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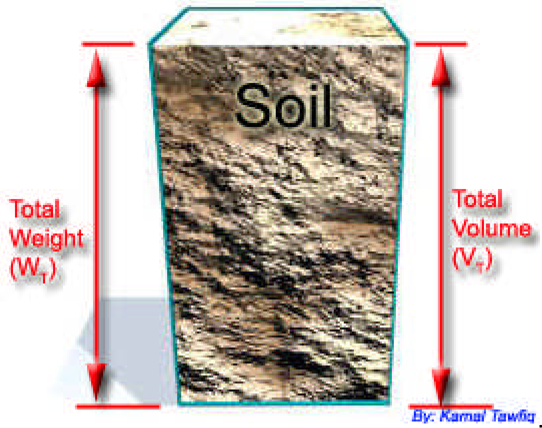
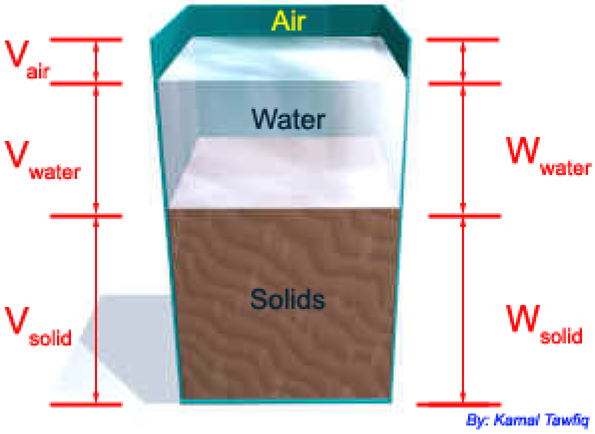
The physical state of a soil sample





Total Volume = $V_T = V_s + V_w + V_a$
Total Weight = $W_T = W_s + W_w$

Porosity (n): is the ratio of void volume.

$$\mathbf{n = V_v/V_T}$$

Void Ratio (e):is the ratio of void volume to solid volume. $\mathbf{e = V_v/V_s}$

now $\mathbf{n = V_v/V_T = V_v/\ V_v + V_s} = \dfrac{V_v/V_s}{V_v/V_s + 1} = \dfrac{e}{e+1}$

Note also that:

$$\mathbf{e = n / (1 - n)}$$
$$\mathbf{v = 1 / (1 - n)}$$

Note:

- ***n < 1 and is expressed as %***
- ***e may be any value greater or smaller than unity.***

Example: A soil has a total volume of 250ml and a void ratio of 0.872. What is the **volume of solids** of the sample?

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s}$$

$$250 - V_s = 0.872 V_s$$

$$250 = 1.872\ V_s$$

$$V_s = 133.55\ ml$$

Example: A soil has a porosity of 0.45. What is the value of its **void ratio**?

$$n = \frac{e}{1+e}$$

$$e = n(1+e) = n + ne \qquad \{e - ne = n\}$$

$$e(1 - 0.45) = 0.45$$

$$e = \underline{\underline{0.818}}$$

Degree of saturation (s):is the ratio of water volume to void volume.

$S_r = V_w/V_v$

if $S = 0$ dry soil ($V_w = 0$)

$S=100$ saturated soil ($V_w = V_v$)

$0 < S < 1 \rightarrow$ the soil is partially sat.

Water Content (w):is the ratio of water weight in a soil sample to the solids weight.

$$w_c = W_w / W_s$$

Specific gravity (GS): specific gravity of *soil solids* of a soil is defined as the ratio of the density of a given volume of the solids to the density of any equal volume of water at 4ºC.

$$G_S = \frac{\text{mass of a soil grain}}{\text{mass of an equal volume of water}}$$

$$G_s = \frac{\gamma_s}{\rho_w} = \frac{M_s / V_s}{\rho_w} = \frac{M_s}{V_s\ \rho_w}$$

Soil type	G
Gravel	2.65-2.68
Sand	2.65-2.68

Silt 2.66-2.7
Clay 2.68-2.8
Organic soils may be less than 2.0

Example: A sample of oven dried soil had a mass of 306g. The soil was broken down and placed into a jar of internal volume 1000 ml. Water at 20C was then poured into the jar and a rubber stopper placed to seal the jar. The soil and water were thoroughly mixed using an end over end shaker until all air had been removed. The jar was then topped up with water to the 1000ml mark and the total mass of the jar and its contents was found to be 1440.5g. The mass of the empty jar was 250g.

Determine the **particle specific gravity** of the soil.

Mass of soil + water = 1440.5 - 250 = 1190.5g
Mass of dry soil = 306g
Mass of water present with soil = 1190.5 - 306 = 884.5g
Mass of water present without soil = ρ_w x vol = 1×10^6 x 1000×10^{-6}
(g/m³) *(m³)*
= 1000g

$\Rightarrow$ Mass of water of same volume as soil = 1000 - 884.5 = $\underline{\underline{115.5g}}$

$$G_s = \frac{\text{Mass of soil}}{\text{Mass of water of same volume as soil}}$$

$$= \frac{306}{115.5}$$

$$G_s = \underline{\underline{2.65}}$$

or...

$$G_s = \frac{\text{Mass of dry soil}}{\{(\text{jar+ water}) - (\text{jar+ water+ soil})\} + \text{dry soil}}$$

$$(\text{jar + water}) = 1000 + 250 = 1250\text{g}$$

$$\Rightarrow G_s = \frac{306}{(1250 - 1440.5) + 306}$$

$$\Rightarrow G_s = \underline{\underline{2.65}}$$

<u>Air Content</u> (A):is the ratio of air volume to total volume.

