

{ \underline{x}^2 † \underline{x} }:

Dialectical Ideography

A Contribution to the Immanent Critique of Arithmetic

Part I. c.

Prolegomena: Briefings on $\underline{\mathbb{H}}$

Distributed «Samizdat» by

Foundation *Encyclopedia Dialectica*



Version: **10.11.12.3871**

Last Updated: **12 November 2006 C.E./B.U.E.**

First Distributed: **25 November 1999 C.E./B.U.E.**

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Note: This essay is a partially self-exemplifying exposition of, and a record of an ongoing self-critique of, the ideas advanced herein. It is a 'meta-dynamical' and 'meta-evolving' conceptual object. Edition index [self-edit iteration number] and last revision date are stated on the title page; the most recent changes are coded as magenta-colored text. The time sequence of changes in the form / content of this essay is predicted to be both an illustration and an instantiation of the meta-model of *ideo-ontological ideo-meta-dynamics* that this essay explores, as well as of the 'homeomorphic defect' of that meta-model. We expect that successive editions of this document will document an 'ideo-onto-dynamasis' rather than an 'ideo-onto-stasis'; a 'meta-evolving ideo-ontology'; a 'multi-meta-ontic idea-cumulum'; an expanding, and ever 'thickening', increasingly inter- and intra-connected, 'inter-acted' network of 'inter-implicatory', 'inter-determinate', 'inter-generative' ideas, elaborated on a mounting count of 'metafinite', 'meta-fractal' scales, all exemplifying a "non-standard", Contra-Boolean logic; the ideo-ontologically dynamical logic of the dialectical "law" of cognition signified by the 'ideo-ontological', "pure-qualitative", Ω -algebraic inequation $x^2 \frac{1}{4} x$.

This writing is an unpublished work, and one which is not sold or exchanged for remuneration or commercial gain of any kind, but is distributed «samizdat» to selected individuals and organizations, on a donation basis, free of charge. This work is a potential contribution to the collective creative property of the Terran human species: assimilate, disseminate, critique, and surpass at will. We, the authors, seek hereby to further neither our monetary riches, nor our public power, nor our personal fame. *What we want, money cannot buy.* We hope, with your help, to build a better us, and to help do our "infinitesimal" part in building a better universe ["Infinitesimal" differences can matter, as nonlinear dynamics demonstrates]. More monetary wealth will not buy that betterment. More political power cannot impose it. More fame would mainly distract from it. We hope that you have chosen, or will choose, to build a better you. We hold that this choice entails the profoundest consequences for one's life, as well as for the lives of others. We also hold that such choices belong to you alone. We wish to share, with you, the forthcoming conceptual riches. We will rejoice, and we will be compensated, if you teach us in turn, help us to correct our errors, and thus advance the common-wealth of all beyond this offering. We also request our readers' forgiveness in the areas of our many shortcomings, some of which, though determined to strive ceaselessly to overcome them, we will never, in a lifetime, overcome. Others' voices need to be raised -- perhaps your own voice -- to check and balance our biases. We, the authors, are not publicly accessible, but will endeavor to continue private transmittals to you if you indicate publicly, however cryptically, and we recommend that it be cryptically, your desire that we should do so. We want not that our existences, let alone our egos, should be an impediment to that great reverberating propagation of new cognitions, and of emerging *new forms* of cognition, of which this essay is, at best, an incomplete, imperfect, transitory, and transitional manifestation. We therefore happily forego personal credit, and, by thus renouncing in advance the [remote] possibility of any notoriety resulting thereby, hope also to retain more lifetime for the continuation of this work. Dialectical ideography as set forth herein is interpreted variously as: (1) a calculus of 'quanto-qualitative change', encompassing an explicit, ideographical arithmetic for the *dimensional unit*[ie]s or metrical "monads" of classical "dimensional analysis", and, thereby, 'semantifying' the "meaningless" singularities [finite-time "infinite" values] of especially the "unsolvable" [in part, because of those very singularities] *nonlinear* integrodifferential equations and their solution-functions through their metrical as well as ontological 're-qualification' using those new, explicit 'metrical qualifiers' of this 'dimensional arithmetic', as well as 'quantifiable' kinds of ontological qualities, concretizing and operationalizing aspects of Plato's «*Arithmoi Monadikoi*» and Diophantus' \mathbb{M} , last extant circa 250 C.E.; (2) an alternative, onto-logical *contra-Boolean algebra*; (3) an ideographic, 'onto-dynamical' "symbolic logic" for the state-space/control-parameter-space 'meta-space' 'meta-dynamics' of 'meta-finite', conversion-singularity 'self-bifurcation'; (4) *A mathematics for modeling the history of mathematical ideas* as well as a [psycho-]historical algebra and arithmetic for modeling the 'meta-evolution' of the sciences generally; an ideography for the [psycho-]history of ideas; an ideography of the 'meta-dynamical' logic of conceptual self-innovation and self-development; a 'philosophical algebra' or trans-Leibnizian «*characteristica universalis*»; an arithmetic and algebra of innovative conception or of the creative conceptual process; (5) a rules-system for an ideographical language of *ontological self-escalation* in self-transcending [meta-]systems; (6) a generic algorithm for the 'meta' operation regress; for a trans-Hegelian, autopoietic version of the 'aufheben' operation; and for a "dynamical", 'temporalized', diachronic, 'meta-evolutionary' version of the Russellian/Gödelian logical types hierarchy; (7) a model for a 'meta-fractal', non-Cantorian *theory of totalities*, of 'meta-finite' arithmetics, and of the "foundations" of mathematics; (8) an arithmetic, algebra, geometry, and analysis built on certain "non-standard natural numbers", i.e., on the 'Gödelian 'meta-natural numbers', a space of 'evolutive' 'meta-numbers' of 2nd degree, 'made up out of' "standard", '1st degree' natural numbers, instantiating those "non-standard models of first order Peano arithmetic" whose possibility is implied by the first-order joint applicability of Gödel's completeness theorem and first incompleteness theorem, as also by the Löwenheim-Skolem theorem, constructing thereby an 'ontologically dynamical', 'de-Parmenideanized' actualization of Plato's "arithmetic of dialectics", the «*Arithmoi Eidetikoi*». This essay, in addition to that of ideogramic, pictogramic, and phonogramic symbolization, draws also upon the power of neo-mythological, allegorical, and mythopoetic symbolism -- that is, of psycho-historical symbolism -- to aid in the conveyance of its most urgent messages. *World-historically consequential universal labor*; the evocation of *effective psycho-historical force*, including individual 'psyche-ological', affective force, requires \mathbb{Q}_{SPR} ; requires that its mythopoetic momenta, denoted \mathbb{R} , be integrated, indeed, *dialectically synthesized*, with its Philosophical and Scientific momenta. Dialectical ideography is, we believe, a humble but potent seed. As with the Riemannian, and the other non-Euclidean geometries that arose from the failed attempts to prove the absoluteness of Euclid's geometry, these *non-Parmenidean, contra-Boolean, and contra-Cantorian onto-logical and onto-dynamical arithmetics* and their *algebras of dialectics* may bear fruit for humanity only if germinated through the intra- and inter-personal dialogue, and dialectic, of assimilation, critique, refutation, and supersession. We have avoided broadcast publication and indiscriminant distribution of this essay. We wish to base its circulation, and the selection of its recipients, upon our best judgment of its potential value to each candidate recipient. The taking to heart of the ideas "graphed", ideographically, 'pictographically', and narratively, herein, can produce profound transformation in the very identity of the person so taking. Panic in response to perception of the early signs of such transformation in others may elicit, from some perceivers, a violent reaction. In particular, the intimations of the 'meta-human', Δh , implications of the 'cumulum' of human[oid] evolution is profoundly disturbing to some. We are therefore directly transmitting this document only to those whom we perceive, via their own published writings, to be already verging on similar or related conceptions as a result of their own protracted 'self-meta-evolution'. We have also decided not to disseminate the most "dangerous" of the results to date. We believe that you are eminently capable of 're'-discovering these results, if you have not yet discovered them already. Should you do so, we urge that you treat them, and their dissemination to others, with utmost care. The system, more accurately, the systems, of *dialectical ideography* glossed herein continue to evolve and 'meta-evolve' rapidly in our research. *They burgeon beneath our feet.* We expect to exercise a similar restraint and discretion in any future progress reports which we may send your way. We therefore lodge the Omni-Copyright statement above together with this countervailing caveat: we recommend, should you choose to disseminate this document, its ideas, and/or related ideas of your own discovery, that you do so with careful judgment as to the recipients you select. *Give the friends of humanity a head start vis-à-vis their adversaries.* Dialectics should inculcate humility. "Perfection" is not a final meta-state that can be finally manifested, but an open-ended, 'uncompleteable', asymptotic process, moving from greater to lesser imperfection. We realize that conceptual 'homeomorphic defect' is inescapable for cognizing beings such as ourselves. Even at best, we must always be partly wrong. Even at best, one cannot be finally, completely, and wholly right. One's mental constructs cannot ever be the truth, the whole truth, and nothing but the truth. But one may be right enough for one's time, for one's moment, for one's role, and for one's part; right enough to help one's contemporaries to live through, and beyond, one's time, and thus, potentially, to enjoy the privilege, the pain notwithstanding, of a vital [life-ful] and willing participation in the succeeding epoch of imperfection.

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Need for Explicit, Ideographic 'Metrical Qualifiers', versus both 'Ontological Qualifiers' and 'Pure Quantifiers'.

[\underline{a}_{MU}] \leftrightarrow \underline{a}_7 : Briefing on \underline{a}_7 ,

Quanto-Qualitative Calculus of Ontologically and Metrically Qualified Quantifiers for Unified, Numerical/Dimensional Analysis and Singularity 'Semantification' [**Re-Qualified Metafinite Analysis**].

The 'Intra-Duality' of 'Quanto-Qualitative' Arithmetic and the Progression from \underline{U} to \underline{a}_7 [and Beyond].

Sketch: Narrative Exposition/Model of the 'Meta-Systematic Dialectic' of \underline{U} through $\underline{U} \rightarrow \underline{M}$ to \underline{a}_7 .

Thesis -- Self-Unity: Initial, Limited Apperception of \underline{U} as Ontology-only Quanto-Qualification.

Partial Antithesis -- Self-Difference: Self-Intra-Differentiation to Self-Intra-Opposition of the Self-Unity.

Some Descriptive Limitations / Inadequacies of the \underline{U} Ideography [Some Aspects of Its 'Homeomorphic Defect'].

Metrical Inspecificity -- Ideographic-Arithmetical Inexpressibility of Explicit Metrical Determinations.

Restricted Repertoire of 'State-Variables' and 'Control Parameters': Confinement to 'Population Cardinometry'.

Need for Explicit, Ideographic 'Metrical Qualifiers', versus both 'Ontological Qualifiers' and 'Pure Quantifiers'.

Self-Exo-Differentiation of, and Exo-Opposition to, this Self-Unity: Emergence / Formation of the \underline{M} Ideography.

[\underline{a}_M] \leftrightarrow \underline{a}_4 : 'Peanic', Un-Quantifiable, 'Contra-Boolean' Arithmetic Mapping Onto-Dynamics from a Single Metric/Unit «Arché».

Some Descriptive Limitations/Inadequacies of the $\underline{a}_M = \underline{M}$ Ideography.

Unquantifiability.

Confinement to the 'Onto-Dynamics' of a Single "Species" of Metric / Metrical Unit[y].

Ambiguity / Multi-Valence of Division/Decomposition [i.e., of 'Metrical Qualifier Factorization'].

Partial Syntheses -- Partial Solutions: Steppingstones and Half-Way Houses on the Way to Full Synthesis.

Progressive Partial Syntheses as also Progressive Partial Antitheses.

