GRAPHICAL APPROACH TO CONTINUITY, INJECTIVITY AND DERIVABILITY OF FUNCTIONS

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ABSTRACT

-all we know that calculus is considered as the complicated part of mathematics due to its importance in the field of engineering, physics and all type of equations which governs the whole of the science.Even great schrodinger have to take tuitions to overcome the complications of differential equations. So limits and continuity is the real key of mathematics .We are here discussing about continuity and differentiality by making use of limits and graphs properly

I.LIMIT OF A FUNCTION:-

Suppose *f* is a real-valued function and *c* is a real number. Intuitively speaking, the expression

Lim f(x)=L

x→c

> means that *f*(*x*) can be made to be as close to *L* as desired by making *x* sufficiently close to *c*. In that case, the above equation can be read as "the limit of *f* of *x*, as *x* approaches *c*, is *L*".

Augustin-Louis Cauchy in 1821,followed by Karl Weierstrass, provided the definition of the limit of a function which is popular as the (ε, δ)-definition of limit. The definition uses ε (the lowercase Greek letter *epsilon*) to represent any small positive number, so that "*f*(*x*)becomes arbitrarily close to *L*" means that *f*(*x*) eventually lies in the interval $(L - \varepsilon, L + \varepsilon)$, which can also be written using the absolute value sign as $|f(x) - L| < \varepsilon$. The phrase "as *x*approaches *c*" then indicates that we refer to values of *x* whose distance from *c* is less than some positive number δ (the lower case Greek letter *delta*)—that is, values of *x* within either $(c - \delta, c)$ or $(c, c + \delta)$, which can be expressed with $0 < |x - c| < \delta$. The first inequality means that the distance between *x* and *c* is greater than 0 and that $x \neq c$, while the second indicates that *x* is within distance δ of *c*.[2]

The above definition of a limit is true even if $f(c) \neq L$. Indeed, the function *f* need not even be defined at *c*.

Injectivity of function:-If we draw a line parallel to x axis and cuts the graph at only one point then function is one-one unless many one

i.e $f(x_1)=f(x_2)$ implies $x_1=x_2$

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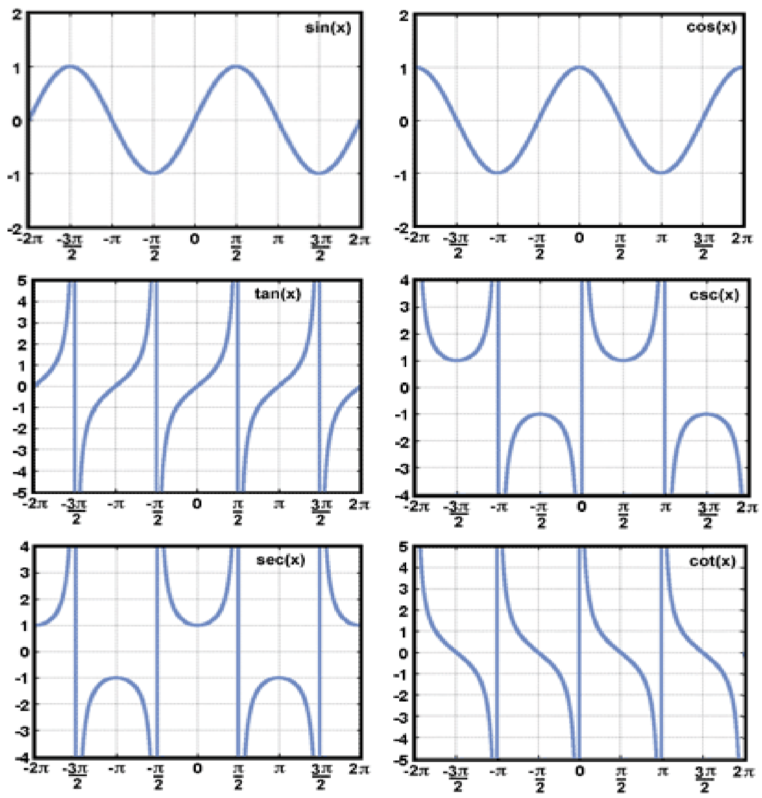
II.CONTINUITY OF FUNCTION:-

