ALFVÉN SURFACE WAVES ALONG CORONAL STREAMERS

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Abstract. The dispersive characteristics of Alfvén Surface Waves (ASW) along a moving plasma surrounded by a stationary plasma is discussed. The stability curves for the symmetric and the asymmetric modes are also discussed.

1. Introduction

The study of Alfvén Surface Waves (ASW) has become very important from an astrophysical point of view, particularly in the solar atmosphere (Parker, 1979). Experimental observations reveal the fact that the solar corona is highly structured with a closed magnetic loop over a wide range of scales (Vaiana and Rosener, 1978). Recently, the surface wave propagation pertaining to the solar surface has been studied with a few approximations (Wentzel, 1979; Uberoi and Somasundaram, 1980; Uberoi, 1981; Somasundaram and Uberoi, 1982; Somasundaram, 1983; and, in specific cases, Rae and Roberts, 1981).

All the above referred works deal with a static boundary. However, in situations such as in coronal streamers, a moving plasma column is surrounded by a stationary plasma. Parker (1963) discussed the stability of the interface when there is a discontinuity in both magnetic field and density. Geronicolas (1977) also discussed a similar problem, where the magnetic field in the stationary plasma medium is absent. His study has also been on the stability of magnetic flux tubes in the photosphere. It is known that a discontinuity either in the magnetic field or in the density at the interface introduces Alfvén surface waves. In this paper, we discuss the surface wave propagation of Alfvén waves for a moving plasma surrounded by a stationary plasma.

2. Dispersion Relation

The linearized equations governing the electromagnetic and hydrodynamic properties of an incompressible and infinitely conducting plasma column (medium 1 in Figure 1) moving with a velocity U are given by

$$\frac{\partial \bar{b}}{\partial t} + U \frac{\partial \bar{b}}{\partial z} = B_{01} \frac{\partial \bar{v}}{\partial z}, \tag{1}$$

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$$\rho_{01}\left(\frac{\partial \bar{v}}{\partial t} + U\,\frac{\partial \bar{v}}{\partial z}\right) = -\nabla\left(\bar{p} + \frac{\bar{B}_{01} - \bar{b}}{4\pi}\right) + \frac{B_{01}}{4\pi}\,\frac{\partial \bar{b}}{\partial z}\,, \tag{2}$$

$$\nabla \cdot \bar{b} = 0\,, \tag{3}$$

where $\bar{v}$, $\bar{p}$, and $\bar{b}$ are the perturbed fluid velocity, pressure, and magnetic field, respectively, while ρ_{01}, B_{01}, and U are the density, magnetic field, and velocity of the basic fluid.

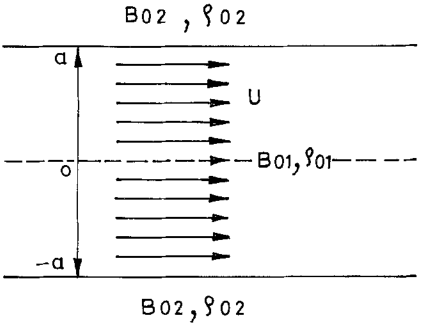

Fig. 1. The geometry.

Taking the divergence of Equation (2) and manipulating with the other equations, we have

$$\nabla^2 \tilde{p} = 0\,, \tag{4}$$

where

$$\tilde{p} = \bar{p} + \frac{\bar{B}_{01} \cdot \bar{b}}{4\pi}\,.$$

If we assume perturbations of the form $f(x, z, t) = f(x) \exp\{i(kz - \omega t)\}$, the solution of Equation (4) can be written as

$$\tilde{p}_1 = A \sinh(kx)\,, \tag{5}$$

where A is an arbitrary constant.

The solution for the pressure field for the stationary plasma (medium 2) surrounding the moving plasma can be shown to be

$$\tilde{p}_2 = B\, e^{-kx}\,, \tag{6}$$

where B is an arbitrary constant. All the other field quantities can be easily calculated from the basic equations.

