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Theoretical Calculations of Cerium Nucleon Densities by Skyrme and Gogny Forces

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Abstract. The ground state properties of the nuclei, especially nucleon densities, are generally calculated using Skyrme and Gogny forces. The nucleon densities of Cerium isotopes calculated by using Hartree-Fock-Skyrme (based Woods-Saxon Potential) (SHF-WS), Hartree-Fock-Skyrme (based Harmonic Oscillator Potential) (SHF-HO), Hartree-Fock-Bogolyubov-Skyrme (HFB-S) and the Hartree-Fock-Bogolyubov-Gogny (HFB-G) methods. In the first two methods, the densities and rms radii for both nucleons of Cerium isotopes were calculated by different Skyrme parameters set. Theoretical calculated charge density compared with experimental data to determine best parameter set for each Cerium isotope. Then all nucleon densities compared each other and all methods gave similar results for all Cerium isotopes.

1 Introduction

The Hartree-Fock (HF) codes [1–3] are a useful tool to describe the ground states properties of the spherical nucleus. Spherical HF codes have been used for 12 years and are developed by deriving gradient iterations [4–7]. In HF approach, equations are solved using an iteration, namely, harmonic oscillator or Wood-Saxon [8] wave functions are recommended for wave function. The density $\rho(0)$ is generated based on these wave functions. Then, iterations are made between the intensity and $U(0)$ energy potential for φ wave function.

The Skyrme force [9, 10] is the effective force for HF calculations. In this way, quantities, which are very important and can be measured experimentally, such as core radii, density distributions and surface thickness can be calculated. HF codes are applied to the specified Skyrme force by fitting the least squares method. It is also fitted to ground states correlations [11] and nucleus excitation studies.

The properties of a nucleus which comprises neutrons and protons are determined by forces which acting between the nucleons. The two-body and three-body interactions potentials can be written as

$$\begin{aligned}\vec{V}_{(i,j)} &= t_0(1+x_0P_x)\delta(\vec{r}_i-\vec{r}_j)\\ &+\frac{1}{2}t_1(1+x_1P_x)[\vec{P'}^2_{12}\delta(\vec{r}_i-\vec{r}_j)+\delta(\vec{r}_i-\vec{r}_j)\vec{P}^2_{12}]\\ &+t_2(1+x_2P_x)\vec{P'}_{12}\delta(\vec{r}_i-\vec{r}_j)\vec{P}_{12}\\ &+it_4\vec{P}_{12}\cdot\delta(\vec{r}_i-\vec{r}_j)(\vec{\sigma}_i+\vec{\sigma}_j)\times\vec{P}_{12} \qquad (1)\\ \vec{V}_{(i,j,k)} &= \frac{1}{6}t_3(1+x_3P_x)\rho(\frac{\vec{r}_i+\vec{r}_j}{2})\delta(\vec{r}_i-\vec{r}_j) \qquad (2)\end{aligned}$$

to calculate Skyrme interaction potential;

$$\vec{V}_{\text{Skyrme}}=\sum_{i<j}\vec{V}_{ij}+\sum_{i<j<k}\vec{V}_{ijk}\,. \qquad (3)$$

In these equations, δ is delta function, $\vec{P}_{12}=\pm(\vec{\nabla}_1-\vec{\nabla}_2)$ is relative momentum operator acting on the wave functions moving to the right or left, P_x is spin exchange operator and σ is Pauli spin matrices. The interaction of two objects depends on seven parameters (t_0, t_1, t_2, x_0, x_1, x_2, t_4) while three-body depends on nine parameters (also, x_3 and t_3).

The zero-range, three-body force in the original Skyrme force is assumed to be equivalent to two-body dependent interaction.

$$\vec{V_{ij}}\backsimeq\vec{V_{ijk}}\,. \qquad (4)$$

For effective interactive HF calculations, the Skyrme interactive HF (SHF) method is best suited for ground state calculations of all nuclei, from light nuclei to heavy nuclei. The SHF method is essentially based on the shell model, which assumes that one nucleon moves independently within an average central potential of other nucleons.

