Generalized Transmission Line Equations

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Abstract: Generalized transmission line equations (GTLEs) are derived by circuit theory, and equation parameters are determined by the method of moments (MoM). In comparison with conventional transmission line equations (CTLEs), the new equations have added two new terms expressed by dependent series voltage and shunt current sources. For an infinite-length uniform transmission line, the GTLEs are the same as the CTLEs since two coefficients for the two added terms in the GTLEs are found to be zero. For a finite-length uniform transmission line or nonuniform transmission line, the GTLEs, however, are quite different from the CTLEs since two coefficients for the two added terms in the GTLEs are found to be nonzero. In words, the GTLEs are modifications to the CTLEs.

Introduction

It is known that the derivation of conventional transmission line equations (CTLEs) is based on such an assumption of an infinite-length transmission line and the CTLEs are extended into an infinite-length nonuniform transmission line without any mathematical derivation. Unfortunately, practical transmission lines are finite-length. When the CTLEs are used in a finite-length unmatched uniform transmission line or arbitrary length nonuniform transmission line, the description of the CTLEs for such line discontinuities needs further scrutiny. The reason is that when the nonuniform transmission line (including continuously varying transmission line) is generally treated as a cascading of many short uniform transmission lines, the discontinuities between any two neighbouring segments are not only generate reflections, but also produce radiations. Although the radiation from the sharp discontinuities is observed for a long time, no one has given us the transmission line equations that can take account of both reflections and radiations. In this paper, based on the finite-length line concept, we derive generalized transmission line equations (GTLEs) by using circuit theory. However, the coefficients of the GTLEs need to be determined by numerical methods, such as moment of methods (MoM). In comparison with the CTLEs, the CTLEs have added two new terms that express dependent series voltage and shunt current sources, respectively. For an infinite-length uniform transmission line, the GTLEs are the same as the CTLEs since two coefficients for the two added terms in the GTLEs are found to be zero. For a finite-length uniform transmission line or nonuniform transmission line, the GTLEs, however, are quite different from the CTLEs since two coefficients for the two added terms in the GTLEs are found to be nonzero. In words, the GTLEs are modifications to the CTLEs.

Generalized Transmission Line Equations

For simplicity, we start with a 1-D finte-length nonuniform transmission line. As mentioned above, the nonuniform transmission line could be regarded as the cascading of infinitely short segments of the uniform transmission line with different characteristic parameters. For each segment, the per-unit length series impedance Z and per-unit length shunt admittance Y are different. For an infinite-length nonuniform transmission line, the CTLEs are

$$\begin{aligned} \frac{\partial v(l)}{\partial l} &= -Z(l)i(l) \\ \frac{di(l)}{dl} &= -Y(l)v(l) \end{aligned} \tag{1}$$

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where the per-unit length series impedance Z and shunt admittance Y are a function of position *l*. If the transmission line is uniform, Z and Y in eq. (1) would be constant. For the nonuniform transmission line, eq. (1) is not perfect because the local radiation generated by the transmission line itself discontinuities has not been considered yet. When working frequency is getting higher, such a radiation becomes significant, especially for interconnect of the high-speed transmission lines.

Now we begin to derive new equations of a finite-length nonuniform transmission line by means of the circuit theory. Let us consider an infinitely short segment with length Δl in the nonuniform transmission line. This segment can be regarded as a circuit with two ports. Thus, the transmission matrix in the circuit theory can be used to express this circuit,

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix} \tag{2}$$

where v_1, i_1 and v_2, i_2 are the voltages and currents in input and output ports, respectively. It should be pointed out that the input current i_1 and output current i_2 defined in (2) have the same reference direction so that it is convenient to derive the current differential equation. Eq. (2) can be rewritten as

$$\begin{cases} \dfrac{v_2 - v_1}{\Delta l} = -\dfrac{B}{\Delta l} i_2 + \dfrac{(1-A)}{\Delta l} v_2 \\ \dfrac{i_2 - i_1}{\Delta l} = -\dfrac{C}{\Delta l} v_2 + \dfrac{(1-D)}{\Delta l} i_2 \end{cases} \tag{3}$$

