# Cavitation Noise

The violent and catastrophic collapse of cavitation bubbles results in the production of noise that is a consequence of the momentary large pressures that are generated when the contents of the bubble are highly compressed. Consider the flow in the liquid caused by the volume displacement of a growing or collapsing cavity. In the far field the flow will approach that of a simple source, and it is clear that equation (Ngb5) for the pressure will be dominated by the first term on the right-hand side (the unsteady inertial term) since it decays more slowly with radius, $r$, than the second term. If we denote the time-varying volume of the cavity by $V(t)$ and substitute using equation (Ngb2), it follows that the time-varying component of the pressure in the far field is given by

$$p_a = \frac{\rho_L}{4\pi\mathcal{R}} \frac{d^2V}{dt^2} \tag{Nhg1}$$

where $p_a$ is the radiated acoustic pressure and we denote the distance, $r$, from the cavity center to the point of measurement by $\mathcal{R}$ (for a more thorough treatment see Dowling and Ffowcs Williams 1983 and Blake 1986b). Since the noise is directly proportional to the second derivative of the volume with respect to time, it is clear that the noise pulse generated at bubble collapse occurs because of the very large and positive values of $d^2V/dt^2$ when the bubble is close to its minimum size. It is conventional (see, for example, Blake 1986b) to present the sound level using a root mean square pressure or *acoustic* pressure, $p_s$, defined by

$$p_s^2 = \overline{p_a^2} = \int_0^\infty \mathcal{G}(f)df \tag{Nhg2}$$

and to represent the distribution over the frequency range, $f$, by the spectral density function, $\mathcal{G}(f)$.

To the researcher or engineer, the crackling noise that accompanies cavitation is one of the most evident characteristics of the phenomenon. The onset of cavitation is often detected first by this noise rather than by visual observation of the bubbles. Moreover, for the practical engineer it is often the primary means of detecting cavitation in devices such as pumps and valves. Indeed, several empirical methods have been suggested that estimate the rate of material damage by measuring the noise generated (for example, Lush and Angell 1984).

The noise due to cavitation in the orifice of a hydraulic control valve is typical, and spectra from such an experiment are presented in figure 1. The lowest curve at $\sigma = 0.523$ represents the turbulent noise from the noncavitating flow. Below the incipient cavitation number (about 0.523 in this case) there is a dramatic increase in the noise level at frequencies of about $5kHz$ and above. The spectral peak between $5kHz$ and $10kHz$ corresponds closely to the expected natural frequencies of the nuclei present in the flow (see section (Ngj)).

Most of the analytical approaches to cavitation noise build on knowledge of the dynamics of collapse of a single bubble. Fourier analyses of the radiated acoustic pressure due to a single bubble were first visualized by Rayleigh (1917) and implemented by Mellen (1954) and Fitzpatrick and Strasberg (1956). In considering such Fourier analyses, it is convenient to nondimensionalize the frequency by the typical time span of the whole event or, equivalently, by the collapse time, $t_{tc}$, given by equation (Ngd13). Now consider the frequency content of $\mathcal{G}(f)$ using the dimensionless frequency, $ft_{tc}$. Since the volume of the bubble increases from zero to a finite value and then returns to zero, it follows that for $ft_{tc} < 1$ the Fourier transform of the volume is independent of frequency. Consequently $d^2V/dt^2$ will be proportional to $f^2$ and therefore $\mathcal{G}(f) \propto f^4$ (see Fitzpatrick and Strasberg 1956). This is the origin of the left-hand asymptote in figure 2.