Depending on single particle states three local density functions

$$\rho_q(\vec{r})=\sum_{i,\sigma}w_i|\varphi_i(\vec{r},\sigma,q)|^2\,, \qquad (5)$$

$$\tau_q(\vec{r})=\sum_{i,\sigma}w_i|\vec{\nabla}\varphi_i(\vec{r},\sigma,q)|^2\,, \qquad (6)$$

$$\vec{J}_q(\vec{r})=(-i)\sum_{i,\sigma,\sigma'}w_i\varphi_i^*(\vec{r},\sigma,q)[\vec{\nabla}\varphi_i(\vec{r},\sigma',q)\times\langle\sigma|\vec{\sigma}|\sigma'\rangle]\,. \qquad (7)$$

Here, ρ_q is the proton density, τ_q is the kinetic energy, $\vec{J}_q$ is the spin-orbital density (wherein $q=n$ and p, represent protons and neutrons). Also, w_i denotes

the occupation probability of the states i and σ is Pauli spin matrix. The SHF method is a self-compatible field method.

The most important advantage of Skyrme interaction is that HF energy is expressed by three density functions

$$E_{\text{HF}} = \int H(\rho_q, \tau_q, \vec{J}_q) d^3r \,, \tag{8}$$

$$E_{\text{HF}} = E_{\text{Skyrme}} + E_{\text{Coulomb}} + E_{\text{pair}} - E_{\text{cm}} \,, \tag{9}$$

where E_{Skyrme} is energy function of Skyrme force, E_{Coulomb} is Coulomb interaction energy, E_{pair} is the coupling interaction energy of nucleons and E_{cm} is the correction term for the center of mass of the mean field [5].

Skyrme type forces are widely used phenomenological forces. With the development of the Skyrme force and the identification of new parameter sets, it is also applied to cores far from the spherical structure. Skyrme force parameters can be adjusted by comparison of experimental data with theoretical results or only by theoretical approaches.

Another phenomenological force is the Gogny force which is a finite-range two-body interaction [12, 13]. The use of the Gogny interaction with the HF method yields a non-relativistic approach like the SHF method.

Another type of HF method is the Hartree-Fock-Bogolyubov (HFB) method [14, 15]. Independent particles moving within an average potential are depicted as single particles [16] Correlations between these particles and incomplete shells give the HFB theory. HFB-S and HFB-G methods [17] were developed by using Skyrme and Gogny forces.

2 Materials and Methods

The nucleon densities of ^{136,138,140,142,144,146,148}Ce isotopes calculation was calculated by using HAFOMN [18] computer program, for HO and WS poten-

Table 1. Various Skyrme parameter sets and coefficients [27]

Set	t_0	t_1	t_2	t_3	x_0	x_1	x_2	x_3	W_0	α
SkA	-1602.8	570.9	-67.7	8000	-0.02	0	0	-0.28	125	1/3
GS6	-1012	209	-76.3	10619	0.14	0	0	1	105	1
SkM*	-2645	410	-135	15595	0.09	0	0	0	130	1/6
SGII	-2645	340	-41.9	15595	0.09	-0.06	1.43	0.06	105	1/6
SLy4	-2488.9	486.8	-546.4	13777	0.83	-0.34	-1	1.35	123	1/6
SLy5	-2483.5	484.2	-556.7	13757	0.78	-0.32	-1	1.26	125	1/6
SLy6	-2479.5	462.2	-448.6	13673	0.83	-0.47	-1	1.36	122	1/6
SLy7	-2480.8	461.3	-433.9	13669	0.85	-0.49	-1	1.39	126	1/6
SLy8	-2481.4	480.8	-538.3	13731	0.81	-0.34	-1	1.31	122.4	1/6
SLy9	-2511.1	510.6	-429.8	13716	0.80	-0.62	-1	1.37	119	1/6
SLy10	-2506.8	430.9	-304.9	13826	1.04	-0.67	-1	1.68	90.7	1/6

tials separately by HF method with Skyrme interaction. TALYS [19] nuclear reaction code was used to calculate Skyrme and Gogny interactions by HFB method in the same manner. Using SkA [20], GS6 [21], SKM* [22], SGII [23], Sly4, Sly5, SLy6, SLy7 [24], SLy8, SLy9 and SLy10 [25] parameters, proton and neutron densities were obtained for each. Theoretical calculated charge density compared with experimental data of Angeli and Marinova [26] works to determine which is the best parameter set in the calculations for Skyrme and Gogny interactions and were plotted in a graphic drawn software. These are the parameters sets of Skyrme force which we used in the calculations given in Table 1.

