Rumi Numeral System Symbols,

Additional characters proposed to Unicode

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August 5, 2006

1 Introduction

A special numeral system *rumi*[1] has been in use in North Africa since the X^{e} century. It remained in use until the XVII^{e} century. This system has been especially used in the administration of the city of Fez in Morocco. It has also been used in Al-Andalusians, Spain, starting from the XII^{e} century. The forms of the digits are quiet different from the Arabic[2] or the Arabic-Indic[2] digits in use today.

"The rumi numeral system originates in the Coptic or at least the Greek-Coptic tradition. The exact date when it was viewed as a system unto itself – that is, not a Coptic or Graeco-Coptic borrowing – has not yet been determined. rumi is used in foliation, chapters, and quire notations in a variety of manuscripts (including religious, scientific, accounting, and mathematical works) and on astronomical instruments. As such, encoding rumi will be very helpful for researchers in the history of mathematics, astronomy, and science in general, particularly as it was used for several hundred years."

The system of numeration wasn't really neither decimal nor positional. rumi use some special symbols (see Table 1, Table 2, Table 3 and Table 4). Some examples are available (see Table 5, Table 6 and Table 7). This system

[1] using Transtec Transliteration
http://www.ucam.ac.ma/fssm/rydarab/doc/communic/transtec.pdf

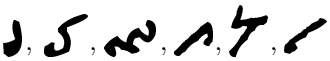
[2] the identifier name used by The Unicode Consortium http://www.unicode.org

is also known as *zimam letters*, *Roman* or *Fez letters* (namely Hrwf al-zmAm, al-rwmy or Hrwf fAs (حروف الزمام، الرومي او حروف فاس) or also rsm al-zmam, al-qlm al-rwmy or al-qlm al-fAsy (رسم الزمام، القلم الرومي او القلم الفاسي)).

2 Description

rumi numeral system has been described by many researchers and there is many studies about it [7], [8], [9], [10], [11], [12], [1] and others in [2]. We have adopted the one described by the mathematician Ibn Al-Banna (1256-1321, Marrakech) in his famous book [7] "Abstract of using rumi in calculus" or "Shortening from the work in the rumi in the account" in Arabic (namely Al-aqtDAb mn al-Eml b-al-rwmI fI al-HsAb (الاقتضاب من العمل بالرومي في الحساب)).

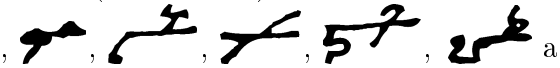
rumi use some special symbols for digits:

- rumi ones are (see Table 1):  ;
- rumi tens are (see Table 2):  ;
- rumi hundreds are (see Table 3):  .

Multiples of thousand are represented by adding bars under the based number:

- rumi thousands are noted by one bar under the number (see Table 5) (ex.,  for three thousands);
- rumi million are noted by two bars under the number (see Table 6) (ex.,  for three million);
- and so on.

Fraction is represented by adding a slash symbol separating the numerator from the denominator:

- the most used rumi fractions: 1/2, 2/3, 3/4, 4/5, 5/6, 6/7, 7/8, 8/9 and 9/10 are noted by (see Table 7):
   and  respectively;

- the special fractions: 1/2, 1/4, 1/3 and 2/3 are also noted by (see Table 4): , , and  respectively.

Generally, there is a generative principle for fractions, similar to building western ones. Notice that the numerator is in the top right and the denominator is in the bottom left, perhaps according to the Arabic writing direction. The numerator and the denominator are separated by almost an horizontal slash without spacing. For example, in rumi system, the fraction 253/500 is represented by . We could were wondering if this position of numerator and denominator are respected in the other writing with left-to-right direction what was adopted? However, according the Ibn Al-Banna manuscript (boat Rabat's and Marrakech's), the only four fractions: 1/2, 1/4, 1/3 and 2/3 have either there building symbols or special symbols pre-composed. We could were wondering if the "special fractions" are used much elsewhere?

3 Proposition

The rumi numeral system symbols are proposed to be included in Unicode Standard.

There names are described in rumi ones digits (see Table 8), rumi tens digits (see Table 9), rumi hundreds digits (see Table 10) and rumi special fractions (see Table 11).

There glyphs are taking from the Rabat copy of Ibn al-Banna manuscript [7]. The rumi glyphs used during its using can't be identical in all manuscripts. Through some available manuscripts, there are some small differences between them. We think that is natural as there are writing by hand from different writers in many environments. We can't recherche to *establishing the canonical pattern of the Rumi figures* but only a *representative glyph* since is what is contained in the Unicode charts. A study is made the have a comparative rumi glyphs scanned from some manuscripts (see Figure 13 such that colones from [7], [8], [9], [10], [11], [11] and [12] respectively) and (see Figure 14).

As many manuscripts in studying use these symbols, we need to encode them. We are working on studying and translating to English the Ibn Al-Banna manuscript. A computer system for transforming numbers from and to rumi numeral system is also in development.

A detailed bibliography and some used examples are presented in the end.

The **rumi** and **AntiSym** fonts available, includes all these characters. In **rumi** font, used here, the shapes of the reference glyphs are scanned

from [7]. It's in OpenType format [3] and converted in METAFONT as a LaTeX package [5]. In **AntiSym** font, glyphs are drawing by hand in METAFONT as a LaTeX package [4].

The shapes of the reference glyphs used are not frozen. They are continually being improved in *Multilingual scientific e-document processing* Project at Al-khawarizmi Atelier.

Some boxes are add to some symbols in Figures in order to emphases them and understand the purpose of the samples.

More information about this presentation is available in [6].

References

[1] Rosa Comes, *Arabic, Rûmî, Coptic, or merely Greek Alphanumerical Notation? The case of a Mozarabic 10^{th} Century Andalusî Manuscript*, Suhayl **3** (2003), 157–185, `http://chama.fltr.ucl.ac.be/chama2/newsletter_3.htm`.

