Digital Circuits and Systems
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Module – 04
Min terms, Max terms, SOP and POS Forms

Welcome to module 4 of week ones lecture. So, till now we have seen some basics of Boolean logic and the basics of operating Boolean expressions and so on. We will now start looking at more machinery required to start from a problem, how to get a truth table, how to get to an expression and from there on how to minimize. So, to do that, we need a few things.

So, this lecture is going to be slightly theoretical in nature, we will not be looking at too many problems. There are lots of definitions and other things that you have to know, so that from the next week onwards, we will know all the terminology that we need to. So, the first thing I want you to know is our change in notation that we are going to have, so far we have been using this dot for AND everywhere.

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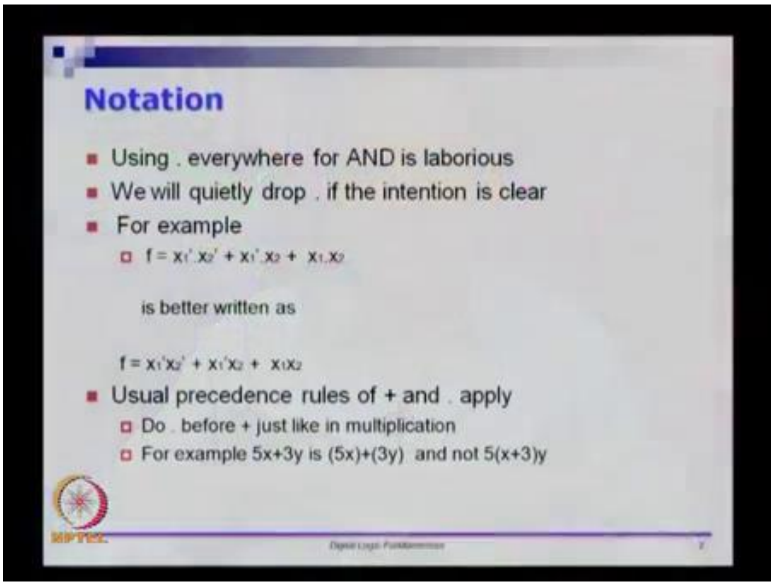

But, using it everywhere is quite laborious, because we will have to remember to put the dot everywhere and visually, it is not very good. So, for example, if I go and look at this expression here x 1 complement and x 2 complement or x 1 complement and x 2 and or x 1 and x 2. So, this is a bit tedious to do every time and in basic algebra, we do not always put this multiplication everywhere. So, just like that, you want to be able to get rid of this dot whenever possible.

So, this is the same expression re written with and remove, so x 1 complement and x 2 complement. So, the AND is implicit here whenever we write something of this form, the AND is implicit, so x 1 complement and x 2 complement or x 1 complement and x 2 or x 1 and x 2. So, this is visually better than doing this and especially if we are going to write a much larger expression with many, many terms and many, many variables, remembering to put that AND is a problem, also it becomes visually slightly more clunky.

So, instead we would drop the dot, if the intention is clear, so we will have to remember about usual presidency rules. So, whenever there is, if we have a plus and an expression which it is has an implied dot or dot you have to be careful about the precedence’s. So, in basic algebra, if you have 5 x plus 3 y, you interpret that as 5 x added with 3 y and not 5 times x plus 3 times y and so on.

So, the same rule applies here, when you read this expression, this is x 1 complement and x 2 complement logically OR with x 1 complement and x 2, it is not x 1 complement and x 2 complement or x 1 complement. So, we have to be careful about, how these things are group, sometimes we will use parenthesis explicitly, sometimes you should use the basic rules of precedence that you have in algebra, just carry it over here.

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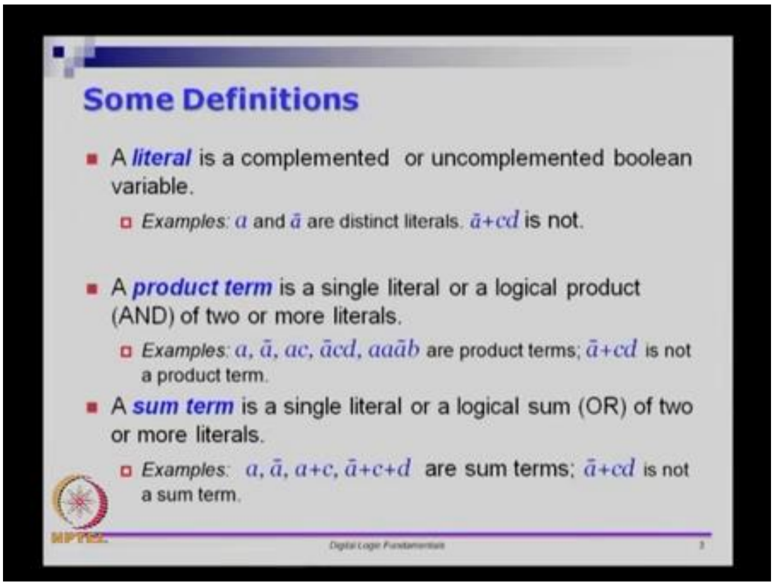

So, now let us move on to some definitions these are fairly simple definitions, but these are necessary, so that whatever we discussed later becomes clearer, so we will start with a single variable or a literal. So, a literal is essentially variable which is either in the complemented or in the uncomplemented form. So, let us look at these examples here, if I have a, it is a single variable and it is in the uncomplemented form, a bar is in the complemented form.

