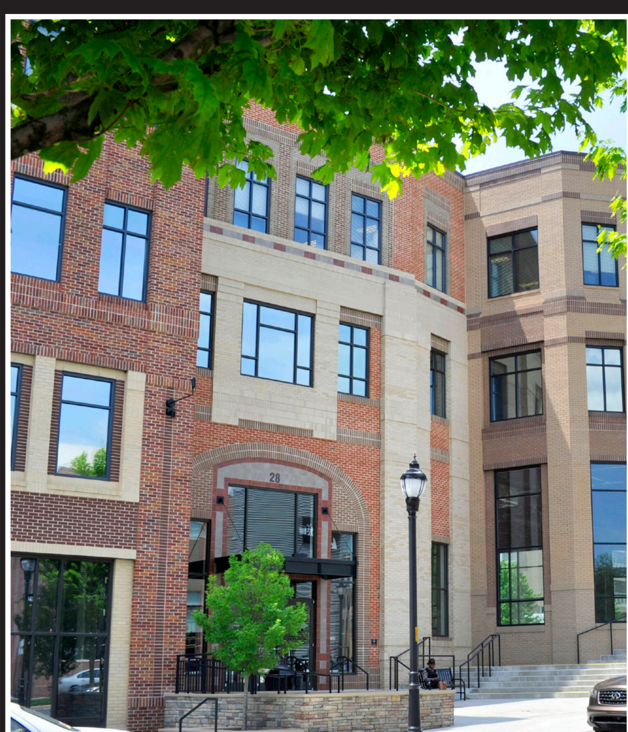
Zassenhaus Group Theory Conference

Hosted by Western Carolina University's Department of Mathematics and Computer Science

May 24 - 26, 2013

WCU Programs at Biltmore Park



Biltmore Park, Asheville, NC

Hans Zassenhaus

Western Carolina University

Friday, May 24, 2013; Afternoon Session
12:30-1:30 PM Registration

2:00-2:20 pm Tuval Foguel (Western Carolina University)	Subloop Lattice and Neck loops
2:30-2:50 pm William DeMeo (University of South Carolina)	A general Dedekind transposition principle and examples of isotopic algebras with non-isomorphic congruence lattices.
3:00-3:20 pm Jay Zimmerman (Towson University)	The symmetric genus of p-groups
3:30-3:50 pm Luis Valero-Elizondo (Universidad Michoacana de San Nicols de Hidalgo)	Minimal groups with isomorphic tables of marks
4:00-4:20 pm Arnold D. Feldman (Franklin & Marshall College)	Subgroup closure in formations
4:30-4:50 pm Alexandre Turull (University of Florida)	The strengthened Alperin Weight Conjecture for p-solvable groups

8:30-9:30 AM Coffee and Pastries

Saturday, May 25, 2013; Morning Session 1

9:30-9:50 am Ryan McCulloch (Binghamton University)	Constructing Chermak-Delgado Lattices
10:00-10:20 am Martha Kilpack (SUNY Oneonta)	Closure Operators on a Subgroup Lattice: Preliminary Report
10:30-10:50 am Joseph Brennan (Binghamton University)	On Finite p-Groups with an Abelian Subgroup of Index p
11:00-11:20 am Hung Ngoc Nguyen (The University of Akron)	Commuting probability in finite groups
11:30-11:50 am Thomas Wolf (Ohio University)	Group actions related to non-vanishing elements

Saturday, May 25, 2013; Morning Session 2

9:30-9:50 am Alexander Gruber (University of Cincinnati)	A 3-coloring of the prime graph of a solvable group
10:00-10:20 am Mark L. Lewis (Kent State University)	Minimal prime graphs of solvable groups
10:30-10:50 am C. B. Sass (Kent State University)	Prime Character Degree Graphs of Solvable Groups with Diameter Three
11:00-11:20 am William Montanaro, Jr. (Kent State University)	Character Degree Graphs of Some Almost Simple Groups
11:30-11:50 am Andrew Misseldine (Brigham Young University)	Counting Schur Rings Over Cyclic Groups

