

Lecture #12

- Critical resolved shear stress
- Slip by dislocation movement
- Deformation produced by motion of dislocations (Orowan's Eq.)

References:

1- Derek Hull, David Bacon, (2001), Introduction to Dislocations, fourth edition, Butterworth-Heinemann.

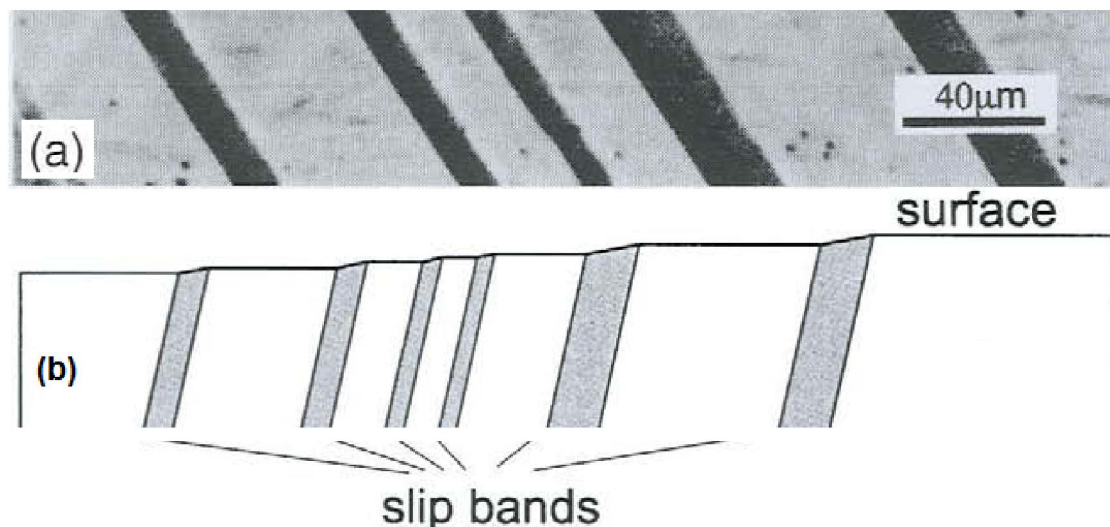
2- Hosford W.F, (2005), Mechanical Behavior of Materials, Cambridge

3- Meyers M.A. and Chawla K.K., (2009). Mechanical Behavior of Materials, Prentice-Hall.

4- Dieter G.E., (1986), Mechanical Metallurgy, McGraw-Hill

Critical resolved shear stress :

Slip results in the formation of steps on the surface of the crystal. These are readily detected if the surface is carefully polished before plastic deformation.



(a) Straight slip bands on a single crystal of 3.25 per cent silicon iron.

(b) Sketch of a section across the slip bands normal to surface shown in (a). Each band is made up of a large number of slip steps on closely spaced parallel slip planes.

Erich Schmid discovered that if a crystal is stressed, slip begins when the shear stress on a slip system reaches a critical value, τ_c , often called the

critical resolved shear stress. In most crystals, slip occurs with equal ease forward or backward, so a characteristic shear stress is required for slip. Consider a crystal with a cross-sectional area A_0 being deformed in tension by an applied force F along the axis of the cylindrical crystal.

Plastic deformation of a single crystal is initiated at a critical stress, the critical resolved shear stress (CRSS). This is the same stress at which dislocations begin to move.

NORMAL Force: $F_N = F \cos \phi$

SHEAR Force: $F_S = F \cos \lambda$.

