Friedman, H. 1986. *Sun and Earth*. San Francisco: Freeman (see especially chap. 5).

QUANTITATIVE DISCUSSIONS

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Priest, E. R. (ed.). 1985. *Solar System Magnetic Fields*. Dordrecht: Reidel (see especially chap. 2, introduction to Alfvén and other MHD waves, and chap. 8, on shocks).

PROBLEMS

2.1. The Lorentz force on a charged particle is

$$\mathbf{F}_L = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

In the absence of an electric field, show that a charged particle's motion can be resolved into two components: one constant, along the magnetic field, and one periodic, perpendicular to the magnetic field. Show that the gyroradius ρ_c is given by

$$\rho_c = \frac{mv_\perp}{qB}$$

and that the gyrofrequency is given by

$$\Omega_c = \frac{qB}{m}$$

Calculate the gyrofrequency (in hertz) of a proton in a 100-nT field, of an electron in a 1,000-nT magnetic field, of a singly ionized oxygen atom in a 50,000-nT field. At roughly what distances from the earth would these gyrofrequencies be found in the equatorial plane? What is the gyroradius of a proton moving transverse to a 100-nT field at 2×10^5 m · s^{-1}? How does this distance compare with the distance (near the equator) over which the earth's dipole field changes by a factor of 2 near the region where $B = 100$ nT? Would you expect the proton to conserve its first adiabatic invariant?

2.2. For each of the following fields, sketch the particle trajectories separately for electrons and protons. Define your coordinate system. Illustrate clearly the direction of the magnetic and electric fields and your coordinate axes. Sketch trajectories in the planes that best illustrate the motions of the charged particles. Describe the form of motion normal to the plane. Explain what you are assuming about a particle's thermal energy, in particular its magnitude relative to the electric-drift speed.

(a) Assume a static uniform magnetic field oriented along the x-axis, with no electric field. The particles have initial velocity $v_x = 0$ and $v_z = v_0$.

(b) Assume a static uniform magnetic field oriented along the z-axis, with no electric field. Charged particles are initially moving with nonvanishing v_x and v_z.

(c) Assume a static uniform magnetic field oriented along the y-axis, with a static electric field along z. Charged particles are initially at rest.

(d) Assume a static uniform magnetic field $\mathbf{B}$ oriented along the y-axis, with a static electric field $\mathbf{E}$ along z. Charged particles are initially moving in the x-direction with velocity $v_0 = v_x + v_\perp$, where

$$v_x = \frac{|\mathbf{E}\times\mathbf{B}|}{B^2} \quad \text{and} \quad v_\perp > v_x$$

(e) Assume a magnetic field along the z-direction increasing in strength with increasing z. Charged particles initially have velocities v_x and v_z, with $v_x \ll v_z$.

(f) Which of the situations (a)–(e) will give rise to currents in plasmas with equal numbers of positive and negative charges?

(g) If in part(d) the electric field is 10 mV · m^{-1} and the magnetic field is 100 nT, how fast will particles drift?

2.3. Consider the situation presented in part (d) of the preceding problem, assuming equal ion and electron temperatures, with a uniform magnetic field $\mathbf{B} = B_y$ and static electric field $\mathbf{E} = E_z$. Suppose that slowly moving neutral atoms are also present (ion thermal speed much greater than neutral velocity, ~0). Sketch qualitatively the drift paths as in Problem 2.2, and calculate ion- and electron-drift velocities (speed and direction) assuming that

(a) collisions occur, on average, once every ion gyroperiod,

(b) collisions occur, on average, twice every gyroperiod,

(c) collisions occur once every five electron gyroperiods.

2.4. A gas in thermal equilibrium has particles at all velocities. The most probable distribution for the particles is that of a Maxwellian

$$f(v) = A\exp(-\tfrac{1}{2}mv^2/kT)$$

in one dimension, where A is a normalization constant. This distribution is graphically depicted in Figure 2.3a. The width of the distribution is characterized by a constant T that we call temperature. The number density of particles n is given by integrating over all velocities. In three dimensions the relation is

$$n = \int_{-\infty}^{+\infty} f(v)\, d^3v$$

(a) Derive the constant A in the distribution function for a one-dimensional Maxwellian and for a three-dimensional Maxwellian.

(b) If the average value of a function $g(v)$ is

$$\langle g(v)\rangle = \frac{\int_{-\infty}^{+\infty} g(v)f(v)\, d^3v}{\int_{-\infty}^{+\infty} f(v)\, d^3v}$$

derive the average (velocity)^2 of a one-dimensional Maxwellian of temperature T and its average kinetic energy. The square root of this (velocity)^2 is sometimes called the thermal velocity, but it differs from the most probable speed in the distribution [equation (2.22)].

2.5. A particle of mass m and charge e, initially at rest at the origin, is subject to constant fields $\mathbf{E} = E_y$ and $\mathbf{B} = B_z$. Show that it moves on the cycloid

$$x = \frac{E}{\Omega B}(\Omega t - \sin \Omega t)$$

$$y = \frac{E}{\Omega B}(1 - \cos \Omega t)$$

in the plane $z = 0$, where $\Omega = eB/m$. The motion is periodic with period Ω. What is the wavelength of the motion?

2.6. The magnetic-field strength in the earth's magnetic equatorial plane is given by

$$B = B_0(R_E/r)^3$$

where $B_0 = 0.3$ G, R_E is an earth radius, and r is the geocentric distance. Derive an expression for the drift period (the time it takes a particle to drift around the earth) of a particle on the equatorial plane with a pitch angle of 90° and energy W. (Hint: Think carefully about what this implies). Evaluate this period for both a proton and an electron of 1 keV energy at a distance of $5R_E$ from the center of the earth. Compare the answer to

(a) the drift induced by the force of gravity on the same particles,

(b) the orbital period of an uncharged particle at the same position.