Applying the boundary conditions that the total pressure and normal component of velocity are continuous, we obtain a dispersion relation which on simplification yields the nondimensional phase velocity as

$$\frac{\omega}{kV_A} = \frac{V \pm \{[1 + \eta \tanh(ka)]\,[1 + \beta^2 \tanh(ka)] - \eta V^2 \tanh(ka)\}^{1/2}}{1 + \eta \tanh(ka)}$$

(symmetric mode), (7a)

$$\frac{\omega}{kV_A} = \frac{V \pm \{[1 + \eta \coth(ka)]\,[1 + \beta^2 \coth(ka)] - \eta V^2 \coth(ka)\}^{1/2}}{1 + \eta \coth(ka)}$$

(asymmetric mode), (7b)

where $\beta = B_{02}/B_{01}$ and $\eta = \rho_{02}/\rho_{01}$ are the interface parameters, $V = U/V_A$ is a nondimensional velocity, V_A is the Alfvén velocity in medium 1 and a is half the width of the moving plasma column.

3. Discussion of the Results

We first discuss the special cases when the moving plasma column is at rest. The dispersion relation for this case, i.e., for $V = 0$, has already been discussed in detail by Uberoi (1981) and Edwin and Roberts (1982).

In the limit $ka \to 0$, Equations (7a, b) with $V = 0$ become

$$\frac{\omega}{kV_A} = 1 \quad \text{for a symmetric mode},$$

$$\frac{\omega}{kV_A} = \sqrt{\beta^2/\eta} \quad \text{for an asymmetric mode}. \tag{8}$$

In this case, the phase velocity of the symmetric mode is independent of the interface parameters β and η, which is not the case for the asymmetric mode, as is seen from Equation (8).

In the limit $ka \to \infty$, both $\tanh(ka)$ and $\coth(ka) \to 1$; so that Equations (7a, b) become

$$\frac{\omega}{kV_A} = \frac{V \pm \{(1 + \eta)(1 + \beta^2) - \eta V^2\}^{1/2}}{(1 + \eta)}. \tag{9}$$

The phase velocity of both the modes coincide unlike in the case $ka \to 0$.

For $V = 0$, Equation (9) becomes

$$\frac{\omega}{kV_A} = \left\{\frac{(1 + \beta^2)}{(1 + \eta)}\right\}^{1/2}, \tag{10}$$

which has been discussed in detail by Hasegawa and Uberoi (1982). For $\beta = 0$ and

$\eta = 1$, Equations (7a, b) become

$$\frac{\omega}{kV_A} = \frac{V \pm \{1 + \tanh(ka) - V^2 \tanh(ka)\}^{1/2}}{1 + \tanh(ka)} \quad \text{(symmetric)},$$
$$= \frac{V \pm \{1 + \coth(ka) - V^2 \coth(ka)\}^{1/2}}{1 + \coth(ka)} \quad \text{(asymmetric)}. \tag{11}$$

These reduce to the values obtained by Parker (1974) for $V = 0$. Roberts (1981) discussed the case when $V = 0$ and $\beta = 0$ so that Equations (7a, b) become

$$\frac{\omega}{kV_A} = \pm \left\{ 1 \Big/ \left[1 + \eta \begin{pmatrix} \tanh \\ \coth \end{pmatrix} ka \right] \right\}^{1/2}, \tag{12}$$

for the symmetric and asymmetric modes, respectively. For a plasma slab in vacuum, $\eta = 0$, so that

$$\frac{\omega}{kV_A} = V \pm \left\{ [1 + \beta^2 \begin{pmatrix} \tanh \\ \coth \end{pmatrix} ka \right] \right\}^{1/2}. \tag{13}$$

Surface Waves

Each of the Equations (7a) and (7b) yields two modes, of which we consider only the modes with positive phase velocity. We call the modes obtained with $(+)$*ve* sign in Equations (7a) and (7b) as the upper branch and those obtained with $(-)$*ve* sign as the lower branch. Figures 2 and 3 give the dispersion curves obtained from Equations (7a) and (7b) for the symmetric (broken lines) and the asymmetric (solid lines) modes for different values of the interface parameter β with $\eta = 0.5$ and $V = 0.2$ and 1.5, respectively.

In Figure 2, only the upper branch exists for both symmetric and asymmetric modes. For values $\beta^2/\eta < 1$, the dispersion curves for asymmetric modes have positive slope and, hence, they have normal dispersion while the symmetric modes have anomalous dispersion ($-$ *ve* slope). This feature for the two modes are seen to be interchanged when $\beta^2/\eta > 1$. This is similar to the modes discussed earlier by Uberoi (1981), Edwin and Roberts (1982), and Somasundaram (1983), when the flow velocity $V = 0$. For values $ka > 2$, it is seen that both the modes tend to the same constant phase velocity. Hence, only those modes having long wavelength such that $ka < 2$ are seen to be affected more.

