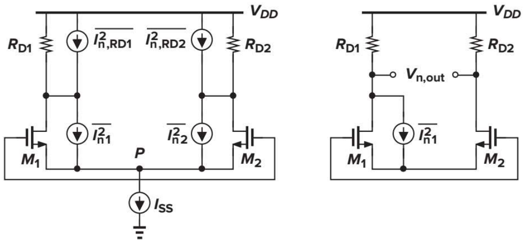
# Noise in Differential Pairs




- To calculate the thermal component $\overline{V^2_{n,in}}$, we first obtain the total output noise with inputs shorted together (left).
- Since In1 and In2 are uncorrelated, node P cannot be considered a virtual ground, so cannot use half-circuit concept
- Derive contribution of each source individually (right fig).

53

53

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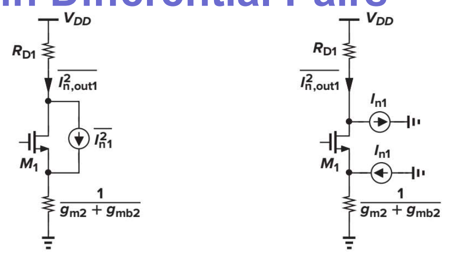
# Noise in Differential Pairs




- In left fig, to calculate the contribution of In1, neglecting channel-length modulation, it can be proven that half of In1 flows through $R_{D1}$ and the other half through $M_2$ and $R_{D2}$ (right fig.).
- The differential output noise due to $M_1$ is equal to (7.119)

$$V_{n,out}|_{M1} = \frac{I_{n1}}{2}R_{D1} + \frac{I_{n1}}{2}R_{D2}$$

54

54

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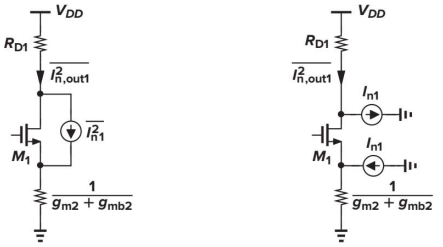
## Noise in Differential Pairs




- If $R_{D1}=R_{D2}=R_D$, $\overline{V^2_{n,out}}\big|_{M1} = \overline{I^2_{n1}}R^2_D$ $\quad \overline{V^2_{n,out}}\big|_{M2} = \overline{I^2_{n2}}R^2_D$
- Thus (7.122) $\overline{V^2_{n,out}}\big|_{M1,M2} = (\overline{I^2_{n1}} + \overline{I^2_{n2}})R^2_D$
- Taking into account the noise of $R_{D1}$ and $R_{D2}$… (7.123, 7.124)

$$\overline{V^2_{n,out}} = \left(\overline{I^2_{n1}} + \overline{I^2_{n2}}\right)R^2_D + 2(4kTR_D)$$

$$= 8kT\left(\gamma g_m R^2_D + R_D\right)$$

- Dividing by the square of the diff gain (7.125)

$$\overline{V^2_{n,in}} = 8kT\left(\frac{\gamma}{g_m} + \frac{1}{g^2_m R_D}\right)$$

55

55

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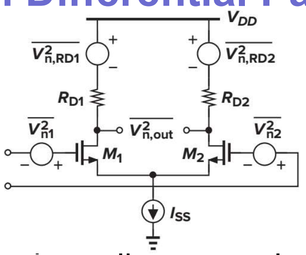
## Noise in Differential Pairs




- Input-referred noise voltage can also be calculated using the previous lemma
- The noise of $M_1$ and $M_2$ can be modelled as a voltage source in series with their gates
- The noise of $R_{D1}$ and $R_{D2}$ is divided by $g^2_m R^2_D$ resulting in previously obtained equation
- Including 1/f noise (7.126)

Yellow: Equation

$$\overline{V^2_{n,in,tot}} = 8kT\left(\frac{\gamma}{g_m} + \frac{1}{g^2_m R_D}\right) + \frac{2K}{C_{ox}WL}\frac{1}{f}$$

56

56

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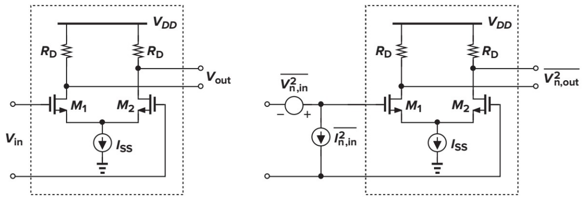
## Noise in Differential Pairs