Saturday, May 25, 2013; Afternoon Session

2:00-2:20 pm Robert F. Morse (University of Evansville)	Capable special p-groups of rank 2: Structure results
2:30-2:50 pm Luise-Charlotte Kappe (Binghamton University)	Capable special p-groups of rank 2: The isomorphism problem
3:00-3:20 pm Arturo Magidin (University of Louisiana)	A nilpotent group of class 2 and prime exponent is capable if it is nonabelian enough
3:30-3:50 pm Petr Vojtěchovský (University of Denver)	Commutator theory in varieties close to groups
4:00-4:20 pm Michael Kinyon (University of Denver)	Automorphic loops
4:30-4:50 pm Derek J.S. Robinson (University of Illinois)	Groups with Few isomorphism Classes of derived Subgroups

Saturday, May 25, 2013; Conference Banquet - Hilton Hotel at Biltmore Park

6:00 PM Cash Bar and Buffet Dinner

Sunday May 26, 2013; Morning Session
8:30-9:30 AM Coffee and Pastries

9:30-9:50 am Ken Johnson (Penn State Abington)	The group matrix, superalgebras and Berezinians?
10:00-10:20 am Cornelius Pillen (University of South Alabama)	Cartan invariants for finite groups of Lie type in the defining characteristic
10:30-10:50 am Zoran Sunic (Texas A & M University)	On groups with polynomial geodesic growth
11:00-11:20 am Adam Glesser (California State University, Fullerton)	Saturated Fusion Systems over Small p-Groups
11:30-11:50 am Daniela Nikolova-Popova (Florida Atlantic University)	Incidence Matrices and the Covering Number of S_{10}

Abstracts

Title: Subloop Lattice and Neck Loops
Presenter: Tuval Foguel
Affiliation: Western Carolina University

Abstract: We will discuss some topics from the theory of subgroup lattices. After giving a general overview, we will look at major open problem: if every finite lattice occurs as an interval in the subgroup lattice of a finite group. Next we investigate lattice of subloops of finite loops that are not lattices of subgroups of a finite group. Finally, we will discuss the Neck loop structure and its lattice.

Title: SA general Dedekind transposition principle and examples of isotopic algebras with non-isomorphic congruence lattices.
Presenter: William DeMeo
Affiliation: University of South Carolina

Abstract: We present a lattice transposition principle that holds in all lattices of equivalence relations, and give an analogous principle that holds in subgroup lattices. We then show how group actions can be used to demonstrate that the difference in size of congruence lattices of isotopic algebras can be arbitrarily large. (Some definitions and motivation for the notion of isotopy will also be provided.)

Title: The symmetric genus of p-groups
Presenter: Jay Zimmerman
Affiliation: Towson University

Abstract: Let G be a finite group. The *symmetric genus* $\sigma(G)$ is the minimum genus of any Riemann surface on which G acts. We show that a non-cyclic p-group G has symmetric genus not congruent to 1 $(mod\ p^3)$ if and only if G is in one of 10 families of groups. The genus formula for each of these 10 families of groups is determined. A consequence of this classification is that almost all positive integers that are the genus of a p-group are congruent to 1 $(mod\ p^3)$. Finally, the integers that occur as the symmetric genus of a p-group with Frattini-class 2 have density zero in the positive integers.

Title: Minimal groups with isomorphic tables of marks
Presenter: Luis Valero-Elizondo
Affiliation: Universidad Michoacana de San Nicols de Hidalgo

Abstract: Using GAP it was proved that the smallest example of non-isomorphic groups with isomorphic tables of marks are two groups of order 96. In this talk we show that for many orders n smaller than 96, non-isomorphic groups of order n cannot have isomorphic tables of marks.

Title: Subgroup closure in formations
Presenter: Arnold D. Feldman
Affiliation: Franklin & Marshall College

Abstract: Subgroup closure is a very convenient property for a formation of finite groups to possess, and many frequently studied formations, including the formations of nilpotent, solvable, and supersolvable groups, have it. But not all interesting formations have this property, so it is desirable not to assume that a formation is subgroup closed. This talk will detail some circumstances in which it is possible to assume weaker properties than subgroup closure and still get results, and some in which the assumption of subgroup closure is very difficult to avoid. The context will be the presenter's work with A. Ballester-Bolinches, J. C. Beidleman, M.C. Pedraza-Aguilera, and M. F. Ragland on generalizing pronormality and subnormality.

