Statistical study of Non Uniform Transmission Lines

I – Summary

The main purpose of this part is the study of the statistical behaviour of the output current of a non-uniform transmission line (NUTL) above a ground plane, submitted to an external field. Non-uniform transmission lines' random geometries are generated through a Fourier transform representation of the height [III-1].

The NUTL is of length L and its projections l_i, resulting from various configurations, are calculated via Cauchy' formula related to the length of a bended wire [III-2].

The lumped elements approach on the MTL theory has been implemented to determine the output current from each configuration of the transmission line. The NUTL is approximated by cascaded series of many short sections of uniform lines having different characteristic impedances and forcing terms on adjacent sections. The contribution of the external waves to a partial section of the line is expressed in terms of equivalent lumped voltage and current sources using the conventional TL theory [II-1, II-2].

Simulations are carried out with FEKO (MoM) and comparisons are made with experiments performed within the TEM cell and the SEC2 (relatively free space environment) for some random configurations.

The statistical behaviour of the output current is then estimated by running Monte-Carlo simulations and the statistical moments are estimated via the stochastic collocation method.

The sensitivity of the output current to the TL's geometry is analysed via a method of "Global sensitivity analysis" in particular Sobol 'indices [VIII-1, VIII-2].

II – Chain parameter Matrix approach for non-uniform transmission lines submitted to an incident field

A simple approximated approach for solving the transmission lines' equations for a non-uniform TL is to approximate it as a discretely uniform TL. This neglects any interaction between the sections, which may be due to fringing of the fields at the junctions between the uniform sections.

The NUTL is broken into a cascade of sections with different characteristic impedances, each of which can be modelled approximately as a uniform line characterized by a chain-parameter matrix Φ_k. The contribution of the external waves to a partial section of the line is expressed in terms of equivalent lumped voltage and current sources.

The overall chain-parameter matrix of the entire line can be obtained as the product of the chain-parameter matrices of the individual uniform sections.

The formulation and analysis method is based on the quasi-TEM approach.
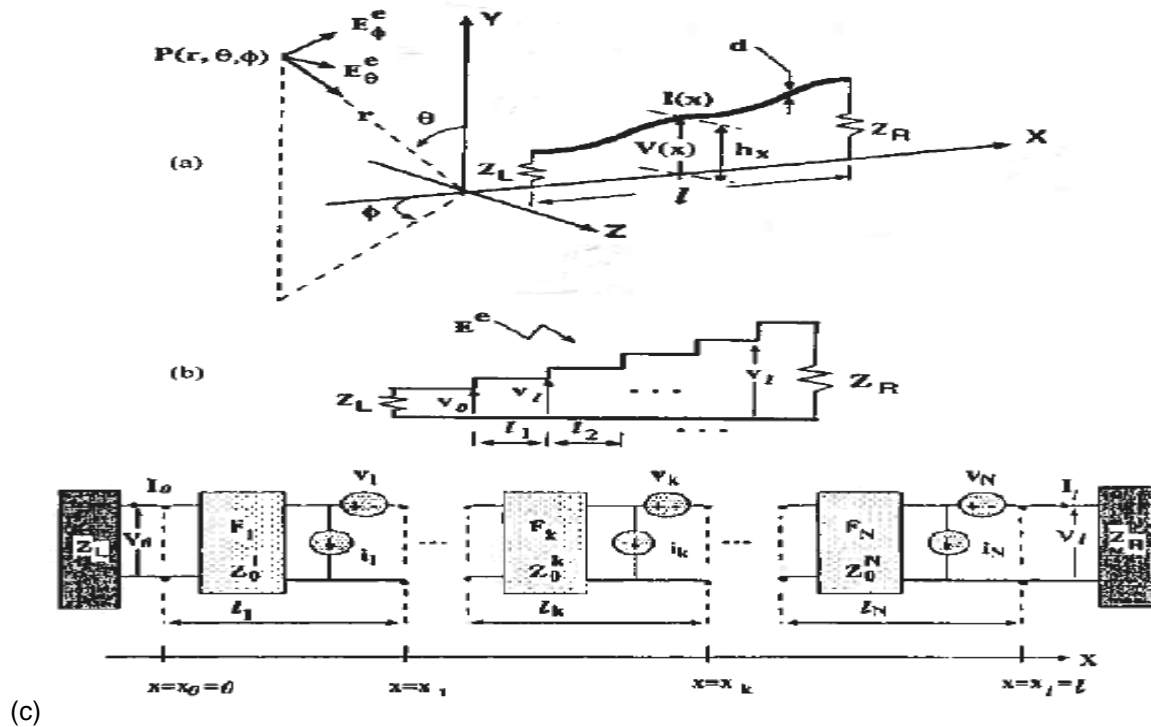
Formulas can be found in [II-1].

Fig1 [II-2]-(a) Geometry of a nonuniform TL excited by an external electromagnetic wave
(b) Piecewise uniform representation
(c) Circuit representation of nonuniform TL

References:
II-1-Clayton Paul: "Analysis of Multiconductor transmission lines-Second edition"
II-2-Omid, Kami: "Field coupling to Non-uniform and uniform transmission lines"

III – Random geometry of the TL

The height is considered via its Fourier representation:

$$h_\alpha(z) = h_{mean} + \sum_{k=1}^{N} \alpha_k \sin[k\pi(z - z_0)] \tag{1}$$

where α_k are uniformly distributed random variables.

The projection l_i of L for each configuration is calculated using Cauchy formula:

$$L = \int_{z_0}^{li} \sqrt{1 + \left(h_\alpha^{'}(z)\right)^2} \, dz \tag{2}$$

Fig2 :Calculation of the length of a bended cable

Since L, the length of the uniform line, is known, a dichotomy process can be used to determine the projection l_i.

Note that for a line with too many curves, this formula will not be applicable anymore.

We considered a cable of length L=30cm (blue in figure 3): the main reason was to make it fit into the TEM cell. We also have to find a compromise between the frequency ranges of the measurements instruments and L.

For this relatively small dimension, many $\alpha_{k=1:12}$ are needed to bend the wire whereas for a length of 1m, $\alpha_{k=1:4}$ could be sufficient.

The method described above has been used to generate random configurations: config3, config4, config5.
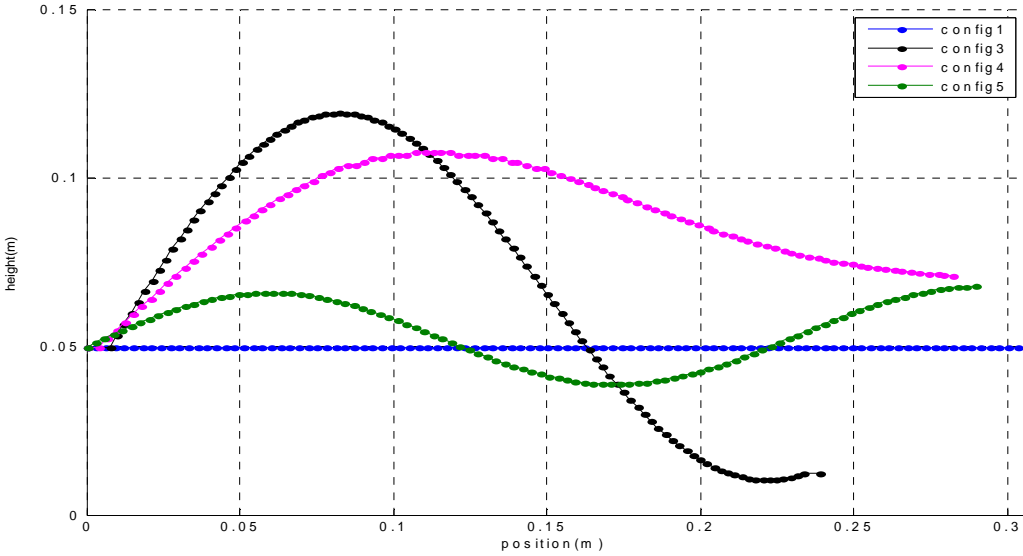
Fig3:Generation of the random configurations

These configurations above a ground plane were reproduced with a copper wire above a metal plate and used for the experimental validation.

config3

config4

config5

config1

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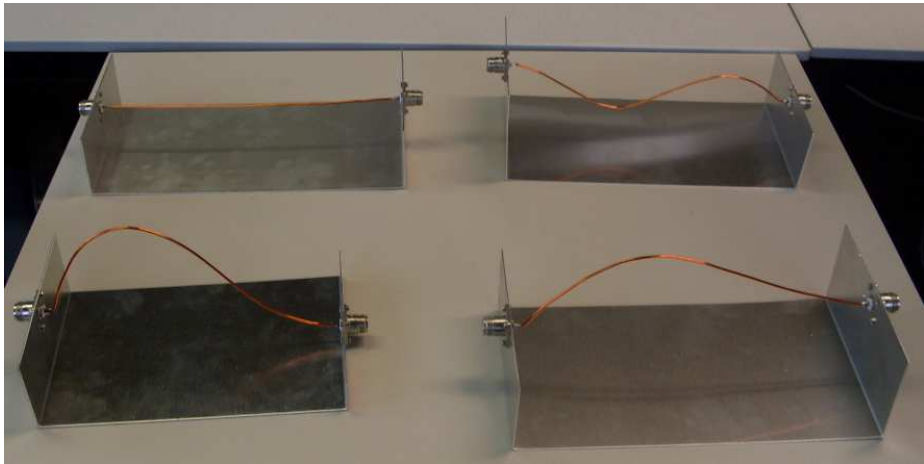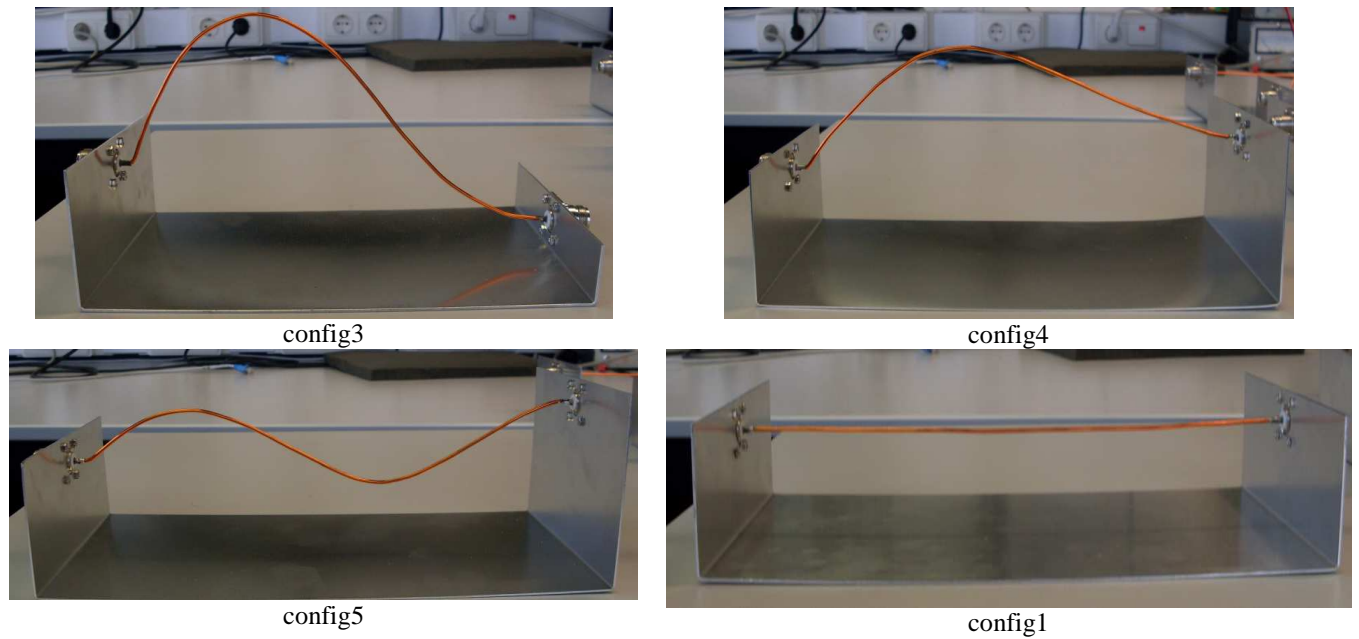
Fig4: Reproduction of the random configurations