[2] A complete reference of rumi system, `http://www.ucam.ac.ma/fssm/rydarab/doc/unicode/referencerumi.doc`.

[3] Rumi Arabic mathematical old symbols font rumi in OpenType, `http://www.ucam.ac.ma/fssm/rydarab/doc/unicode/rumi.ttf`.

[4] Arabic mathematical old symbols package antisym for LaTeX, `http://www.ucam.ac.ma/fssm/rydarab/system/zip/antisym.zip`.

[5] Rumi Arabic mathematical old symbols package rumi for LaTeX, `http://www.ucam.ac.ma/fssm/rydarab/system/zip/rumi.zip`.

[6] Arabic mathematical symbols for Unicode, `http://www.ucam.ac.ma/fssm/rydarab/english/unicode.htm`.

[7] Ibn Al-Banna (1256-1321, Marrakech), Shortening from the work in the rumi in the account, Manuscript in the national library in Rabat. It was copied after the death of Ibn Al-Banna, without noting the writer name. In the end, it notes the date of the end of the manuscript writing is Friday 3/10/908 hijer witch will be 31/3/1503.

[8] Ibn Al-Banna (1256-1321, Marrakech), Shortening from the work in the rumi in the account, Manuscript in the Ibn Youssef library in Marrakesh. It was copied after the death of Ibn Al-Banna, without noting neither the name nor the date to copy it.

[9] Abderrahman Ibn Mohammed Al-Fasi known by Ibn Al-Arabi, Design of Zimam, Manuscript in the Ibn Youssef library in Marrakesh. It was copied after the death of Ibn Al-Arabi, without noting neither the name nor the date to copy it.

[10] Narrow faces in the account and Zimam, Manuscript in the Ibn Youssef library in Marrakesh. Manuscript without noting neither the author name nor the date to copy it.

[11] Ahmed Ibn Al-Ayashi Skyrj, Direction of learning and that which forgot the form and the characteristics of the Al-Fasi style, Edition of Fez, 1316h. Skyrj said that Fasi system is used by Fasi people (Moroccan town Fez) and came from the old Rumi system. He presents the two systems. He presents a table of the nine unite fractions (the unite symbol with a tail sign without knowing witch one!.)

[12] Qasem Ibn Ahmed Al-Samarie, The numbers Arabic carpenter in the East and Sanskrit Indian gown Western Europe, Word of the books, the binding nineteenth, question fifth and sixth doubled about the Arabic number, 1998. (Fasi numbers as appear in the introduction in "Kudira" for the "Link" book of Ibn Pashkwal, Madrid 1882.)

1	2	3	4	5	6	7	8	9
								

Table 1: rumi ones symbols

10	20	30	40	50	60	70	80	90
								

Table 2: rumi tens symbols

100	200	300	400	500	600	700	800	900
								

Table 3: rumi hundreds symbols

1/2	1/4	1/3	2/3
			

Table 4: rumi special fractions symbols

1000	2000	3000	4000	5000	6000	7000	8000	9000
								
10000	20000	30000	40000	50000	60000	70000	80000	90000
100000	200000	300000	400000	500000	600000	700000	800000	900000

Table 5: rumi thousands examples

1000000	2000000	3000000	4000000	5000000	6000000	7000000	8000000	9000000
								
10000000	20000000	30000000	40000000	50000000	60000000	70000000	80000000	90000000
								
100000000	200000000	300000000	400000000	500000000	600000000	700000000	800000000	900000000
								

Table 6: rumi millions examples

1/2	2/3	3/4	4/5	5/6	6/7	7/8	8/9	9/10
								

Table 7: rumi fractions examples

 RUMI DIGIT ONE
≈ 0031 1 digit one
≈ 0661 ١ Arabic-Indic digit one

 RUMI DIGIT TWO
≈ 0032 2 digit two
≈ 0662 ٢ Arabic-Indic digit two

 RUMI DIGIT THREE
≈ 0033 3 digit three
≈ 0663 ٣ Arabic-Indic digit three

 RUMI DIGIT four
≈ 0034 4 digit four
≈ 0664 ٤ Arabic-Indic digit four

 RUMI DIGIT FIVE
≈ 0035 5 digit five
≈ 0665 ٥ Arabic-Indic digit five

 RUMI DIGIT SIX
≈ 0036 6 digit six
≈ 0666 ٦ Arabic-Indic digit six

 RUMI DIGIT SEVEN
≈ 0037 7 digit seven
≈ 0667 ٧ Arabic-Indic digit seven

 RUMI DIGIT EIGHT
≈ 0038 8 digit eight
≈ 0668 ٨ Arabic-Indic digit eight

 RUMI DIGIT NINE
≈ 0039 9 digit nine
≈ 0669 ٩ Arabic-Indic digit nine

Table 8: rumi ones digits


RUMI DIGIT TEN
• used as a symbol with a numeric value of 10


RUMI DIGIT TWENTY
• used as a symbol with a numeric value of 20


RUMI DIGIT THIRTY
• used as a symbol with a numeric value of 30


RUMI DIGIT FORTY
• used as a symbol with a numeric value of 40


RUMI DIGIT FIFTY
• used as a symbol with a numeric value of 50


RUMI DIGIT SIXTY
• used as a symbol with a numeric value of 60


RUMI DIGIT SEVENTY
• used as a symbol with a numeric value of 70


RUMI DIGIT EIGHTY
• used as a symbol with a numeric value of 80


RUMI DIGIT NINETY
• used as a symbol with a numeric value of 90

Table 9: rumi tens digits

RUMI DIGIT HUNDRED
• used as a symbol with a numeric value of 100


RUMI DIGIT TWO HUNDRED
• used as a symbol with a numeric value of 200


RUMI DIGIT TREE HUNDRED
• used as a symbol with a numeric value of 300


RUMI DIGIT FOUR HUNDRED
• used as a symbol with a numeric value of 400


RUMI DIGIT FIVE HUNDRED
• used as a symbol with a numeric value of 500


RUMI DIGIT SIX HUNDRED
• used as a symbol with a numeric value of 600


RUMI DIGIT SEVEN HUNDRED
• used as a symbol with a numeric value of 700


RUMI DIGIT EIGHT HUNDRED
• used as a symbol with a numeric value of 800


RUMI DIGIT NINE HUNDRED
• used as a symbol with a numeric value of 900

Table 10: rumi hundreds digits

RUMI FRACTION ONE HALF
• used as an other symbol with a numeric value of 1/2
≈ 00BD 1/2 vulgar fraction one half

