Space-charge-limited conduction mechanism II

Yifan Yuan
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D. R. Lamb, Electric Conduction Mechanisms in Thin Insulating Films (Methuen, London, 1967)

1. Space-charge-limited flow

- One-carrier space-charge-limited flow without traps. (electrons)
- One-carrier space-charge-limited flow with traps.
- Two-carrier space-charge-limited flow without traps or recombination centers. (cathode electrons, anode holes)
- Two-carrier space-charge-limited flow with recombination centers

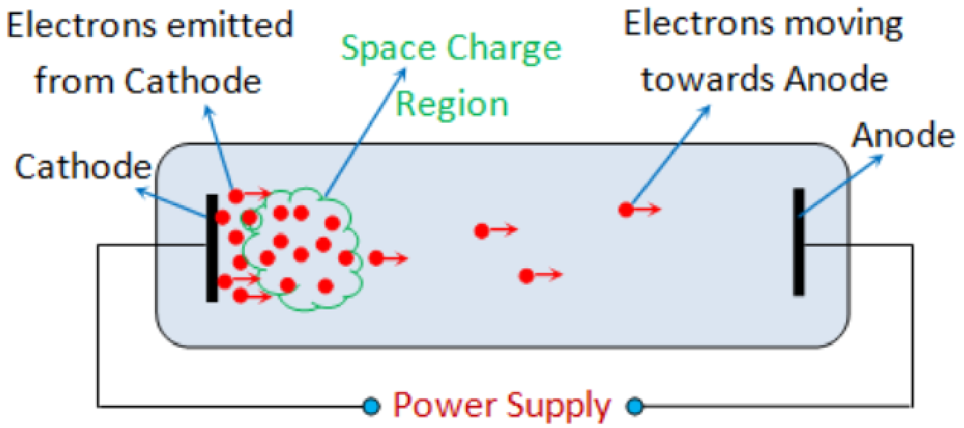
2. One-carrier space-charge-limited

Definition: if an electron injecting contact is applied to an insulator, electrons will travel from the metal into the conduction band of the insulator and form a space-charge similar to that of a vacuum diode.

Accumulation of charges in a particular region is referred to as **space charge**.

In vacuum (Child's law):

$$J = \frac{I_a}{S} = \frac{4\epsilon_0}{9}\sqrt{2e/m_e}\frac{V_a^{3/2}}{d^2}.$$



3. Theory

- At low voltages where the injected carrier density is less than n_0, which is the thermally generated free carrier density, Ohm's law will be obeyed:

$$J = en_0\mu\frac{V}{s} \quad (1)$$

- At transition voltage, V_{tr}, the transition from Ohm's law to Mott and Gurney law takes place:

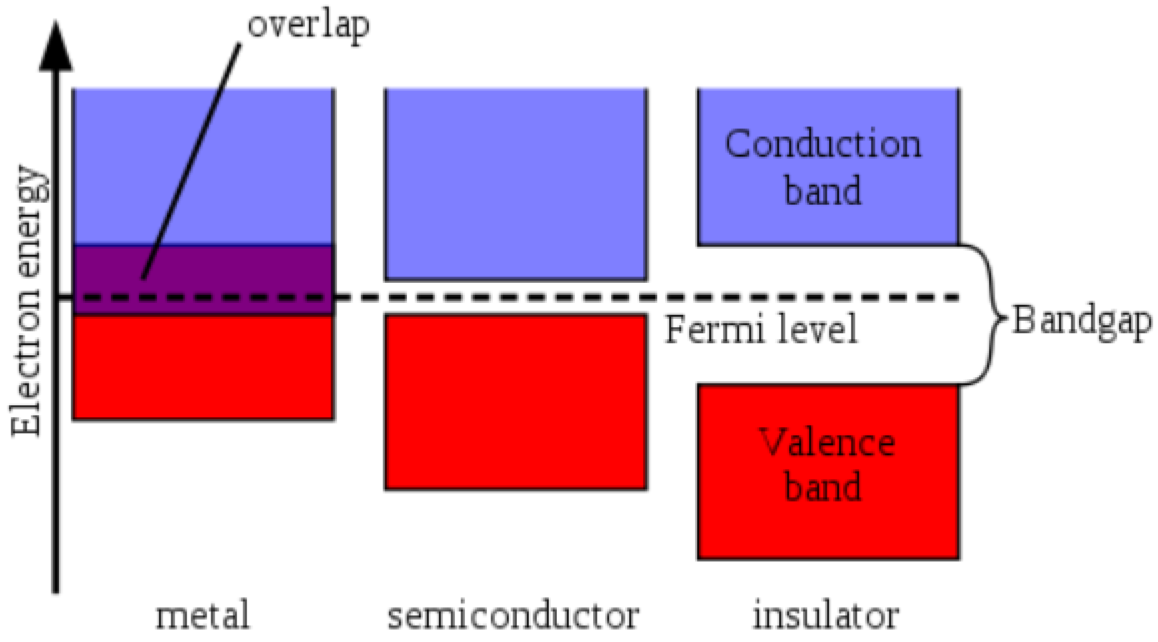
$$J = \frac{9}{8}k\mu\frac{V^2}{s^3} \quad (2)$$

