Randomly stopped sum of distributions with dominatingly varying tails

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Heavy tailed random variables are useful in the insurance stochastic models. Usually such random variables describe a series of claim amounts. Various subclasses of heavy tailed random variables are considered. The best known subclasses are $\mathcal{L}$, $\mathcal{D}$ and $\mathcal{S}$. It should be recalled that:

• distribution function (d.f.) $F = 1 - \overline{F}$ is said to be heavy-tailed ($F \in \mathcal{H}$) if $\lim_{x\to\infty} \overline{F}\mathrm{e}^{\delta x} = \infty$ for an arbitrary positive δ;

• d.f. F is said to be long-tailed ($F \in \mathcal{L}$) if $\overline{F}(x+y) \sim \overline{F}(x)$ for every positive y;

• d.f. F has dominatingly varying tail ($F \in \mathcal{D}$) if $\limsup_{x\to\infty}(\overline{F}(xy)/\overline{F}(x)) < \infty$ for some $y \in (0,1)$;

• d.f. F is subexponential ($F \in \mathcal{S}$) if $\overline{F_+ * F_+}(x) \sim 2\overline{F}(x)$, where F_+ denotes the positive part of d.f. F.

It is known (see, for instance, [3]) that $\mathcal{L} \cap \mathcal{D} \subset \mathcal{S} \subset \mathcal{L} \subset \mathcal{H}$ and $\mathcal{D} \subset \mathcal{H}$.

Various properties of classes $\mathcal{L}$, $\mathcal{D}$ and $\mathcal{S}$ have been considered by many authors. For instance, in [1] the problem of max-sum equivalence and the problem of convolution closure were considered, while in [4] the problem of random convolution closure was investigated.

Particulary, in [4] conditions were obtained under which d.f. of random sum of independent and identically distributed random variables $\xi_1 + \xi_2 + \ldots + \xi_\eta$ belongs to the class $\mathcal{D}$. One can show that the similar results can be obtained in the case when random variables $\xi_1, \xi_2, \ldots$ are independent, but not necessary identically distributed. The exact formulations of the results together with its detailed proofs can be found in [2].

References

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