References:
III-1-Sy and al.: "Probabilistic study of the coupling between deterministic electromagnetic fields and a stochastic thin-wire over a PEC plane".
III-2-Cauchy

IV – Experiments

So far, experiments have been performed within the TEM cell and the SEC2: deterministic field.
Experiments within the VIRC will be done as soon as the VIRC will be available.

1-TEM cell

Frequency range: 30MHz-1GHz
Tests performed for the sidefire, broadside and endfire configurations of the electric field.
However, the length of the cables, makes it difficult to have reliable results.
Indeed, for the sidefire excitation, the cables are almost in contact with the roof of the TEM cell whereas it is recommended to put the equipment under test at maximum of H/3. The drawback with the broadside and endfire excitation is the use of the cable to connect the output the cable to the feed through panel.
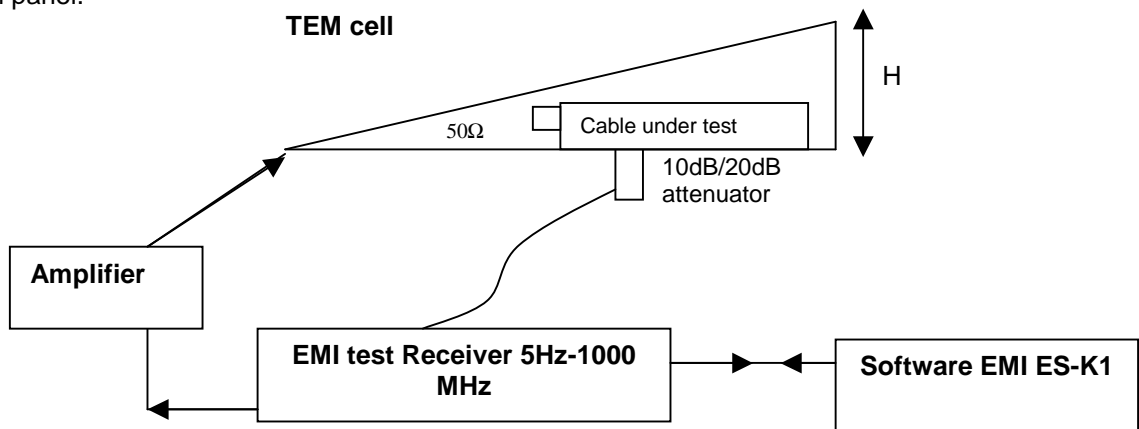
Fig5: Set-up for the experiments in the TEM cell
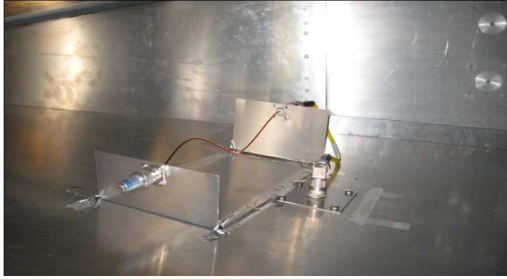
Broadside excitation
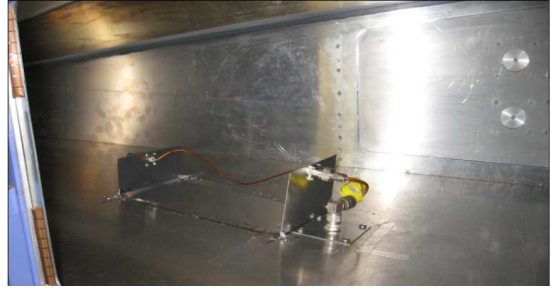
Endfire excitation

Sidefire excitation
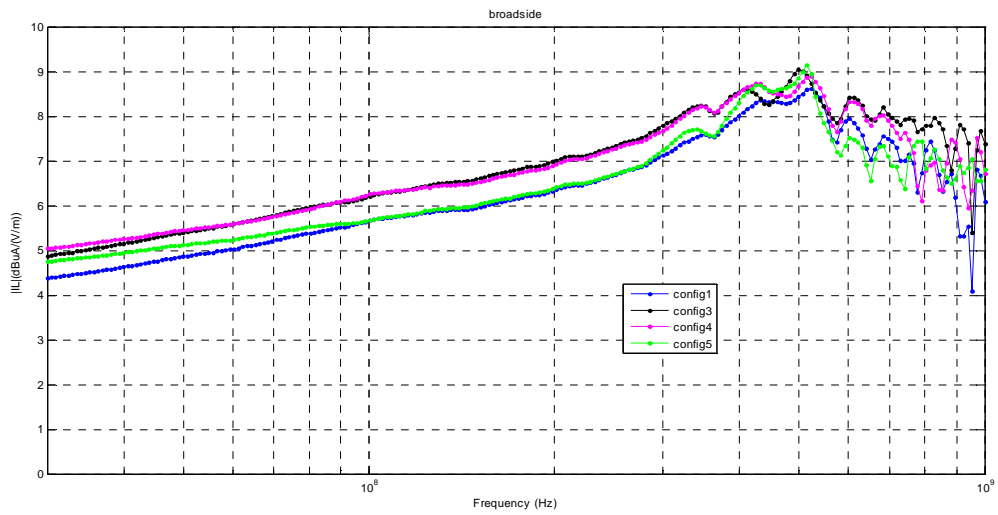
Fig. Broadside TEM cell
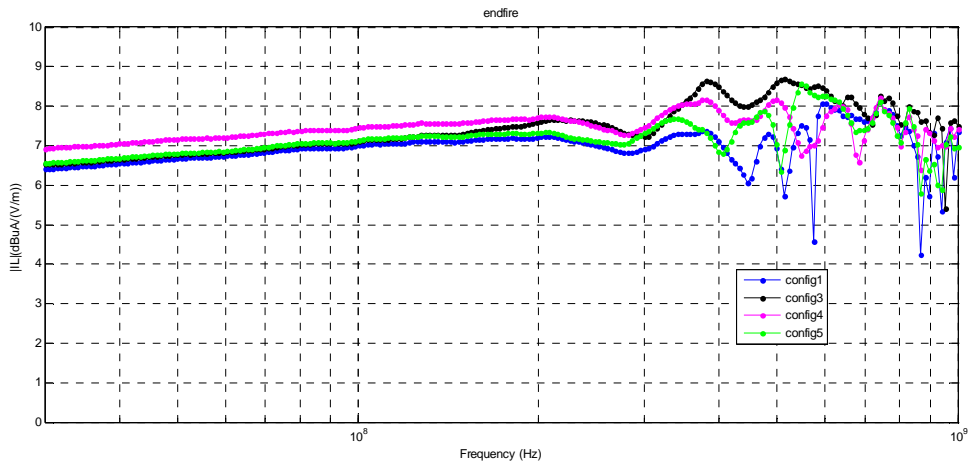
Fig. Endfire TEM cell
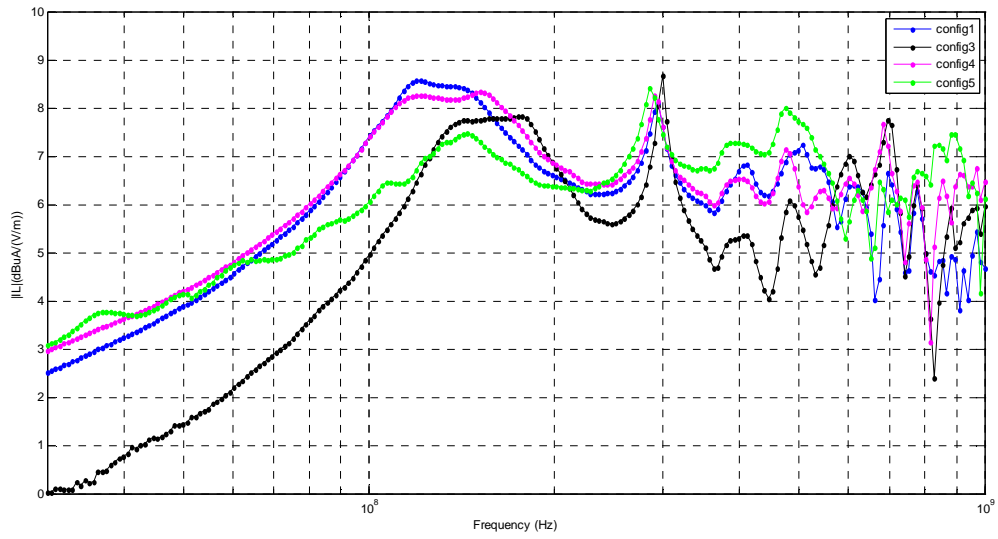
Fig. Sidefire TEM cell

2-Free space

Substitution method has been used for the calibration of the field.