Definition: A function f is continuous at x_o in its domain if for every sequence (x_n) with x_n in the domain of f for every n and lim x_n= x_o, we have lim f(xn) = f(x_o). We say that f is continuous if it is continuous at every point in its domain. What does this say? It says that any time a sequence converges in the domain, the image of the sequence in the range also converges. In other words, we could either take the limit first, and then apply the function, or apply the function first, and then take the limits. Informally, f is continuous if limf(x_n) = f(lim x_n) This is a powerful definition due its practical use in the. In particular, we can use all the limit rules to avoid tedious calculations. However, there is a $\epsilon - \delta$ definition, similar to the definition of a limit, which goes as follows:

Definition: A function f is continuous atx_o in its domain if for every $\epsilon > 0$ there is a $\delta > 0$ such that whenever x is in the domain of f and $|x - x_o| < \delta$, we have $|f(x) - f(x_o)| < \epsilon$. Again, we say f is continuous if it is continuous at every point in its domain.

But the most reliable method is the graphical method due its direct and impressive results and gives the correct result.By drawing graph we there is no need of picking the hand then that function is continuous otherwise discontinuous here we are providing some graphs to confirm the things



Here wecan see that in case of sin and cosine function there is no need of picking the hand so these are continuous functions while tanx ceases to give values at 90 degree and hence discontinuous

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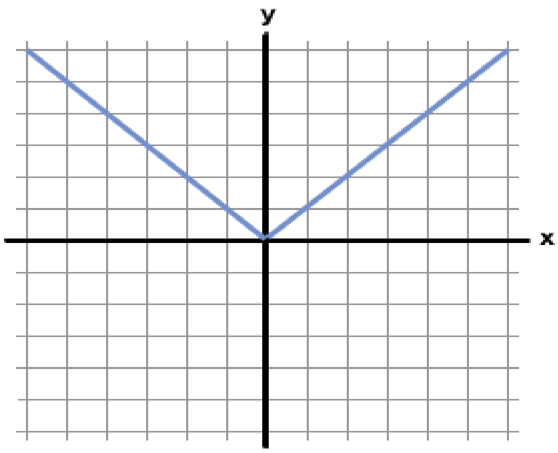
Similarly the modulus function Graphing the Absolute Value Function

The graph of the absolute value function *f* (*x*) = | *x*| is similar to the graph of *f* (*x*) = *x* except that the "negative" half of the graph is reflected over the *x*-axis. Here is the graph of *f* (*x*) = | *x*|:

F(x)= x if x≥0

-x if x<0

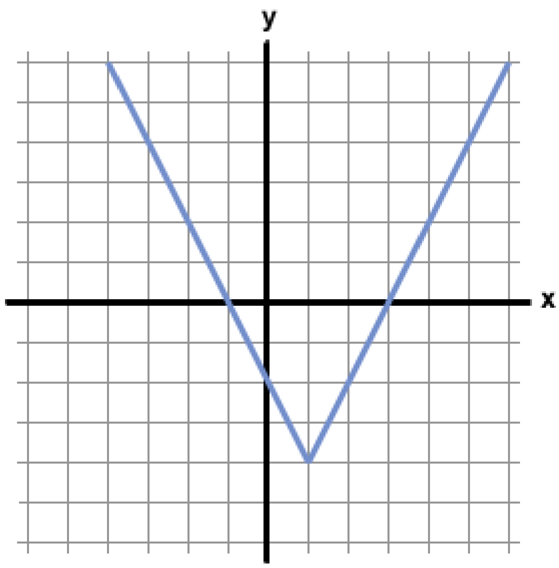


f (x) = | x|

The graph looks like a "V", with its vertex at (0, 0). Its slope is *m* = 1 on the right side of the vertex, and *m* = -1 on the left side of the vertex.