So, a and a bar are actually two different literals of the same variable a. So, you use the variable a, a and a bar are two different literals. This expression here a bar plus c d is not a literal, because it is not a single variable in the complemented or uncomplemented form, it is a combination of multiple variables. So, this is not a literal, so single variable either in complemented or uncomplemented form gives you the literal.

A product term is essentially either a single literal by itself or a logical product or and of two or more literals. So, will start using then... So, instead of saying and every time we can start using the term product now, because it is just like multiplication. So, I told you the analogy earlier that you can think of and as multiplication. So, we say product of two terms, products of two expressions and so on in the integer and real number fields is same thing we can use it in same terminology we can use in Boolean algebra.

So, we will use the term product for AND and a product term is either a single literal like this. So, a bar are single literals, so they are by definition product terms also. So, product term includes literals or you should have something of this kind. So, a c is actually a and c, so it is a product of a and c, a bar c d is the product of a bar c and d and so on. Even, this is a product term a, a bar b is a product term a bar plus c, d is not a product term, because this involves a plus here.

So, this is not a product term, because there is a plus symbol here, it means is a bar or c d, we cannot rewrite this using only single literals and their products. So, all these are product terms, this is not a product term. Similarly, a sum term is a single literal or a logical sum of two or more literals. So, a a bar a plus c, so this is a logical OR of 2 literals, this is a logical OR of three literals and so on.

So, these are all valid sum terms and a bar plus c d is not a valid sum term, because it has a bar plus c d and this is not a single literal. So, if I had given you a bar plus c, that is actually a sum term, but a bar plus c d actually involves a product also. So, a product term should not involve sums and a sum term should not involve products.

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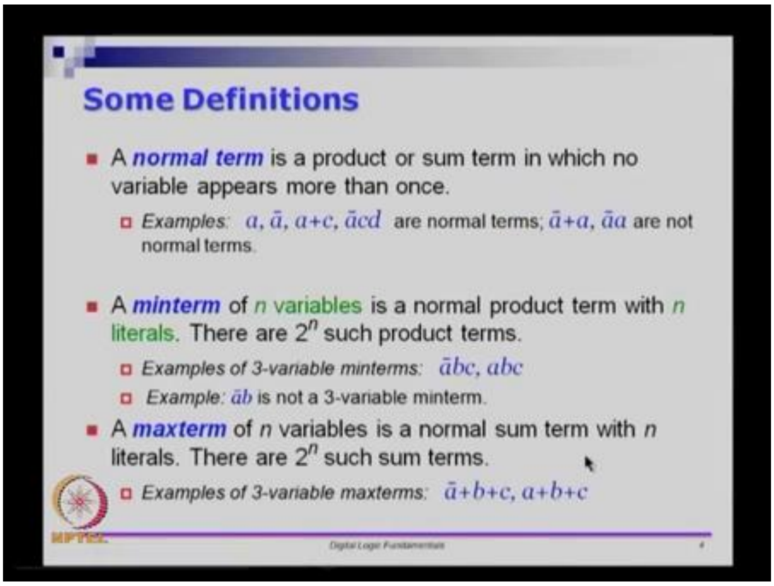

So, now let us go and look at what a normal term is, so normal term is usually a product term in which no variables appear more than once. So, let us look at this, a is a literal, it is actually a sum term as well as a product term and there is nothing repeating in it. So, it is a normal term. Similarly a bar, a plus c is you have a single literal with a plus in between, you do not see any variable which is repeating more than once in each of the terms. So, a plus c is a valid normal term, a bar c d is also a valid normal term.

So, normal term is either a product or a sum term in which variables appear no more than once. So, if you go and look at this you have a bar plus a, so if you take this as a sum a bar plus a is a same variable appearing twice. So, it is not a normal term, similarly earlier I showed you this is a, a bar b, it is a same variable appearing twice in the uncomplemented form and once in the complemented form. So, this is not a normal term.

So, a normal term is a product or sum in which no variable appears more than once, even if it is a complemented and uncomplemented form like this a bar a, it is not a valid normal term. Now, what we really need is, the definition of min terms and max terms, these are the things that we need. So, min term is defined on n variables, it is a normal product term.