RUMI FRACTION ONE QUARTER
• used as a symbol with a numeric value of 1/4
≈ 00BC 1/4 vulgar fraction one quarter

RUMI FRACTION ONE THIRD
• used as a symbol with a numeric value of 1/3
≈ 2153 1/3 vulgar fraction one third

RUMI FRACTION TWO THIRDS
• used as an other symbol with a numeric value of 2/3
≈ 2154 2/3 vulgar fraction two thirds

Table 11: rumi special fractions

Figure 1: rumi numeral system in [7] page 1

بسم الله الرحمن الرحيم

صلى الله على سيدنا محمد وعلى آله وصحبه وسلم تسليما

الاقتضاب من العمل بالرومي في الحساب، تأليف الشيخ الفقيه الجليل الفاضل المشارك الأكمل المرحوم أبي العباس أحمد بن محمد بن عثمان الأزدي، عرف بابن البناء العددي المراكشي، قدس الله روحه ورحمه بمنه وفضله وطوله[1].

الحمد لله حمدا لا انقطاع[2] لعده[3] ولا نهاية لحده والصلاة على محمد نبيه وعبده وعلى آله وسلم وشرفه كثيرا[4]. وبعد، فهذا اقتضاب من العمل بالرومي في الحساب على نحو ما وقع[5] اختيار الرؤساء[6] من العمال، ومن الله اسأل حسن التوفيق. باب في أسماء مراتب الأعداد وصفة الرشم بالرومي. اعلم أن المرتبة الأولى هي الآحاد التسعة والثانية هي العشرات التسع والثالثة هي المؤن التسع والرابعة هي الآلاف التسعة والخامسة هي عشرات الآلاف التسعة[7] والسادسة هي مئوا الآلاف التسع والسابعة هي آلاف الآلاف التسعة[8] وهكذا إلى غير نهاية. ولكل عدد من أعداد المراتب الثلاث الأول رشم يخصه أولها واحد وآخرها تسع مائة ورشوم الآلاف وعشراتها ومئيها كرشوم الآحاد وعشراتها ومئيها والفرق بينهما بالتكرار وشكل التكرار خطة[9] تحت العدد وهذه صورة ذلك كله:

وكذلك آلاف الآلاف وعشراتها ومؤها ترجع إلى رشوم التي قبلها والفرق بينهما التكرار. لكل نوع ما يجب له منه[10] كذلك إلى غير نهاية. وأما رشوم الكسور فرشم العدد الذي يشتق منه الكسر[11] ويخط فوقه خط يسمى كرسيا ويكتب فوقه عدة الأجزاء التي تشتق منه وكذلك كسور الكسور وهي أمثلة منها رشم نصف وثلثين وثلاثة أرباع وأربعة أخماس وخمسة أسداس وستة أسباع وسبعة أثمان وثمانية أتساع[12] وتسعة أعشار وقد اصطلح أصحاب العمل على أن يرشموا النصف هكذا والربع هكذا والثلث هكذا والثلثين[13] هكذا ولا يستعملون الأجزاء التي مقاماتها أكثر من عشرة. وان وقع لهم شيء منها في العمل صرفوه على ما ستعرفه في باب الجمع بعد هذا. والكسور المستعملة عندهم نوعان مضافة ومختلفة فأما المضافة فيتقدم الكسر الذي مقامه أعظم ويكون الأقل عن يمينه منخفضا منه قليلا مثل خمسة أثمان وثلاثة أثمان ثمن وثلث ثمن ثمن رشمها[14] هكذا وأما المختلفة فبعضها تحت بعض مثل خمسة أثمان وستة أسباع رشمها هكذا

[1] ... في النسخة ... : قال الشيخ الأجل الأستاذ ... أبو العباس أحمد بن محمد بن عثمان الأزدي رحمه الله ورضي عنه

[2] ورد ... "انقاع" ... "انقطاع" ... في النسخة ...

[3] في النسخة ... : بعده

[4] في النسخة ... : والصلاة والسلام على سيدنا محمد نبيه وعبده وعلى آله وشرف كثيرا

[5] في النسخة ... : عليا

[6] ورد ... "الروسا" ... "الرؤساء" ... في النسخة ...

[7] في النسخة ... : التسع

[8] في النسخة ... : والسابعة هي آلاف الآلاف التسعة ...

[9] ورد ... "خطة" ... "بخطة" ... في النسخة ...

Figure 2: rumi numeral system in [7] page 1 printed

In the name of The God, the most Merciful, the most Compassionate
The God prayed on Mohammed reigned and on his family and accompanied him
and peace of delivery

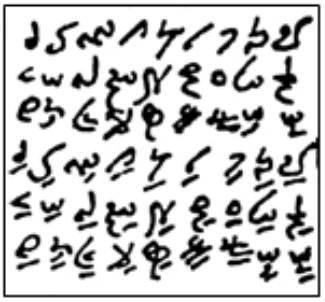
The shortening from the work in the Rwmy in the calculation, formation of the participant virtuous of the magnificent jurist generous associated complete Abou AlEbas Ahmed Ben Mohammed Ben Othman al'azdy. He was introduced as mason's son numerical Almraakshy. The God sanctified went him and his uterus in his blessing and his casual dress and lengthens him.

The God praised neither his break-off for acne nor his end for border and the prayer on Mohammed is discerning and worshipped him and on his family and his peace and honor a lot. After, so this shortening from the work in the Rwmy in the account on towards what choice of the heads from the laborers fell, and from The God asks the good success.

