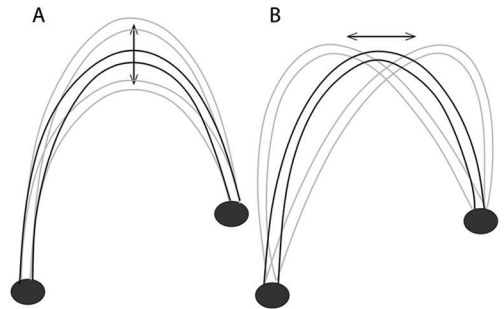
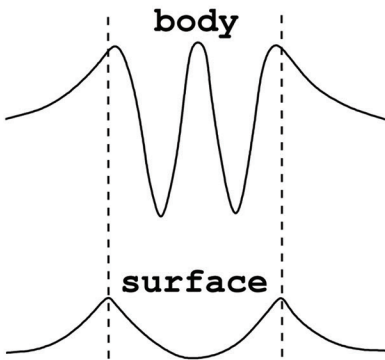
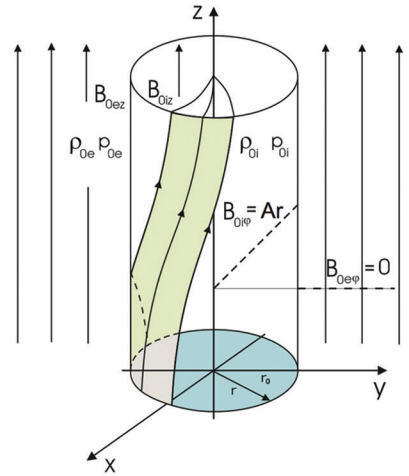
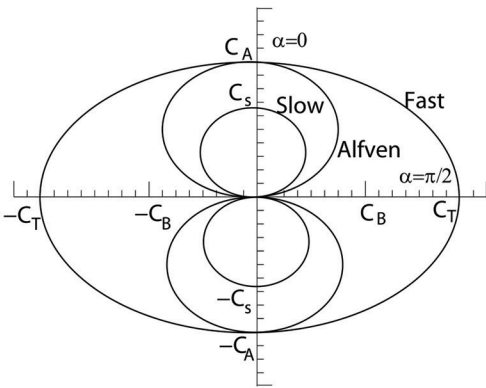


WAVES AND OSCILLATIONS IN NATURE

— An Introduction —



A. Satya Narayanan
Swapan K. Saha



CRC Press
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Principal Symbols

a	Acceleration
A	Ampere
<i>A</i>	Complex amplitude of the vibration
A	Vector potential
B	Magnetic induction
B (r , <i>t</i>)	Time dependent magnetic field
B ₀	Amplitude of the magnetic field vector
<i>C</i>	Capacitance
C	Coulomb
<i>c</i>	Velocity of light
<i>D</i>	Diameter of the aperture
D	Electric flux density
E	Electric field vector
E (r , <i>t</i>)	Time dependent electric field
E ₀	Amplitude of the electric field vector
<i>ℰ</i>	Electromotive force
F	Force
<i>F</i>	Farad
<i>f</i>	Electrical frequency
<i>G</i>	Gain
H	Magnetic field
<i>H</i>	Henry
<i>ħ</i> (= <i>h</i> /2π)	Reduced Planck constant
<i>i</i>	Current
<i>I</i>	Intensity of light
ℑ	Imaginary part of the quantity
<i>J</i>	Jones matrix
J	Current density
J _{<i>c</i>}	Conduction current density
J _{<i>v</i>}	Convection current density
<i>j</i>	= 1, 2, 3 (indices)
J	Joules
<i>J</i> ₀ (<i>x</i>)	Bessel function of the first kind and order zero
<i>J</i> ₁ (<i>x</i>)	Bessel function of the first kind and order one
<i>L</i>	Inductance
<i>l</i> _{<i>c</i>}	Coherence length
<i>M</i>	Mueller matrix
M	Magnetization
<i>m</i>	Mass