Title: The strengthened Alperin Weight Conjecture for p-solvable groups
Presenter: Alexandre Turull
Affiliation: University of Florida

Abstract: The Alperin Weight Conjecture is a well-known conjecture, and it is central to the modern representation theory of finite groups. It is known to be true for many types of finite groups, but it remains open. In particular, the conjecture is known to be true for finite p-solvable groups. We prove that a very strong form of the conjecture holds for all finite p-solvable groups. It follows that the strengthening of the Alperin Weight Conjecture that includes Galois automorphisms over the p-adic numbers holds for all finite p-solvable groups. This strengthening of the Alperin Weight Conjecture was suggested in an earlier work of Navarro on the McKay Conjecture. We will present the first direct evidence (beyond the calculation of individual cases and the case of groups of odd order) for this strengthening.

Title: Constructing Chermak-Delgado Lattices
Presenter: Ryan McCulloch
Affiliation: Binghamton University

Abstract: Given a finite group, G, and $H \leq G$, the Chermak-Delgado measure of H in G, denoted $m_G(H)$, is the quantity $|H||C_G(H)|$. The number $m^*(G) = \max\{m_G(H) \,|\, H \leq G\}$, and $\mathcal{CD}(G) = \{H \leq G \,|\, m_G(H) = m^*(G)\}$ is the Chermak-Delgado lattice of G. This lattice has nice properties, some of which are that it is modular and self-dual. I would like to construct groups for which (1) I can actually compute their Chermak-Delgado lattices, and (2) the lattices that I compute are interesting. For certain groups satisfying $m^*(G) = |G|$, there is a method for accomplishing (1). By stitching together examples in an appropriate way, I hope to accomplish (2).

Title: Closure Operators on a Subgroup Lattice: Preliminary Report
Presenter: Martha Kilpack
Affiliation: SUNY Oneonta

Abstract: Starting with a lattice which is isomorphic to a subgroup lattice, $Sub(G)$, we take all the closure operators on that lattice and create a new lattice, the lattice of closure operators of $Sub(G)$, $c.o.(Sub(G))$. The question we will consider is $c.o.(Sub(G))$ also isomorphic to a subgroup lattice.

Title: On Finite p-Groups with an Abelian Subgroup of Index p
Presenter: Joseph Brennan
Affiliation: Binghamton University

Abstract: Many of the existing determinations of prime-powered groups takes advantage of the existence of a subgroup whose index is a prime and the fact that all such subgroups are normal. The class $\mathfrak{A}_p$ consisting of finite p-groups with an abelian maximal subgroup have been extensively investigated. In particular, the class was classified up to isoclinism by Phillip Hall in 1939.
In this talk, I will discuss the structure of groups in $\mathfrak{A}_p$ including the abelian invariants of their abelian maximal subgroups. Additionally, I will present the classification of $\mathfrak{A}_p$ up to isomorphism (building upon the work of George Szekeres).

Title: Commuting probability in finite groups
Presenter: Hung Ngoc Nguyen
Affiliation: The University of Akron

Abstract: For a group G, let $d(G)$ denote the probability that a randomly chosen pair of elements of G commute. The study of commuting probability of finite groups dates back to work of Gustafson in the seventies. He showed that $d(G) = \frac{k(G)}{|G|}$, where $k(G)$ denotes the number of conjugacy classes of G. It turns out that $d(G)$ provide a lot of information on the structure of G. For instance, it is known that if $d(G) > 1/2$ then G must be nilpotent.
We will present some study on the structure of finite groups with commuting probability at least $1/3$. In particular, we show that the groups isoclinic to A_4 are the only non-supersolvable groups with commuting probability at least $1/3$. This is a joint work with Paul Lescot and Yong Yang.

Title: Group actions related to non-vanishing elements
Presenter: Thomas Wolf
Affiliation: Ohio University

Abstract: It was conjectured in a joint paper with Isaacs and Navarro that a non-vanishing element of a solvable group G must lie in the Fitting subgroup of G. In studying this, one encounters the following situation: A solvable group G acts faithfully and irreducibly on a vector space V and there exists a non-identity element x in the Fitting subgroup of G that centralizes an element of each G-orbit of V. We characterize all such actions.