Chapter in names ranks of the numbers and prescription glyphs in Rwmy.

Knows that the rank first is the ones nine and the second is the tens nine and the third is the hundreds nine and the fourth is the thousands nine and the fifth is the tens of thousands nine and the sixth is the hundreds of thousands nine and the seventh is the thousands thousands nine and so on until the infinite one.

For each number of the first three numbers ranks a sign witch distinguishes it, the first is one and the last is nine hundred. The draws of thousands and its tens and its hundreds are the same as the draws of ones and tens and hundreds and the difference between them is the repetition. The form of the repetition is a bar under the number and these images for all them:



Likewise thousands of thousands and its tens and its hundreds return to glyphs before them and the difference between them the repetition. For each kind what be necessary for him likewise so on until the infinite one.

When the fractions are to be drawn, we write the base number which derives the fraction and draws above him a line called chair and writes above him the parts which derives from him, and as the fraction's fraction. Here are examples of draw: one half , two thirds , tree quarters , four fifths , five sixths , six sevenths , seven eighths , eight ninths , nine tenths . And leads the man working in make up provided that draw the one half in this way , and one quarter in this way and one third in this way and the two third in this way . They don't use fractions which there based are more than ten. If they arrive to have some of them they transform them to what you will know in the addition chapter after this one.

The fractions they used stubborn two kinds added and different. So for added fraction, the fraction whose based is great advances and the little about right and little low from him, example five eighth and three eighth and third eighth in this way . As for different fraction, parts are under others parts example five eighths and six sevenths in this way

Figure 3: rumi numeral system in [7] page 1 translated in English

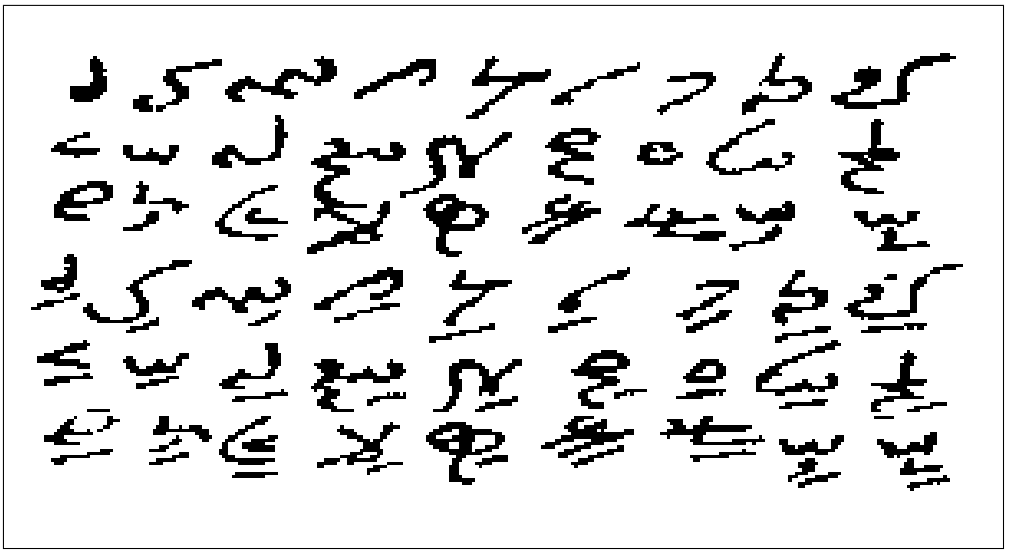


Figure 4: Integer rumi symbols in [7]

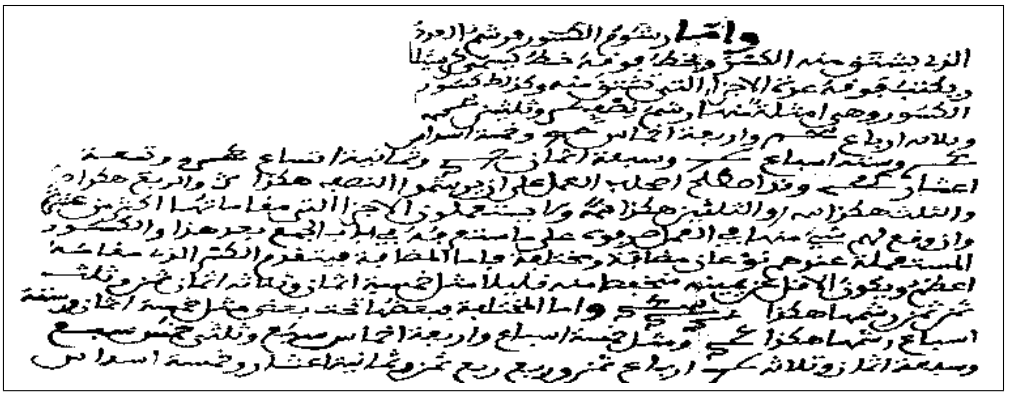


Figure 5: Fraction rumi symbols in [7]

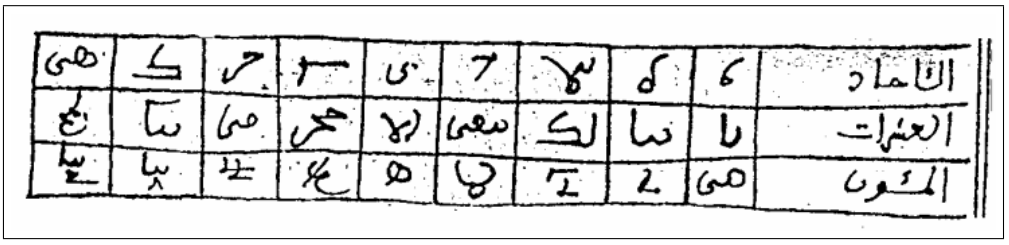


Figure 6: rumi numbers from Skyrj rumi in [11]