Calibration (10V/m)

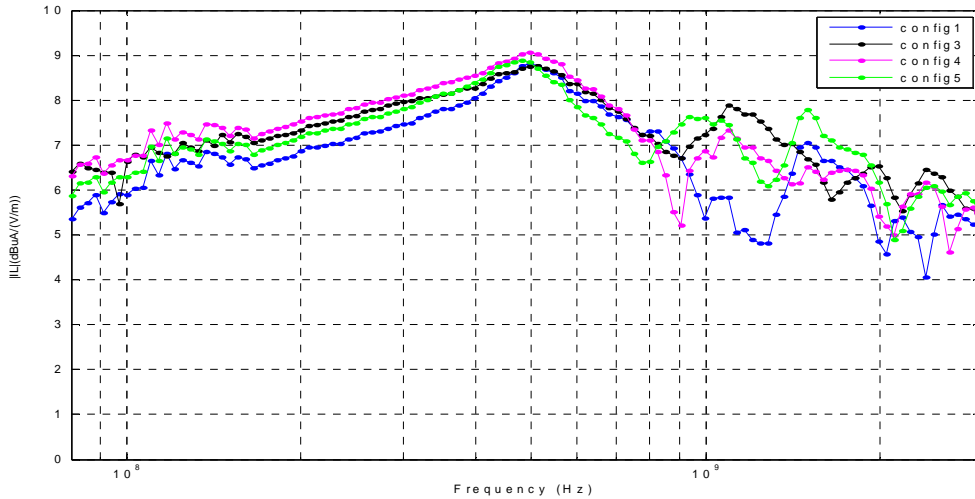Horizontal polarization
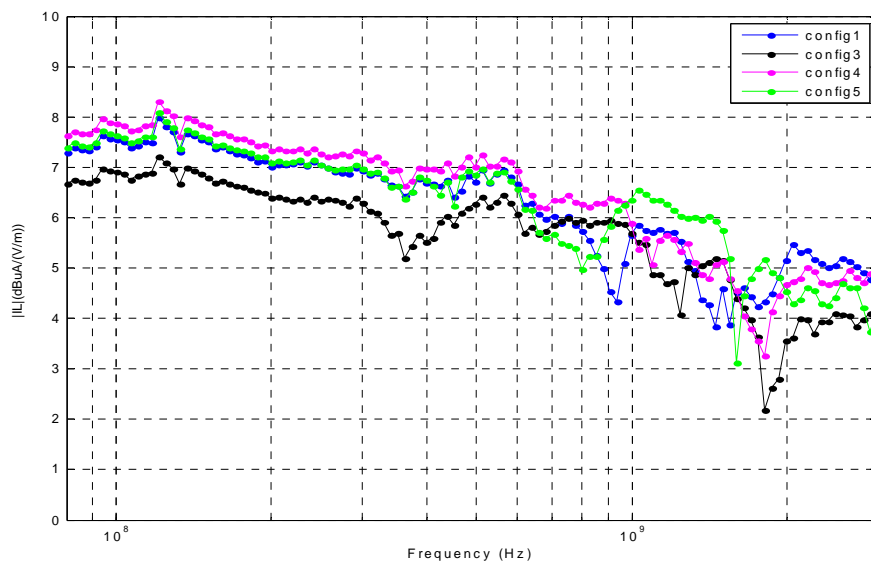
Fig.Horizontal polarization

Fig.Vertical polarization

V- Simulation with Feko

Simulation were carried out with two types of source:
- Plane wave illumination
- Simulation of the Log-P

Note that Feko doesn t have a tool to define non-uniform lines yet.
So the cables were drowned by defining the coordinates of 100 points and the software linked them by short segments. This might explain some of the results.
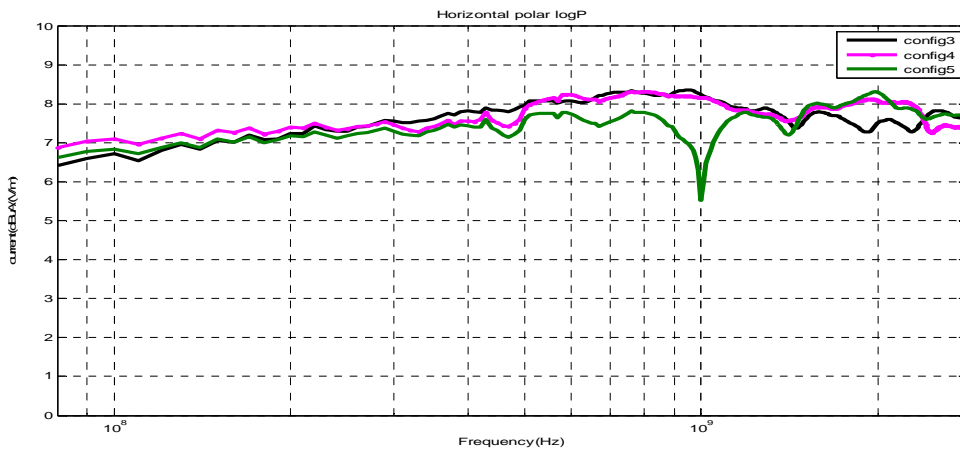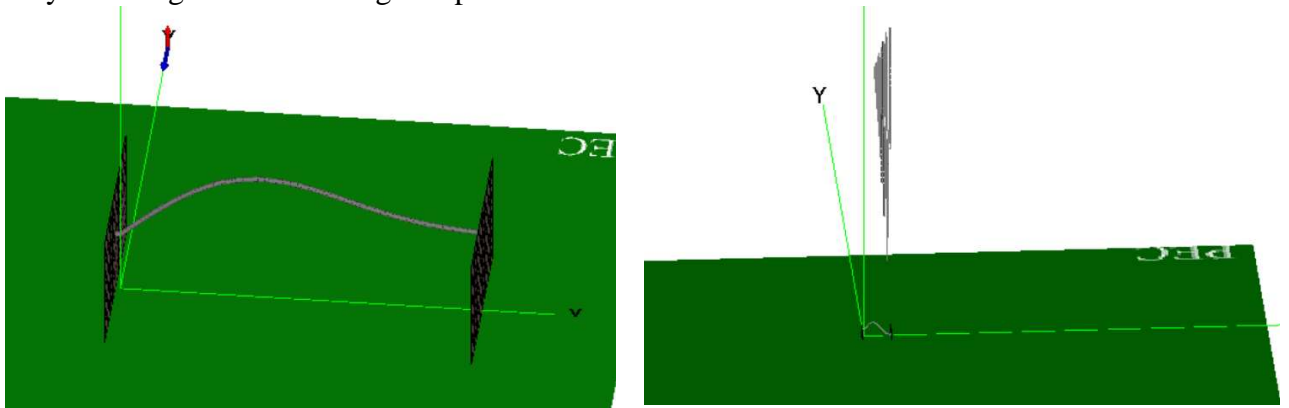
Fig. Horizontal polarization feko LogP
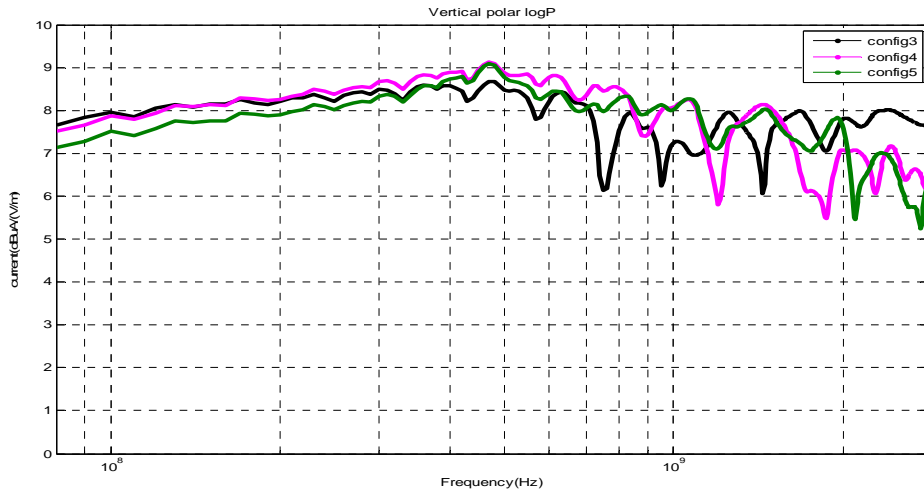
Fig. Vertical polarization feko LogP
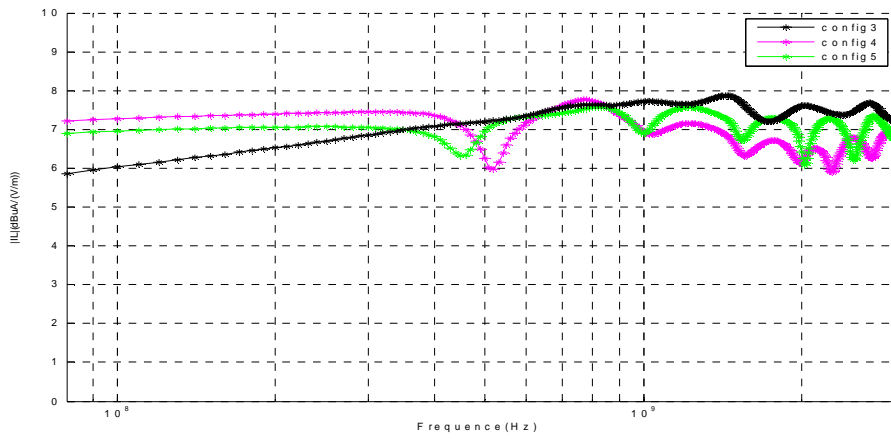
Fig. Horizontal polarization feko plane wave
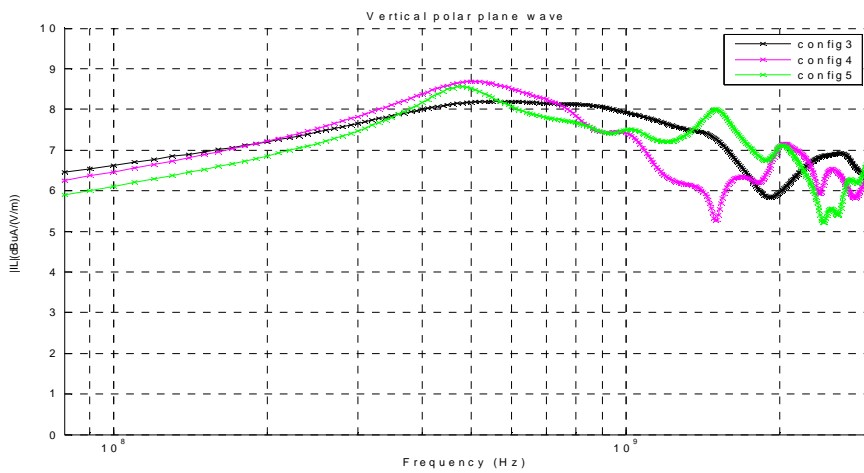
Fig. Vertical polarization feko plane wave

