Homological Algebra

Course	Sec	CRN	Meeting	
601A: Topics in Topology	01	11361	MWF 10:50-11:50	LN 2201

Homological algebra is a tool to prove results in algebra, algebraic topology and algebraic geometry. In the introduction of [W] illustrates this with the following elementary example: Let A be an Abelian group, $B \leqslant A$ a subgroup and n a natural number. Clearly, $nB \leqslant nA \cap B$ – but when is this, in fact, an equality? A central topic of elementary homological algebra is the functor Tor – and we shall see that the above question is equivalent with understanding when $\mathrm{Tor}(A/B, \mathbb{Z}/n) \} \neq 0$.

Prerequisites. Homological algebra grew out of algebraic topology. Today the scope is more general, but algebraic topology is still a major source for motivation and understanding – also in this course: we shall occasionally refer to elementary combinatorial-topological concepts for illustration and examples but do not assume knowledge from a course in topology. On the algebraic side it is assumed that the audience has solid knowledge of and experience with groups, rings, ideals and modules, and is prepared to swallow some abstract language and ideas from the theory of categories and functors.

Aim. The course aims to present a passable trail into the theory which enables the audience to read more advanced texts, like [3] and [4] in the list below which contain a wealth of deeper results and fascinating applications.

Contents

1. **Modules:** Homomorphism groups and tensor products of modules over a non-commutative ring. Free, projective, injective and flat modules
2. **Chain complexes:** the homology functor, long exact sequence, chain homotopy, projective and injective resolutions, derived functors.
3. **Homological dimension.**
4. **Homology and Cohomology of groups.**

Literature

1. Joseph J. Rotman: *An Introduction to Homological Algebra* (Academic press 1979).
2. Peter J. Hilton and Urs Stammbach: *A Course in Homological Algebra* (Graduate Texts in Mathematics 4, Springer 1970).
3. Charles A Weibel: *An introduction to homological algebra* (Cambridge studies in advanced mathematics 38, Cambridge University Press 1994).
4. Kenneth S. Brown: Cohomology of Groups (Graduate Texts in Mathematics 87, Springer 1982).

Robert Bieri, August 10th 2011