

Development of the Manufacturing Geometry of Conical and Cylindrical Worms by Analysing of Their Axoids

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In this paper an examination has been outlined a possibility of further generalization respecting the Dudás's kinematic-mathematical model, which is suitable for the production geometric development of elements of helicoid drive pairs. The positioning of the rolling surfaces of the cylindrical and conical surfaces, as axoids of worms, in the extended model to the projective space under the right conditions, based on the surfaces invariant to projective transformations can be a right extension of the model, as the projective space model includes the Euclidean space model in full. The machining of the conical worm surfaces modelled in this method involves the clarification of the kinematic-geometric relations of production geometry in the Euclidean space model, which shows the possibility of further development of production geometry.

The goal of generalization of the mathematical description is the systematization by creating central collinear projective connection between axoids (rolling cones and rolling cylinders). Managing the production geometry of the worms in a system is also a further improvement in manufacturing precision.

Keywords: Conical worm, kinematical-mathematical model, axoid, rolling surface, central collinear connection

1 Introduction

The developments of construction, quality assurance and manufacturing geometry in the University of Miskolc has been produced a significant field of worm drivings research for more than half a century. One of the outstanding topics of the Worm Scientific Team lead by Professor Dudás is the examination of the meshing of cylindrical and conical worm gear drives. In order to examine the geometry of the cylindrical and conical worms and their tools, the known Dudás's integrated general kinematic-mathematical model can be seen on Figure 1.

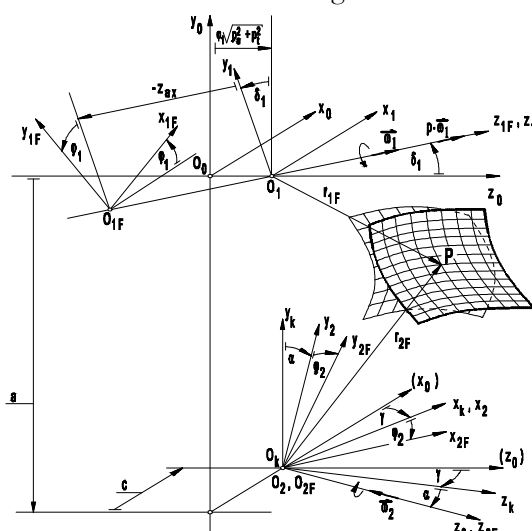


Fig. 1 General kinematic-mathematical model for the methods of the manufacturing technology [4]

The coordinate systems and applied geometrical parameters for the production geometrical analysis of the cylindrical and conical worms with the axis coinciding z_{1F} , furthermore the worm-gears and the tools of worms with the axis coinciding z_{2F} have been shown in Figure 1 above, where:

a, c ...Distances of axes [mm],

α ...Forming, tilting angle of tool to the helical surface in a characteristic section, e.g. grinding of an evolvent worm surface with a flat operating surface grinding wheel [°],

γ ...Lead angle on the worm's reference surface [°],

$K_0 (x_0, y_0, z_0)$...Stationary coordinate system [-],

$K_{1F} (x_{1F}, y_{1F}, z_{1F})$... Rotating coordinate system affixed helicoid surface [-],

$K_1 (x_1, y_1, z_1)$... Coordinate system connected to linear moving table [-],

$K_{2F} (x_{2F}, y_{2F}, z_{2F})$... Rotating coordinate system fixed to tool [-],

$K_2 (x_2, y_2, z_2)$... Stationary coordinate system fixed to tool [-],

$K_{20} (x_{20}, y_{20}, z_{20})$... Coordinate system of the generating curve of a tool of surface of revolution [-],

$K_k (x_k, y_k, z_k)$... Auxiliary coordinate system [-],

$O_0, O_1, O_2, O_{1F}, O_{2F}, O_k$... Origins of coordinate systems related to their subscripts [-],

p ... Screw parameter of the helix on worm [-],

p_a ... Axial screw parameter [-],

p_r ... Radial screw parameter [-]

φ_1 ...Angle rotation of the helical surface [°],

φ_2 ...Angle rotation of the tool surface [°],

ω_1 ...Angular velocity of the worm [s⁻¹],
 ω_2 ...Angular velocity of the tool [s⁻¹].