[\underline{a}_{MN}] \leftrightarrow \underline{a}_5 : 'Peanic', Quantifiable, 'Contra-Boolean' Arithmetic Mapping Multiples of the Metrical Units arising from the

'Onto-Dynamasis' of a Single Metric/Unit «Arché».

[\underline{a}_{MQ}] \leftrightarrow \underline{a}_6 : 'Peanic', Un-Quantifiable, 'Boolean' Units Mapping 'Diophantine Monads' for an Indefinite Multiplicity of

Qualitatively/Ontologically Distinct Monadic Species.

Synthesis -- Self-Re-Unity ['Meta-Finite', 'Meta-Fractal', 'Self-Subsumptive' Self-Re-Unification]: \underline{a}_{MQN} or \underline{a}_{MLU} as the \underline{a}_7 Ideography.

Some Aspects of 'Quanto-Qualitative Computation' and 'Quanto-Qualitative Modeling' in \underline{a}_7 .

Interpretation of $\sum_{\underline{U}} \underline{U}_x$ 'Cumula' as Models of 'Meta-System' 'Meta-Dynamics' -- of Sequences of Systems separated by 'Meta-Fractal', 'Meta-Finite', Self-Bifurcation 'Conversion Singularities', Modeled via Unified State-Space/Control-Space 'Meta-Spaces', describing such 'Meta-Evolution' via Ontological/Dimensional/Axial/Metrical ['Quanto-Qualitative'] Net Self-Expansion of the 'Meta-Space'.

The Problem of Zero Division.

The 'Paradox of Singularity' in Dynamical 'Total-Differential' and 'Partial-Differential' Systems.

The Paradox of Infinite Error: "Instantaneous" Transition from Micro-Finite Residuals to Infinity Residuals.

The Metrico-Ontological 'Fixity' of Dynamical Systems Models -- Corollary of 'The Parmenidean Postulate'?

Singularity and 'The Nonlinearity Barrier'.

The Two Components of "Unsolvability".

Differential Equation Higher Degree / Degree > 1 Nonlinearity ties to Proneness to 'Moveable-Pole' Singularity, due to 'denominatorized' / "negative degree" dynamical finite differences -- time-varying functions minus control parameters.

The 'Conversion Meta-Dynamic' and Zero Division.

Physical Meanings of [Meta-]Dynamical 'Conversion-Singularity' as Guide to its Mathematical Representation.

The «a posteriori», "synthetic", inductive, empirical Principle of 'Metafinity'.

Singularity and Onto-Dynamasis.

Pure-Quantitative Zeros versus Pure-Qualitative Zeros versus Quanto-Qualitative Zeros.

The 'Method of [Ontological-Metrical] Re-Qualification' and 'Singularity Semantification'.

Computation of Singularities via 'The Method of Re-Qualification' -- Some Illustrations.

The Rocket Equation: «Gedanken» Experiment.

The Newtonian Many-Body Problem.

The Ultraviolet Catastrophe.

The 'Meta-Dynamics' of Stellar 'Meta-Evolution': Conversion Singularity Nucleosynthesis Inter-Epochal Transitions.

Transition to \underline{a}_7 and Beyond: Some Descriptive Limitations / Inadequacies of the \underline{a}_7 Ideography.

[q_{MU}] ↔ **q_7** : Briefing on **$a_{\underline{U}}$** ,

Quanto-Qualitative Calculus of Ontologically and Metrically Qualified Quantifiers for Unified, Numerical/Dimensional Analysis and Singularity 'Semantification' ['Re-Qualified Metafinite Analysis'] [continued & concluded].

Transition to the Next Section: The Precocity of these Briefings and the Work of Part II.

The Scale of *Systematic-Dialectical 'Ideo-Meta-Evolution'* from **N** to **W** to **Z** to **Q** to **R** and Beyond.

The Scale of *'Ideo-Meta-Evolution'* from **$N_{\underline{A}_\tau}$** to **$W_{\underline{A}_\tau}$** to **$Z_{\underline{A}_\tau}$** to **$Q_{\underline{A}_\tau}$** to **$R_{\underline{A}_\tau}$** , . . . within each **\underline{A}_τ** .

The *Psycho-Historical Ideo-Dialectic* of **N**.

Transition In: Connotational Calculative Derivation of the [Rules-]System / Ideo-Ontological Category denoted $\mathbf{N}\mathbf{Q}$ \leftrightarrow \hat{q}_2 --

A rules-system of operatorial ideograms representing generic 'kind-of-being qualifiers' -- generic 'ontological qualifiers' -- as the '[meta-]numerals' denoting the 'unquantifiable' or 'purely-qualitative' '[meta-]numbers' of a dialectical arithmetic which provides a 'meta-dynamical', «auto-kinesis» version of the Platonic "arithmetic of dialectics", the «arithmos eidetikos», or «arithmos noetikos».

Section III. of Dialectical Ideography, entitled *The Arithmetics of Meta-Evolution*, is designed to comprehensively present an $\mathbf{N}\mathbf{Q}$ model of the 'meta-system' of the progression of the *dialectical ideographies*; of the 'meta-systematic dialectical', categorial-self-progression, systems-self-progression, 'connotative-calculative' derivation of the various "epochs", or stages, of the *dialectical ideographies*, in which the $\mathbf{N}\mathbf{Q}$ system itself arises within the second stage, $\tau = 1$, immediately after the first, $\tau = 0$ stage, and in which each successor system of *dialectical arithmetic* is *more concrete* and *more complex* -- richer in determinations -- in terms of its capability for description of both the dynamics & the 'meta-dynamics' of natural & 'human-natural' systems. That is, this 'meta-systematic dialectic' of the *dialectical arithmetics* is modeled, in Section III., by means of the second-arising system of *dialectical arithmetic* in that systems-progression, the $\mathbf{N}\mathbf{Q}$ *dialectical arithmetic*, which arises immediately after the "Natural numbers" system of arithmetic. This yields a *model*, thus written in the language of one of the *dialectical arithmetics* itself, of the dialectic of the *dialectical arithmetics*. By assigning the «arché» system of arithmetic of this 'meta-system' -- namely, the "first-order" rules-system of the "Natural" numbers, \mathbf{N} , represented by the first four of the five Peano Postulates, which [rules-]system we denote by \mathbf{N} -- to the first of the $\mathbf{N}\mathbf{Q}$ meta-numbers, denoted by \hat{q}_1 [symbolizing this act of assignment via the expression $\mathbf{N} \leftrightarrow \hat{q}_1$], we obtain the following 'connotative calculation', or 'intensional, heuristic derivation', of the $\mathbf{N}\mathbf{Q}$ system of rules of dialectical arithmetic, part of "number-system one" [$\hat{\#}_1$], from "number-system zero" [$\hat{\#}_0$] --

unqualified quantifiers as numbers \rightarrow (unqualified quantifiers as numbers)² = unqualified quantifiers as numbers \oplus unquantifiable qualifiers as 'meta-numbers', or:

$$\hat{\#}_0 = (\mathbf{N})^2 = (\mathbf{N})^1 = \mathbf{N} \rightarrow \hat{\#}_1 = (\mathbf{N})^2 = (\mathbf{N})^2 = \mathbf{N}^2 = \mathbf{N} \circ \mathbf{N} = \mathbf{N}(\mathbf{N}) = \mathbf{N} \text{ of } \mathbf{N} = \mathbf{N} \dashv \oplus \Delta\mathbf{N} = \mathbf{N} \oplus \mathbf{N}\mathbf{Q} \leftrightarrow \hat{q}_1 \rightarrow \hat{q}_1^2 = \hat{q}_1 \boxplus \hat{q}_1 = \hat{q}_1 \boxplus \hat{q}_2.$$

The "Natural" numbers space, $\mathbf{N} = \{1, 2, 3, \dots\}$, in this derivation, is stipulated as basing the first-"thesis" [rules-]system, or 'ideo-ontological' category, of arithmetic, \mathbf{N} . This \mathbf{N} connotes an arithmetic of 'Peanic' [=first-order-Peano-Postulates-compliant] 'unqualified quantifiers' as numbers. This first 'thesis' rules-system, or category, of arithmetic, as a result of its 'intra-duality', its harboring within itself of "non-standard models" of the "Standard Natural numbers" -- i.e., by what we term 'ideo-auto-kinesis' -- also gives rise, by 'self-reflexion', to a first 'contra-thesis' rules-system, or category, of arithmetic. This 'contra-thesis' connotes a rules-system with 'negated' or 'opposite' connotations with respect to those of \mathbf{N} , namely, those of a still 'Peanic' first-order rules-system, but one of 'not-unqualified not-quantifiers', or 'unqualified \neg quantifiers', i.e., of 'unquantified qualifiers' as '[meta-]numbers' [expressed via a system of higher, meta-"Natural" number units, $\{\hat{q}_n\}$ [with n denoting a "Standard Natural number"]. Its symbols/'meta-numerals' are made up out of multiplicities of "Natural" numbers' numerals, via a 'self-internalization'/'self-subordination'/'self-subscript-ization' of their arithmetic, and is denoted herein by $\Delta\mathbf{N}$ or $\mathbf{N}\mathbf{Q}$, whose 'number-space' is $\mathbf{N}\mathbf{Q} = \{\hat{q}_1, \hat{q}_2, \hat{q}_3, \dots\}$. Together, \mathbf{N} and $\mathbf{N}\mathbf{Q}$ form an 'antithesis-sum', denoted $\mathbf{N} \oplus \mathbf{N}\mathbf{Q}$ [a 'non-reductionist', 'non-collapsing', "inhomogeneous" [i.e., "heterogeneous", or "non-amalgamative" sum [cf. Musès], à la the proverbial "sum" of qualitatively different terms/kinds, "apples + oranges"]].

The Briefings of this sub-section excerpt from, and gloss, the first few derivations of Section III. The first Briefing, starting just below, glosses the second system of dialectical arithmetic, denoted $\mathbf{N}\mathbf{Q}$, which relates as 'contra-thesis' to the first, «arché» thesis -- 'vestigially-dialectical' -- arithmetic, denoted \mathbf{N} .

Introducing the $\mathbf{N}\mathbf{Q}$ system/'ideo-ontological category' of the dialectical arithmetics. Before we plunge ahead into Section II., the next stage of this excursion, a *Psycho-Archaeological* excavation of *The Meta-Evolution of Arithmetics*, let us tarry long enough to tell you, in the most direct way we can, the most basic rules of the 'first contra-thesis' *'Dialectical Ideographic Language'*; of its incipient "Non-Standard Natural", 'contra-Boolean [*Onto*]-Logical Arithmetic', and 'Ontological Algebra', about whose bush we have been beating, and to which we have been led by that very *'The Meta-Evolution of Arithmetics'*.

