Structural Geology

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Lecture 06 - Concept of Strain and Deformation (Part - 2)

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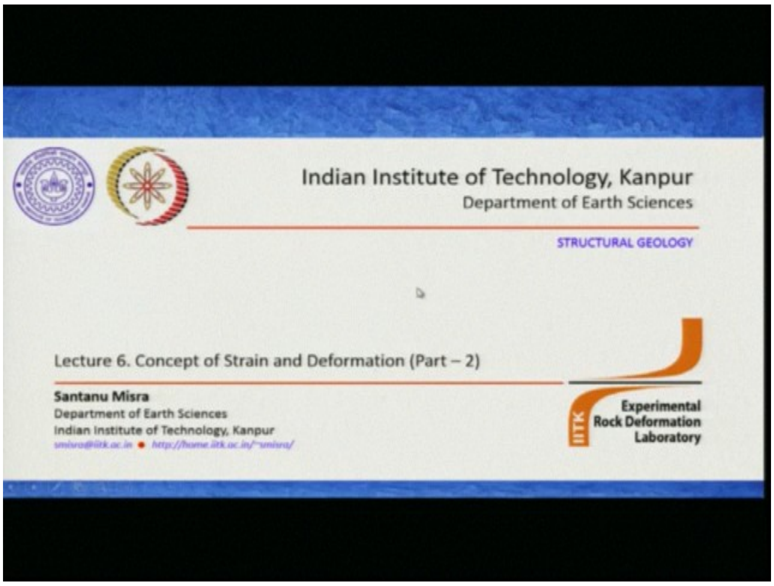

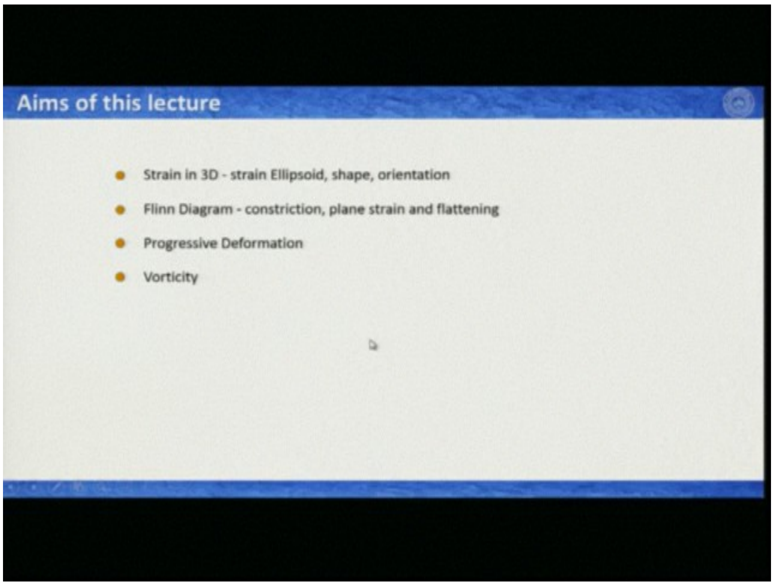

Hello everyone! Welcome back again, we are in our lecture number 6 and we will continue what we have learned in the last lecture, so it is a topic, concept of strain and deformation and we are in part 2. In this lecture will cover strain in 3 dimensions and we will mostly look at strain ellipsoids, their shapes and orientations, then we will move to Flinn diagram and there we will see constriction, plane strain and flattening types of deformation. After that we will see the different ways to look at progressive deformation from where we will switch to vorticity and conclude the lecture.

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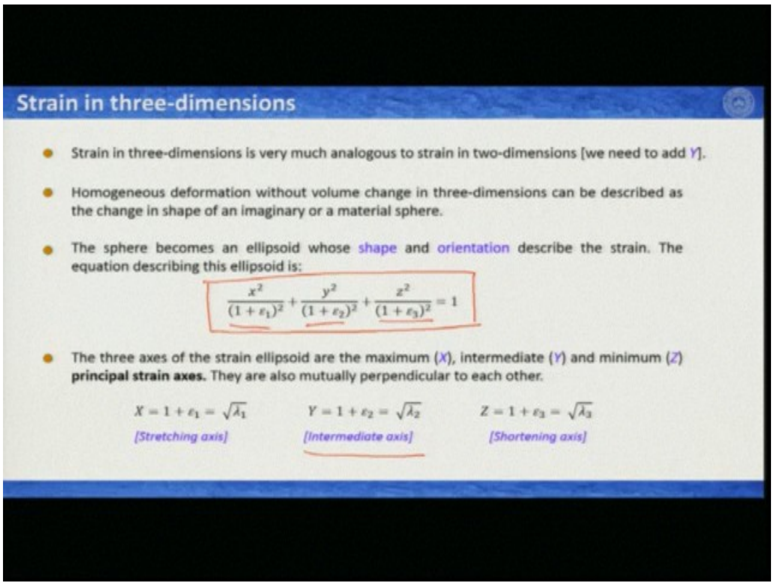

This above mentioned slide shows strain in three dimensions is very similar to which we have learned when we discussed the strain in two dimensions, there we had X and Z and this time because we will be dealing with three dimensions we will add Y, so homogeneous deformation without volume change in three dimensions generally like we already described previously with strain ellipse, presently we will described as a strain ellipsoid which is a shape change from an imaginary or a material sphere. Now the sphere becomes ellipse, so we have to define its shape and orientation to describe the strain and the ellipsoid can be described by this equation.