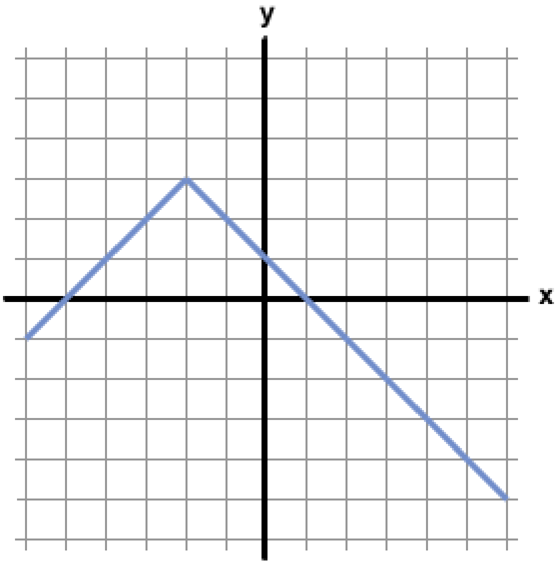
We can translate, stretch, shrink, and reflect the graph.

Here is the graph of *f* (*x*) = 2| *x* - 1| - 4:



f (x) = 2| x - 1| - 4

Here is the graph of $f(x) = -|x + 2| + 3$:



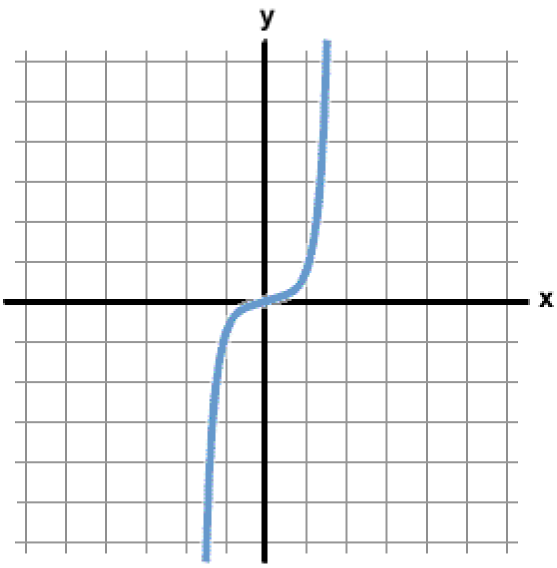
f (x) = - | x + 2| + 3

In general, the graph of the absolute value function $f(x) = a|x - h| + k$ is a "V" with vertex (h, k), slope $m = a$ on the right side of the vertex $(x > h)$ and slope $m = -a$ on the left side of the vertex $(x < h)$. The graph of $f(x) = -a|x - h| + k$ is an upside-down "V" with vertex (h, k), slope $m = -a$ for $x > h$ and slope $m = a$ for $x < h$.

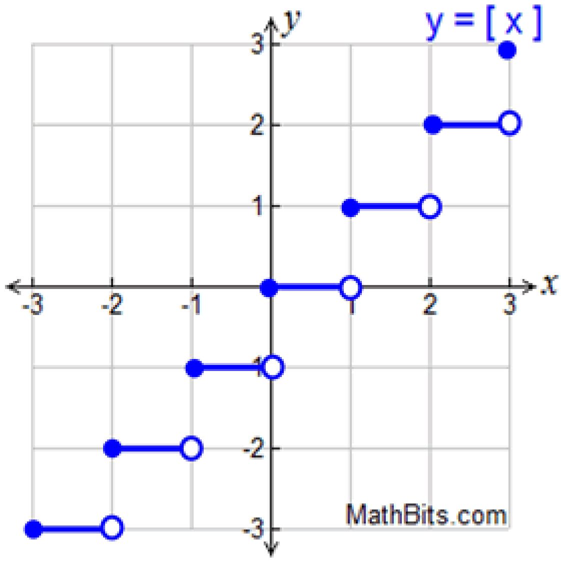
If $a > 0$, then the lowest y-value for $y = a|x - h| + k$ is $y = k$. If $a < 0$, then the greatest y-value for $y = a|x - h| + k$ is $y = k$.

