PHY3145 Topics in Theoretical Physics

Astrophysical Radiation Processes

3: Relativistic effects I

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Course structure

1. **Radiation basics**. Radiative transfer.
2. **Accelerated charges produce radiation**. Larmor formula. Acceleration in electric and magnetic fields – non-relativistic bremsstrahlung and gyrotron radiation.
3. **Relativistic modifications I**. Doppler shift and photon momentum. Thomson, Compton and inverse Compton scattering.
4. **Relativisitic modifications II**. Emission and arrival times. Superluminal motion and relativistic beaming. Gyrotron, cyclotron and synchrotron beaming. Acceleration in particle rest frame.
5. **Bremsstrahlung and synchrotron spectra.**

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Photon scattering by electrons - Overview

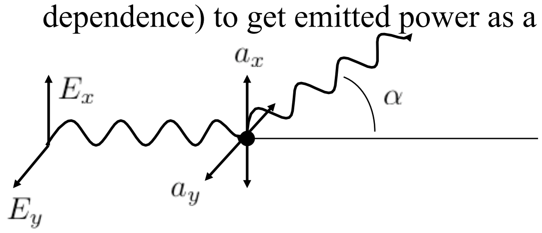
	Low energy photons $\hbar\omega \ll m_e c^2$	High energy photons $\hbar\omega \geq m_e c^2$
$v \ll c$	Thomson scattering Classical treatment frequency unchanged	Compton scattering Quantum treatment incorporating photon momentum frequency decreases
$v \sim c$	$\gamma\hbar\omega \ll m_e c^2$ Inverse Compton Photons gain energy from relativistic electrons Approximate with classical treatment in electron rest frame Frequency increases	$\gamma\hbar\omega \geq m_e c^2$ Inverse Compton Quantum treatment in electron rest frame Photons gain energy from relativistic electrons

Thomson scattering (from last lecture)

$\hbar\omega \ll m_e c^2$
$v \ll c$

Aim: Derive angular dependence of power for classical scattering of EM field by a non-relativistic electron.

Method: Treat photon as EM wave accelerating electron. Consider acceleration in plane of scattering and perpendicular to plane of scattering separately. Use Larmor's formula (with angular dependence) to get emitted power as a function of angle.



x-component of accel. (in plane of α) => power $\propto \cos^2\alpha$

y-component of accel. (perp. to plane of α) => power independent of α

Result: (derivation in lectures)

$$\frac{dW}{dt\,d\Omega} = \frac{e^4}{16\pi^2\epsilon_0 c^3 m_e^2}\left(|\langle E_x\rangle|^2\cos^2\alpha + |\langle E_y\rangle|^2\right)$$

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Compton scattering

$$\hbar\omega \geq m_e c^2$$
$$v \ll c$$

In Compton scattering the photon loses momentum to the electron, reducing the wavelength of the scattered light.

Example: X-ray scattering from Beryllium

Pattison et al. 1981, J. Phys. E: Sci. Instrum., Vol. 14, 1981

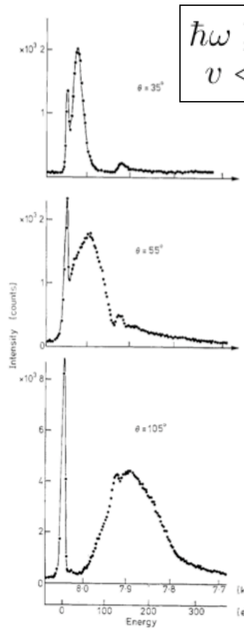


Figure 2 The spectra of CuKα₁ x-rays scattered by a Be target for scattering angles of 35°, 65° and 105°. The intensity of the scattered x-rays is shown both as a function of their final energy, and the energy loss suffered in the collision.

Compton scattering

$$\hbar\omega \geq m_e c^2$$
$$v \ll c$$

In Compton scattering the photon loses momentum to the electron, reducing the wavelength of the scattered light.

Aim: Derive the wavelength shift.

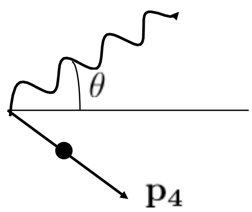
Method: Conserve 4-momentum and exploit norms.

$$\mathbf{p_1} = (p_1, 0, 0, p_1)$$

$$\mathbf{p_2} = (0, 0, 0, m_e c)$$

Before

$$\mathbf{p_3} = (p_3 \cos\theta, p_3 \sin\theta, 0, p_3)$$



After

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Compton wavelength shift

$\hbar\omega \geq m_e c^2$
$v \ll c$

Conservation of 4-momentum: $\mathbf{p_1} + \mathbf{p_2} = \mathbf{p_3} + \mathbf{p_4}$

For photons: $\mathbf{p_1} \cdot \mathbf{p_1} = \mathbf{p_3} \cdot \mathbf{p_3} = 0$

