Stokes drift and large scale ocean circulation

CS Jones

On surface drift currents in the ocean

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The Wave-Driven Ocean Circulation

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(Manuscript received 3 June 1998, in final form 4 November 1998)

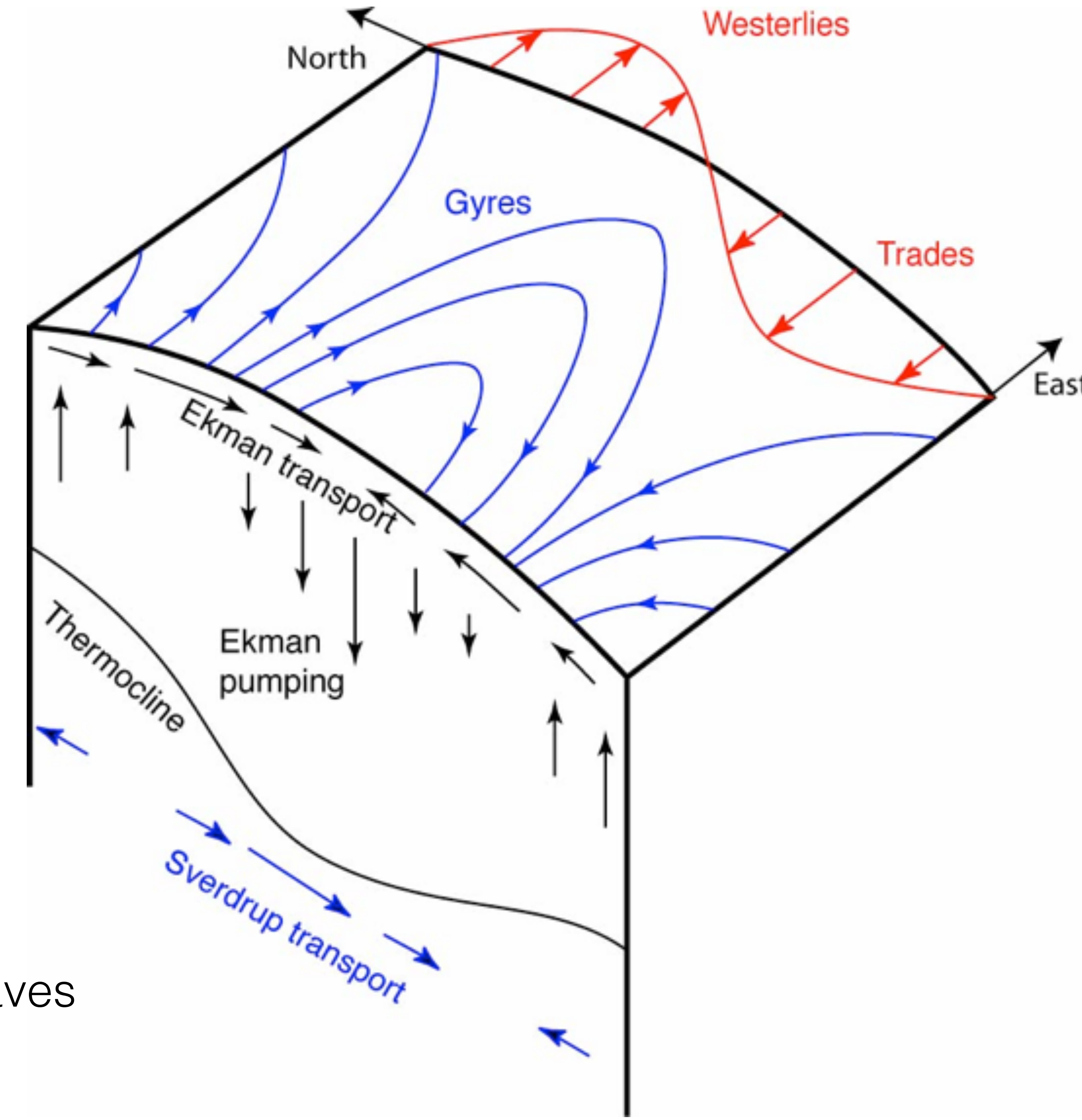
ABSTRACT

Traditional models of large-scale ocean circulation do not include surface waves

Ocean circulation is usually considered to be driven directly by the wind stress at the surface, without thinking about the role of the Stokes drift due to surface waves.

It is impossible to directly incorporate surface waves into numerical models of the ocean, because they require high spatial and temporal resolution.

$$\left.\begin{array}{l}\Delta t \sim 10\mathrm{s} \\ \Delta x \sim 100\mathrm{m}\end{array}\right\} \quad \text{for long waves}$$



Talley, Pickard, Emery and Swift (2011)

Huang (1979)

$$\frac{\partial \boldsymbol{\varpi}}{\partial t} + \nabla \times (\boldsymbol{\varpi} \times \mathbf{q}) + \frac{2\Omega}{\sigma_0} \nabla \times (\mathbf{e} \times \mathbf{q}) = \frac{\nu_e k_0^2}{\sigma_0} \nabla^2 \boldsymbol{\varpi},$$

Coriolis term

Follow Craik-Leibovich

$$-\frac{\nu_e k_0^2}{\sigma_0 \epsilon^2} \nabla^2 \overline{\boldsymbol{\varpi}}_0 = (\overline{\boldsymbol{\varpi}}_0 \,.\, \nabla)(\bar{\mathbf{v}}_0 + \mathscr{U}_s) - (\bar{\mathbf{v}}_0 + \mathscr{U}_s) \,.\, \nabla \overline{\boldsymbol{\varpi}}_0 + \frac{2\Omega}{\sigma_0 \epsilon^2} (\mathbf{e} \,.\, \nabla)(\bar{\mathbf{v}}_0 + \mathscr{U}_s).$$

McWilliams and Restrepo (1999)

$$\int_{-D}^{0} \mathbf{v}\,\mathrm{d}z + \int_{-D}^{0} \mathbf{u^s}\,\mathrm{d}z = \hat{\mathbf{z}} \times \nabla_h \Psi$$

$$\beta\Psi = \int_{X_{east}}^{x} \nabla \times \frac{\tau}{\rho}\,\mathrm{d}x'$$

If $\tau = 0$, then we get the Ursell case

$$\int \mathbf{v}\,\mathrm{d}z = \int -\mathbf{u^s}\,\mathrm{d}z$$

If $\mathbf{u^s} = 0$, then we get Sverdrup balance

$$\int_{-D}^{0} \mathbf{v}\,\mathrm{d}z = \hat{\mathbf{z}} \times \nabla_h \Psi$$

Boussinesq equations, multiple timescales

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}\cdot\boldsymbol{\nabla})\mathbf{q} + \boldsymbol{\Omega}\times\mathbf{q} - \tilde{b}\hat{\mathbf{z}} + \frac{1}{\rho_0}\boldsymbol{\nabla}\tilde{p} = \nu\nabla^2\mathbf{q},$$

$$\boldsymbol{\nabla}\cdot\mathbf{q} = 0,$$

$$\mathbf{q} = \epsilon[\underbrace{\mathbf{u}^w(\mathbf{x}, t)}_{\text{Irrotational}} + \underbrace{\delta\mathbf{v}(\mathbf{x}, t_s, t)}_{\text{Rotational}}],$$

$$2\boldsymbol{\Omega} = [0, f^{(y)}(y), f^{(z)}(y)].$$

$$t_s = \gamma t$$

$$\gamma = \epsilon\delta$$

Set

$$\Omega_0,\ \nu_0,\ N_0 = O(\gamma);$$

$$B_0,\ \tau_0,\ \mathcal{T}_0 = O(\gamma^2).$$

Broken promise: what is δ?

$$\frac{\partial\boldsymbol{\omega}}{\partial t} + \gamma\frac{\partial\boldsymbol{\omega}}{\partial t_s} = \frac{\Omega_0}{\epsilon\delta}\boldsymbol{\nabla}\times[\epsilon(\mathbf{u}^w + \delta\mathbf{v})\times 2\boldsymbol{\Omega}] + \nu_0\nu\nabla^2\boldsymbol{\omega} + \boldsymbol{\nabla}\times[\epsilon(\mathbf{u}^w + \delta\mathbf{v})\times\boldsymbol{\omega}] + \left(\frac{B_0}{\epsilon\delta}\right)\boldsymbol{\nabla}\times\hat{\mathbf{z}}\left[b + \left(\frac{B_0^w}{B_0}\right)b^w\right],$$

(same as CL and Huang)

Boussinesq equations, multiple timescales

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \gamma \frac{\partial \boldsymbol{\omega}}{\partial t_s} = \frac{\Omega_0}{\epsilon\delta}\boldsymbol{\nabla} \times [\epsilon(\mathbf{u}^w + \delta\mathbf{v}) \times 2\boldsymbol{\Omega}] + \nu_0\nu\nabla^2\boldsymbol{\omega} + \boldsymbol{\nabla} \times [\epsilon(\mathbf{u}^w + \delta\mathbf{v}) \times \boldsymbol{\omega}] + \left(\frac{B_0}{\epsilon\delta}\right)\boldsymbol{\nabla} \times \hat{\mathbf{z}}\left[b + \left(\frac{B_0^w}{B_0}\right)b^w\right],$$

$$\mathbf{v} = \mathbf{v}_0 + \epsilon\mathbf{v}_1 + \epsilon^2\mathbf{v}_2 + \cdots,$$

$$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \epsilon\boldsymbol{\omega}_1 + \epsilon^2\boldsymbol{\omega}_2 + \cdots,$$

$O(1)$

$$\boldsymbol{\omega}_0 = \boldsymbol{\omega}_0(\mathbf{x}, t_s).$$

$O(\epsilon)$

$$\boldsymbol{\omega}_1 = \boldsymbol{\nabla} \times (\mathbf{U} \times \boldsymbol{\omega}_0) + \boldsymbol{\nabla} \times (\mathbf{U} \times 2\boldsymbol{\Omega}), \quad \text{where} \quad \mathbf{U} \equiv \int^t \mathbf{u}^w(\mathbf{x}_h, s)\, ds.$$