The mathematical analysis required for manufacturing geometry of elements of conical or cylindrical worm gear drives can be done in this model. During the *manufacturing process*, from the kinematic substitution aspect, all continuous machining can be seen as *mapping of relative motions or motion informations*, so the theorem of motion mapping, which is the production method of the related surfaces of special kinematic pairs, can be considered an organic part of manufacturing geometry. The analysis of the relative motions of the drive pair's elements can be done by rigid space systems of elements, which leads to the examination of the axoids. Due to the handling of rolling cones and rolling cylinders of worms in one model, the use of projective geometry has been introduced as a generalization of the Euclidean geometry, which includes the Euclidean geometric system. In the case of modeling manufacturing geometry, in several specific cases,

the Euclidean spatial model should be complemented by infinite distant space elements. [1, 4, 8, 14] The geometric analysis is performed through the invariants of the transformation groups, such as the fitting, or distance and angle. For the analysis of technical constructions, similarly to other researchers, a synthetic form of geometry has been selected to stimulate thinking, with the support by analytic geometry, continuing Petrich's professional work, who has been founded the Department of Descriptive Geometry in University of Miskolc [12].

2 Development task

During machining of the conical worm surface on a traditional machine with apex adjustment, the straight-line creator of the cone has been setted into parallel situation to the axis of the grinding wheel as shown in Figure 2.

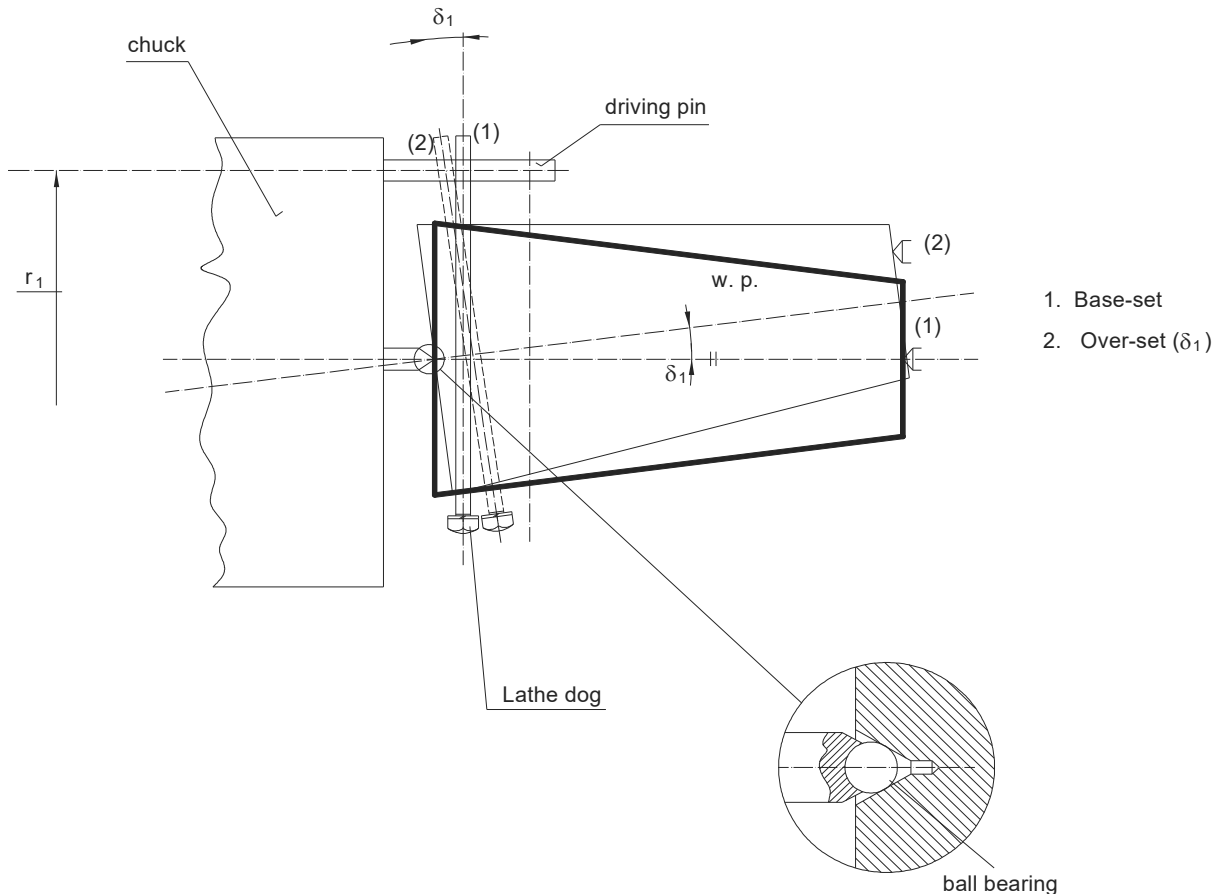


Fig. 2 Shifting of the apex at the conical worm manufacturing [10]

