The International Standard Atmosphere (ISA)

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Nomenclature

a = speed of sound, m/sec
g = acceleration of gravity, m/sec^2
h = altitude, m or ft
p = pressure, N/m^2 or hPa
R = real gas constant for air, 287.04 m^2/°Ksec^2
T = temperature, °K or °C
ρ = density, kg/m^3

Subscripts

0 = standard sea level conditions
11 = tropopause caonditions

Abbreviations

ICAO = International Civil Aviation Organization
ISA = International Standard Atmosphere
MSL = Mean Sea Level
PA = Pressure Altitude

1. Standard Atmosphere Modeling

For purposes of pressure altimeter calibrations, aircraft and rocket performance and their design, and so forth, knowledge of the vertical distribution of such quantities as pressure, temperature, density, and speed of sound is required. Since the real atmosphere never remains constant at any particular time or place, a hypothetical model must be employed as an approximation to what may be expected. This model is known as the standard atmosphere. The air in the model is assumed to be devoid of dust, moisture, and water vapor and to be at rest with respect to the Earth (that is, no winds or turbulence). [1]

The first standard atmospheric models were developed in the 1920's in both Europe and the United States. The slight differences between the models were reconciled and an internationally accepted model was introduced in 1952 by the International Civil Aviation Organization (ICAO). [1] The International Standard Atmosphere is defined in ICAO Document 7488/2. The ISA assumes the mean sea level (MSL) conditions as given in Table 1.

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Table 1 International Standard Atmosphere, Mean Sea Level Conditions

Pressure	$p_0 = 101\ 325\ \text{N/m}^2 = 1013.25\ \text{hPa}$
Density	$\rho_0 = 1.225\ \text{kg/m}^3$
Temperature	$T_0 = 288.15°\text{K}\ (15°\text{C})$
Speed of sound	$a_0 = 340.294\ \text{m/sec}$
Acceleration of gravity	$g_0 = 9.80665\ \text{m/sec}^2$

1.1. Temperature Modeling

The following diagram (Figure 1) illustrates the temperature variations in the standard atmosphere:

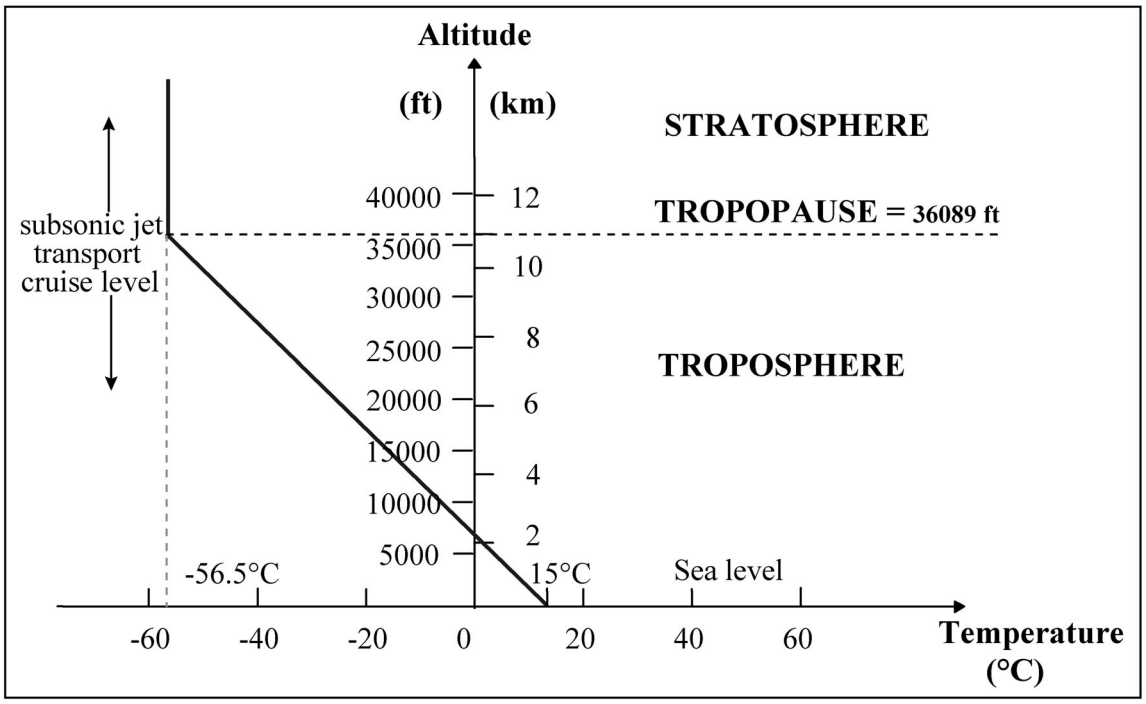

Figure 1 International Standard Atmosphere temperature variation [2].