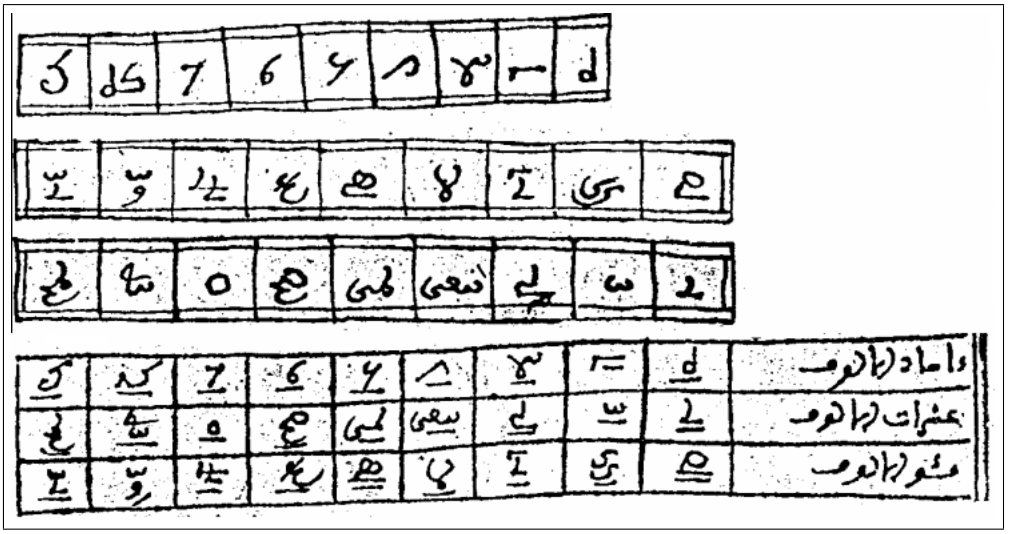


Figure 7: rumi numbers from Skyrj Fasi in [11]

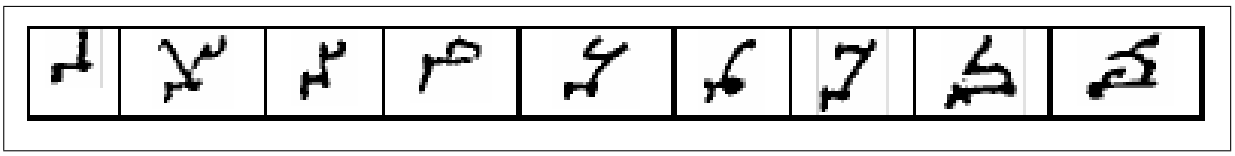


Figure 8: Nine unite rumi fractions from Skyrj Fasi [11]

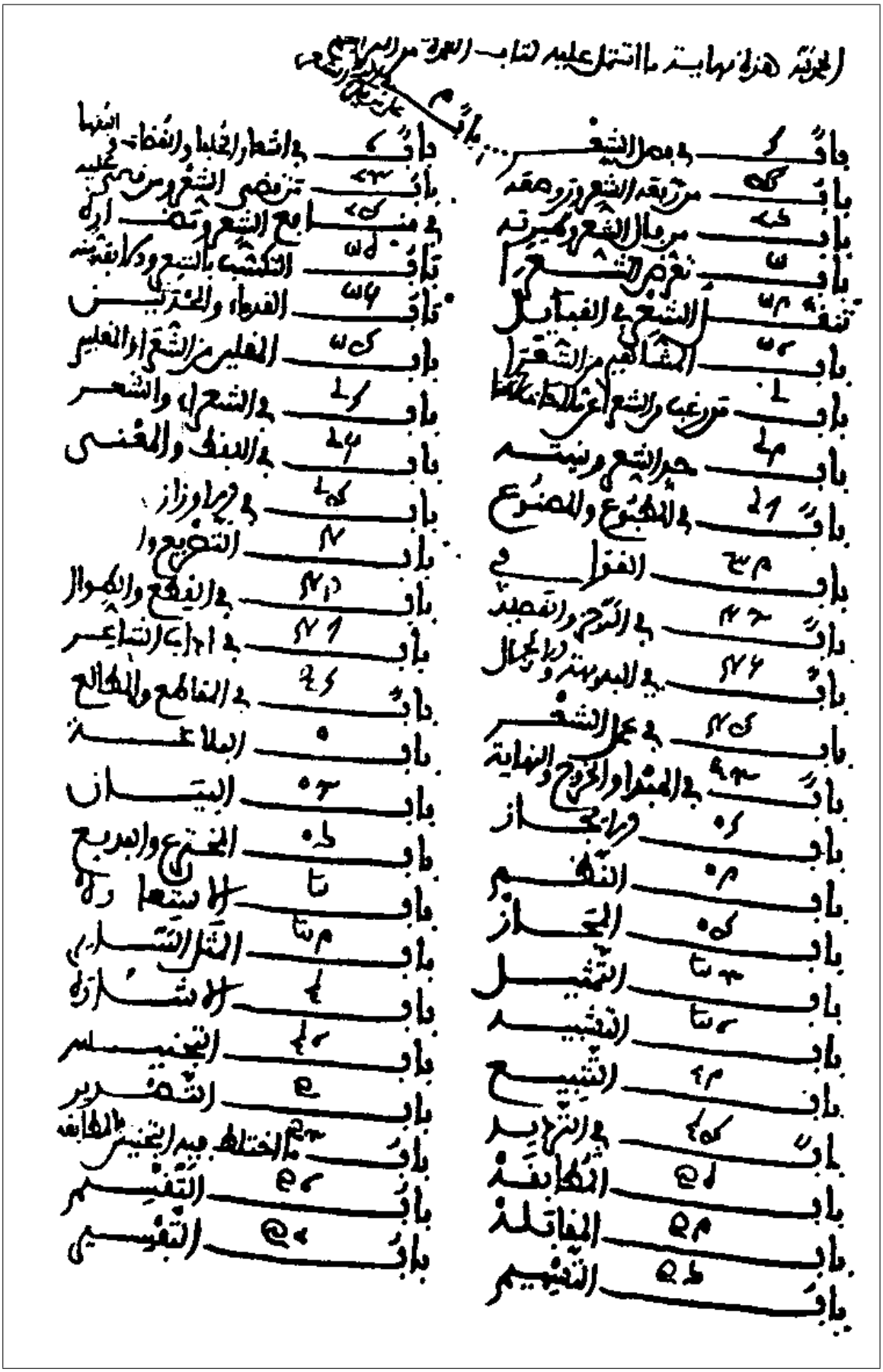
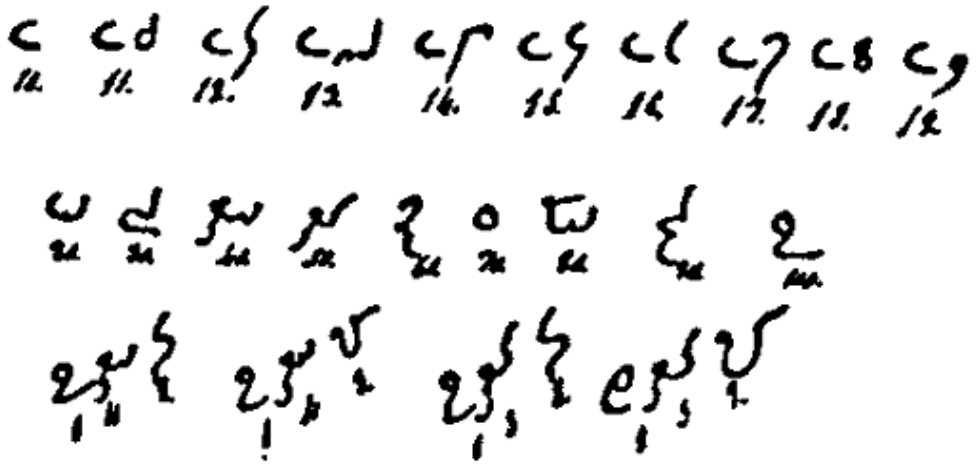