Therefore, time averaging gives

$$\gamma\frac{\partial \boldsymbol{\omega}_0}{\partial t_s} = \epsilon^2\boldsymbol{\nabla} \times \langle \mathbf{u}^w \times \boldsymbol{\omega}_1 \rangle + \gamma\boldsymbol{\nabla} \times (\mathbf{v}_0 \times \boldsymbol{\omega}_0) + \gamma\nu\nabla^2\boldsymbol{\omega}_0$$

$$+ \gamma\boldsymbol{\nabla} \times (\mathbf{v}_0 \times 2\boldsymbol{\Omega}) + \gamma\boldsymbol{\nabla} \times (b_0\hat{\mathbf{z}}),$$

Some more manipulation gives CLH

$$\frac{\partial \boldsymbol{\omega}_0}{\partial t_s} - \boldsymbol{\nabla} \times [\mathbf{V} \times \boldsymbol{Z}] - \boldsymbol{\nabla} \times b_0\hat{\mathbf{z}} = \nu\nabla^2\boldsymbol{\omega}_0, \quad \text{where}$$

$$\mathbf{z} = 2\boldsymbol{\Omega} + \omega_0$$

$$\mathbf{V} = \mathbf{v}_0 + \xi\mathbf{u}^s,$$

Boundary conditions

$$w = \frac{D\eta}{Dt}, \quad \text{at } z = \eta$$

Expanding about $z = 0$

$$w + \eta w_z = \eta_t + u\eta_x + v\eta_y + O(\epsilon^2)$$

at $z = 0$

$$w = \eta_t + (u\eta)_x + (v\eta)_y + O(\epsilon^2)$$

at $z = 0$

$$\mathbf{M} \equiv \langle \mathbf{u}^w(\mathbf{x}_h, 0, t)\eta^w(\mathbf{x}_h, t)\rangle.$$

$$< w >= \gamma \cancel{<\eta} >_{ts} + \xi\nabla\cdot M$$

$$w_0 = \xi\boldsymbol{\nabla}\cdot\mathbf{M} \quad \text{at } z = 0,$$

$$\tilde{p} = g\rho_0\eta + \tilde{p}_a, \quad \text{at } z = \eta$$

Expanding about $z = 0$

$$p + \eta p_z = g\rho_0\eta + p_a + O(\epsilon^2)$$

$$p_0 = \eta_0 + p_{a0} - \xi P \quad \text{at } z = 0,$$

$$P \equiv \langle p_z^w(\mathbf{x}_h, 0, t)\eta^w(\mathbf{x}_h, t)\rangle = \langle(\eta_t^w)^2\rangle = \langle(w^w)^2\rangle,$$

$$\xi = \frac{\epsilon^2}{\gamma}$$

Boundary conditions

$$\nu \frac{\partial \mathbf{q}}{\partial z} = \frac{1}{\rho_0} \boldsymbol{\tau},$$

Is this the correct boundary condition anyway? Most momentum is transferred by a correlation between p_a and η

Expanding about $z = 0$

$$\nu \left(\gamma \frac{\partial \mathbf{v_0}}{\partial z} + \epsilon \frac{\partial \mathbf{u^w}}{\partial z} + \epsilon \frac{\partial^2 \mathbf{u^w}}{\partial z^2} \eta \right) + O(\epsilon^2) = \tau$$

Boundary conditions

$$\nu \frac{\partial \mathbf{q}}{\partial z} = \frac{1}{\rho_0} \boldsymbol{\tau},$$

Is this the correct boundary condition anyway? Most momentum is transferred by a correlation between p_a and η

Expanding about $z = 0$ and averaging over a wave period

$$\nu \left(\gamma \frac{\partial \mathbf{v_0}}{\partial z} + \epsilon \cancel{\frac{\partial \mathbf{u^w}}{\partial z}} + \epsilon \frac{\partial^2 \mathbf{u^w}}{\partial z^2} \eta \right) + O(\epsilon^2) = \tau$$

$$\nu \left(\frac{\partial \mathbf{v}_0}{\partial z} + \xi \mathbf{S} \right) = \tau \quad \text{at } z = 0, \qquad \mathbf{S} \equiv \left\langle \frac{\partial^2 \mathbf{u}^w(\mathbf{x}_h, 0, t)}{\partial z^2} \eta^w(\mathbf{x}_h, t) \right\rangle$$

New equations, including surface wave effects

Momentum $\frac{\partial \mathbf{v}_0}{\partial t_s} - \mathbf{V} \times \mathbf{Z} + \boldsymbol{\nabla}\Phi - b_0\hat{\mathbf{z}} = \nu\nabla^2\mathbf{v}_0,$

Vorticity $\frac{\partial \boldsymbol{\omega}_0}{\partial t_s} - \boldsymbol{\nabla} \times [\mathbf{V} \times \mathbf{Z}] - \boldsymbol{\nabla} \times b_0\hat{\mathbf{z}} = \nu\nabla^2\boldsymbol{\omega}_0,$

Pressure $\nabla^2\Phi = \boldsymbol{\nabla} \cdot (\mathbf{V} \times \mathbf{Z} + b_0\hat{\mathbf{z}} + \nu\nabla^2\mathbf{v}_0).$

Tracers $\frac{\partial \theta_0}{\partial t_s} + \mathbf{V} \cdot \boldsymbol{\nabla}\theta_0 = \kappa\nabla^2\theta_0.$

$$\mathbf{q} \rightarrow \mathbf{V}$$

$$\mathbf{V} = \mathbf{v}_0 + \xi\mathbf{u}^s$$

Surface velocity $w_0 = \xi\boldsymbol{\nabla} \cdot \mathbf{M}$ at $z = 0,$

Surface pressure $p_0 = \eta_0 + p_{a0} - \xi P$ at $z = 0,$

Surface stress $\nu\left(\frac{\partial \mathbf{v}_0}{\partial z} + \xi\mathbf{S}\right) = \boldsymbol{\tau}$ at $z = 0,$

Surface tracer flux $\kappa\frac{\partial \theta_0}{\partial z} = \mathcal{T}$ at $z = 0.$

$$\mathbf{M} \equiv \langle \mathbf{u}^w(\mathbf{x}_h, 0, t)\eta^w(\mathbf{x}_h, t)\rangle.$$

$$P \equiv \langle p_z^w(\mathbf{x}_h, 0, t)\eta^w(\mathbf{x}_h, t)\rangle = \langle(\eta_t^w)^2\rangle = \langle(w^w)^2\rangle,$$

Modified boundary conditions

$$\mathbf{S} \equiv \left\langle \frac{\partial^2\mathbf{u}^w(\mathbf{x}_h, 0, t)}{\partial z^2}\eta^w(\mathbf{x}_h, t)\right\rangle$$

Entering section 3: more assumptions

Hydrostatic
Ignore advective terms in the momentum equation
Neglect horizontal component of the Coriolis vector

$$\int_{-D}^{0} \mathbf{v}\,\mathrm{d}z + \int_{-D}^{0} \mathbf{u^s}\,\mathrm{d}z = \hat{\mathbf{z}} \times \nabla_h \Psi$$

$$\beta\Psi = \int_{X_{east}}^{x} \nabla \times \frac{\tau}{\rho}\,\mathrm{d}x'$$

If $\tau = 0$, then we get the Ursell case

$$\int \mathbf{v}\,\mathrm{d}z = \int -\mathbf{u^s}\,\mathrm{d}z$$

If $\mathbf{u^s} = 0$, then we get Sverdrup balance

$$\int_{-D}^{0} \mathbf{v}\,\mathrm{d}z = \hat{\mathbf{z}} \times \nabla_h \Psi$$

Wind strength

(b)

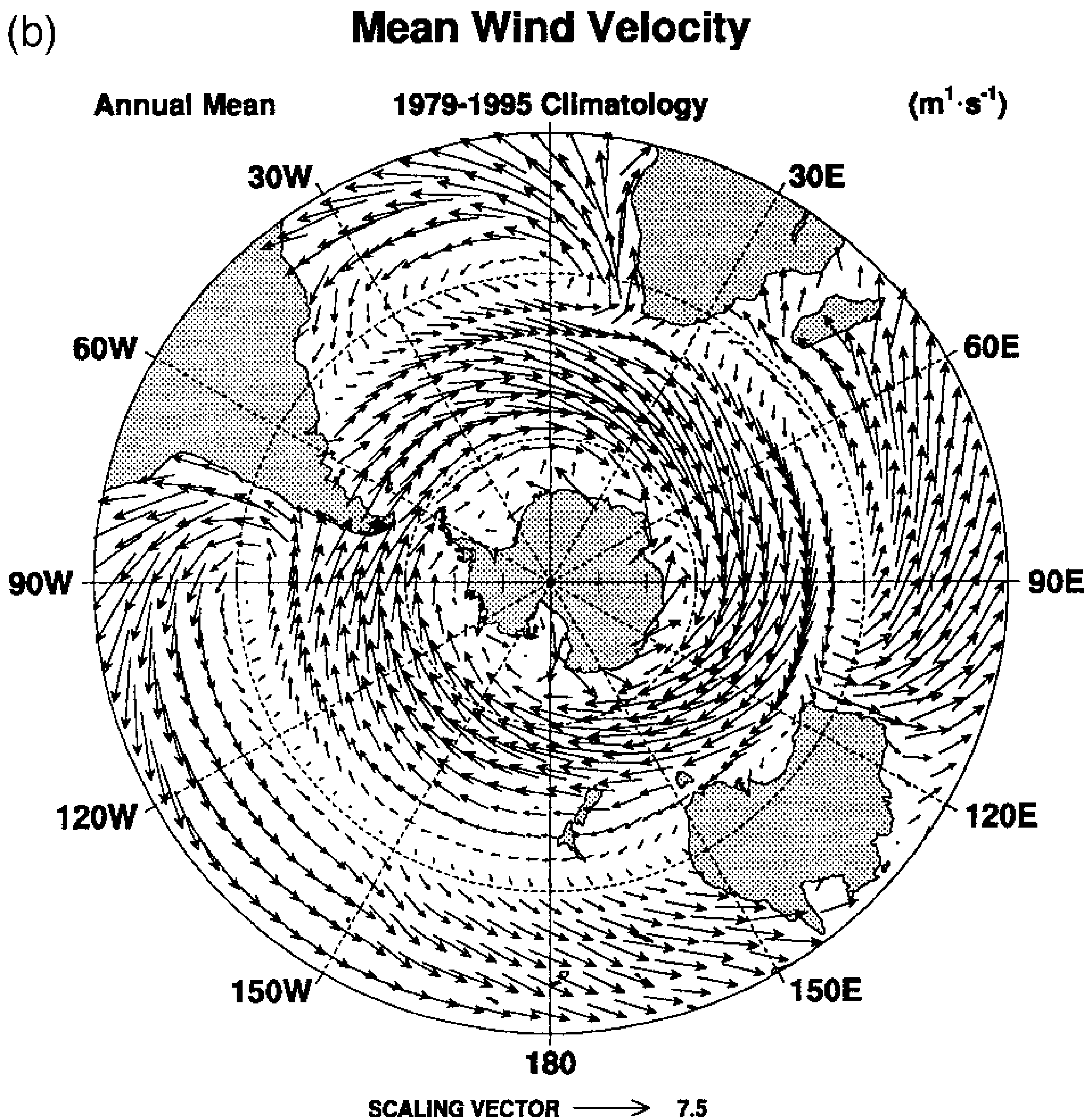


FIG. 1. Annual-mean wind, $\mathbf{U}_a$, at a height of 10 m above the sea surface: (a) Northern Hemisphere; (b) Southern Hemisphere.