Temperature decreases with altitude at a constant rate of -6.5°C/1000m (-1.98°C/1000ft) up to the tropopause. The standard tropopause altitude is 11,000 m (36,089 ft). Therefore, the air which is considered as a perfect gas in the ISA model presents the following characteristics within the troposphere:

$$T = T_0 - 6.5\frac{h(\text{m})}{1000} \tag{1}$$

or

$$T = T_0 - 1.98\frac{h(\text{ft})}{1000} \tag{2}$$

For simple estimations, Equation (2) can be assumed

$$T = T_0 - 2\frac{h(\text{ft})}{1000} \tag{3}$$

The temperature remains at a constant value of -56.5°C (216.65°K) from the tropopause up to 20,000 m (65,600 ft).

This ISA model is used as a reference to compare real atmospheric conditions and the corresponding engine/aircraft performance. The atmospheric conditions will therefore be expressed as ISA +/- ΔISA at a given flight level [2].

Example:

Let's consider a flight in the following conditions:

Altitude = 31,000 feet
Actual Temperature = -37ºC

The standard temperature at 31,000 feet is: $T = 15 - 2 \times 31 = -47$ ºC, whereas the actual temperature is -37ºC, i.e. 10ºC above the standard.

Conclusion: The flight is operated in ISA+10 conditions

1.2. Pressure Modeling

To calculate the standard pressure p at a given altitude, the temperature is assumed standard, and the air is assumed as a perfect gas. The altitude obtained from the measurement of the pressure is called pressure altitude (PA). Both Table 2 and Figure 2 show variation of the pressure altitude as a function of the pressure. The last column of Table 2 shows corresponding flight levels for the given pressure altitudes. The flight level is the altitude expressed in hundreds of feet.

Table 2 Pressure altitude versus pressure [2].

Pressure (hPa)	Pressure altitude (PA)		FL= PA/100
	(feet)	(meters)	
200	38661	11784	390
250	34000	10363	340
300	30066	9164	300
500	18287	5574	180
850	4813	1467	50
1013	0	0	0