So the three principal axes of strain are here and then it is a Cartesian coordinate system XYZ, so this is the equation of the strain ellipsoid which is essentially the function of your three principal axes of strain, now the three axes of strain we know, the principal axes of strain are the maximum, intermediate and minimum principal strain axes, so they are also mutually perpendicular to each other and like we have described the X and Z axes in the 2-D, we have to add here the intermediate axis which is Y equal to 1 plus epsilon 2 or root over of lambda 2. The other 2 remains the same that is X equal to 1 plus epsilon 1 and Z equal to 1 plus epsilon 3.

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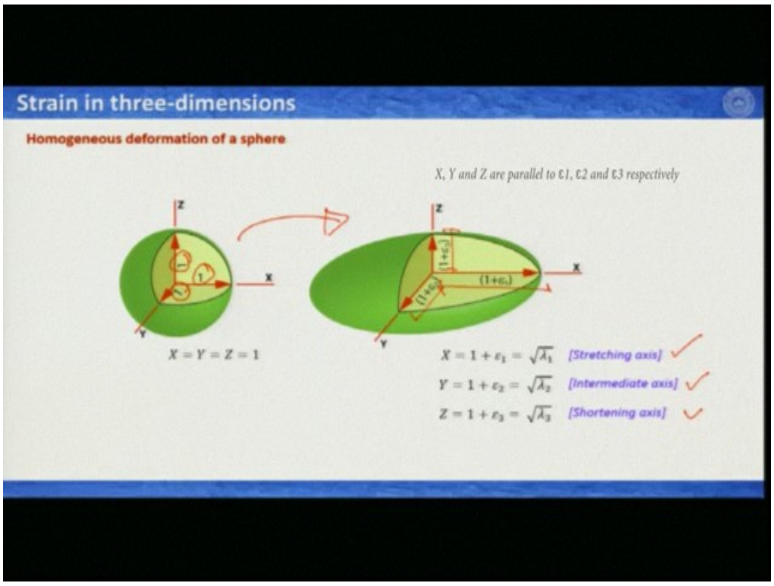

Now let us have a look of what do we mean by strain ellipsoid which is a transformation from a sphere. So if we deform a sphere homogeneously then ideally it should take a shape of an ellipse, what we see here, this is a sphere and if we cut it perpendicularly from the middle then we can get the three ready of the sphere, in this case all these are units, so this is one, this is one and this is one. Therefore this is un-deformed and XYZ are all equal to 1, if it transformed to an ellipse due to homogeneous deformation, then this X, Y and Z would change and therefore their values would change as well and the sphere would take form of an ellipse.

In that case Z would be 1 plus epsilon 3, X would be 1 plus epsilon 1 and Y would be 1 plus epsilon 2 and therefore at least from this visualisation as X has elongated most that is our considerations, so X is the stretching axis, Y is the intermediate one compared to Z, so Y is the intermediate axis and Z which got compressed most, so therefore Z is the shortening axis, so these XYZ these three play a very important role, their deformations, their shortening, their extension relative to other two.

Now based on these relative elongations or relative shortenings of XYZ, we can actually describe a number of possibilities or end members of strain ellipsoid, but this time instead of ellipses we looked with a unit cube.

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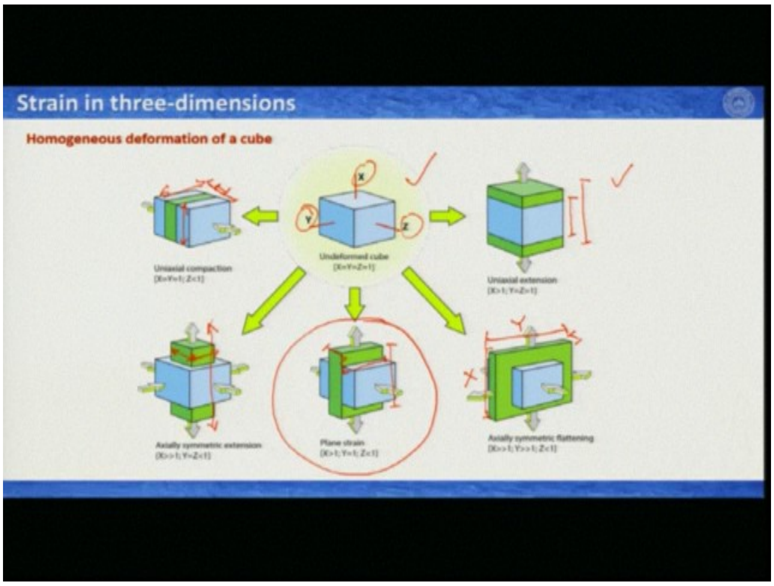

This diagram presents, this is the unit cube highlighted by a circle where X is vertical, Y and Z are oriented horizontally, this is un-deformed cube therefore XYZ equal to 1, now one can deform this cube in many different ways, but here what is shown by these five illustrations at these five sides, these are the some sort of end members of the deformation in three dimensions.

