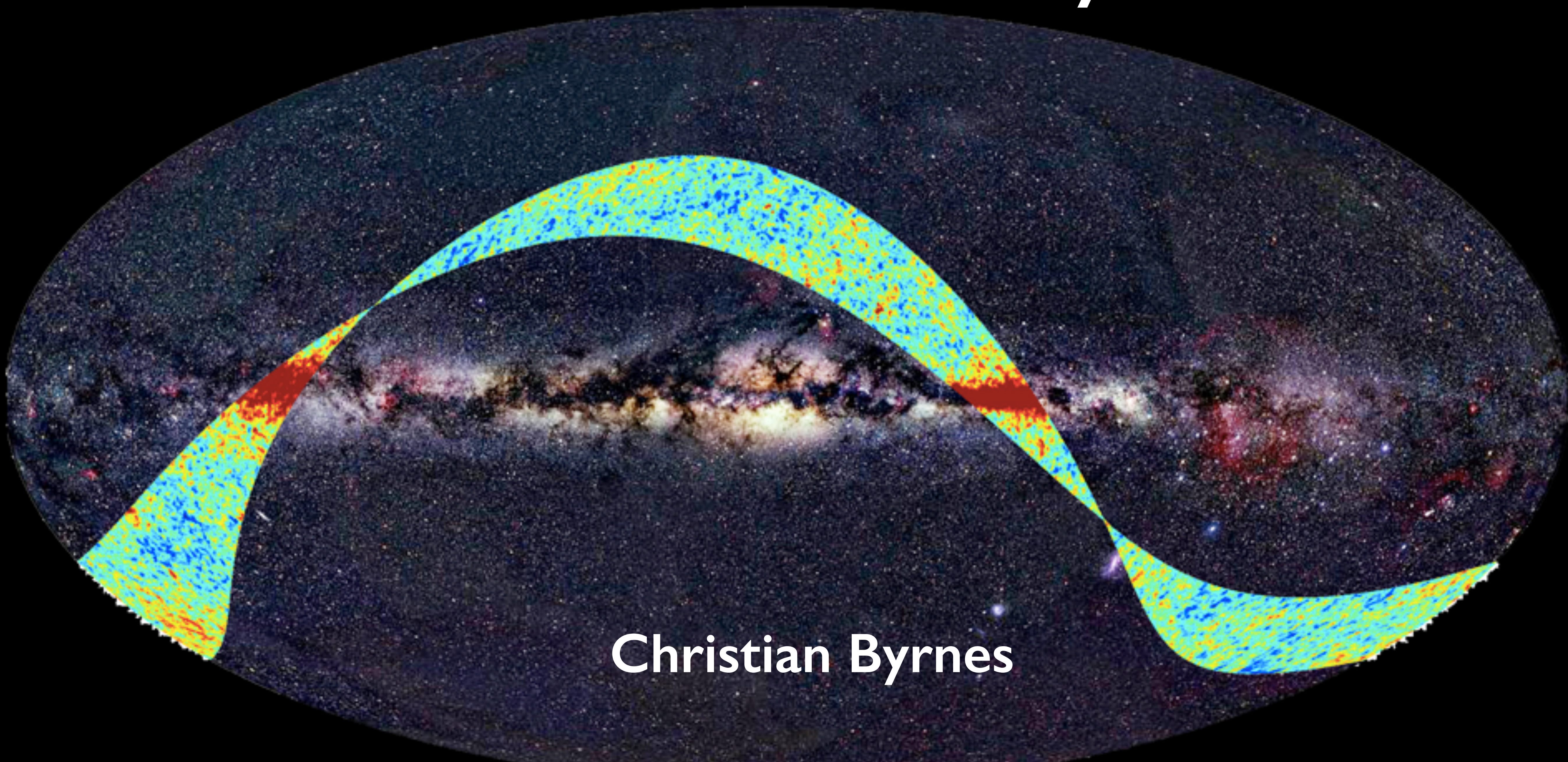
Non-Gaussianity



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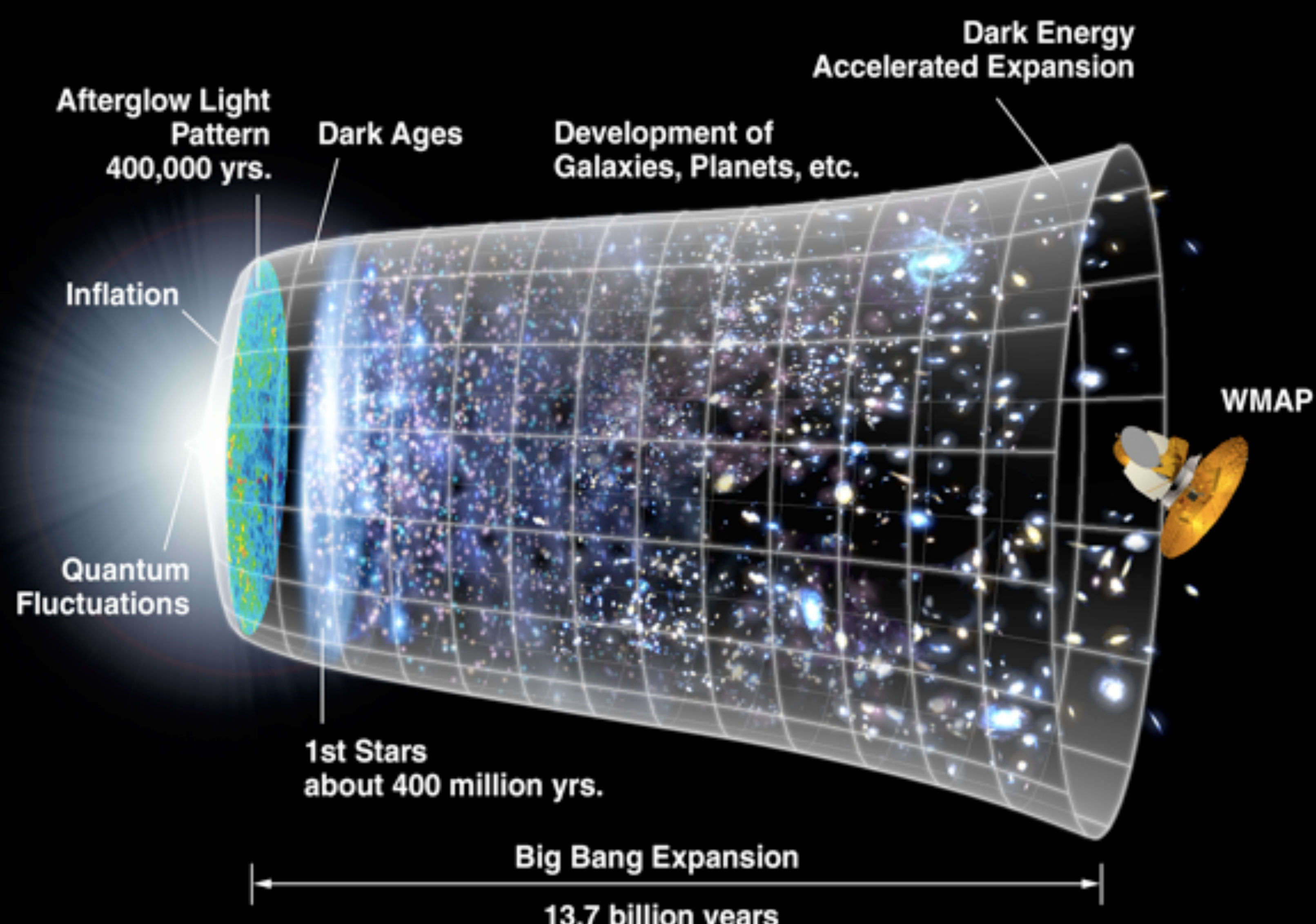
Aims

- To motivate the subject
- To provide a broad overview
- Non-Gaussianity is often seen as a technical subject, it is the study of non-linear perturbation theory
- Physical intuition can still be provided, at the cost of losing the details (e.g. neglecting some terms or numerical factors)
- I aim to provide details of some research areas and techniques, at the cost of focusing the overview at a few small areas
- I will frequently use the whiteboard, so please choose a suitable seat
- Please ask questions, especially students! This is a school, not a conference.

A tale of the early universe: inflation and the CMB, plus LSS

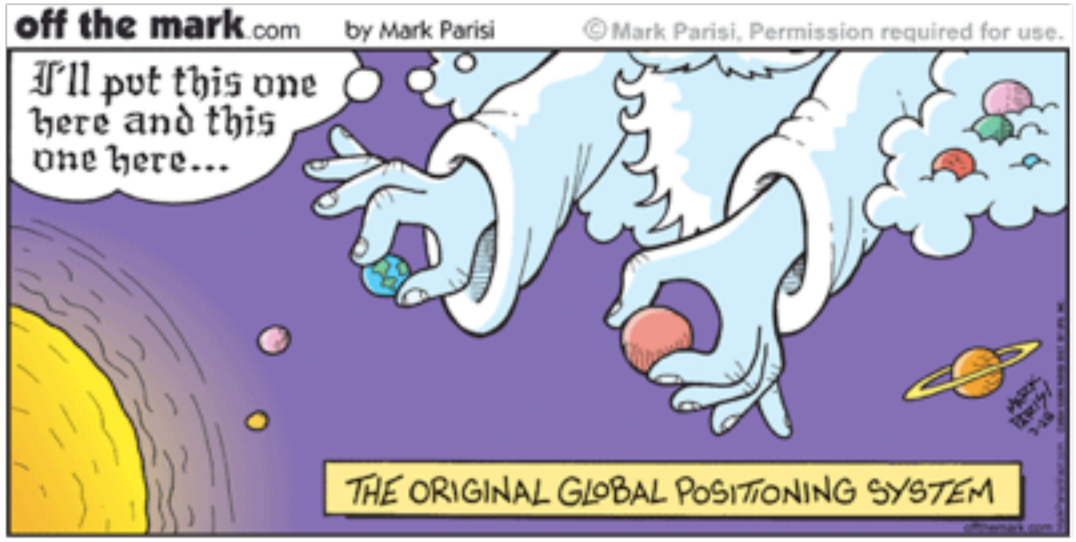


now Planck

Inflation

- Believed to have generated the primordial curvature perturbation, the seed from which all structures grow
- The earliest epoch of the universe which we can observationally probe
- It tends to erase all memory of the initial conditions
- Again by design, (almost) all models give rise to an identical, spatially flat background
- Only the perturbations can discriminate between models

We are interested in statistical properties of the CMB



- The temperature in specific directions is just down to chance
- But the statistical properties are crucial, this is what we use to compare theory and observation
- The most convenient way to parametrise the curvature perturbation is zeta, which we can relate to the perturbations during inflation and to temperature perturbations on the CMB

Linear perturbations

- These are the dominant, Gaussian perturbations
- They have been measured extremely accurately on CMB scales
- Give us information about the primordial power spectrum
- See David and Jerome's courses

CMB measurements: Power spectrum

- We measure the statistical pattern of the tiny temperature perturbations in the CMB
- Main observational tool is the power spectrum (2-point function)
- Measure the amplitude of perturbations at a given scale (in Fourier space)

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = P_\zeta(k)(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2), \qquad k = |\mathbf{k}_1| = |\mathbf{k}_2|$$

$$\mathcal{P}(k) = A \left(\frac{k}{k_p} \right)^{n-1} \equiv k^3 P(k), \qquad k \sim \frac{1}{physical\ scale}$$

- Angle brackets denote an average
- Two observable parameters, amplitude and spectral index
- If n=1 then scale invariant, i.e. perturbations have the same amplitude on every scale
- Planck observations show $A \simeq 10^{-10}, \qquad n - 1 \simeq -0.04$