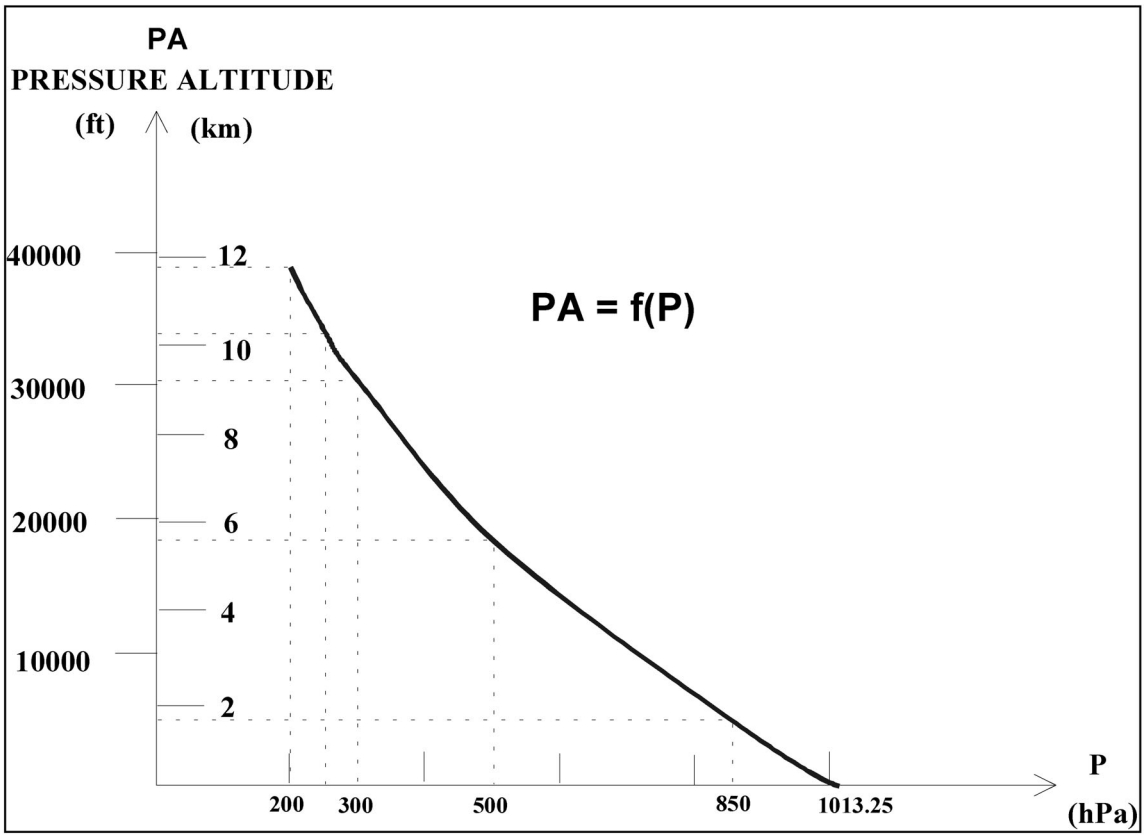

Figure 2 Pressure altitude versus pressure [2].

The pressure variations for the International Standard Atmosphere can be calculated by using the hydrostatic equation, perfect gas law and the temperature lapse rate equation. The hydrostatic equation for a column of air (Figure 3):

$$dp = -\rho g dh \tag{4}$$

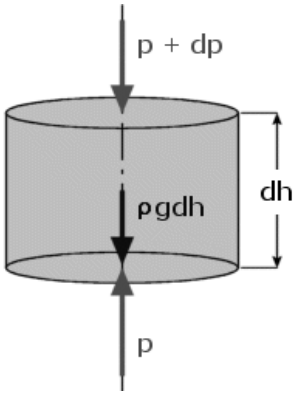

Figure 3 A small atmosphere element.

The equation of state for the perfect gas:

$$p = \rho RT \tag{5}$$

where R is the real gas constant for the air. Dividing the hydrostatic equation by the equation of state gives:

$$\frac{dp}{p} = -\frac{\rho g dh}{\rho RT} = -\left(\frac{g}{RT}\right)dh \tag{6}$$

The relationship between the pressure at a troposphere altitude and sea level pressure can be obtained by integrating equation (1) between $h_0 = 0$ and h:

$$\int_{p_0}^{p} \frac{dp}{p} = -\frac{g}{R}\int_{h_0=0}^{h} \frac{dh}{T_0 - 0.0065h}$$

Performing the above integration, we obtain:

$$p = p_0\left(1 - 0.0065\frac{h}{T_0}\right)^{5.2561} \tag{7}$$

In equation (7), the unit of T_0 is °K, and h is in meters.

Pressure above the tropopause

For the altitudes above the tropopause, the temperature is constant, so that integrating equation (6) from the tropopause to an altitude above the tropopause:

$$\int_{p_{11}}^{p} \frac{dp}{p} = -\frac{g}{RT_{11}}\int_{h_{11}=11000}^{h} dh$$

results in

$$p = p_{11}e^{-\frac{g}{RT_{11}}(h-h_{11})} \tag{8}$$

where the parameters with subscript "11" correspond to the values at the tropopause, and $p_{11} = 226.32$ hPa, $T_{11} = 216.65$ °K, and $h_{11} = 11{,}000$ m

1.3. Density Modeling

Since the pressure and standard temperature are known for a given altitude, the standard density can easily be calculated from the perfect gas equation (5):

$$\rho = \frac{p}{RT} \tag{9}$$

2. International Standard Atmosphere (ISA) Table [2]

The International Standard Atmosphere parameters (temperature, pressure, density) can be provided as a function of the altitude under a tabulated form, as given in Table 3:

Table 3 International Standard Atmosphere [2]