The **air-voids volume, V_a**, is that part of the void space not occupied by water(is the ratio of air volume to total volume).

$$A_v = V_a / V$$
$$V_a = V_v - V_w$$
$$= e - e.S_r$$
$$= e.(1 - S_r)$$

Air-voids content, A_v

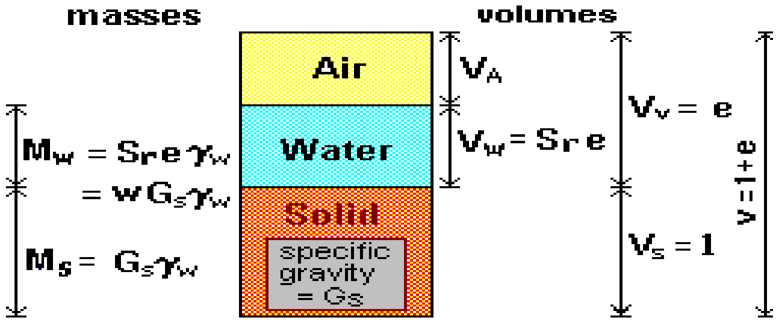
A_v = (air-voids volume) / (total volume)

$$= Va / V$$
$$= e.(1 - S_r) / (1+e)$$
$$= n.(1 - S_r)$$

For a perfectly *dry* soil: $A_v = n$
For a *saturated* soil: $A_v = 0$

<u>e in term of V, Ws, Gs</u>

$$e=\frac{V_v}{V_s}=\frac{V-V_s}{V_s}=\frac{V}{V_s}-1=\frac{V}{\frac{W_s}{G_s\gamma_w}}-1=\frac{G_s\gamma_w V}{W_s}-1$$



Bulk (Total)density (ρt)and Dry density (ρ_d):

$$\text{Dry density, } \rho_d = \frac{\text{Mass of solids}}{\text{Total volume}} = \frac{M_S}{V} = \frac{G_s\rho_w}{1+e}$$

$$\text{Bulk density, } \rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{M_S + M_w}{V} = \frac{G_s\rho_w + S_r e\rho_w}{1+e}$$

Bulk (Total)unit weight (γt)and Dry Unit weight (γ_d):

$$\text{Dry unit weight, } \gamma_d = \frac{\text{Dry weight}}{\text{Total volume}} = \frac{W_s}{V} = \frac{G_s\gamma_w}{1+e} = 9.81\,\rho_d$$

$$\text{Unit weight, } \gamma = \frac{\text{Total weight}}{\text{Total volume}} = \frac{W_s + W_w}{V} = \frac{G_s\gamma_w + S_r e\gamma_w}{1+e} = 9.81\,\rho$$

Saturated Unit weight (γ_s):for sat .soil S= 100% =1

$$\gamma_s=\frac{e+G_s}{1+e}\gamma_w$$

submerged unit weight γ' (or **buoyant unit**

$$\gamma' = \gamma_t - \gamma_w = \frac{G_s + Se}{1+e}\gamma_w - \gamma_w = \frac{G_s - 1 - e(1-S)}{1+e}\gamma_w \text{ (for **partially saturated**)} \quad (2.11)$$

$$\gamma' = \gamma_t - \gamma_w = \frac{G_s + e}{1+e}\gamma_w - \gamma_w = \frac{G_s - 1}{1+e}\gamma_w \text{ (for **fully saturated**)} \quad (2.12)$$

Ex1:

For a given soil, $w = 25\%$ and $\gamma_t = 18.5$ kN/m³ are measured. Determine void ratio e and degree of saturation S. Assume that G_s is 2.70.

Solution (a):

First assume $W_s = 100$ kN as shown in Figure 2.7a. Then, $W_w = 100 \times 0.25 = 25$ kN.

Calculate $V_s = W_s/G_s\gamma_w = 100/(2.7 \times 9.81) = 3.775$ m³.

Calculate $V_w = W_w/\gamma_w = 25/9.81 = 2.548$ m³.

Since $\gamma_t = 18.5$ kN/m³ $= (W_s + W_a)/(V_s + V_w + V_a) = (100 + 25)/(3.775 + 2.548 + V_a)$, thus, $V_a = 0.434$ m³.

Now, all components in the three phases are obtained as shown in Figure 2.7a and,

$e = (V_w + V_a)/V_s = (2.548 + 0.434)/3.775 = \mathbf{0.790}$ ←

$S = V_w/(V_w + V_a) = 2.548/(2.548 + 0.434) = 0.854 = \mathbf{85.4\%}$. ←

First assume $V = 10$ m³ as seen in Figure 2.7b.

From $W_s + W_w = W_s + wW_s = (1 + w)W_s = V\gamma_t = 10 \times 18.5 = 1\ 85$ kN,

$W_s = 185/(1 + 0.25) = 148$ kN, and $W_w = 185 - 148 = 37$ kN.

Using Gs as a bridge value, $V_s = W_s/G_s\gamma_w = 148/(2.7 \times 9.81) = 5.588$ m³.