The first one will take over is uniaxial compression or compaction, so, before I go to this, go to the description of this deformation as you see this blue cube is the un-deformed cube and what we see in the green colours these are deformed shaped or from the cube, it got transformed to the shape of this and these are all in three dimension. So what we see here, uniaxial compaction as Z is the shortening axis, so the compaction happened along Z, X and Y remain same, as we can see this is the X direction, so X is constant, this is the Y direction, Y is constant, but Z instead of this length, it is now got shortened and it got shortened in one direction along one axis, Z axis, so therefore this is uniaxial compaction.

Now in this side, you can do also uniaxial extension, in that case, X is the maximum stretching direction, so extension should happen along the X direction, so this was your initial length along X, but now it got changed to this magnitude, whereas Z and Y remained same. Therefore this is uniaxial extension, now there are three other possibilities, one is axially symmetric extension, that means it gets extended in one direction and shortened in other two directions.

And therefore it got extended along X direction, but got shortened along Y and along Z direction, the other possibility is plane strain, this is very important term that you need to learn. Plane strain is when in one direction there is no strain but in other directions yes, in this case, this illustration what we see here, this unit cube got extended along X direction, it got shortened along Z direction, but it is Y direction remained constant length as it was, so then it is plane strain that is along one direction, there is no strain.

And the final one is axially symmetric flattening, what happens in this case that only one axis shortened very much and other two axes they extend highly, so what we see here as you can see the Z direction, it got highly shortened and this is the X and Y both, this is Y and this is X, they both got extended significantly compared to Z and this is known as axially symmetric flattening.

Now before we switch to the next slide there are many books or many texts or manual materials, you may find these X, Y and Z these are oriented different ways, so you may find that X and Y these are placed horizontally and Z is vertical. We may use in some cases these illustrations as well, but whenever you look at such diagrams, it is very important that you first make sure you know where is your X, where is your Y and where is your Z compared to the deformation pattern.

Now based on this idea of deformation of an un-deformed cube to this different possibilities, you can understand that once these materials they do deform different ways, you would produce different types of strains, that means the rocks would deform differently and at the same time, because the rocks would be deforming differently, you will see different types of features in your rocks and we will see this now.

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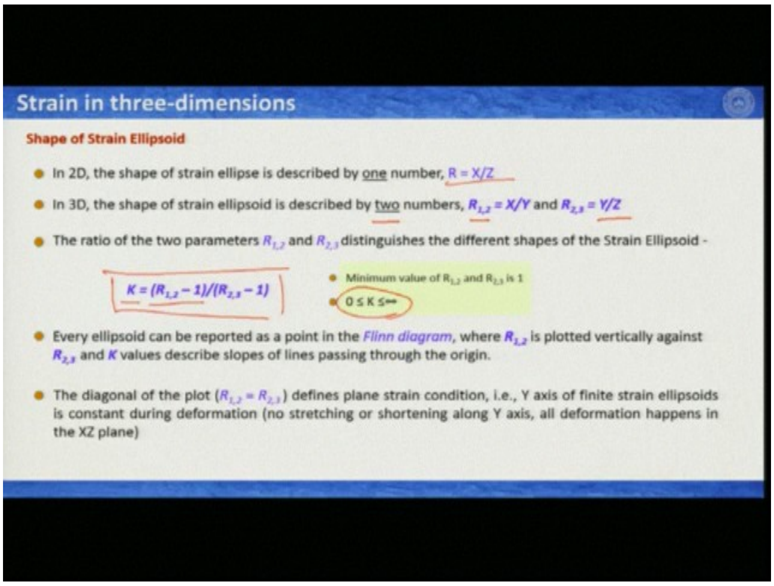

So the best way to explain these different types of deformation is known as Flinn diagram, the description of three-dimensional strain ellipsoid is best represented in Flinn diagram. So, in two dimension the shape of the strain ellipse that we have described by only one number, if you remember that was R which was the ratio of long axis versus short axis but in three-dimension the shape of the strain ellipsoid is described by two numbers, one is R1,2, which is the ratio of long axis versus intermediate axis and another one is R2,3 which is the ratio of intermediate axis versus short axis.

Now the ratio of these two parameters that is ratio of R1,2 and R2,3 describes and distinguishes the different shapes of the strain ellipsoid and this number is known as K, where K is defined as R1,2 minus 1 divided by R2,3 minus 1, so from this equation and our understanding on XYZ we can figure out that minimum values of R1,2 and R2,3 possible is 1 and K can be from or could be equal or greater than 0 and equal or less than infinity.