First off, let us take this chance to say that just because we call this dialectical calculus 'contra-Boolean' does not mean that we are 'contra-Boole'; that we harbor some overriding animus toward the man himself. On the contrary, we hold his pioneering contributions, and his exemplary life of universal labor, in high esteem [In this season of unceasing incivility such sensibilities can no longer be assumed to go without saying]. The several sub-sections below describe rules of operation for a 'multi-unit-intervals meta-number-space', or "-set", that we call $\mathbf{N}\mathbf{Q}$. This 'unit-intervals-restricted' ideography is that part of this dialectical ideography which has the most in common with Boole's *original* arithmetic/algebra of logic, as set forth in his *The Laws Of Thought*. However, the space $\mathbf{N}\mathbf{Q}$ has "nothing" in common with the realm of arithmetic most familiar today, that of the so-called "**R**real" numbers, formally denoted by **R** or **R**¹. Nor has $\mathbf{N}\mathbf{Q}$ much in common with the unit interval "sub-space" of that space, $[0, 1] \subset \mathbf{R}$, or even with the end-points of that interval, $\{0, 1\} \subset \mathbf{R}$, often taken to be the space of Boolean arithmetic, which we denote herein by **E**. That is, the intersection of $\mathbf{N}\mathbf{Q}$ and **R** is \emptyset : $\mathbf{N}\mathbf{Q} \cap \mathbf{R} = \emptyset$, and also $\mathbf{N}\mathbf{Q} \cap \mathbf{E} = \emptyset$. The empty set, \emptyset , is thus, in a sense, all that they have in common. [Herein, we denote the set of **W**hole Numbers -- the **N**atural Numbers, **N**, with the adjunction of 0 -- by **W**]. Yet we will see that, with the expansion from $\mathbf{N}\mathbf{Q} = \{ \hat{q}_1, \hat{q}_2, \hat{q}_3, \dots \}$, to $\mathbf{w}\mathbf{Q} = \{ q_0, \hat{q}_1, \hat{q}_2, \hat{q}_3, \dots \}$, q_0 bridges the $\mathbf{w}\mathbf{Q}$ 'evolutive' 'pure' qualifier meta-numbers' to the 'convolute' numbers, including **R** & **C** [**C** \supset **R**]. Indeed, $\mathbf{w}\mathbf{Q}$ 'meta-finitely contains' **R** & **C** -- **R**, **C**, **H**, **O**, ... $\subset \mathbf{w}\mathbf{Q}$ -- because **R**, **C**, **H**, **O**, **K**, **G**, ... $\subset q_0$, and because $q_0 \subset \mathbf{w}\mathbf{Q}$, notwithstanding that **R**, **C**, ... $\not\subset \mathbf{w}\mathbf{Q}$ & **R**, **C**, ... $\cap \mathbf{w}\mathbf{Q} = \emptyset$ [Even as $\{a\} \subset \{\{\dots\{a\}\dots\}$, and yet $\{a\} \cap \{\{\dots\{a\}\dots\} = \emptyset$, & also although $\{a\} \not\subset \{\{\dots\{a\}\dots\}$, & $\{a\} \notin \{\{\dots\{a\}\dots\}$. This is because '**A** \subset **B**' means '**A** is a component/sub-system/constituent/of **B**' at some/any/at least one 'scale'/layer' of **B**'s possibly 'meta-fractal', multi-dimensional, multi-logical type, 'multi-ontic cumulum' internal composition].

We call the individual 'meta-numbers' which are the 'elements' or 'constituents' of [most variants of] the "set" or "space" denoted by $\mathbf{N}\mathbf{Q}$, by the name 'evolutive qualifier meta-numbers', to distinguish them from familiar kinds of numbers, which they presuppose, but also "transcend". We employ ' $\mathbf{N}\mathbf{Q}$ ', to symbolize this space or "set" of meta-numbers, because we interpret our 'Rules Of Operation' for that space as an 'Arithmetic of Qualities'; of 'Ontological Qualifiers'; of 'ontological categories', or 'ontos', for short; of *ontic* «*monads*» or *unit*[ie]s. We will often denote any one of the individual 'meta-numerals' for the 'meta-numbers' that reside in $\mathbf{N}\mathbf{Q}$ via the generic format ' \hat{q}_k '. The underscored lower-case ' \hat{q}_k ' component of that 'compound symbol' identifies \hat{q}_k as a member of $\mathbf{N}\mathbf{Q}$. The [post-]subscript, here denoted generically by 'k', can vary over a space of specific values, e.g., Ordinal, Cardinal ["**N**atural"], "**W**hole", "Rational", or "**R**real" number, etc., values, but is here constrained to vary over the space of "**N**atural" numbers, denoted by the symbol **N**. The specific value, **k**, identifies the individuality of a given \hat{q}_k within $\mathbf{N}\mathbf{Q}$; the unique member of $\mathbf{N}\mathbf{Q}$ which \hat{q}_k represents; for $k \in \mathbf{N}$, **W**, or **Z**, the ordinal position or 'order of appearance' of that unique 'onto' in the progression of 'onto'-representing/denoting ontological qualifiers within $\mathbf{N}\mathbf{Q}$, $\mathbf{w}\mathbf{Q}$, or $\mathbf{z}\mathbf{Q}$, respectively. We apply $\mathbf{N}\mathbf{Q}$ to describe 'onto-dynamical', 'meta-evolutionary' processes in which a succession of new 'ontological categories', or 'ontos' -- new 'taxons' or 'taxa'; qualitatively unprecedented [ev]entity/activity categories -- 'arise', 'appear', or 'manifest', via the inter-operation/self-operation of previously-extant 'ontos'.

The 'sequence of counting numbers' of the number-space \mathbf{N} , conceived, since Simon Stevin's circa 1600 works, as "pure" [i.e., 'unqualified'] 'quantifiers', the so-called "**N**atural numbers" **1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ...**, has become paradigmatic for the accounting processes so fundamental to proto-human society -- especially to monetized society. Likewise, we hypothesize that the 'sequence of "pure" [i.e., 'unquantified'] qualifiers' inhering in $\mathbf{N}^{\hat{Q}}$ -- i.e., $\mathbf{N}^{\hat{Q}}_1, \mathbf{N}^{\hat{Q}}_2, \mathbf{N}^{\hat{Q}}_3, \mathbf{N}^{\hat{Q}}_4, \mathbf{N}^{\hat{Q}}_5, \mathbf{N}^{\hat{Q}}_6, \mathbf{N}^{\hat{Q}}_7, \mathbf{N}^{\hat{Q}}_8, \mathbf{N}^{\hat{Q}}_9, \mathbf{N}^{\hat{Q}}_{10}, \mathbf{N}^{\hat{Q}}_{11}, \dots$, -- may, in the future, prove paradigmatic for the modeling of generic processes of 'self-bifurcation' and 'meta-evolution'; of qualitative/ontological-revolutionary change, in this universe, including in its sub-[component]universes [of discourse]. It may so do, we hold, for an emerging form of human mentation and an affine technology in tune with the ubiquity of 'meta-dynamism' in this cosmos. We can draw upon an indefinite "number" of "**N**atural" Numbers -- as many as required by a given process of *measuring-by-counting* that we may be imagining or actually conducting. Similarly, we can marshal an indefinite number of the $\mathbf{W}^{\hat{Q}}_k$ -- as many as the given 'meta-dynamical', 'meta-evolutionary', 'onto-dynamical' process that we wish to model has, or is expected to have, ontos, per our analysis -- within the range of the history of that process which we wish to cover. No ending to the sequence of the $\mathbf{W}^{\hat{Q}}_k$ within $\mathbf{W}^{\hat{Q}}$ in pre-posit. For every k in \mathbf{W} , sub- $\mathbf{W}^{\hat{Q}}_k$ is also in $\mathbf{W}^{\hat{Q}}$, or, to employ the standard, more "fully ideographic" symbolism, $[\forall k \in \mathbf{W}][\mathbf{W}^{\hat{Q}}_k \in \mathbf{W}^{\hat{Q}}]$.

This exposition is heading toward 'quanto-qualitative', 'full-multiplicity' arithmetics of 'explicitly quantified qualifiers', or, equivalently, of 'explicitly and ideographically qualified quantifiers', with their "'quantifiers'" not restricted to the unit interval. We denote the first-two-to-emerge of those 'quanto-qualitative', or, equivalently, 'qualo-quantitative' arithmetics herein by \mathbf{U} and by $\mathbf{a}\mathbf{U}$. But the $\mathbf{N}^{\hat{Q}}$ arithmetic, like that of the Boolean "'Elector"' arithmetic, \mathbf{E} , is a unit-interval-restricted arithmetic. Every $\mathbf{N}^{\hat{Q}}$ 'meta-number' is, in effect, quantified by '1', by the 'unit quantifier'. Better still is to conceive that each \hat{q}_k is quantified by 'no quantifier', is 'unquantified', in a sense reminiscent of the ancient Greek, and Aristotelian, idea that number connotes an aggregation of units, or «*monads*» -- an «*arithmos monadikos*» -- not a single, isolated unit in itself, so that "quantity" begins with **2**, not with **1**. The \hat{q}_k are 'unit qualifiers', or better, 'unquantifiable', "additively idempotent" [each-metainumber-its-own-additive-identity-element], 'non-additive', 'un-addable', "pure" generic qualifiers, with no meaning distinct from that assigned to their 'singleness' assigned to their multiplicity: for the $\mathbf{N}^{\hat{Q}}$, if $\mathbf{N} \ni n \geq 2$, then ' n ' $\hat{q}_k = '2$ ' $\hat{q}_k = '1$ ' $\hat{q}_k = \hat{q}_k$. Boolean arithmetic posits, at most, a single unit interval, denoted $[0, 1]$. Its upper end-point, **1**, is interpreted as representing the "universal set", the "logical quantity" "**All**". Its lower end-point, **0**, is interpreted as representing the logical quantity "**None**". The entire 'interior' of that unit-interval, denoted $(0, 1)$ -- the interval without either of its end-points, **0** or **1** -- might connote the 'fuzzy' logical quantity 'Some(-but-not-All)' = neither "All" nor "None", which Boole does not explicitly allow, but invokes implicitly, via the 'logical quantity' he denotes by **0/0**, which 'ambiguously' represent "none, some, or all" of the universe of discourse. The special, 'onto-dynamical' analytical geometry of the $\mathbf{W}^{\hat{Q}}$ arithmetic maps a continual, self-propelled proliferation of mutually perpendicular, qualitatively/directionally distinct unit-length, unit-interval directed line-segments, represented generically as $[q_0, \hat{q}_k]$, each one -- after, and with the exceptions of, q_0 and \hat{q}_1 -- 'generate-able' by mutual-interactions and self-interactions of its predecessor unit-intervals, or "'dimensions'". As interpreted herein, this self-proliferation yields a continually expanding "ontology set" of ontological qualities, 'qualifiers', 'ontic unit[ie]s', or categorial 'monads'; a self-growing, non-amalgamative 'universal qualities sum', describing an 'ontological cumulum', as a self-expanding 'possibility-space' of a generic universe of discourse.

That is, that 'universal set of ontos' or 'universal sum of ontos', denotes and describes the potential momentaneous ontological contents of the universe being modeled at each 'stage' or 'epoch' of a 'meta-evolutionary' succession of such 'ontology-stages' of that universe. This 'expanding-dimensionality domain', 'self-growing manifold', or '[dimensionally] self-expanding space' is the 'possibility-space' for the universe [of discourse] being modeled. This is because, by 'ontological contents' we mean the possible -- not necessarily even the probable, let alone the actualized -- forms of existence/activity for and within that universe[of-discourse] at that stage or epoch of its self-driven/other-driven 'meta-evolution'.

$\mathbb{N}\underline{\mathbb{Q}}$ Arithmetic [Statics]. [Note – this ' $\mathbb{N}\underline{\mathbb{Q}}$ ' is *not* the same as ' \mathbb{Q} ', traditionally denoting the "Rational Numbers": $\mathbb{N}\underline{\mathbb{Q}} \not\equiv \mathbb{Q}$].

Rule 0. [The Rule of Ontological Diversity]: for every \hat{q}_i and \hat{q}_j in $\mathbb{N}\underline{\mathbb{Q}}$, if $i \neq j$, then $\hat{q}_i \not\equiv \hat{q}_j$;

or, in "fully" -- and more formally -- ideographical symbolism: $[\forall \hat{q}_i, \hat{q}_j \in \mathbb{N}\underline{\mathbb{Q}} \ \& \ \forall i, j \in \mathbb{N} \ | \ i \neq j][\hat{q}_i \not\equiv \hat{q}_j]$.

This means that each \hat{q}_k , for each distinct value of k , denotes a unique '*qualifier*', interpretable as connoting a distinct *ontological quality*, *ontic category*, or '*onto*', and hence differing *qualitatively* from all of the others.