So, let us see, what that means. Let us start with normal, normal means you it can... So, let us start with product, product means, it should be concatenation a logical AND of n variables. We want n literals and we want n of them to be concatenated and it should be normal. So, let us see examples, so a bar b c it has... So, if I say that a bar b c is a three variable min term on a b and c let us see why a bar b c is a min term.

So, first of all, if I say that, it is a three variable term, I should have three variables in it. So, I have a b and c, so these are three variables and what are the literal forms, a is in, it is complemented form a bar. So, that is a literal, b is another literal and c is another literal and I have a product of these three. So, it takes cares of product part of it and finally, it is normal, because none of a b or c is appearing more than once.

So, a bar b c is a min term, a b c is also a min term, a bar b cannot be a three variable min term, because a three variable min term should contain three variables a bar b contains only two variables, even though it is a product of literal a a bar and literal b. So, it is a valid product term it is a valid normal term also. But, it is not a valid three variable min term, it is a valid two variable min term however. Similarly, a max term on n variables is a normal sum term with n literal.

So, this definition that it has to, if it is a max term of n variables, it should have n literals, similarly a min term on n variables should have n literals. So, let us look at three variable max terms for example, a bar plus b plus c has first of all, it is a sum term. Because, it has only sum between different literals and the literals do not repeat and all the variables appear at least once.

And in fact, all the variables appear exactly once, a appears as a bar b appears as b itself, variable c appears as a literal c itself. So, a bar plus b plus c is a valid max term, a plus b plus c is also a valid max term in three variables, like here a bar plus c cannot be a valid max term in three variables, because it needs one more variable. Whereas, a bar plus b is a valid two variable max term.

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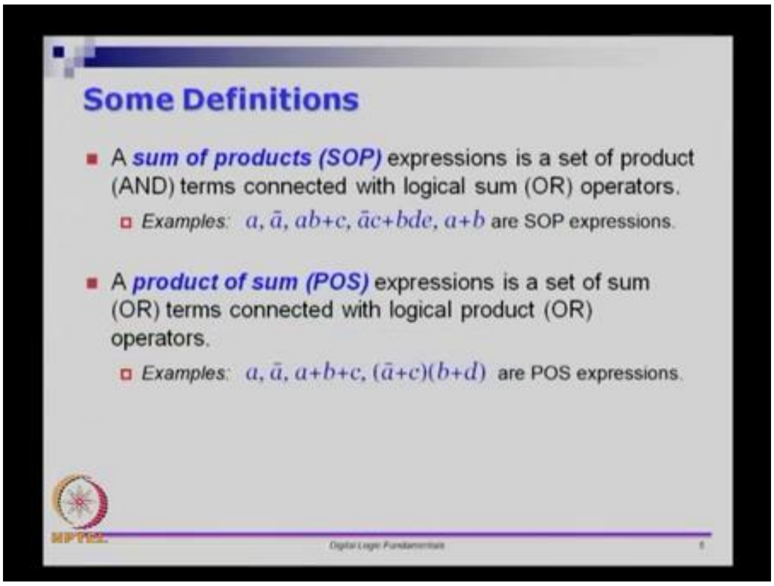

Now, let us move one step further, a sum of products also called SOP is a set of product terms, which means, it is a collection of terms that are connected with logical OR. So, let us pass this sentence sum of products. What do we know about products? The product terms will only have AND in between them. So, if I have a product term of n literals, I will have only AND connecting those and a sum of products should be a summation of product terms.

So, let us go and look at this, so let us starts with this expression first, a bar c plus b d e a bar c is a valid product term, b d e is a valid product term. So, a bar c plus b d e is sum of products, so it is a valid SOP. Similarly, a b plus c, so this is a product term and that is a product term. So, a single term is also a product term remember a b plus c, so these both are product terms connecting that with plus is ok.

So, by definition single literals are sum of products, because there is no product, there no sum there, single literals have to be sum of products. This a plus b is also a valid sum of products, because you can think of a as a product of just a that is it one variable, one literal and b is just one. So, it is a summation of product a and product b. So, if you understand that a itself is a valid product term and b itself is a valid product term, a plus b is a valid sum of products.

Similarly, a product of sum expression is a set of sum terms or odd terms which are connected with logical AND operator, this must have been AND, they are connected with logical AND operator. So, a a bar a plus b plus c all these are valid product of sum terms.

So, if I look at this a plus b plus c is a valid sum term, it is not multiplied with anything, it is not there is no product of it with anything else.

So, a single term is a valid product, this is a more good example a bar plus c is a valid sum term b plus d is a valid sum term and there is an implied AND between these two, so a bar plus c and b plus d, so that is a valid product of sum terms. So, this is a product of sum expression.