Planck measured power spectrum

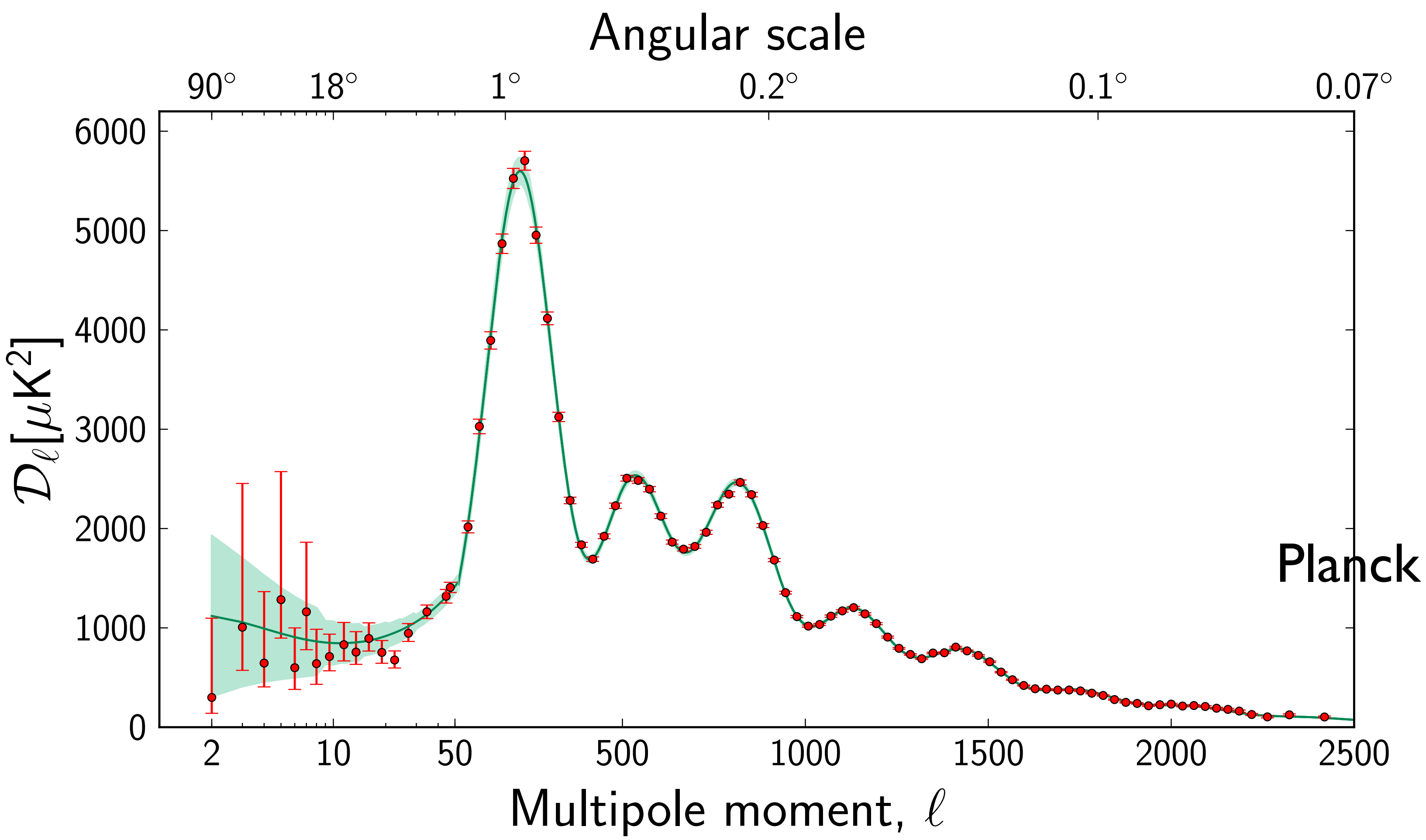


Figure 37. The 2013 *Planck* CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude is indicated by the green shaded area around the best fit model. The low-ℓ values are plotted at 2, 3, 4, 5, 6, 7, 8, 9.5, 11.5, 13.5, 16, 19, 22.5, 27, 34.5, and 44.5.

Planck collab 2013 XV

Looks complicated, but all this can be fit by a primordial power law spectrum with just two input parameters

The range of scales probed is $2500/2=10^3=e^7$ - corresponds to about 7 efoldings of inflation

Enormous data compression

- Planck observes ~10^7 pixels in the CMB sky
- Reduced to ~10^3 Cl
- Further reduced to A and n_s-1
- Can only be justified if the perturbations are Gaussian
- Then by Wicks theorem, the odd point correlators are zero, the even ones are reducible to products of two points functions - i.e. all information is contained in the power spectrum

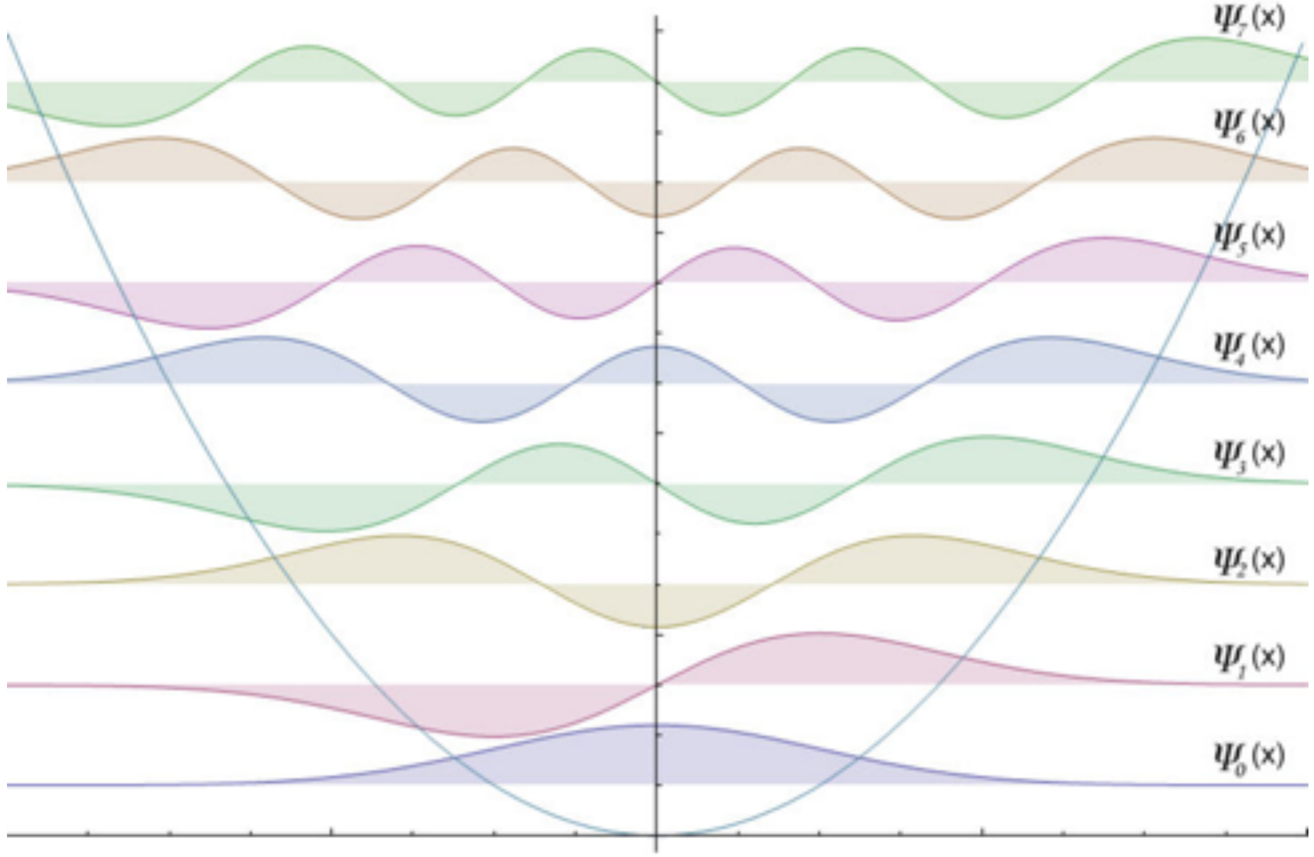
Simplicity of Gaussianity

$$\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$

- The mean x0 can be set to zero, because we define perturbations as deviation from the background
- There is no information to be gained by not doing so, for example the average temperature of the CMB changes (extremely slowly) with time, due to photon red shifting
- This leaves us with only one free parameter, the variance. The variance may depend on scale
- A non-Gaussian distribution may have any number of free parameters

Why Gaussian?

- Gaussian perturbations are found everywhere in nature
- Often due to the central limit theorem
- The ground state of the simple harmonic oscillator is Gaussian - quantum origin of perturbations



Conditions for Gaussianity

The primordial density perturbation will be indistinguishably close to Gaussian if inflation is

1. single field (only one light field present)
2. slow roll (for all slow-roll parameters)
3. canonical kinetic term
4. perturbations start in the Bunch Davies vacuum (the usual ground state)

Breaking any condition makes generating large non-Gaussianity possible, but it may also remain small

Perturbations remain nearly Gaussian

- Gaussianity is only preserved under linear transformations
- The square of a Gaussian is a chi-squared distribution
- Gravity is non-linear (thats why GR is so hard), but the tiny amplitude of perturbations mean that only negligible non-Gaussianity is generated on the CMB
- The part which always arises is known as the secondary non-Gaussianity, anything else is primordial non-Gaussianity
- Secondary non-Gaussianities could also teach us about fundamental physics, e.g. if gravity is not GR

Late time secondary non-Gaussianity

- As gravitational collapse becomes more effective, at later times (lower z) and shorter scales, the secondary non-Gaussianities grow

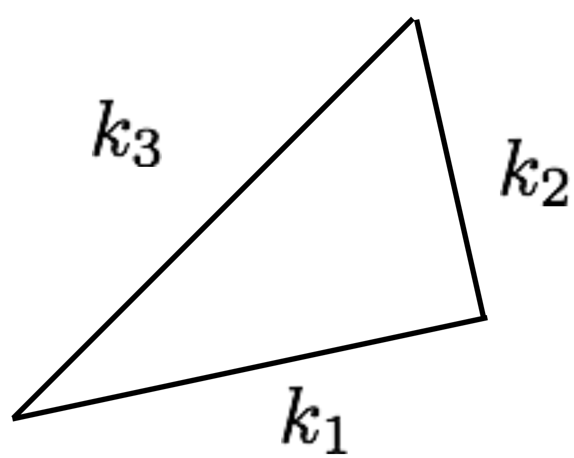
$$-1 \leq \frac{\delta\rho}{\bar{\rho}} = \frac{\rho - \bar{\rho}}{\bar{\rho}} \leq \infty$$

- This is not symmetric, unlike a Gaussian distribution
- This makes the CMB a cleaner and easier probe, subtracting secondary non-Gaussianities is hard work with LSS
- The fact that LSS provides 3D information and more modes makes this hard task ultimately worthwhile
- The best future constraints will be LSS/21cm line

Non-Gaussian information

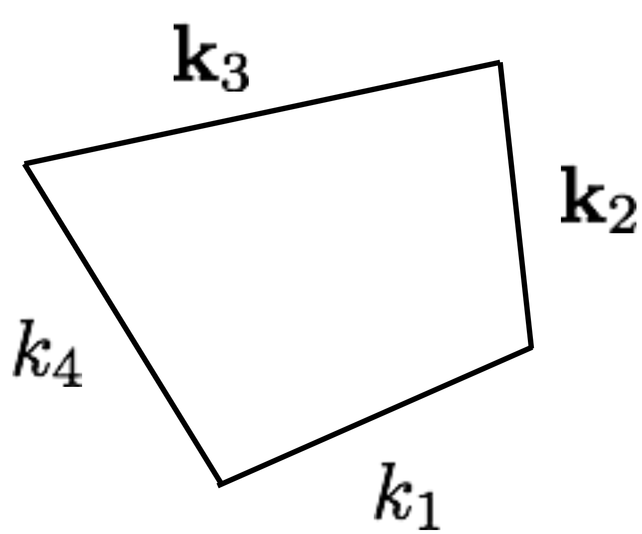
The bispectrum and trispectrum, connected 3 and 4-point functions

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = B_\zeta(k_1, k_2, k_3)(2\pi)^3 \delta^3(k_1 + k_2 + k_3)$$



3 parameters - function of 3 lengths

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle_c \equiv T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)(2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4)\,.$$



7 parameters in 3 dimensions
5 parameters in 2 dimensions
function of 1 wavelength and 2 wave vectors

Power spectrum is a function of just one wavelength

Too much information

- In practice, the signal-to-noise ratio for the bispectrum is always tiny for any given triangle
- Instead one usually considers a single shape (B as a function of 3 k’s) and only allow its amplitude to vary
- Infinitely many shapes can be chosen, but fortunately a few shapes are enough to cover a large class of models
- Blind searches over a whole basis of shapes can be made, but we must be very cautious if we detect a shape without prior theoretical motivation
- Even for Gaussian perturbations 1% of shapes will be detected at the 99% confidence level
- Beware of posterior detections, i.e. anomalies. But also beware of taking theoretical prejudices too seriously

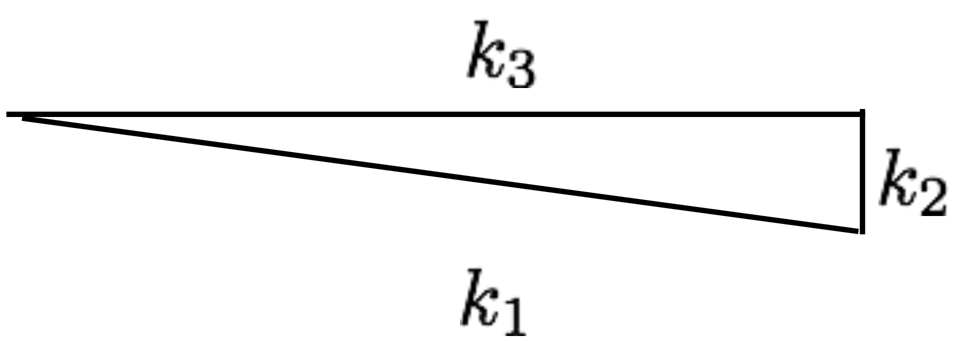
Bispectral shapes correspond to physical models

- A big advance in this field from the past decade is the realisation that large classes of models can all be characterised by a few shapes
- Non-Gaussianity is not just "it could be anything else"
- The degeneracy within a class of models, corresponding to a given shape can be broken (in principle) by measures of its scale dependence, and the trispectrum which contains even more information
- Of course all constraints should also be considered together with the power spectrum and gravitational wave limits

Komatsu et al; Decadel review 2009

The local model

- Examples include the local model which arises from super-horizon evolution of the curvature perturbation
- Zeta is conserved in single-field models on large scales, therefore this model only arises in models with multiple light fields present during inflation
- This shape has its largest signal in the squeezed limit, when one wavelength is very large
- Because a detection of a squeezed limit bispectrum would rule out all single-field models, the local model has been studied in great depth