ALTITUDE (Feet)	TEMP. (°C)	PRESSURE			PRESSURE RATIO δ = P/Po	DENSITY σ = ρ/ρo	Speed of sound (kt)	ALTITUDE (meters)
		hPa	PSI	In.Hg				
40 000	- 56.5	188	2.72	5.54	0.1851	0.2462	573	12 192
39 000	- 56.5	197	2.58	5.81	0.1942	0.2583	573	11 887
38 000	- 56.5	206	2.99	6.10	0.2038	0.2710	573	11 582
37 000	- 56.5	217	3.14	6.40	0.2138	0.2844	573	11 278
36 000	- 56.3	227	3.30	6.71	0.2243	0.2981	573	10 973
35 000	- 54.3	238	3.46	7.04	0.2353	0.3099	576	10 668
34 000	- 52.4	250	3.63	7.38	0.2467	0.3220	579	10 363
33 000	- 50.4	262	3.80	7.74	0.2586	0.3345	581	10 058
32 000	- 48.4	274	3.98	8.11	0.2709	0.3473	584	9 754
31 000	- 46.4	287	4.17	8.49	0.2837	0.3605	586	9 449
30 000	- 44.4	301	4.36	8.89	0.2970	0.3741	589	9 144
29 000	- 42.5	315	4.57	9.30	0.3107	0.3881	591	8 839
28 000	- 40.5	329	4.78	9.73	0.3250	0.4025	594	8 534
27 000	- 38.5	344	4.99	10.17	0.3398	0.4173	597	8 230
26 000	- 36.5	360	5.22	10.63	0.3552	0.4325	599	7 925
25 000	- 34.5	376	5.45	11.10	0.3711	0.4481	602	7 620
24 000	- 32.5	393	5.70	11.60	0.3876	0.4642	604	7 315
23 000	- 30.6	410	5.95	12.11	0.4046	0.4806	607	7 010
22 000	- 28.6	428	6.21	12.64	0.4223	0.4976	609	6 706
21 000	- 26.6	446	6.47	13.18	0.4406	0.5150	611	6 401
20 000	- 24.6	466	6.75	13.75	0.4595	0.5328	614	6 096
19 000	- 22.6	485	7.04	14.34	0.4791	0.5511	616	5 791
18 000	- 20.7	506	7.34	14.94	0.4994	0.5699	619	5 406
17 000	- 18.7	527	7.65	15.57	0.5203	0.5892	621	5 182
16 000	- 16.7	549	7.97	16.22	0.5420	0.6090	624	4 877
15 000	- 14.7	572	8.29	16.89	0.5643	0.6292	626	4 572
14 000	- 12.7	595	8.63	17.58	0.5875	0.6500	628	4 267
13 000	- 10.8	619	8.99	18.29	0.6113	0.6713	631	3 962
12 000	- 8.8	644	9.35	19.03	0.6360	0.6932	633	3 658
11 000	- 6.8	670	9.72	19.79	0.6614	0.7156	636	3 353
10 000	- 4.8	697	10.10	20.58	0.6877	0.7385	638	3 048
9 000	- 2.8	724	10.51	21.39	0.7148	0.7620	640	2 743
8 000	- 0.8	753	10.92	22.22	0.7428	0.7860	643	2 438
7 000	+ 1.1	782	11.34	23.09	0.7716	0.8106	645	2 134
6 000	+ 3.1	812	11.78	23.98	0.8014	0.8359	647	1 829
5 000	+ 5.1	843	12.23	24.90	0.8320	0.8617	650	1 524
4 000	+ 7.1	875	12.69	25.84	0.8637	0.8881	652	1 219
3 000	+ 9.1	908	13.17	26.82	0.8962	0.9151	654	914
2 000	+ 11.0	942	13.67	27.82	0.9298	0.9428	656	610
1 000	+ 13.0	977	14.17	28.86	0.9644	0.9711	659	305
0	+ 15.0	1013	14.70	29.92	1.0000	1.0000	661	0
- 1 000	+ 17.0	1050	15.23	31.02	1.0366	1.0295	664	- 305

References

[1] Talay, T.A., *Introduction to the Aerodynamics of Flight*, NASA SP-367, National Aeronautics and Space Administration, Washington, D.C., 1975, p. 6-9.

[2] Airbus, *Getting to Grips with Aircraft Performance*, Airbus Industrie, Customer Services, Blagnac, 2000, p. 11-16.