This is depicted, geometrically, via the 'perpendicularity-structure' of $\mathbb{W}\underline{\mathbb{Q}}$ space.

Each unit interval line-segment $[q_0, | \hat{q}_k |]$ is perpendicular to every other, thus forming an n -dimensional space of unit-length line-segments, where n denotes the maximum extant cardinal/ordinal subscript of the \hat{q}_k in use for a given dialectical model. Interpreted geometrically, then:

$$[\forall \hat{q}_j, \hat{q}_k \in \mathbb{W}\underline{\mathbb{Q}} \ \& \ \forall j, k \in \mathbb{W} - \{0\} \ | \ j \neq k][\hat{q}_j \perp \hat{q}_k];$$

Rule 1. [The Non-Amalgamation Rule]: for every \hat{q}_i and \hat{q}_j in $\mathbb{N}\underline{\mathbb{Q}}$, such that $i \neq j$,
there does not exist \hat{q}_n in $\mathbb{N}\underline{\mathbb{Q}}$ such that $\hat{q}_i + \hat{q}_j = \hat{q}_n$,

or, in "fully" ideographic symbolism -- $[\forall m \in \mathbb{N} \ \& \ \hat{q}_i, \hat{q}_j \in \mathbb{N}\underline{\mathbb{Q}} \ | \ i \neq j][\nexists \hat{q}_n \in \mathbb{N}\underline{\mathbb{Q}} \ | \ \hat{q}_i \boxplus \hat{q}_j = \hat{q}_n]$.

This means, e.g., that "apples and oranges, *as such*, do not add". The sum of **2** (or more) qualitatively-distinct $\mathbb{N}\underline{\mathbb{Q}}$ *qualifiers* does not *reduce* to a single *qualifier* within $\mathbb{N}\underline{\mathbb{Q}}$, for any given interpretation of $\mathbb{N}\underline{\mathbb{Q}}$. Such sums, which we term herein '*poly-qualinomials*', illustrated above via a '*bi-qualinomial*', are traditionally termed "complexes", "inhomogeneous sums", "heterogeneous sums", or "non-amalgamative sums". In Gibbs/Heaviside vectorial arithmetic, modeling **3**-dimensional physical spaces, where \hat{x} , \hat{y} , and \hat{z} denote "unit vectors" -- unit lengths directed in each of **3** distinct respective mutually-perpendicular spatial directions -- the vector $3\hat{x} + 5\hat{y} + 9\hat{z}$ does not reduce to $(3+5+9)?s = 17$ *all-the-same-somethings*. Likewise, in "Complex" arithmetic -- the '*arithmetic of "complexes"*' -- where r denotes **+1**, the unity of the "**Real**" numbers, and where i denotes the unity of the "**Imaginary**" numbers, the "complex" $7r + 11ri$ does not "simplify", or amalgamate down to **18** *somethings*. So it is also with the \hat{q}_k . The [interpreted] sum of any **2** (or more) qualitatively distinct 'ontos' does not *reduce*, or "simplify", to any *single* 'onto'.

Rule 2. [The Rule of Ontological Parsimony]: for every \hat{q}_j in $\mathbb{N}\underline{\mathbb{Q}}$, $\hat{q}_j + \hat{q}_j = \hat{q}_j$,
or, more formally, $[\forall \hat{q}_j \in \mathbb{N}\underline{\mathbb{Q}}][\hat{q}_j \boxplus \hat{q}_j = \hat{q}_j]$, or, w.r.t. Rule 1,
 $[\forall \hat{q}_i, \hat{q}_j \in \mathbb{N}\underline{\mathbb{Q}} \ | \ i = j][\exists \hat{q}_k \in \mathbb{N}\underline{\mathbb{Q}} \ | \ \hat{q}_i \boxplus \hat{q}_j = \hat{q}_k = \hat{q}_i = \hat{q}_j]$.

In other words, ' $2\hat{q}_k = 1\hat{q}_k$ ', or, by induction, ' $n\hat{q}_k = 1\hat{q}_k = \hat{q}_k$ ', for $\mathbb{N} \ni n \geq 2$. The mere or singular assertion of the *possible* "existence" or 'extant-ness' of a given 'onto', denoted \hat{q}_k , is sufficient. *Multiplicity is redundant* in the semantical, conceptual context of $\mathbb{N}\underline{\mathbb{Q}}$, just as, in that of sets, $\{a, a, \dots, a\} = \{a, a\} = \{a\}$.

This kind of behavior is also known as "*additive idempotency*". Each \hat{q}_k is its own '*additive identity*' element. The \hat{q}_k are thus, essentially, '*non-additive*', that is, '*non-addable*' and therefore '*unquantifiable*'. Thus, "*Homogeneous sums*" or "*non-heterogeneous sums*" in $\mathbb{N}\underline{\mathbb{Q}}$ -- '*mono-qualinomials*' -- *do* amalgamate; they in fact 'hyper-amalgamate', to unit monomials. Seeming '*poly-quant-mono-quali-nomials*' reduce to *non*-multiplicity.

Rule 3. [The «Aufheben» Evolute Product-Rule]: for any \hat{q}_i & \hat{q}_j in $\mathbf{N}\underline{Q}$, if $i \neq j$, then $\hat{q}_i[\hat{q}_j] = \hat{q}_j + \hat{q}_{i+j}$,
 or, more generally $[\forall \hat{q}_i, \hat{q}_j \in \mathbf{N}\underline{Q}][\hat{q}_i \boxplus \hat{q}_j] = \hat{q}_j \boxplus \hat{q}_{i+j}$;

That is, in the "generalized multiplication", or '*flexion*', of *qualifiers*, defined herein for the $\mathbf{N}\underline{Q}$, the "starting point" -- in the above, \hat{q}_i -- does not disappear into the 'stopping point', or outcome -- in the above, \hat{q}_{i+j} . Among the $\mathbf{N}\underline{Q}$, "multiplication" is thus 'pathway-preserving', 'pathway-exhibiting', or 'pathway-recording'. The $\mathbf{N}\underline{Q}$ product is an '*evolute*' ["nonlinear"] product, rather than a '*convolute*' ["linear"] one, as these terms are defined in the prequel. Also, $\mathbf{N}\underline{Q}$ "multiplication" is a '*multiplication [increase] of qualities*' rather than a "multiplication" (increase) of quantities. We also term this syntactical behavior '*multiplicative hyper-potency*', '*super-potency*', or '*meta-potency*'. The "offspring" *quality* produced by the *qualities* \hat{q}_i and \hat{q}_j is *qualitatively* different from either: $\hat{q}_i \neq \hat{q}_{i+j} \neq \hat{q}_j$.

Corollary: $[\forall \hat{q}_k \in \mathbf{N}\underline{Q}][\hat{q}_k \boxplus \hat{q}_k = \hat{q}_k^2 = [\hat{q}_k \boxplus \hat{q}_{k+k}] = [\hat{q}_k \boxplus \hat{q}_{2k}] \neq \hat{q}_k]$,

which is an instance of the 'contra-Boolean' fundamental "law", or "rule", itself:

$$\hat{q}_k^2 \neq \hat{q}_k; \quad \hat{q}_k^2 = [\hat{q}_k \boxplus \hat{q}_k] = [\hat{q}_k \boxplus \hat{q}_{2k}] \neq \hat{q}_k.$$

To interpret or partially map this new Product-Rule and its '[Meta-]Number-Space' [in]to a more familiar mathematical idiom, we also present this Product-Rule as a new *Vector* Product-Rule -- a '*fourth* Vector Product' if we put it in sequence after the "Scalar", "Vector", and "Tensor" Products of "Vectors". We term it the 'Dialector Product-Rule', or, in dynamical contexts, the '*history-revealing*', 'history-preserving', '*path-positing*', '*past-disclosing*' Product-Rule. We represent this 'Dialector Product-Rule' in form which makes use of a 'Metavector Product' *operation sign* via the picto-ideogram \uparrow . This operation involves the conception of a '[self-]*expanding* [Meta-]Vector-Space', or of a "pre-existent [potentially] *infinite-dimensional* Vector Space" [but different from Hilbert Space]. Also, though this Product-Rule involves the escalation of the dimensionalities of its "[Meta-]Vector-Spaces" ['Dialector Spaces'], it is distinct from the Grassmann Outer Product. Per this interpretation, the following kinds of similes arise, wherein \hat{e}_k denotes the "orthonormal", unit-length vector pointing in[to] or along the *k*th orthogonal dimension/direction [and wherein the *T* exponents call for the transposition of row[-vector]s into column[-vector]s]:

$$\begin{aligned} [\hat{q}_1 \hat{q}_2 = \hat{q}_2 \boxplus \hat{q}_3] &\leftrightarrow [\hat{e}_1 \uparrow \hat{e}_2 = \hat{e}_2 + \hat{e}_3], \text{ or } [1]^T \uparrow [0 \ 1]^T = [0 \ 1]^T + [0 \ 0 \ 1]^T = [0 \ 1 \ 1]^T \neq \\ [\hat{q}_2 \hat{q}_1 = \hat{q}_1 \boxplus \hat{q}_3] &\leftrightarrow [\hat{e}_2 \uparrow \hat{e}_1 = \hat{e}_1 + \hat{e}_3], \text{ or } [0 \ 1]^T \uparrow [1]^T = [1]^T + [0 \ 0 \ 1]^T = [1 \ 0 \ 1]^T \neq \\ [\hat{q}_1 \hat{q}_1 = \hat{q}_1 \boxplus \hat{q}_2] &\leftrightarrow [\hat{e}_1 \uparrow \hat{e}_1 = \hat{e}_1 + \hat{e}_2], \text{ or } [1]^T \uparrow [1]^T = [1]^T + [0 \ 1]^T = [11]^T \neq \\ [\hat{q}_2 \hat{q}_2 = \hat{q}_2 \boxplus \hat{q}_4] &\leftrightarrow [\hat{e}_2 \uparrow \hat{e}_2 = \hat{e}_2 + \hat{e}_4], \text{ or } [0 \ 1]^T \uparrow [0 \ 1]^T = [0 \ 1]^T + [0 \ 0 \ 0 \ 1]^T = [0101]^T. \end{aligned}$$

The expressions $\hat{e}_1 \uparrow \hat{e}_2$ or $[1 \ 0]^T \uparrow [0 \ 1]^T$ may be read off as ' \hat{e}_1 lift \hat{e}_2 ' or ' $[1]^T$ escalate $[0 \ 1]^T$ '; $\uparrow[\hat{e}_1; \hat{e}_2]$ as *transcend* $[\hat{e}_1; \hat{e}_2]$. Note that this Product is *non-commutative*, because it is *evolute*. In '*meta-dynamical*', e.g., "'historical-dialectical'"/"dialectic of nature" contexts, we say that this Product-Rule/Definition is '*history-disclosing*', '*pathway-disclosing*', '*ancestry-disclosing*', '*meta-genealogy-disclosing*', or '*source/origin/causation-disclosing*'. The result of an \uparrow operation discloses '*from whence to whence*': *from whence* [from what "point"/'metavector'] its movement began as well as *to whence* [to what "point"/'metavector'] it arrived. Note too that, for these "'unit-vectors'" too, as for the \hat{q}_i : $[\forall \hat{e}_j, \hat{e}_k \in \mathbf{V} \ \& \ \forall j, k \in \mathbf{W} - \{0\} | j \neq k][\hat{e}_j \uparrow \hat{e}_k]$.