By producef adjusting on the tailstock, the thread of the conical worm has been machined on a plane perpendicular to main spindle's axis on an elliptical orbit, due to an setted angle between the rotating main spindle and the lathe dog. In case of conical worm's machining the thread is influenced by the peripheral

speed. The peripheral velocity of the chuck denoted v_t can be written in the next form

$$v_t = r \cdot \omega \tag{1}$$

The constant velocity of drive pin contact point denoted v_m can be written in the next form based on Figure 3.

$$\mathbf{v}_m = r_m \cdot \boldsymbol{\omega}_m \quad (2)$$

The angular velocity of workpiece denoted $\boldsymbol{\omega}_{mwp}$ can be specified as

$$\boldsymbol{\omega}_{mwp} = \frac{\mathbf{v}_m}{r_2} = \frac{r_1 \cdot \boldsymbol{\omega}_m}{r_2} \quad (\text{variable quantity}) \quad (3)$$

Variable distance from the axis changed the peripheral

velocity and caused a H pitch error among other problems

$$H = p \cdot \boldsymbol{\omega}_{mwp} \quad (4)$$

Depending on the angular position of the workpiece, sometimes the pitch is smaller, sometimes greater, that is, it fluctuates around a nominal value based on $p \cdot \boldsymbol{\omega}_1$ value.