m_0	Rest mass
N	Integer value
N	Newton
n	Refractive index
$\hat{\mathbf{n}}$	Unit vector
\mathbf{p}	Momentum
P	Pressure
\mathcal{P}	Polarization
q	Electron charge
$\mathbf{r}(= x, y, z)$	Position vector of a point in space
R	Resistance
\Re	Real part of the quantity
R_L	Load resistance
S	Stokes parameter
$\hat{\mathbf{s}}$	Unit vector
$S(\mathbf{x})$	Point spread function
\mathcal{S}	Surface
$\mathbf{S}(\mathbf{r}, t)$	Poynting vector
S_r	Strehl's ratio
sr	Steradian
t	Time
T	Period
$U(\mathbf{r}, t)$	Complex representation of the analytical signal
$\mathbf{u} = (u, v)$	Spatial frequency vector
V	Electrostatic potential
$V(\mathbf{r}, t)$	Monochromatic optical wave
v	Velocity
\mathcal{V}	Visibility
V	Volume
v_g	Group velocity
v_p	Phase velocity
W	Work
$\mathbf{x}(= x, y)$	Two-dimensional (2-D) position vector
$*$	Complex operator
\star	Convolution operator
$\langle \rangle$	Ensemble average
$\hat{}$	Fourier transform operator

Greek Symbols

β	Orientation angle
γ	Lorentz factor
δ	Phase difference
$\delta(x)$	Dirac delta function
ν	Optical frequency
$\Delta\nu$	Spectral width (Optical)
Δf	Electrical bandwidth
Δt	Integration time
$\Delta\varphi$	Optical path difference
ϵ	Permittivity or dielectric
ϵ_0	Permittivity in vacuum
θ	Angular diameter
(θ, ϕ)	Polar coordinates
κ	Wavenumber
$\boldsymbol{\kappa}$	Wave vector
λ	Wavelength
λ_0	Wavelength in vacuum
μ	Permeability of the medium
μ_0	Permeability in vacuum
ν	Frequency
$\boldsymbol{\xi}(\xi, \eta)$	2-D position vector
ρ	Charge density
σ	Specific conductivity
τ_c	Coherence time
ϕ	Rotating angle
Φ_B	Magnetic flux
Φ_E	Electric flux
φ	Electrostatic potential
χ	Ellipticity angle
$\psi(\mathbf{r}, \nu)$	Phase function
Ψ	Time-dependent wave-function
Ω	Solid angle
ω	Angular frequency
∇	Linear vector differential operator
∇^2	Laplacian operator

Some Numerical Values of Physical and Astronomical Constants

c	Speed of light in free space	$3 \times 10^8 \text{ m.s}^{-1}$
eV	Electron volt	$1.60 \times 10^{-19} \text{ J}$
G	Gravitational constant	$6.672 \times 10^{-11} \text{ kg}^{-1}.\text{m}^3.\text{s}^{-2}$
h	Planck's constant	$6.626096 \times 10^{-34} \text{ J.s}$
q	Elementary charge	$1.6 \times 10^{-19} \text{ C}$
ϵ_0	Permittivity constant	$8.8541 \times 10^{-12} \text{ F.m}^{-1}$
μ_0	Permeability constant	$1.26 \times 10^{-6} \text{ H.m}^{-1}$

List of Acronyms

AC	Alternating current
BC	Babinet compensator
CCD	Charge coupled device
dB	Decibel
DC	Direct current
EMF	Electromotive force
EUV	Extreme ultraviolet
FT	Fourier transform
FWHM	Full width at half maximum
HF	High frequency
Hz	Hertz
IF	Intermediate frequency
IR	Infrared
keV	Kilo electron-volt
Laser	Light amplification by stimulated emission of radiation
LCP	Left-handed circular polarizer
LED	Light emitting diode
LF	Low frequency
LHS	Left-hand side
LO	Local oscillator
MHz	Megahertz
MMF	Magnetomotive force
nm	Nanometer
NUV	Near UV
OPD	Optical path difference
PSF	Point spread function
RCP	Right-handed circular polarizer
RHS	Right-hand side
RMS	Root-mean-square
UV	Ultraviolet

Preface

Waves and oscillations are found everywhere in nature. They are present at large scales (galactic), as well as at microscopic scales (neutrino). The dynamics and behavior of these oscillations depend a lot on the nature of the medium through which they propagate. For example, we have hydrodynamic waves (water waves), shock waves in a medium which is compressible. We also come across MHD (magneto hydrodynamic) waves, present in a magnetized medium. When the matter is ionized, one encounters plasma waves and so on.