Title: A 3-coloring of the prime graph of a solvable group
Presenter: Alexander Gruber
Affiliation: University of Cincinnati

Abstract: In this talk, we introduce an orientation of the complement of the prime graph of a solvable group based on Frobenius action in its Hall subgroups. We then demonstrate how this may be applied to characterize the prime graphs of solvable groups. In particular, we show that a graph is isomorphic to the prime graph of a solvable group if and only if its complement is 3-colorable and triangle-free. (This is joint work with Thomas Keller, Keeley Naughton, Benjamin Strasse, and Mark Lewis, the latter of whom will discuss an application of this theorem in a later talk.)

Title: Minimal prime graphs of solvable groups
Presenter: Mark L. Lewis
Affiliation: Kent State University

Abstract: (This is joint work with Alexander Gruber, Thomas M. Keller, Keeley Naughton, and Benjamin Strasser.) In his talk, Alexander Gruber, will characterize the prime graphs of solvable groups. Based on this characterization, the idea of a minimal prime graph will be defined. In this talk, we will show that if G is a solvable group whose prime graph is minimal, then the Fitting height of G is at most 4.

Title: Prime Character Degree Graphs of Solvable Groups with Diameter Three
Presenter: C. B. Sass
Affiliation: Kent State University

Abstract: Let G be a finite solvable group and (G) the set of character degrees of G. The prime character degree graph $\Delta(G)$ of G is the graph whose vertices are the primes dividing the degrees in (G), and there is an edge between the primes p and q if pq divides some degree in (G). When $\Delta(G)$ has diameter three we can partition the vertices $\rho(G)$ into four non-empty disjoint subsets $\rho_1 \cup \rho_2 \cup \rho_3 \cup \rho_4$ where no prime in ρ_1 is adjacent to any prime in $\rho_3 \cup \rho_4$, no prime in ρ_4 is adjacent to any prime in $\rho_1 \cup \rho_2$, every prime in ρ_2 is adjacent to some prime in ρ_3, every prime in ρ_3 is adjacent to some prime in ρ_2, and $|\rho_1 \cup \rho_2| \leq |\rho_3 \cup \rho_4|$. In this paper we show that if $\Delta(G)$ has diameter three and $\rho_1 \cup \rho_2$ contains n primes, then ρ_3 must contain at least $2^n - 1$ primes. In particular, if $\rho_1 \cup \rho_2$ has size n, then $\rho_3 \cup \rho_4$ has size at least 2^n.

Title: Character Degree Graphs of Some Almost Simple Groups
Presenter: William Montanaro, Jr.
Affiliation: Kent State University

Abstract: In a series of recent articles, D. L. White and M. L. Lewis have studied properties of the set of degrees of irreducible complex characters of nonsolvable groups. One of the properties they have considered is the character degree graph. For a finite group G, the character degree graph $\Delta(G)$ is the (simple) graph whose vertices are the primes which divide an irreducible character degree of G and two vertices p and q are adjacent provided pq divides an irreducible character degree of G. We say G is almost simple provided $S \leqslant G \leqslant \mathrm{Aut}(S)$ for some simple group S. Here we determine the character degree graphs of almost simple groups containing $\mathrm{PSL}_3(q)$ or $\mathrm{PSU}_3(q^2)$.

Title: Counting Schur Rings Over Cyclic Groups
Presenter: Andrew Misseldine
Affiliation: Brigham Young University

Abstract: A Schur ring is a special subalgebra of a group algebra afforded by a partition of the group. Schur rings have been a useful tool in the study of combinatorics, graph theory, and representation theory, especially Schur rings over a cyclic group. There was much work in the 90's toward the classification of Schur rings over cyclic groups, until it was finally reached in 1996 by Leung and Man. I will present a new method to classify all Schur rings over cyclic groups using methods from Galois theory. These new methods will also lead to a formula which counts the number of Schur rings over particular cyclic groups.

Title: Capable special p-groups of rank 2: Structure results
Presenter: Robert F. Morse
Affiliation: University of Evansville

Abstract: A finite p-group G such that $G' = Z(G)$ and G' is an elementary abelian p-group of rank 2 is called special of rank 2. A group G is capable if there exists a group H such that $H/Z(H)$ is isomorphic to G. The goal of this research is to classify up to isomorphism all of the capable special p-groups of rank 2. In this talk we will determine the structure of these groups, give a parameterized presentation for each group and provide a criterion for exactly when a special p-group of rank 2 and exponent p^2 is capable.