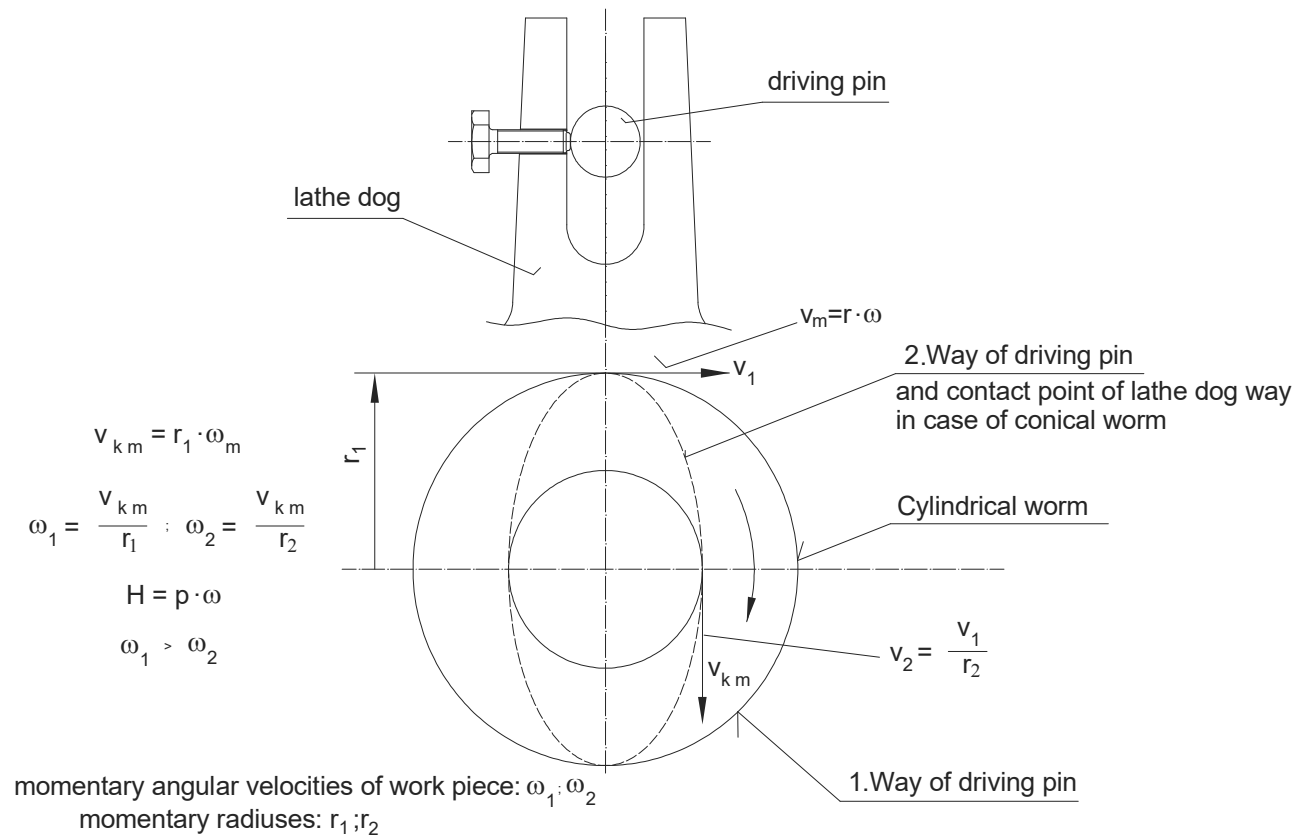


Fig. 3 Mathematical layout of the thread in case of conical worm manufacturing [10]

3 About the axoids

During the development of manufacturing geometry of helicoidal surfaces and their tools the necessity to investigate kinematic relationships and to treat rolling surfaces in one model is well established. [2, 4, 5, 6, 7, 9, 10, 13]

In case of the moving of the rigid plane system in its own plane, the relationship between it's two different positions can always be given by rotation around a point, or by a parallel offset to a straight line, in the case of the infinite instantaneous rotation point.

In case of the moving of the rigid three dimensional system in its own space, the relationship between it's two different positions can always be given by a helical movement along a straight line. The spatial rigid system has been described by the Descartes' right-handed mutually-orthogonal normalized axis system, which is defined by it's own three non-collinear points, such as the origo O , as well as the unit points E_x and E_y on the x and y axes. The spatial movement of triangle OE_xE_y between it's position $O_1E_{x1}E_{y1}$ and

$O_1(\varphi)E_{x1}(\varphi)E_{y1}(\varphi)$, can be replaced by one helical movement along the helical axis f_i . In the first step the direction of the helical axis f_i can be determined as shown in Figure 4. In case of parallel offsetting with itself on the two triangles $O_1E_{x1}E_{y1}$ and $O_1(\varphi)E_{x1}(\varphi)E_{y1}(\varphi)$ onto a point in the space in so way, that one of the matched pointpairs, such as the O_1 and $O_1(\varphi)$ overlap each other, the triangles $O_1E_{x1}E_{y1}$ and $\underline{O}_1(\varphi)\underline{E}_{x1}(\varphi)\underline{E}_{y1}(\varphi)$ can be placed in an overlap position with one rotation. The axis f_i of rotation passes through on point $O_i = \underline{O}_i(\varphi)$. The rotation-axis f_i is created by the intersection of two planes, of which the first plane is the bisector perpendicular plane of the section $E_{x1}\underline{E}_{x1}(\varphi)$, and the second plane is the bisector perpendicular plane of the section $E_{y1}\underline{E}_{y1}(\varphi)$. In the second step, it is necessary to determine the exact position of the helical movement's axis f_i . The triangles $O_1E_{x1}E_{y1}$ and $O_1(\varphi)E_{x1}(\varphi)E_{y1}(\varphi)$ are projected from their original positions parallel to the direction of rotation axis f_i , onto the plane \underline{K} , perpendicular to the rotation axis f_i , as shown in Figure 4.

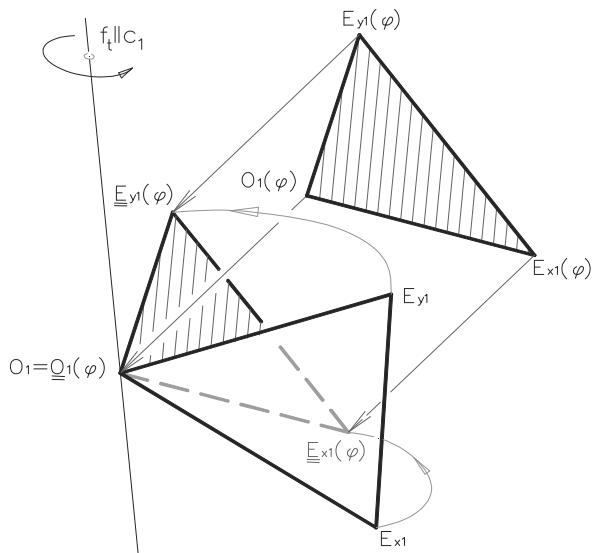


Fig. 4 Determination of the direction f_i of the torsion axis c_1

Since the planes of the congruent triangles $O_1E_{x1}E_{y1}$ and $O_1(\varphi)E_{x1}(\varphi)E_{y1}(\varphi)$ creates an equal angle with the plane \underline{K}_s , the projections of triangles will be congruent also. In the plane \underline{K}_s the congruent projections of triangles can be rotated into overlap with an angle φ , around an point O . The axis of the helical movement is a straight line c_i parallel to the the rotation axis f_i and passing through point O .