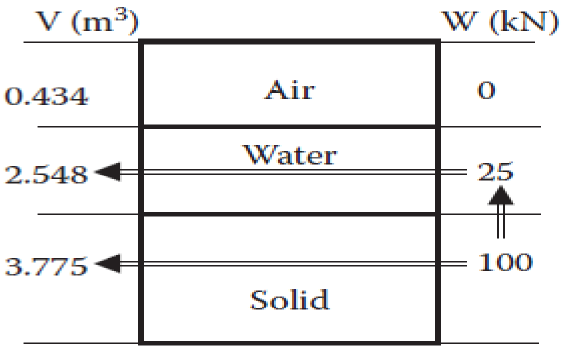
Using γ_w as a bridge value, $V_w = W_w/\gamma_w = 37/9.81 = 3.772$ m³.

Thus $V_a = V - (V_s + V_w) = 10 - (5.588 - 3.772) = 0.641$ m³.

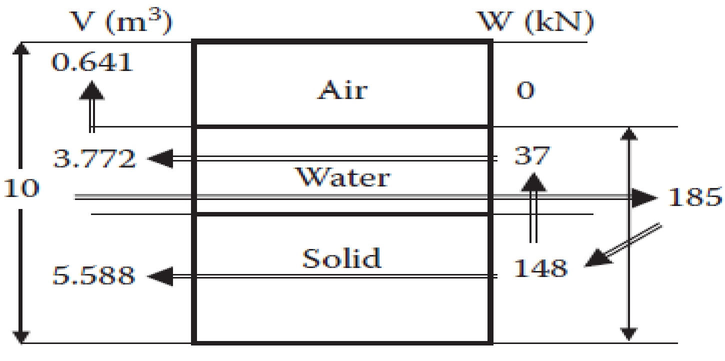
Now, all components in the three phase are obtained as shown in Figure 2.7b and,

$e = (V_w + V_a)/V_s = (3.772 + 0.641)/5.588 = \mathbf{0.789}$. ←

$S = V_w/(V_w + V_a) = 3.772/(3.772 + 0.641) = 0.855 = \mathbf{85.5\%}$. ←



Solution (a)



Solution (b)

EX2.

In a fill section of a construction site, 1500 m^3 of moist compacted soils is required. The design water content of the fill is 15%, and the design unit weight of the compacted soil is 18.5 kN/m^3. Necessary soil is brought from a borrow site, with the soil having 12% natural water content, 17.5 kN/m^3 wet unit weight of the soil, and $G_s = 2.65$. How much (in cubic meters) of the borrow material is required to fill the construction fill section? And how heavy is it?

Solution:

Draw three-phase diagrams of the fill site and the borrow site in Figure 2.8a and b, respectively.

First for the fill site in Figure 2.8a, $V = 1500$ m^3 so that $W_s + W_w = V\gamma_t = 1500 \times 18.5 = 27750$ kN.

$$W_s + W_w = (1 + w)W_s = 27750 \text{ kN, so that } W_s = 27750/(1 + 0.15) = 24130 \text{ kN.}$$

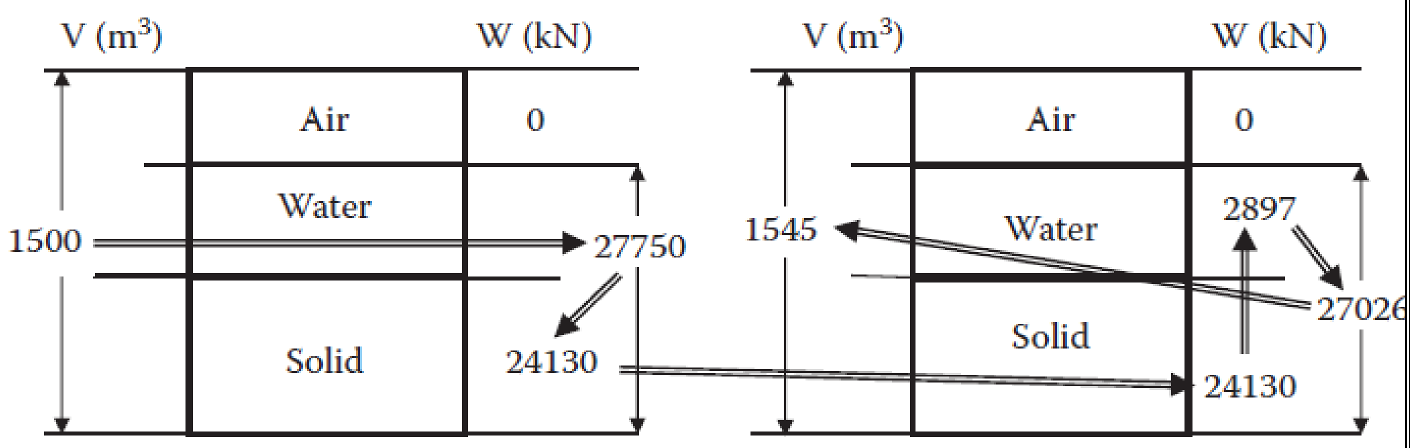
This much solid weight of the soil is required at the fill site.

At the borrow site, the same solid weight 24130 kN is needed as shown in Figure 2.8b.

Thus, $W_w = wW_s = 0.12 \times 24130 = 2897$ kN, and $W_s + W_w = 24130 + 2897 =$ **27026 kN**. ←

Since $\gamma_t = (W_s + W_w)/V = 17.5$ kN/m^3, $V = 27026/17.5 =$ **1545 m^3**. ←

Thus, 1545 m^3 of the borrow material is needed for the project carrying a total weight of 27026 kN.



(a) Fill site (b) Borrow site

Problem 3.1 A soil sample was collected for laboratory examination. It has a wet mass of 5.2 kg, bulk density of 1.65 g cm^{-3}, dry density of 1223 kg m^{-3} and degree of saturation of 82%. Determine the density of solids.