Figure 3 gives the dispersion curves when the flow velocity is greater than the Alfvén wave velocity. Here, we get all the four branches. The upper branch or the asymmetric mode has a normal dispersion while the symmetric mode has an anomalous dispersion for $\beta^2/\eta < 1$ and to a certain extent for $\beta^2/\eta > 1$. Thus, the increase in the flow velocity changes the dispersive nature of the ASW. However, an increase in the magnetic field in the stationary regions increases the phase velocity of the both modes. The lower branch of the symmetric mode which was totally absent when $V = 0.2 (U < V_A)$ appears

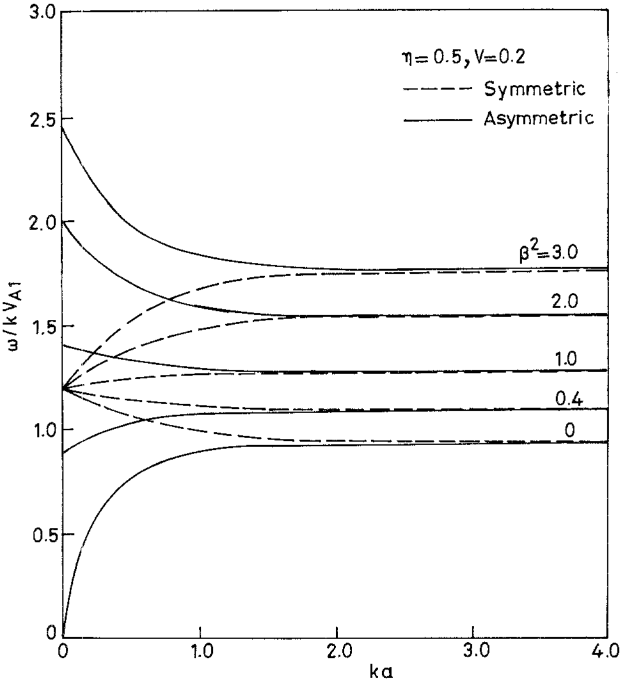

Fig. 2. Dispersion curves of ASW for $\eta = 0.5$, $V = 0.2$.

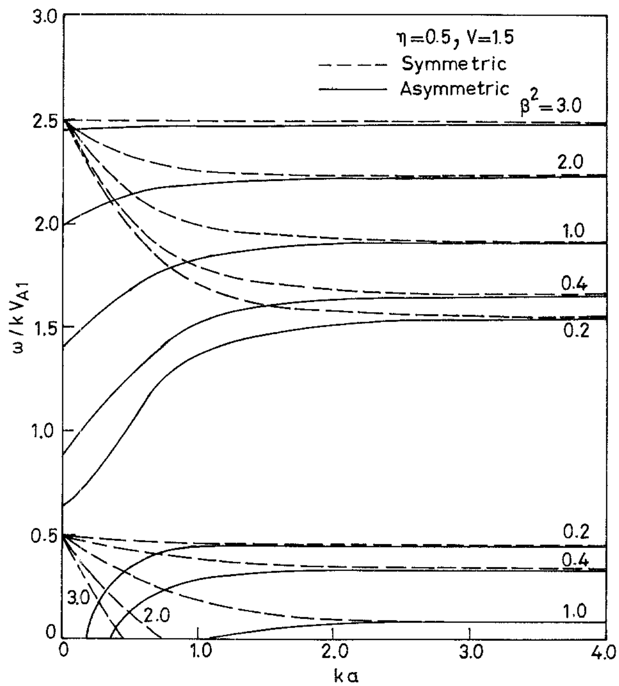

Fig. 3. Dispersion curves of ASW for $\eta = 0.5$, $V = 1.5$.

when $V = 1.5\,(U > V_A)$. The lower branches of the symmetric and asymmetric modes suffer upper cut-off and lower cut-off, respectively, in their wavenumber k, as the magnetic field ratio β^2 increases. Thus, the presence of the lower branches of the symmetric modes not only depend on the flow velocity of the coronal streamer, but also on the environment of the surrounding plasma.

In Figure 4, the dispersion curves for $\beta^2 = 0.0$, $V = 0.2$, and for various values of the density, ratio η are given. In comparison with Figure 2, it should be noted that an increase in η decreases the phase velocity of both the modes (upper branches) while the magnetic field ratio enhances the phase velocities.