3 Results and Discussion

The calculated charge radius of nucleus was compared with experimental data of Angeli and Marinova [25], then best compatible set of Skyrme parameters were determined for Skyrme interaction using WS and HO. SLy10 for ^{136}Ce, SkA for ^{138}Ce, SLy4 for ^{140}Ce, SL710 for ^{142}Ce, ^{144}Ce, ^{146}Ce and SLy9 for ^{148}Ce using WS potential; SLy4 for ^{136}Ce, ^{144}Ce, SLy5 for ^{138}Ce, SLy7 for ^{142}Ce, SGII for ^{146}Ce and SLy10 for ^{148}Ce using HO given in Table 2 and Table 3.

Table 2. Charge radius of ^{136,138,140,142,144,146,148}Ce isotopes were calculated by different parameter sets using WS method. Comparison of calculated charge radii with experimental values of [26].

R_{charge}	^{136}Ce	^{138}Ce	^{140}Ce	^{142}Ce	^{144}Ce	^{146}Ce	^{148}Ce
EXP	4.8739	4.8737	4.8771	4.9063	4.9303	4.9590	4.9893
SkA	4.8667	4.8753	4.8841	4,8964	4.9087	4.9208	4.9328
GS6	4.4604	4.4699	4.4793	4.4981	4.5172	4.5366	4.5568
SkM*	4.8539	4.8616	4.8695	4.8814	4.8933	4.9050	4.9166
SGII	4.8469	4.8561	4.8655	4.8789	4.8921	4.9052	4.9183
SLy4	4.8620	4.8706	4.8793	4.8923	4.9051	4.9177	4.9301
SLy5	4.8525	4.8609	4.8694	4.8822	4.8948	4.9072	4.9194
SLy6	4.8504	4.8592	4.8682	4.8816	4.8949	4.9080	4.9209
SLy7	4.8521	4.8609	4.8698	4.8834	4.8967	4.9099	4.9229
SLy8	4.8534	4.8620	4.8706	4.8836	4.8963	4.9089	4.9212
SLy9	4.9297	4.9395	4.9493	4.9632	4.9769	4.9905	5.0039
SLy10	4.8731	4.8841	4.8952	4.9099	4.9244	4.9388	4.9530

Then neutron and proton density graphs versus nucleus radii were plotted according to best Skyrme parameter set for SHF-WS and SHF-HO calculations. The theoretically calculated proton and neutron densities of used Ce isotopes by HAFOMN code were given in Figures 1–7, respectively. Neutron and proton densities also have been theoretically evaluated using Gogny and Skyrme interactions in HFB model by TALYS nuclear reaction code and given in Figures 1–7.

Table 3. Charge radius of ^{136,138,140,142,144,146,148}Ce isotopes were calculated by different parameter sets using HO methods. Comparison of calculated charge radii with experimental values of [26].