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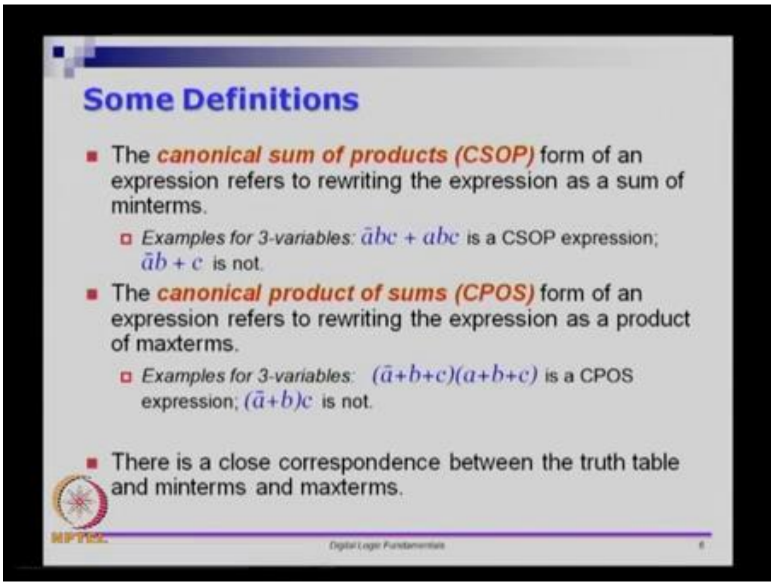

And finally, we get to what is called canonical sum of products and this refers to expression which is written in terms of min terms. So, if I go and look at the expression a bar plus c, it is a valid sum of products a bar b is a product, c is a product, a bar b plus c is a valid sum of products. However, in the canonical sum of products each product should be a min term. So, in this case, a bar b c is a min term a b c is a min term, so when I combine these two, it is a valid canonical sum of product expression.

The reason, why it is canonical is, each of the product term is actually a min term, so a min term is one in which it is first of all a normal term, no variable can repeat and all the variables should appear at least once. So, all the variables are appearing at least once here, all the variables here are appearing at least once here and there is no variable that is repeated. So, equivalently we have the canonical product of sums, where each of the product...

So, there is a product between sums each of the sum term should be a max term, so this is a max term and this is a max term. So, this is a canonical product of sums, so I am not implying that this expression is actually equivalent to this expression, I did not imply that this is a different expression. This is a different expression, this is a valid CSOP expression, this is a valid SOP expression, but it is not a CSOP expression.

Similarly, this is a valid CPOS or Canonical Product of Sums; this is not a valid canonical product of sums. So, the reason why we have this max terms, min terms, canonical sum of products and all of these mambo jumbo is because, there is a direct correspondence between the truth tables and min terms and max terms.

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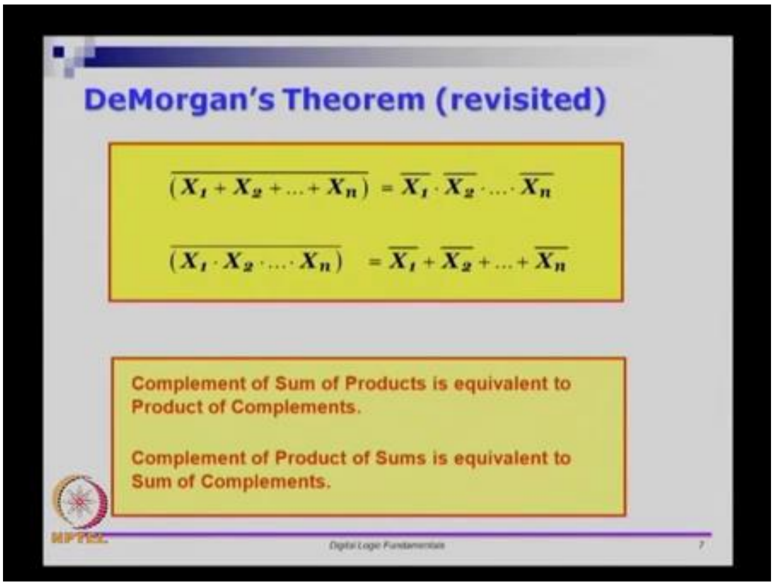

So, let us take a look at DeMorgan's theorem I showed this in the previous module X 1 plus X 2 and so, on up to X n the complement of that is X 1 complement and X 2 complement and so, on. So, what it essentially says is, if I start with a sum of products, so let us say express of product term instead of individual variables X 1, X 2 and so on assume that X 1 itself is a product term, X 2 itself is a product term and so, on.

If I take individual product terms and I do a summation of those things and if I take the complement of that. So, the complement of sum of products is the product of the complements, so you can see that each term got complemented. So, complement of sum of products is equivalent to product of complements and complement of product of sums. So, you have product of let us say X 1 is a sum term X 2 is a sum term and so on up to X n if I take the product of those and complement it is a same as the summation of the complements.