• Area of slip-plane: $A_S = A_0 / \cos \phi$

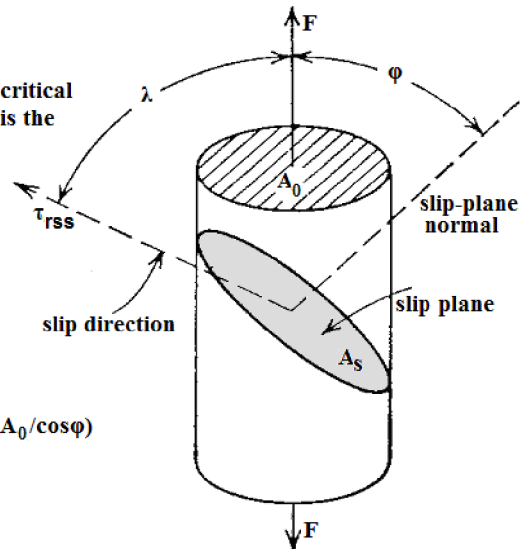
NORMAL Stress to the ϕ -plane: $\sigma = F_N / A_S$

$$= (F \cos \phi) / (A_0 / \cos \phi) = \sigma \cos^2 \phi$$

SHEAR Stress in the ϕ -plane: $\tau_s = F_S / A_S = (F \cos \lambda) / (A_0 / \cos \phi)$
 $= \sigma \cos \phi \cos \lambda$.

The shear stress causes slip to occur.

$$\tau_{crss} = \sigma \cos \phi \cos \lambda$$



λ is the angle between the slip direction and the tensile axis
 ϕ is the angle between the tensile axis and the slip-plane normal

where $\cos \phi \cos \lambda$ is **Schmid Factor**. The active slip system will have the largest Schmid factor.

$$\tau_{crss} = \tau_{max} = \sigma / 2 \quad \text{for} \quad \lambda = \phi = 45^\circ$$

$$\tau_{crss} = 0 \quad \text{for} \quad \lambda = \phi = 90^\circ \quad (\text{NO slip, so fracture})$$

Example 1: A tensile stress of $\sigma = 5$ kPa is applied parallel to the $[432]$ direction of a cubic crystal. Find the shear stress, τ , on the $(11\bar{1})$ plane in the $[011]$ direction.

SOL. First find $m = \cos \lambda \cos \phi$ for $(11\bar{1})[011]$ slip. In cubic crystals, the normal to a plane has the same indices as the plane, so the normal to $(11\bar{1})$ is $[11\bar{1}]$. Also, in cubic crystals, the cosine of the angle between two directions is given by the dot product of unit vectors in those directions. Therefore
 $\cos \phi = (4 \cdot 1 + 3 \cdot 1 + 2 \cdot \bar{1}) / [(4^2 + 3^2 + 2^2)^{1/2} (1^2 + 1^2 + \bar{1}^2)^{1/2}]$
 $= 5 / (\sqrt{29}\sqrt{3}),$

$$\cos \lambda = (4 \cdot 0 + 3 \cdot 1 + 2 \cdot 1) / [(4^2 + 3^2 + 2^2)^{1/2} (0^2 + 1^2 + 1^2)^{1/2}]$$

$$= 5 / (\sqrt{29} \sqrt{2}),$$

$$m = \cos \lambda \cos \phi = 25 / (29 \sqrt{6}) = 0.352,$$

$$\tau = m \sigma = 0.352 \times 5 \text{ kPa} = 1.76 \text{ KPa}.$$

Example 2 Determine the tensile stress that is applied along the $[1\bar{1}0]$ axis of a silver crystal to cause slip on the $(1\bar{1}\bar{1})[0\bar{1}1]$ system. The critical resolved shear stress is 6 MPa.