VI- Results

1-Config1
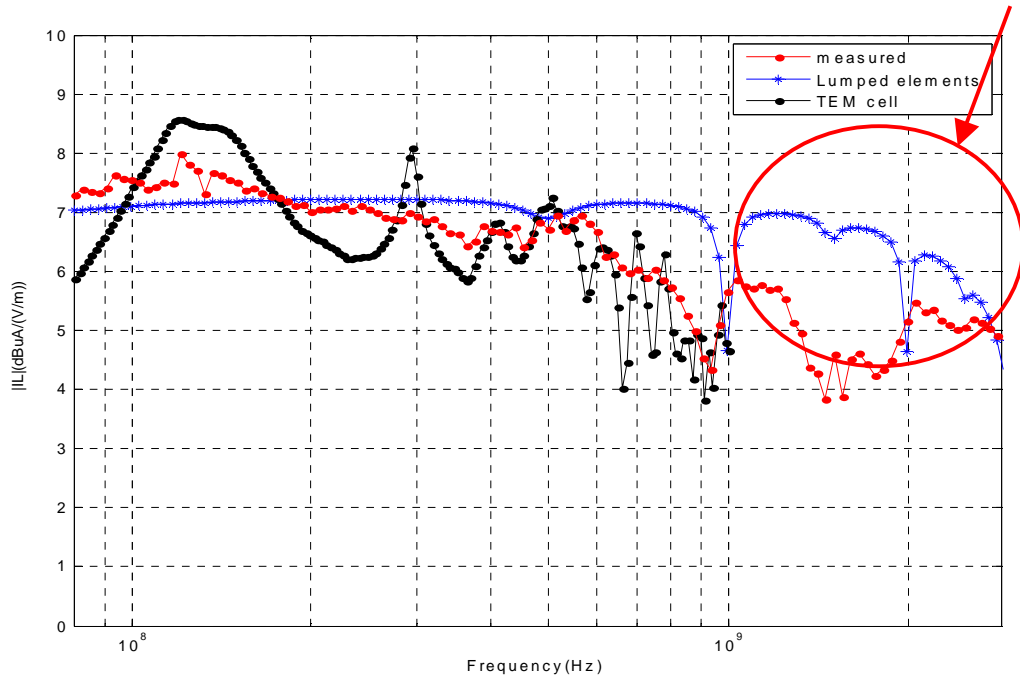
Fig.Horizontal polarization config1
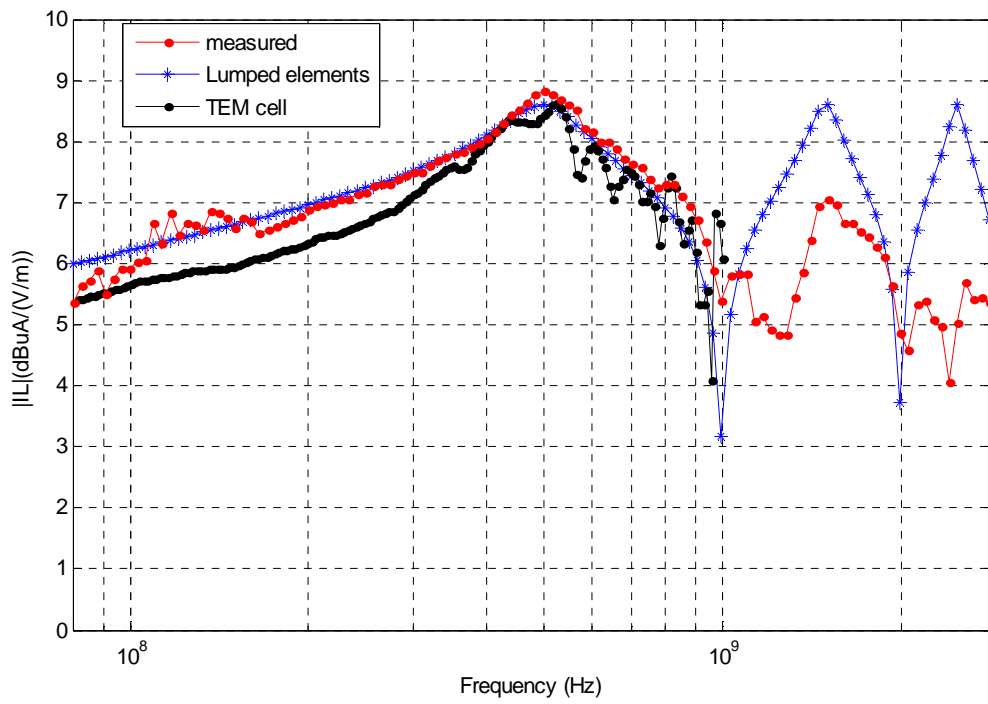
Fig.Vertical polarization config1

2-Config3
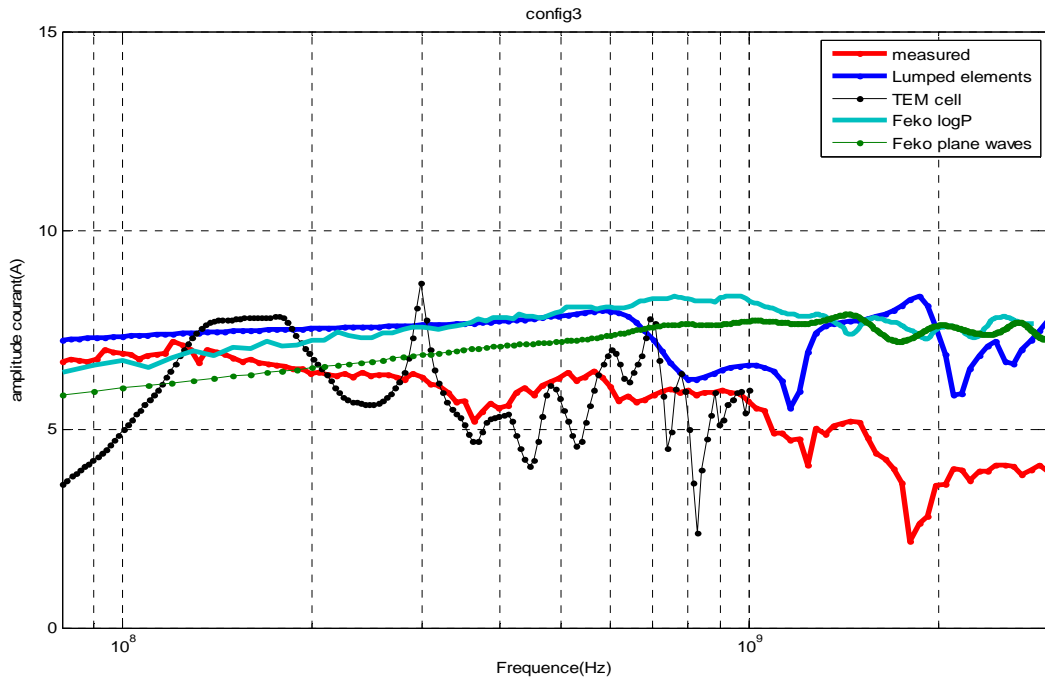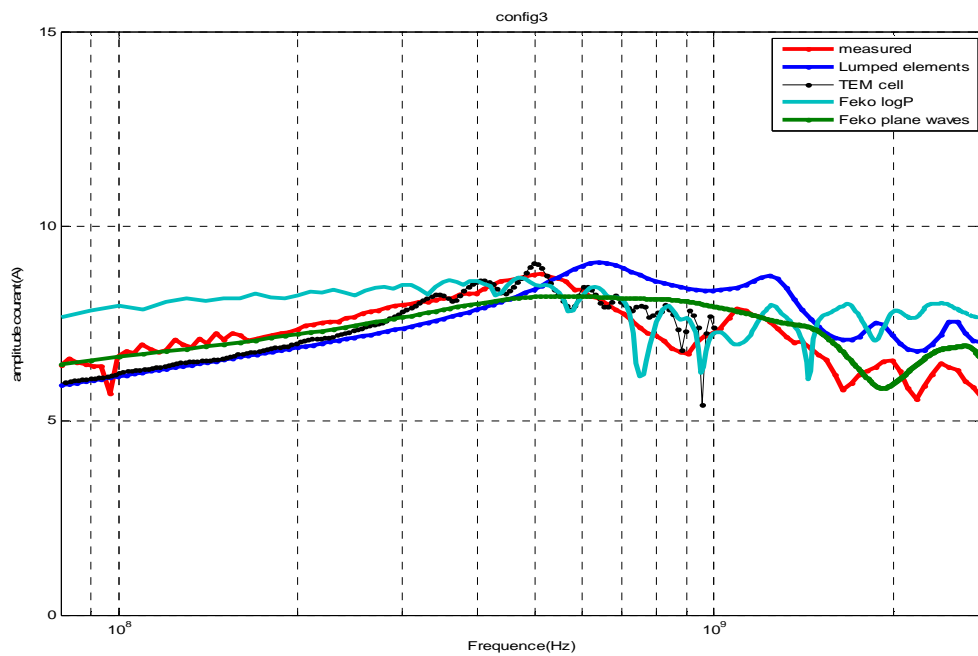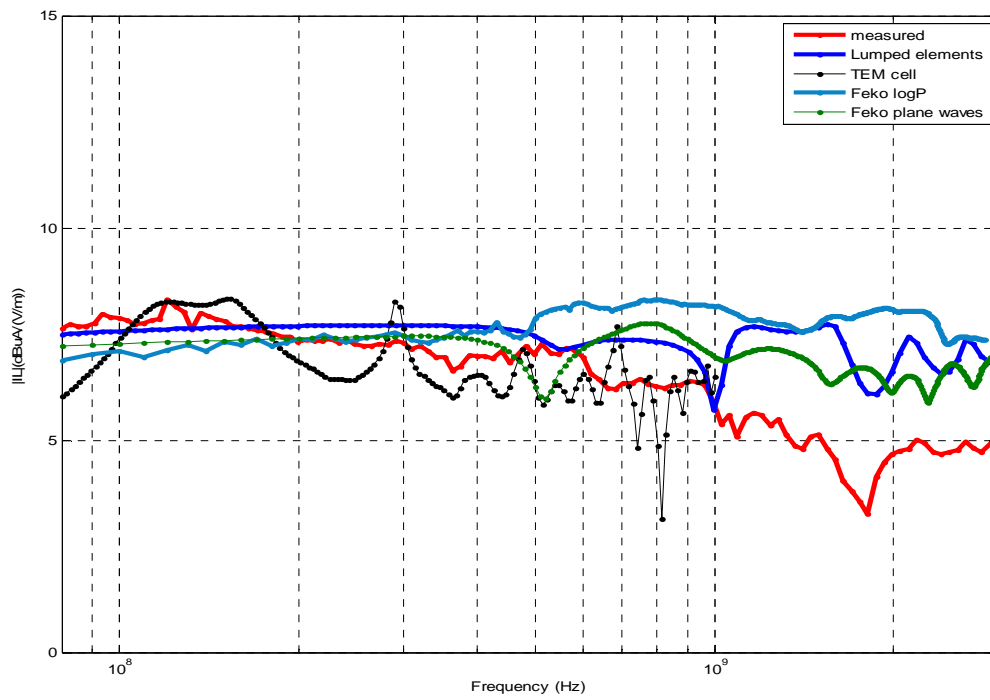