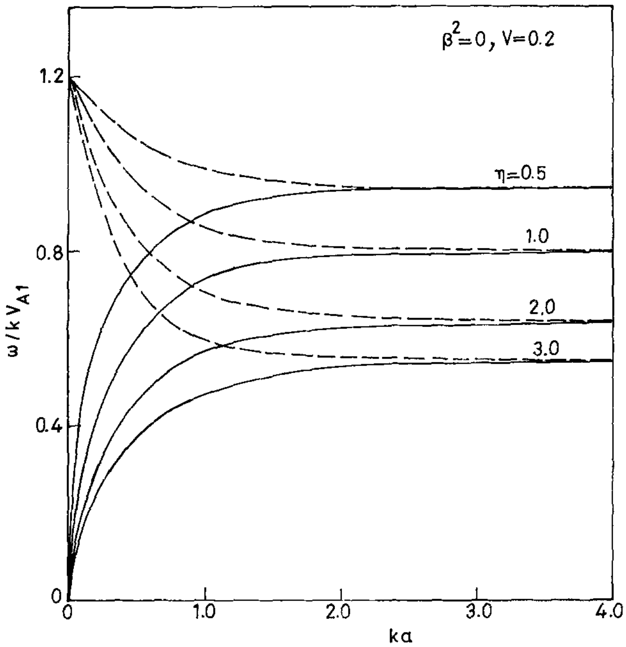

Fig. 4. Dispersion curves of ASW for $\beta^2 = 0.0$, $V = 0.2$.

Stability

Figure 5 presents the stability curves for the symmetric and asymmetric modes for $\eta = 0.5$ and various other values of β^2. The symmetric and asymmetric modes are similar to the sausage and serpentine modes, respectively, discussed by Parker (1963). The dispersion relation derived in the previous section can be shown to be similar to the one derived by Parker (1963). The important point which Parker has missed in his discussion is the combined effect of the interface parameters η and β^2. An interesting feature, as can be seen in Figure 5, is that for $\eta\beta^2 = 1.0$, the stability curves of both the modes coincide. In other words, if the square of the magnitude of discontinuity in the magnetic field is equal to the reciprocal of the magnitude of discontinuity in the density, then both the symmetric and asymmetric modes coincides. The stability curve for the

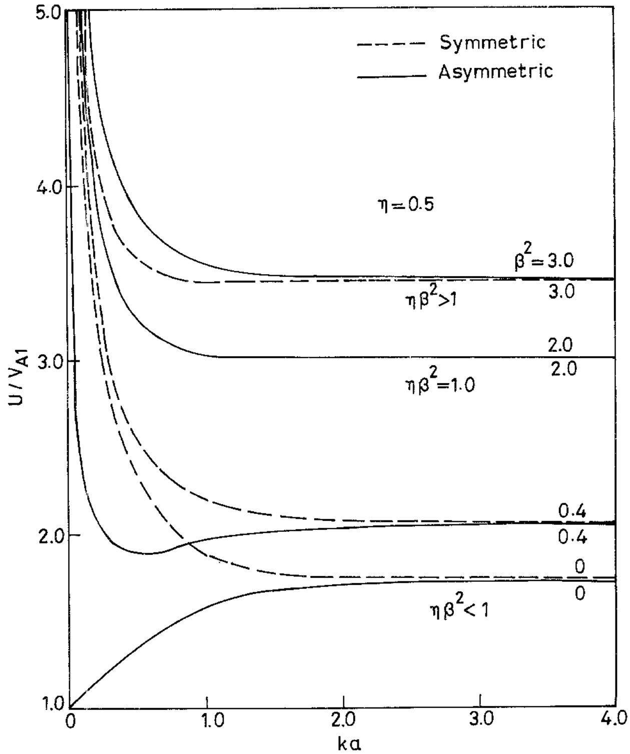

Fig. 5. Stability curves for $\eta = 0.5$.

asymmetric mode when $\beta^2 = 0.4$, i.e., $\eta\beta^2 < 1$ has a kink which is not present for the other curves. This means that the dispersion relation has a minimum at $ka = \tanh^{-1}(\sqrt{\beta^2\eta})$. Finally, the stability results obtained by Geronicolas (1977) follow when $\beta^2 = 0.0$ in our present study.

4. Conclusions

The dispersive characteristics of ASW for a moving plasma surrounded by a stationary plasma reveal very interesting results for the symmetric and asymmetric modes. In particular, the interface parameters η and β^2 significantly affect the phase velocities of these modes. The stability properties of these waves are very much dependent on the combined effect of the interface parameters. A similar study with compressibility effects taken into consideration is in progress and will be reported shortly.

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