Thus, the movement denoted $[0 \ 1]^T \uparrow [1 \ 0]^T$, which starts at $[1 \ 0]^T$ and stops at $[1 \ 0 \ 1]^T$, is qualitatively different from that denoted $[1 \ 0]^T \uparrow [0 \ 1]^T$, which starts at $[0 \ 1]^T$ and ends at $[0 \ 1 \ 1]^T$. This model maps $\mathbf{N}\underline{Q}$ as an "orthonormal" *Meta-Basis* of a *unit-metavectors-confined*, i.e., *additively idempotent*, [self-]expandable '*Metavector Space*'. The latter rule makes this \mathbf{V}_T a less 'natural' model of '*Generalized Onto-Dynamasis*', or \underline{Q}_x , than is \underline{Q} . It demands, e.g., that $\hat{e}_1 + \hat{e}_1 = \hat{e}_1 = [1 \ 0 \ . \ .]^T$, vs. $\hat{e}_1 + \hat{e}_1 = 2\hat{e}_1 = [2 \ 0 \ . \ .]^T$, the more traditional result for standard vectors. This violates our expectations of vector arithmetic, as does $1 + 1 = 1$ when ordinary 1 is used to stand for the logical quantity '**All**' in Boolean arithmetic. It may thus be better to craft *new [meta-]numerals*, fitted from their outset to their defined -- new and unprecedented -- roles. The latter approach leads to $\mathbf{N}\underline{Q}$.

$\mathbf{N}\underline{Q}$ *Statical Algebra and Statical Geometry*. As detailed above, the *arithmetic* of $\mathbf{N}\underline{Q}$ involves formulas written in terms of "*constants*", namely, the "constant" 'meta-numerical' values, each one denoted by one of the 'meta-numerals' of the set of all $\mathbf{N}\hat{q}_k$, for the given universe of the k , here \mathbf{N} :

$$\mathbf{N}\underline{Q} = \{ \sum_{\mathbf{N}} \hat{q}_k \mid k \in \mathbf{N} \} = \{ \mathbf{N}\hat{q}_1, \mathbf{N}\hat{q}_2, \mathbf{N}\hat{q}_3, \mathbf{N}\hat{q}_4, \mathbf{N}\hat{q}_5, \mathbf{N}\hat{q}_6, \mathbf{N}\hat{q}_7, \mathbf{N}\hat{q}_8, \mathbf{N}\hat{q}_9, \mathbf{N}\hat{q}_{10}, \mathbf{N}\hat{q}_{11}, \dots \}.$$

The *algebra* of the $\mathbf{N}\underline{Q}$ abstracts from and generalizes upon this *arithmetic*, via the use of '*qualifier variables*'.

A $\mathbf{N}\underline{Q}$ -algebraic variable may denote, generically, a single constant in a context where that constant's specific identity is unknown. Alternatively, it may denote many possible such values, which the variable "ranges over", that is, characterizes univocally [as in the complex variable $\mathbf{C}z = xr + yri, r = 1$], or takes on in succession [as, e.g., in a τ -ordered \mathbf{x}_τ , where $\mathbf{x}_1 = 1, \mathbf{x}_2 = 1, \mathbf{x}_3 = 2, \mathbf{x}_4 = 3, \mathbf{x}_5 = 5$, etc.]. The "domain" of such constant values which the variable can denote may include all of the units, or «*monads*», in the *potentially infinite space* $\mathbf{N}\underline{Q}$, or just a proper subset thereof.

One species of such variables is exemplified above, in the ideographic statement of the four Rules. This type of variable-symbol denotes generically "any" or "every" [\mathbf{V}] [*logical-]individual unit/«monad»*, constituent of $\mathbf{N}\underline{Q}$. It does so by using a literal *subscript-variable*, such as i, j, k, ℓ, m , or n , to denote a specific, e.g., "**N**atural" number subscript-constant, e.g., $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots$, viz., $\mathbf{N}\hat{q}_i, \mathbf{N}\hat{q}_j, \mathbf{N}\hat{q}_k, \mathbf{N}\hat{q}_\ell, \mathbf{N}\hat{q}_m, \mathbf{N}\hat{q}_n$. This variable-denotation strategy is designed to represent only '*mono-qualinomials*', single constituents/units/«*monads*», of $\mathbf{N}\underline{Q}$. This strategy does not encompass inhomogeneous sums of two or more 'metanumber' unit[ie]s or 'unit qualifiers', i.e., it cannot cover '*poly-qualinomials*'. A second species of \underline{Q} -algebraic variable represents, indifferently, either 'mono-qualinomial' or 'poly-qualinomial' $\mathbf{N}\underline{Q}$ values, depending upon the equational or other formulaic context in which it appears. This type of variable symbol is exemplified throughout this section, viz. -- $\underline{x}, \underline{y}, \underline{z}$, etc. -- where $\underline{x}^2 \hat{=} \underline{x}$; $\underline{y}^2 \hat{=} \underline{y}$, and $\underline{z}^2 \hat{=} \underline{z}$, etc. A third species of $\mathbf{W}\underline{Q}$ -algebraic variable, not previously exemplified, is key to the '*Onto-Dynamical Equations*' from which *ontic* dialectical models of '*meta-evolutionary process*' are formed. This type of $\mathbf{W}\underline{Q}$ -algebraic variable is a '*poly-qualinomial variable*', which employs "**Whole numbers**" subscripts to denote specific subsets of $\mathbf{W}\underline{Q}$, viz., $\mathbf{W}\underline{Q}_1$. Each of these subsets is an unbroken, consecutive, finite subsequence of the full, potentially infinite $\mathbf{W}\underline{Q}$ sequence, starting with $\mathbf{W}\hat{q}_0$, and/or with $\mathbf{W}\hat{q}_1$, and ending with some $\mathbf{W}\hat{q}_w$, such that $\mathbf{W} \ni w \geq 1$. These $\mathbf{W}\hat{q}_k$ -sequences succeed one another by means of *self-[re-]flexion*, that is, by the self-"multiplication" of each $\mathbf{W}\underline{Q}_k$. They thus form a '*meta-sequence*' of their own: $\mathbf{W}\underline{Q}_0 \hat{=} \mathbf{W}\underline{Q}_1 \hat{=} \mathbf{W}\underline{Q}_2 \hat{=} \mathbf{W}\underline{Q}_3 \dots$

The $\{\mathbf{W}_{Q_k}\}$ 'meta-sequence' provides an abstract, generic image for the 'meta-dynamical', 'self-conversion singularity', 'self-bifurcating systems' paradigm of self-developing process, *at that level of complexity/concreteness/specificity of ontological, 'possibility-space' description*. Every \mathbf{W}_{Q_k} subset, restricted by the non-negative integer subscript k , is a proper subset of \mathbf{W}_Q as a whole: $\mathbf{W}_{Q_k} \subset \mathbf{W}_Q$. That subset can be "interpreted for" or "applied to" the myriad contexts of our 'psycho-historical materialism' paradigm, e.g., *concrete*, "'external"/physical, 'physio-ontological', or *abstract*, conceptual mimetic ['ideo-ontological'] 'meta-evolutions', thus forming *specific-process 'onto-dynamical' models* for specific universes of discourse. The ordinary \mathbf{W} -numbers, $0, 1, 2, 3, \dots$ are vastly generic, precisely *because* they are 'unqualified', or 'purely-quantitative', 'de-qualified'; because they abstract/simplify/de-specify from any determinations -- from any ontological, metrical, or other qualifications/'qualifiers' -- that might be included in more specific enumerations, and are thus "'assignable" to countings of $0, 1, 2, 3, \dots$ anythings, from cauliflowers to cannibals to kingdoms. Just so, we hold, can $\mathbf{W}_{Q_0}, \mathbf{W}_{Q_1}, \mathbf{W}_{Q_2}, \mathbf{W}_{Q_3}, \dots$ be assigned to the sequences, successions, or progressions of emergent *ontological qualities/categories*, or '*ontos*' observed in 'self-developing processes' as diverse as: (1.) the 'meta-evolutions' of *atomic species* and of planets in the *star-and-planet-birthing interstellar cloud cumula* of the galactic interstellar medium; (2.) the *nebular/planetary 'meta-evolutions'* of *molecular species* by/in "molecular clouds"; (3.) the *planetary, biospheric and then 'bio-noospheric' 'meta-evolution'* of *humanoid species*, and beyond.

Each \mathbf{W}_{Q_k} denotes an [the k th] 'meta-evolutionary epoch' of an 'ontologically-expanding', 'onto-dynamical' 'universe of discourse'. The 'meta-sequence' to which that \mathbf{W}_{Q_k} belongs is used to model the 'meta-dynamics' of that universe as a succession/progression of such epochs. Each \mathbf{W}_{Q_k} in this \mathbf{W}_{Q_k} 'meta-sequence' denotes a diverse or "heterogeneous", "'non-reductionist'", 'non-collapsing', "non-amalgamative" 'sum of [ontic] qualifiers' characterizing the possibilities of the given 'onto-dynamical' universe of discourse during the given epoch. This characterization is achieved by means of the interpretation of that sum as a special kind of 'universal ontology-set' for that universe of discourse -- a *set of 'ontological qualifiers'* or '*ontos*' defining the current-epoch *potential ontology* of that universe. This 'ontology set' defines what may be actually extant, but, essentially, what is *possibly extant* for/in that universe during that epoch. This 'onto-list' is the set of all ontological categories or qualities which that epochal universe contains or *could* contain; the ontological '*possibility-space*' or '*possibility-metastate*' of that universe in that epoch. It is an enumeration of the categories of what activities/[ev]entities *can* exist in that universe at that "time". As the *value* of the 'self-bifurcation index', or whole-number 'epoch-index', k , increases or advances, the '*self-[re-]flexion*' of the presently-extant \mathbf{W}_{Q_k} 'meta-state' generates the next 'ontology meta-state' of that universe of discourse as the next value in the 'meta-sequence', namely $\mathbf{W}_{Q_{k+1}}$. The 'universe-progression' or 'onto-dynamical' process modeled by the \mathbf{W}_{Q_k} 'meta-sequence' is thus that of a '*self-expanding ontology-set*', or of a '*self-expanding possibility-space*', grounding both probability and actualization in each such universe-epoch. Only that which is first at least *possible* can become also *probable* and, perhaps, even *actual*[-ized]. This dynamical image of universal autopoiesis is thus an '*Onto-Dynamical*' Model, departing from traditional 'Onto-Static', Parmenidean paradigms.