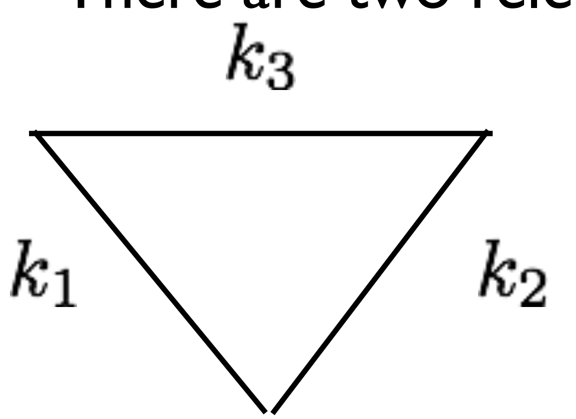


$$k_2 \ll k_1 \simeq k_3$$

Reviews include: Byrnes & Choi 2010; Wands 2010

The equilateral and orthogonal models

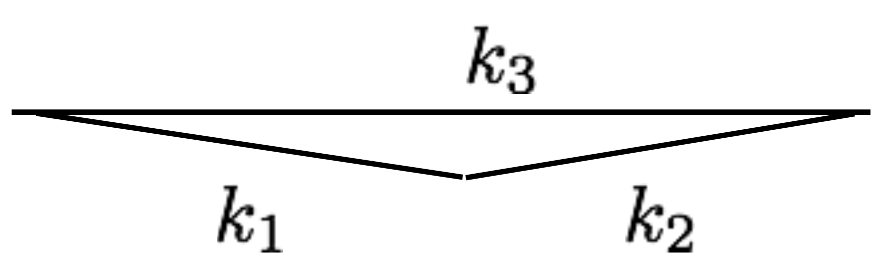
- If the inflaton field has a non-canonical kinetic term, the sound speed of perturbations becomes less than the speed of light
- E.g. k-inflation (Armendariz-Picon et al 1999) and DBI inflation
- This corresponds to large non-linearity/interaction terms in their equations of motion
- The dominant effect arises around horizon crossing, and so it acts to correlate modes of the same size (because modes cross when k=aH)
- The dominant signal therefore arises in the equilateral limit
- There are two relevant shapes, equilateral and orthogonal



$$k_1 \simeq k_2 \simeq k_3$$

The flattened/folded model

- If the initial vacuum state was not the simplest adiabatic vacuum state, known as the Bunch Davies vacuum state, then even the initial quantum field perturbations were not Gaussian
- This gives rise to a signal which is maximised in the limit of flattened/folded triangles
- Warning: This and many other models are named after the configuration which has the largest signal, but all models have a value for all shapes (i.e. for all possible values of k_i). This has caused a lot of confusion



$$k_1 \simeq k_2 \simeq k_3/2$$

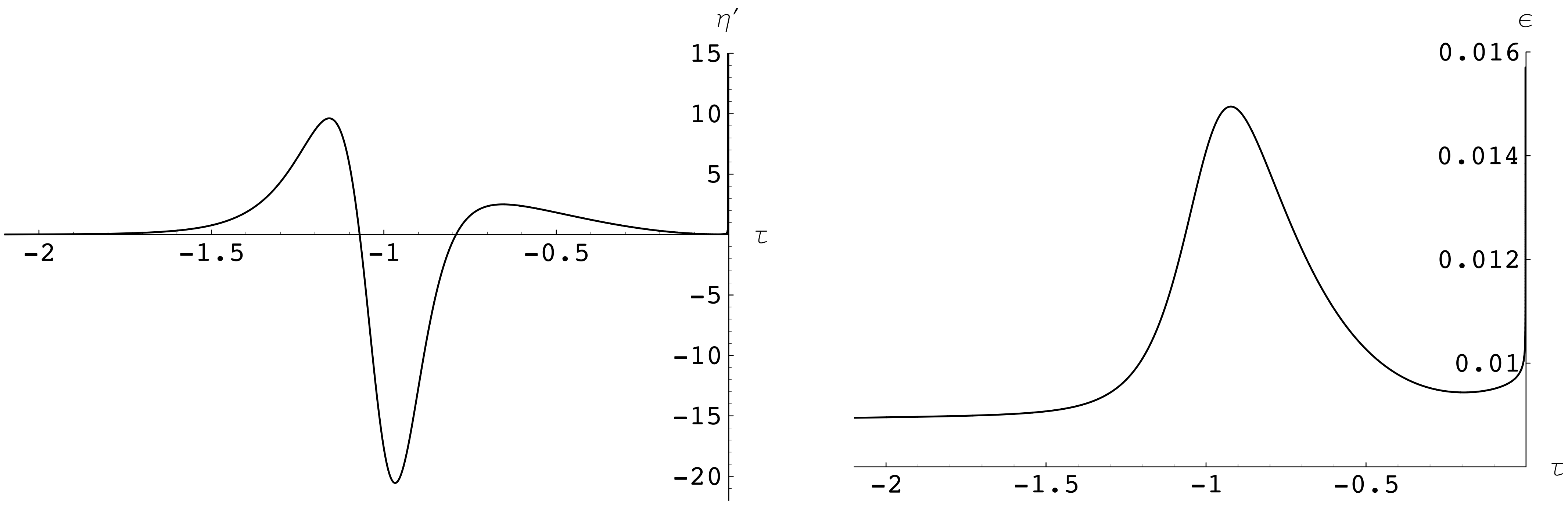
“Feature” models

- If there is a temporary breakdown in slow-roll during inflation, the modes which cross the horizon around this time can become strongly non-Gaussian
- The epsilon parameter cannot become large (without stopping inflation), but derivatives of this parameter can become much larger than unity for a short period (~ 1 e-folding)
- Only the modes crossing while these parameters are large are disturbed, so the non-Gaussianity is localised in Fourier space to the relevant scales
- These models can take almost any shape, normally oscillate and are hard to search for. Fortunately they also give rise to patterns in P(k) at the same scales, so combined P and B searches are possible

“Feature” model example

$$V(\phi) = \frac{1}{2}m^2\phi^2\left[1 + c\tanh\left(\frac{\phi - \phi_s}{d}\right)\right] \qquad \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Notice how the derivative of eta becomes very large, while epsilon remains small
Normally non-Gaussianity is suppressed by the smallness of the slow-roll parameters, here it can become large



Chen, Easther & Lim 2006

There are plenty of other models

- Models in which non-Gaussianity is localised in real space
- Active source models, such as magnetic fields or cosmic strings/ topological defects. These are not really primordial non-Gaussianity, but still of interest for studying fundamental physics
- Secondary non-Gaussianity could potentially also be used as a probe of fundamental physics, since if GR is not the correct gravity theory, the alternative might be more non-linear and generate a larger amplitude
- Combinations of all previously mentioned models
- Still many more exist, there are plenty of review articles
- But three shapes, local, equilateral and orthogonal have dominated, based on their theoretical motivation (perhaps also their simplicity)

Lecture 1 summary
Why study primordial non-linearities?

Even in the “golden era” of cosmology, there is a lot we don't understand

The LCDM “standard model” of cosmology is phenomenologically simple but not motivated by theory

The inflationary paradigm is still successful after decades, but has hundreds of models, non are compelling

Success of the many new surveys, both CMB and LSS, must be utilised and interpreted in terms of realistic models

We need as many observables as possible

Non-linear perturbations may contain much more information

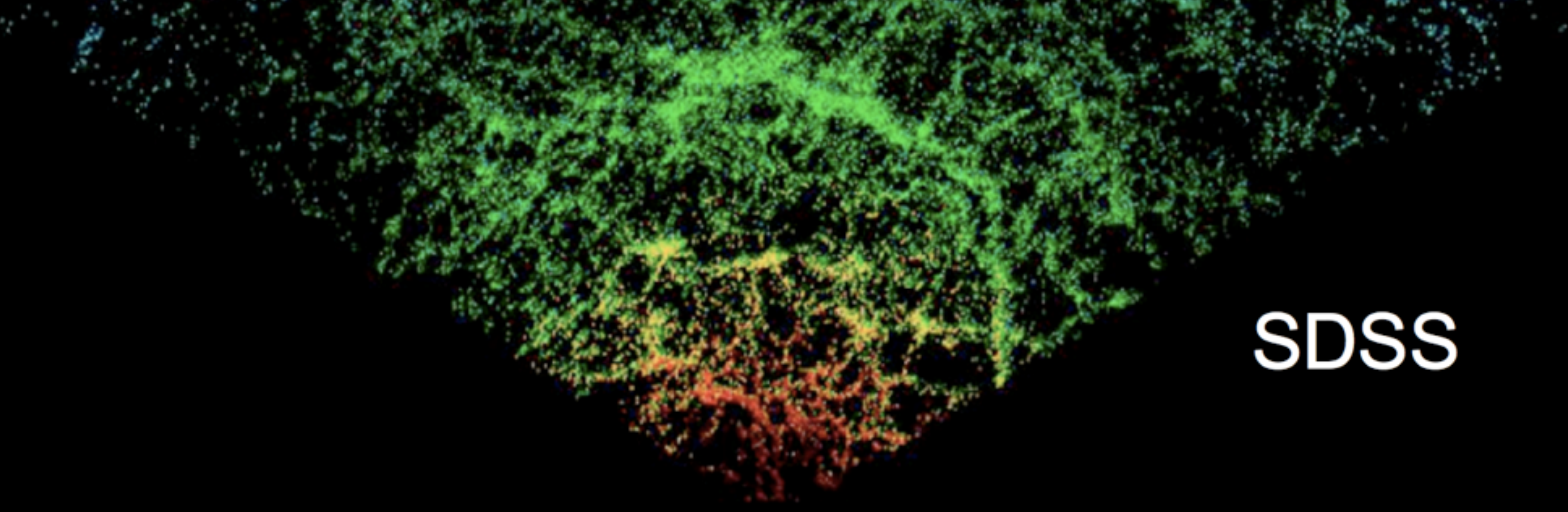
Many models: Encyclopaedia Inflationaris: Martin, Ringeval & Vennin 2013
368 pages, and its only about single field models. Poor referee

With large data sets, its neither practical nor desirable to search for every possible signal

To avoid endless discussions about posterior detections and anomalies

Theorists are needed to motivate template searches

We all know theorists need observers, but observers also need theorists



The local model of non-Gaussianity

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}(\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2(\mathbf{x}) \rangle)$$

- This name comes because it is a local function in real space. The annoying 3/5 factor due to original definition in terms of Newtonian potential, Phi=3 zeta/5 during matter era. The constant term is subtracted such that the expectation value is zero

- In Fourier space, locality is lost due to the convolution

$$\zeta(\mathbf{k}) = \zeta_G(\mathbf{k}) + \frac{3}{5} f_{\rm NL} \frac{1}{(2\pi)^3} \int d^3\mathbf{q} \zeta_G(\mathbf{q}) \zeta_G(\mathbf{k} - \mathbf{q})$$

- Notice that the second order term is very small, we may expect an excellent convergence. Fortunately this simple model has good theoretical motivation

- We will study this model in depth, including an inflationary scenario giving rise to large local non-Gaussianity in the next lecture

The local model: definitions

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}(\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2(\mathbf{x}) \rangle)$$

- f_{NL} is usually defined by

$$f_{NL} = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + 2\text{perms}}$$

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = P(k)(2\pi)^3 \delta^3(\mathbf{k} + \mathbf{k}')$$

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = B_\zeta(k_1, k_2, k_3)(2\pi)^3 \delta^3(k_1 + k_2 + k_3)$$

- We now show that its consistent with the definition of the local model
- Beware that the two definitions are not equivalent, even for local non-Gaussianity

The local bispectral shape

- The bispectral shape is

$$B^{\rm local} = 2\mathcal{P}^2 f_{\rm NL} \left\{ \frac{1}{k_1^3 k_2^3} + 2{\rm perms} \right\}$$

- Notice that this is largest in the squeezed limit, when one of the k's->0
- The Planck constraint (and WMAP9 in brackets) are

$$f_{\rm NL} = 2.7 \pm 5.8 \qquad (37.2 \pm 19.9)$$

- Notice the strong improvement, non-Gaussianity results were eagerly awaited from the Planck satellite

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}(\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2(\mathbf{x}) \rangle)$$

- Using the power spectrum amplitude, we see that the CMB is at least 99.9% Gaussian for this model.

A practical way to calculate perturbations

- Perturbation theory is complicated, and non-linear perturbation theory even more so. Fortunately for many models, we can use a much simpler technique to calculate the curvature perturbation, een at non-linear order

- This is the delta N formalism, based on the separate universe approach. It allows on to relate perturbed variables and background variables.

- It is only valid on super horizon scales (when the gradient terms are negligible), but a great deal of the interesting evolution of zeta does take place on very large scales

- We will just provide a sketch derivation of this technique, with references provided for those who want to see a formal derivation. The aim is to provide a "working knowledge" of one of the most powerful techniques in non-Gaussianity

delta N formalism: I

The flat, unperturbed FRW metric is given by

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j,$$

and neglecting vector and tensor perturbations, the perturbed space-space part is given by

$$g_{ij} = a(t)^2 e^{2\zeta(t,\mathbf{x})} \delta_{ij}$$

Therefore, the curvature perturbation ζ is the difference between the local expansion rate to the global expansion rate

$$\zeta(t, \mathbf{x}) = \delta N = N(t, \mathbf{x}) - N(t),$$

where

$$N(t) = \ln\left(\frac{a(t)}{a_{\text{initial}}}\right) = \int \frac{da}{a} = \int H(t) dt$$

N should be evaluated from a spatially flat hypersurface shortly after horizon crossing, to a final uniform energy density (or uniform Hubble) hypersurface.

delta N formalism: II

During inflation, the scalar fields provide the only contribution to the energy density, and within the slow-roll approximation their time derivatives do not provide a second degree of freedom. Therefore

$$\zeta = N(\phi_a + \delta\phi_a) - N(\phi_a),$$

where a labels the fields, and we may expand this as a Taylor series to find the key result

$$\zeta = N_a \delta\phi_a + \frac{1}{2} N_{ab} \delta\phi_a \delta\phi_b + \cdots$$

where the field perturbations should be evaluated at the initial time (horizon crossing), summation convention is used and

$$N_a = \frac{\partial N}{\partial \phi_{a*}}.$$

Notice that the derivatives of N depend only on background quantities, so provided that the statistical distribution of the field perturbations is known at horizon crossing, we can do perturbation theory using background quantities.

delta N formalism: Results

Assuming canonical kinetic terms, Bunch Davies vacuum and slow roll, the initial conditions are very simple. The field perturbations are Gaussian, and

$$\langle \delta\phi_a(\mathbf{k})\delta\phi_b(\mathbf{k}')\rangle = \delta_{ab}P_*(k)(2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}'),$$

where

$$\mathcal{P}_*(k) = \frac{4\pi k^3}{(2\pi)^3}P_*(k) = \left(\frac{H_*}{2\pi}\right)^2 .$$

Using these results, we may calculate (using the whiteboard) the power spectrum and amplitude of the local bispectrum

$$P_\zeta(k) = N_a N_a P_*(k) .$$

$$f_{\mathrm{NL}} = \frac{5}{6}\frac{N_a N_b N_{ab}}{(N_c N_c)^2} .$$

This result was first derived by Lyth and Rodriguez 2005, and is very useful since it allows us to calculate the bispectrum amplitude using only background quantities (and we know it has the local shape).