The angle between tensile axis $[1\bar{1}0]$ and normal to $(1\bar{1}\bar{1})$ is

$$\cos \phi = \frac{(1)(1) + (-1)(-1) + (0)(-1)}{\sqrt{(1)^2 + (-1)^2 + (0)^2} \sqrt{(1)^2 + (-1)^2 + (-1)^2}} = \frac{2}{\sqrt{2} \sqrt{3}} = \frac{2}{\sqrt{6}}$$

The angle between tensile axis $[1\bar{1}0]$ and slip direction $[0\bar{1}\bar{1}]$ is

$$\cos \lambda = \frac{(1)(0) + (-1)(-1) + (0)(-1)}{\sqrt{2} \sqrt{(0)^2 + (-1)^2 + (-1)^2}} = \frac{1}{\sqrt{2} \sqrt{2}} = \frac{1}{2}$$

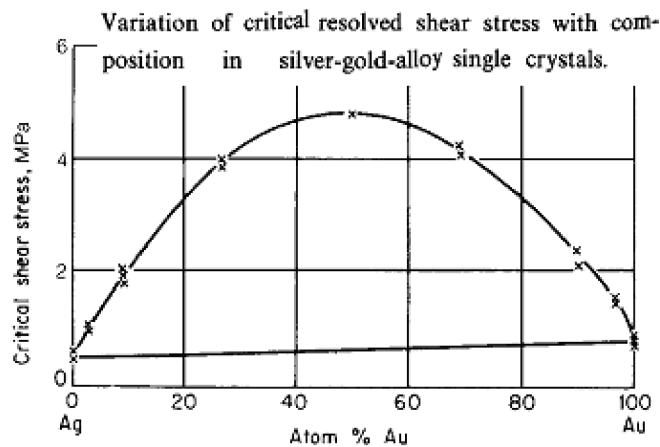
So:
$$\sigma = \frac{\tau_{\text{crss}}}{\cos \phi \cos \lambda} = \frac{6}{2/\sqrt{6} \times \frac{1}{2}} = 6\sqrt{6} = 14.7 \text{ MPa}$$

- Impurities **increase** critical resolved shear stress (see Cu & Ag)

Room-temperature slip systems and critical resolved shear stress for metal single crystals

Metal	Crystal structure	Purity, %	Slip plane	Slip direction	Critical shear stress, MPa
Zn	hcp	99.999	(0001)	$[11\bar{2}0]$	0.18
Mg	hcp	99.996	(0001)	$[1120]$	0.77
Ag	fcc	99.99	(111)	$[110]$	0.48
		99.97	(111)	$[110]$	0.73
		99.93	(111)	$[110]$	1.3
Cu	fcc	99.999	(111)	$[110]$	0.65
		99.98	(111)	$[110]$	0.94
Fe	bcc	99.96	(110)	$[111]$	27.5
			(112)		
			(123)		

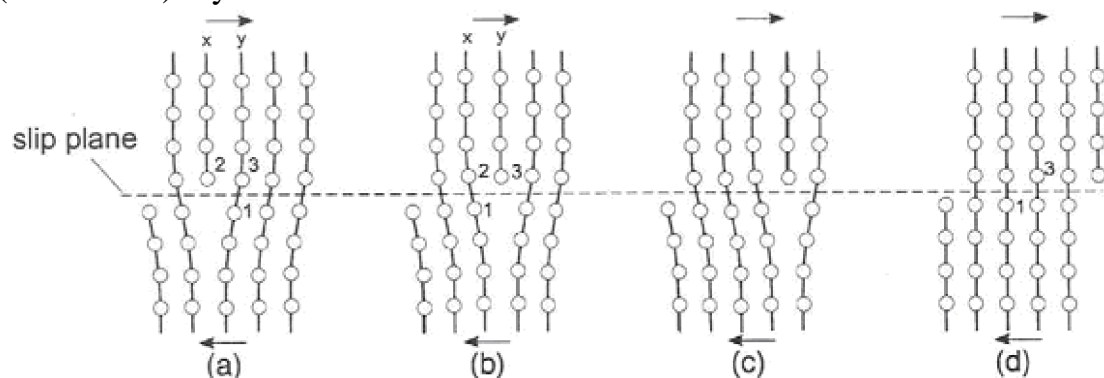
- Alloying element additions (to produce a solid solution) have their effect on **increasing** the critical resolved shear stress (see Au & Ag).