Solution

There are different ways to solve this problem; we will use the definitions of soil constituents. We will first find the mass of solids (i.e., the mass of dry soil), then the volume of solids and finally its density.

Volume of soil sample, $V = \frac{M}{\rho} = \frac{5.2}{1650} = 0.00315\,\text{m}^3$

Mass of dry sample, $M_d = \rho_d \cdot V = 1223 \cdot 0.00315 = 3.85\,\text{kg}$

Mass of water, $M_w = M_{soil} - M_d = 5.2 - 3.85 = 1.35\,\text{kg}$

Volume of water, $V_w = \frac{M_w}{\rho_w} = \frac{1.35}{1000} \approx 0.00135\,\text{m}^3$

From $S = \frac{V_w}{V_v} = 0.82$

We will obtain the volume of voids as

$$V_v = \frac{V_w}{S} = \frac{0.00135}{0.82} = 0.00164\,\text{m}^3$$

Then, the volume of solids equals

$$V_s = V - V_v = 0.00315 - 0.00164 \approx 0.0015\,\text{m}^3$$

Therefore, the density of solids is

$$\rho_s = \frac{M_d}{V_s} = \frac{3.85}{0.0015} = 2{,}566\,\text{kg m}^{-3}$$

Problem 3.2 Site investigation was performed to study soil conditions at a construction site in a new development area. A cylindrical soil sample (height = 100 mm, diameter = 50 mm) was collected at a depth of 1.5 m below the ground. The following soil characteristics were obtained: soil density was 1.52 t m^{-3}, moisture content was 68.2% and density of solid particles was 2.53 g cm^{-3}. Determine:

a) Weight of solids (in *N*)
b) Volume of air (in m^3).

Solution

Similar to Problem 3.1, there are different ways to solve it; we will use the definitions of soil constituents.

Volume of soil sample, $V = \pi r^2 h = 0.000196\,\text{m}^3$

Unit weight of soil, $\gamma = 1.52 \cdot 9.81 = 14.9\,\text{kN m}^{-3}$

Weight of soil, $W = \gamma \cdot V = 14.9 \cdot 1000 \cdot 0.000196 = 2.92\,\text{N}$

Weight of solids, $W_s = \dfrac{W}{1+w} = \dfrac{2.92}{1+0.682} \approx 1.74\,\text{N}$

Specific gravity, $G_s = \dfrac{\rho_s}{\rho_w} = \dfrac{2.53}{1} = 2.53$

Volume of solids, $V_s = \dfrac{W_s}{\gamma_s} = \dfrac{W_s}{G_s \cdot \gamma_w} = \dfrac{1.74}{2.53 \cdot 9.81 \cdot 1000} \approx 7 \cdot 10^{-5}\,\text{m}^3$

Weight of water, $W_w = W_s \cdot w = 1.74 \cdot 0.682 = 1.19\,\text{N}$

Volume of water, $V_w = \dfrac{W_w}{\gamma_w} = \dfrac{1.19}{9.81 \cdot 1000} \approx 0.00012\,\text{m}^3$

Volume of air, $V_a = V - V_s - V_w \approx 5.2 \cdot 10^{-6}\,\text{m}^3$

Problem 3.3 Soil excavated from a borrow pit is being used to construct an embankment (Fig. 3.3). The soil sample from the borrow pit has a specific gravity of 2.7 and unit weight of 17.8 kN m^{-3}. The weight of the sample was 3.5 N. The sample was then placed in an oven for 24 h at 105°C and its weight reduced to a constant value of 2.9 N.

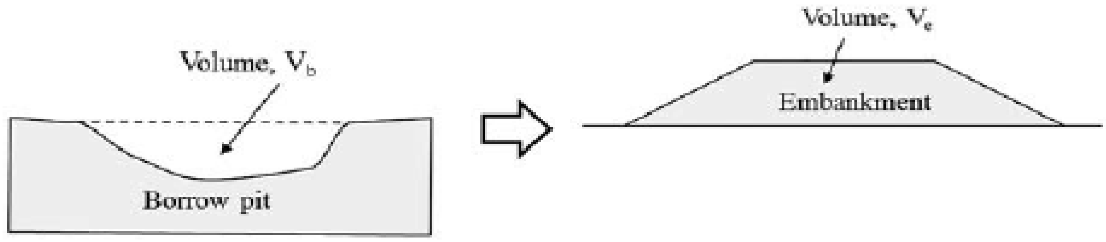


Figure 3.3 Borrow pit and embankment for Problem 3.3.

The soil at the embankment is required to be compacted to a void ratio of 0.71. If the finished volume of the embankment (V_e) is 80,000 m^3, what would be the volume of the soil (V_b) excavated at the borrow area?

Solution

Please note that there are different ways to solve this problem. This solution will deal with the volume of soil in the embankment (V), in the borrow pit (V_p) and soil void ratios (e_e and e_p, respectively). From the three phase diagram (Fig. 3.1), we can derive that the total volume of soil can be written as V = 1 + e.

Question: *How is the total volume (V) related to the void ratio (e)?*
Answer: For many problems related to soil constituents, it can be assumed that the volume of solids (V_s) is equal to 1 m^3 as it makes the solution work-out much easier. Then, from the definition of void ratio (Equation 3.1), the volume of voids (V_v) will be equal to e and thus the total volume of soil will be V = 1 + e.