Wave strength

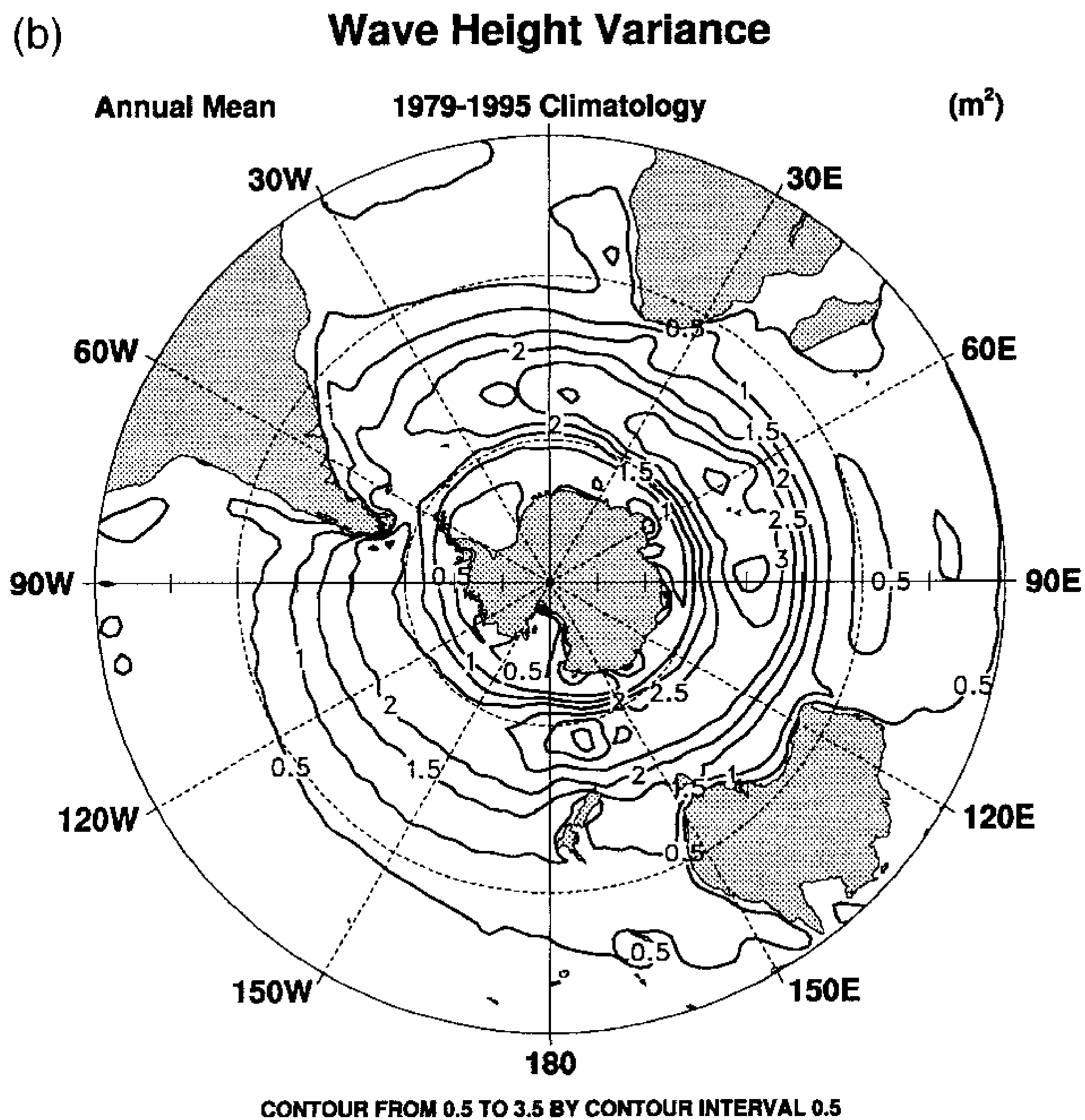


FIG. 2. Annual-mean wave height variance, $\langle(\eta^w)^2\rangle$, from (66): (a) Northern Hemisphere; (b) Southern Hemisphere.

Waves are strongest far from land, and where the winds are strong and westerly

How big is the correction due to the surface pressure?

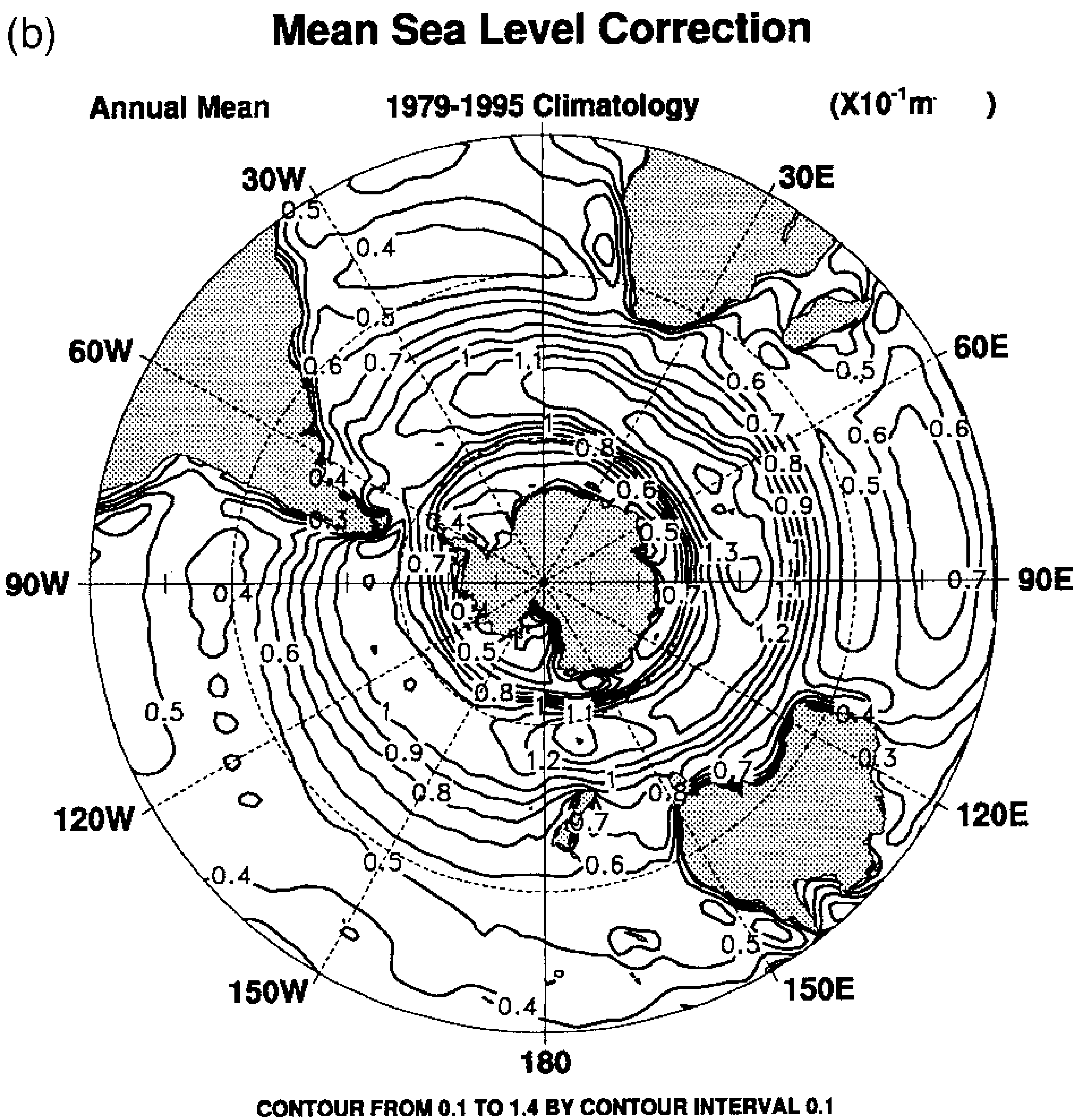


FIG. 3. Annual-mean increment to the surface-pressure boundary condition, that is, $g^{-1}P$ from (66): (a) Northern Hemisphere; (b) Southern Hemisphere.

$$p_0 = \eta_0 + p_{a0} - \xi P \quad \text{at } z = 0,$$

Strongest near antarctica

$$P = \langle (\eta_t^w)^2 \rangle$$

How big is the correction due to Stokes drift?