Fig.Horizontal polarization config3

Fig.Vertical polarization config3
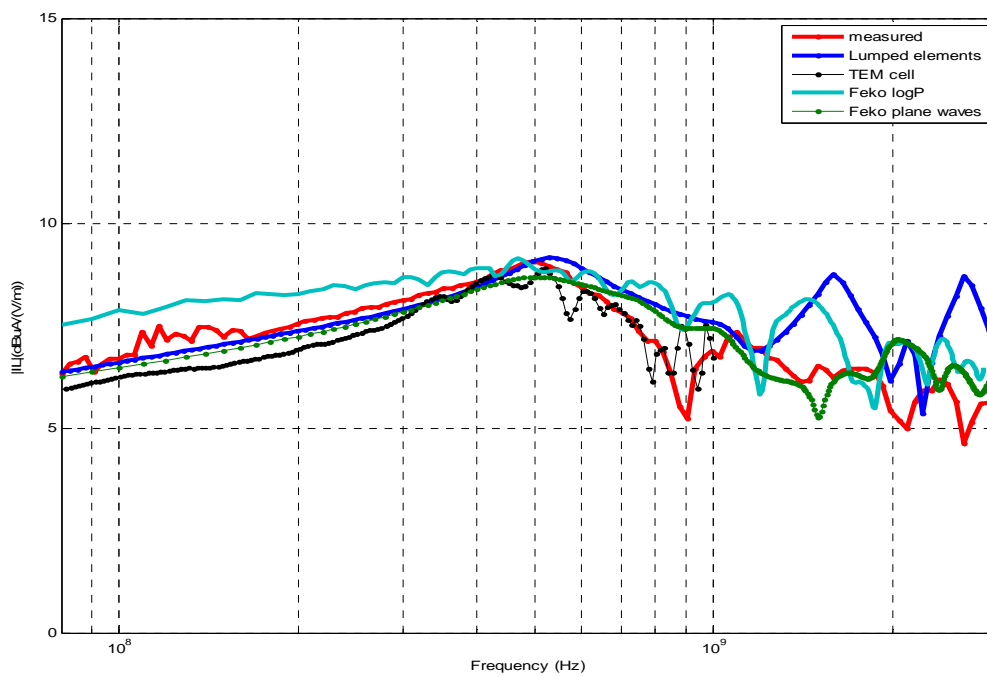
3-Config4

Fig.Horizontal polarization config4

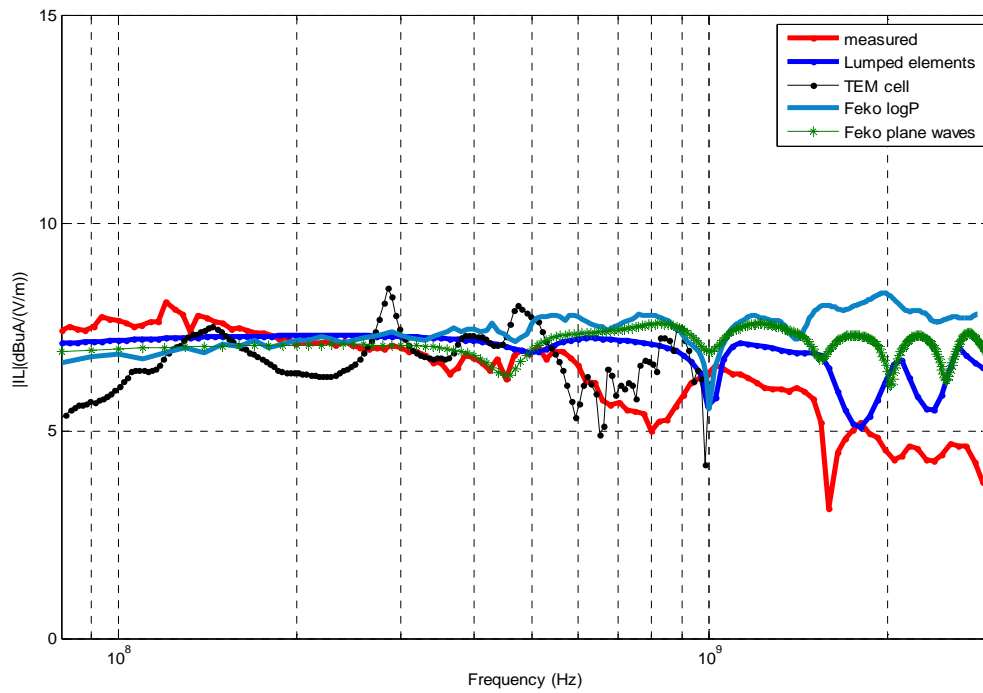*Fig.Vertical polarization config4*

4-Config5

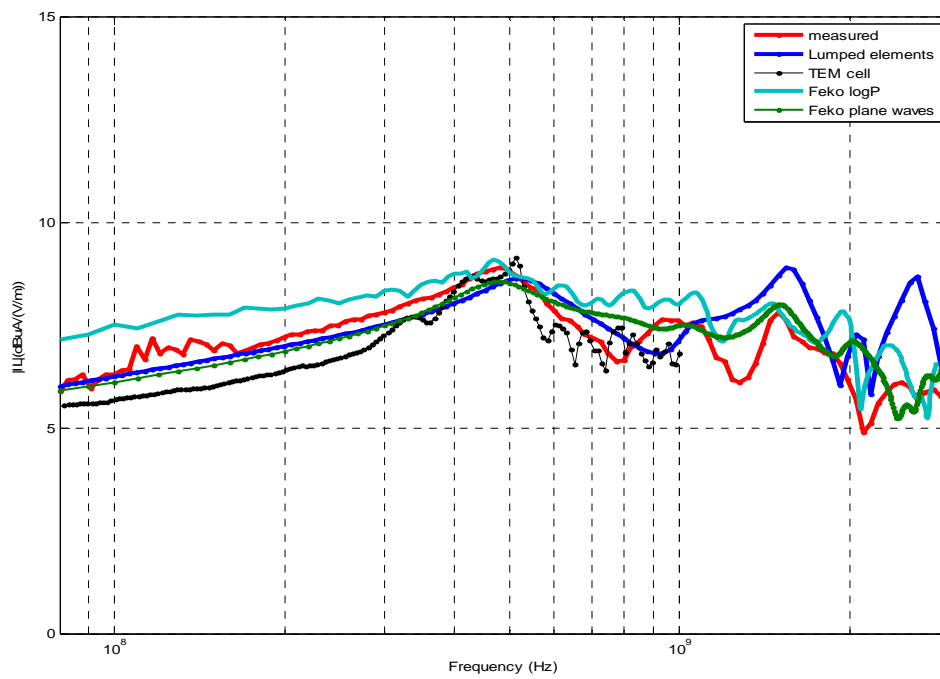*Fig.Horizontal polarization config5*

Fig.Vertical polarization config5

VII- Monte-Carlo simulations

Almost 2000 runs were executed, each corresponding to a unique configuration of the NUTL.

$\alpha_{k=1:12}$ are uniformly distributed between $[a_k b_k]$. a_k, b_k were chosen to avoid $h_\alpha(z)$ to be negative or to reach the ground.

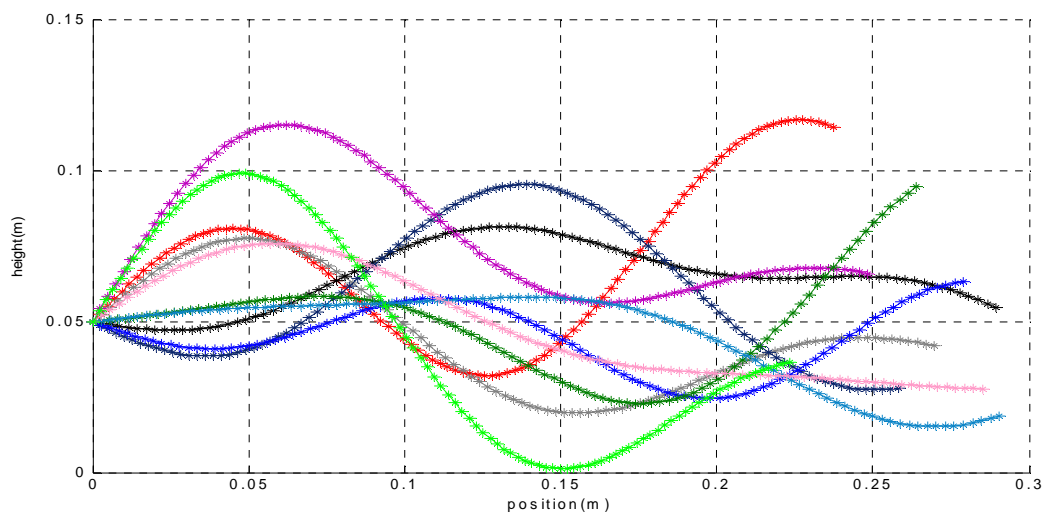Some of the configurations are represented in the following figure.