Single-field inflation

In the case of single-field inflation, the derivatives of N are given by

$$
\begin{aligned}
N' &\simeq \frac{\bar{H}}{\dot{\bar{\varphi}}} \simeq \frac{1}{\sqrt{2}} \frac{1}{M_{\rm Pl}} \frac{1}{\sqrt{\epsilon}} \sim \mathcal{O}\left(\epsilon^{-\frac{1}{2}}\right) , \\
N'' &\simeq -\frac{1}{2} \frac{1}{M_{\rm Pl}^2} \frac{1}{\epsilon} (\eta - 2\epsilon) \sim \mathcal{O}(1) ,
\end{aligned}
$$

where the slow-roll parameters are defined by

$$
\epsilon = \frac{M_{\rm Pl}^2}{2} \left(\frac{V'}{V}\right)^2 , \qquad \eta = M_{\rm Pl}^2 \frac{V''}{V} .
$$

This suggests that

$$
f_{NL} = \frac{5}{6} \frac{N''}{N'^2} = \frac{5}{6} (\eta - 2\epsilon)
$$

but since $f_{\rm NL}$ is slow-roll suppressed for this model, we should have also included the equally small non-Gaussianity of the field perturbations at horizon exit.

The final result, known as the Maldacena consistency relation, states that

$$
f_{\rm NL} \equiv \frac{5}{12} \lim_{k_1 \to 0} \frac{B(k_1, k_2, k_3)}{P(k_1) P(k_2)} = \frac{5}{12} (1 - n_s).
$$

See Creminelli and Zaldarriaga 2004 for a general proof, valid even for any single field model (even with non-canonical kinetic terms, breaking slow roll and a non Bunch-Davies vacuum state). The exciting result is that a detection of the bispectrum in the squeezed limit (similar to local non-Gaussianity) would rule out all single field models. A detection of any different shape of non-Gaussianity would not do this.

Single-source inflation

If we instead assume that a single-field generated the primordial curvature perturbation, which was not the inflaton field, then large local non-Gaussianity is possible. Many models in the literature fit into this case, for example

- the curvaton scenario (to be studied in depth)
- modulated (p)reheating (the duration of reheating varies with position)
- inhomogeneous end of inflation (the duration of inflation varies with position)

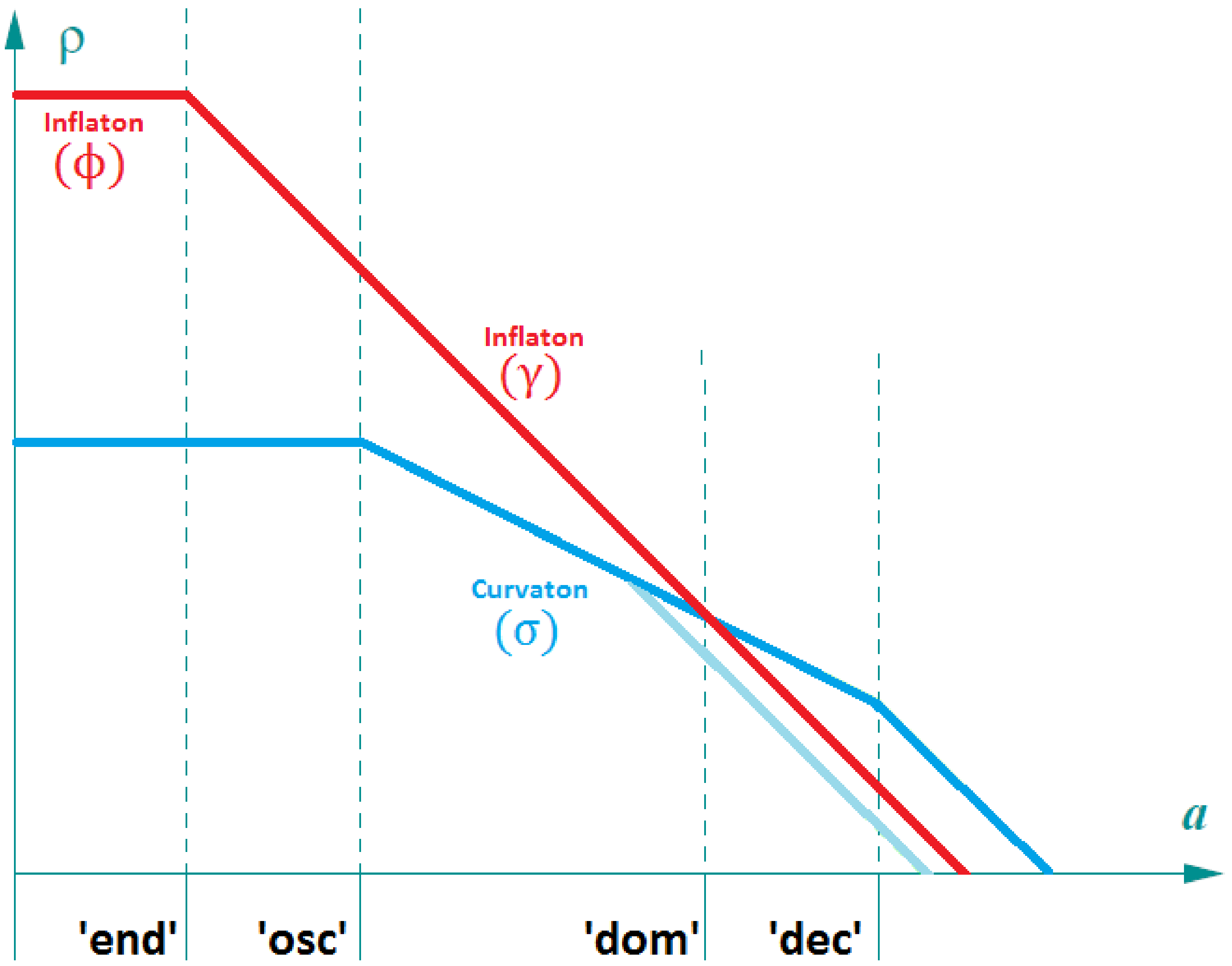
What they all have in common is that the duration of periods with differing equations of state varies with position. This is required in order that N becomes perturbed, since it only depends on the amount of expansion, i.e. H. In modulated reheating, the equation of state is 0 while the inflaton oscillates in a quadratic potential, but jumps to $1/3$ after the inflaton has decayed into radiation. This means that the varying the time of reheating will change the expansion history, and hence N and ζ.

The curvaton scenario

- So far, non-Gaussianity will have appeared quite abstract, with only the broadest reference made to inflationary models
- We will study a concrete scenario in some depth, and learn lessons which also apply to other scenarios
- The curvaton scenario is a simple physical model, in which there are two light fields present during inflation. Both fields are perturbed at Hubble exit, with field perturbations of order H
- One field drives inflation, the inflaton. A second field, the curvaton, generates the primordial curvature perturbation. This liberates the inflaton, because its perturbation spectrum no longer needs to match observations
- By definition, because the inflaton field drives inflation, at first it has the dominant energy density
- If the curvaton decays later than the inflaton, its "importance" grows in time

Curvaton background evolution:

Log of scale factor versus log of energy density



$$r_{\rm dec} \equiv \left. \frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma} \right|_{\rm decay}$$

Here we assume that the inflaton decays instantaneously into radiation, and that the curvaton has a quadratic potential. The general picture remains the same if you drop these assumptions.
After the curvaton decays, we have only radiation, which has perturbations imprinted onto it from the curvaton (and inflaton)

Curvaton evolution

$$V = \frac{1}{2} m_\sigma^2 \sigma^2$$

- For simplicity, we initially assume a quadratic potential for the curvaton, most papers in the literature do so

$$\ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0,$$
$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + V_{,\sigma\sigma}\delta\sigma = 0.$$

- Just for a quadratic potential, the two evolution equations are the same. This implies that the ratio of the two solutions is constant in time. The second equation also neglects back reaction from gravity, accurate as long as its energy is subdominant

Curvaton perturbations

$$\frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \frac{V(\sigma+\delta\sigma)-V(\sigma)}{V(\sigma)} = 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2$$

- This is a constant, see the previous slide
- The truncation at second order follows because we assumed a quadratic potential
- The above formula follows the local model, and if the above was the final result result for zeta we would have f_{NL}~1
- We should consider that the curvaton is not the only component of the universe

$$\zeta = \Omega_\sigma \zeta_\sigma \qquad \Omega_\sigma = \rho_\sigma / \rho_{tot}$$

$$f_{\rm NL} \propto \zeta^{(2)}/\zeta^{(1)2} \propto 1/\Omega_\sigma$$

Corrections

- The basic result is correct, the less efficient the transfer from the curvaton perturbation to total curvature perturbation, the larger the non-Gaussianity becomes. This holds quite generally

- This calculation made many approximations and assumptions. If we keep the assumptions, but drop the approximations, the full result is

$$f_{NL} = \frac{5}{4r_{\rm dec}} - \frac{5}{3} - \frac{5}{6}r_{\rm dec} \qquad r_{\rm dec} \equiv \left.\frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma}\right|_{\rm decay}$$

- If f_{NL} is large, $f_{\rm NL} \propto \frac{1}{r_{\rm dec}} \propto \frac{1}{\Omega_\sigma}$

- The Planck constraint, $f_{NL}<10$, tells us $r_{dec}>0.1$. A priori, 10^{-5} was possible.

- If the curvaton dominates before it decays $f_{NL}=-5/4$

Checking assumptions

- So far, we have neglected the inflaton field perturbations, phi=inflaton
- But remember that all light scalar fields are equally perturbed during inflation, at horizon crossing

$$\delta\phi = \delta\sigma = \frac{H}{2\pi}$$

$$\zeta \sim (1 - \Omega_\sigma)\zeta_\phi + \Omega_\sigma(\zeta_\sigma + \zeta_\sigma^2)$$

$$\zeta_\phi \sim \frac{\delta\phi_*}{\sqrt{\epsilon_*}}, \;\; \zeta_\sigma \propto \frac{\delta\sigma_*}{\sigma_*}$$

- If the curvaton is subdominant at decay, which is what we require for large non-Gaussianity, it also needs to have a very small initial vev in order that its perturbations are large compared to the inflatons

Mixed inflaton-curvaton scenario

The power spectra due to the two fields is

$$P_\phi \sim \frac{1}{\epsilon}\left(\frac{H_*}{2\pi}\right)^2, \qquad P_\sigma \sim \Omega_\sigma^2 \frac{1}{\sigma_*^2}\left(\frac{H_*}{2\pi}\right)^2,$$

and the total power spectrum is

$$P_\zeta = P_\phi + P_\sigma = (1+\lambda)P_\sigma, \qquad \lambda = \frac{P_\phi}{P_\sigma}.$$

The bispectrum is unchanged from the pure curvaton limit $(\lambda = 0)$,

$$B_\zeta = B_\sigma = \frac{1}{\Omega_\sigma} P_\sigma^2$$

but $f_{\rm NL}$ is reduced because the power spectrum is enhanced by the Gaussian inflaton field perturbations

$$f_{\rm NL} \sim \frac{B_\zeta}{P_\zeta^2} = \frac{B_\sigma}{P_\zeta^2} = \frac{1}{\Omega_\sigma}\frac{1}{(1+\lambda)^2}.$$

How can we distinguish Ω_σ and λ if $f_{\rm NL}$ is detected?