Now given this condition that K could be 0 to infinity, so you can report the all possibilities of strain ellipsoids in the Flinn diagram and the Flinn diagram we will see in the next slide, it is actually a sort of two-dimensional Cartesian plot where X axis is represented by R2,3 and Y axis is represented by R1,2 and you can plot a number of K values in between, so the diagonal of this plot as we will see soon that is where R1,2 equal to R2,3 is actually defining the plane strain condition, that is Y axis of the finite strain ellipsoid is constant during the deformation, that means we have learned it before that there is no stretching or no shortening along the Y axis. Whatever deformation happens that happens along the X and Z axis and therefore on the XZ plain.

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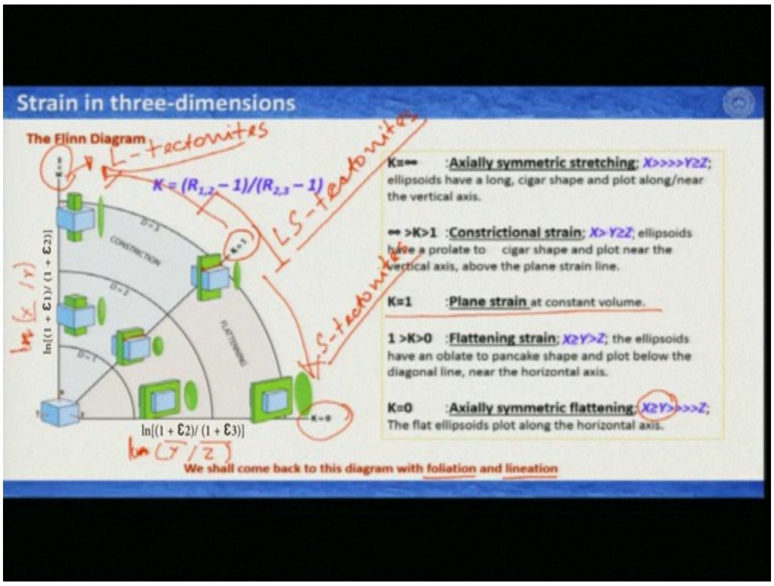

And this is the Flinn diagram we are talking about, as you can see that along the X axis we have plotted these and in this case, this is again long axis which is X and here, this is Y and again all these things is under log, now this log is used, sorry, this is LN, so this log is used to accommodate different shapes or sizes of the measurements. So different magnitudes of the measurement if you have a high value and you have a small value of the measurements, then you can plot everything in the same diagram.

Let us try to understand this diagram in detail, so as we said that along this line which is running 45 degrees with respect to this X and Y axis, this is where K equal to 1 and as we have said K equal to 1 you have plane strain at constant volume, as you can see here at this magnitude of strain the Y axis remained constant, even I increase the strain magnitude Y axis remained constant, but it extends along X and shortens along Z direction. So whatever left on both sides of this K1 value, if I go to the top side that says that it is under the constriction domain, that means that in this domain everything would deform, I mean I am sorry, along X axis it would extend and other two directions it would shrink.

Therefore with progressive deformation you would produce something which is like a pencil or like a long object, on the other hand if we go to the flattening side, which is a downside of this K equal to 1 direction, there the deformation would happen in axially symmetric manner, or axially symmetric flattening manner where X is greater than or equal to Y and these XY values are much, much greater than Z, therefore you would produce something like a pancake or like a flattened disk.

So if I go towards this side K equal to infinity then I produce rocks characteristically with some sort of linear features and if I come to this side K equal to 0, the deformed rock would show a lot of flattened objects or disk shaped objects. Now considering this idea, you can figure out, if I am in the constriction domain, then the rocks would tend to produce more linear features through deformation and at K equal to 0, the rocks would tend to produce more flattened or planer fabrics.

Therefore these rocks here would be prone with schistosity and therefore K equal to 0 or in this domain whatever we produce towards this side, we call it S-tectonites and here because we will be producing lineations dominantly these are known as L-tectonites and whatever stays in the middle it would have both lineations and schistosity, therefore these are known as LS-tectonites. We will learn more about L-tectonites, LS-tectonites and S-tectonites when will study foliation and lineation in one of the next lectures, so we will come back to this Flinn diagram again and again for more interpretation of deformed rocks.