The aim of this book is to present, at an introductory level, the different types of waves and oscillations that one observes and studies, from macroscopic to microscopic scales. The first chapter deals with an introduction to electromagnetism, the different types of electromagnetic spectra, wave and its characteristics such as phase velocity, group velocity and dispersion relation, applicable to all types of waves. Emphasis on application to astrophysics is introduced at this stage for setting up the theory with possible applications in astrophysics. The notion of interference, diffraction, and their importance in observational astronomy is mentioned briefly. The second chapter is a discussion on Maxwell's equations, used to study electromagnetic waves.

An introduction to waves in a uniform medium is presented in Chapter 3. Topics such as simple harmonic motion, simple pendulum, forced and free oscillations, along with resonance are developed from first principles. Damped and coupled oscillators are discussed briefly. The mathematical description of these waves is introduced and the corresponding solutions are derived briefly. The notions of normal mode eigenvalue problem and dispersion of waves are discussed in subsequent sections. A brief discussion on solitons is introduced at the end of the chapter. The concept of waves and oscillations in hydrodynamics is introduced in Chapter 4. The basic equations in rotating and nonrotating fluids are discussed. The effect of gravity, stratification, Coriolis force, and long wavelength approximation are studied for small and finite amplitude waves. Nonlinear aspects, which lead to the KdV equation (solitary waves), are discussed in brief.

Waves in a magnetized medium, for both homogeneous and non-homogeneous media, are discussed in Chapters 5 and 6. The linearized equations of MHD waves are derived from Maxwell's equations and the dispersion relation for the Alfvén, fast and slow magnetoacoustic waves, is derived from first principles. Effects of uniform flow, gravity, and density stratification are dealt with briefly. Application to solar physics and nonlinear aspects including finite amplitude effects are discussed briefly. The notion of resonant absorp-

tion and phase mixing as possible mechanisms for coronal heating are also introduced.

Different types of shock waves, such as normal and oblique shocks, are studied in Chapter 7. The concept of blast wave and the solution from Taylor and Sedov is introduced in this chapter. Weakly nonlinear aspects of shock waves are also discussed briefly. An application of shock waves in the sun is presented here. Shocks in MHD and collisionless plasmas are also dealt with briefly. Nonlinear studies, which include an introduction of Burger's equation, stationary, and single-hump solutions, are also presented. The notion of the Planar N-wave and the Backlund transformation is found at the end of the chapter. Chapter 8 deals with waves in optics. The notion of classical and modern optics is presented briefly. Nonmonochromatic fields and their properties are discussed at length. One of the important concepts of waves in optics is the notion of polarization. A discussion on Stokes parameters and their measurements is presented. Topics which deal with polarization from experimental and observational points of view are presented in detail. For example, the notions of polarizer, retarder, and rotator are discussed briefly.

Chapter 9 deals with waves and oscillations in plasmas (the fourth state of matter). It starts with the basic definition of a plasma, the different plasma parameters such as Debye shielding, plasma frequency, gyro frequency, and collision frequency, etc. A discussion on electrostatic waves in a cold as well as in a normal plasma is presented from first principles. The effect of an magnetic field, which transforms an electrostatic wave to an electromagnetic wave, is presented. Langmuir waves (warm plasma) and ion-acoustic waves which arise due to compressibility and charge effects are discussed in brief. Quasi-linear as well as nonlinear aspects of plasma waves are presented at the end of the chapter. For example, the nonlinear Schrodinger equation and the Zakharov–Shabat equation and their solutions are presented briefly. The final chapter deals with the notion of instabilities in hydrodynamics and plasmas. Some of the important instabilities, such as the Kelvin–Helmholtz instability, Taylor–Goldstein instability, and parametric instabilities, are presented in this chapter. Also plasma instabilities such as the two-stream, sausage, kink, and ballooning are presented at the end of the chapter.

We have provided some simple exercises at the end of each chapter. This will enable the student to have a good grasp of the basics involved in each of the topics covered. The book is far from complete. In fact, books and monographs have been written in the past for each of the types of the waves presented in this book. These works are more technical and exhaustive. One would agree that a work which covers several aspects of waves and oscillations is a very ambitious project and it would be impossible to do a good justification in bringing out such a work. However, this is an earnest effort by us to write a book at an introductory level on the various types of waves and oscillations that one encounters in nature. The long list of bibliography sources will enable the interested reader to get into more technical aspects. We hope that this book will be a welcome introduction to researchers working in different areas of physics and hopefully serve as a good reference book.

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