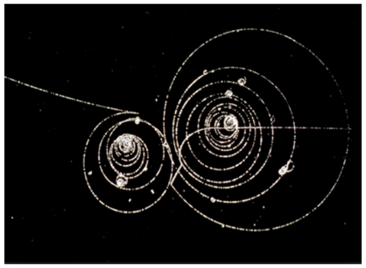
Lecture 15

University of Massachusetts
UMASS Lowell
Physics II



Chapter 32

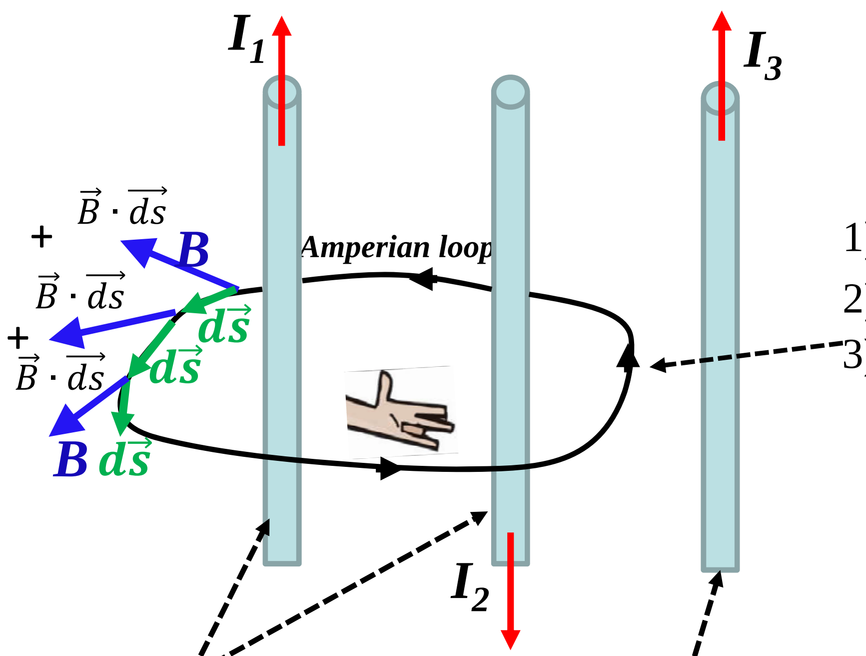
Ampere's law

Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Ampere's Law

Electric Field	Magnetic Field
From Coulomb's law $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\hat{r}}{r^2}$	Bio-Savart law $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$
Gauss's Law $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$	***There must be something similar for B***

Ampere's Law



These currents pass through the area bounded by the loop, so they are enclosed, I_{in}

I_3 doesn't pass through the enclosed area.

The line integral of the magnetic field around the curve is given by Ampère's law:

$$\oint_{\substack{\text{Closed loop}\\ \text{(Amperian)}}} \vec{B}\cdot d\vec{s} = \mu_0 I_{in}$$

1) An Amperian loop is imaginary
2) It is a closed loop (any path can be used)
3) Choose direction (up to you).
   It gives us which current is positive/negative
   (use a right-hand rule:

curl your fingers in a chosen direction and an outstretched thumb shows a positive current direction)

So I_1 is positive; I_2 is negative

$$\oint_{\text{Amperian loop}} \vec{B}\cdot d\vec{s} = \mu_0 (I_1 - I_2)$$

Ampère's law is very useful for a problem with a high degree of symmetry.

ConcepTest 1 Ampere's Law

- The line integral of B around the loop is $\mu_0 \cdot 7.0$ A. Current I_3 is

A) 0 A
B) 1 A out of the screen
C) 1 A into the screen
D) 5 A out of the screen
E) 5 A into the screen