The behavior at intermediate frequencies for which $ft_{tc} > 1$ has been the subject of more speculation and debate. Mellen (1954) and others considered the typical equations governing the collapse of a spherical bubble in the absence of thermal effects and noncondensable gas (equation (Ngd9)) and concluded that, since the velocity $dR/dt \propto R^{-\frac{3}{2}}$, it follows that $R \propto t^{\frac{2}{5}}$. Therefore the Fourier transform of $d^2V/dt^2$ leads to the asymptotic behavior $\mathcal{G}(f) \propto f^{-\frac{2}{5}}$. The error in this analysis is the neglect of the noncondensable gas. When this is included and when the collapse is sufficiently advanced, the last term in the square brackets of equation (Ngd9) becomes comparable with the previous terms. Then the behavior is quite different from $R \propto t^{\frac{2}{5}}$. Moreover, the values of $d^2V/dt^2$ are much larger during this rebound phase, and therefore the frequency content of the rebound phase will dominate the spectrum. It is therefore not surprising that the $f^{-\frac{2}{5}}$ is not observed in practice. Rather, most of the experimental results seem to exhibit an intermediate frequency behavior like $f^{-1}$ or $f^{-2}$. Jorgensen (1961) measured the noise from submerged, cavitating jets and found a behavior like $f^{-2}$ at the higher frequencies (see figure 2). However, most of the experimental data for cavitating bodies or hydrofoils exhibit a weaker decay. The data by Arakeri and Shangumanathan (1985) from cavitating headform experiments show a very consistent $f^{-1}$ trend over almost the entire frequency range, and very similar results have been obtained by Ceccio and Brennen (1991).

Ceccio and Brennen (1991) recorded the noise from individual cavitation bubbles in a flow; a typical acoustic signal from their experiments is reproduced in figure 3. The large positive pulse at about 450 $\mu s$ corresponds to the first collapse of the bubble. This first pulse in figure 3 is followed by some facility-dependent oscillations and by a second pulse at about 1100 $\mu s$. This corresponds to the second collapse that follows the rebound from the first collapse.

A good measure of the magnitude of the collapse pulse is the acoustic impulse, $I$, defined as the area under the pulse or

$$I = \int_{t_1}^{t_2} p_a dt \qquad \text{(Nhg3)}$$

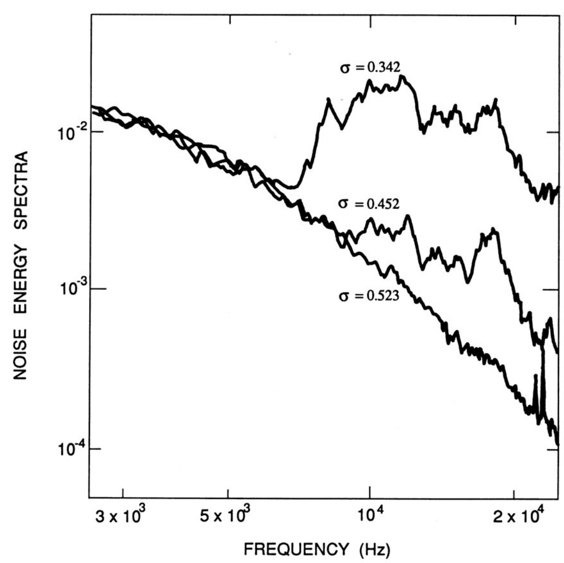


Figure 1: Acoustic power spectra from a model spool valve operating under noncavitating ($\sigma = 0.523$) and cavitating ($\sigma = 0.452$ and $0.342$) conditions (from the investigation of Martin *et al.* 1981).

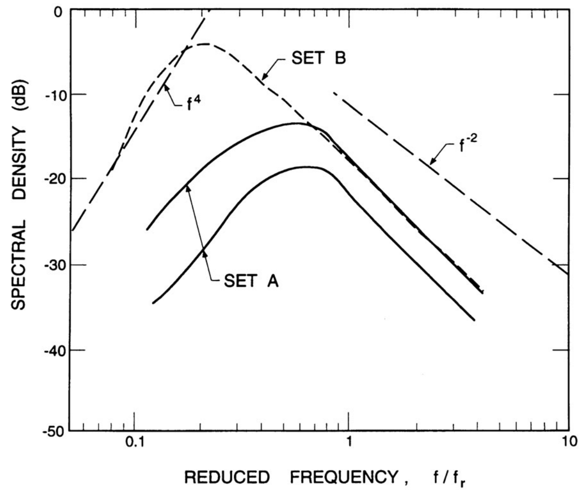


Figure 2: Acoustic power spectra of the noise from a cavitating jet. Shown are mean lines through two sets of data constructed by Blake and Sevik (1982) from the data by Jorgensen (1961). Typical asymptotic behaviors are also indicated. The reference frequency, $f_r$, is $(p_\infty/\rho_L d^2)^{\frac{1}{2}}$ where $d$ is the jet diameter.