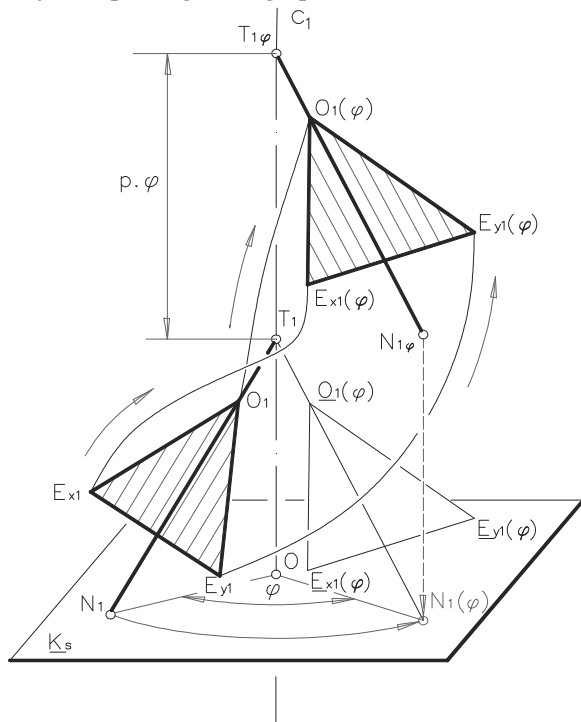


Fig. 5 The instantaneous torsion axis of motion between two spatial position

In Figure 5 the planes of the triangles $O_1E_{x1}E_{y1}$ and $O_1(\varphi)E_{x1}(\varphi)E_{y1}(\varphi)$ intersect the axis c_i at points T_1 and T_2 . The measure of displacement between points T_1 and T_2 is $p \cdot \varphi$. The triangle $O_1(\varphi)E_{x1}(\varphi)E_{y1}(\varphi)$ can be offset parallel to direction of rotation axis f_i and the size of the

section $\overline{T_1T_2}$ into the intermediate position $\underline{Q_1}(\varphi)\underline{E_{x1}}(\varphi)\underline{E_{y1}}(\varphi)$, which can be rotated into position $O_1E_{x1}E_{y1}$ around axis c_i , by angle φ . The resulting movement follows the helical lines described by the points O_1, E_{x1} and E_{y1} , the common axis of the helical lines is the c_i , and the common parameter is the p . (If we interpret rotation angle $\varphi+2n\pi$ instead of rotation with angle φ , then with a constant amount of offset, we get a solution with different pitch helices.)

A machine tool stand and it's any moving unit, a workpiece and tool fixed in the tool machine one by one may be considered belonging to a rigid space system.

The most general motion of the moving rigid space system Σ_1 correlated to the fixed rigid space system Σ_0 is an instantaneous screw motion. The instantaneous screw motion is determined by the instantaneous position of the instantaneous axis c_{10} , furthermore the instantaneous angular velocity ω_{10} of the Σ_1 space system correlated to Σ_0 space system, and it's instantaneous screw parameter $p_{10} \neq 0$. If the position of the screw axes c_{10} in the space system Σ_0 , the angular velocity ω_{10} and the screw parameter p_{10} are constant, the screw motion is named elementary. [3]

If a space system Σ_1 and another spaces system Σ_2 are in the relative motion each other, the instantaneous screw axes of this relative motion create a line surface α_1 in space system Σ_1 and an another line surface α_2 in space system Σ_2 . An axoid is the family of instantaneous axes of the relative motion of Σ_1 and Σ_2 three dimensional systems, that is generated in three dimensional systems Σ_1 or Σ_2 . [8] The axoids have always appeared in pairs, they are connected (conjugated). [13]

In the space system Σ_0 fixed to the drive mechanism casing the screw motion space system Σ_1 with axis c_{10} , angular velocity ω_{10} , parameter p_{10} and also screw motion space system Σ_2 with axis c_{20} , angular velocity ω_{20} , parameter p_{20} are in relative motion, that is generally between in them a continuous motion of screw motions. The instantaneous screw axes create the axoid α_1 in Σ_1 and axoid α_2 in Σ_2 , which are slipping and rolling on a distorted screw surface on each other in the most comprehensive case. These are touched together in a Σ_0 space system in an instantaneous relative screw axis $c_{12} = c_{21}$. In special cases the element of the axoid pairs may be a distorted screw surface, closed or opened and sharp or flat screw surfaces, a developable helicoid, a rotary distorted hyperboloid, a cylinder generated by rotation, a cone generated by rotation, ect. .