Figure 9: Uses example taking from [12]

parum dirutus, necesse fuit abscidere folia quæ inutilia evaserant; hæc suspitio confirmatur ex facto, quod folia octogesimo posteriora morsus vermium majores habent præcipue circa folium centesimum tertium; et notandum est hos morsus jam in codice existentes esse cum collatio, saltem secunda fiebat, nam quidam restaurati fuere, ut videtur ab antiquo et verba a vermibus plene corrosa in margine explicantur: charta antiquioris codicis major erat, nam a folio trigesimo nono usque ad centesimum quintum vestigia antiquioris foliorum numerationis existunt, nam numeratio ex magna parte abscisa fuit: in foliis secundæ partis nihil hujuscemodi numerationis distinguitur, et breves notæ marginales, præcipue verbum بلغت pluries in margine primæ partis scriptum fere evanuit.

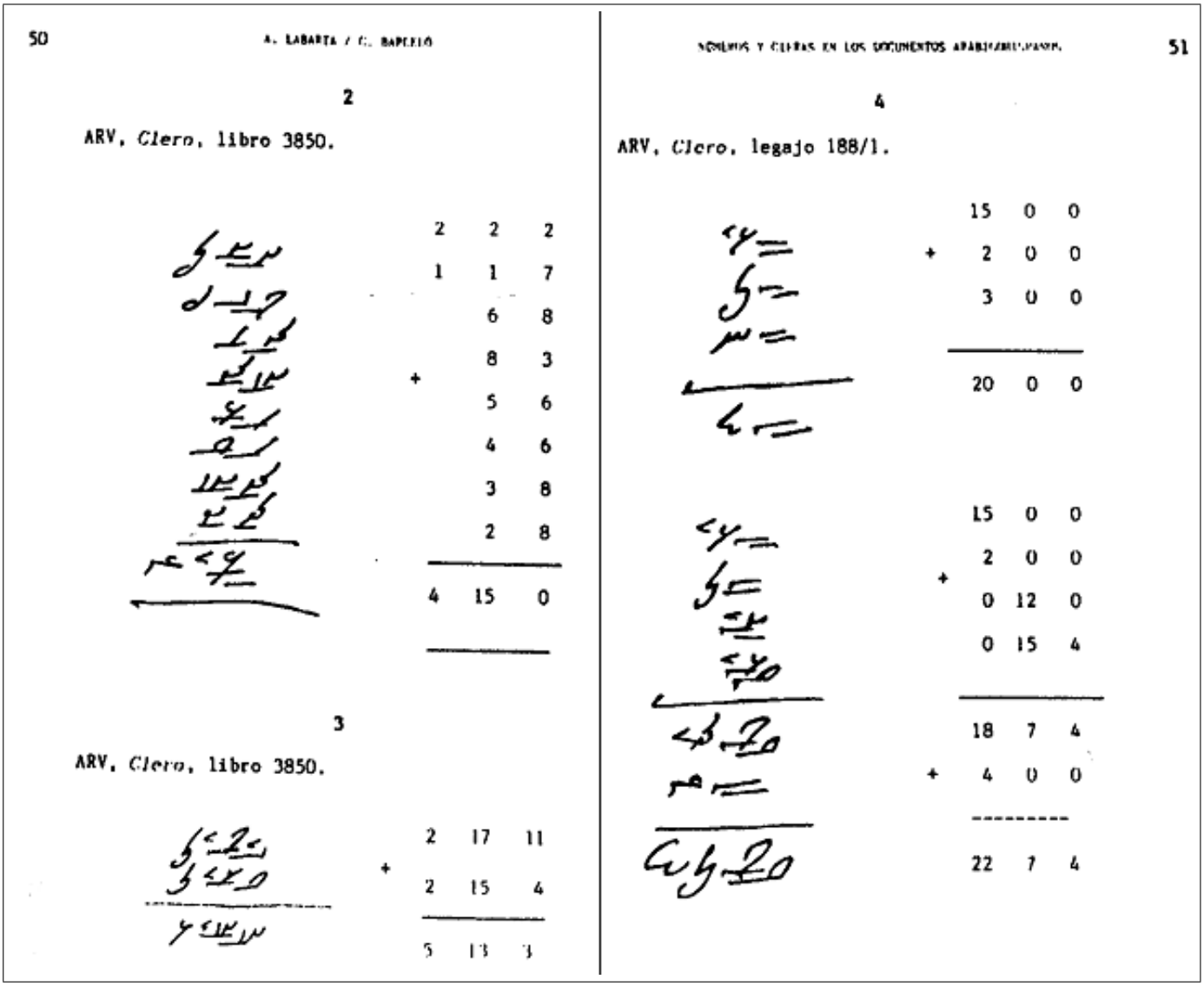
SPECIMEN NUMERATIONIS FOLIORUN IN CODICE ESCURIALENSI.