It can be stated that

$$\frac{V_p}{V_e} = \frac{1+e_p}{1+e_e}$$

To find e_p, the following calculations involving soil water content and unit weight should be done:

$$\text{Weight of water, } W_w = W - W_s = 3.5 - 2.9 = 0.6\,\text{N}$$

$$\text{Water content, } w = \frac{W_w}{W_s} = \frac{0.6}{2.9} = 0.21$$

$$\text{Dry unit weight, } \gamma_d = \frac{\gamma}{1+w} = \frac{17.8}{1+0.21} = 14.7\,\text{kN m}^{-3}$$

$$\text{From } \gamma_{dry} = \frac{G_s \cdot \gamma_{water}}{1+e}$$

We will find that the void ratio of soil from the borrow pit equals

$$e = e_p \approx 0.796$$

Finally, we will have

$$\frac{V_p}{80,000} = \frac{1+0.796}{1+0.71}$$

Giving the volume of soil from the borrow pit

$$V_p \approx 84,019\,\text{m}^3$$

Problem 3.4 A 1 m thick soil with the initial void ratio of 0.94 was compacted by a roller and its thickness reduced by 0.09 m (Fig. 3.4). The specific gravity of this soil was 2.65. A 178 g soil sample was collected from the compacted soil mass to examine the degree of compaction; it was dried in an oven for 24 h and it had a dry mass of 142.4 g. Determine the degree of saturation after the compaction.

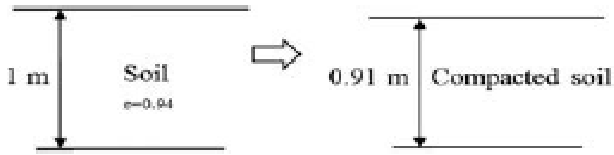


Figure 3.4 Changes in the soil layer thickness in Problem 3.4

Solution

Assume that a width of the soil mass before compaction is 1 m, then its volume is V = 1 m³

From the definition of void ratio

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s}$$

We will find the volume of solids as

$$V_s = \frac{V}{1+e} = \frac{1}{1+0.94} = 0.515\,\text{m}^3$$

It is logical to assume that the volume of solids remains the same after the compaction; however, the volume of voids would likely decrease.

The new volume of the compacted soil mass equals

$$V_{new} = 0.91 \cdot 1 = 0.91\,\text{m}^3$$

Then, the volume of voids will become

$$V_v = V_{new} - V_s = 0.91 - 0.515 = 0.395\,\text{m}^3$$

The new void ratio of compacted soil equals

$$e_{new} = \frac{0.395}{0.515} = 0.765$$

The water content will change to

$$w = \frac{178 - 142.4}{142.4} = 0.25$$

And the degree of saturation will become

$$S = \frac{w \cdot G_s}{e} = \frac{0.25 \cdot 2.65}{0.765} \approx 0.866 \text{ or } 86.6\ \%$$

Problem 3.5 A laboratory specimen of soil has a volume of 2.3 m^3. The void ratio of the sample is 0.712 and water content is 16.1%. The specific gravity of the solid particles is 2.7. Determine:

a) Volume of water
b) Mass of solids
c) Dry density
d) Bulk density

Solution

Volume of soil (V) consists of the volume of voids (V_v) and volume of solids (V_s), i.e.,

$$V = V_v + V_s = 2.3\,\text{m}^3 \qquad \text{(a)}$$

We also know (Equation 3.1) that

$$e = \frac{V_v}{V_s} = 0.712 \qquad \text{(b)}$$

Substituting V_s from Equation (b) to Equation (a), we get

$$1.712 V_v = 1.64$$

Therefore,

$$V_v = 0.96\,\text{m}^3 \text{, and } V_s = 1.34\,\text{m}^3$$

From $w = \dfrac{S \cdot e}{G_s}$

We will get the degree of saturation (S) as

$$S = \frac{wG_s}{e} = \frac{0.161 \cdot 2.7}{0.712} = 0.61$$

From $S = \dfrac{V_w}{V_v}$

We will find the volume of water

$$V_w = S \cdot V_v = 0.61 \cdot 0.96 \approx 0.59\,\text{m}^3$$

Mass of solids equals

$$M_s = \rho_s \cdot V_s = G_s \cdot \rho_w \cdot V_s = 2.7 \cdot 1000 \cdot 1.34 \approx 3618\,\text{kg}$$

Dry density (ρ_d) of soil will be

$$\rho_d = \frac{M_s}{V} = \frac{3618}{2.3} = 1573\,\text{kg/m}^3 = 1.57\,\text{g/cm}^3$$

Mass of water equals

$$M_w = \rho_w \cdot V_w = 1000 \cdot 0.59 \approx 590\,\text{kg}$$

From the definition of soil density, we have

$$\rho = \frac{M_s + M_w}{V} = \frac{3618 + 590}{2.3} \approx 1829.6\,\text{kg/m}^3$$

Problem 3.6 A cylindrical sample of clay, 50 mm (diameter) × 100 mm long, had weight of 3.5 N. It was placed in an oven for 24 h at 105°C. The sample weight reduced to a constant value of 2.9 N. If the specific gravity is 2.7, determine:

a) Void ratio
b) Dry unit weight
c) Degree of saturation

Solution

This problem will be solved using the aforementioned equations/relationships between the soil constituents.