R_{charge}	^{136}Ce	^{138}Ce	^{140}Ce	^{142}Ce	^{144}Ce	^{146}Ce	^{148}Ce
EXP	4.8739	4.8737	4.8771	4.9063	4.9303	4.9590	4.9893
SkA	4.9775	4.9711	4.9824	4.9966	5.0129	5.0317	5.0477
GS6	4.4605	4.4773	4.4937	4.5086	4.5232	4.5377	4.5520
SkM*	4.8557	4.8644	4.8731	4.8904	4.9049	4.9318	4.9349
SGII	4.8672	4.8764	4.8861	4.9085	4.9262	4.9438	4.9611
SLy4	4.8777	4.8857	4.8939	4.9115	4.9276	4.9435	4.9598
SLy5	4.8678	4.8758	4.8840	4.9006	4.9160	4.9316	4.9474
SLy6	4.8674	4.8762	4.8851	4.9039	4.9213	4.9385	4.9556
SLy7	4.8698	4.8784	4.8875	4.9062	4.9237	4.9410	4.9582
SLy8	4.8691	4.8772	4.8856	4.9028	4.9189	4.9347	4.9509
SLy9	4.9467	4.9566	4.9664	4.9886	5.0074	5.0257	5.0435
SLy10	4.8871	4.8984	4.9101	4.9357	4.9561	4.9758	4.9944

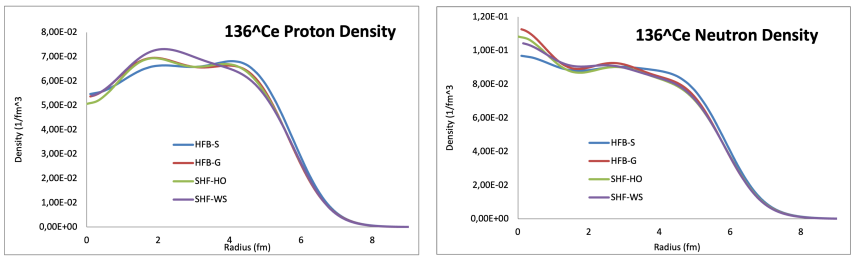

Figure 1. The nucleon densities of ^{136}Ce.

In Figure 1, the effects of neutron and proton energy wells are clearly seen. After 3 fm all models gave similar results.

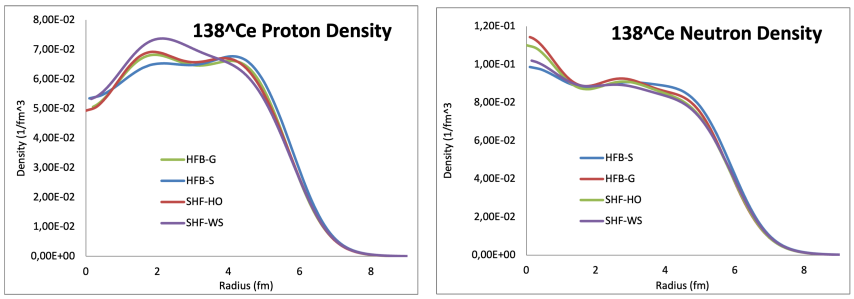

Figure 2. The nucleon densities of ^{138}Ce.

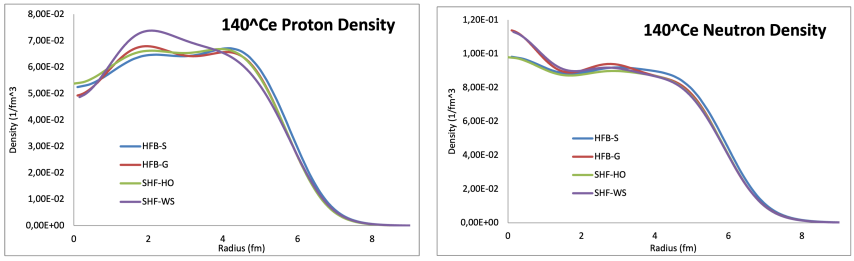

Figure 3. The nucleon densities of ^{140}Ce.

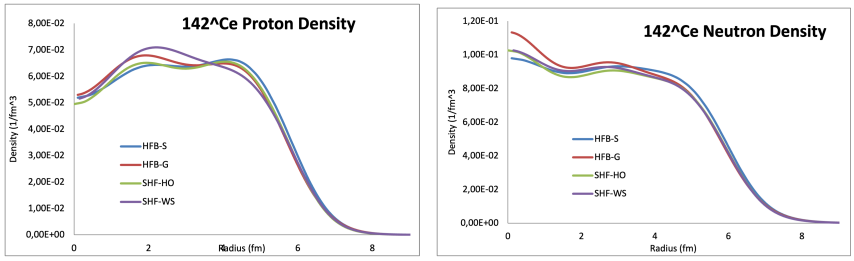

Figure 4. The nucleon densities of ^{142}Ce.