Scale-dependence of f_{NL}

$$n_{f_{NL}} \equiv \frac{\partial \log |f_{NL}|}{\partial \log k}$$

- Analogously to the power spectrum, f_{NL} is expected to have some scale dependence. This reflects evolution during inflation, e.g. it ends, so it cannot be exactly de Sitter
- It can distinguish between different non-Gaussian scenarios, not just between Gaussian and non-Gaussian models
- The amplitude of f_{NL} can be tuned in most non-Gaussian models, so a precise measurement of f_{NL} wont do this
- In contrast the scale dependence often can not be tuned independently of:
    1. f_{NL}
    2. spectral index of the power spectrum
- We should seek consistency relations between observables, test or rule our whole classes of models

Curvaton scale-dependence of f_{NL}

$$f_{\rm NL} \propto \frac{1}{\Omega_\sigma}\left(\frac{P_\sigma}{P_\zeta}\right)^2 \propto k^{2(n_\sigma - n_s)} \qquad n_\sigma - 1 = \frac{\partial \ln \mathcal{P}_\sigma}{\partial \ln k}$$

- In the curvaton limit, where we neglect the inflaton field perturbations

$$\lambda \to 0, \qquad n_{f_{\rm NL}} \to 0$$

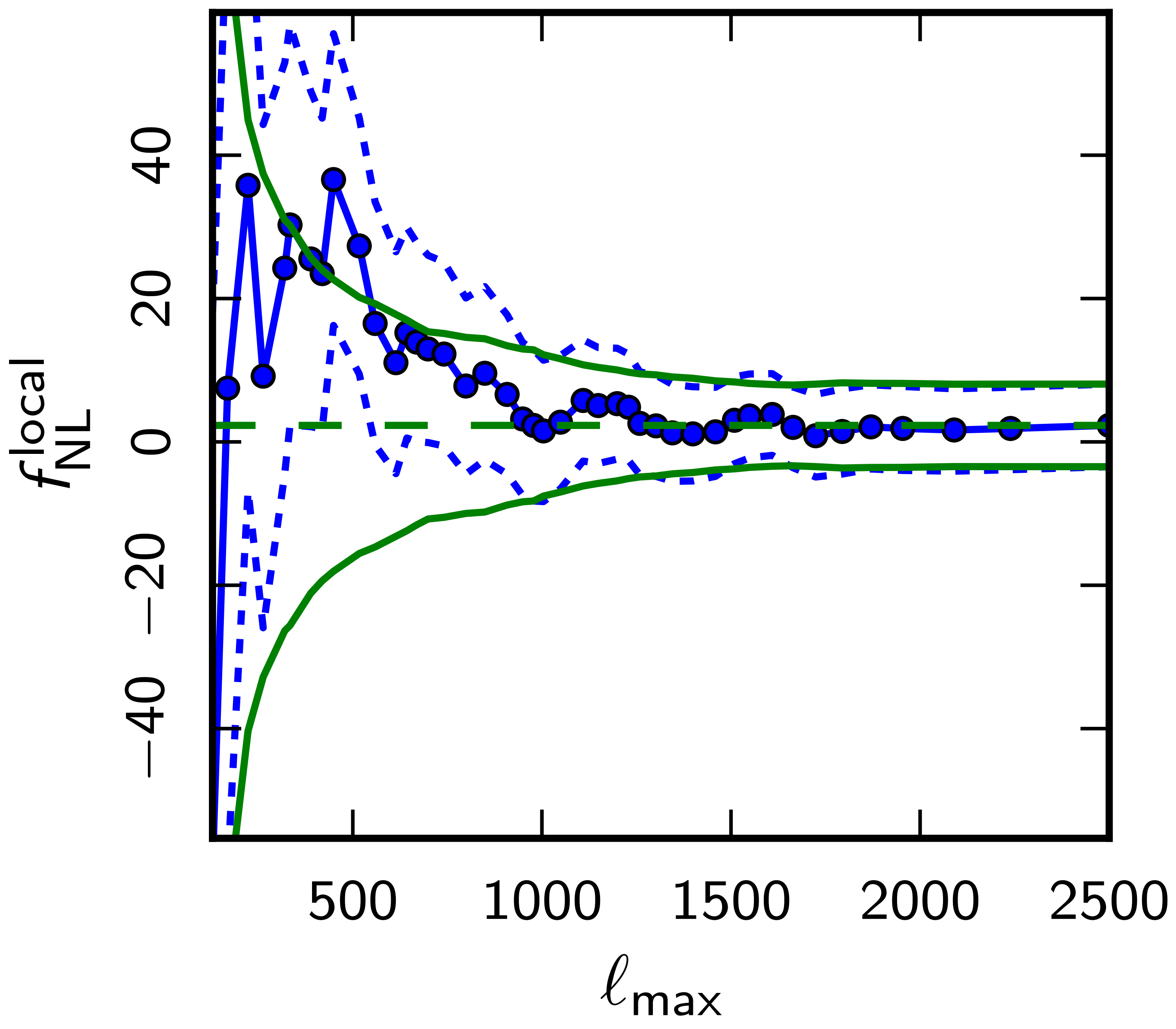
- If the inflaton perturbations are important, and the curvaton perturbations are scale invariant

$$n_\sigma - 1 \to 0, \qquad n_{f_{\rm NL}} \to -2(n_s - 1)$$

- However more generally, in the single source limit, scale independence only follows when there is a quadratic potential

$$n_{f_{NL}} \sim \frac{\sqrt{r_T}}{f_{NL}} \frac{V'''}{3H^2} \qquad r_T = \frac{P_T}{P_\zeta}$$

Planck and scale dependence



- WMAP had consistently found a preference for positive f_{NL}. Planck is consistent with this, because the low l modes do prefer a positive value

Introducing the trispectrum

ζ is a function of multiple Gaussian fields

Bispectrum and trispectrum

$$B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{6}{5} f_{NL}\left[P_\zeta(k_1)P_\zeta(k_2) + P_\zeta(k_2)P_\zeta(k_3) + P_\zeta(k_3)P_\zeta(k_1)\right]$$

$$T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \tau_{NL}\left[P_\zeta(k_{13})P_\zeta(k_3)P_\zeta(k_4) + (11 \text{ perms})\right] + \frac{54}{25} g_{NL}\left[P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + (3 \text{ perms})\right]$$

There are 3 k independent parameters $\quad k_{ij} = |\mathbf{k}_i + \mathbf{k}_j|$

$$f_{NL} = \frac{5}{6}\frac{N_A N_B N^{AB}}{(N_C N^C)^2} \qquad g_{NL} = \frac{25}{54}\frac{N_{ABC}N^A N^B N^C}{(N_D N^D)^3} \qquad \tau_{NL} = \frac{N_{AB}N^{AC}N^B N_C}{(N_D N^D)^3}$$

If only 1 field generates the primordial curvature perturbation,
2 independent parameters remain

$$f_{NL} = \frac{5}{6}\frac{N''}{(N')^2} \qquad g_{NL} = \frac{25}{54}\frac{N'''}{(N')^3} \qquad \tau_{NL} = \frac{(N'')^2}{(N')^4} = \frac{36}{25} f_{NL}^2$$

A general test of single-source models

- For all models in which only one field generates the primordial curvature perturbation (not the inflaton)

$$\tau_{\rm NL} = \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- In models where multiple fields contribute there is instead the Suyama-Yamaguchi inequality

$$\tau_{\rm NL} \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- For the mixed inflaton curvaton scenario

$$\tau_{\rm NL} = \frac{P_\zeta}{P_\sigma}\left(\frac{6 f_{\rm NL}}{5}\right)^2 \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- From Planck, $\mathrm{tau}_{\rm NL}<2800$ (95% confidence)

The second trispectrum parameter, g_{NL}

$$\frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \frac{V(\sigma+\delta\sigma)-V(\sigma)}{V(\sigma)} = 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2$$

- For a quadratic potential, we may truncate at second order, which implies $g_{NL}=0$. Quadratic potentials are simple to calculate with, but not preferred by fundamental theories. So g_{NL} has been unfairly neglected.
- $|g_{NL}| >> f_{NL}^2$ is possible with non-quadratic potentials
- Unfortunately g_{NL} is very hard to constrain, because its shape is maximised in fewer configurations than tau_{NL}. The current bound is $|g_{NL}|<10^5$, Planck has not yet produced a constraint

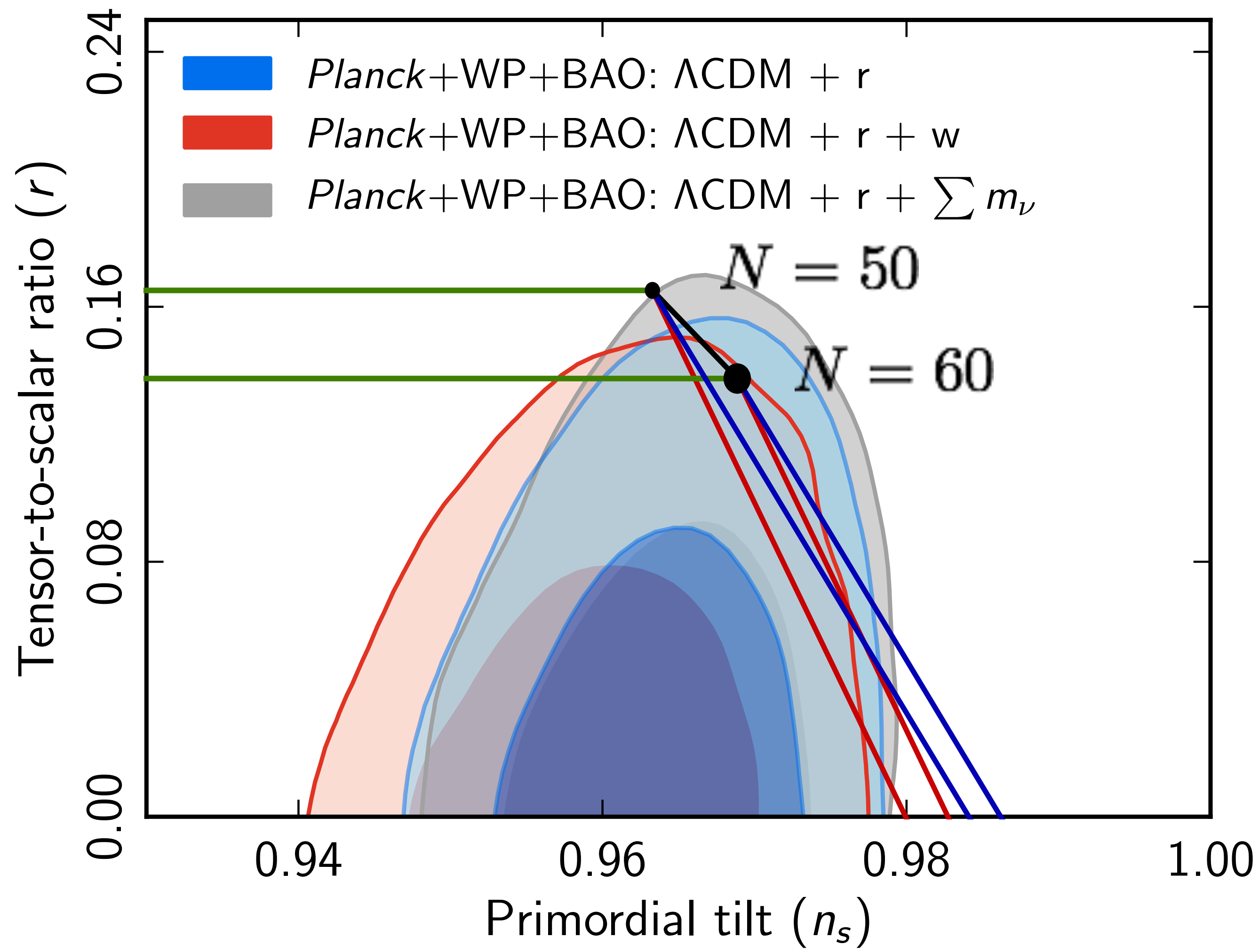
Curvaton summary

- Single-source scenario if $\tau_{\rm NL} = \left(\frac{6 f_{\rm NL}}{5}\right)^2$, i.e. the curvaton perturbations dominate
- Then detection of a large g_{NL} or scale dependence of f_{NL} would tell us the potential is non-quadratic
- Both the inflaton and curvaton field perturbations must contribute if

$$\tau_{\rm NL} \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- Again a large g_{NL} would signal a non-quadratic potential for the curvaton. The scale dependence of f_{NL} will not be zero, and provides further information
- An explicit example of how much we can learn (in principle) from non-Gaussianity

Curvaton post Planck



Red lines are for negligible curvaton mass, blue lines have m_sigma=m_phi/2. Green lines are the inflating curvaton regime, where it drives a second period of inflation.

Curvaton scenario has a lower bound on r_{dec} from the Planck satellite via f_{NL}. But only a detection of f_{NL}<-5/4 would rule it out. However, the simplest curvaton scenario, where both it and the inflaton field have quadratic fields may soon be ruled out. Changing the inflaton potential changes the quadratic curvaton predictions.

Work in progress with Marina Cortes and Andrew Liddle (to appear on the arxiv very soon)

General lessons learnt via the curvaton scenario

Not only for the curvaton scenario, for two field models of inflation, where one field dominates during inflation (the inflaton) one can often write zeta in the form:

$$\zeta \sim (1-r)\zeta_\phi + r(\zeta_\chi + \zeta_\chi^2)$$

Gaussian inflaton field   subdominant non-Gaussian field

$$\zeta_\phi \sim \frac{\delta\phi_*}{\sqrt{\epsilon_*}}, \ \zeta_\chi \propto \frac{\delta\chi}{\chi}, \ V(\chi) \propto \chi^2 \Rightarrow \zeta_\chi^{(2)} \propto \left(\zeta_\chi^{(1)}\right)^2 = constant$$

Curvaton scenario: $r\zeta_\chi \gg \zeta_\phi, \ r \simeq \Omega_\chi|_{\rm decay}, \ f_{\rm NL} \propto \frac{1}{r} \gtrsim 1, \ \tau_{\rm NL} = \left(\frac{6f_{\rm NL}}{5}\right)^2$

Mixed scenario: $|f_{\rm NL}| \propto \frac{1}{r}\left(\frac{P_\chi}{P_\zeta}\right)^2 \propto k^{2(n_\chi - n_s)}, \ \tau_{\rm NL} = \frac{P_\zeta}{P_\chi}\left(\frac{6f_{\rm NL}}{5}\right)^2 \geq \left(\frac{6f_{\rm NL}}{5}\right)^2$

Dominant quadratic curvaton: $f_{\rm NL} = -\frac{5}{4}, \ g_{\rm NL} = \frac{9}{2}$

r measures the efficiency of the transfer from the initially subdominant field, which is isocurvature during inflation

The less efficient the transfer, the more non-Gaussian the perturbations, and tau$_{\rm NL}$ is relatively more important

However the Gaussian inflaton perturbations are more likely to dominate in this limit

- Previous slide made several assumptions:
  - 2 fields, one of which is Gaussian
  - Quadratic potential (implies negligible g_{NL})
  - Conversion takes place after the end of inflation (important, things work differently if during slow roll, and often get slow-roll value of f_{NL})
- Apart from the third assumption, prediction of (local) $|f_{NL}|>1$ is quite generic.
- Can we observe $f_{NL}=1$, if so, when???