Fig. Several configurations of the cable coming from the MC runs

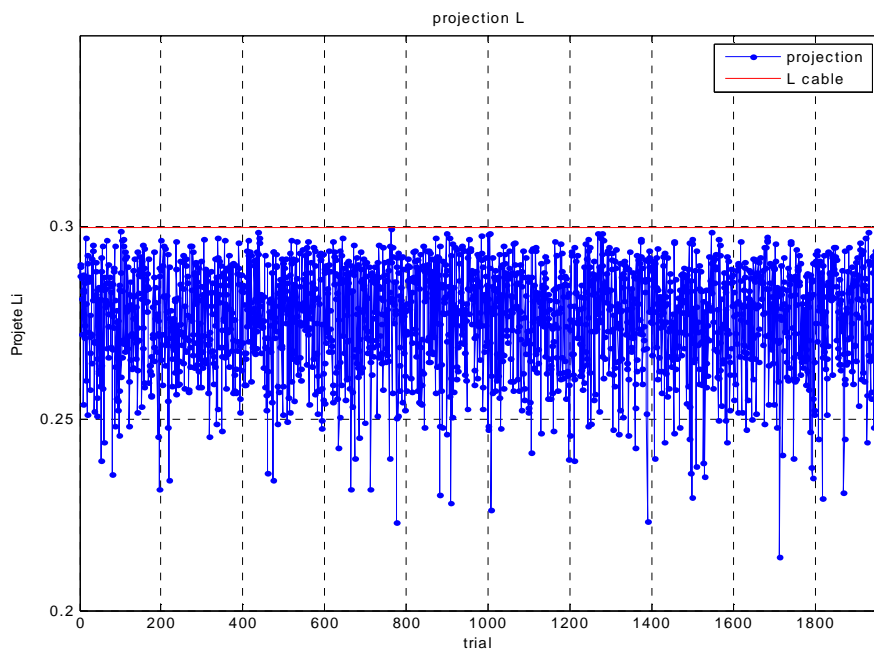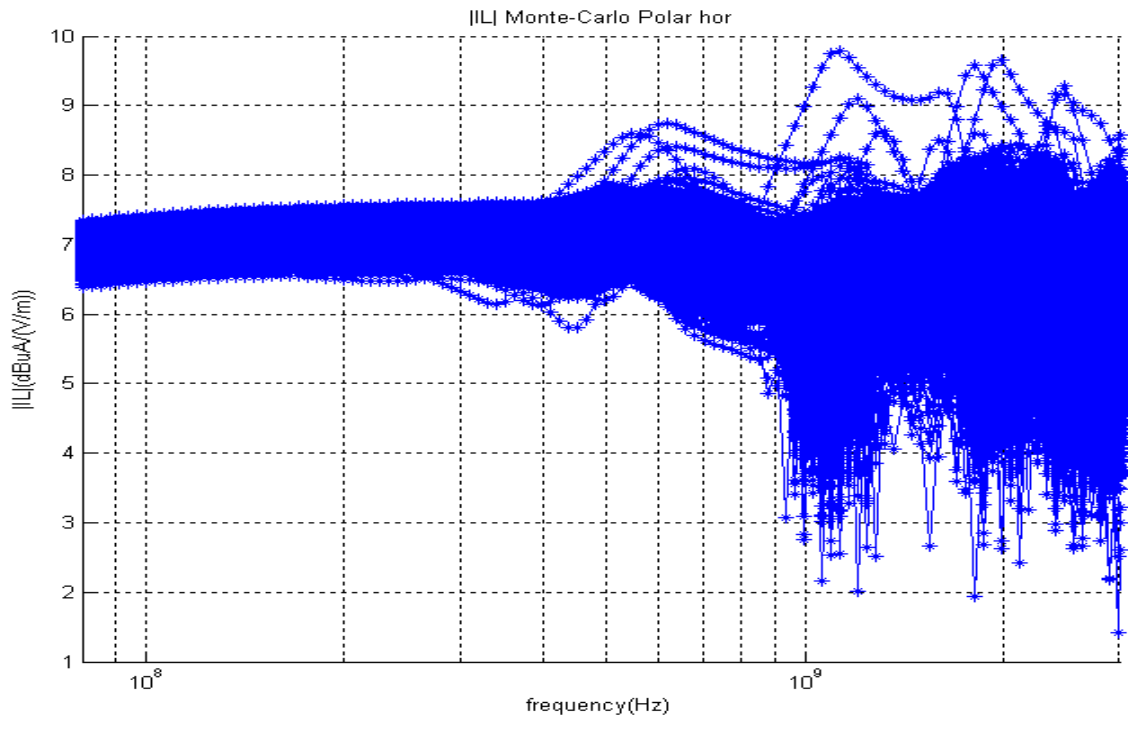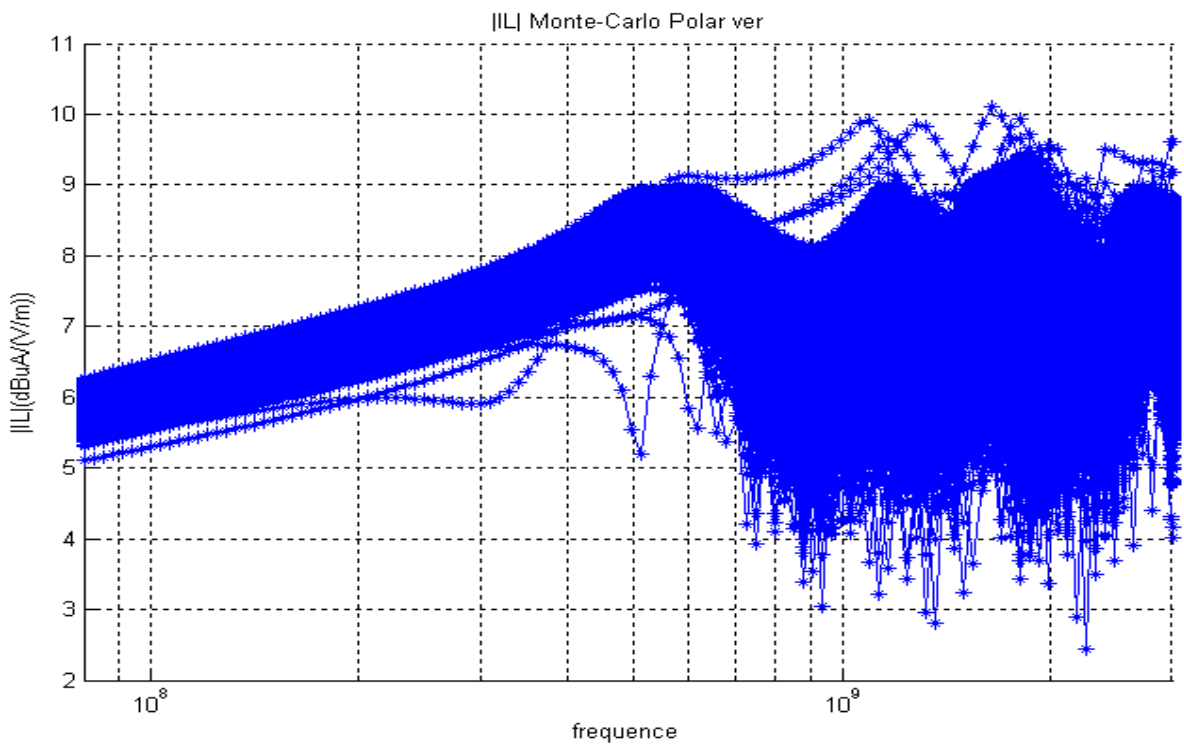The projection of L, l_i are represented in the figure below.

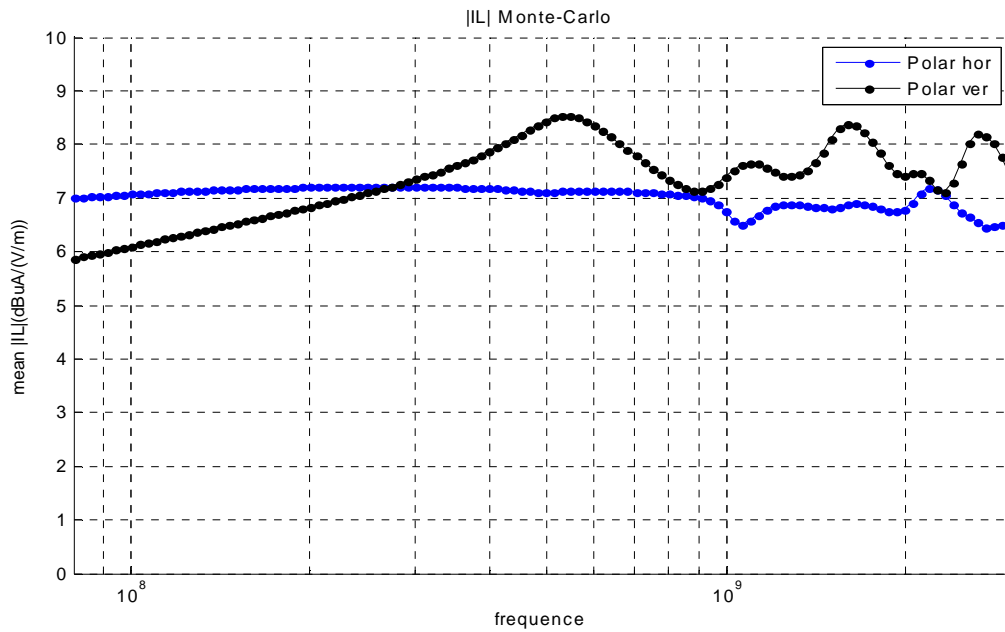*Fig. Projection of L for 2000 Monte-Carlo simulations*