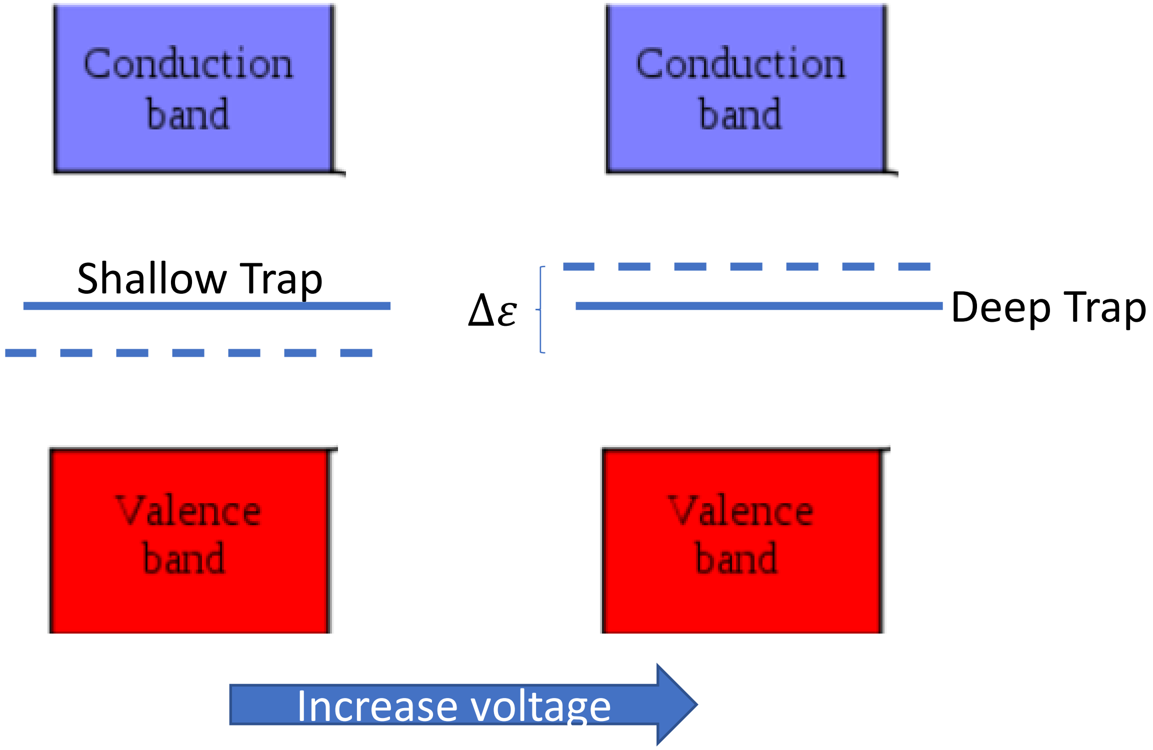
The theory is based on purely field driven currents and diffusion current:

$$J = ne\mu E - De(\frac{dn}{dx}) \quad (3)$$

s: film thickness
μ: mobility
V: voltage
k: dielectric constant
n: free electron density
D: diffusion coefficient

