

Course ID 030253

Representation and Characters of Groups

MATH 42061

Unit coordinator: Peter Symonds

Credit rating 15
ECTS credits 7.5

Semester 1

School of Mathematics
Undergraduate

Level 4

FHEQ level ' Masters/Integrated Masters P4'

Marketing course unit overview

In the second and third year course units on group theory we have seen that abstract groups are quite complicated objects. One of the most fruitful approaches to studying these objects is to embed them into groups of matrices (to "represent" the elements of an abstract group by matrices). The advantage of this approach lies in the fact that matrices are concrete objects, and explicit calculations can easily be performed. Even more importantly, the powerful methods of linear algebra can be applied to matrices. The course is devoted to representations of finite groups by matrices with entries in the field of complex numbers.

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Aims

To introduce students to representations of groups over the field of complex numbers.

Learning outcomes

On successful completion of this course unit students will

- know the basic properties of complex representations of finite groups and be able to use them in examples;
- understand the relationship between a representation and its character;
- know the basic properties of characters and use them in examples;
- know the basic properties of a character table and be able to calculate character tables for certain small groups.

Syllabus

- Informal introduction to matrix representations, permutation representations and G-sets.

[3 lectures]

- Definition and basic properties of complex representations of a finite group. Maschke's Theorem, Characters and character tables. [6]
- The special cases of: cyclic groups, abelian groups, 1-dimensional representations. [6]
- Schur's Lemma, orthogonality of characters, the number of irreducibles, the character degree divides the order of the group. [6]
- Induced representations, Frobenius Reciprocity, double coset formula, methods of calculation. Transitive and 2-transitive permutation representations and their characters. [6]

Assessment methods

Other	20%
Written exam	80%

Mid-semester coursework: weighting 20% End of semester examination: three hours weighting 80%

Feedback methods

Tutorials will provide an opportunity for students' work to be discussed and provide feedback on their understanding.

Requisites

MATH32001 Group Theory Pre-Requisite Compulsory
Students are not permitted to take, for credit, MATH42061 in an undergraduate programme and then MATH62061 in a postgraduate programme at the University of Manchester, as the courses are identical.

Academic programs

MMath (Hons) Mathematics with Financial Mathematics	Fourth Year	Optional
MMath(Hons) Mathematics	Fourth Year	Optional
MMath (Hons) Mathematics and Statistics	Fourth Year	Optional

Available as free choice? N

Recommended reading

- B. Steinberg, *Representation Theory of Finite Groups*, Springer Verlag 2012
- J. P. Serre, *Linear Representations of Finite Groups*, GMT 42, Springer-Verlag
- G. James and M. Liebeck, *Representations and Characters of Groups*, CUP, 1993

Scheduled activity hours

Lectures	22
Tutorials	11

Independent study hours 117 hours

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