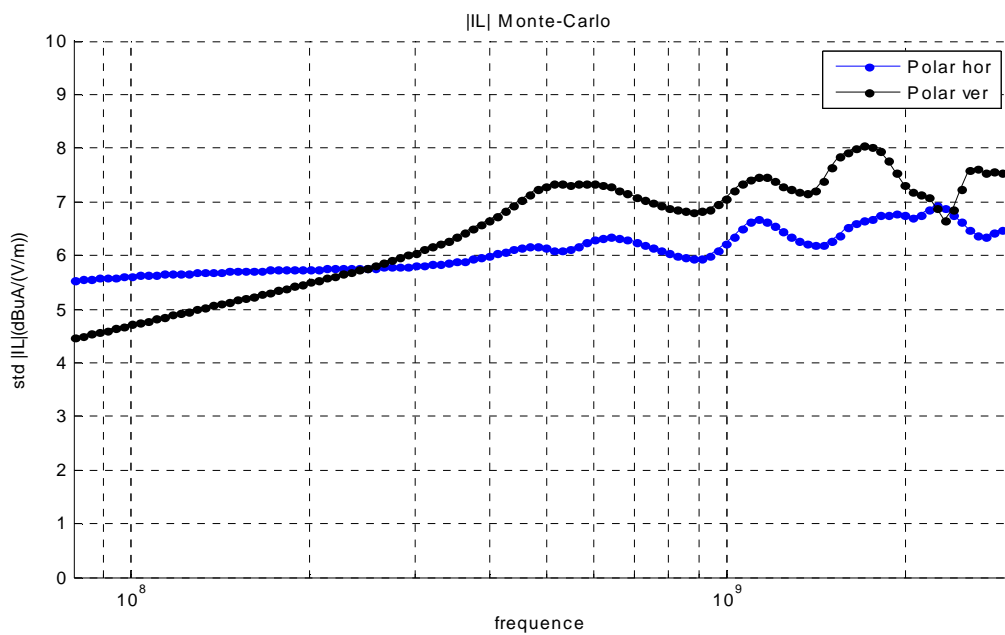*Fig.Monte-Carlo Horizontal polarization*

Fig.Monte-Carlo vertical polarization

1-Statistical parameters

The second order statistical parameters i.e the mean value and the standard deviation are depicted on figure x:

Fig. Mean value of the output current

Fig. Standard deviation of the output current

The coefficient of variation which represents the ratio between the standard deviation and the mean value is depicted on figure x:
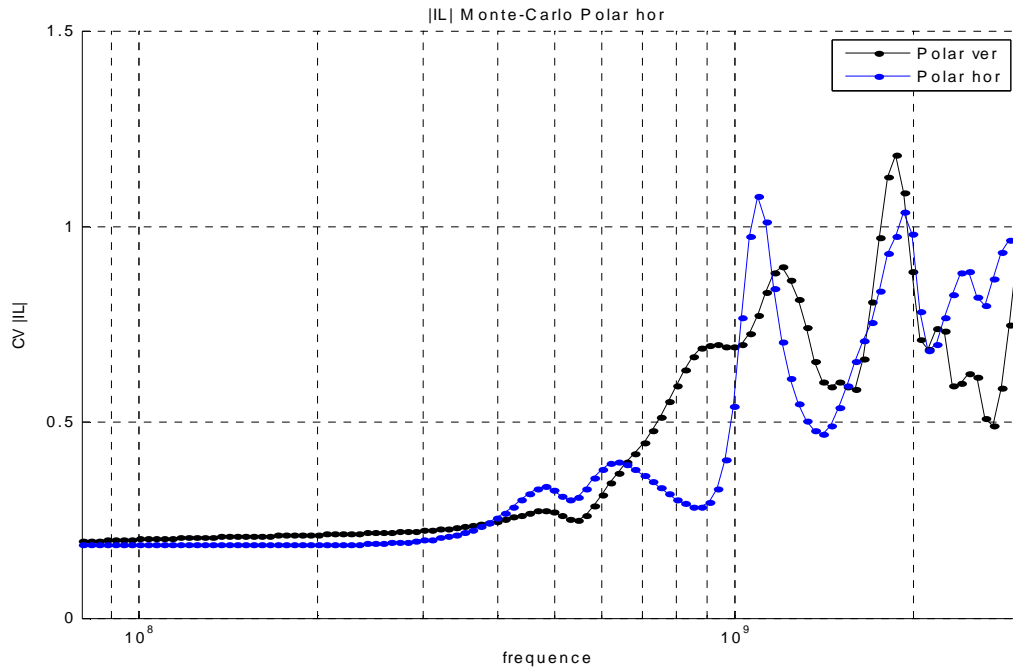
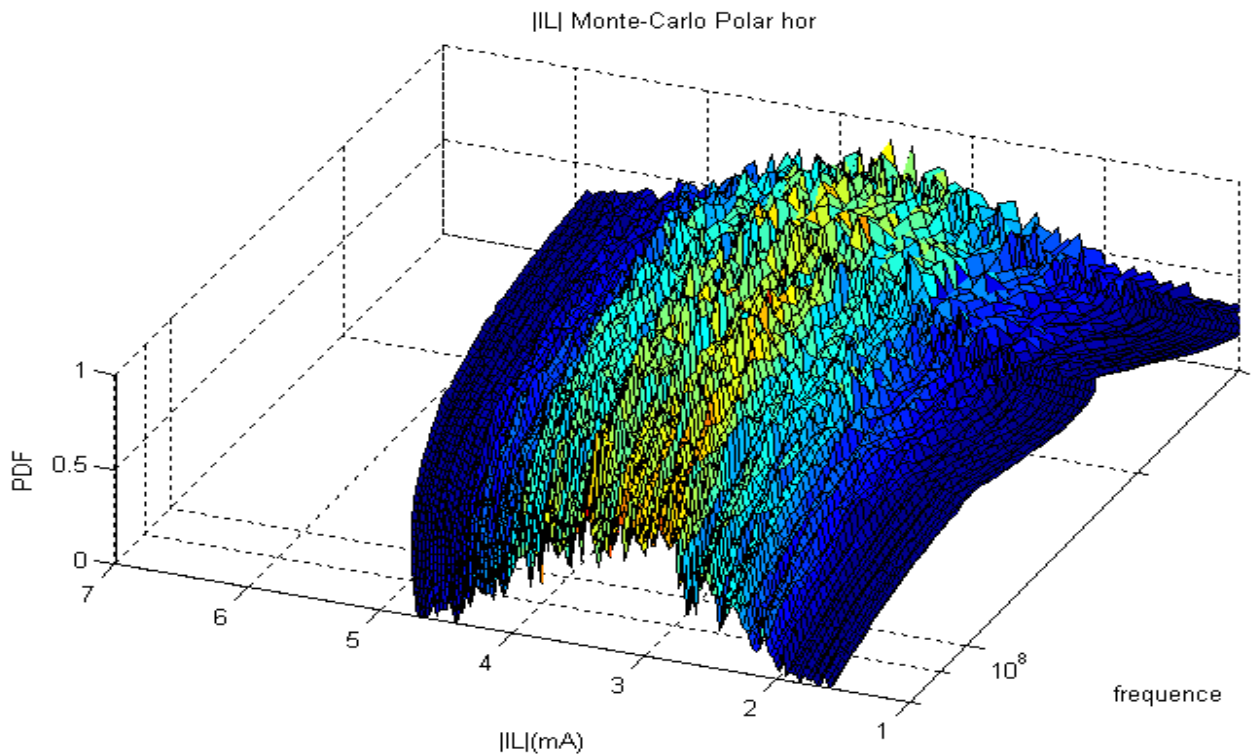*Fig. Coefficient of variation of the output current*