So, this is DeMorgan's theorem actually, so this is an easier way to put it in English.

Complements of sum products is equivalent to product of complements and complement of product of sums is equivalent to sum of complements.

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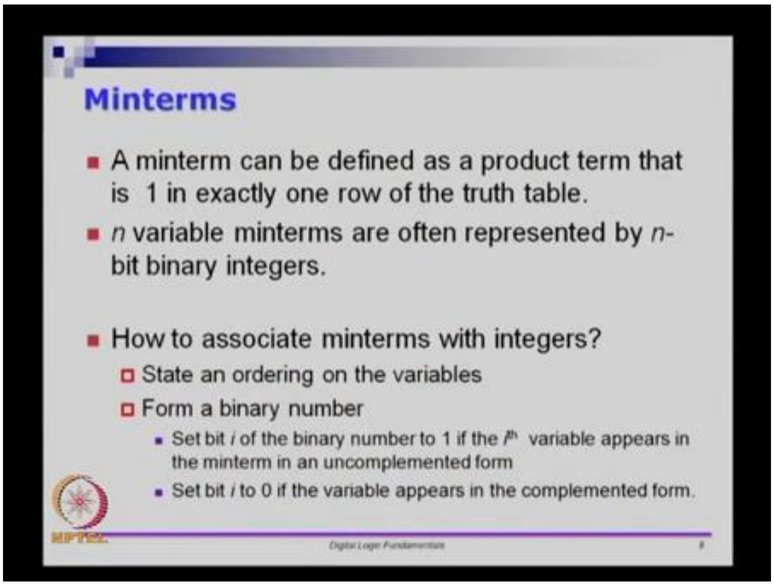

So, let us start with a min term again, a min term can be defined as a product term; that is 1 in exactly 1 row of the truth table. So, now we will take it to truth tables and usually you go and represent a n variable min term using n bit binary integers, let see how to get n min term with an integer expression. So, you start with some ordering on the variables and you form a binary number and you follow these two rules. So, they may look like it is a bit complicated, but we will see with examples it will become clear. We will said bit i of the binary number to 1 if the i'th variable appears in the min term, in the uncomplemented form and we set it to 0 it appears in the complemented form.

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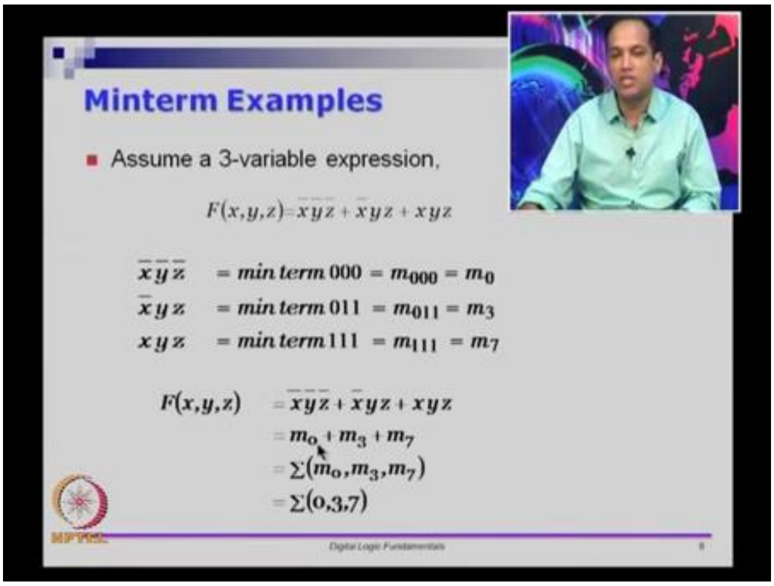

So, let us look at this three variable expression here, this is x bar y bar z bar x bar y z, x y z it is a function on three variables x, y and z. So, can you tell me whether this is valid sum of product expression, just take a while and think about whether it is a valid sum of product expression. So, if we think about it, so there is a sum separating three terms and each of these terms is a product, so it is a valid sum of product expression.

So, if I ask you this question is it a valid canonical sum of products, the answer would be true for that also. Because, this is a function on three variables, in a canonical sum of products each of the product terms cannot have any repetitions and all the variables must appear. So, this term has x y z, this term has x y z and so is this term all of them have x y z in some form in either the complemented or uncomplemented form, all the literals x y z are available, so this is a valid canonical sum of product.

The way we are going to write this is as follows, so if we take x bar y bar z bar, we assume that the ordering is x y and z. Wherever I see x in the complemented form I will put a 0, wherever I see x in uncomplemented form I will put a 1. So, if I take this product term x bar y bar z bar, it is actually the min term 0 0 0 the reason is x is appearing in the complemented form, y is appearing in the complemented form and z is appearing in the complemented form. We assume that the ordering is x y and z from the left to right.