Below is depicted how this generic-dialectical process looks per the accompanying 'dynamic-geometric' or 'dynamic-pictographic' formulae which these ideographic formulae, per this 'topo-metrical interpretation', encode, imaging a *self-expanding* [possibility-]space with ever more new mutually-perpendicular unit-interval line-segment directions, axes, or arrows sprouting from the w^q_0 origin as the epoch-index advances [in the depictions below, only

the end-points of the directional, unit-length line-segments are labeled with an $w^{\hat{q}}_k$ 'meta-number' value, but each of these labels actually applies to the entire directed line-segment, including the starting-point of each of these directed line-segments, always labeled w^q_0]:

$$w^q_0 = [w^{\hat{q}}_1], \text{ ideographically; } w^q_0 = \begin{array}{c} \bullet \cdots \bullet \\ w^q_0 \quad [w^{\hat{q}}_1] \end{array} \quad [\text{vs. } E \leftrightarrow \bullet \cdots \bullet \approx [0, 1] \subset \mathbf{R}^1],$$

pictographically;

$$w^q_1 = [w^{\hat{q}}_1 + w^{\hat{q}}_2]; \quad w^q_1 = \begin{array}{c} \bullet \cdots \bullet \\ w^{\hat{q}}_2 \quad [w^{\hat{q}}_1 + w^{\hat{q}}_2] \\ \bullet \cdots \bullet \\ w^q_0 \quad w^{\hat{q}}_1 \end{array} \quad \text{pictographically;}$$

$$w^q_2 = [w^{\hat{q}}_1 + w^{\hat{q}}_2 + w^{\hat{q}}_3 + w^{\hat{q}}_4]; \quad w^q_2 = \begin{array}{c} \bullet \cdots \bullet \\ w^{\hat{q}}_2 \quad [w^{\hat{q}}_1 + w^{\hat{q}}_2 + w^{\hat{q}}_3 + w^{\hat{q}}_4] \\ \bullet \cdots \bullet \\ w^q_0 \quad w^{\hat{q}}_1 \end{array} \quad \text{pictographically;}$$

[the 4th dimension, the $w^{\hat{q}}_4$ axis, is not depicted];

$$w^q_3 = [w^{\hat{q}}_1 + w^{\hat{q}}_2 + w^{\hat{q}}_3 + w^{\hat{q}}_4 + w^{\hat{q}}_5 + w^{\hat{q}}_6 + w^{\hat{q}}_7 + w^{\hat{q}}_8];$$

$$w^q_4 = [w^{\hat{q}}_1 + w^{\hat{q}}_2 + w^{\hat{q}}_3 + w^{\hat{q}}_4 + w^{\hat{q}}_5 + w^{\hat{q}}_6 + w^{\hat{q}}_7 + w^{\hat{q}}_8 + w^{\hat{q}}_9 + w^{\hat{q}}_{10} + w^{\hat{q}}_{11} + w^{\hat{q}}_{12} + \dots + w^{\hat{q}}_{16}]; \dots;$$

Each $w^{\hat{q}}_k$ in the '[poly]-qualinomials' above whose subscript is a **W**hole-number power of 2 is interpretable as, and assignable to, the 'contra-thesis', 'qualitative increment', or 'ontological increment', denoted by the 'symbol-complex' $\Delta_w^{\hat{q}}_{k/2}$; the 'new onto yield' of the self-product 'self-hybridization' or 'self-reflexion' of an earlier 'onto' whose subscript is 1/2 the value of the subscript of the 'meta-numeral' assigned to that earlier 'onto'. Thus $w^{\hat{q}}_8$ is the $\Delta_w^{\hat{q}}_{k/2}$ of $w^{\hat{q}}_4$ of $w^{\hat{q}}_4$; the $w^{\hat{q}}_4$ of 2nd degree' -- the '2nd degree' of earlier 'onto' $w^{\hat{q}}_4$. Such a $w^{\hat{q}}_k$ value is interpretable and assignable as representing the 'meta-onto' of that 'onto'. Each of the new units [new "'logical individuals'"], or 'neo-«monads», of this 'meta-onto' are made up out of a heterogeneous multiplicity of the old units, or «monads», of that predecessor 'onto', via an «aufheben» operation instantiating 'meta-fractal', 'meta-finite' 'self-subsumption', 'self-interiorization', or 'self-internalization' 'meta-dynamic', i.e., wherein molecules are 'meta-atoms made up out of a heterogeneous multiplicity of atoms', or are 'atoms of second degree'; wherein prokaryotic cells are 'meta-molecules made up out of a heterogeneous multiplicity of molecules', or are 'molecules of second degree', wherein eukaryotic cells are 'prokaryotic cells made up out of a heterogeneous multiplicity of prokaryotic cells', or are 'prokaryotic cells of second degree', etc.

$$\text{Such } w^{\hat{q}}_k \text{ come under the Corollary of Rule 3, } [\forall \hat{q}_j \in w^q_0] [\hat{q}_j \hat{q}_j - \hat{q}_j^2 = [[\hat{q}_j + \hat{q}_{j+1}] = [[\hat{q}_j + \hat{q}_{2j}] \frac{1}{2} \hat{q}_j].$$

Each $w^{\hat{q}}_k$ in the '[poly]-qualinomials' above whose [post-]subscript, **k**, is not a positive integral power of 2 denotes a 'hybrid onto'. Each such 'hybrid onto' stands for a 'joint product', and, usually, for an 'ontological [self-]conversion process', transforming, to the [meta-]«monads» of the 'onto' denoted by the left-most interpreted subscript, the 'fuel' [sub-]«monads» of the [ev]entity denoted by the "'complex unity'" connoted by all of the interpreted subscripts to the right of that left-most subscript -- thereby producing the new, qualitatively distinct offspring of the "mutual interactions" of the one or more other, *distinct*, *hybrid* and/or *non-hybrid* or 'self-hybrid' 'ontos' so denoted. The contemporary atmosphere, ocean, and soils of planet Earth are combined results of the mutual activities of 'eventities' belonging to the atomic, molecular, prokaryotic cellular, eukaryotic cellular, multicellular a-social, social, and "'human[oid]-social'" or 'meta-social' 'ontos'. Each of these media requires a distinct 'hybrid-onto qualifier' in its own right, representing a 'hybrid ontological category', distinct from those of each of its progenitor 'ontos'. Such hybrids also connote categorial 'non-empty boundaries' or 'interfaces' [Σ "leidenfrost layers"], and 'ontological [self-]conversion formations', situated "'between'" their parent 'ontos'.

Another way to understand these terms is as denoting *mutual existential adjustments* or "syntheses" that non-hybrid or 'self-hybrid' 'ontos' make to one another as a result of their *co-existence* and ensuing *co-activity*.

Such \hat{Q}_k come under **Rule 3**, $[\forall \hat{Q}_i, \hat{Q}_j \in \underline{Q} \mid i \neq j][\hat{Q}_i \hat{Q}_j = \hat{Q}_j + \hat{Q}_{i+j}]$.

Note that this Rules-System for \underline{Q} implies that the relations among the \hat{Q}_k meta-numbers include "*linear independence*" but also '*nonlinear dependence*'. **Rule 0**, and the generic algebraic characterization $\{\hat{X} \perp \Delta \hat{X}\}$, assert that \hat{Q}_m and \hat{Q}_n are "*linearly independent*" if $n \neq m$. **Rule 3** asserts that, e.g., whenever $n = 2m$, \hat{Q}_m and \hat{Q}_n are '*nonlinearly dependent*', '*self-reflexively inter-dependent*', or '*self-application inter-dependent*', i.e., that $\hat{Q}_n = \hat{Q}_{2m} = \hat{Q}_m^2 - \hat{Q}_m$. In summary, $\{\hat{X}^2 \neq X\}$ and $\{\hat{X}^1 \perp \Delta \hat{X} = \hat{X}^2 - \hat{X}^1\}$ together connote the *linear independence* relationship, and, together with $\{\hat{X}^2 \neq X\}$, also the '*qualitative disproportionality*', that is, the '*meta-nonlinear*' nature, of the \hat{Q}_k 'meta-numbers'. Note also that the self-product rules, in their geometric interpretation as given above, follow a pattern we term '*meta-diagonalization*', viz., each \hat{Q}_k denotes, geometrically, the *diagonal* 'meta-vector' of a 2^k -dimensional *unit* ['hypo-' or "hyper-"]*cube*, and denotes the *intensional-semantic quality* of each such *interpreted* \hat{Q}_k 'ontological meta-state', as mapped via a metaphor of *dimensionally-distinctive directionality* --

$$\begin{aligned} \underline{\text{diagonal}}_k &\rightarrow \underline{\text{diagonal}}_k [\underline{\text{diagonal}}_k] = -[\underline{\text{diagonal}}_k] = [\underline{\text{diagonal}}_k]^2 = \underline{\text{diagonal}}_k + \Delta [\underline{\text{diagonal}}_k] \\ &= [\underline{\text{diagonal}}_k + \underline{\text{meta-diagonal}}_k] = \underline{\text{diagonal}}_{k+1} \neq \underline{\text{diagonal}}_{k'} \end{aligned}$$

and, using the Euclidean [Pythagorean] metric, we can prove that the 'pure quantifiers' of the lengths of these '*hypo-diagonals*' [for $k = 0$], '*diagonals*' [for $k = 1$], & '*hyper-diagonals*' [for $k \geq 2$] bear the following relations --

$$\|\underline{\text{diagonal}}_{k+1}\| > \|\underline{\text{diagonal}}_k\|, \text{ as } \sqrt{2^{k+1}} > \sqrt{2^k}.$$

This also suggests the possibility that each '*possibility-space meta-system meta-state*' \hat{Q}_k for *interpreted* \underline{Q} can be specified *uniquely* by a single \mathbb{R}^1 scalar pure-quantifier value, qualified by the radians metrical qualifier, namely, the scalar quantifier value quantifying the *angle* of the **2-D** projection of the k th '*hyper-diagonal*' upon the $\hat{Q}_1 \perp \hat{Q}_2$ plane.

[**Note:** In the rest of this sub-section, we will usually omit "number-space" designating '*pre-subscripts*', writing \hat{Q}_k & \hat{Q}_k as simply \underline{Q}_k & \hat{Q}_k , with the value of **X** -- either **N, W, Z, Q, R, C, H, O, K, G**, or beyond -- left unspecified explicitly, but made clear implicitly via context.

Q '*Meta-Dynamical*' Algebra. Interpreting subscripts **k** of the meta-sequence $\{\underline{Q}_k\}$ as indices of discrete 'meta-time' in the form of the *meta-evolutionary epoch index*, τ , we see that the $\underline{Q}_k = \underline{Q}_\tau$ 'multi-meta-ontic' and 'multi-meta-monadic' 'cumulum'-descriptors evolutely regenerate themselves and generate each other via what we term 'meta-dynamical self-reflexion', per **Rule 3**, *vide* --

$$\underline{Q}_\tau \rightarrow \sim[\underline{Q}_\tau] = \underline{Q}_\tau[\underline{Q}_\tau] = \underline{Q}_\tau^2 = \underline{Q}_\tau + \square\underline{Q}_\tau = \underline{Q}_{\tau+1} \stackrel{\hat{A}}{\neq} \underline{Q}_\tau; \quad \underline{Q}_{\tau+1} = \sim[\underline{Q}_\tau] = \underline{Q}_\tau[\underline{Q}_\tau] = \underline{Q}_\tau^2;$$

$$\underline{Q}_0 = [\hat{q}_1];$$

$$\underline{Q}_{0+1} = \underline{Q}_1 = \underline{Q}_0^2 = \underline{Q}_0[\underline{Q}_0] = [\hat{q}_1][\hat{q}_1] = [\hat{q}_1 + \hat{q}_{1+1}] = [\hat{q}_1 + \hat{q}_2];$$

$$\underline{Q}_{1+1} = \underline{Q}_2 = \underline{Q}_1^2 = \underline{Q}_1[\underline{Q}_1] = [\hat{q}_1 + \hat{q}_2][\hat{q}_1 + \hat{q}_2] = [\hat{q}_1 + \hat{q}_2 + \hat{q}_{1+2} + \hat{q}_{2+2}] = [\hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4];$$

$$\underline{Q}_{2+1} = \underline{Q}_3 = \underline{Q}_2^2 = \underline{Q}_2[\underline{Q}_2] = [\hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4][\hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4] = [\hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4 + \hat{q}_5 + \hat{q}_6 + \hat{q}_7 + \hat{q}_8];$$

$$\underline{Q}_{3+1} = [\hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4 + \hat{q}_5 + \hat{q}_6 + \hat{q}_7 + \hat{q}_8 + \hat{q}_9 + \hat{q}_{10} + \hat{q}_{11} + \hat{q}_{12} + \hat{q}_{13} + \hat{q}_{14} + \hat{q}_{15} + \hat{q}_{16}]; \dots$$