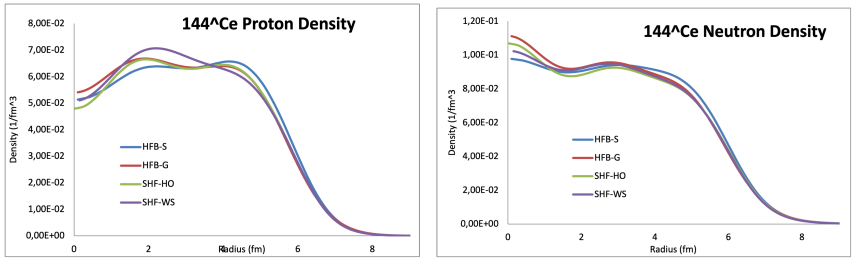

Figure 5. The nucleon densities of ^{144}Ce.

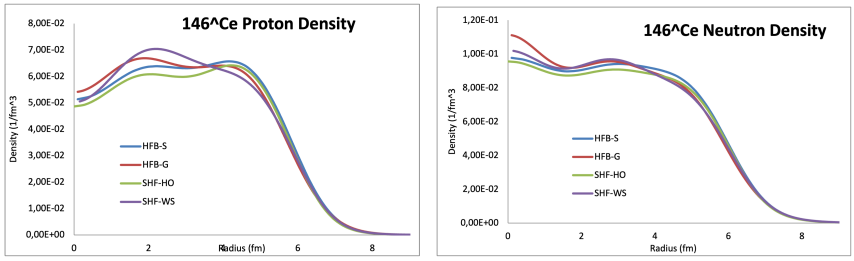

Figure 6. The nucleon densities of ^{146}Ce.

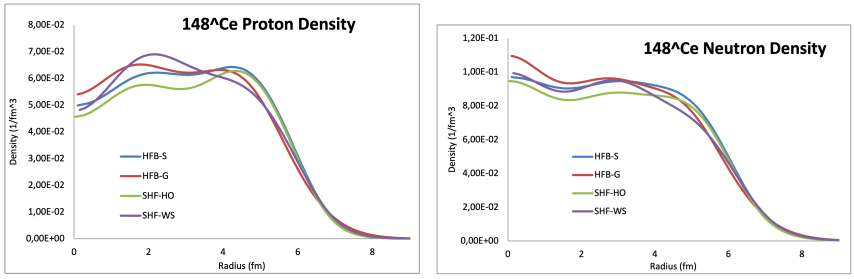

Figure 7. The nucleon densities of ^{148}Ce.

4 Conclusion

The nucleon densities of Ce isotopes were calculated by four different models. SHF-WS and SHF-HO models are useful to calculate spherical nuclei. HFB-S and HFB-G calculations were done by using TALYS code for rotating nuclei by addition of angular momentum. One can see that, in Figures 1–7 the four models almost same after 2–3 fm.

While the proton densities of the nuclei are approximately constant around 2 to 3 fm, the density values sharply drop to zero after these distance values. The calculated neutron densities were similar to the proton densities. At around 2 fm in Ce nuclei, the neutron distribution reaches its maximum values. The charge density distributions were obtained very close to each other at the centers of the nuclei ($r = 0$). For the same nuclei, after 3–3.5 fm, as the radial distance increases, the density values quickly decrease to zero.

Proton density values among the Ce isotopes, in which the number of protons remained constant, changed with the increase in the number of neutrons. The reason for this is that as the number of neutrons increases, the distribution of matter in the nucleus changes, and this change allows the protons to spread over a larger volume.

In shell model calculations, it is necessary to use spectrum for energy levels that accurately predict the magic numbers of nucleons in the Woods-Saxon potential well to obtain accurate results.

Especially the changing up to 3 fm will be important in the calculations of nuclear potentials which will be next step of this study.

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