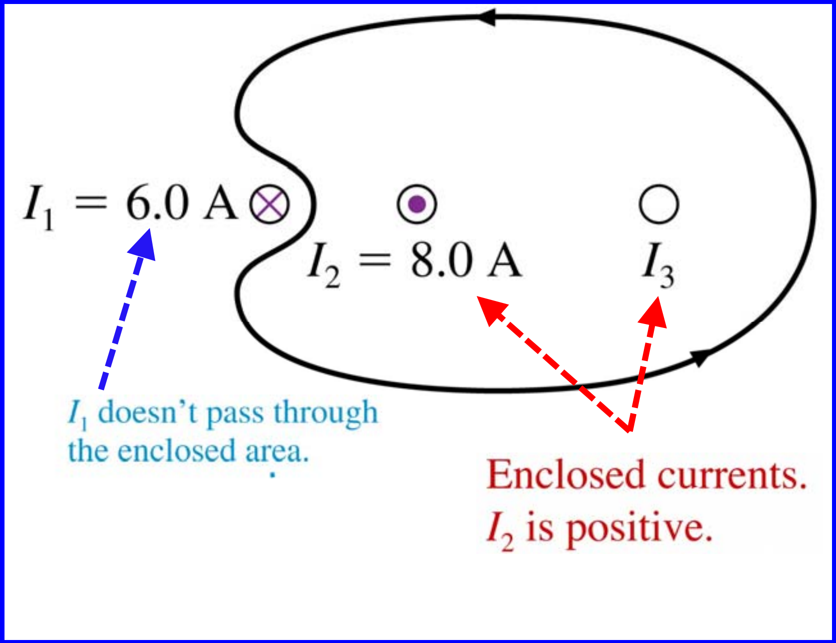
$$\oint_{\substack{Closed\ loop\\(Amperian)}} \vec{B} \cdot d\vec{s} = \mu_0 I_{in}$$

Assume I_3 is out of the page

$$\oint \vec{B} \cdot d\vec{s} = \mu_0(+8 + I_3) = 7\ \mu_0$$

$$I_3 = -1A$$

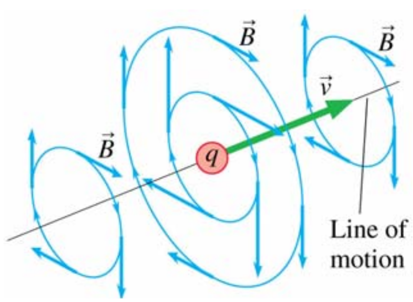
Minus means our original assumption was wrong, it is into the screen



Magnetic field of a current-carrying wire

The wire has cylindrical symmetry so that we can easily use Ampere's law.

One moving charge creates magnetic field lines centered on the motion line:



Now we have many moving charges (not just one). The field pattern must be the same.

So we'll take our Amperian loop to be a concentric circles of r.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 \cdot I_{encl}$$

$$I_{encl} = +I$$

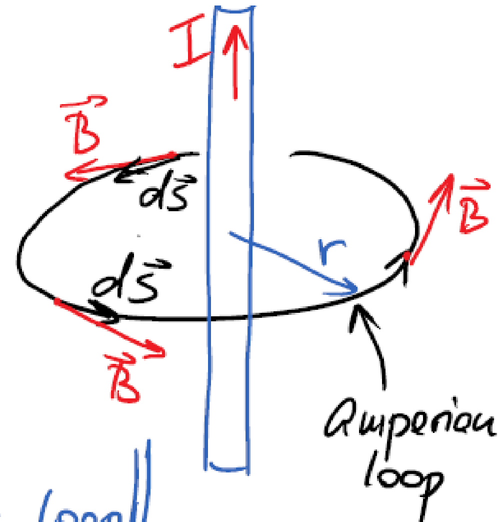
$$\oint \vec{B}\cdot d\vec{s} = \left\| \begin{matrix} \vec{B}\uparrow\uparrow d\vec{s} \\ \theta = 0 \end{matrix} \right\| =$$

$$= \oint B\,ds = \left\| \begin{matrix} B = \text{const} \\ \text{on the Amperian loop} \end{matrix} \right\|$$

$$= B \underbrace{\oint ds}_{2\pi r} = \underbrace{2\pi r \cdot B = \mu_0 \cdot I}$$