Foliorum numeratio. Codex habet foliorum numerationem modernam, factam postquam codex a librario numerationis arabicæ imperito compactus est; a folio enim secundo transilire necesse fuit ad folium decimum septimum et a folio vigesimo secundo iterum ad tertium recedere.

Ex foliorum numeratione antiqua codex centum sexaginta

Figure 10: Uses example taking from [12]



50 A. LABARTA / C. BARCELÓ

2

ARV, *Clero*, libro 3850.

	2	2	2
	1	1	7
		6	8
		8	3
+		5	6
		4	6
		3	8
		2	8
	4	15	0

3

ARV, *Clero*, libro 3850.

	2	17	11
+	2	15	4
	5	13	3

NOMBRES Y CIFRAS EN LOS DOCUMENTOS ARÁBIGOANDALUSÍES 51

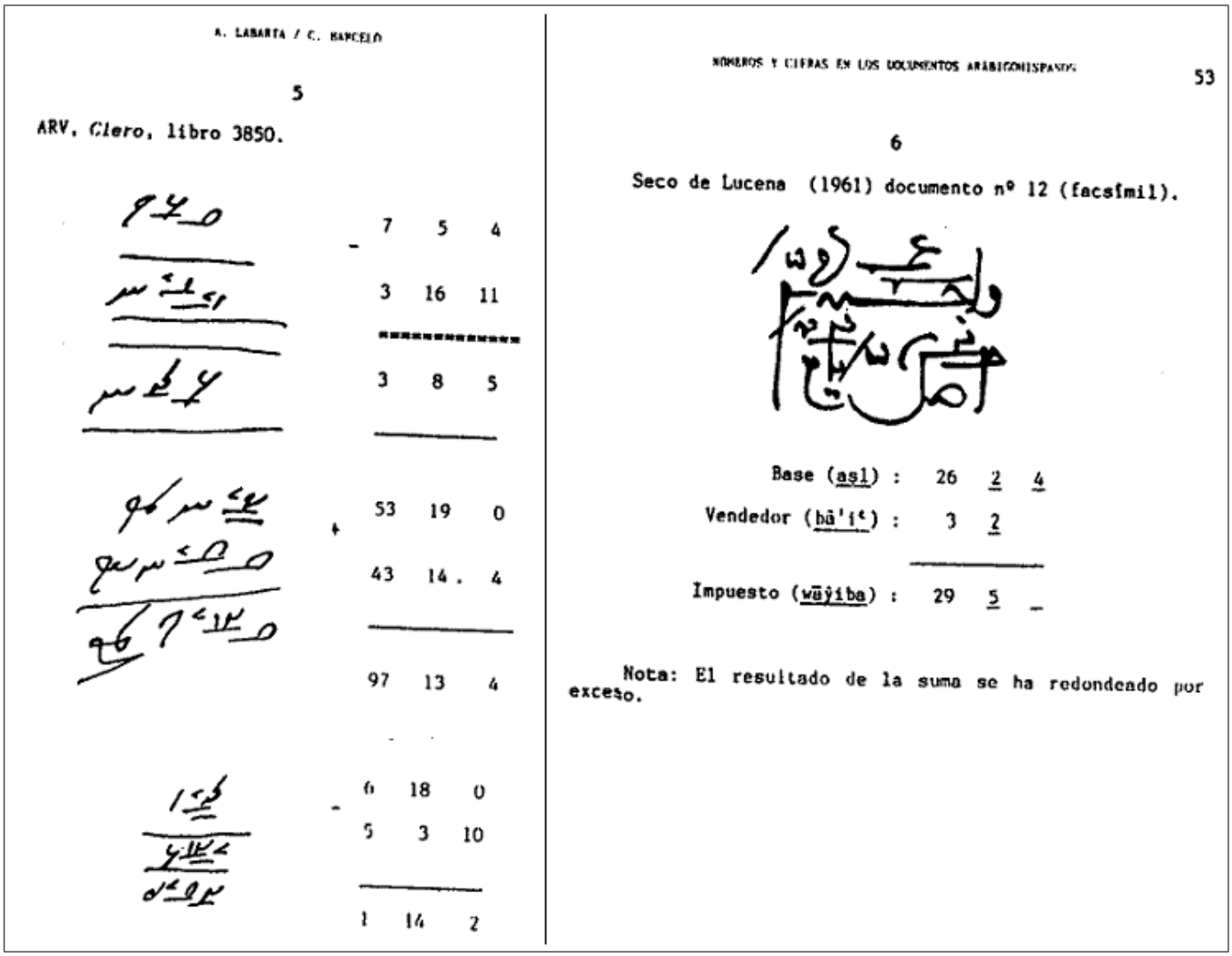
4

ARV, *Clero*, legajo 188/1.

	15	0	0
+	2	0	0
	3	0	0
	20	0	0

	15	0	0
	2	0	0
+	0	12	0
	0	15	4
	18	7	4
+	4	0	0
	22	7	4

Figure 11: Uses example taking from [12]



A. LABARTA / C. BARCELÓ

5

ARV, *Clero*, libro 3850.