3. Shallow and deep trapping





Free electron density: $n(x) = N_c \exp[\frac{(\epsilon_F - \epsilon_c)}{k_B T}]$

N_c: effective density of states in the conduction band
ϵ_F: Fermi level
ϵ_t: trap level
ϵ_c: bottom of conduction band

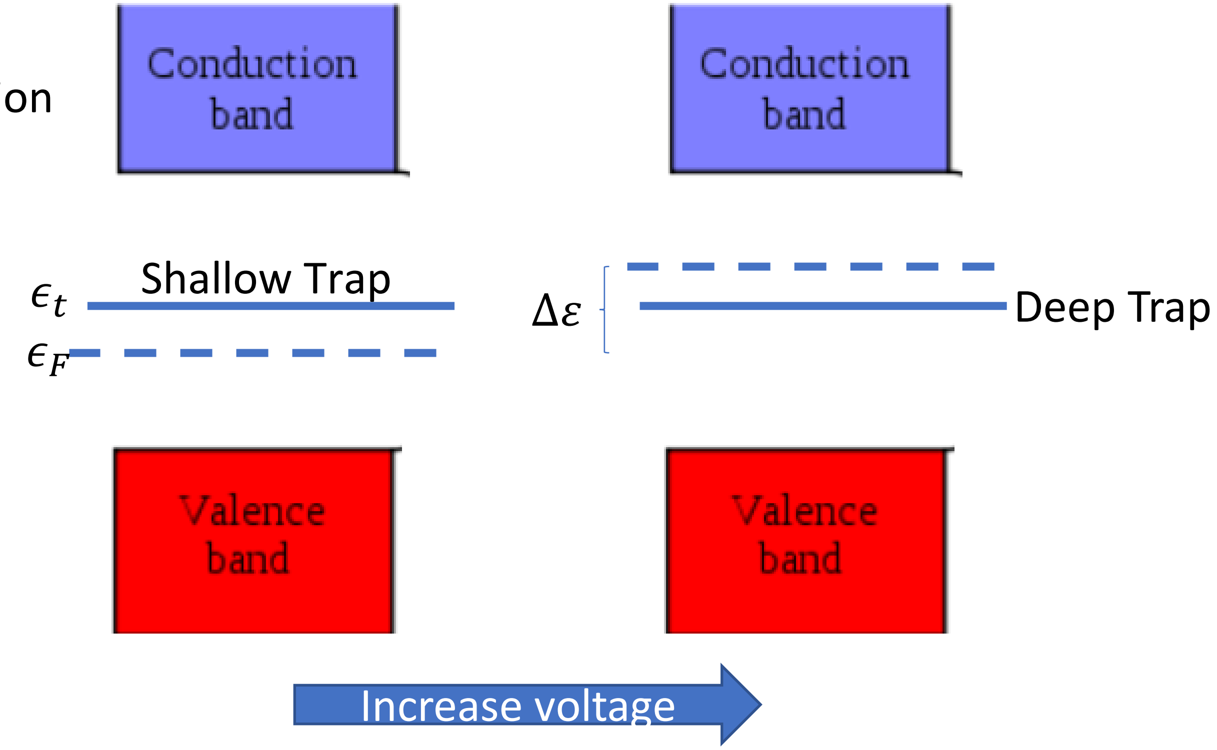
3. Shallow and deep trapping

Trapped charge density (occupancy of trap):

$$n_t = \frac{N_t}{1 + \frac{1}{g}\exp[(\epsilon_t - \epsilon_F)/k_B T]}$$ Fermi–Dirac distribution

For shallow trap: $n_t \approx \frac{N_t}{\frac{1}{g}\exp[(\epsilon_t - \epsilon_F)/k_B T]}$

For deep trap: $n_t = N_t$



N_t: trap density
ϵ_F: Fermi level
ϵ_t: trap level
ϵ_c: bottom of conduction band