4 About the projective spaces model

For the analytical description of the Euclidean space, it's points are assigned a triplet of real numbers (x, y, z) in the Descarte coordinate system. The homogeneous coordinates of points of the Euclidean space

model are obtained with the following identities

$$x = \frac{x_1}{x_4}, y = \frac{x_2}{x_4}, z = \frac{x_3}{x_4}, x_4 \neq 0, \quad (5)$$

where the rang $(x_1, x_2, x_3, x_4) = 1$.

Two points are identical, if their corresponding coordinates are product of scalar to each other. In case of $x_4=0$, the coordinates $(x_1, x_2, x_3, 0)$ represent the point in the infinite distance in direction (x_1, x_2, x_3) .

The homogeneous coordinates of the points fitting to the second-order surface of the projective space fulfil the next form

$$a_{IK} \cdot x_I \cdot x_K = 0 \quad (I, K = 1, 2, 3, 4) \quad (6)$$

By successive application of projective transformations, it is always possible that the matrix of the second order surfaces can be made into symmetric matrix, which only has the elements in the diagonal that are different from 0, which can be +1 and -1. The range and signature of the matrix of the second-order surface are invariant with projective transformation. The second-order surfaces are classified according to the range and signature of their matrix representation.

Classes of second-order surfaces include surfaces with same range, denoted with R, and with same signature, denoted with S.

In case of R= 3 and S= 1, the second-order surface can be written by homogeneous coordinates in the next form

$$x_{12}^2 + x_{22}^2 - x_{32}^2 = 0 \quad (7)$$

and the same surface can be written by Descartes coordinates in the Euclidean space

$$x^2 + y^2 - z^2 = 0 \quad (8)$$

The class of the real cone surface generated by rotation also includes the rotated surface, which means that in the projective spatial model has the same qualities. Since their qualities are invariant with projective transformation, therefore projective transformation

can be created between them.

The matrix form of all second order surface

$$\mathbf{X}^T \cdot \mathbf{A} \cdot \mathbf{X} = 0 \quad (9)$$

where the \mathbf{X}^T is transposed of matrix \mathbf{X} .

Developments in the projective space model can be linked to the Dudás' ProMat model [5].

5 The central collineation relation between the rolling surfaces of cylindrical and conical worms

Based on the Dudás mathematical-kinematical model, the Euclidean space model has been complemented by the infinite distant plane, so the rolling cone and rolling cylinder of the worms have been located in the projective space model in a central collineation relationship. In the projective space model the cylinder is a cone with an infinity vertex. To create a central collineation relationship between the two surfaces, the cone generated by rotation and the cylinder generated by rotation have been placed on the common axis z_{IF} with the following correspondence: the finite vertex M_k of the cone on the axis z_{IF} and the infinite distance vertex M_h of the cylinder on the axis z_{IF} , correspond to each other, the plane of the common circle of the cone and cylinder is the base plane \underline{A}_s of the central collinearity, and the C center of the central collinearity may be any point of the axis z_{IF} without the axis point on the base plane and without the vertex of the cone and cylinder. The straight line lying on the P_h point of the cylinder and on the P_k point of the cone has been passed through the center C. These points have been assigned to each other in the central collinear connection. The disappearing planes have been positioned, that one of the vanishing planes has been as far as away from the center C as the other vanishing plane from the base plane \underline{A}_s .