7 5 4

3 16 11

3 8 5

53 19 0

43 14 . 4

97 13 4

6 18 0

5 3 10

1 14 2

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6

Seco de Lucena (1961) documento nº 12 (facsímil).

Base (aṣl) : 26 2 4

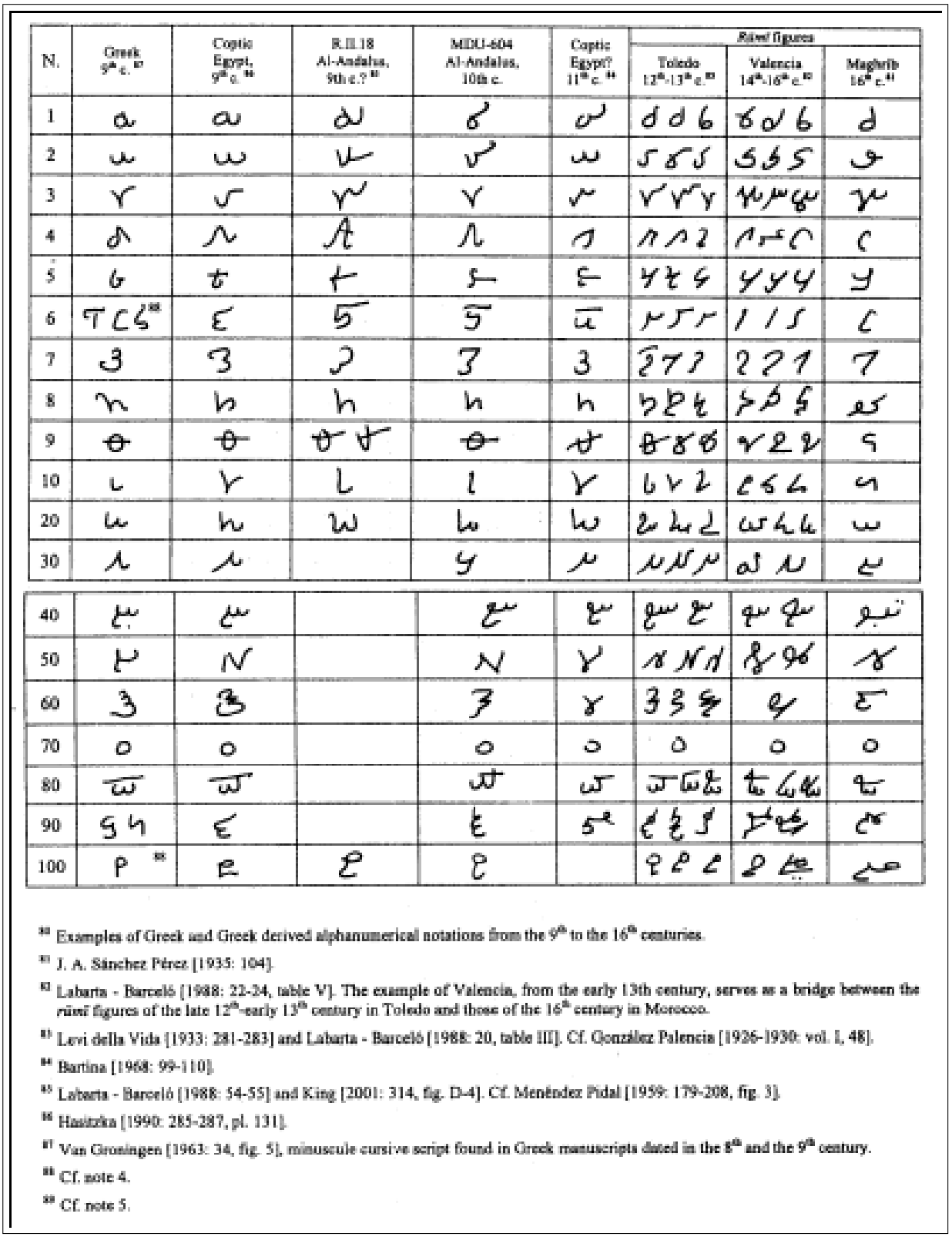
Vendedor (bā'iᶜ) : 3 2

Impuesto (wāẏiba) : 29 5 _

Nota: El resultado de la suma se ha redondeado por exceso.

Figure 12: Uses example taking from [12]

Figure 13: Comparised of some uses rumi system



N.	Greek 9th c. [80]	Coptic Egypt, 9th c. [81]	R.II.18 Al-Andalus, 9th c.? [82]	MDU-604 Al-Andalus, 10th c.	Coptic Egypt? 11th c. [83]	*Rūmī* figures		
						Toledo 12th-13th c. [82]	Valencia 14th-16th c. [82]	Maghrib 16th c. [83]
1								
2								
3								
4								
5								
6	[84]							
7								
8								
9								
10								
20								
30								
40								
50								
60								
70								
80								
90								
100	[85]							

[80] Examples of Greek and Greek derived alphanumerical notations from the 9th to the 16th centuries.

[81] J. A. Sánchez Pérez [1935: 104].

[82] Labarta - Barceló [1988: 22-24, table V]. The example of Valencia, from the early 13th century, serves as a bridge between the *rūmī* figures of the late 12th-early 13th century in Toledo and those of the 16th century in Morocco.

[83] Levi della Vida [1933: 281-283] and Labarta - Barceló [1988: 20, table III]. Cf. González Palencia [1926-1930: vol. I, 48].

[84] Bartina [1968: 99-110].

[85] Labarta - Barceló [1988: 54-55] and King [2001: 314, fig. D-4]. Cf. Menéndez Pidal [1959: 179-208, fig. 3].

[86] Hasitzka [1990: 285-287, pl. 131].

[87] Van Groningen [1963: 34, fig. 5], minuscule cursive script found in Greek manuscripts dated in the 8th and the 9th century.

[88] Cf. note 4.

[89] Cf. note 5.

Figure 14: Comparised of some uses rumi system taking from [1]