Graphing the Cubic Function

Here is the graph of $f(x) = x^3$:

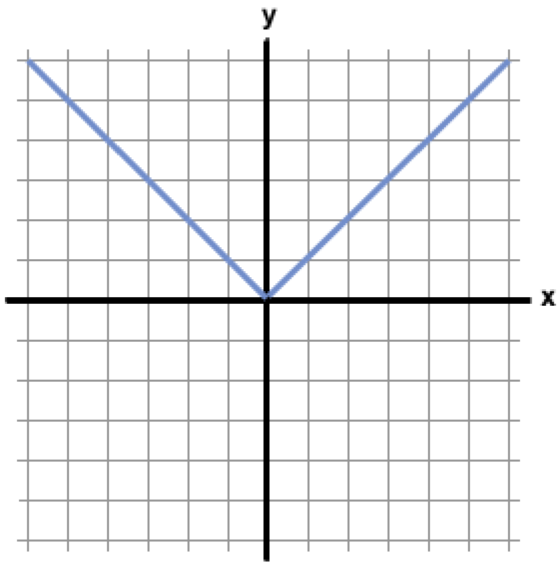


Greatest integer function :-



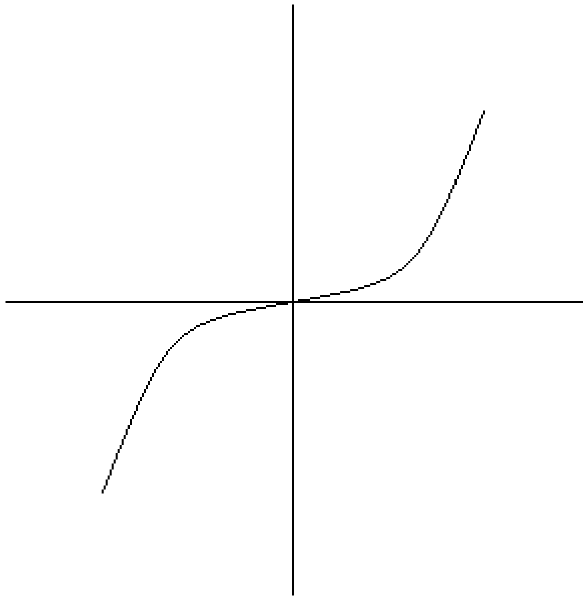
Clearly this graph shows that function is function is discontinuous at the integers and not contionuous means not derivable at the integers .

Now we add a shortcut to the differentiation if there is an sharp edge at graph at that point function is not derivable .modulus /absolute function is the best example of this shortcut.



Here function is continuous at the all the points where as due to the sharp edge at the x=0 the function is not differentiable at x=0.

But it is not necessary that modulus does this job as the graph of x|x| is not having the sharp edging property



As x|x| is the symmetrical graph and it also fail the theory that modulus functions do not shows the property of injectivity .where as is not |x| is not injective(one one function.

At the end wepay attention towards l hospitals rule which is very popular among teachers and students due its direct approach of calculations over continuous functions but it also fails where limit does not exists and forcefully still students tries it even if limit are nor equal i.e left hand limit and right hand limit

For example

Lim $\frac{\sqrt{1-cos2x}}{x}$

x→0 for the solution of such problems students apply L.hospital rule and find that

Lim $\sqrt{2}$sinx/x and applying l hospital rule this becomes

x→0

Lim $\sqrt{2}$cosx and ans is $\sqrt{2}$ which is wrong

x→0

due to fact that while using short cut methods we looses our real concepts and misses the real solution its right solution is $\frac{\sqrt{1-cos2x}}{x}$= |sinx|/x gives right hand limit=1 and left hand limit =-1 which is not contionuous and hence not derivable .

III.HYPOTHESIS AND RESULTS:

1. By making graphs we have shown there is no need of picking hand for sinx ,cosx and modulus function so these are continuous whereas tanx is discontinuous at right angle.
2. At the sharp edge points function is not derivable and hence |x| is not derivable at the x=0
3. If we draw a line parallel to x axis and cuts the graph at only one point then function is one-one unless many one so sinx ,cosx and |x| is many one function and not injective while cubic function is one one
4. L hospital methd is applicable only where function is contionuous and limit exists

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