2-Statistical behaviour

2 frequency ranges will be considered:
-80MHz-400MHz: where the current is more or less linear
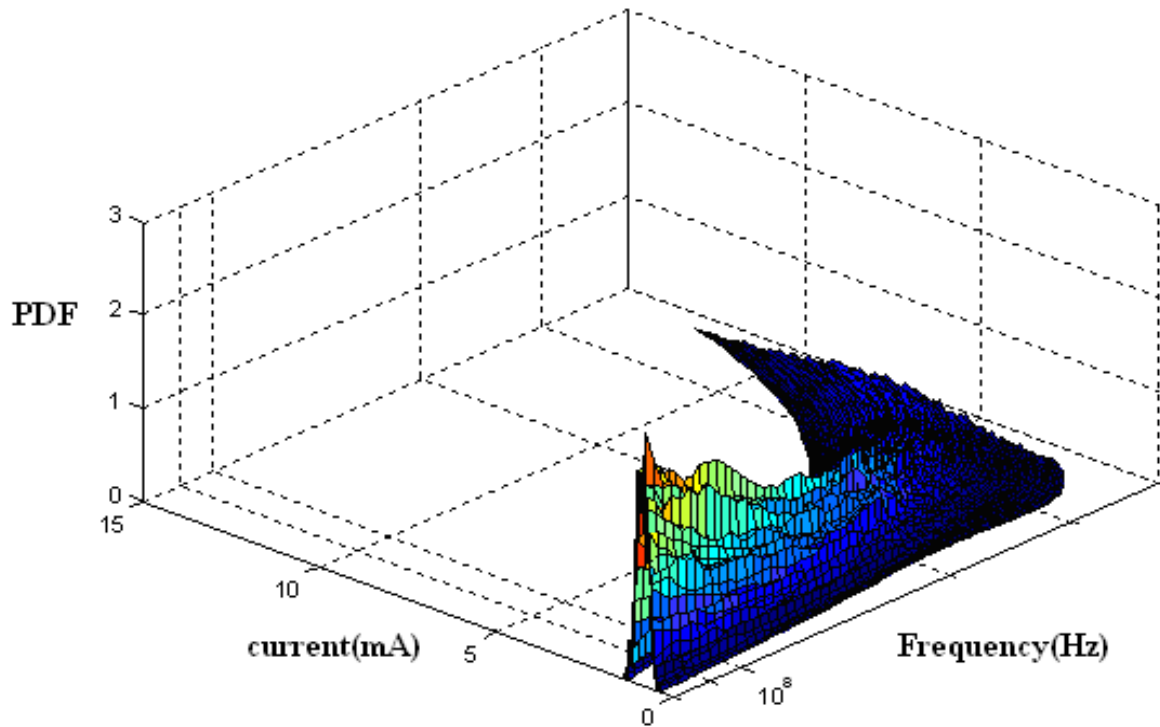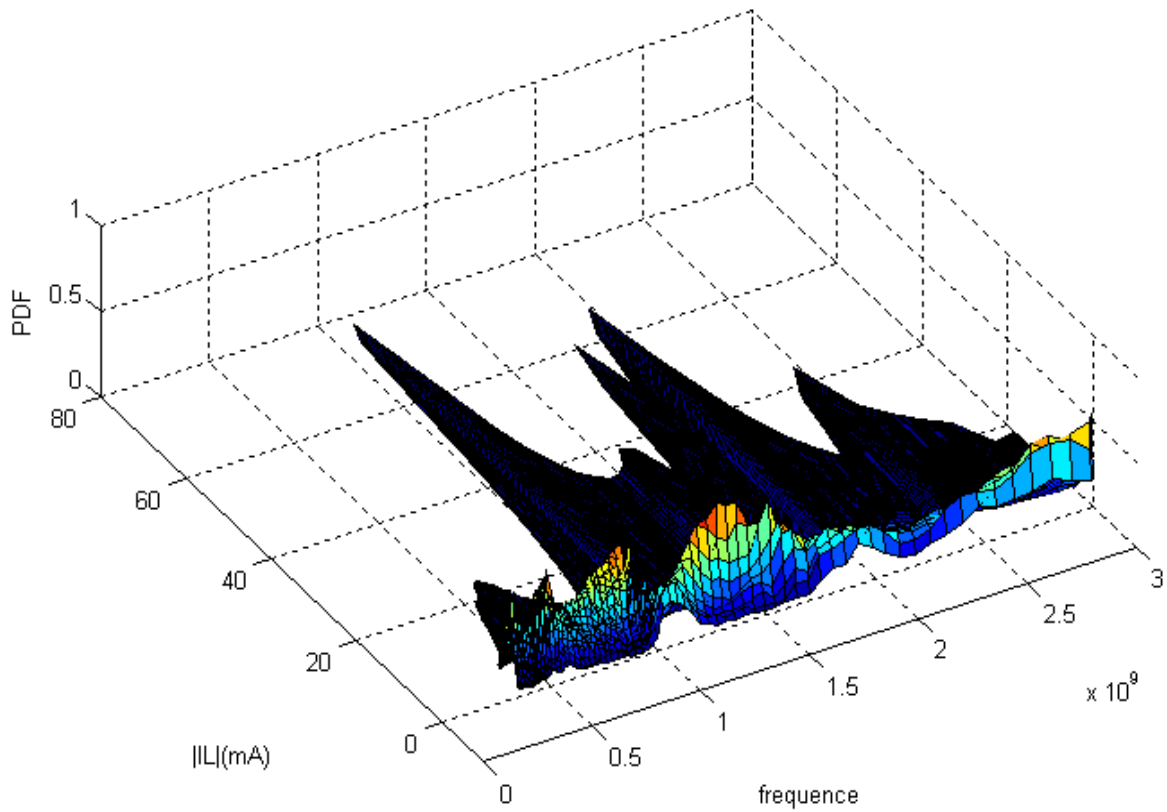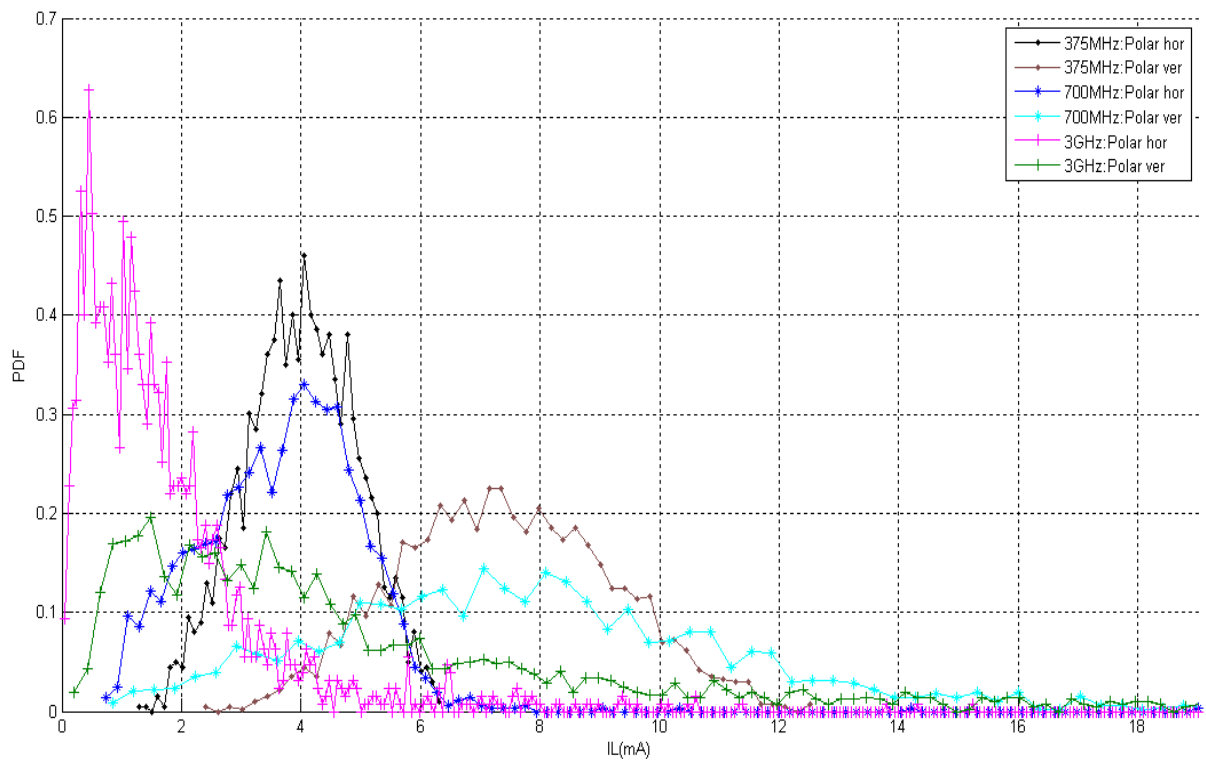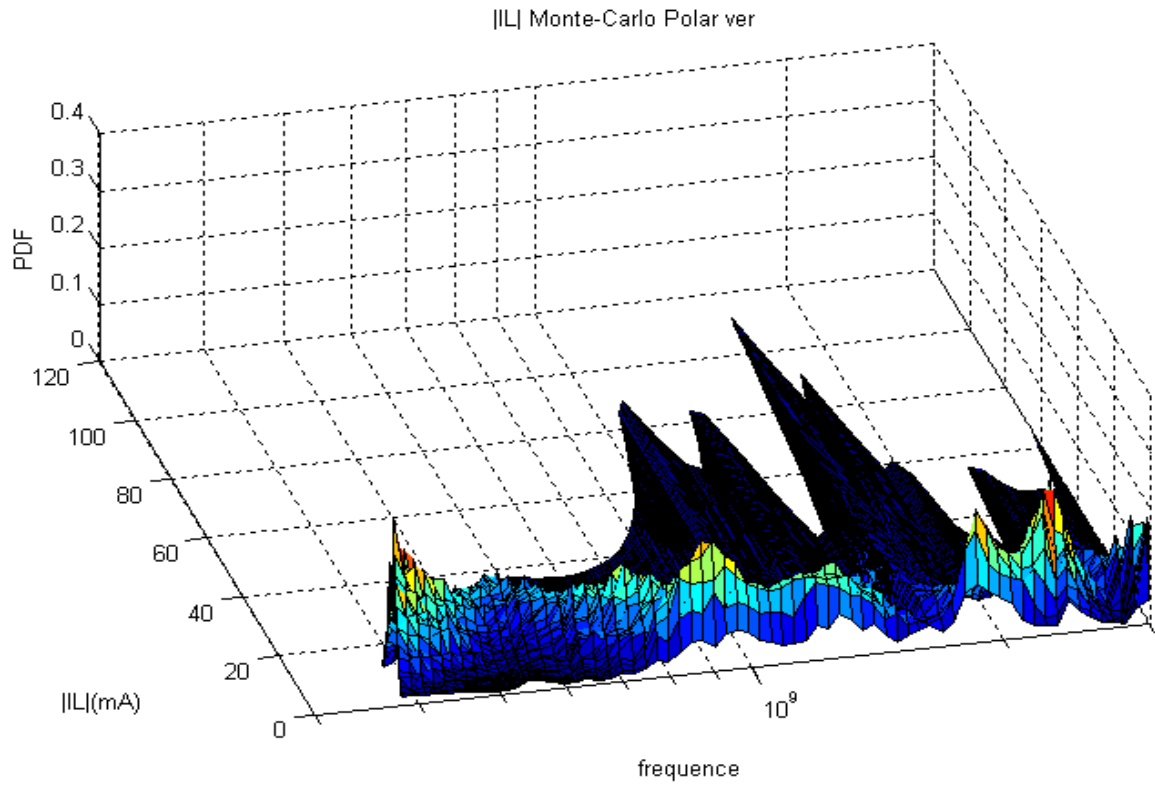-400MHz-3GHz: where the resonance occur

a-80MHz-400MHz

Fig. PDF of the output current between 80MHz and 400MHz(horizontal polarization and vertical polarization)

b-400MHz-3GHz

Fig. PDf of the output current between 400MHz and 3GHz

At low frequencies, the distribution tends to be a Gaussian whereas at higher frequencies, one notices a change in the behaviour.
Let us see, in the next paragraph, the parameters that influence more the behaviour of the output current

VIII- Sensitivity analysis

The sensitivity analysis study how disturbances on the inputs of the model produce disturbances on the outputs.

For a given model, global sensitivity analysis evaluates the impact of the inputs variabilities on the variability of the response of the model.

The method consists in determining which part of the output variance is due to the variance of each input.

References:
VIII-1-Julien Jacques and al.: *"Analyse de sensibilite globale"*
VIII-2-I.Dimov, R.Geogieva: *"Monte-Carlo algorithms for evaluation Sobol'sensitivity indices"* Math. Comput.Simul (2009)

In order to reduce the number of random variables α_k, a cable of length L=1m will be considered. This wire can be easily bended with $\alpha_{k=1:4}$ i.e with 4 independent random variables.

The simulations are running