- If the differential input is zero and the circuit is symmetric, the noise in $I_{SS}$ divides equally between $M_1$ and $M_2$, and produces only a common-mode noise voltage at the output
- For a small differential input $\Delta V_{in}$, we have (7.129, 7.130)

$$\Delta I_{D1} - \Delta I_{D2} = g_m \Delta V_{in} = \sqrt{2\mu_n C_{ox} \frac{W}{L} \left( \frac{I_{SS} + I_n}{2} \right)} \Delta V_{in}$$

- $I_n$ denotes the noise in $I_{SS}$ and we have $I_n \ll I_{SS}$
- As circuit departs from equilibrium, $I_n$ is more unevenly divided, generating differential output noise

57

57

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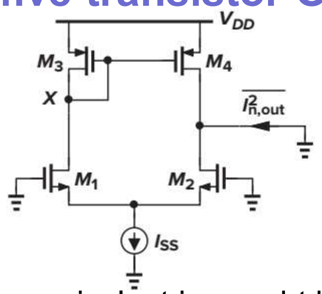
## Noise in five-transistor OTA




- The Norton noise equivalent is sought by first computing the output short-circuit noise current. This is then multiplied by the output resistance and divided by the gain to get input-referred noise voltage
- Transconductance is approximately $g_{m1,2}$
- Output noise current due to $M_1$ and $M_2$ is this transconductance multiplied by gate-referred noises of $M_1$ and $M_2$, i.e. $g^2_{m1,2}(4kT\gamma/g_{m1} + 4kT\gamma/g_{m2})$

58

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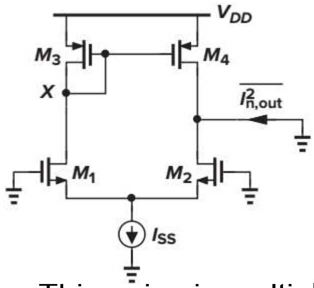
## Noise in five-transistor OTA




- The noise current of $M_3$ primarily circulates through the diode-connected impedance $1/g_{m3}$, producing a voltage at the gate of $M_4$ with spectral density $4kT\gamma/g_{m3}$
- This noise is multiplied by $g_{m4}^2$ as it emerges from the drain of $M_4$; the noise current of $M_4$ also flows directly through the output short-circuit thus (7.141) $\overline{I^2_{n,out}} = 4kT\gamma(2g_{m1,2} + 2g_{m3,4})$
- Multiplying this noise by $R^2_{out} \approx (r_{O1,2}||r_{O3,4})^2$ and dividing the result by $A^2_v = G^2_m R^2_{out}$ the total input-referred noise is

$$\overline{V^2_{n,in}} = 8kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g^2_{m1,2}}\right)$$

Yellow: Equation

59

59

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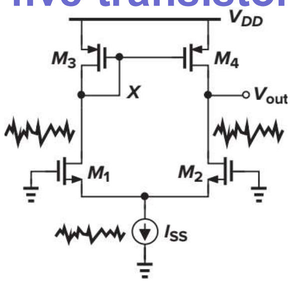


- The output voltage in the OTA *Vout* is equal to *Vx*
- If $I_{SS}$ fluctuates, so do $V_X$ and $V_{out}$
- Since the tail noise current *In* splits equally between $M_1$ and $M_2$, the noise voltage at *X* is given by $\overline{I^2_n}/(4g^2_{m3})$ and so is the noise voltage at the output

60

60

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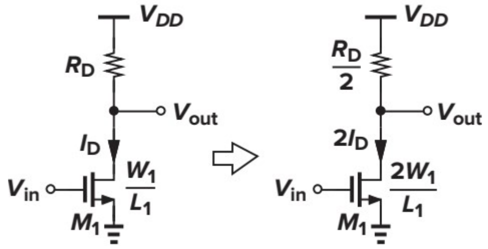
## Noise-Power Trade-off

- Noise contributed by transistors "in the signal path" is inversely proportional to their transconductance
  - Suggests a trade-off between noise and power consumption




- In the simple CS stage, we double $W/L$ and $I_{D1}$ and halve $R_D$, maintaining voltage gain and output swing
- Input-referred thermal and flicker noise power is halved, at the cost of power consumption