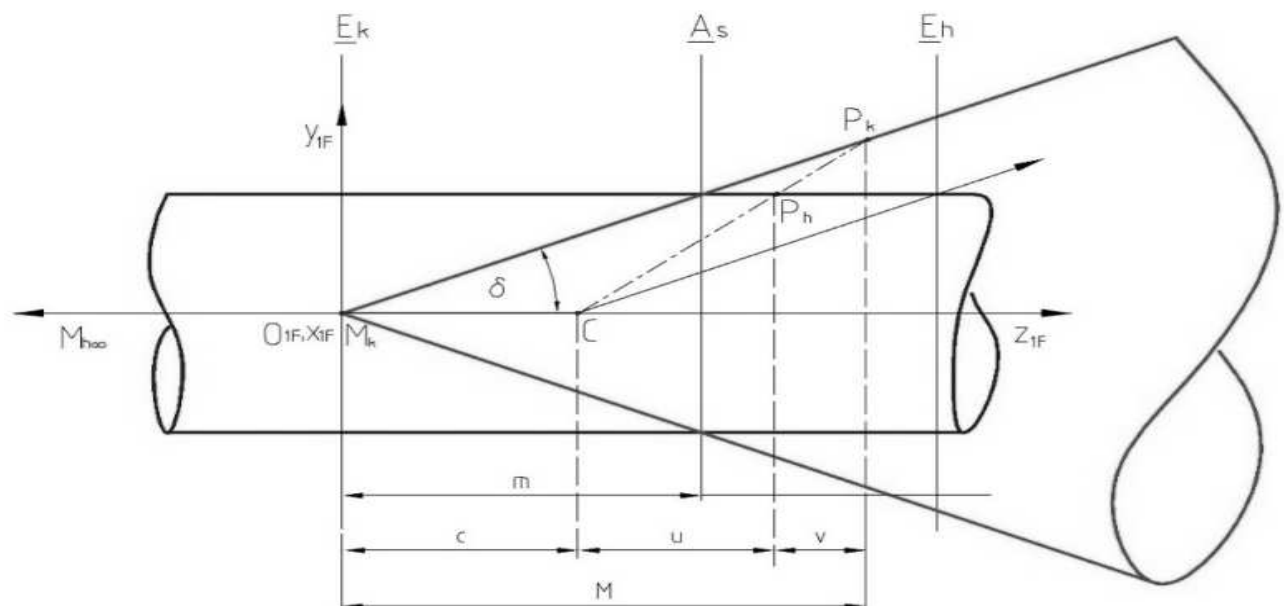


Fig. 6 Central collineation relation between the rolling cone and rolling cylinder of worms [11]

The vanishing plane E_k of the cone fits the cone vertex M_k , as it can be seen on Figure 6. The coordinates of the cylinder's points (marked by x) satisfy the next equation

$$h_{IK} \cdot x_I \cdot x_K = 0 \quad (I, K = 1, 2, 3, 4) \quad (10)$$

that can be written in the following matrix format

$$X^T \cdot H \cdot X = 0 \quad (11)$$

$$X^T \cdot K \cdot X = (X^T \cdot C^T) \cdot H \cdot (C \cdot X) = X^T \cdot \{C^T \cdot H \cdot C\} \cdot X \quad (14)$$

where the C matrix represents the central collinear relationship.

6 Creating of helicoid surfaces

The rolling surfaces of the conical and cylindrical worms have been located in the projective space model based on their central collinearity relationship. Than the worm's teeth surfaces produced by a rotary motion and a tangential and a radial displacement in the Euclidean space model, as the Euclidean geometry is the theorem of the transformation group's invariants.

The creation of the helicoid surfaces, and it's matrix representation can be done into relation with analytical modelling of Euclidean space. An important goal is to provide usability in a single model. In the following the connection is worth examining between the locating in the projective space model of the worms' rolling surfaces and the manufacturing of the teeth surfaces in the Euclidean space model along it's matrix representations as well. The combined motion can be described by a 4×4 matrix. Placing behind the 3×3 matrix of the rotating motion the column matrix of the translation, so the 3×4 matrix has to be complemented in the bottom line with the row $[0, 0, 0, 1]$ to a 4×4 matrix.

Handling of the coordinates of the points is also important to consider the theoretical clarification. The rolling surface of the worms can be written in the projective space using a 4×4 matrix, to which can be used the homogeneous coordinates of points. The homogeneous coordinates $(\lambda \cdot x_1, \lambda \cdot x_2, \lambda \cdot x_3, \lambda \cdot x_4)$, $(\lambda \neq 0)$ of points can be formed by (x, y, z) Descartes coordinates of points according to formula (1).

The description of the combined motions can be done with a part homogeneous coordinates $(x, y, z, 1)$ formed from the (x, y, z) Descartes coordinates of points in the Euclidean space model, using only $x_4 = 1$ from the endless possibilities.