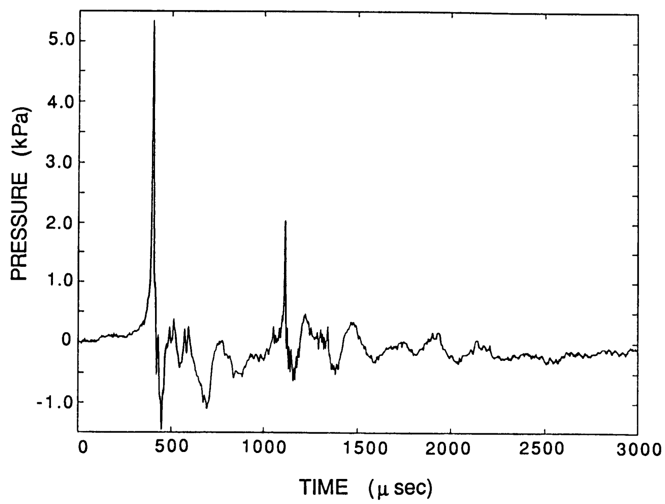


Figure 3: A typical acoustic signal from a single collapsing bubble. From Ceccio and Brennen (1991).

where $t_1$ and $t_2$ are times before and after the pulse at which $p_a$ is zero. For later purposes we also define a dimensionless impulse, $I^*$, as

$$I^* = 4\pi I \mathcal{R}/\rho_L U \ell^2 \quad \text{(Nhg4)}$$

where $U$ and $\ell$ are the reference velocity and length in the flow. The average acoustic impulses for individual bubble collapses on two axisymmetric headforms (ITTC and Schiebe headforms) are compared in figure 4 with impulses predicted from integration of the Rayleigh-Plesset equation. Since these theoretical calculations assume that the bubble remains spherical, the discrepancy between the theory and the experiments is not too surprising. Indeed one interpretation of figure 4 is that the theory can provide an order of magnitude estimate and an upper bound on the noise produced by a single bubble. In actuality, the departure from sphericity produces a less focused collapse and therefore less noise.

The next step is to consider the synthesis of cavitation noise from the noise produced by individual cavitation bubbles or events. If the impulse produced by each event is denoted by $I$ and the number of

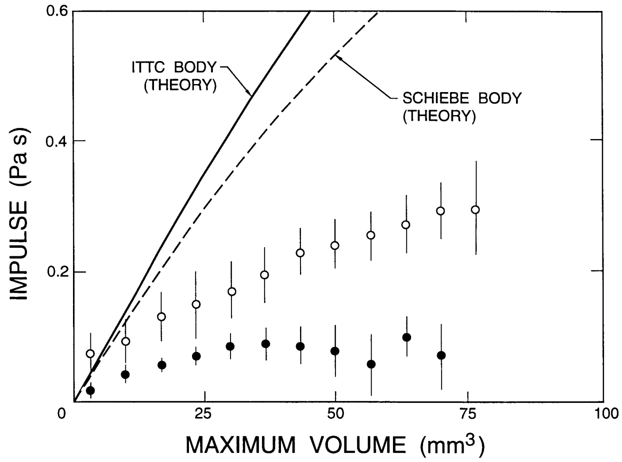


Figure 4: Comparison of the acoustic impulse, $I$, produced by the collapse of a single cavitation bubble on two axisymmetric headforms as a function of the maximum volume prior to collapse. Open symbols: average data for Schiebe headform; closed symbols: ITTC headform; vertical lines indicate one standard deviation. Also shown are the corresponding results from the solution of the Rayleigh-Plesset equation. From Ceccio and Brennen (1991).