This is joint work with Hermann Heineken and Luise-Charlotte Kappe.

Title: Capable special p-groups of rank 2: The isomorphism problem
Presenter: Luise-Charlotte Kappe
Affiliation: Binghamton University

Abstract: A finite p-group G such that $G' = Z(G)$ and G' is an elementary abelian p-group of rank 2 is called special of rank 2. A group G is capable if there exists a group H such that $H/Z(H)$ is isomorphic to G. A result of H. Heineken shows the capable special p-groups of rank 2 have order at most p^7. Of such groups of exponent p we know from published classifications that there is a constant number of isomorphism classes. Experiments with GAP show the number of isomorphism classes of special p-groups of rank 2 and exponent p^2 grows with p for groups of order p^5, p^6, and p^7. However, the number of capable special groups of each order is constant. That gave us the idea that perhaps capable special p-groups of rank 2 and exponent p^2 cannot only be characterized by a structure description but can actually be classified. With the help of some linear algebra and ideas from projective geometry we were finally able to establish what GAP helped us conjecture, namely that for odd p there are three isomorphism classes each for capable special p-groups of rank 2 exponent p^2 and order p^5 and p^6 and one such class for order p^7.

This is joint work with Hermann Heineken and Robert F. Morse.

Title: A nilpotent group of class 2 and prime exponent is capable if it is nonabelian enough
Presenter: Arturo Magidin
Affiliation: University of Louisiana

Abstract: A group G is capable if and only if $G \cong K/Z(K)$ for a suitable group K. Sufficient conditions for capability are generally hard to come by. I will discuss the situation in which G is a nilpotent group of class 2 and prime exponent, in which I have been able to prove that if $[G,G]$ is sufficiently large (larger than a bound that depends only on the rank of $G/Z(G)$), then G is capable.

Title: Commutator theory in varieties close to groups
Presenter: Petr Vojtěchovský
Affiliation: University of Denver

Abstract: Freese and McKenzie developed commutator theory for congruence modular varieties. It is instructive to see how their general theory specializes in the variety of groups, resulting in the familiar and simple formula for the commutator of two normal subgroups. As an intermediate step, we will show what happens in loops, where, until now, it was not known how the commutator of normal substructures should be calculated efficiently. This is joint work with David Stanovský.

Title: Automorphic loops
Presenter: Michael Kinyon
Affiliation: University of Denver

Abstract: The multiplication group $\mathrm{Mlt}(G)$ of a group G is just the permutation group generated by all left and right translations. The inner automorphism group $\mathrm{Inn}(G)$ can be characterized as the stabilizer in $\mathrm{Mlt}(G)$ of the identity element of G. In a loop Q, the same definitions give the multiplication group $\mathrm{Mlt}(Q)$ and the inner mapping group $\mathrm{Inn}(Q)$, but the latter does not necessarily act as automorphisms of Q. A loop is said to be *automorphic* if every inner mapping is an automorphism. Thus automorphic loops include groups, but also other classes of loops such as commutative Moufang loops.
Modulo the need for more constructions and a deeper understanding of Sylow theory, commutative automorphic loops are now rather well understood, and I will briefly summarize what is known about them. In the general case, the main open problem is the existence or nonexistence of a finite, simple, nonassociative, automorphic loop. I will discuss recent progress. For instance, if Q is such a loop, then $|Q|$ cannot be odd, that is, there is an Odd Order Theorem for automorphic loops. Also, $\mathrm{Mlt}(Q)$, which must be nonsolvable and primitive, cannot be of affine or twisted affine type. The main problem has a purely group-theoretic formulation which I hope will interest the audience.