$$\boxed{B(r) = \frac{\mu_0 \cdot I}{2\pi r}}$$

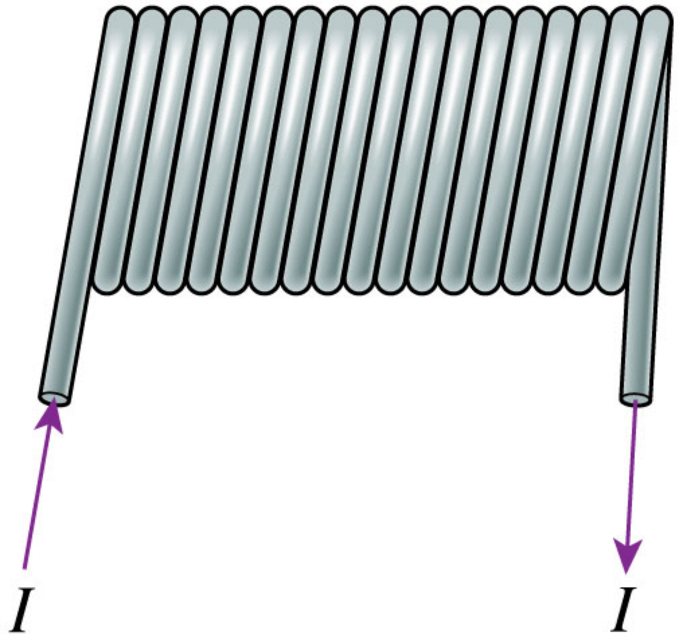
r - $\perp$ distance to the wire



$\oint ds$ - simply the circumference of the Amperian loop, which is $2\pi r$.

Solenoid

Solenoid

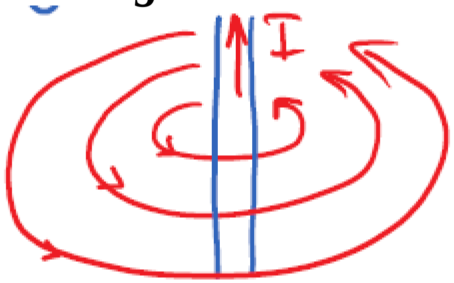


A solenoid is a helical coil of wire with the same current I passing through each loop in the coil.

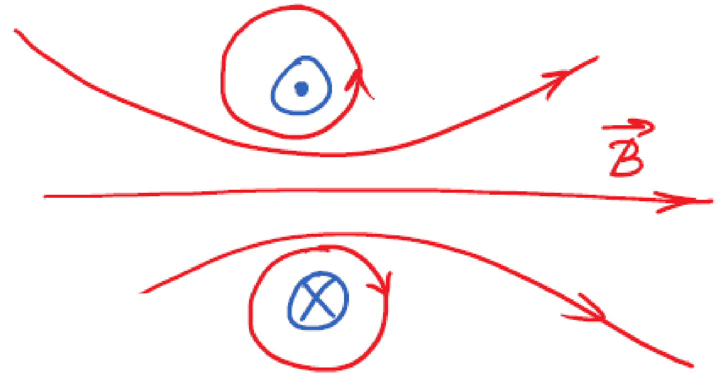
A **uniform magnetic field** can be generated with a **solenoid.**

Steps to make a solenoid

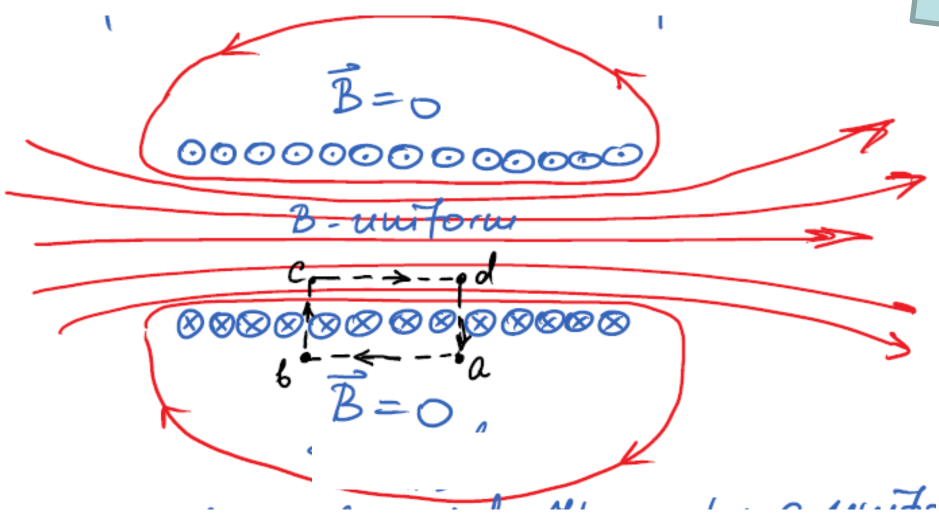
Magnetic field lines produced with a straight wire