Weight of water, $W_w = 3.5 - 2.9 = 0.6\,\text{N}$

Bulk unit weight, $\gamma_{bulk} = \dfrac{W}{V} = \dfrac{3.5 \cdot 10^{-3}}{196.4 \cdot 10^{-6}} \approx 17.8\,\text{kN m}^{-3}$

Water content, $w = \dfrac{W_w}{W_s} = \dfrac{0.6}{2.9} \approx 0.207\ or\ 20.7\ \%$

Dry unit weight, $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{17.8}{1+0.207} \approx 14.7\,\text{kN m}^{-3}$

From $\gamma_d = \dfrac{G_s}{1+e} \cdot \gamma_w$

We will get that

$$e \approx 0.8$$

Finally, the degree of saturation equals

$$S = \frac{w \cdot G_s}{e} = \frac{0.207 \cdot 2.7}{0.8} \approx 0.7\ or\ 70\ \%$$

Tutorial

1. For a given soil , show that $Gs = \dfrac{\gamma_{sat}}{\gamma_w - \omega_{sat}(\gamma_{sat} - \gamma_w)}$

Solution $Gs = \dfrac{\gamma_{sat}}{\gamma_w - \omega_{sat}(\gamma_{sat} - \gamma_w)}$

$\because \ \gamma_b = \gamma_{sat} - \gamma_w$

$$\therefore \ Gs = \frac{\gamma_{sat}}{\gamma_w - \omega_{sat}\gamma_b} = \frac{\frac{e+Gs}{1+e}\gamma_w}{\gamma_w - \omega_{sat}(\frac{Gs-1}{1+e}\gamma_w)}$$

$$Gs = \frac{\gamma_w \ \frac{e+Gs}{1+e}}{\gamma_w(1-\omega_{sat}(\frac{Gs-1}{1+e})} = \frac{e+Gs}{1+e-\omega_{sat}.Gs+\omega_{sat}}$$

For γ_{sat} $\rightarrow$ S = 100%

$\because \ Se = G_s\omega \rightarrow$ e = ω_{sat} Gs

$$\therefore \ Gs = \frac{e+Gs}{1+e-e+\omega_{sat}} = \frac{e+Gs}{1+\omega_{sat}} = \frac{\omega_{sat}Gs+Gs}{1+\omega_{sat}}$$

$$\therefore Gs = \frac{Gs(\omega_{sat}+1)}{(1+\omega_{sat})}$$

$\therefore$ Gs = Gs o.k.

2. For a given soil , show that $\omega_{sat} = \dfrac{n\gamma_w}{\gamma_{sat} - n\gamma_w}$

Solution $\omega_{sat} = \dfrac{n\gamma_w}{\frac{e+Gs}{1+e}\gamma_w - n\gamma_w}$

$\because n = \dfrac{e}{1+e}$

$$\omega_{sat} = \frac{\frac{e.\gamma w}{1+e}}{\frac{e+Gs}{1+e}\gamma w - \frac{e.\gamma w}{1+e}} = \frac{\frac{e.\gamma w}{1+e}}{\frac{\gamma w.e+Gs\gamma w-e.\gamma w}{1+e}}$$

$$\omega_{sat} = \frac{e.\gamma_w}{Gs.\gamma_w}$$

At $\omega_{sat} \rightarrow \omega_{sat}.Gs =$

$$\therefore \omega_{sat} = \frac{e}{Gs}$$

$\therefore \ \omega_{sat} = \omega_{sat}$ o.k.

3. For a given soil , the following are given : Gs= 2.67 ; moist. Unit weight $\gamma = 112 Ib/ft^3$; moisture content $\omega = 10.8\%$. Determine :

a-Dry unit weight
b-Void ratio
c-Porosity
d-Degree of saturation

Solution

$$\gamma_t = \frac{w_c + 1}{1+e}\gamma_w Gs$$

$$112 = \frac{0.108+1}{1+e}(62.4)(2.67)$$

$$\therefore \quad e = \frac{184.6}{112} - 1 = 0.6482$$

$$n = \frac{e}{1+e} = \frac{0.6482}{1+0.6482} = 0.393$$

$$\gamma_d = \frac{Gs.\gamma_w}{1+e} = \frac{(2.67)(62.4)}{1+0.6482} = 101.08 lb/ft^3$$

$$\because Se = G_s \omega$$

$$\therefore \; S = \frac{\omega_c .Gs}{e} = \frac{(0.108)(2.67)}{0.6482} = 44.48\%$$

4. For the soil describe in problem 2.6 , determine the weight of water , in pounds to be added per ft^3 of soil for saturation ?

Solution

1 ft^3 of soil weight 112 lb
Before saturation ω_c =0.108

$\because \; \omega_c = \frac{W_w}{W_s} \Rightarrow W_w = (\omega_c)(W_s)$ = 0.108 x 112 = 12.096 lb

After saturation S = 100% → $\because \; Se = G_s \omega$

$$\therefore \; e = G_s \omega \Rightarrow \omega_{cforsaturation} = \frac{e}{Gs} = \frac{0.6482}{2.67} = 0.2427$$

After saturation Ww = ω_c x Ws = 0.2427 x 112 = 27.1824 lb
∴ $Ww_{after} - Ww_{before}$ = 27.1824 – 12.096 = 15.08 lb added .