Non-canonical models

- Models with a non-canonical kinetic term give rise to non-Gaussianity, related to the reduced sound speed of perturbations

$$\mathcal{L} = P(X, \phi) \qquad\qquad X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- Canonical case: $\mathcal{L} = -X/2 - V(\phi)$
- Sound speed: $$c_s^2 = \frac{P_X}{P_X + 2XP_{XX}}$$

- Can generate two bispectral shapes, both maximised in the equilateral limit, and both satisfy $f_{NL} \sim 1/c_s^2$

Equilateral and orthogonal non-Gaussianity

$$B^{\text{equil}} = 6A^2 f_{\text{NL}}^{\text{equil}} \left\{ -\left(\frac{1}{k_1^3 k_2^3} + 2\text{perms} \right) - \frac{2}{(k_1 k_2 k_3)^2} + \left(\frac{1}{k_1 k_2^2 k_3^3} + 5\text{perms} \right) \right\}$$

$$B^{\text{ortho}} = 6A^2 f_{\text{NL}}^{\text{ortho}} \left\{ -3\left(\frac{1}{k_1^3 k_2^3} + 2\text{perms} \right) - \frac{8}{(k_1 k_2 k_3)^2} + \left(\frac{1}{k_1 k_2^2 k_3^3} + 5\text{perms} \right) \right\}$$

The orthogonal model was designed to be "orthogonal" to the equilateral model.
This means they won't be confused with each other by observations

How correlated different shapes are is defined by [Fergusson & Shellard 2008]

$$S(k_1, k_2, k_3) = \frac{1}{f_{\text{NL}}} (k_1 k_2 k_3)^2 \, B(k_1, k_2, k_3)$$

$$F(S, S') = \int_{V_k} S(k_1, k_2, k_3) S(k_1, k_2, k_3) \frac{1}{k_1 + k_2 + k_3} dV_k$$

$$\mathcal{C}(S, S') = \frac{F(S, S')}{\sqrt{F(S,S) F(S', S')}}$$

Non-Gaussianity constraints

- Constraints on the "headline" parameters are given, (WMAP9 in brackets)

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8 \qquad (37.2 \pm 19.9),$$
$$f_{\rm NL}^{\rm equil} = -42 \pm 75 \qquad (51 \pm 136),$$
$$f_{\rm NL}^{\rm ortho} = -25 \pm 39 \qquad (-245 \pm 100).$$

- A factor of 2-4 improvement with Planck
- All central values are close to zero
- For models with non-canonical kinetic terms, leading to a sound speed different from the speed of light: Planck => c_s>0.02
- One big implication is that single-field DBI inflation is (probably) ruled out, by the constraint on equilateral non-Gaussianity
- An extremely popular string motivated model of inflation

Comparing the constraints

$$f_{NL}^{\rm local} = 2.7 \pm 5.8 \qquad (37.2 \pm 19.9),$$
$$f_{NL}^{\rm equil} = -42 \pm 75 \qquad (51 \pm 136),$$
$$f_{NL}^{\rm ortho} = -25 \pm 39 \qquad (-245 \pm 100).$$

$$B^{\rm local} = 2\mathcal{P}^2 f_{\rm NL} \left\{ \frac{1}{k_1^3 k_2^3} + 2{\rm perms} \right\}$$

$$B^{\rm equil} = 6A^2 f_{\rm NL}^{\rm equil} \left\{ -\left(\frac{1}{k_1^3 k_2^3} + 2{\rm perms} \right) - \frac{2}{(k_1 k_2 k_3)^2} + \left(\frac{1}{k_1 k_2^2 k_3^3} + 5{\rm perms} \right) \right\}$$

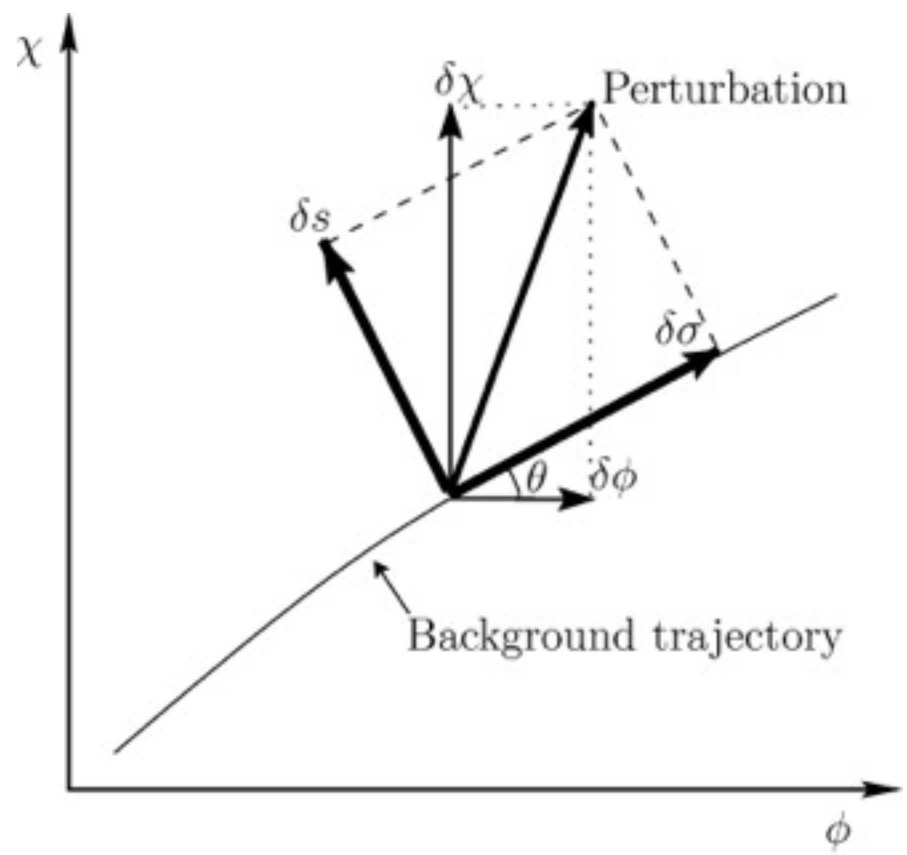
$$B^{\rm ortho} = 6A^2 f_{\rm NL}^{\rm ortho} \left\{ -3\left(\frac{1}{k_1^3 k_2^3} + 2{\rm perms} \right) - \frac{8}{(k_1 k_2 k_3)^2} + 3\left(\frac{1}{k_1 k_2^2 k_3^3} + 5{\rm perms} \right) \right\}$$

- All bispectral shapes are normalised to give the same amplitude for an equilateral triangle. This is "unfair" to models which have the largest signal-to-noise ratio in this configuration (e.g. equilateral and orthogonal), while the local model is minimised for an equilateral triangle.

- Therefore, the difference in the error bars is arguably just an artefact of the chosen normalisation.

Large non-Gaussianity from slow-roll inflation?

- Yes — Byrnes, Choi and Hall '08
- Impossible with single field, so multi-field



Adiabatic perturbations parallel to background trajectory

Isocurvature perturbations perpendicular to background trajectory

The perturbations are correlated and zeta evolves if the trajectory curves

Theta measures angle between background trajectory and axes

Need to track the large scale evolution of zeta up to second order
Tractable for a sum or product separable potential

Vernizzi and Wands '06; Choi, Hall and van de Bruck '07,
see also Rigopoulos, Shellard & van Tent '05

Focus on two-field product separable potential, sum potential are analogous

$$W(\varphi,\chi) = U(\varphi)V(\chi)$$

f_{NL} depends on 5 slow-roll parameters and initial and final theta - complicated

$$f_{NL} = \sum_{i=1}^{5} f_i(\sin^2\theta^*, \sin^2\theta^e)\mathrm{s.r.}_i$$

$$\sin^2\theta = \frac{\dot{\chi}^2}{\dot{\phi}^2+\dot{\chi}^2} = \frac{\epsilon_\chi}{\epsilon}$$

$$\epsilon_\chi = \frac{M_P^2}{2}\left(\frac{W_\chi}{W}\right)^2, \quad \epsilon = \epsilon_\phi + \epsilon_\chi$$

If $f_i \sim \mathcal{O}(1)$ assumed, then f_{NL} is slow-roll suppressed

Region with large non-Gaussianity

$$f_{NL} \simeq \frac{5}{6} \frac{\sin^6 \theta^e}{\left(\sin^2 \theta^* + \sin^4 \theta^e\right)^2} \left(2\eta^e_{\chi\chi} - \eta^*_{\chi\chi}\right)$$

Often $2\eta^e_{\chi\chi} - \eta^*_{\chi\chi} \simeq \eta_{\chi\chi}$

Then $|f_{NL}| \gtrsim 1$ provided that:

$$\sin^2 \theta^* < \eta^2_{\chi\chi}, \qquad \sin^2 \theta^e < |\eta_{\chi\chi}|, \qquad \frac{\sin^2 \theta^e}{\sin^2 \theta^*} > \frac{1}{|\eta_{\chi\chi}|}$$

Requires a trajectory with one field very subdominant, but that grows during inflation (and remains subdominant). Trajectory must curve.

All slow-roll parameters remain small through inflation, the large fNL is not associated with a breakdown in slow roll
Only possible for some potentials

Can interpret this as at least a 1% tuning of the initial conditions

Byrnes, Choi and Hall '08

Quadratic * exponential potential

Byrnes, Choi & Hall '08

$$W = \frac{m^2}{2}\varphi^2 e^{-\lambda\chi^2/M_P^2} \simeq \frac{m^2}{2}\varphi^2\left(1 - \frac{\lambda\chi^2}{M_P^2}\right)$$

Interested in the regime where the exponential is order unity so the phi field drives and ends inflation

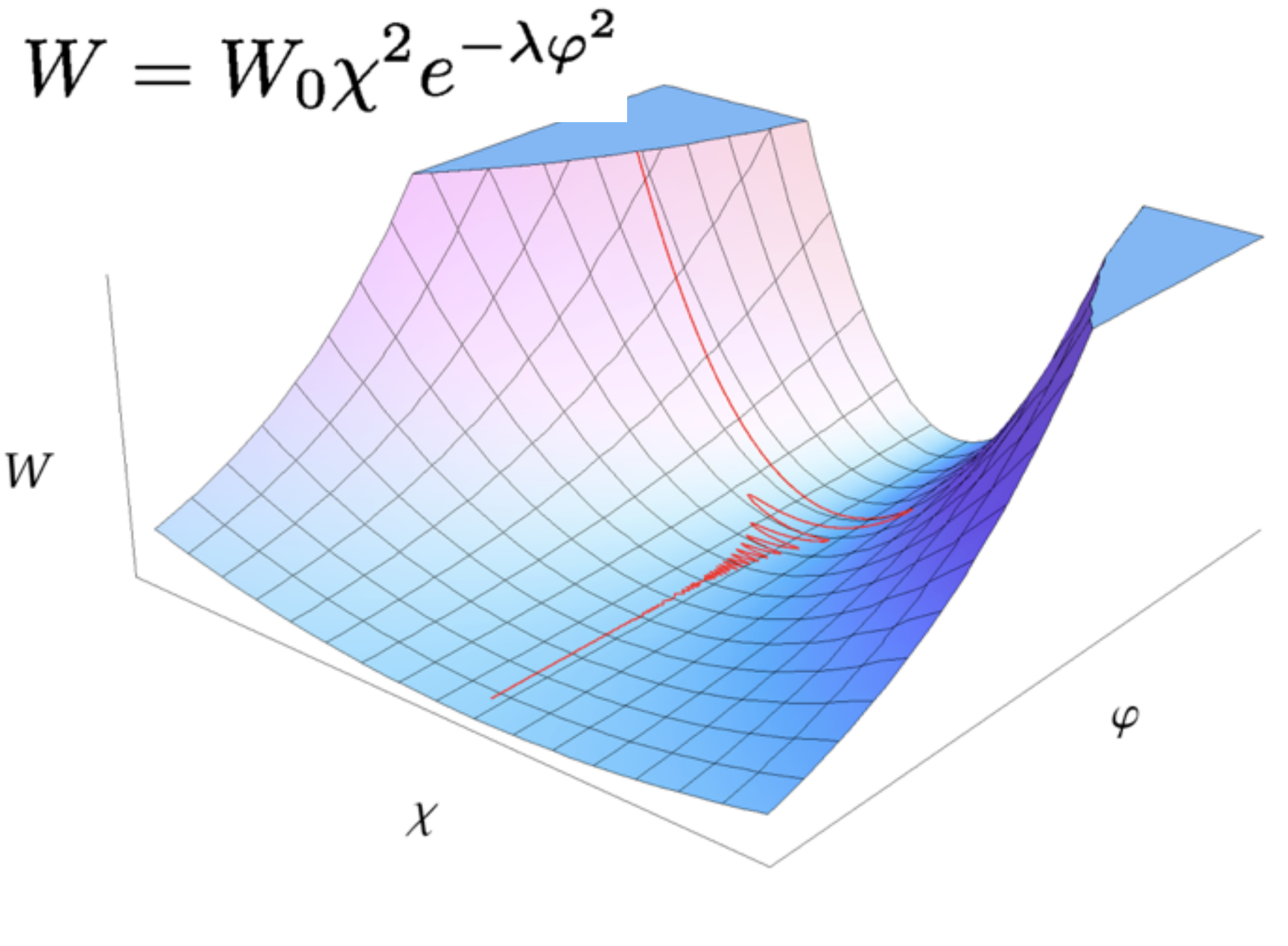
$$\epsilon_\chi = 2\lambda^2\frac{\chi^2}{M_P^2}, \quad \epsilon_\varphi = \frac{2M_P^2}{\varphi^2} \quad \sin\theta^2 \simeq \frac{\epsilon_\chi}{\epsilon_\varphi} \ll 1$$