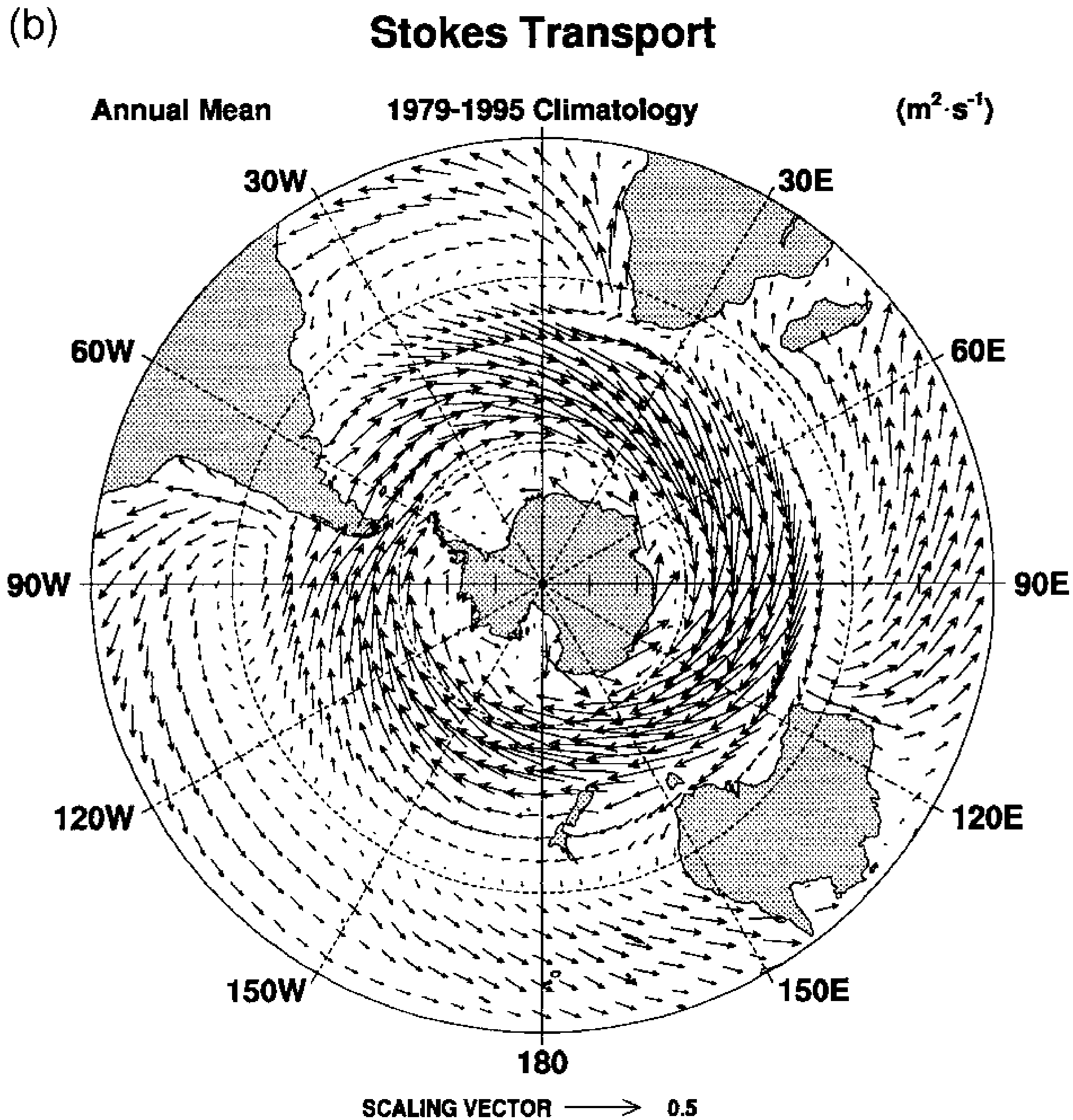


FIG. 4. Annual-mean Stokes transport, $\mathbf{T}_{st}$ from (67): (a) Northern Hemisphere; (b) Southern Hemisphere.

$$\mathbf{T}_{st} = \int_{-D}^{0} \mathbf{u}^s \,\mathrm{d}z$$

Strongest in westerly wind regimes

Mainly in the direction of the wind

How big is the Ekman transport?

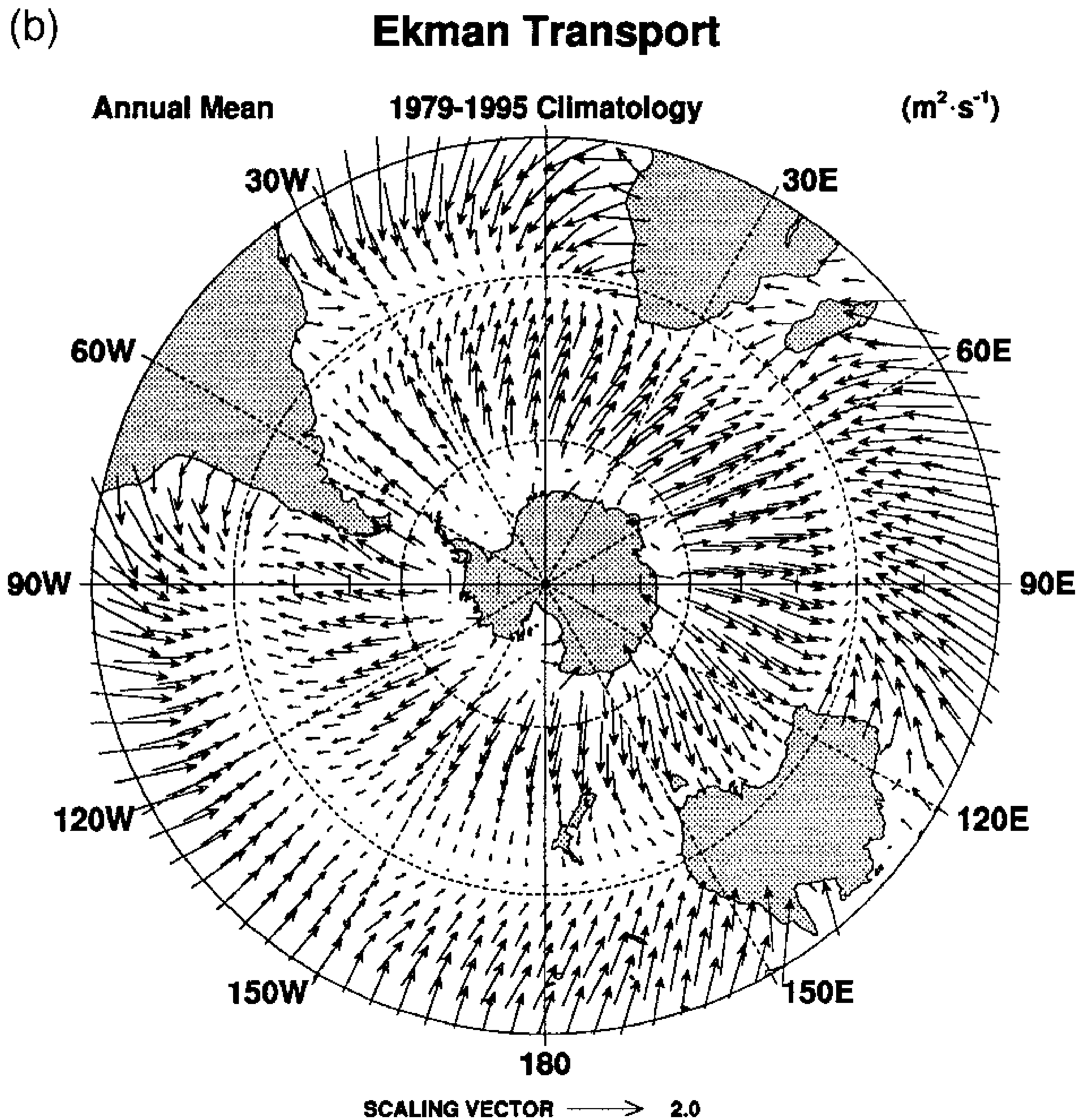


FIG. 5. Annual-mean Lagrangian Ekman transport $-\hat{\mathbf{z}} \times (1/f\rho_0)\boldsymbol{\tau}^t$ from (68): (a) Northern Hemisphere; (b) Southern Hemisphere.

Mainly perpendicular to the wind

About 4x the size of the Stokes transport

How big is the correction due to Stokes drift?