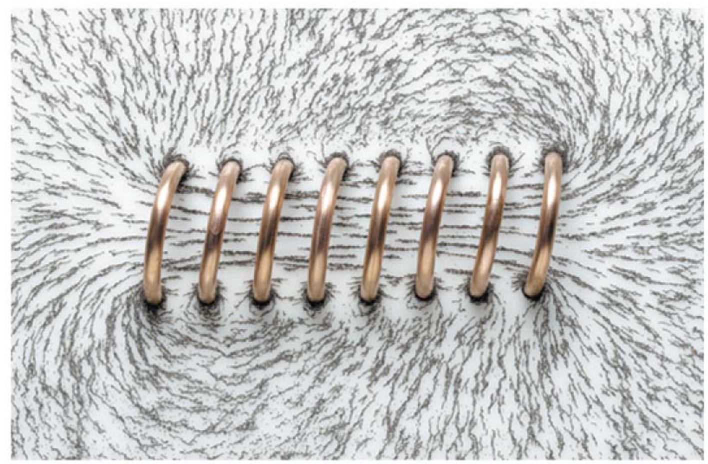


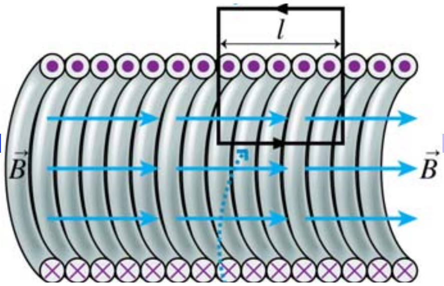
Let's bend the wire into a loop



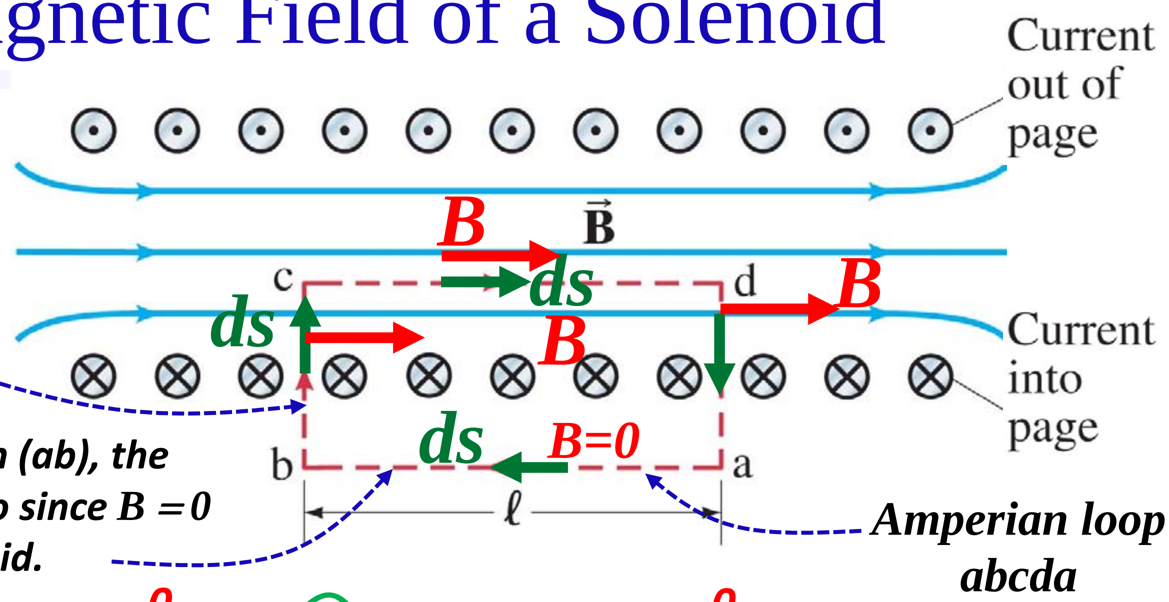
Now, let's add more loops







The Magnetic Field of a Solenoid



- ***Along the sides (bc, da), the line integral is zero since the field is perpendicular to the path.***