The above portrays, in skeletal outline, the 'Evolute Onto-Logic' of this 'meta-model' of self-expanding universes of existential possibility. The $\{\underline{Q}_\tau\}$ 'meta-sequence' thus models 'meta-evolving' universe progressions as *self-extending similarity structures*, self-growing 'quasi-/meta-fractal' rheid crystals. We say 'quasi-fractal' or 'meta-fractal' because, at each epoch, that 'meta-sequence' involves a *finite* forward *qualitative*-scales-regress, rather than a supposedly/potentially *infinite* *quantitative*-scales-regress, as do mathematical/idealized "fractals", and one in which the next higher 'meta-scale' in the 'meta-scales sequence' is constructed from, and by, the inherent activity of immediately preceding 'meta-scale-level', with the cumulative participation of all previous 'meta-scale-levels'. This 'meta-sequence' is designed to capture *the self-similarity invariant* of these structures, relative to their stipulated origination, \hat{q}_1 . This 'meta-sequence' locates the source of qualitative novelty, or of 'ontological innovation', in the mutual "interaction" and in the "self-interaction", that is, in the "self-reflexion", of previous 'self-innovation'. The following pair of *nonlinear* 'meta-dynamical'-algebraic equations, which we term '*Meta-Rules*', summarize its '*Onto-Dynamical Logic*':

Meta-Rule 1. [Meta-Evolution Equation]: Next universe "equals" [or 'results from'] self-interaction of current universe,

$$\text{or, ideographically, } \underline{Q}_{\tau+1} = \underline{Q}_\tau^2 = \underline{Q}_\tau[\underline{Q}_\tau] = \sim[\underline{Q}_\tau];$$

Meta-Rule 2. [Generating Equation, closed-form general solution of the Pure-Qualitative-nonlinear 'Meta-Evolution Equation']:

$$\underline{Q}_\tau = \underline{Q}_0^{2^\tau} = [\hat{q}_1]^{2^\tau}.$$

The "Right-Hand-Side" [RHS] of the above solution-equation is a 'meta-exponential', 'hyper-exponential', or algebraically 'hyper-nonlinear' function. This RHS involves two tiers of superscription, denoting two levels of exponentiation. This RHS is a term of 'degree' 2^τ in \underline{Q}_0 , a degree which escalates with the advance of the *epoch-index*, τ . That ' 2^τ degree' is an arithmetically and algebraically *nonlinear* degree, a degree greater than **1**, when $\tau > 0$. That ' 2^τ degree' is of the linear degree, the degree **1**, only when $\tau = 0$, at the *stipulated origination* of the initial ontology of the *universe of discourse*. The 'sub-meta-sequence' \underline{Q}_τ will have 2^τ terms. It will be a 2^τ '*poly-qualinomial*'. Its terms span all \hat{q}_k s having consecutive "Natural" number subscripts from **1** to 2^τ inclusive, *with no gaps*, no \hat{q}_k s missing in-between. The "cardinality" of the 'ontology-set' at stage τ , the number of 'ontological quality categories', or '*ontos*', which it contains [i.e., whose *possible* existence during that epoch it *asserts*], is 2^τ . The maximal ordinal subscript among the \hat{q}_k extant at stage τ is also 2^τ . The minimum subscript is always, at every stage, $2^0 = 1$.

The 'meta-sequence' denoted by $\{\underline{Q}_\tau\}$ is a *nested, cumulative, or 'evolutive' meta-sequence*: $\forall \tau \in \mathbf{o} \mid \Delta\tau = 1$, $\underline{Q}_\tau \subset \underline{Q}_{\tau+\Delta\tau}$, where \mathbf{o} denotes the set of "**N**atural" ordinal numbers, as interpreted, in this case, to represent ordinal discrete epochs, τ . Thus this '*onto-dynamics*' follows a kind of '**2** ^{τ} Dynamical Combinatorics'.

Note the equation of **Meta-Rule 2** enshrines the 'doubling function', $f(\tau) = 2^\tau$, which pops up so ubiquitously in nonlinear dynamics and in other key areas of modern mathematics, describing (a.) the "period-doubling route to [so-called] chaos", (b.) the number of sub-segments of the unit-interval segment remaining after the completion of the τ th step of the *Cantor middle-thirds process*, whose potentially transfinite iteration approaches the fractal *Cantor Set*, (c.) the *Cantorian cardinality of the power-set, or set of all subsets*, for a finite [or potentially "transfinite"] set of cardinality τ , etc., etc.

Four 'Meta-Dynamical' Products and Their 'Gödel Numbering Subscript-Rule' Variants. Besides the '*Aufheben*' Evolute Product' of **Rule 3**, we also partially explore, in the Section entitled *The Arithmetics of Meta-Evolution*, three other alternative product rules. We also explore a 'Gödelian' variant of each of these four product rules. The latter variants employ a subscript rule inspired by "Gödel numbering" -- the use Kurt Gödel made of the *Fundamental Theorem of ["Natural" Number] Arithmetic* in his *Incompleteness Theorem*. Gödel applied the former *Theorem* to the construction of the latter *Theorem*, in such a way as to form a unique mapping/encoding of the formulae of symbolic logic to elements of **N**. The function $p(n)$ selects the n th prime number, for $n \in \mathbf{N}$. In summary, we explore the following eight product rules --

1. The «Aufheben» Evolute Product Rule: $\hat{q}_j[\hat{q}_k] = [\hat{q}_k + \hat{q}_{j+k}]; \quad \hat{q}_k[\hat{q}_j] = [\hat{q}_j + \hat{q}_{j+k}];$
2. The 'Meta-Catalysis' Evolute Product Rule: $\hat{q}_j[\hat{q}_k] = [\hat{q}_j + \hat{q}_{j+k}]; \quad \hat{q}_k[\hat{q}_j] = [\hat{q}_k + \hat{q}_{j+k}];$
3. The 'Meta-Genealogical' Evolute Product Rule: $\hat{q}_j[\hat{q}_k] = [\hat{q}_j + \hat{q}_k + \hat{q}_{j+k}];$
4. The 'Meta-Heterosis' Convolute Product Rule: $\hat{q}_j[\hat{q}_k] = [\hat{q}_{j+k}].$

The 'Gödelian' variants of these product rules are designed to achieve partial 'de-confounding', or greater distinguishability of distinct 'ontic' interaction-products from one another. This entails an even stronger form of *non*-commutativity than that of the first two '*non*-Gödelian' product rules stated above. In these 'Gödelian' variants, the 'index' or subscript of the 'qualitative increment' portion of a product is a 'Gödel number' encoding the syntax of the 'multiplication' formula from which that 'qualitative increment' or 'ontological increment' arose. Thereby, each 'evolutive' product *reveals, contains, or records its path-of-formation, origin, ancestry, or 'meta-genealogy'*, and is thus 'evolutive', or 'ontology-conserving' in the «aufheben» sense, in a yet deeper way. Given that $p(k)$ denotes the k th "**N**atural" prime number, s.t. $p(1) = 2$, and that $j < k$, we obtain:

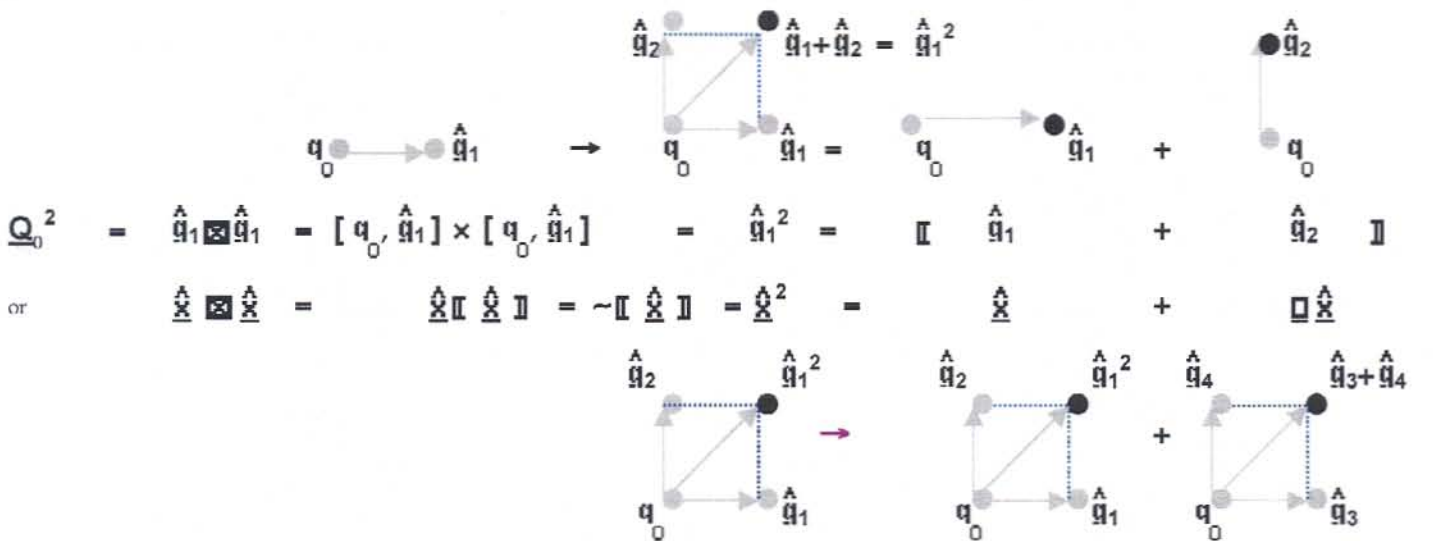
- 1g. 'Gödelian' «Aufheben» Evolute Product: $\hat{q}_j[\hat{q}_k] = \hat{q}_k + \hat{q}_{p(j)^j \cdot p(k)^k}; \quad \hat{q}_k[\hat{q}_j] = \hat{q}_j + \hat{q}_{p(k)^j \cdot p(j)^k};$
- 2g. 'Gödelian' 'Meta-Catalysis' Evolute Product: $\hat{q}_j[\hat{q}_k] = \hat{q}_j + \hat{q}_{p(j)^j \cdot p(k)^k}; \quad \hat{q}_k[\hat{q}_j] = \hat{q}_k + \hat{q}_{p(k)^j \cdot p(j)^k};$
- 3g. 'Gödelian' 'Meta-Genealogical' Evolute Product: $\hat{q}_j[\hat{q}_k] = \hat{q}_j + \hat{q}_k + \hat{q}_{p(j)^j \cdot p(k)^k}; \quad \hat{q}_k[\hat{q}_j] = \hat{q}_k + \hat{q}_j + \hat{q}_{p(k)^j \cdot p(j)^k};$
- 4g. 'Gödelian' 'Meta-Heterosis' Convolute Product: $\hat{q}_j[\hat{q}_k] = \hat{q}_{p(j)^j \cdot p(k)^k}; \quad \hat{q}_k[\hat{q}_j] = \hat{q}_{p(k)^j \cdot p(j)^k};$

Per «aufheben» versions of the 'meta-vector' or 'dialector' product, $\{\underline{Q}_\tau\}$ denotes a *sequence = series* s.t.:

$$\begin{aligned}
 \underline{Q}_0 &\leftrightarrow \hat{\Psi}_0 = [100000000000000000\dots]^T = \sqrt{\hat{\Psi}_0} \uparrow \sqrt{\hat{\Psi}_0}; \\
 \underline{Q}_1 &\leftrightarrow \hat{\Psi}_1 = [110000000000000000\dots]^T = \hat{\Psi}_0 \uparrow \hat{\Psi}_0; \\
 \underline{Q}_2 &\leftrightarrow \hat{\Psi}_2 = [111100000000000000\dots]^T = \hat{\Psi}_1 \uparrow \hat{\Psi}_1; \\
 \underline{Q}_3 &\leftrightarrow \hat{\Psi}_3 = [111111110000000000\dots]^T = \hat{\Psi}_2 \uparrow \hat{\Psi}_2; \\
 \underline{Q}_4 &\leftrightarrow \hat{\Psi}_4 = [1111111111111111\dots]^T = \hat{\Psi}_3 \uparrow \hat{\Psi}_3; \dots
 \end{aligned}$$

In contrast to the dual *consecutivities*, first of an all 1s sub-string, then followed by an all 0s sub-string, evident above, 'Gödelian' versions of the «*aufheben*» 'evolute' product, per the *potentially-infinite-dimensional 'meta-vector'* model above, exhibit a *widening sequence of '0 sub-string gaps'* between each successive pair of 1s.