3. Shallow and deep trapping

Thus the ratio of free to trapped charge is

$$\theta = \frac{n(x)}{n_t} = \frac{\dfrac{N_c \exp[\frac{(\epsilon_F - \epsilon_c)}{k_B T}]}{N_t}}{1 + \frac{1}{g}\exp[(\epsilon_t - \epsilon_F)/k_B T]}$$

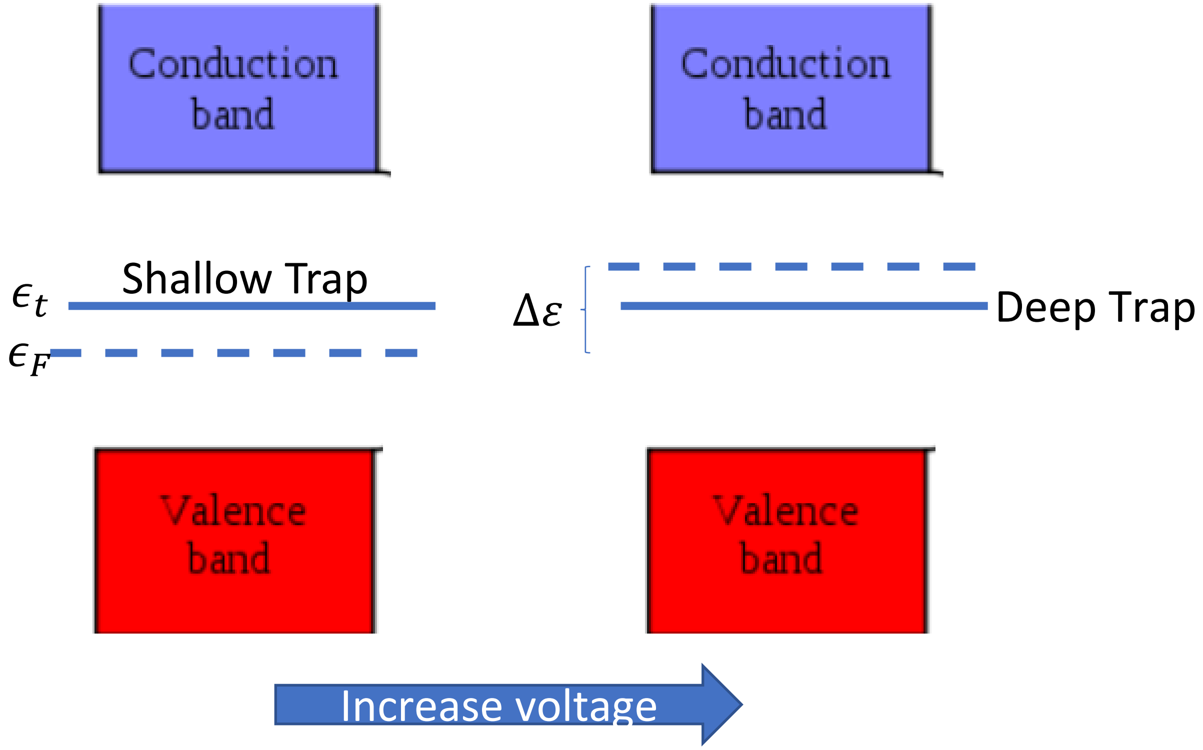
θ can be as low as 10^{-7}, very large effect.

For shallow trap: $\theta \approx \dfrac{N_c \exp[\frac{(\epsilon_t - \epsilon_c)}{k_B T}]}{g N_t}$

For deep trap: $\theta = N_c \exp[\frac{(\epsilon_F - \epsilon_c)}{k_B T}]/N_t$

Therefore,

$$J = \frac{9}{8} k\mu \frac{V^2}{s^3} \quad \Longrightarrow \quad J = \frac{9}{8} k\mu\theta \frac{V^2}{s^3}$$



3. Shallow trapping

The charge which has been injected into the insulator can be distributed in three parts:

(1) Free charge in the conduction band
(2) Trapped charge above the Fermi level
(3) Trapped charge in the states between the initial Fermi level and the final Fermi level.