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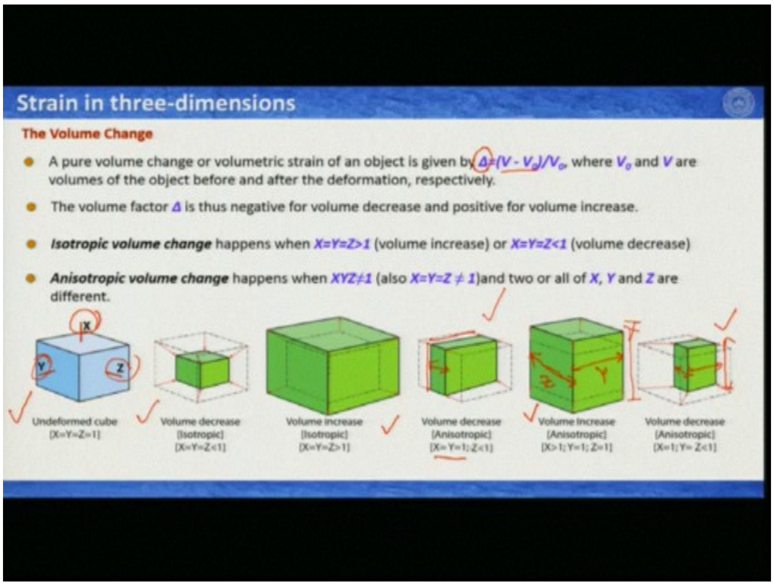

Now let us talk about the volume change, so we have discussed so far it is homogeneous deformation without volume change, but volume change like it happens in two dimensions, it can also happen in three dimensions. So pure volume change or volumetric strain of an object as we, it is a very similar equation that we have seen, so it is a ratio of the change of the volume with respect to the initial volume. So you can say that volume change or if you say this is delta then it is they define as V minus V0 by V0, where V0 and V are volumes of the object before and after the deformation respectively.

Thus the volume factor delta is thus negative for volume decrease and positive for volume increase, now volume decrease and increase whatever happens, it can happen either isotopically or anisotropically, if it happens isotopically means that all principal axes of strain are either extending or shortening equally and if that does not happen, then it is anisotropic volume change. So isotropic volume change therefore could happen when XYZ, they are all greater than, there all equal and greater than 1, therefore you would have volume increase and if these are all equal, but the values are less than 1, then you have volume decrease.

So here I have tried to show these with some illustrations, so again we have these un-deformed cubes, un-deformed cube here, you have X, you have Y, you have Z direction here. As you can see in the first illustration where volume has decreased isotopically, all these directions the shortening was equal and along all these directions the shortenings were equal. In the second diagram where the volume has increased, again along all directions the extensions or elongations are equal.

Now anisotropic volume decreases essentially where your XYZ would not remain same of values, so they could be either greater than 1 or less than 1, they could be equal to 1 or not equal to 1, does not matter, but they cannot be equal to each other. So here there are three examples, so in the first case what we see that volume has decrease anisotopically where X and Y did not change, so therefore your X remain constant, Y remain constant but along Z it decreased its volume.

In this case, the second example, which is also anisotropic volume increase but in this case, X has extended but Y and Z, this is your Y and this is your Z, I am sorry, this is Y, this is Z and this is your X, so you can see X has extended but Y and Z they remain constant. So therefore volume has increased but anisotopically and in the third and final example what we see here, volume has decreased again anisotopically where nothing has changed along the X direction, but it got shortened along Y direction and also it got shortened along Z direction.

Now, so we have learned so far that homogeneous deformation in three-dimension and also volume deformation that means you can change your volume in three-dimension as well.

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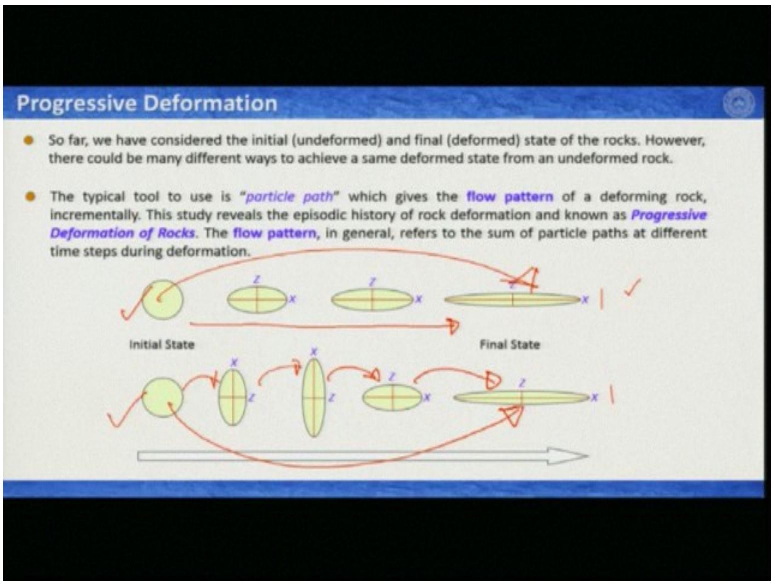

The next topic we discussed in this lecture is progressive deformation, now if you remember the first lecture of strain we talked about initial position, final position and then one set of examples we talked about displacement vectors, where we corrected the initial position with the final position and added these two points by an arrow and from initial to final position we added that arrow heads and that was a vector.