For electrons: $\mathbf{p_2} \cdot \mathbf{p_2} = \mathbf{p_4} \cdot \mathbf{p_4} = m_e^2 c^2$

Norms: $|\mathbf{p_1} + \mathbf{p_2} - \mathbf{p_3}|^2 = |\mathbf{p_4}|^2$

Remember: $p = \hbar k = \dfrac{h}{\lambda}$

Results (see lectures):

$$\lambda_3 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta)$$

Compton wavelength shift

Inverse Compton scattering

$v \sim c$
$\gamma\hbar\omega \ll m_e c^2$

Inverse Compton scattering is scattering of "low"-energy photons $\gamma\hbar\omega \ll m_e c^2$
which are boosted to higher energy by relativistic electrons

Aim: Derive approximate factor in frequency shift / energy gain
Method: Consider Doppler shifts of photons as we L.T. from observer's to electron rest frame and back. Use the relativistic Doppler factor (see lectures)

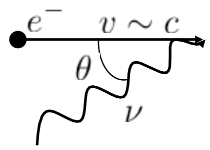
$$K = \gamma(1 + \beta\cos\theta')$$

Inverse Compton frequency shift

$v \sim c$
$\gamma\hbar\omega \ll m_e c^2$

Before

S



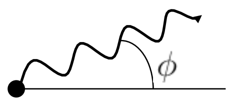
S'

$$\nu' = \nu\gamma(1 - \beta\cos\theta)$$

In electron rest frame, collision is almost head-on.

After

S'



S

$$\nu'' = \nu'\gamma(1 + \beta\cos\phi)$$
$$= \nu\gamma^2(1 + \beta\cos\phi)(1 - \beta\cos\theta)$$

factor γ^2 increase in frequency/energy

Assume Thomson scattering (no frequency change) as

$$\gamma\hbar\omega \ll m_e c^2$$

Exact solution: $$\frac{dE}{dt} = 4/3\sigma_T c(\gamma^2 - 1)U_{\text{light}}$$

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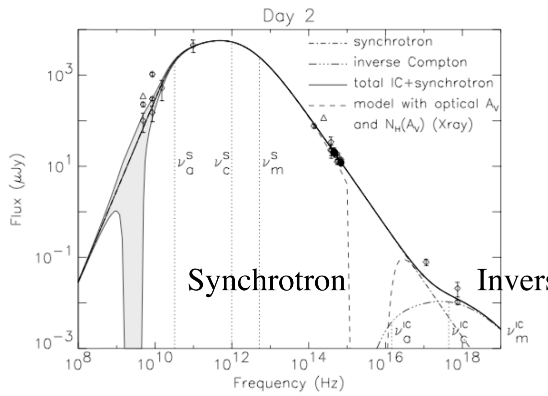
Inverse Compton scattering

$v \sim c$
$\gamma\hbar\omega \ll m_e c^2$

Example: Synchrotron self-Compton (SSC)



Synchrotron and self-Compton X-ray spectrum from GRB 000926
Harrison et al. 2001 Astrophysical Journal 559 123

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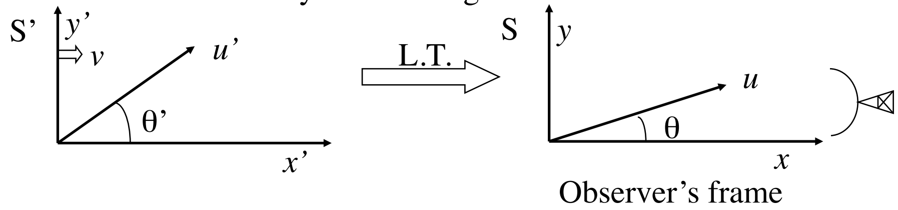
Relativistic aberration

Used for (a) relativistic Doppler effect (b) relativistic beaming

Aim: apparent change of direction of motion between frames

Method: L.T. 4-velocity for u' at angle θ' to observer's frame.



4-velocities

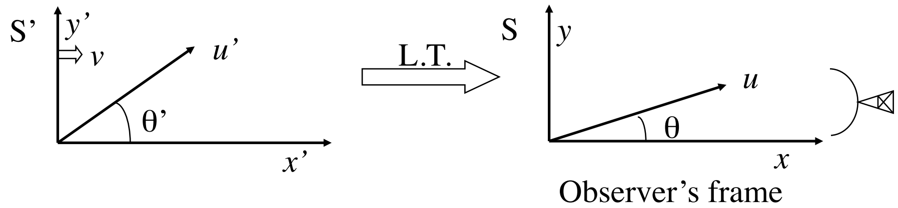
$(\gamma'_u u' \cos\theta',\ \gamma'_u u' \sin\theta',\ 0,\ \gamma'_u c)$ $\qquad$ $(\gamma_u u \cos\theta,\ \gamma_u u \sin\theta,\ 0,\ \gamma_u c)$