Title: Groups with Few isomorphism Classes of derived Subgroups
Presenter: Derek J.S. Robinson
Affiliation: University of Illinois

Abstract: By a *derived subgroup* in a group G is meant the derived (or commutator) subgroup of a subgroup of G. It is a natural question: how important are the derived subgroups within the lattice of all subgroups? In this talk we are concerned with groups for which the set of isomorphism types of derived subgroups is small. If n is a positive integer, let

$$\mathfrak{D}_n$$

denote the class of groups whose derived subgroups fall into at most n isomorphism classes. Clearly $\mathfrak{D}_1$ is the class of abelian groups. The class $\mathfrak{D}_2$ has been investigated and a classification of locally finite $\mathfrak{D}_2$-groups using finite fields can be given.
Recently, in joint work with P.Longobardi and M. Maj, a complete classification of locally finite $\mathfrak{D}_3$-groups has been found. This is a much more challenging task than for $\mathfrak{D}_2$-groups: it emerges that there are nine different families of these groups. The analysis falls naturally into two cases, namely nilpotent groups, where the so-called generalized Camina groups are important, and non-nilpotent groups.

Title: The group matrix, superalgebras and Berezinians?
Presenter: Ken Johnson
Affiliation: Penn State Abington

Abstract: The group matrix of a finite group is an encoding of the group operation under one-sided division, and appeared at the foundations of representation theory. It appears in applications in several contexts (random walks, control theory, wavelets...). Further work associated a group matrix to any representation of a group. The Grothendieck ring of a finite group is the ring of virtual representations, but it appears hard to express this in terms of group matrices. However, if ideas coming from the superalgebras (first investigated by physicists) are used, such a representation can be obtained. It seems to be interesting to investigate whether the interaction with superalgebras can be used in the applications.

Title: Cartan invariants for finite groups of Lie type in the defining characteristic
Presenter: Cornelius Pillen
Affiliation: University of South Alabama

Abstract: Let G be a simple, simply connected algebraic group defined over an algebraically closed field k of positive characteristic p. Assume that G is defined and split over the field $\mathbb{F}_p$ and let F denote the standard Frobenius morphism on G. For $r > 0$, let $G(\mathbb{F}_{p^r})$ denote the finite subgroup consisting of the fixed points of the r-th iterate of F and let G_r be the r-th Frobenius kernel.
Given a pair of simple $G(\mathbb{F}_{p^r})$-modules L and L', the corresponding Cartan invariant is the multiplicity of L' as a composition factor of the injective hull $U(L)$ of L. In this talk we show that there exists an upper bound $N(\Phi, r)$ for the Cartan invariants of $G(\mathbb{F}_{p^r})$, depending only on the underlying root system Φ of G and r. These bounds do not depend on p. We actually prove a stronger result. Namely, that there exists an upper bound $k(\Phi, r)$, not depending on p, for the total number of composition factors of a $U(L)$. Similar results are obtained for the Frobenius kernel G_r.

Title: On groups with polynomial geodesic growth
Presenter: Zoran Sunic
Affiliation: Texas A & M University

Abstract: We present some observations concerning geodesic growth functions. If a nilpotent group is not virtually cyclic then it has exponential geodesic growth with respect to all finite generating sets. On the other hand, if a finitely generated group G has an element whose normal closure is abelian and of finite index in G, then G has a finite generating set with respect to which the geodesic growth is polynomial. This is a joint work with Bridson, Burillo, and Murray. Title: On groups with polynomial geodesic growth

Title: Saturated Fusion Systems over Small p-Groups
Presenter: Adam Glesser
Affiliation: California State University, Fullerton

Abstract: The study of fusion systems, as a bridge between group theory, block theory, and algebraic topology, and as a new approach to the classification of finite simple groups, is currently incredibly active. In this talk, we discuss the current state of the classification of saturated fusion systems on 2-groups of 2-rank 2.

Title: Incidence Matrices and the Covering Number of S_{10}
Presenter: Daniela Nikolova-Popova
Affiliation: Florida Atlantic University

Abstract: We say that a group G has a finite covering if G is a set theoretical union of finitely many proper subgroups. The minimal number of subgroups needed for such a covering is called the covering number of G denoted by $\sigma(G)$.
Let S_n be the symmetric group on n letters. For even n Maroti showed that $\sigma(S_n)2^{n-2}$, i.e. $\sigma(S_{10}) \leq 256$. Using *GAP* calculations, as well as incidence matrices we have shown that $\sigma(S_{10}) = 221$.
This is a joint work with Luise-Charlotte Kappe.