Let the short section length $\Delta l \to 0$, the new nonuniform transmission line equation can be approximately obtained,

$$\begin{cases} \dfrac{dv}{dl} = -Z(l)I(l) + a(l)v(l) \\ \dfrac{di}{dl} = -Y(l)v(l) + b(l)i(l) \end{cases} \tag{4}$$

where v and i are the average voltage and current for each infinite short segment of the transmission line. For the lossless case, $Z = \dfrac{B}{\Delta l} = jwL$ and $Y = \dfrac{C}{\Delta l} = jwC$. L and C are the per unit length series inductance and shunt capacitance. $a(l)$ and $b(l)$ are the coefficients for the per unit length series dependent voltage source and shunt dependent current source. Compared with eq.(1), two additional terms are added into eq.(4), which stand for the local radiation of the nonuniform transmission line. However, for an infinite uniform transmission line, the radiation parameter a and b should be found to be zero. It should be emphasized that the transmission line parameters will be directly extracted by the transmission line equation in (4) rather than by the transmission matrix parameters in (2). For an arbitrary shaped lossless nonuniform transmission line, all the parameters, L, C, a, and b can be determined by solving eq.(4) two times through the MoM program to obtain two linear independent solutions of the voltage and current. In practice, two solutions of current and voltage distributions along the transmission line are first calculated from two arbitrary loads and then substituted into eq.(4) to find parameters L, C, a, and b. Usually, one solution is for a short load, another for an open load.

Examples

The validity of the new telegrapher equation can be verified by the following examples. In all the examples, the transmission line is assumed to be lossless. The first example is for a uniform microstrip line. The second example is for a right angle bend built up by the above microstrip line. For both structures, relative dielectric constant e_r is 9.8, height h between metal strip and metal ground plate is 0.635 mm, thickness t of the metal strip is 2 mm, width w of the metal strip is 0.6 mm. The MoM software of the Zeland IE3D has been used to calculate the currents and voltages of the above two structures, respectively. For the uniform microstrip line, the extracted parameters L and C are constant and the radiation parameters a and b equal zero. For the right angle microstrip bend, the extracted parameters L and C along the bend are vary significantly and the radiation parameters a

and b are not equal to zero. Fig.1 shows comparison of the inductance between the uniform line and right angle bend. Fig.2 shows comparison of the capacitance between the uniform line and right angle bend. For the uniform transmission line in figures 1 and 2, the per unit length inductance and per unit length capacitance extracted by the new equation are compared with the data calculated by TEM semi-experimence formulas and are of agreement well. Fig.3 shows comparison of the radiation parameter a between the uniform line and right angle bend. Fig.4 shows comparison of the radiation parameter b between the uniform line and right angle bend. The above figures express that the right angle bend