5. For a moist soil , given the following : V = 0.25 ft^3 ; W = 30.75 lb ; ω = 9.8% ; Gs = 2.66 . determine the following :

a. γ (lb/f^3)
b. γ_d(lb/f^3)
c. e
d. n
e. S
f. Volume occupied by water

Solution

a. $\gamma_t = \frac{W}{V} = \frac{30.75}{0.25} = 123 lb/ft^3$

c. $\because \gamma_t = \frac{\omega_c + 1}{1+e}\gamma_w Gs \Rightarrow 123 = \frac{0.098+1}{1+e}(62.4)(2.66)$

$\therefore$ e =0.4817

b. $\gamma_d = \frac{Gs}{1+e}\gamma_w = \frac{2.66}{1+0.4817}(62.4) = 112.02 lb/ft^3$

d. $n = \frac{e}{1+e}$ $\quad \therefore n = \frac{0.4817}{1+0.4817} = 0.325 = 32.5\%$

e. $Se = G_s\omega \rightarrow$ S x 0.4817 =2.66 x 0.098   S= 0.5411 =54.11%

f. $n = \frac{V_v}{V} \rightarrow 0.325 = \frac{V_v}{0.25} \Rightarrow V_v = 0.08125 ft^3 \quad \because \quad S = \frac{V_w}{V_v} \Rightarrow 0.5411 = \frac{V_w}{0.08125}$

$\therefore V_w = 0.0439 ft^3$

6. For a soil , given ρ_d = 1712 Kg/m^3 ; e = 0.51 determine
   a. n
   b. Gs

Solution

$$n = \frac{e}{1+e} = \frac{0.51}{1+0.51} = 0.3377 = 33.77\%$$

$$\gamma_d = \rho_d(9.81) = (1.712)(9.81) = 16.794 KN/m^3$$

$$\gamma_d = \frac{Gs\gamma_w}{1+e} \Rightarrow 16.794 = \frac{(Gs)(9.81)}{1+0.51}$$

$$Gs = 2.585$$

7. A soil has unit weight of 126.8 lb/ft^3. Given Gs = 2.67 and ω= 12.6% determine
a. dry unit weight (lb/ft^3)
b. void ratio
c. porosity
d. The weight of water in (lb/ft^3) of soil needed for full saturation .

Solution

$$\gamma_t = \frac{\omega_c + 1}{1+e}(Gs)(\gamma_w) \Rightarrow 126.8 = \frac{0.126+1}{1+e}(2.6)(62.4)$$

$$\therefore e = 0.4407$$

$$\gamma_d = \frac{Gs}{1+e}\gamma_w = \frac{2.6}{1+0.4407}(62.4) = 112.61 lb/ft^3$$

$$n=\frac{e}{1+e}=\frac{0.4407}{1+0.4407}=0.3058=30.58\%$$

for S = 100%

$$Se=\omega_c Gs \Rightarrow (1)(0.4407)=\omega_c(2.6)$$

$$\omega_c=0.1695=16.95\%$$

1 lb/ft3 of dry soil has

$$\omega_c=\frac{W_w}{W_s}\Rightarrow \because Ws=1lb$$

$$\therefore 0.1695=\frac{W_w}{1}\Rightarrow W_w=0.1695lb$$

8. The saturated unit weight of soil is 20.12 kN/m^3 . Given Gs = 2.74 , determine
   a. γ_{dry}
   b. e
   c. n
   d. ω_c

   Solution

$$\gamma_{sat}=\frac{e+Gs}{1+e}\gamma_w \Rightarrow 20.12=\frac{e+2.74}{1+e}(9.81)$$

$\therefore$ e = 0.657

$$\gamma_d=\frac{Gs}{1+e}\gamma_w=\frac{0.657+2.74}{1+0.657}(9.81)=16.22kN/m^3$$

$$n=\frac{e}{1+e}=\frac{0.657}{1+0.657}=0.3964=39.64\%$$

$$\because Se=\omega_c Gs$$

$\because$ S = 100%

$$\therefore (1)(0.6574)=\omega_c(2.74)\Rightarrow \omega_c=0.24=24\%$$

9. For a soil given e = 0.86 , ω_c = 28% and Gs = 2.72 determine
   a. moist unit weight (lb/ft3)
   b. degree of saturation (%)

Solution

$$\gamma_t=\frac{\omega_c+1}{1+e}(Gs)(\gamma_w)=\frac{0.28+1}{1+0.86}(2.72)(9.81)=18.362kN/m^3$$

$$\because Se=\omega_c Gs$$

S x 0.86 = 0.28 x 2.72

$\therefore$ S = 0.8855 = 88.55%

10. For a saturated soil ; given γ_d = 15.29 kN/m^3 ; and ω_c =21% ; determine
    a. $\gamma_{sat.}$
    b. e
    c. Gs
    d. γ_{moist} when the degree of saturation is 50%.

Solution

$$\gamma_{sat} = \frac{e + Gs}{1+e}(\gamma_w)$$

$\because Se = \omega_c Gs$ for S = 100%

1 x e = 0.21 x Gs

$$\therefore G_s = \frac{e}{0.21}$$

$$\gamma_d = \frac{Gs}{1+e}\gamma_w \Rightarrow \gamma_d = \frac{\frac{e}{0.21}}{1+e}(9.81) \Rightarrow 15.29 = \frac{\frac{e}{0.21}}{1+e}(9.81)$$

$\therefore$ e = 0.4865

$$\because G_s = \frac{e}{0.21} = \frac{0.4865}{0.21} = 2.316$$

$$\gamma_{sat} = \frac{e + Gs}{1+e}(\gamma_w)$$

$$\gamma_{sat} = \frac{0.4865 + 2.316}{1+0.4865}(9.81) = 18.49 kN/m^3$$

For 50% = S

$\because Se = \omega_c Gs$

$\therefore$ 0.5 x 0.4865 = ω_c x 2.316

$\therefore$ ω_c = 0.105 = 10.5%

$$\gamma_t = \frac{\omega_c + 1}{1+e} Gs \gamma_w = \frac{0.105+1}{1+0.4865}(2.316)(9.81) = 16.889 kN/m^3$$

Or:

$$\gamma_t = \frac{Gs + se}{1+e}\gamma_w = \frac{2.316 + (0.5)(0.4865)}{1+0.4865}(9.81) = 16.889 kN/m^3$$

Measurement of Soil Properties

1. The in-situ density of a soil is 1.85 Mg/m³. A moisture content determination test on a sample of the soil gave the following results.