Assumption that all injected charge will in fact be trapped in (3).

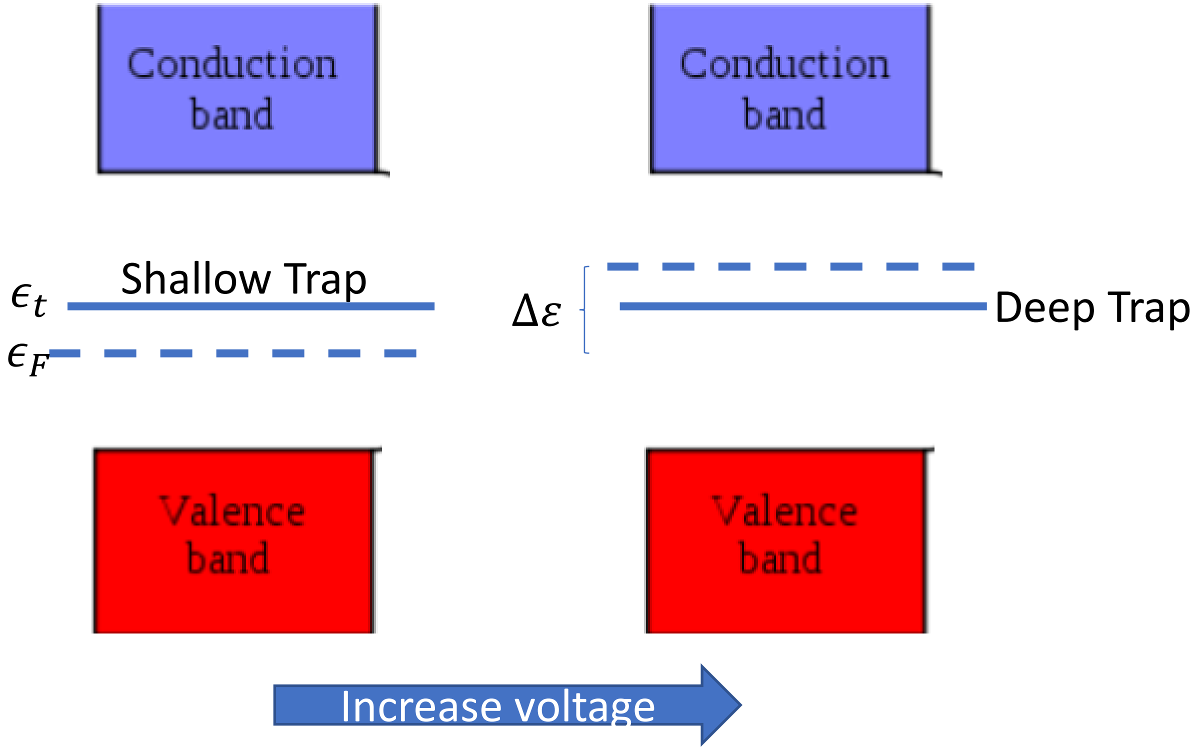
$$\Delta\varepsilon = \frac{Q}{eN_t s} \approx VC/(eN_t s)$$

$$n_t = \frac{Q}{es} = \frac{VC}{es}$$

The free carrier density is given by

$$n = N_c \exp[\frac{(\epsilon_F - \epsilon_c)}{k_B T}] \exp[\frac{\Delta\varepsilon}{k_B T}]$$

$$= n_0 \exp[\frac{\Delta\varepsilon}{k_B T}] = n_0 \exp[\frac{VC}{eN_t s k_B T}] = n_0 e^{tV}$$



Q: injected charge
ϵ_F: Fermi level
ϵ_t: trap level
ϵ_c: bottom of conduction band

Hence $\theta = \frac{n}{n_t} = \frac{n_0 e^{tV}}{VC/es} = \frac{n_0 es}{VC} e^{tV}$

Therefore, $J = \frac{9}{8} k\mu\theta \frac{V^2}{s^3} = \frac{9}{8} k\mu \frac{V}{s^3} \left(\frac{n_0 e}{C}\right) e^{tV}$