the chi field is subdominant throughout inflation but grows, the conditions for large non-Gaussianity can be satisfied, provided that:

$$\lambda \sim 10^{-2}, \quad \chi_*^2 \ll M_P^2$$

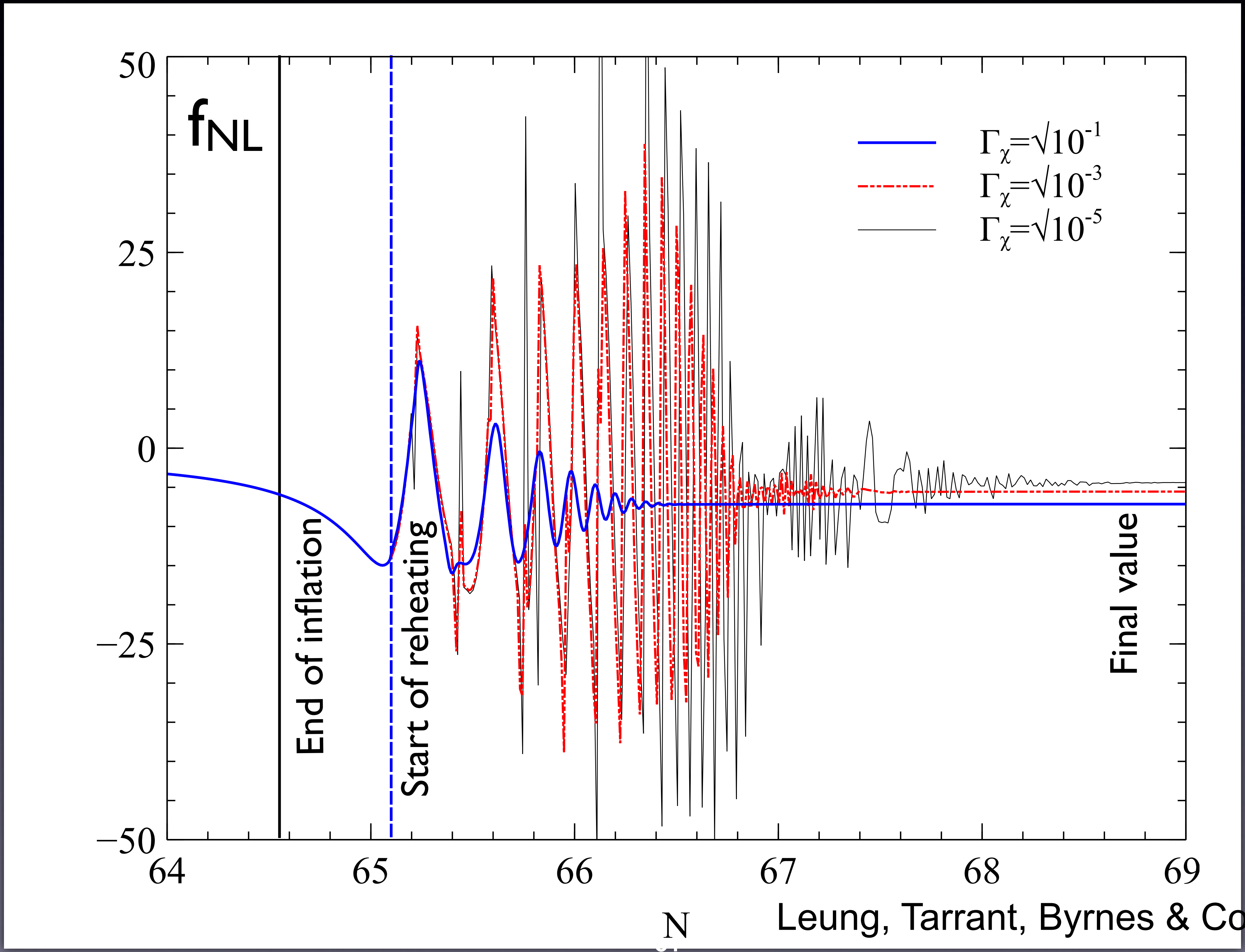
The curvaton scenario also requires a small initial vev, in order to generate large perturbations. This is quite generic.
The calculation performed so far is only valid during inflation, while isocurvature perturbations exist, zeta can continue to evolve

Reheating may change the result



Evolution of f_{NL}

$$\ddot{\phi}_i + (3H + \Gamma_{\phi_i})\dot{\phi}_i + W_{,\phi_i} = 0$$



Leung, Tarrant, Byrnes & Copeland '12

The difficult topic of reheating

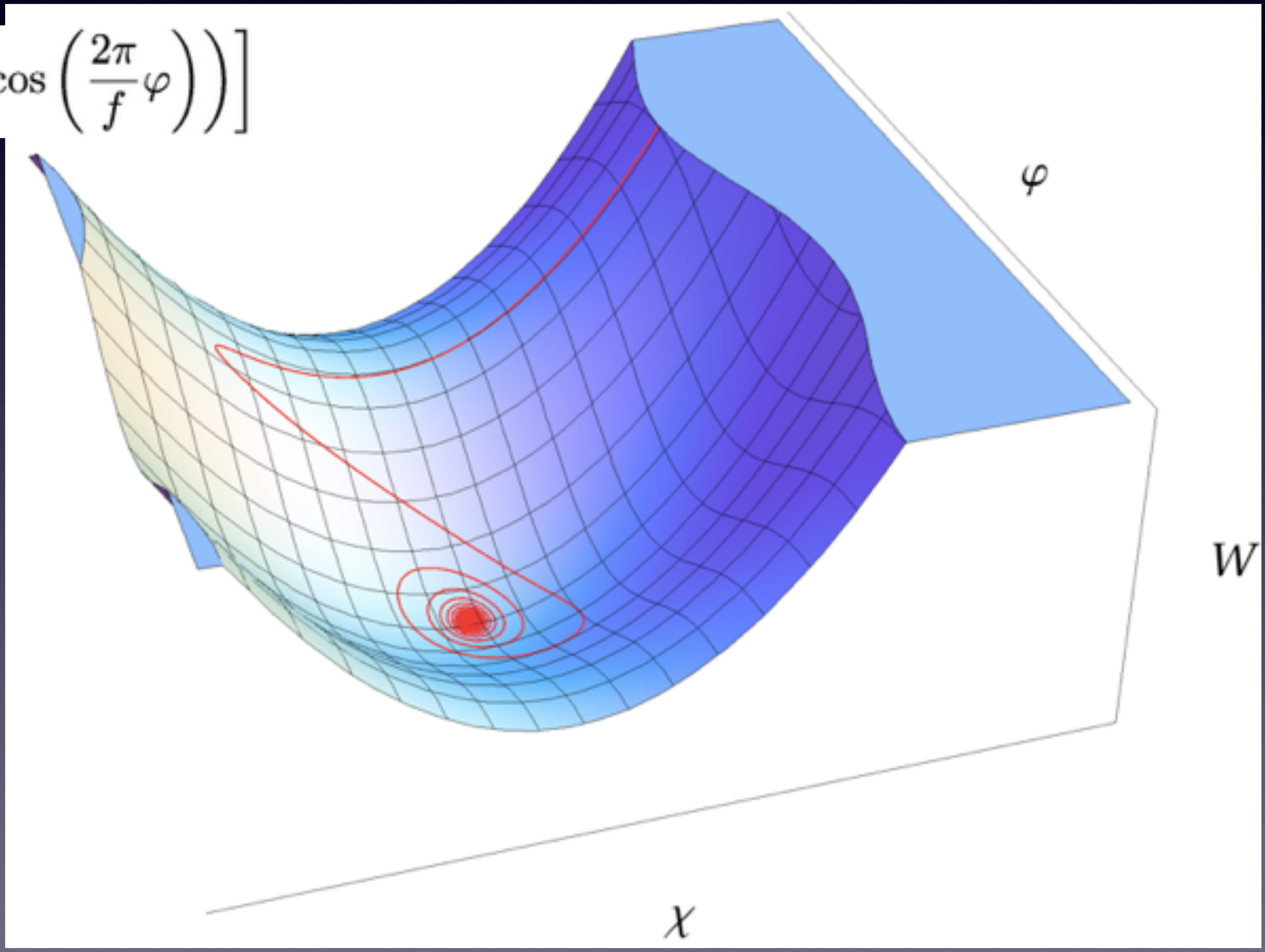
- Its a somewhat neglected topic, but reheating happened
- In single field models, zeta is conserved (on super horizon scales), so not so important
- But even in single field, affects the value of "N" we should use (number of e-foldings), biggest uncertainty for predictions of "chaotic inflation"
- Multifield models, the curvature perturbation continues to evolve on all scales, no excuse to ignore this!
- Inflation alone doesn't specify observables, need to keep calculating
- Exception if adiabatic attractor reached during inflation, f_{NL} decays to a small value except in special cases. This is not typically not the case with reheating

Leung et al '12, Meyers & Sivandam '10, Kim, Liddle & Seery '10; Watanabe '11

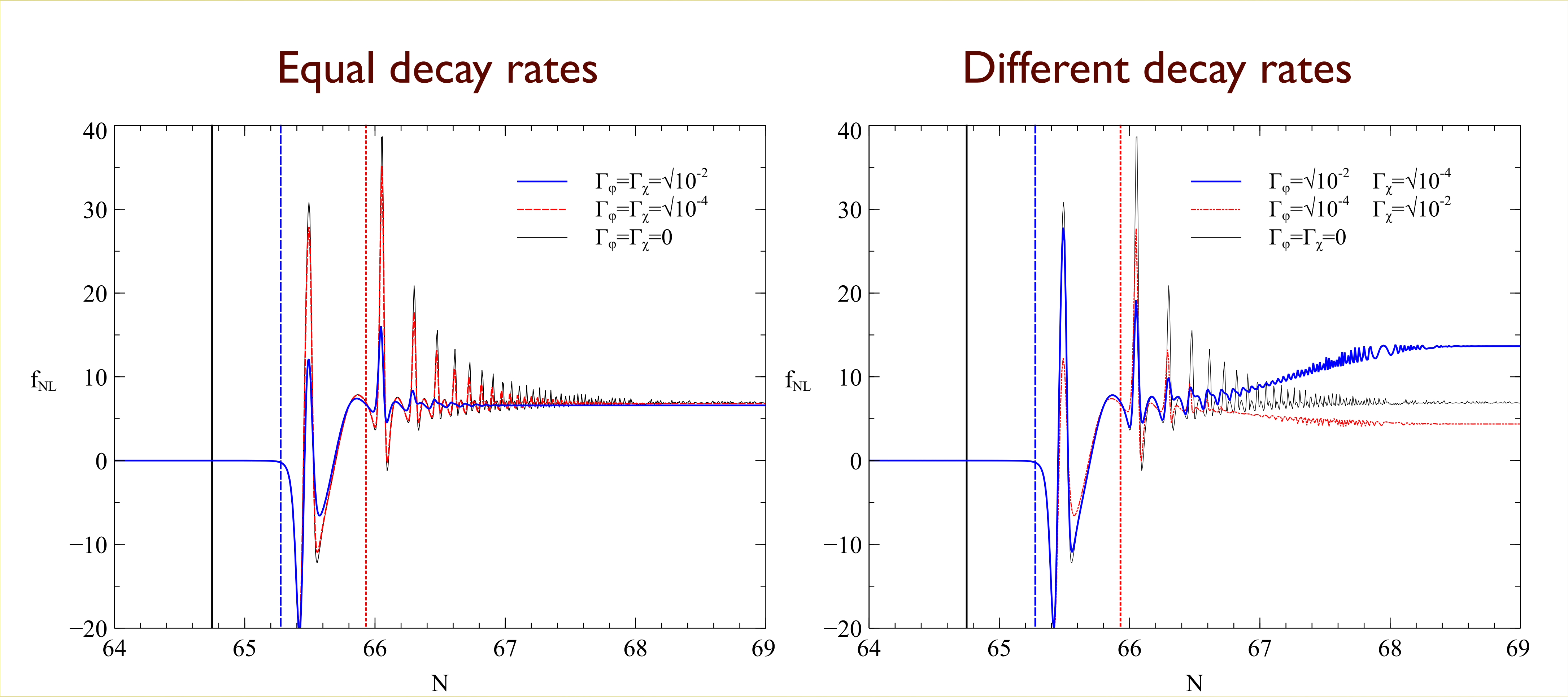
Two fields oscillating

$$W = W_0 \left[\frac{1}{2}m^2\chi^2 + \Lambda^4 \left(1 - \cos\left(\frac{2\pi}{f}\varphi\right)\right)\right]$$

Credit to Ewan Tarrant for figure



f_{NL} evolution



Notice that the final value of f_{NL} doesnt change if the decay rates of the two fields are equal