61

61

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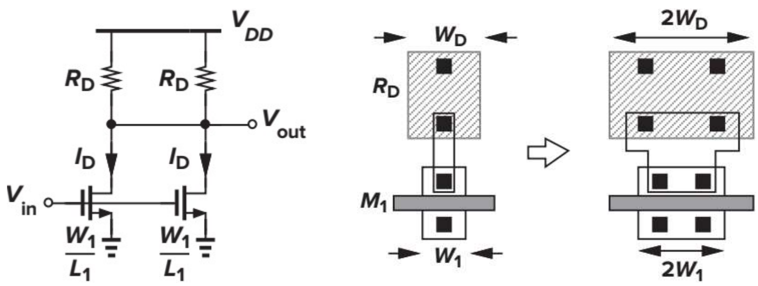
## Noise-Power Trade-off




- Called "linear scaling", the earlier transformation can be viewed as placing two instances of the original circuit in parallel
- Alternatively, it can be said that the widths of the transistor and the resistor are doubled

62

62

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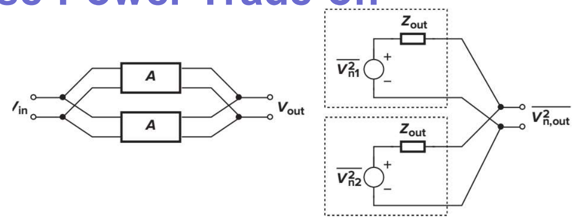
## Noise-Power Trade-off




- In general, if two instances of a circuit are placed in parallel, the output noise is halved
- Proven by setting the input to zero and constructing a Thevenin equivalent for each
- Since Vn1,out and Vn2,out are uncorrelated, we can use superposition of powers to write (7.143, 7.144)

$$\overline{V^2_{n,out}} = \frac{\overline{V^2_{n1,out}}}{4} + \frac{\overline{V^2_{n2,out}}}{4} = \frac{\overline{V^2_{n1,out}}}{2}$$

63

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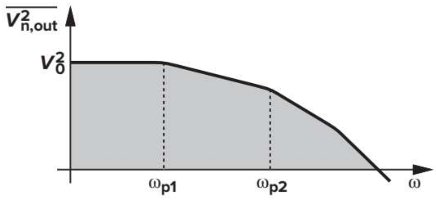
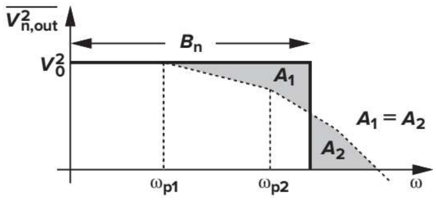
## Noise Bandwidth



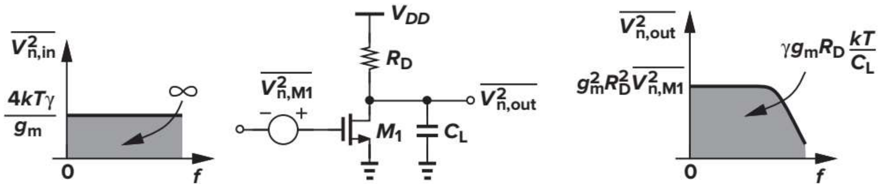




- Total noise corrupting a signal in a circuit results from all frequency components in the bandwidth of the circuit
- For a multipole system with noise spectrum as in left figure, total output noise is (7.145) $\overline{V^2_{n,out,tot}} = \int_0^\infty \overline{V^2_{n,out}} df$
- As shown in right figure, the total noise can also be expressed as $V_0^2 \cdot B_n = \int_0^\infty \overline{V^2_{n,out}} df$
- , where the bandwidth Bn, called the "noise bandwidth" is chosen such that (7.146)

64

64

# Problem of Input Noise Integration




- In the CS stage above, it is assumed that λ=0 and noise of $R_D$ is neglected with only thermal noise of $M_1$ considered.
- Output noise spectrum is the amplified and low-pass filtered noise of $M_1$, easily lending to integration
- Input-referred noise voltage, however, is simply $\overline{V^2_{n,M1}}$, carrying an infinite power and prohibiting integration
- For fair comparison of different designs, we can divide the integrated output noise by the low-frequency gain, for example

(7.147,7.148)

$$\overline{V^2_{n,in,tot}} = \gamma g_m R_D \frac{kT}{C_L} \cdot \frac{1}{g_m^2 R_D^2} = \frac{\gamma}{g_m R_D} \frac{kT}{C_L}$$

65

65