- The magnitude of the critical resolved shear stress is **determined** by the **interaction** of its dislocations with each other and with defects (as vacancies, interstitials, and impurity atoms), so critical resolved shear stress **decreases** as the density of defects **decreases**, **provided that the total number of imperfections is not zero** (critical resolved shear stress of soft metals can be reduced to less than one third by increasing the purity). At the other extreme, tensile tests on whiskers (dislocation-free) have given strengths which are approximately equal to the calculated strength of a perfect crystal.

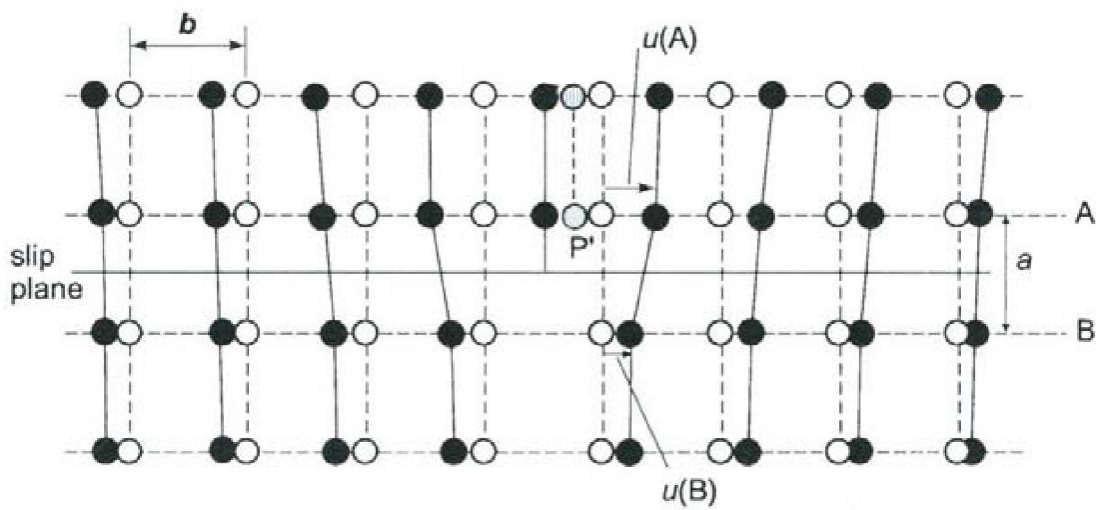
Slip by dislocation movement :

Dislocation motion works as a deformation mechanism because relatively little atomic motion is required compared with the process of slip for perfect (defect free) crystals.



Movement of an edge dislocation: the arrows indicate the applied shear stress. When the extra half plane of atoms reaches a free surface, it results in a slip step of one Burgers vector or one atomic distance for the simple cubic lattice.

The applied stress required to overcome the lattice resistance to the movement of the dislocation is the **Peierls-Nabarro stress**. It is the applied resolved shear stress required to make a dislocation glide in an otherwise perfect crystal. It arises as a direct consequence of the periodic structure of the crystal lattice and depends sensitively on the form of the force-distance relation between individual atoms, i.e. on the nature of the interatomic bonding. It is a function of the core structure and, for this reason; a unique analytical expression for the Peierls stress cannot be derived.



$$\tau_{P-N} = \frac{2G}{(1-\nu)} \exp\left(\frac{-2\pi w}{b}\right)$$

$$\approx \frac{2G}{(1-\nu)} \exp\left(\frac{-2\pi a}{(1-\nu)b}\right)$$

When: w - width of dislocation; a - distance between slip planes and b - distance between atoms in the slip direction.

τ_{P-N} vary with crystal structure. In general $\tau_{P-N} \ll \tau_{theoretical}$

The Peierls stress is very sensitive to the atomic position at the core of dislocation (affected by the width in the equation above), and as the

positions are not known with a high degree of accuracy the equation cannot be used for precise calculation. The equation is accurate enough to show that the stresses needed to move the dislocation in a metal is quite low.