But then there was also another column that was particle path where we actually could track that how this point was moving from its initial position to the next position or to the final position and depending on whether you are doing rigid body rotation or translation or deforming the material by simple shear, pure shear or any other ways we can see or we can understand that, this displacement vectors are not necessarily equal or they are not necessarily similar to those of the particle paths. Now the study of these particle paths actually falls in the domain of progressive deformation.

So there could be many different ways to achieve the same deformed state that is the point, what we see here? We have initial state in these two rows, both are circles and the final stage both got in 2 dimension we are looking at, and we have got a strain ellipse. Now what we see here, from here to here, if we look directly then they look similar. So the displacement vectors if we try to draw from this position to this position in both cases, it would be similar but in between there could be many different possibilities.

The first example is very straightforward that I had a circle then little more strain, even more strain and further more strain I achieved to this state. But in the second row what we see that it took a different shape, it deformed differently from here to here, from here to here, from here to here and finally we could reach here. So displacement vectors here in these two cases will be very much similar but not the particle paths, again the study is progressive deformation.

Now in the next three slides, we will have a look that what could be the possibilities or what could be the characteristic particle paths or flow patterns of deforming rock in terms of pure shear, simple shear and the combination of these two and before that what is flow pattern? A flow pattern of a deforming rock is actually, it refers the sum of particle paths at different times steps during deformation, so it is you add each and every particle path and cumulate them, what you get is your characteristic flow pattern of a particular type of deformation.

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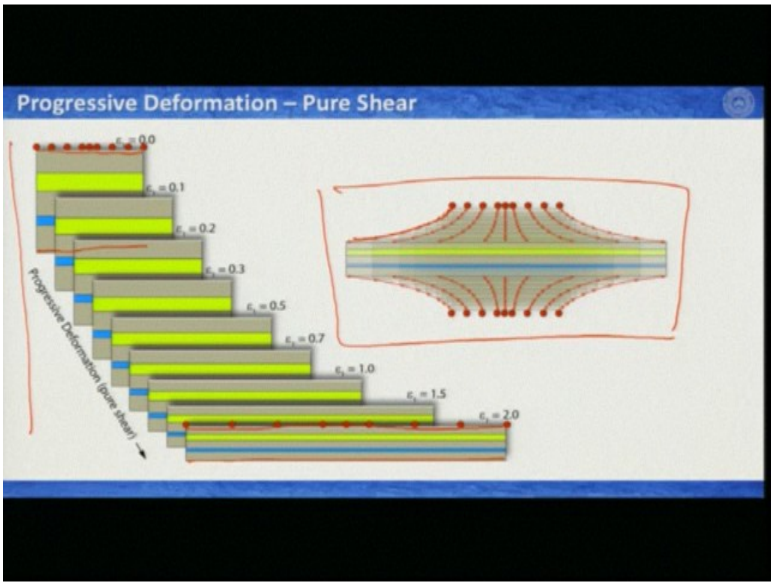

So what do you see in this slide on the left side, I particularly would request you to focus in this side, that here I had in 2 dimensions I had a square and then progressively I deform this square to achieve a finite elongation or finite deformation along X direction of two, of course you do not see the individual squares and later rectangles just because I had to fit them in a single slide, but they are cascaded in this way. So the strain progressed this way, so first one it is undeformed and then slowly 0.1, 0.2, 0.3, 0.5, 0.7, 1, 1.5 and 2, so this is how it progressively deformed in a pure shear manner.

Now if I club them together all these images, then they would appear something like that and I try to track this little red points in the form of particle paths, then this point would move to here and then so on. So characteristically all these points, at least from the two sides, the top side and the bottom side and how they have arrived to their N shape N deformed shape, if we track each and every instances of deformation, then the characteristic flow path would be like this, what you see here in this diagram by red arrows. So tiny red arrows heads and finally you achieve a pattern, we will analyse this later.

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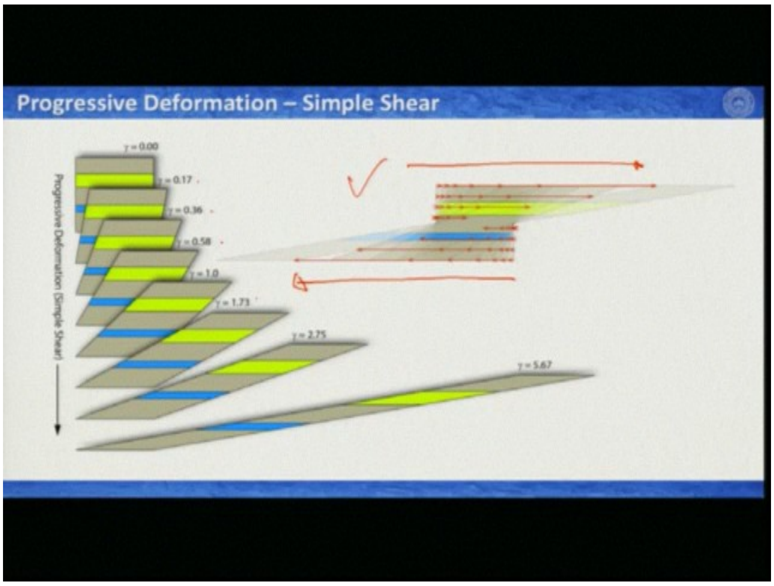

At this time let us go to what happens if you have simple shear deformation, again the plots are in a very similar way. So I have a square and then I deform this square with progressive simple shear, so here at the shear strain magnitudes given one after another and again I can club them together as you can see here and I see the particle paths or the flow patterns if I try to connect each and every point from one stage to the next stage then it would be given as you can see here with this red arrows in this side and this side. And what is interesting that if you remember the previous slide then it is characteristically different from pure shear.