(b) **Stokes/Ekman Transport Ratio (Magnitude)**

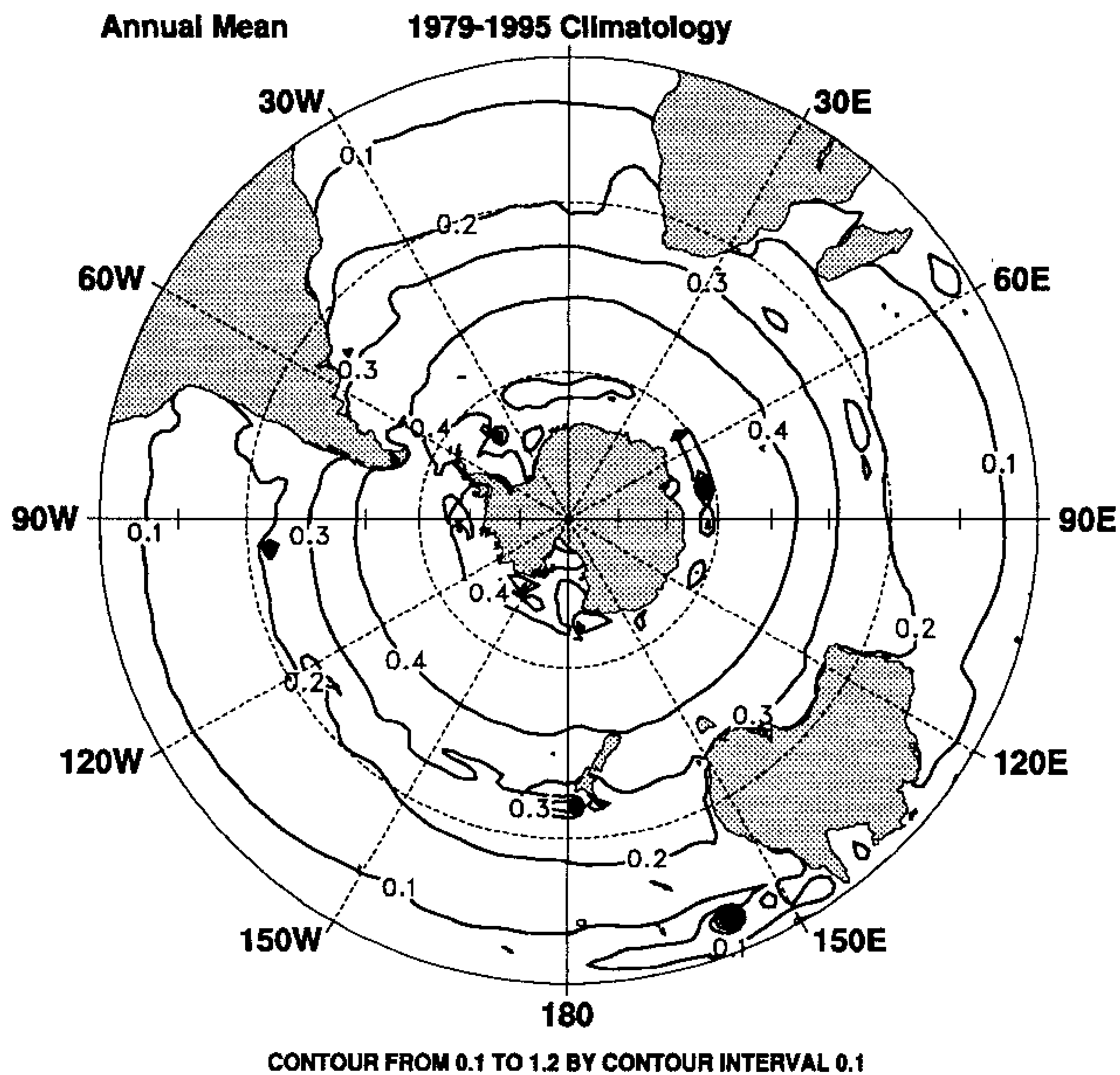
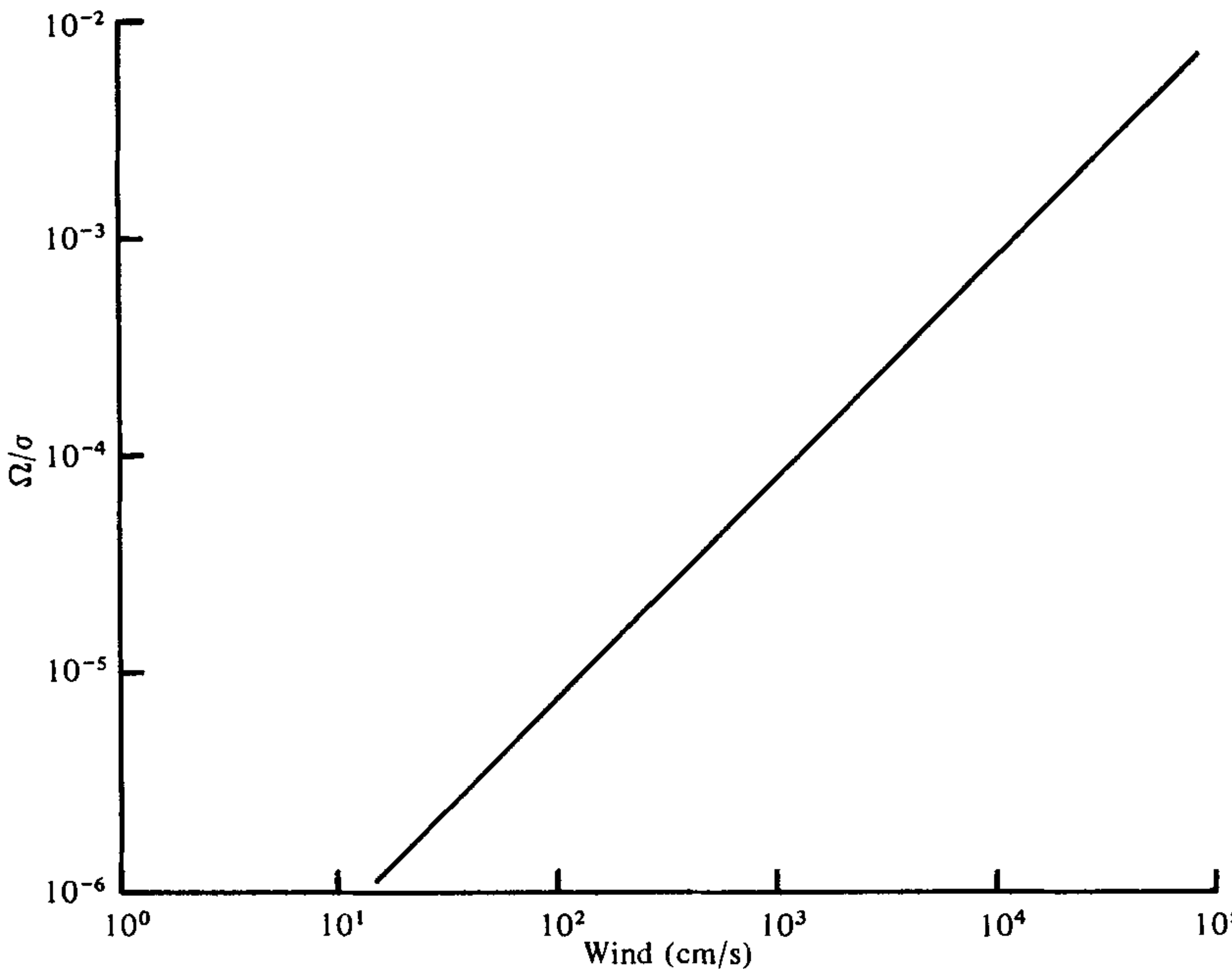


FIG. 6. The ratio of the wave- and wind-driven components in (51), $1/\mathcal{R}$ from (54), using the fields in Figs. 4–5. The contour interval is 0.1. The largest contour of 1.0 is approached only where $|\mathbf{U}_a| \rightarrow 0$.

Wave effects are biggest at higher latitudes

For the terms to be of comparable size, $\epsilon \approx 10^{-2}$

$$-\frac{\nu_e k_0^2}{\sigma_0 \epsilon^2} \nabla^2 \overline{\boldsymbol{\varpi}}_0 = (\overline{\boldsymbol{\varpi}}_0 \,.\, \nabla)(\bar{\mathbf{v}}_0 + \mathscr{U}_s) - (\bar{\mathbf{v}}_0 + \mathscr{U}_s) \,.\, \nabla \overline{\boldsymbol{\varpi}}_0 + \frac{2\Omega}{\sigma_0 \epsilon^2} (\mathbf{e} \,.\, \nabla)(\bar{\mathbf{v}}_0 + \mathscr{U}_s).$$



$$\sigma_0 = g/W.$$

$$\Omega/\sigma_0 \epsilon^2 = \Omega W / g\epsilon^2.$$

if $\Omega/\sigma_0\epsilon^2 \approx 1$

$$\epsilon < 10^{-2}$$

This is true in the open ocean for low frequency waves

Low frequency waves are more important for Stokes drift?

$$\Delta \mathscr{U}_s = [\Delta \overline{\zeta^2}]\, k\sigma = [\Delta \overline{\zeta^2}]\, \sigma^3/g, \qquad \sigma = \sqrt{gk}$$

(using Phillips spectrum assumption)

$$< \zeta^2 >= \int \frac{g^2}{\sigma^5}\, \mathrm{d}\sigma$$

$$[\Delta \overline{\zeta^2}] = \frac{\alpha g^2}{\sigma^5}\, 2\Delta\sigma,$$

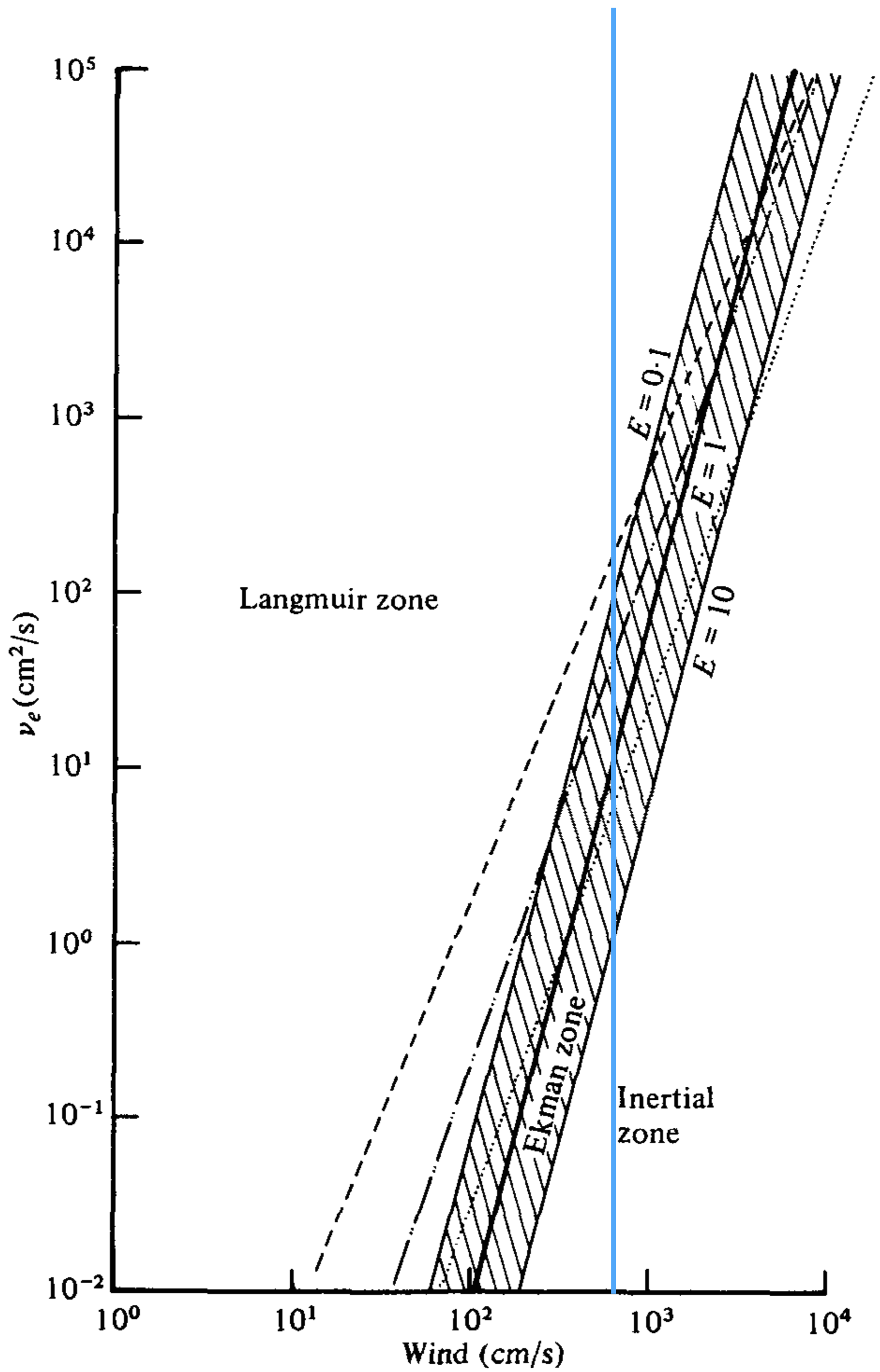
$$\Delta \mathscr{U}_s = \frac{\alpha g}{\sigma^2}\, 2\Delta\sigma.$$