So, this 0 means x is a complemented, this 0 means y is complemented and this 0 means z is complemented and a min term with 0 0 0 means that we have the AND of x complement y complement and z complement. Let us try and interpret this one, x should be in complemented form, y should be in regular form and z should be in regular form and it is a min term, which means it is a product of these three literals x bar y and z that is what we have here.

Finally, the last one has all of the literals in there uncomplemented version, so it is x y z. So, the way we will use this is, we will call this term m 0 0 0 term, we will call this term as m 0 1 1 term and we will call this term as m 1 1 1 term. So, wherever it is complement you put a 0, wherever it is not complement you put a 1. So, this is 0 0 0 term, this is 0 1 1 term and this is 1 1 1 term that is what we have here.

So, this is in the binary form, if you take this into equivalent decimal expression, this 0 0 0 is actually decimal 0, 0 1 1 is decimal 3 and 1 1 1 is decimal 7. So, one concise way in which we can write this expression f of x y z is to write it as the min term m naught or the min term m 3 or the min term m 7 which is the same as will use sigma notation to say that it is sum, we know that for regular algebra, whenever we have sum of different things we can put a sigma outside.

So, sigma of m naught comma m 9 comma m 7, so you can see that it is a comma separating these three. You can think of it as a set of terms m naught m 3 m 7 and you have to logically or those things that is what the sigma implies. And this m naught m 3 and m 7 it is all in subscript form and we can drop this whenever we know that is a sigma outside, we can just drop the small case m and just write it as 0, 3, 7. So, if you see a sigma of 0, 3, 7 it means it is the sum of min term 0, min term 3 and min term 7 and min term 0 as if I give you the variable ordering x, y, z min term 0 would be x bar y bar z bar, min term 3 would be x bar y and z and so on.

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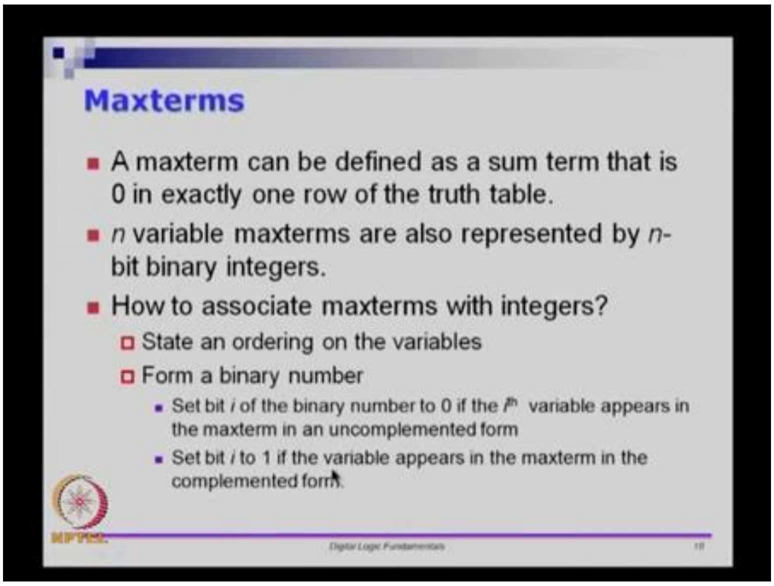

Let see the max terms, a max term can be defined as a sum term that is 0 and exactly one row of the truth table. So, if I give you truth table I go and look at one of the 1s or 0, so I go and look at the output column, wherever there is a 0 the corresponding term is what I am going to look at. So, the n variable max terms are represented by binary numbers as I showed earlier for min terms and this is the way we get the max terms is slightly different from the way we get the min terms.

So, we state an ordering of the variables, the last example we had x, y, z we form a binary number by concatenating several things. We will set bit i of the binary number to 0, if the ith variable is in uncomplemented form and we set it to 1, if it is in the complemented form. So, this was the reverse of what we did for min term.

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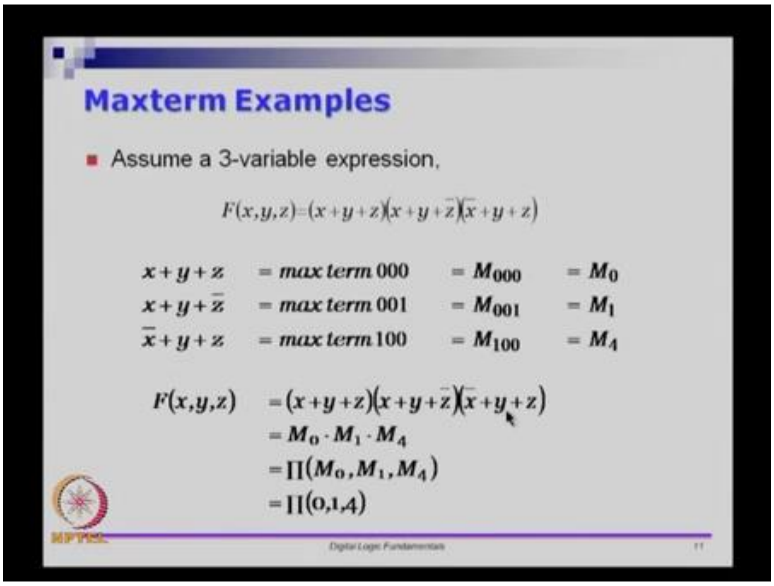

