Math 150 Exam 1 Form A
September 25, 2015

Choose 5 from the following 7 problems. Circle your choices: 1 2 3 4 5 6 7
You may do more than 6 problems in which case one of your two unchosen problems can replace your lowest problem at 4/5 the value (or more) as discussed in class.

1a.) The number of permutations of $n$ objects is ______________.

1b.) The number of ways to place $n$ non-attacking rooks on an $n \times n$ chessboard is ______________.

1c.) The number of $r-$permutations of $n$ objects is ______________.

1d.) The number of ways to place $r$ nonattacking rooks on an $r \times n$ chessboard, $r \leq n$ is ______________.

1e.) The number of ways to place $r$ nonattacking rooks on an $n \times n$ chessboard, $r \leq n$ is ______________.

1f.) $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$. The number of $r$ permutations of $A$ is ______________.

1g.) Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$. The number of $r$ combinations of $A$ ______________.

1h.) The number of subsets of $\{1, 2, ..., n\}$ containing exactly $r$ elements is ______________.

1i.) The number of subsets of $\{1, 2, ..., n\}$ is ______________.

2a.) In how many ways can you place 12 identical rooks in non-attacking position on a $20 \times 20$ chessboard?

2b.) In how many ways can you place 5 red rooks and 7 blue rooks in non-attacking position on a $20 \times 20$ chessboard?

2c) In how many ways can you place 5 red rooks and 7 blue rooks in non-attacking position on a $20 \times 20$ chessboard so that there is a rook in the 2nd and 4th columns?

3.) Suppose we have a group of 15 people that we label with the letters, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O.

3a.) Find the number of arrangements of 10 of these 15 people seated around a circular table.

3b.) What is the probability that A and B are among the 10 people chosen to sit at this table, but A does not sit next to B?

4.) Find the number of integral solutions to $y_1 + y_2 + y_3 = 21$ such that $y_1 \geq -1$, $y_2 \geq 0$, and $y_3 \geq 3$. Explain your answer including relating it to the number of permutations of $\{k \cdot 1, n \cdot +\}$.

5.) Circle $T$ for true and $F$ for false. **If the statement is false, prove it by giving a counter-example.**

5a.) The function $f : R \to R$, $f(x) = x^2 + 3$ is onto T F

5b.) The function $f : R \to R$, $f(x) = x^2 + 3$ is 1:1. T F

5c.) If the statement $p$ implies $q$ is true, then its contrapositive, $\sim q$ implies $\sim p$, is also true.
T F

5d.) If the statement $p$ implies $q$ is true, then its converse, $q$ implies $p$, is also true.
T F

6.) Prove that for any $n+1$ integers, $a_1, a_2, ..., a_{n+1}$, there exist two of the integers $a_i, a_j$ with $i \neq j$ such that $a_i - a_j$ is divisible by $n$.

7) Let $B = \{n_1 \cdot 1, n_2 \cdot 2, ..., n_k \cdot k\}$. Let $n = n_1 + n_2 + ... + n_k$. Prove that the number of permutations of $B = \frac{n!}{n_1! n_2! \cdots n_k!}$.