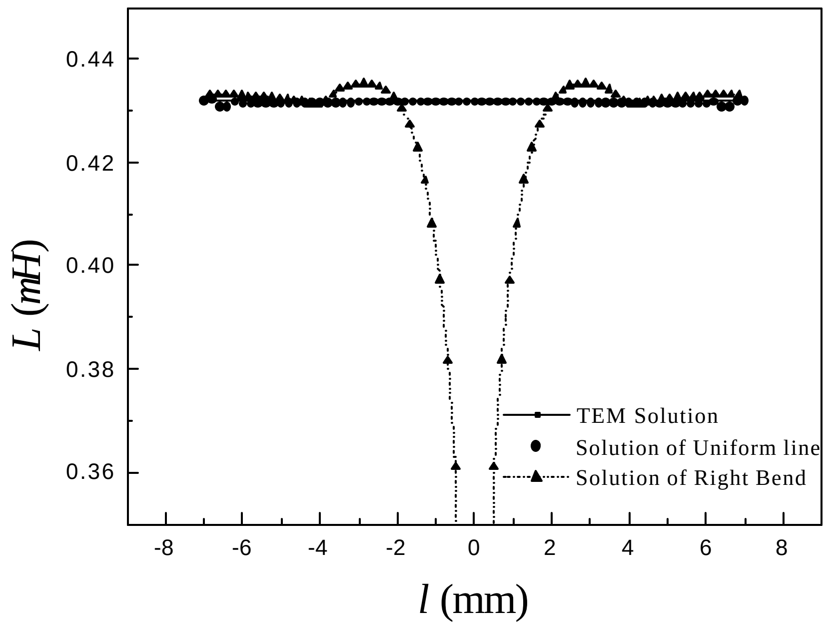

Figure 1. Comparison of inductance between uniform line and right angle bend

not only produces the reflection of the current and voltage due to the variation of L and C, but also generates the local radiation due to the values of a and b. All the data for Fig.1 - Fig.4 were calculated under the frequency of 1.1 GHz. Figures 5 and 6 show the variation of radiation parameters a and b with frequency. It is obvious that the radiation parameters a and b increase with the frequency. These results coincide well with the physical phenomena of radiation.

Conclusions

The GTLEs for a finite-length transmission line is derived by circuit theory and the coefficients of the GTLEs need to be determined by the MoM. In comparison with conventional transmission line equation, the new equation has added two terms representing the local radiation of the transmission line itself.

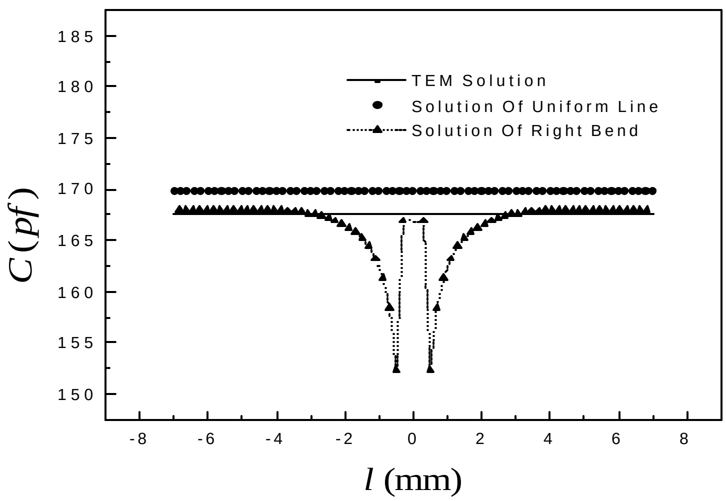

Figure 2. Comparison of capacitance between uniform line and right angle bend

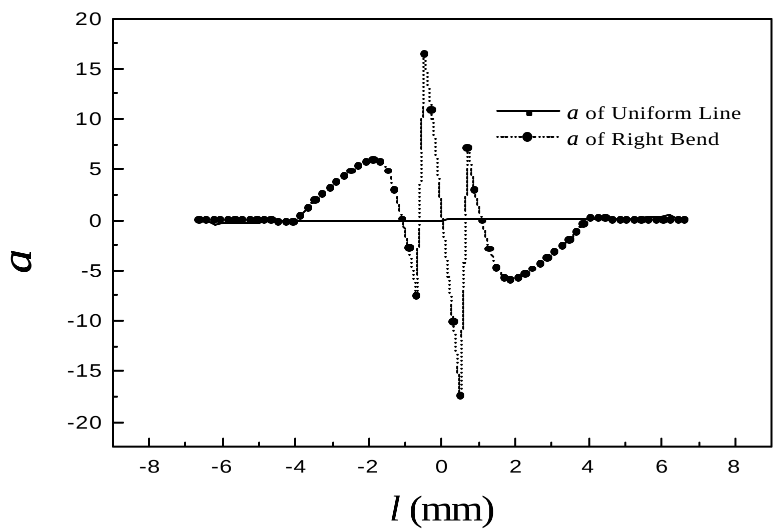

Figure 3. Comparison of a between uniform line and right angle bend