One of the important direction of the mathematical development is to set the goal of systematization, to improve the manageability of the task in the system.

The coordinates of the cone's points (marked by x) satisfy the next equation

$$k_{IK} \cdot x_I \cdot x_K = 0 \quad (I, K = 1, 2, 3, 4) \quad (12)$$

that can be written in the following matrix format

$$X^T \cdot K \cdot X = 0 \quad (13)$$

In case of transformation from rolling cone to rolling cylinder the required identical has been fulfilled, as it can be seen in the following equation

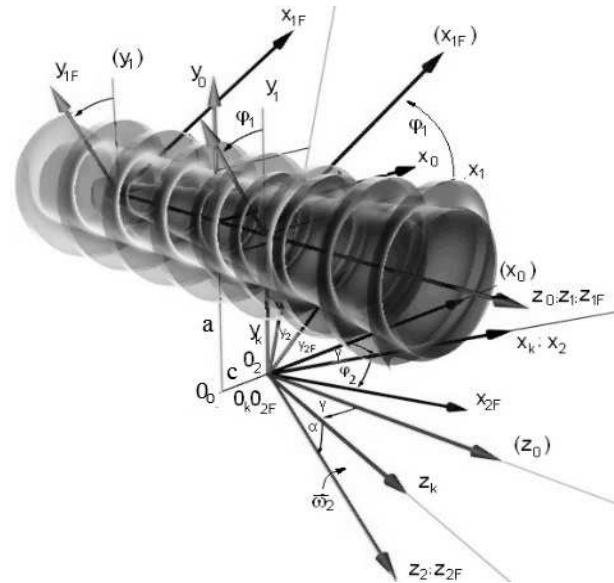


Fig. 7 The mathematical-kinematical model for the theory of manufacturing of the cylindrical and conical worms and their tools for uniaxial conical and cylindrical rolling surfaces [11]

7 Result

Worms from worm gearboxes for high power transmission have been machined by grinding wheel in a modern machining plants. From the rolling surfaces of the helicoidal elements of worm gear drive pairs according to their projective classification, the members of a class in the projective space have the same behavioral characteristics, so they were placed in a central collinear relationship, which further developed the mathematical-kinematical model for unification. All this has been already described in the Euclidean space model, the geometrically correct machining leads to increased accuracy informing the operating surface of the conical worms.

In our further developed mathematical-kinematical geometric model of manufacturing process in case of changing distance of axes $(a_0 \pm p \cdot \phi_1)$, the elliptical errors have been eliminated. The geometrically correct manufacturing of conical worm surfaces, so elements of spiroid drives has become possible, as it can be seen in Figure 8.

The above model's geometrical development requires the construction of a new NC machine, which is also suitable of transverse movement with the grinding wheel in case of manufacturing.

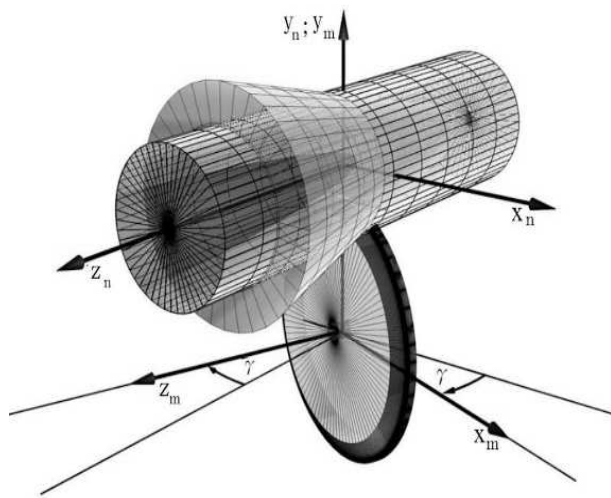


Fig. 8 The locating of the rolling surfaces of the cylindrical and conical worms mounted the same shaft in case of manufacturing with grinding wheel [11]

8 Summary

Based on the knowledge of projective geometry - using the Dudás's mathematical-kinematical model, the further developed using the projective geometrical model has been demonstrated. The necessary examination has been outlined about axoides of conical and cylindrical worms gear drives. The central collinear relationship has been determined between the rolling cone and the rolling cylinder in case of a rotating common axis. The examination has been performed to confirm that the matrix representation of the central collinearly relationship between the rolling cone and rolling cylinder surfaces is suitable to fit ProMat model. The improved mathematical-kinematical model ensures geometrically accurate production of conical worm.

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