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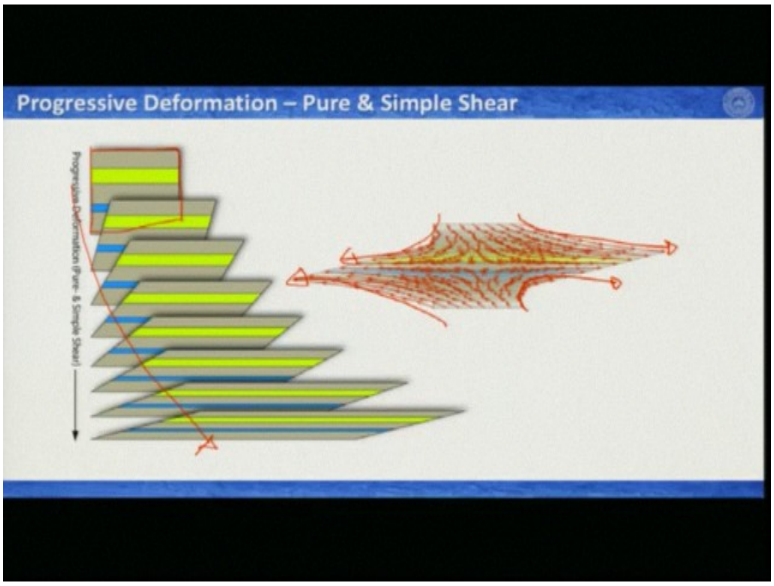

Now if we deform the same square, this square in both pure shear or simple shear manner, that means I combine pure and simple shear, so here again in a similar way we have the cascade of these rectangles and then what we see here again, if we club them and try to see the flow patterns then it is extremely different. We see that these things are going this way; these things are going this way and so on, so again, this is very different to what we have seen with pure shear and simple shear separately.

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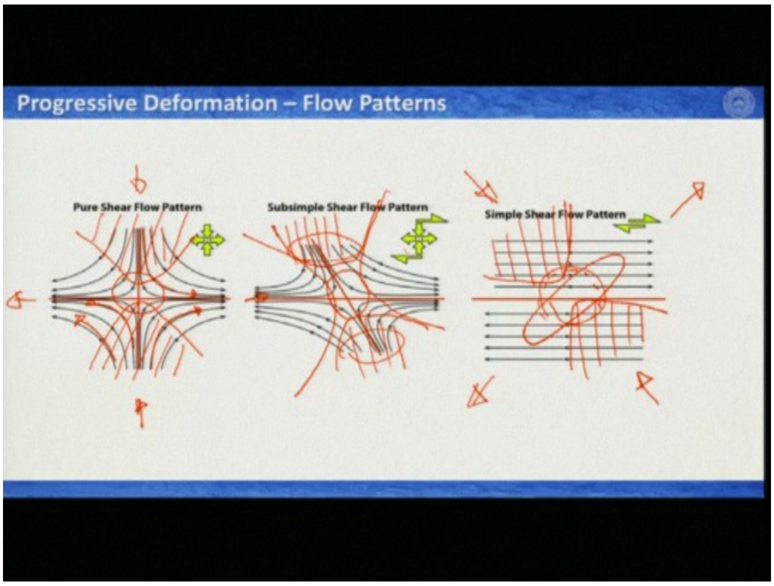

So what do these flow patterns tell us? Let us have a look. So if we see the pure shear, this is the simple form, these yellow arrow heads are showing that this is your compression direction and this is your extension direction in pure shear. What we see here that any material point here is moving and slowly it is going towards the extension direction and while doing so initially this material line or points are in the compression domain, in this direction they are in compression and similarly this is symmetric along the axis. These are your compression domain and on the other side, this is your extension domain.

In simple shear it is not so easy to apprehend the compression or extension looking at the flow pattern, but you can see the flow patterns are very much straight and they are moving opposite to other side of the flow lines. Now in this case because if I had a circle here, the circle would deform to an ellipse of this and therefore I can assume that or I can conclude that this would be my compression domain, that means compressions are coming from this direction, and this would be my extensional domain.

At this moment if you bring in between this combined pure and simple shear then it would look like this, here I have both symbols for pure shear and simple shear. What we see here that, unlike pure shear here the material lines initially have some sort of compression but then again they flow towards the extension and try to achieve something that we see in the simple shear pattern. Now here again, it probably would have a compression direction in this side as we can see here or compressive domain like this and these are extensive domain.