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

- ***Along the bottom (ab), the line integral is zero since B = 0 outside the solenoid.***

$$\oint_{abcda} \vec{B} \cdot d\vec{s} = \underbrace{\int_a^b \cancel{\vec{B} \cdot d\vec{s}}^{\,0}}_{\vec{B}=0} + \underbrace{\int_b^c \cancel{\vec{B} \cdot d\vec{s}}^{\,0}}_{\vec{B} \perp d\vec{s}} + \underbrace{\int_c^d \vec{B} \cdot d\vec{s}}_{\substack{\vec{B} \parallel d\vec{s} \\ \vec{B} = const}} + \underbrace{\int_d^a \cancel{\vec{B} \cdot d\vec{s}}^{\,0}}_{\vec{B} \perp d\vec{s}} = \mu_0 I_{in}$$

There are N loops with current I enclosed by an Amperian loop, so $I_{in} = N \cdot I$

$$B \int_c^d ds = Bl = \mu_0 NI \quad \Rightarrow \quad B = \mu_0 \frac{NI}{l} = \|n = N/l\| = \mu_0 nI$$

where n = N/l is the number of turns per unit length.

Uniform field

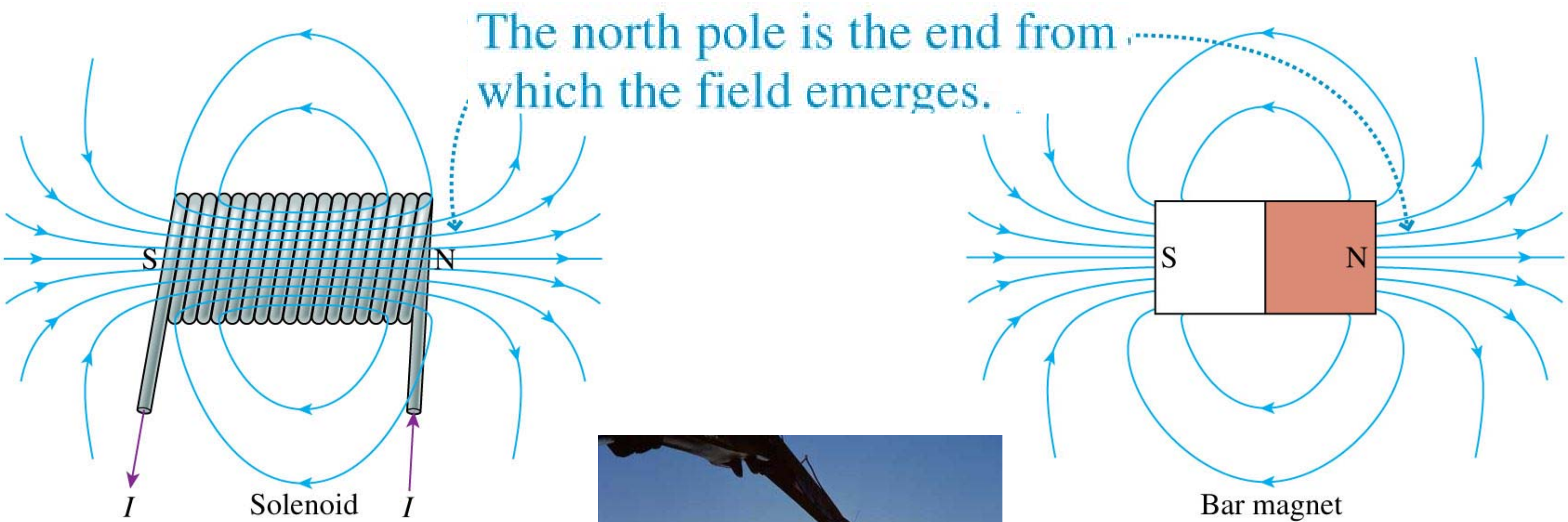
$$B_{solenoid} = \mu_0 nI$$

This patient is undergoing magnetic resonance imaging (MRI). The large cylinder surrounding the patient contains a solenoid that is wound with superconducting wire to generate a strong uniform magnetic field.