events per unit time is denoted by $\dot{n}$, the sound pressure level, $p_s$, will be given by

$$p_s = I\dot{n} \tag{Nhg5}$$

Consider the scaling of cavitation noise that is implicit in this construct. Both the experimental results and the analysis based on the Rayleigh-Plesset equation indicate that the nondimensional impulse produced by a single cavitation event is strongly correlated with the maximum volume of the bubble prior to collapse and is almost independent of the other flow parameters. It follows from equations (Nhg1) and (Nhg3) that

$$I^* = \frac{1}{U\ell^2}\left\{\left(\frac{dV}{dt}\right)_{t_2} - \left(\frac{dV}{dt}\right)_{t_1}\right\} \tag{Nhg6}$$

and the values of $dV/dt$ at the moments $t = t_1, t_2$ when $d^2V/dt^2 = 0$ may be obtained from the Rayleigh-Plesset equation. If the bubble radius at the time $t_1$ is denoted by $R_x$ and the coefficient of pressure in the liquid at that moment is denoted by $C_{px}$, then

$$I^* \approx 8\pi \left(\frac{R_x}{\ell}\right)^2 (C_{px} - \sigma)^{\frac{1}{2}} \tag{Nhg7}$$

Numerical integrations of the Rayleigh-Plesset equation for a range of typical circumstances yield $R_x/R_m \approx 0.62$ where $R_m$ is the maximum volumetric radius and that $(C_{px} - \sigma) \propto R_m/\ell$ (in these calculations $\ell$ was the headform radius) so that

$$I^* \approx \beta \left(\frac{R_m}{\ell}\right)^{\frac{5}{2}} \tag{Nhg8}$$

The aforementioned integrations of the Rayleigh-Plesset equation yield a factor of proportionality, $\beta$, of about 35. Moreover, the upper envelope of the experimental data of which figure 4 is a sample appears to correspond to a value of $\beta \approx 4$. We note that a quite similar relation between $I^*$ and $R_m/\ell$ emerges from the analysis by Esipov and Naugol'nykh (1973) of the compressive sound wave generated by the collapse of a gas bubble in a compressible liquid.

From the above relations, it follows that

$$I \approx \frac{\beta}{12}\rho_L U R_m^{\frac{5}{2}}/\mathcal{R}\ell^{\frac{1}{2}} \tag{Nhg9}$$

Consequently, the evaluation of the impulse from a single event is completed by an estimate of $R_m$ such as that of equation (Nhb4). Since that estimate has $R_m$ independent of $U$ for a given cavitation number, it follows that $I$ is linear with $U$.

The event rate, $\dot{n}$, can be considerably more complicated to evaluate than might at first be thought. If all the nuclei flowing through a certain, known streamtube (say with a cross-sectional area, $A_n$, in the upstream flow) were to cavitate similarly, then the result would be

$$\dot{n} = n A_n U \tag{Nhg10}$$

where $n$ is the nuclei concentration (number/unit volume) in the incoming flow. Then it follows that the acoustic pressure level resulting from substituting equations (Nhg10), (Nhg9) into equation (Nhg5) and using equation (Nhb4) becomes

$$p_s \approx \frac{\beta}{3} \rho_L U^2 A_n n \ell^2 (-\sigma - C_{pmin})^{\frac{5}{2}} / \mathcal{R} \tag{Nhg11}$$

where we have omitted some of the constants of order unity. For the relatively simple flows considered here, equation (Nhg11) yields a sound pressure level that scales with $U^2$ and with $\ell^4$ because $A_n \propto \ell^2$. This scaling with velocity does correspond roughly to that which has been observed in some experiments on traveling bubble cavitation, for example, those of Blake, Wolpert, and Geib (1977) and Arakeri and Shangumanathan (1985). The former observe that $p_s \propto U^m$ where $m = 1.5$ to 2.

Different scaling laws will apply when the cavitation is generated by turbulent fluctuations such as in a turbulent jet (see, for example, Ooi 1985 and Franklin and McMillan 1984). Then the typical tension experienced by a nucleus as it moves along a disturbed path in a turbulent flow is very much more difficult to estimate. Consequently, the models for the sound pressure due to cavitation in a turbulent flow and the scaling of that sound with velocity are less well understood.