Low frequency waves are more important in generating the Stokes drift.
Is this true???

Regimes

$$\nu_e = \kappa W_* z.$$

based on an inverted atmospheric boundary layer model



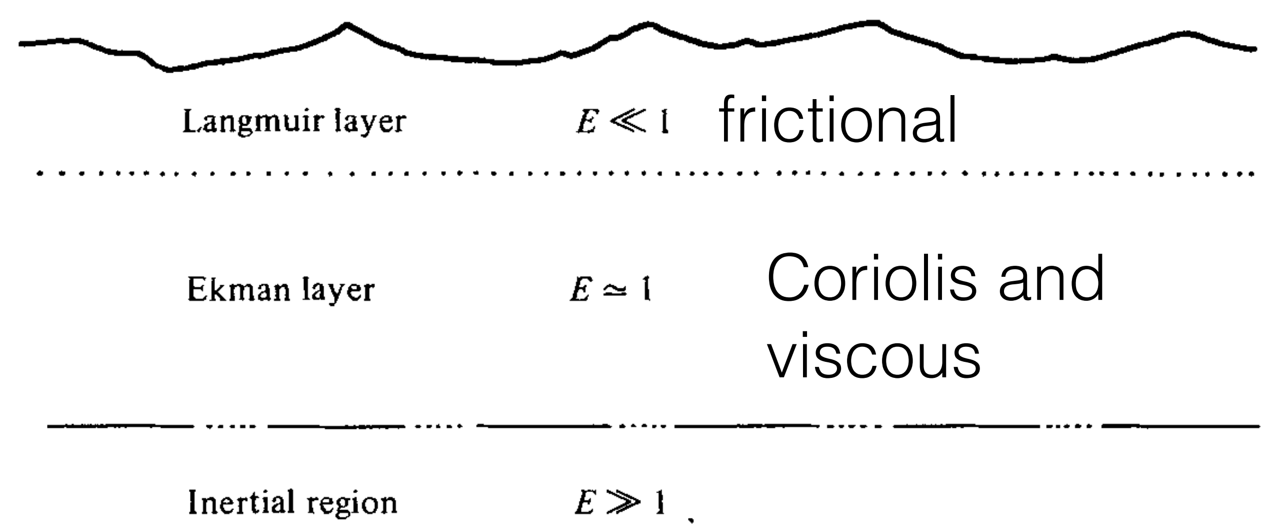


FIGURE 3. A proposed model of surface-layer drift currents.

$$E = \Omega/\nu_e k_0^2.$$

$$-\frac{\nu_e k_0^2}{\sigma_0 \epsilon^2}\nabla^2 \overline{\boldsymbol{\varpi}}_0 = (\overline{\boldsymbol{\varpi}}_0 \,.\, \nabla)\,(\bar{\mathbf{v}}_0 + \mathcal{U}_s) - (\bar{\mathbf{v}}_0 + \mathcal{U}_s)\,.\,\nabla \overline{\boldsymbol{\varpi}}_0 + \frac{2\Omega}{\sigma_0 \epsilon^2}(\mathbf{e}\,.\,\nabla)\,(\bar{\mathbf{v}}_0 + \mathcal{U}_s).$$

Predicted velocity and length scales for Stokes drift

$$\langle(\eta^w)^2\rangle = \int_0^\infty f(\sigma)\, d\sigma.$$

$$\mathbf{u}^s = \hat{\mathbf{e}}_h \frac{2}{g}\int_0^\infty f(\sigma)\sigma^3 \exp\left[\frac{2\sigma^2 z}{g}\right] d\sigma,$$ Kenyon (1969)

$$f_n(\sigma) = \frac{a_n g^2}{\sigma^5}\exp\left[-b_n\left(\frac{g}{W\sigma}\right)^n\right],$$ Pierson & Moskovitz (1964)

$$\mathbf{u}^s(z) = 0.04\mathbf{U}_a \exp\left[-\frac{4\sqrt{g|z|}}{W}\right].$$

$$W = |\mathbf{U}_a|$$

for $W = 10\,\mathrm{m/s}$, depth scale $d = \dfrac{W^2}{16g} \approx \dfrac{5}{8}\mathrm{m}$

$$u^s \approx 0.4\,\mathrm{m/s}$$

Conclusions

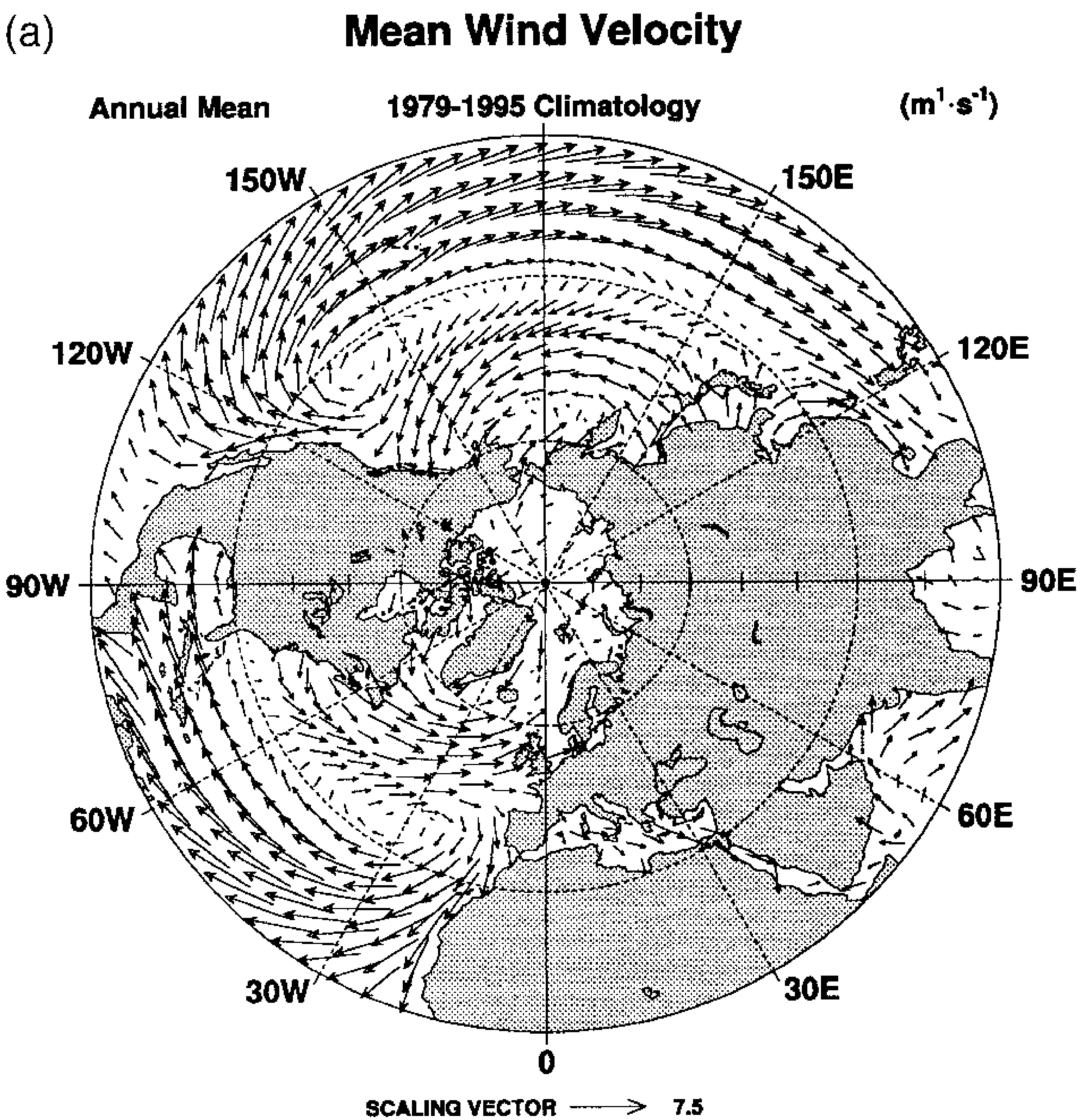
- Stokes drift should be include when modeling the Ekman layer
- Stokes drift might well be important in modeling the ocean, especially at high latitudes. Waves both cause the Stokes drift and alter the boundary conditions.

$$\int_{-D}^{0} \mathbf{v}\,\mathrm{d}z + \int_{-D}^{0} \mathbf{u}^{\mathbf{s}}\,\mathrm{d}z = \hat{\mathbf{z}} \times \nabla_h \Psi$$