Now because I have compression from one side and extension on the other side and also all materials as we see that in these three very basic examples, they tend to flow towards the extension direction. From here, if I consider a material is sitting in between or then it may have a rotation or it may stay stationary. So based on a flow pattern and nature of deformation if we try to analyse whether the materials would stay stationary or materials would rotate, based on this study we can go to another topic which is known as vorticity.

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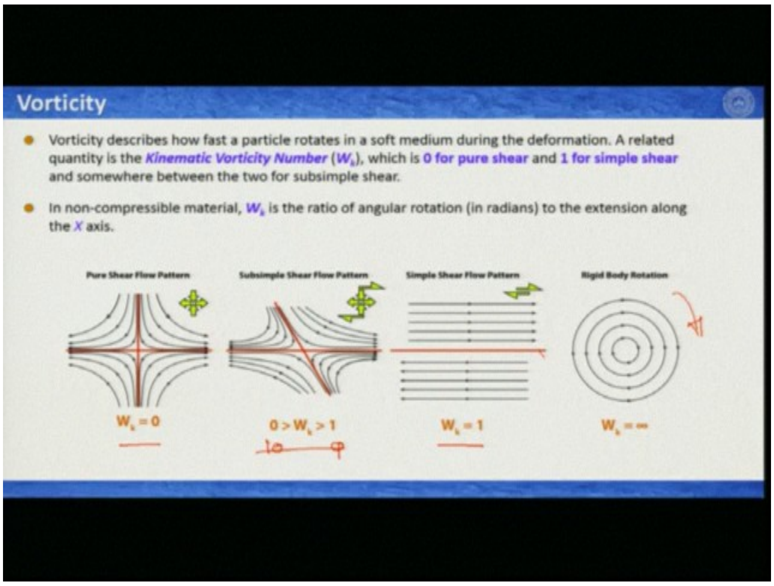

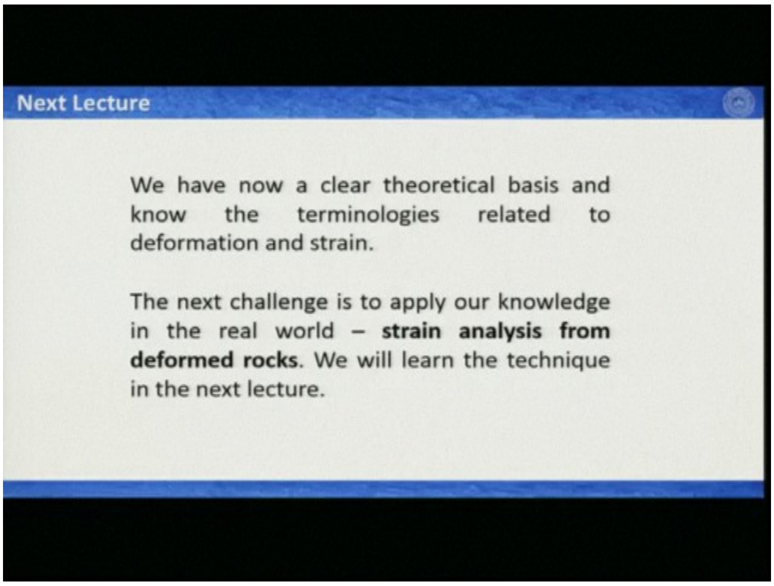

So vorticity is actually the study of, it describes how fast a particle do rotate in a soft medium during deformation and there is a term which is known as kinematic vorticity number. In mathematical expressions, it is expressed as Wk and Wk is, as I said, it is kinematic vorticity number is assigned for pure shear as 0 and 1 for simple shear.

So whatever stays in between is the combination of pure shear and simple shear, so again we can have a look of the same diagram that here Wk. is 0, so this is pure shear, here Wk. is 1, this is simple shear and whatever stays in between from 0 to 1, this is your sub simple shear flow pattern. Okay and if I increase the Wk. value, then actually I achieve something which is flow pattern is very similar to rigid body rotation and we see the effects of rigid body rotations in many different geological structures, including the shears on features like delta structures we have seen it in one lectures and will see it more in when you study the ductile shears.

Now I conclude this lecture, but before concluding this is important to remind you that all this strain and deformation that we have learned so far, these are just not the descriptions, this include mathematical analyses which is not the scope of this lecture series. But I recommend you to read these books I recommended and also look and search in the web to have at least some basic ideas of the mathematical descriptions of strain. So in this lecture, I believe and together with the previous lecture we have some sort of theoretical basis and more or less know the terminologies that we deal with deformation and strain, particularly in the context of structural geology.

At this moment the challenge is how to apply this knowledge to the natural field that means if I see a deformed structure then how to analyse or how to get strain or how to measure strain out of it and this is the topic of the next lecture. Thank you very much, meet you in the next lecture. Bye.