B=1.2 T, I=100 A

The Magnetic Field Outside a Solenoid

- ***The magnetic field outside a solenoid looks like that of a bar magnet.***
- ***Thus a solenoid is an electromagnet***



Electric Field	Magnetic Field
From Coulomb's law $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q\hat{r}}{r^2}$	Biot-Savart law $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$
Gauss's Law $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$	Ampere's Law $\oint \vec{\boldsymbol{B}}\cdot \boldsymbol{d}\vec{\boldsymbol{s}} = \boldsymbol{\mu_0 I_{in}}$

So, now we know how to find magnetic fields using Bio-Savart and Ampere's laws.

Now, the question is

"how does a magnetic field interact with material (which consists of charges and current)?"

Magnetic force on a moving charge

Magnetic force on current

Magnetic force on a moving charge

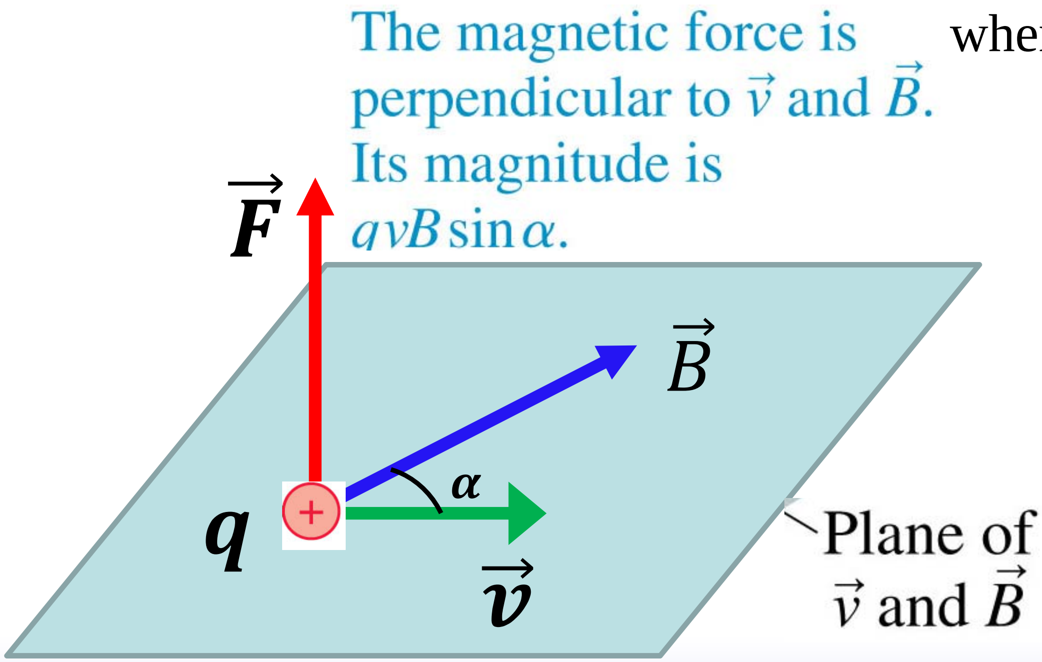
The Magnetic Force on a Moving Charge

After Oersted's discovery, there were many other experiments with magnetic fields, currents, charges, etc. It was found that B exerts a force on a moving charge.

The magnetic force on a charge q as it moves through a magnetic field $\boldsymbol{B}$ with velocity $\boldsymbol{v}$ is:

$$\vec{F}_{\text{on } q} = q\vec{v} \times \vec{B} = (qvB\sin\alpha, \text{ direction of right-hand rule})$$

where α is the angle between $\boldsymbol{v}$ and $\boldsymbol{B}$.



What you should read

Chapter 32 (Knight)

Sections

- ***32.6***
- ***32.5 (skip)***

Thank you

See you in a week.

Enjoy your spring break