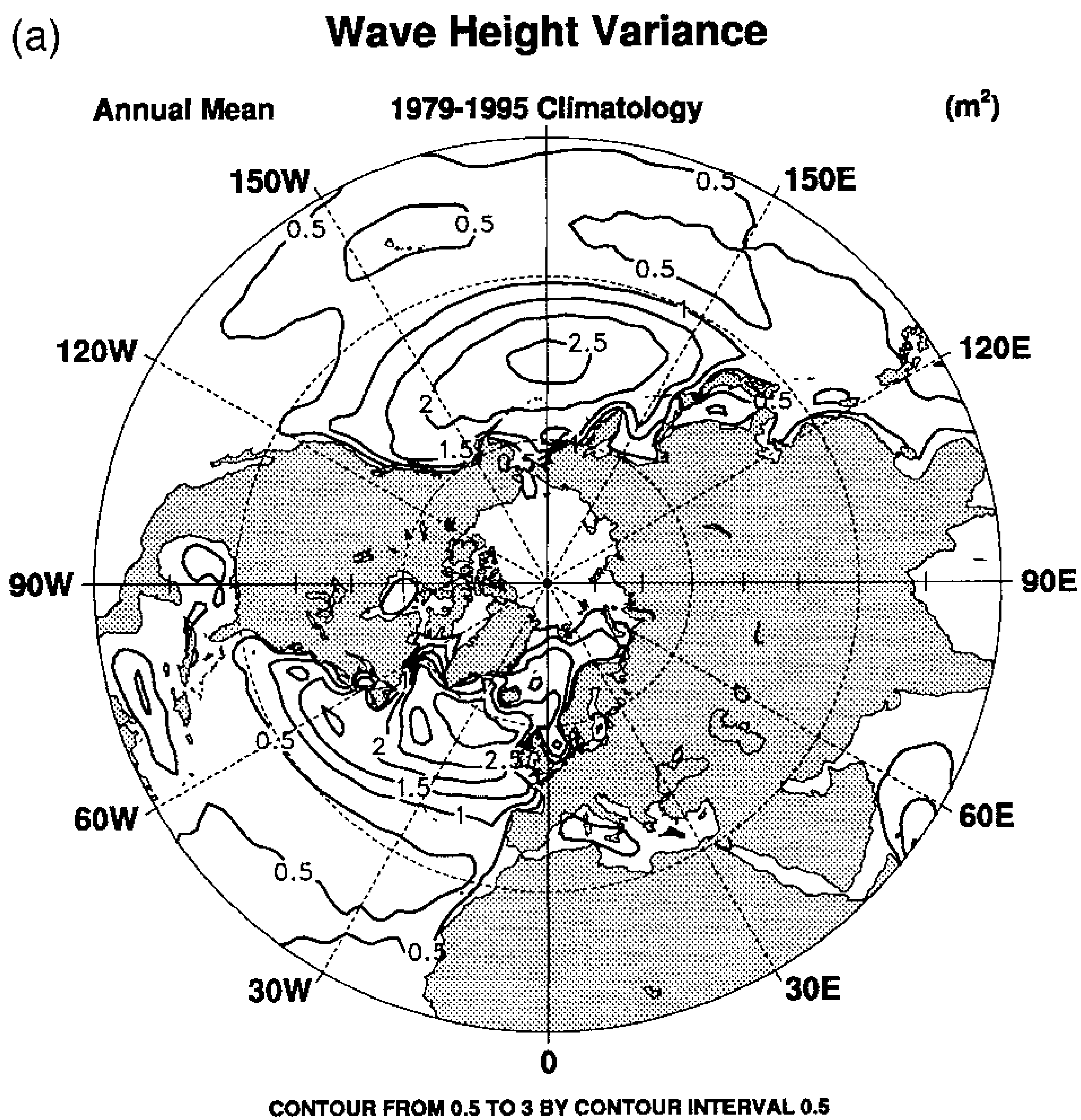
$$\beta\Psi = \int_{X_{east}}^{x} \nabla \times \frac{\tau}{\rho}\,\mathrm{d}x'$$

Wind strength

(a)

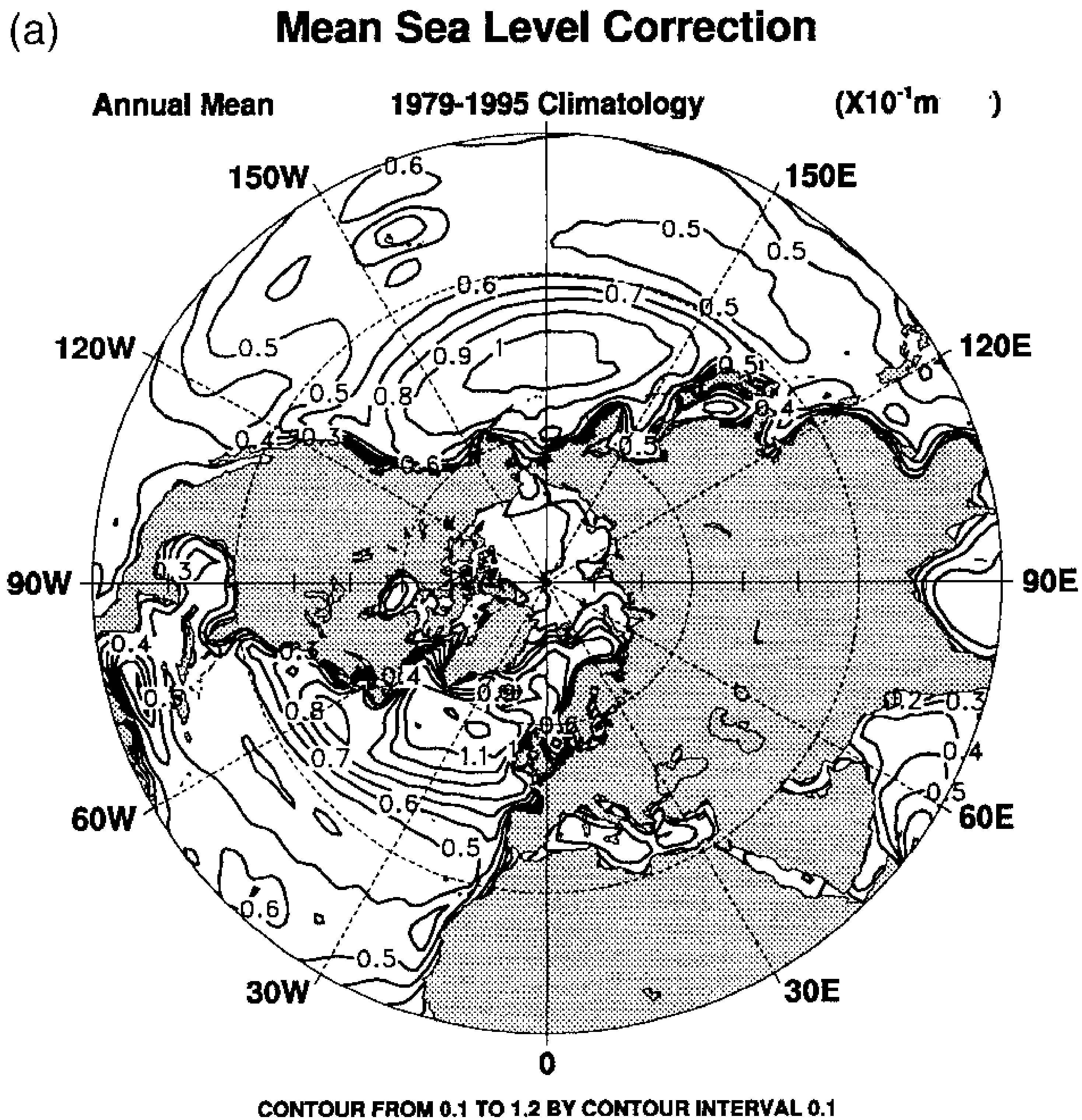


Wave strength



Waves are strongest far from land, and where the winds are strong and westerly

How big is the correction due to the surface pressure?