So, let us look at the example here, if I go and look at these three things, it looks like it is a valid product of sums, each one of them is a sum and it is a product of sums. In fact, it is a valid canonical product of sum expression, because each of these things is a valid max term. So, a max term is one in which all the variables appear either in the complemented or the uncomplemented form.

So, we have x plus y plus z, x plus y plus z bar and x bar plus y plus z, if you want to find out this expression, if I want to write this expression in a slightly simpler manner, we want to define the notion of max terms. So, we have x or y or z what we are going to do is, whenever a variable is first of all we stick the ordering x y z we cannot change the ordering x y z, if I give you x y z here, the interpretation for this 0 is attach with z, x this is with y and this is with z.

So, x is in the uncomplemented form for max terms we will put 0, this is in the uncomplemented form. So, again we put a 0, this is in the uncomplemented form again we put a 0. So, this corresponds the max term 0 0 0 and as a shortcut notation we use capital M to say max term and small case m for min terms. So, the capital M says it is a max term 0 0 0 and again this binary number we use M 0 as a shortcut, so this is an integer 0 which is actually for 0 0 0.

So, let us look at this one which is slightly more interesting, so x plus y plus z bar, x is appearing in the uncomplemented form, we associate this bit 0 for that this is y in the uncomplemented form. So, we associate 0 for that and z bar we associate 1 for that, because it is in the complemented form. So, max term 0 0 1 the shortcut notation we will use this capital M subscript 0 0 1 which is the same as capital m integer 1. So, this expression is the product of M naught, M 1 and M 4 that is what we have here x plus y plus z into x plus y plus z bar into x bar plus y plus z is a product of M naught M 1 and m 4.

And again from algebra we usually use pi for product we use sigma for sum and pi for product. So, we will use pi of M naught comma M 1 comma M 4, so remember there is no AND anymore, because pi already implies that we are anding. So, it is a comma separated list, so is a product of max term 0, max term 1 and max term 4 and whenever we have a pi implicitly we assume that the terms inside or already max terms. So, we can drop the capital M and write it as 0, 1, 4.

So, it is wrong to write pi of small case M naught and things like that, so we always associate pi with max terms and we associate sigma with min terms. So, whenever we write something like 0, 1, 4 here it assumes that we are already having a capital M and implied max term which is being multiplied with each other.

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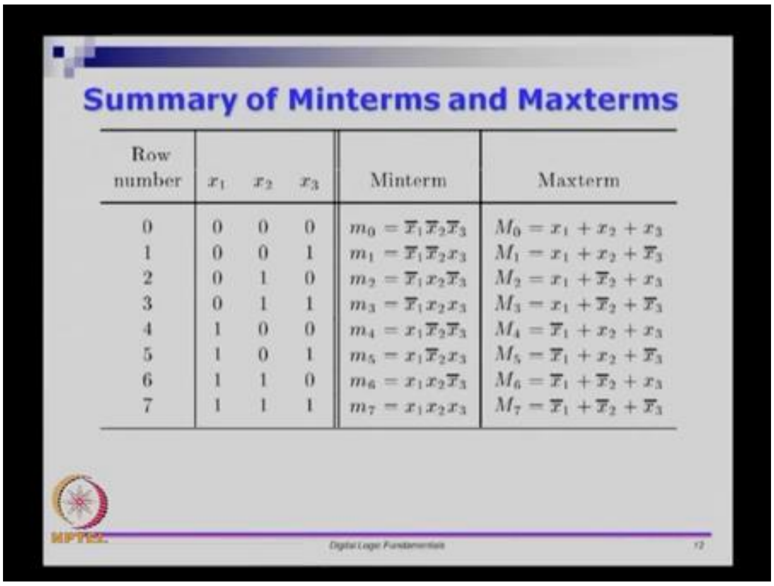

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1x_2x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

So, let us look at a quick summary this slide shows you quick summary of min terms and max terms. So, if I have x 1, x 2, x 3 and if I fix the ordering from left to right, these are the possible choices for x 1, x 2, x 3 you can see that this is number 0, number 1, 2, 3, 4, 5, 6, 7 and so on. So, usually in the Boolean word we start the numbering from 0, so this is row number 0, this is row number 1 and so on. The row number is actually the integer representation of the x 1, x 2, x 3 in the binary form.