Wide dislocation \Rightarrow low stress to move it (as in ductile metals)

Narrow dislocation \Rightarrow high stress to move it (as in ceramics).

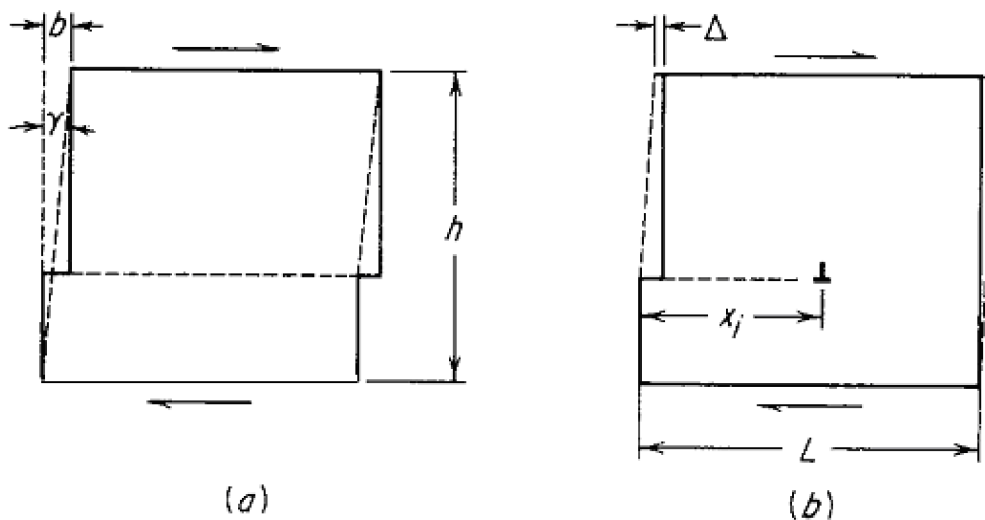
The fact that slip occurs in close-packed directions means that b is minimized, and the Peierls stress will be lower. If $a < b$, as for loosely packed planes, the Peierls stress would be high. This provides a basis for the observation that slip occurs most readily on close-packed planes in the close-packed directions. When the crystal structure complex, without highly close-packed planes and directions, dislocations tend to be immobile. This causes the brittleness and high hardness of intermetallic compounds.

Deformation produced by motion of dislocations (Orowan's Eq.)

(a)- If a single dislocation passes through a cubic crystal, the resulting strain:

$$\text{Shear strain, } \gamma = b/h$$

With N dislocations, $\gamma = N.b/h$ this expression assumes that all dislocations pass all of the way through the crystal. In reality we can not make that assumption. Thus we need to revise our estimate.



(b)- When dislocations pass partially through a crystal:

Assuming that all of the dislocations move a distance x_i along the crystal length, the strain equations will be:

The displacement for a dislocation at an intermediate position between $x_i = 0$ and $x_i = L$ would be proportional to the fractional displacement x_i/L .

So :

- single dislocation: $\gamma = \frac{x_i b}{Lh}$

- multiple dislocation: $\gamma = \frac{b}{Lh} \sum_i^N x_i$

If N dislocations move an average distance \bar{x} then,

$$\gamma = \frac{N\bar{x}b}{Lh}$$

$L * h =$ area of the end of the crystal

$$\text{Dislocation density} = \rho_{\perp} = \frac{N}{Lh} = \frac{\# \perp \text{ lines}}{\text{area}}$$

$$\therefore \gamma = \rho_{\perp} b \bar{x}$$

ρ_{\perp} - The total dislocation line length per unit volume or the number of dislocation lines that cut a unit cross-sectional area.

The shear strain rate associated with this type of motion is:

$$\dot{\gamma} = \frac{d\gamma}{dt} = \rho_{\perp} b \frac{dx}{dt} = \rho_{\perp} b v$$

Where: v is the dislocation velocity. This equation is the Taylor- Orowan relation, which relates dislocation motion to strain rate.