L.T. x and y components and divide

$$\gamma_u u \cos\theta = \gamma_v(\gamma'_u u' \cos\theta' + \beta\gamma'_u c)$$
$$\gamma_u u \sin\theta = \gamma'_u u' \sin\theta'$$

Relativistic aberration

Results:



Aberration formula
$$\tan\theta = \frac{u'\sin\theta'}{\gamma(u'\cos\theta' + v)}$$

For photons *u'=c*
$$\tan\theta = \frac{\sin\theta'}{\gamma(\cos\theta' + v/c)}$$

also
$$\cos\theta = \frac{\cos\theta' + \beta}{(1 + \beta\cos\theta')}$$

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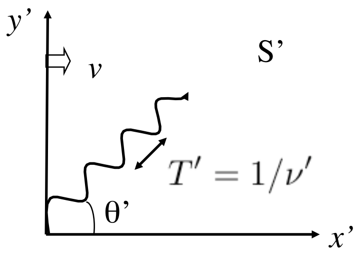
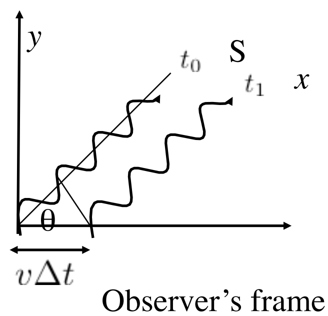
Relativistic Doppler effect

Used for inverse Compton effect
Aim: transform frequency for radiation at an arbitrary angle
Method: emission and arrival times in observer's frame, then LT period and angle to particle rest frame



Particle rest frame



Observer's frame

In observer's frame, first photon travels an extra distance $v\Delta t\cos\theta$

$$t_1 - t_0 = \Delta t(1 - v/c\cos\theta)$$

In observer's frame, first photon travels an extra distance $v\Delta t\cos\theta$

$$t_1 - t_0 = \Delta t(1 - \beta\cos\theta)$$

Substitute for cos θ (aberration) $\cos\theta = \dfrac{\cos\theta' + \beta}{(1 + \beta\cos\theta')}$
and Δt = γT where T is period

$$t_1 - t_0 = \frac{\gamma T'(1 + \beta\cos\theta' - \beta\cos\theta' + \beta^2)}{(1 + \beta\cos\theta')}$$

$$T = \frac{T'}{\gamma(1 + \beta\cos\theta')}$$

In terms of frequency

$$\nu = \gamma\nu'(1 + \beta\cos\theta') = K\nu'$$

where $K = \gamma(1 + \beta\cos\theta')$ is the **relativistic Doppler factor**.

Example: inverse Compton scattering

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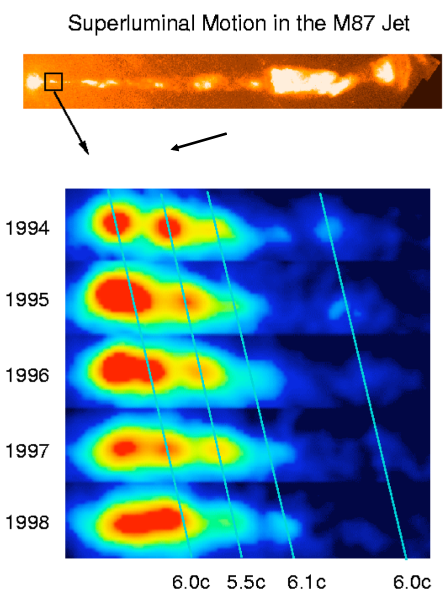


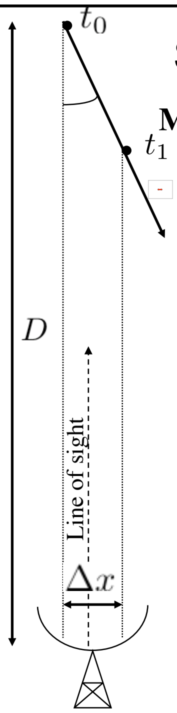
Biretta et al. ApJ 520, 621

Emission and arrival times: Superluminal motion

$D = 11\,\mathrm{Mpc}$

$v_\perp \sim 100 \text{ mas yr}^{-1} \sim 6c$

Jet blobs appear to move faster than speed of light!



Superluminal motion: theory

Method: consider emission and arrival times in lab. frame

Emission time	Arrival time
t_0	$t_0 + D/c$
t_1	$t_1 + (\frac{D - v(t_1 - t_0)\cos\alpha}{c})$

Apparent transverse component of velocity

$$\frac{\Delta x}{\Delta t} = \frac{v(t_1 - t_0)\sin\alpha}{(t_1 - t_0) - v/c(t_1 - t_0)\cos\alpha}$$

$$\boxed{\frac{\Delta x}{\Delta t} = \frac{v\sin\alpha}{(1 - v/c\cos\alpha)}}$$

N.B. working entirely in lab. frame - $t_1 - t_o = \gamma(\Delta t')$