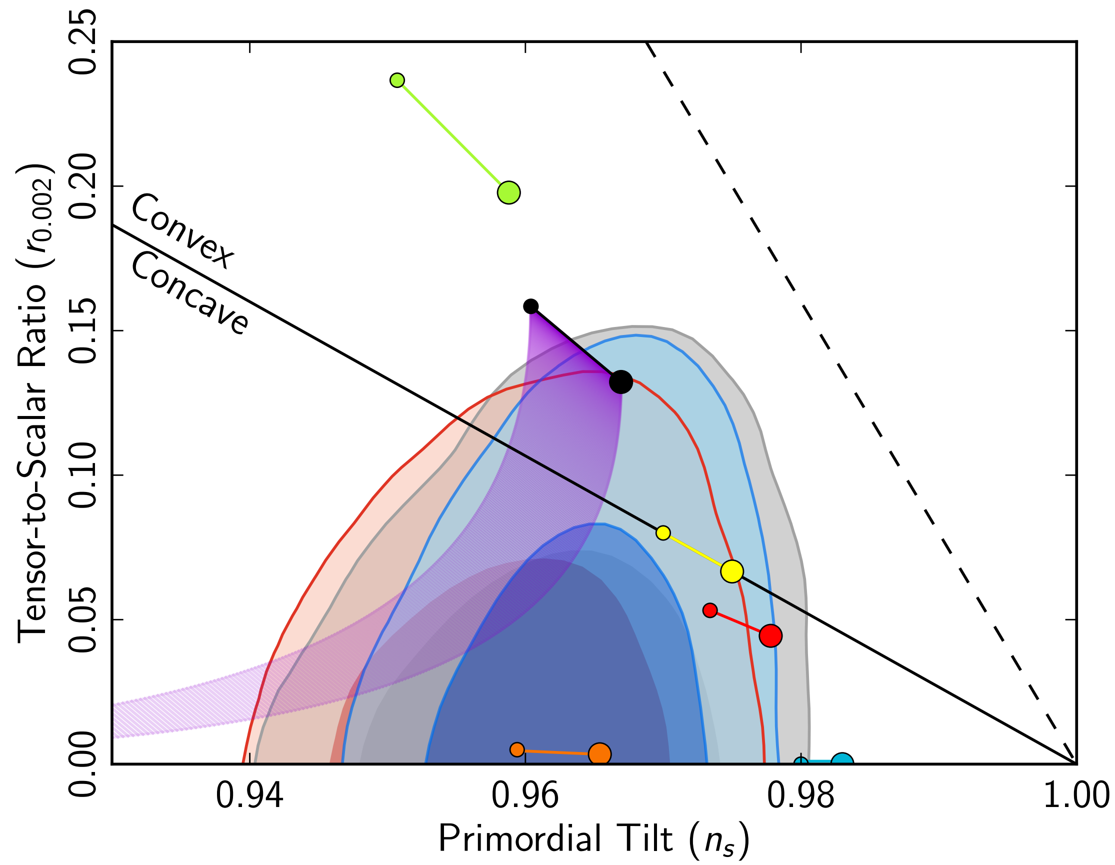
In all cases, f_{NL} is nearly zero until reheating

A calculation performed until the end of inflation would give completely wrong answers

Some "take home" messages

- In multiple field inflation, it is "difficult" rather than "easy" to generate observable non-Gaussianity
- We dont have a fundamental theory to tell us the parameter values and initial conditions, but for many choices f_{NL} is slow-roll suppressed. Inflation is not very predictive
- The curvaton scenario and modulated reheating, etc, do predict f_{NL}~1 or larger (potentially even 10^5). f_{NL}~10 is popular today…
- In general, neither Planck nor any other foreseeable experiment can rule out multiple field inflation, or even push it into a finely tuned regime
- Accurate calculations are hard, even numerically it is quite difficult and hard to scan large parameter spaces.
- Observables may continue to evolve during reheating. The inflaton should couple to something in order to decay

Planck results: Vanilla rules

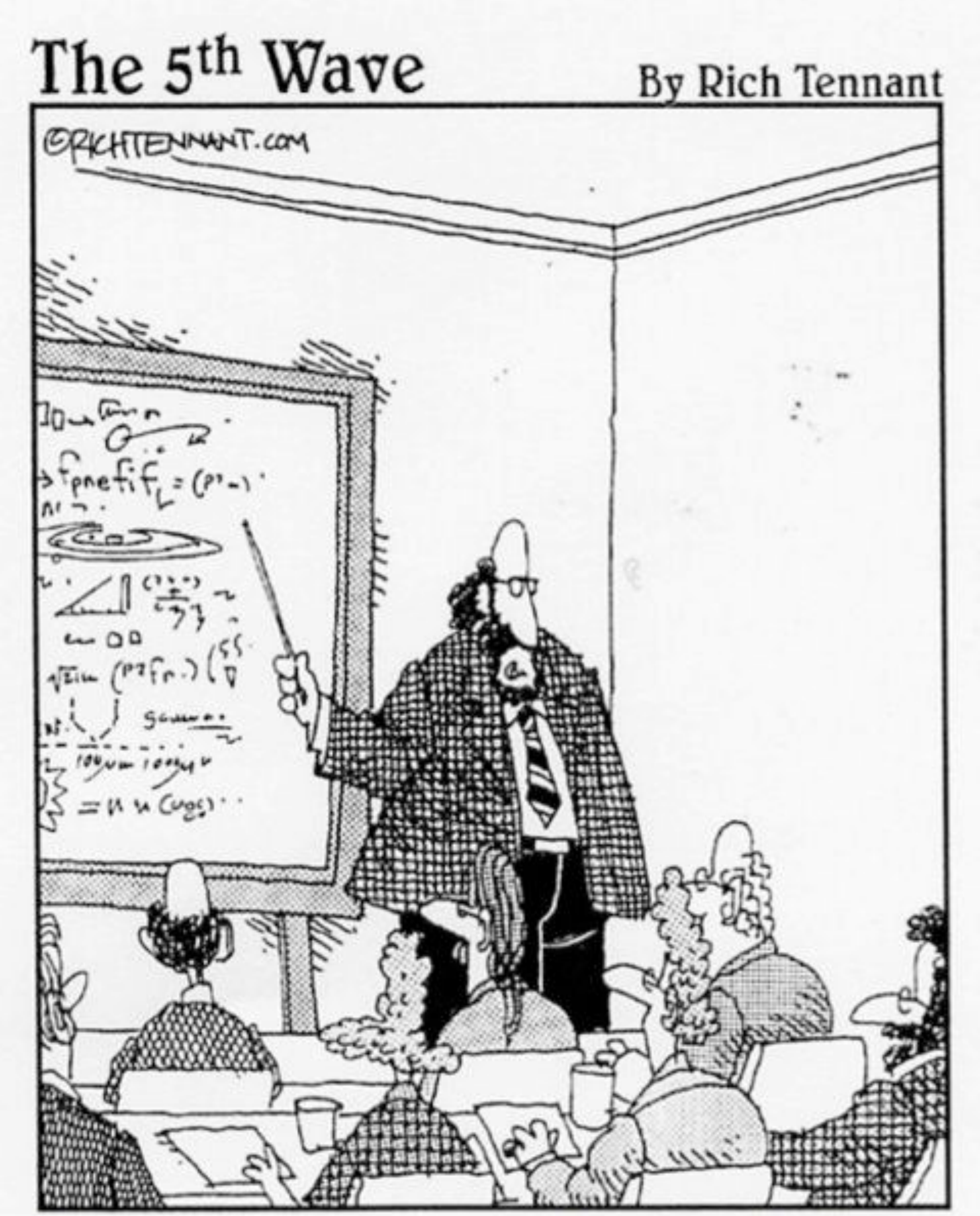


- Except for anomalies at under the 3 sigma level, do they point to anything primordial?
- For example, the power spectrum amplitude is not quite isotropic and there are some hints of "wiggles" in the power spectrum
- Is this a surprise?

Only two measured inflationary parameters

- The spectral index, all other parameters consistent with zero (tensors, isocurvature modes, non-Gaussianity, running of spectral index, cosmic string contribution,…)
- We know the amplitude of perturbations since COBE, but for all models this is an overall scaling of the potential, which is not predicted
- Planck does find preference for a concave potential
- Hence a negative mass squared at horizon crossing, but must have a positive mass squared at the minimum, if the potential gives a "graceful exit" from inflation - alternatively could have multiple fields so direction of slow roll changes
- Non-trivial evolution of the potential during inflation, monomial potentials (chaotic inflation) are disfavored - need an extra model parameter
- However, notice that it was only in combination with the non-detection of gravitational waves that one finds evidence for a concave potential
- Shows that measuring a parameter to be close to zero is still a measurement, and may have important implications

Even things which may never be discovered are important



"After the discovery of 'antimatter' and 'dark matter', we have just confirmed the existence of 'doesn't matter', which does not have any influence on the Universe whatsoever."

Is there anything which Planck did not do?

- Non-Gaussianity could be anything, so infinitely many things left to do!
- But of the "mainstream" targets, g_{NL} is the only obvious missing target
- In fact, τ_{NL} was the only trispectral shape to be constrained so far, huge range left to do (but difficult)
- τ_{NL} is large in the squeezed and collapsed limits, g_{NL} only in the squeezed limit
- WMAP and LSS constraints are weak, $|g_{NL}|<\text{few}*10^5$

Future prospects

- More shapes to be searched for with Planck, lots to do especially with the trispectrum
- For shapes already constrained, the local model has the best prospects (scale dependent bias)
- The galaxy bispectrum is quite poorly explored
- Don't expect significant observational improvements before Euclid
- Higgs field is likely to be a second light degree of freedom during inflation (unless itself the inflaton, requires huge non-minimal coupling to gravity)
- Anomalies such as power spectrum modulation may be non-Gaussian signatures (wait for polarization)
- Large scale magnetic fields definitely exist and are non-Gaussian

Non-Gaussianity FAQs

Personal opinions follow

Do the Planck non-Gaussianity constraints imply that there is negligible non-Gaussianity?

Not really. For the local model of non-Gaussianity, they do imply the sky is over 99.9% Gaussian, which is a remarkable result. For other templates, the constraint could be much weaker. But the constraint $|f_{\rm NL}| \lesssim 10$ are still two to three orders of magnitude larger than the single-field consistency relation for the squeezed limit, $f_{\rm NL} \simeq n_s - 1$. Clearly a large window is left for models which deviate from this consistency relation, but have a level of non-Gaussianity which is not yet be detectable.

Do the Planck non-Gaussianity constraints imply that alternatives to single field inflation are strongly disfavoured?

No. Single field inflation remains consistent with the observations, which does suggest they should be preferred from a Bayesian/Occams razor perspective. This was also true before we had Planck results. However its important to bear two points in mind: 1) A model which is parametrised with the fewest parameters might not be the simplest or most natural from a model building perspective, (we know little about physics at the inflationary energy scale) and 2) there are many multiple field models which predict non-Gaussianity with $|f_{\rm NL}| \ll 1$, and hence are far from ruled out.

Is there a natural target for future non-Gaussianity experiments?

Yes. Several models which convert an isocurvature perturbation present during inflation into the primordial adiabatic perturbation after inflation have a large parameter range in which $f_{\rm NL} \sim 1$. For example, the simplest version of the curvaton scenario, quadratic potential plus dominant at the decay time (which it will be the case if it decays sufficiently late) makes a definite prediction, $f_{\rm NL} = -5/4$. Similarly, a particularly simple realisation of modulated reheating predicts $f_{\rm NL} = 5/2$. Hence having an experiment which is capable of discriminating between $f_{\rm NL} = 1$ and $f_{\rm NL} = 0$ would have great value in disfavouring popular non-Gaussian models.

What are the prospects for future non-Gaussianity measurements?

The final Planck data release, which will contain double the observation time compared to the first release as well as Planck polarisation data, is expected to only lead to a relativitely modest improvement to the $f_{\rm NL}$ constraints, about 20%, compared to a factor of two for several other cosmological paremeters including the spectral index. The next significant improvement in the constraint for $f_{\rm NL}^{\rm local}$ is expected in about a decade from the Euclid survey, which is forecasted to reach and error bar of around 2. Beyond this, there is no clear timeline to future experiments which will have even tighter constraints, although several experiments have been proposed, for example Core, Pixie, etc.

Which forms of non-Gaussianity can we best constrain with future experiments?

Currently, the only concrete expectation for a significant improvement in non-Gaussianity constraints comes from the Euclid satellite. The forecasts have mainly been made for the scale dependent halo bias, which is sensitive to the squeezed limit of the bispectrum and hence primarily to local non-Gaussianity. The prospects for the other shapes is weaker, but limited work has been done on the galaxy bispectrum and using this as an estimator could potentially improve sensitivity to all shapes of the bispectrum. This work is very challenging since the secondary signal from non-linear collapse is much larger than the primordial signal (implying observations will have to deal with many potentially large systematic effects), and even with Gaussian initial conditions, structure formation is a hard topic.

Conclusions

- Even today, non-Gaussianity arguably remains the best window onto the early universe. It has the potential to provide far more information than the power spectrum
- Constraining a parameter to be close to zero is an important measurement. Non-Gaussianity is very well constrained, the local model must produce less than 0.1% non-Gaussianity
- Even tight non-Gaussianity constraints wont rule out multi-field inflation. Reheating and the Higgs discovery may even prefer it. A way to theoretically discriminate between the plethora of surviving models is required
- A few bispectral shapes cover the predictions of many classes of models
- When can we discriminate between $f_{NL}=1$ and 0? Important target
- Progress is needed on top-down theories, reheating, initial conditions for multi field models

Muito obrigado aos organizadores