4. Experiments

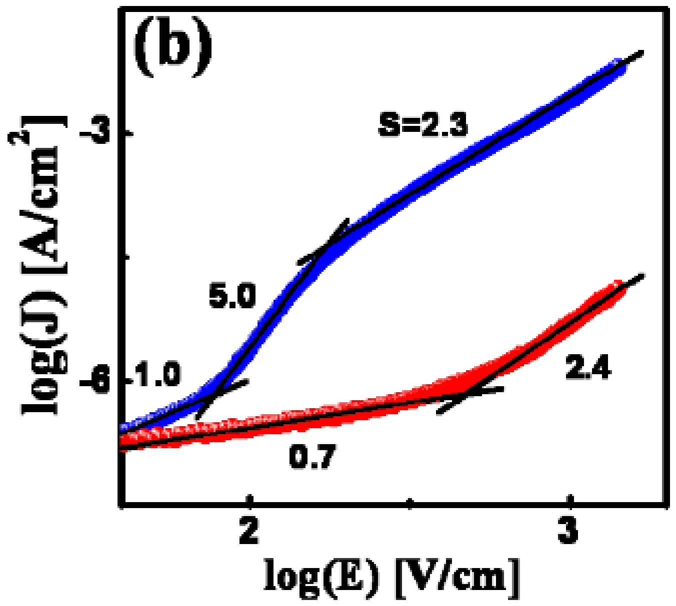


Fig. S2 Typical J-E characteristics of a Au/BFO/Au structure (BFO1) at 300 K, (b) SCLC.