So, if I take this binary number as 0 0 0 its integer form is 0, if I take 1 0 0 it is 1 into 2 square plus 0 into 2 power 0 plus 0 into 2 power 1 plus 0 into 2 power 0 that is 4 and so on. So, we associated min term if I want m naught, so this is actually x 1 bar x 2 bar x 3 bar, if I want max term m naught that is x 1 plus x 2 plus x 3. Because, remember all the three variables if I have m naught m subscript 0 here like here is m 0 0 0.

So, 0 0 0 means x 1 should appear in the complemented form, x 2 should appear in the complemented form, x 3 should also appear in the complemented form and we need a product of these three that is what a min term needs let us pick this row. If I want small case m 4 that is the min term 4, min term 4 would be I want x 1 in the uncomplemented form, x 2 in the complemented form and x 3 in the complemented form, so that is what you have here and we have a product of those.

If I want capital M 4 I look at the row and then I do this, so capital M 4 is m 1 0 0 0. So, the interpretation for capital M 1 0 0 is this 1 means x 1 should be in the complemented form, for max terms we have x 1 complement and x 2 should be in the uncomplemented form and x 3 should be in the uncomplemented form. So, we have those two and then max term is a sum term, so you put plus. So, this is a quick summary of min terms and max terms.

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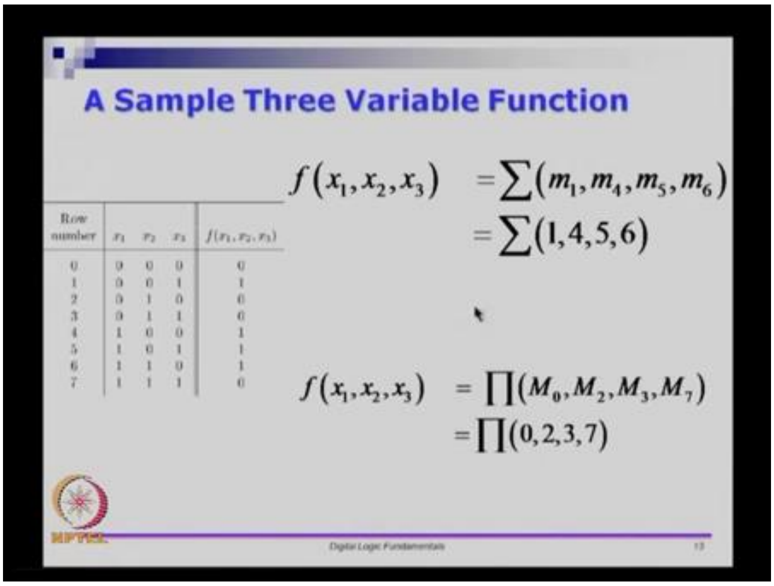

So, let us look at this simple expression here, if I have this expression 0, 1, 2, 3, 4, 5, 6, 7 x 1, x 2, x 3 are the three variables, the row numbers are 0 to 7 and I look at this function f of x 1, x 2, x 3 we have 8 terms here. If I want to go and write it in terms of product terms or sum terms, there is a nice and simple way to do it. So, for sum terms remember it is you can go and look at all the 1's, so there is row number 1 which has a 1, row number 4 which has a 1, row number 5 which has a 1 and row number 6 which has a 1.

So, what we are going to write is f of x 1 comma x 2 comma x 3 is take the rows in which you have them has ones, row number 1 has a 1, row number 4 has a 1, row number 5 has a 1 and row number 6 has a 1. And we pick the corresponding min terms and add them up or put a logical OR of those. So, that is the sigma of m 1 comma m 4 comma m 5 comma m 6 and I told you that whenever there is a sigma and a small case m you can implicitly drop m there.

So, it is the sum of the row 1, row 4, row 5 and row 6. The English way of explaining this is f is a function which is on when the condition in row 1 is true or row 4 is true or row 5 is true and row 6 is true. So, one of these conditions should be true and when is this row 1 will be true, row 1 will be true if x 1 is 0, x 2 is 0 and x 3 is 1. So, that is when row 1 will be exercised and so on.

Similarly, if I want to write this as a POS form a canonical product of sum in fact, what I do is you collect all the 0s, the 0s are in row number 0, row number 3, row number 2 and row number 7. So, you collect all the 0s, so they are in row number 0, 2, 3 and 7 and you put a pi which implies you are actually taking these as max terms first and then you are actually doing a product of the max terms. And since pi goes with capital M you can silently drop the capital M and you have it as pi of 0, 2, 3, 7.

So, given a truth table you can write in canonical sum of product using this sigma notation and canonical product of sum using the pi notation. So, this brings me to the end of module 4, in the next module what we do is, we will try and rewrite various sum of products and product of sums and so on, in various different formats. So, that all these definitions that I gave you they should stick in your mind. So, I will do that in the next module.

Thank you.