Test No.	**Mass of tin (g)**	**Tin + wet soil (g)**	**Tin + dry soil (g)**
1	20.24	30.61	28.73
2	20.36	32.44	30.28

Determine the **moisture content** and **dry density** of the soil.

$$w = \frac{(wet+tin) - (dry+tin)}{(dry+tin) - (tin)} = \frac{30.61 - 28.73}{28.73 - 20.24} = 22.1\%$$

$$w = \frac{(wet+tin) - (dry+tin)}{(dry+tin) - (tin)} = \frac{32.44 - 30.28}{30.28 - 20.36} = 21.8\%$$

Average, $w = \underline{\underline{22\%}}$

$$\rho_d = \frac{\rho_b \times 100}{100 + w} = \frac{185 \times 100}{122} = \underline{\underline{1.52\ \text{Mg/m}^3}}$$

2. The bulk density of a soil sample was found to be 1.90 g/ml and the moisture content 12%.

Determine the **dry density**, **void ratio** and **degree of saturation** if the particle specific gravity was 2.68.

What would the **moisture content** be if the soil were completely saturated at the same void ratio?

$$\rho_d = \frac{\rho_b \times 100}{100 + w} = \frac{1.9 \times 100}{112} = 1.7 \text{ g/ml} = \underline{\underline{1.7 \text{ Mg/m}^3}}$$

$$\rho_d = \frac{\rho_w G_s}{1 + e}$$

$$1 + e = \frac{\rho_w G_s}{\rho_d}$$

$$e = \frac{2.68}{1.7} - 1$$

$$\text{e} = \underline{\underline{0.58}}$$

$$\rho_b = \frac{G_s + eS_r}{1 + e} \qquad (\text{N.B. } \rho_w = \text{ density of water } = 1 \text{ Mg/m}^3)$$

$$1.9 = \frac{(2.68 + 0.58S_r)}{1.58}$$

$$S_r = \frac{3 - 2.68}{0.58} = \underline{\underline{56\%}}$$

$$\text{Saturated, } \Rightarrow e = wG_s$$

$$w = \frac{e}{G_s} = \frac{0.58}{2.68} = \underline{\underline{21.6\%}}$$

3. A sample of saturated clay has a volume of 245ml and, after oven drying, has a mass of 453g.

If the particle specific gravity of the soil is 2.75, determine the **dry** and **saturated unit weights** of the soil in its natural state.

$$G_s = \frac{M_s}{V_s \rho_w}$$

$$2.75 = \frac{453 \times 10^{-6}}{V_s \times 1}$$

$$V_s = \frac{453 \times 10^{-6}}{2.75} = 164.7 \text{ x } 10^{-6} \text{ m}^3 = \underline{\underline{164.7 \text{ ml}}}$$

$$\text{Now, } V_v = V - V_s = 245 - 164.7 = \underline{\underline{80.3 \text{ ml}}}$$

$$e = \frac{V_v}{V_s} = \frac{80.3}{164.7} = 0.488$$

$$\gamma_d = \frac{\gamma_w G_s}{1+e} = \frac{9.81 \times 2.75}{1.488} = 18.1 \ \underline{\underline{\text{kN/m}^3}}$$

$$\gamma_{sat} = \gamma_w \left(\frac{G_s + e}{1+e} \right) = 9.81 \left(\frac{2.75 + 0.488}{1.488} \right) = 21.4 \ \underline{\underline{\text{kN/m}^3}}$$

4. During a particle specific gravity test on a soil sample the following masses were recorded:

Mass of dry soil sample = 450g

Mass of pycnometer when full of water = 1875g

Mass of pycnometer + soil sample and full of water = 2160g

Determine the **particle specific gravity** of the soil.

$$G_s = \frac{\text{Mass of dry soil}}{\{(\text{jar+ water}) - (\text{jar+ water+ soil})\} + \text{dry soil}}$$

$$\Rightarrow G_s = \frac{450}{(1875 - 2160) + 450}$$

$$\Rightarrow G_s = \underline{\underline{2.73}}$$

5. A sand deposit was found to have a bulk density of 1.93 Mg/m³ and a moisture content of 16%. Laboratory tests established that the maximum and minimum void ratio values were 0.75 and 0.48 respectively. If the particle specific gravity was 2.65, determine the **void ratio**, the **degree of saturation** and the **relative density** of the deposit.

$$\rho_d = \frac{\rho_b \times 100}{100 + w} = \frac{1.93 \times 100}{116} = 1.66 \text{ Mg/m}^3$$

$$\& \ \rho_d = \frac{\rho_w G_s}{1 + e}$$

$$\Rightarrow e = \frac{2.65}{1.66} - 1 = \underline{\underline{0.596}}$$

$$\rho_b = \rho_w \left(\frac{G_s + eS_r}{1 + e} \right)$$

$$1.93 = \frac{2.65 + 0.596 S_r}{1.596}$$

$$\Rightarrow S_r = \underline{\underline{72.2\%}}$$

$$R.D. = \frac{e_{\max} - e}{e_{\max} - e_{\min}} = \frac{0.75 - .596}{0.75 - 0.48} = \underline{\underline{0.57}}$$