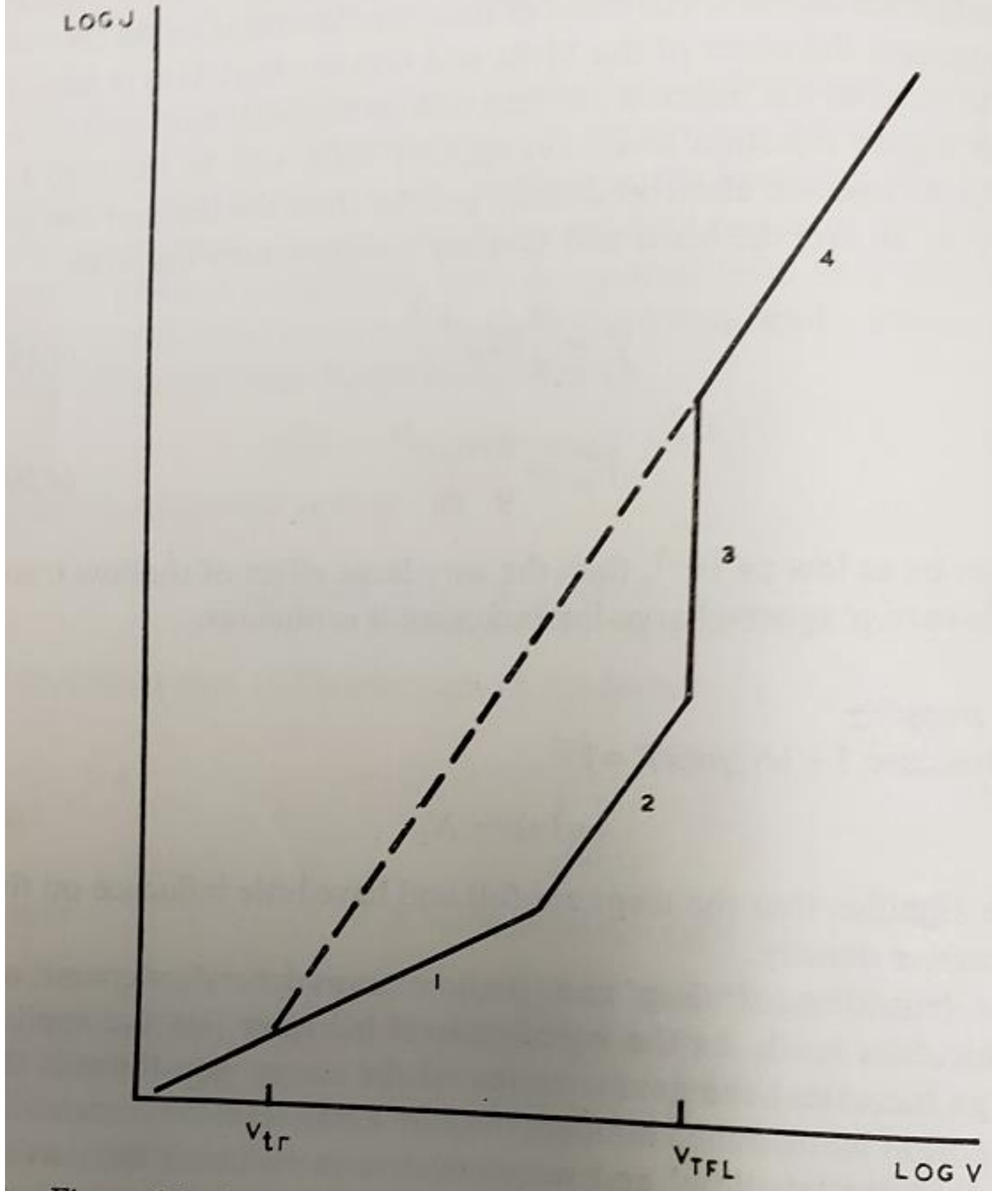


Figure 4.1. One-carrier space-charge-limited current-voltage characteristic for an insulator with a single trap level. (1) Ohm's law. (2) Modified Mott and Gurney law due to the traps. (3) Trap-filled-limit region. (4) Trap free Mott and Gurney law.

Choi, T., et al. "Switchable ferroelectric diode and photovoltaic effect in BiFeO3." *Science* 324.5923 (2009): 63-66. (Supporting information)

4. Experiments

Amorphous selenium (20 u)/ tin oxide / glass substrate

For film 2, the dependence of current on voltage was between V and V^2 at lower voltages.

$$I = 2.2 \times 10^{-11} V e^{V/31.1}$$

$$I = 1.3 \times 10^{-11} V e^{V/57.0}$$

For voltages less than 10 v the current was probably a mixture of ohmic and SCLC. This suggests that the thermal equilibrium Fermi level was less than kT above a uniform distribution of hole capture levels