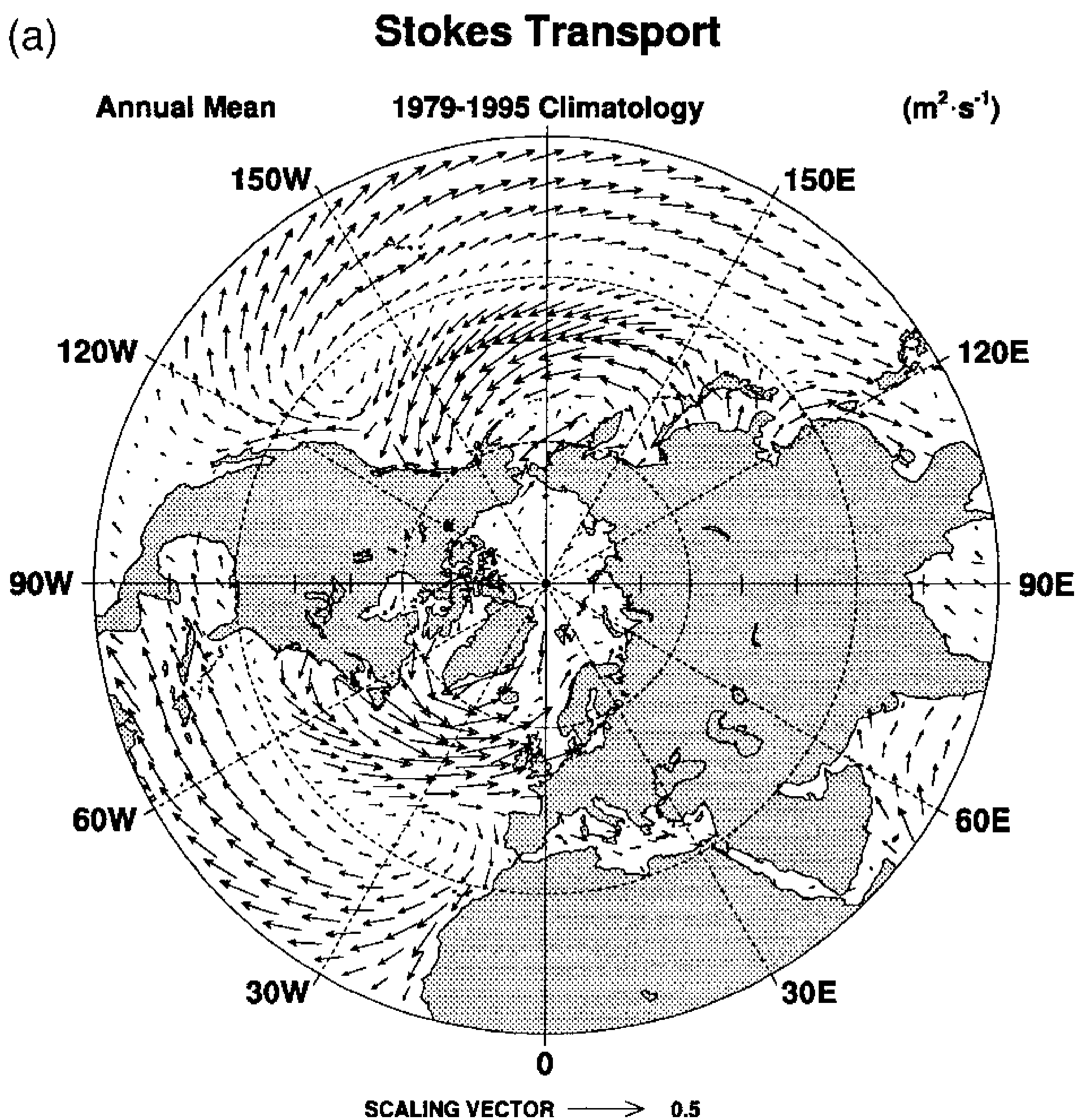


$$p_0 = \eta_0 + p_{a0} - \xi P \quad \text{at } z = 0,$$

Strongest near antarctica

$$P = \langle (\eta_t^w)^2 \rangle$$

How big is the correction due to Stokes drift?

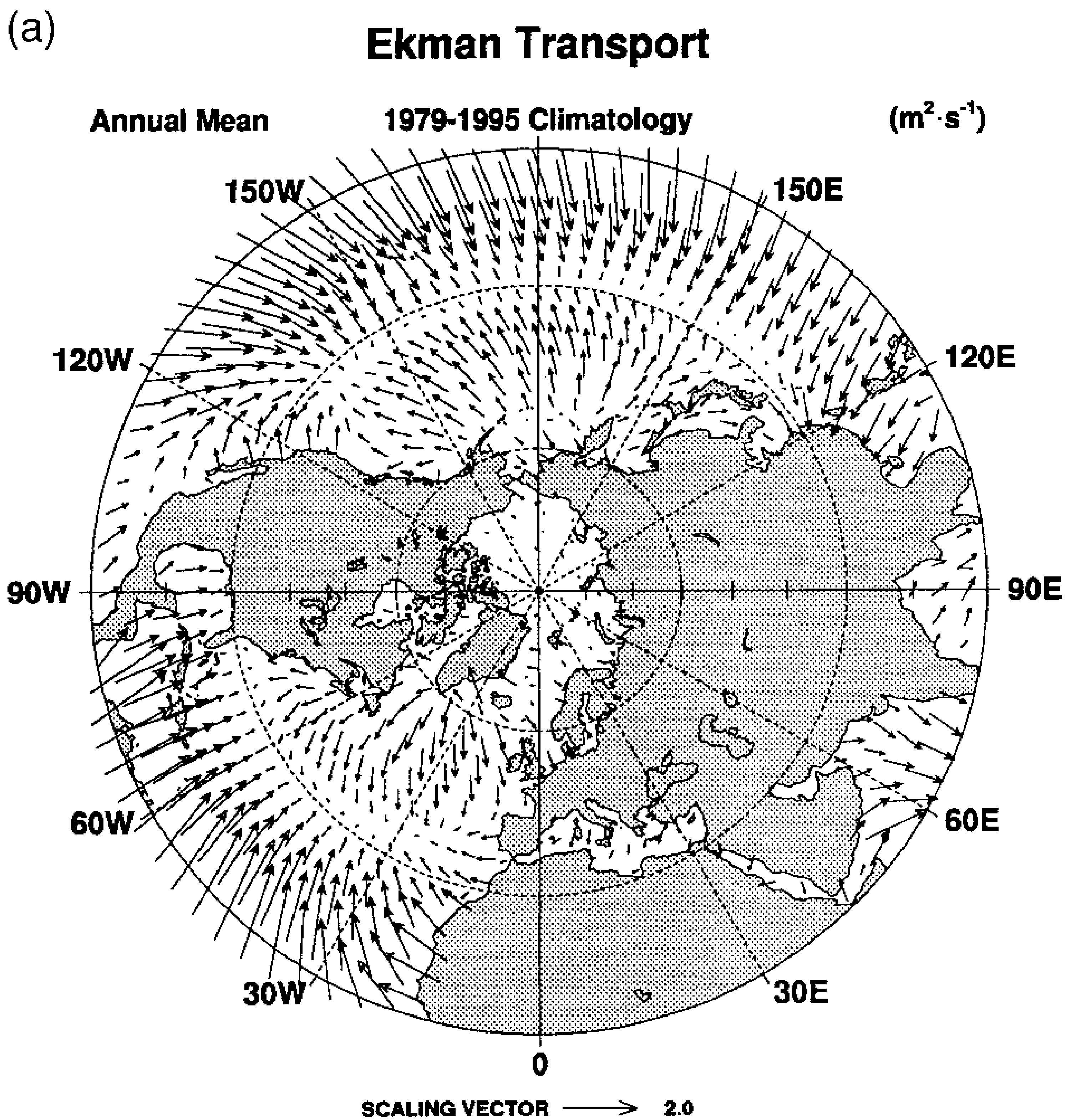


$$\mathbf{T}_{st} = \int_{-D}^{0} \mathbf{u}^s \, \mathrm{d}z$$

Strongest in westerly wind regimes

Mainly in the direction of the wind

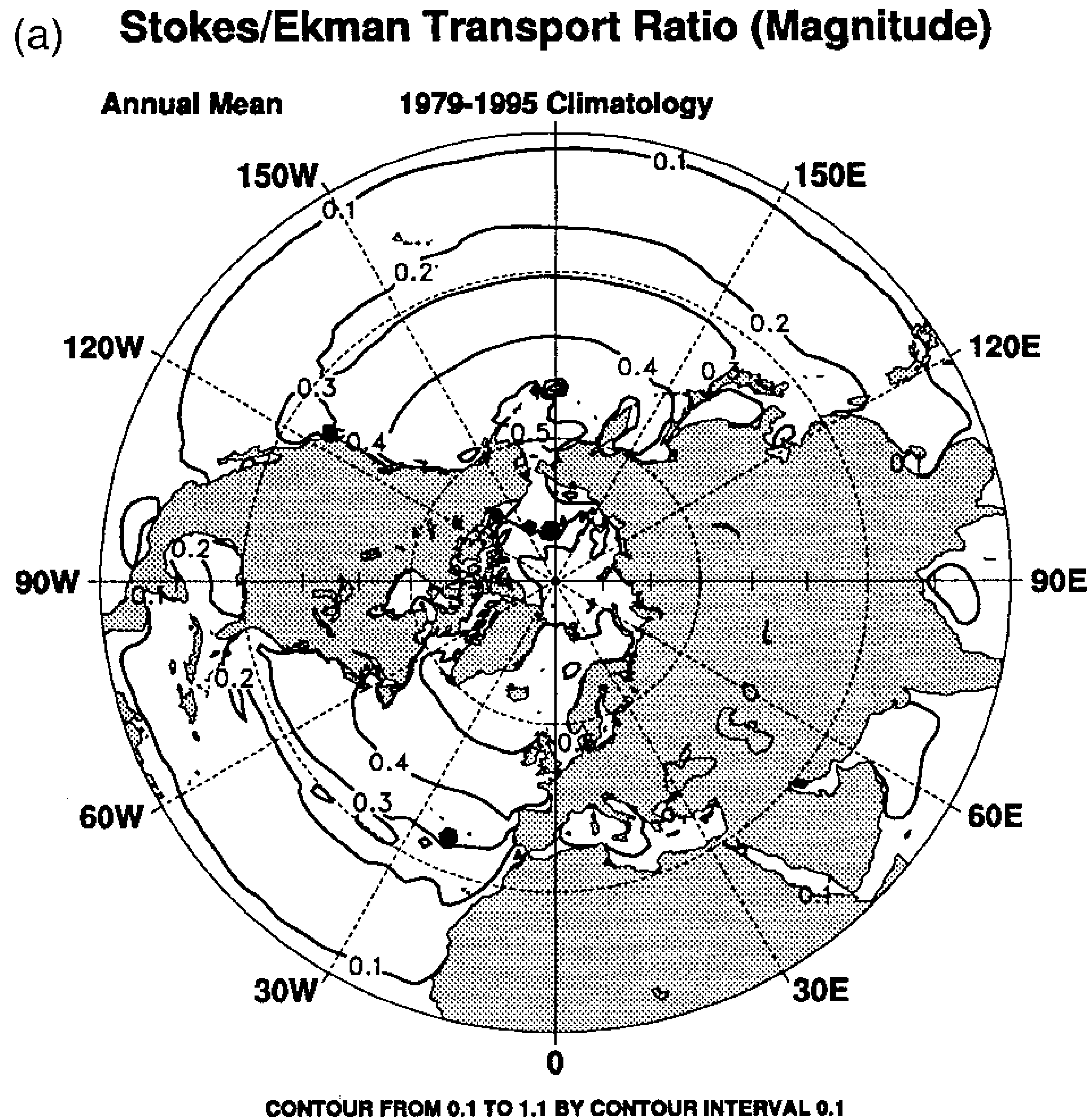
How big is the Ekman transport?



Mainly perpendicular to the wind

About 4x the size of the Stokes transport

How big is the correction due to Stokes drift?



Wave effects are biggest at higher latitudes