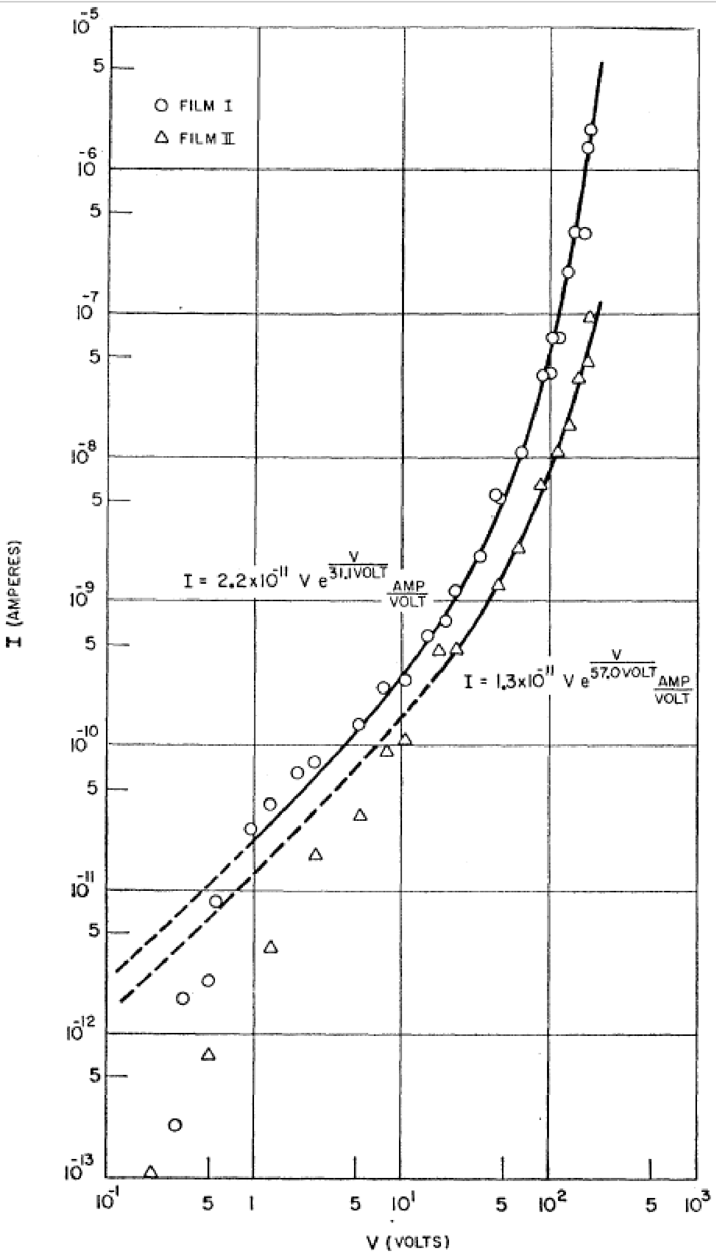


FIG. 11. Space-charge-limited currents in amorphous selenium films having gold hole-injecting contacts.

Hartke, Jerome Luther. "Drift mobilities of electrons and holes and space-charge-limited currents in amorphous selenium films." *Physical Review* 125.4 (1962): 1177.

Thanks and questions?

Derivation of Mott and Gurney law

The theory is based on purely field driven currents and diffusion current:

$$J = n\mu E - D(\frac{dn}{dx}) \quad (3)$$

Using Gauss's law and

$$\frac{n}{k} = \frac{dE}{dx}$$

$$J = k\mu E\left(\frac{dE}{dx}\right) - kD\left(\frac{d^2E}{dx^2}\right)$$

Using Einsteins' relation

$$D = \frac{\mu}{e} k_B T$$

$$J = k\mu E\left(\frac{dE}{dx}\right) - k\mu k_B T\left(\frac{d^2E}{dx^2}\right)$$

Taking $\frac{dE}{dx} \approx \frac{E}{s}$, if $k_B T \ll eEs$, one can neglect the diffusion term

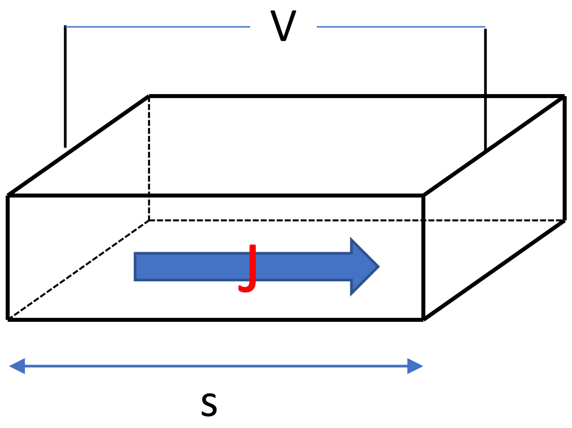
$$J = k\mu E\left(\frac{dE}{dx}\right)$$

And integrating

$$E = \sqrt{\frac{2J}{k\mu}(x + x_0)}$$

where x_0 is a constant

μ: mobility
n: charge carrier density
D: diffusion coefficient
k: dielectric constant



Derivation of Mott and Gurney law

$$V = \int_0^s E dx = \int_0^s \sqrt{\frac{2J}{k\mu}(x + x_0)}\, dx$$

$$= \frac{2}{3}\sqrt{\frac{2J}{k\mu}}\left[(s + x_0)^{3/2} - x_0^{3/2}\right]$$

Thus, for $x_0 << s$, neglecting x_0

$$\boldsymbol{J = \frac{9}{8} k\mu \frac{V^2}{s^3}}$$

The theory in fact cannot give an accurate description of the physical situation near the injecting cathode where the field will be zero and the current must be a pure diffusion current.

μ: mobility
n: charge carrier density
D: diffusion coefficient
k: dielectric constant