Q. 'Meta-Dynamical' Analytical Geometry. The 'meta-dynamical-geometry' or 'meta-analytical-geometry' of the $\{\underline{Q}\}$ space is the 'meta-dynamical' geometry of a 'Self-Expanding', 'Self-Replicating', 'Self-Reproducing', 'Self-Mirroring', 'Self-Reflecting', or 'Self-Reflexive' Space. This geometrical 'Self-Replication' takes the form of a spatial 'Self-Doubling' with each incrementation of the 'epoch-index' or 'self-bifurcation' index-value, τ . This process can be visualized as follows --



$$\underline{Q}_0^2 = \hat{q}_1 \boxtimes \hat{q}_1 = [q_0, \hat{q}_1] \times [q_0, \hat{q}_1] = \hat{q}_1^2 = \llbracket \hat{q}_1 \rrbracket + \hat{q}_2 \rrbracket$$

or

$$\hat{x} \boxtimes \hat{x} = \hat{x} \llbracket \hat{x} \rrbracket = \sim \llbracket \hat{x} \rrbracket = \hat{x}^2 = \hat{x} + \hat{y}$$

$$\underline{Q}_1^2 = \llbracket \hat{q}_1 + \hat{q}_2 \rrbracket \boxtimes \llbracket \hat{q}_1 + \hat{q}_2 \rrbracket = \llbracket \hat{q}_1 + \hat{q}_2 \rrbracket^2 = \llbracket \hat{q}_1 + \hat{q}_2 \rrbracket + \llbracket \hat{q}_3 + \hat{q}_4 \rrbracket$$

or

$$\underline{y} \boxtimes \underline{y} = \sim \llbracket \underline{y} \rrbracket = \underline{y} + \underline{y}$$

-- depicting the $\hat{q}_1 \llbracket \hat{q}_1 \rrbracket$ "self-reflexion" as 'copying' the 1-D finite-length, unit-length 'hypo-cube' line-space \hat{q}_1 axis, and attaching it perpendicularly back to that \hat{q}_1 axis at its origin-point, q_0 , as the directionally, perpendicularly, qualitatively distinct \hat{q}_2 finite, unit-length axis, yielding the "discretized", interval-notation *Cartesian Product* \boxtimes plane-space $[q_0, \hat{q}_1] \times [q_0, \hat{q}_2]$ as the implicit backdrop of the diagonal, finite-length, $+\sqrt{2}$ units-in-length, *directed line-segment* product $\underline{Q}_1 = \llbracket \hat{q}_1 + \hat{q}_2 \rrbracket$. Next, the 'self-product', or "squaring", of that product, the $\llbracket \hat{q}_1 + \hat{q}_2 \rrbracket \boxtimes \llbracket \hat{q}_1 + \hat{q}_2 \rrbracket$ "self-reflexion", 'copies' that 2-D 'hypo-cubic' plane-space, denoted in interval notation by $[q_0, \hat{q}_1] \times [q_0, \hat{q}_2]$, attaching it back to $[q_0, \hat{q}_1] \times [q_0, \hat{q}_2]$ as the qualitatively distinct plane-space $[q_0, \hat{q}_3] \times [q_0, \hat{q}_4]$, forming the 4-D hypercube-space $[q_0, \hat{q}_1] \times [q_0, \hat{q}_2] \times [q_0, \hat{q}_3] \times [q_0, \hat{q}_4]$ as the implicit backdrop of the diagonal, finite-length, $+\sqrt{4} = 2$ units-in-length *directed line segment* product $\underline{Q}_0^{2^2} = \underline{Q}_2 = \llbracket \hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4 \rrbracket$, and so on, with each successive, iterated 'spatial self-reflexion'.

The 'contental logic', 'existential logic', or 'Onto-Logic' of these 'meta-numbers' may be regarded as a 'potentially *infini-valent logic*'. This contrasts with the "bi-valent" or 2-valued logic of later Boolean Algebra [Boole's original Algebra having partially admitted 'tetra-valence', with 1/1 as 1, or the logical quantity "All", with 0/1 as 0, or the logical quantity "None", plus with 0/0 as the logical quantity 'Indefinite' ["None, Some, or All"], and with 1/0 as logical "Infinity" ["Singularity", or "Impossibility"]. Each \hat{q}_k of the potentially-infinite sequence of \hat{q}_k within \underline{Q} may be interpreted as denoting a qualitatively distinct *existential [ontological] "truth-value" or "existence-value" [existential possibility-value]*. As a "number/numeration *system*", or "numeral scheme", the \underline{Q} arithmetic is, in a sense -- at the 'scriptal' level, as opposed to at the 'sub-scriptal' level -- a *unary*, rather than binary, decimal, duodecimal, vigesimal, or sexagesimal, etc., *system*. That is, it forms, in a sense, a [meta-]numeral system to the base 1. Each [meta-]numeral, \hat{q}_k -- lacking "place-value" in taking but one "place" -- denotes a *unique value*: neither a combination of earlier values in the values-sequence, nor a value which reappears later in that values-sequence, in combinations with other such [meta-]numbers, in the symbolic formation of any single unit -- unlike 1; 12; 101; 1,100; ..., etc.

The $\{\sum \hat{q}_k\}$ can also be grasped as a *unified space* of [transcendental] "functions", "transformations", or of "operations" [of 'operators'], in which "functions", "arguments", & "function-values" all coexist as a single space:

$$[\forall f_j, f_k \in \{\sum f_j\} = \mathbb{Q} \mid f_j[f_k] = \underline{Y}] [[f_j: \mathbb{Q} \mapsto \mathbb{Q}] \& [Y \in \mathbb{Q}] \& [Y = f_j + f_{j+k}]]$$

I.e., each "function" \hat{q}_k in \mathbb{Q} is an element of the "domain-space" of its "arguments", and is also an element of the "range-space" of its 'products' or "function-values", both of which are identically the space in which the "function" itself also resides. For varying values of k , each "function" \hat{q}_k can also serve as an "argument" of other "functions" \hat{q}_k , and, as well, as a "function-value" [except for \hat{q}_1 in \mathbb{N}] of "functions", \hat{q}_k , operating upon [or "multiplying"] other "arguments", \hat{q}_k , or operating upon themselves.

Note also that the \hat{q}_k are all 'meta-transcendental numbers'. In that phrase, the epithet 'transcendental' means that these numbers cannot be "roots" of, or *solve*, any 'fini-nomial' -- meaning any finite-terms-count, finite degree, i.e., "algebraic" polynomial -- with "Rational" coefficients. We write 'meta-transcendental numbers', because mere "transcendental" numbers connote *pure-quantifier* "Real" numbers like e and π . We call the \hat{q}_k 'meta-transcendental' precisely because of their 'contra-Boolean' behavior -- $\underline{x^2} \not\equiv \underline{x}$ -- i.e., because $\hat{q}_k^2 \not\equiv \hat{q}_k^1$, coupled with their "additive idempotency", or 'unquantifiability'. This means that they are 'qualitatively disproportionate' [ϕ], i.e., *highly nonlinear*, with respect to the "Rational Numbers", \mathbb{Q} , or even to the "Reals", \mathbb{R} ; that there is no "Quotient" ["Rational"] number [or Real number], no matter how large, or how small, such that this number as a "coefficient" of \hat{q}_k^1 can equate the thus "multiplied" \hat{q}_k^1 to \hat{q}_k^n where $n > 1$:

$$\forall \hat{q}_k \in \mathbb{N}, \forall k \in \mathbb{N} \ni n > 1, \exists r \in \mathbb{Q}, \mathbb{R} \mid \hat{q}_k^n = r \hat{q}_k^1; \text{ i.e., } \forall r \in \mathbb{Q}, \mathbb{R}; \forall k \in \mathbb{N} \ni n > 1, \hat{q}_k^n \not\equiv r \hat{q}_k^1.$$

Therefore, degree n 'fini-nomial' or 'poly-qualinomial' equations like

$$r_n \mathbb{Q}_k^n + r_{n-1} \mathbb{Q}_k^{n-1} + r_{n-2} \mathbb{Q}_k^{n-2} + \dots + r_1 \mathbb{Q}_k^1 + r_0 \mathbb{Q}_k^0 = q_0$$

are 'non-algebraic polynomial equations' or "[meta-]transcendental functions", with no solutions if the parameters, r_j , are restricted to the "Rational numbers", or even to the "Real numbers", except for the null solution $\{r_j = 0\}$. The 'meta-models' of universe 'meta-evolution' that can be constructed using the apparatus set forth above are, of course, '[meta-]arithmetical', 'meta-numerical' models, and are also both highly *simplified* models and highly *abstract* models. Indeed, that is their goal: to extract and to exhibit in idealized form a universal principle; a 'meta-dynamical' essence-pattern of 'meta-evolutionary' or 'ontological-revolutionary' process in general; a generic ideographical image of the dialectic. The relative algebraic simplicity of these 'meta-models' is bought at the price of great generality; of large 'homeomorphic defect', that is, of the very high ratio of the *multitude* of features of the sensuous world *mapped to* but *single features* in the ideographical 'meta-model' of that world; of features in the world not found in the 'meta-model' ['Type I homeomorphic defect'], as well as of some 'extranea' or 'artifacts' -- features in the 'meta-model' not found in the world ['Type II homeomorphic defect']. The 'Onto-Dynamical' equations above describe, in an ideographical language, the *diachronics* of 'onto-dynamical' universes of discourse, of *self-evolving existences / activities*; *self-evolving ontologies*, with about as much detailed coverage as Boole's Algebra Of Logic describes propositions and the interrelationships of 'onto-statical' *synchronic* classes. The advance from the 'unit-interval restricted' \mathbb{Q} algebras to the 'full-multiplicity' \mathbb{U} and $\alpha \mathbb{U}$ algebras, and beyond, described in the *Briefings* that follow this one, as well as in The Arithmetics Of Meta-Evolution, Section III., below, begins to redress these grievances, at the cost of a more elaborate, more complex, more "concrete" syntactical and semantical rules-apparatus. The sub-section next-below summarizes the applicability of the $\{\mathbb{Q}\}$ arithmetics to the 'meta-modeling' of *historical-dialectical* processes -- to the formation of 'ideometric' and ideographical -- and, in that sense, "mathematical" -- 'historical-dialectical meta-models': 'historical-dialectical-mathematical meta-models'.

Q Arithmetic and Historical Dialectics. By way of a brief evocation of the dialectical character of models made in the ideographic language of **Q**, we reproduce, below, a passage by neo-Hegelian philosopher Errol E. Harris, in which he summarizes, in word-text -- in phonogramic/phonetic symbols -- the principal characteristics of dialectical process. Further, we 'commentarize' that text with annotations illustrating the way in which and the degree to which those characteristics of dialectical process per Harris' account can be concisely "captured" in the ideographic language of the **Q** arithmetic and algebra. The passage is as follows --

"Each *level* provides the basis for that which succeeds, yet on every *level* the characteristic properties of the appropriate entities depend upon their total structure. They are "cooperative properties" *impossible* for less complex entities. Atoms have properties *impossible* for free electrons, and molecules evince chemical affinities which are dependent solely upon the pattern of combination of their constituent atoms and are not characteristic of any atom in isolation. This is especially true of the macromolecules involved in the activities of living matter, which are *not feasible* at the inorganic level . . . Consequently the cosmic organism, while it is one and indivisible, is at the same time a *range of developing phases*, which can be represented, and which display themselves, as a *dialectical scale* [or a graduated 'consecuum' made up of/generating a self-extending diachronic sequence of mutually, 'quanto-qualitatively' similar/dis-similar 'quanto-qualitative' 'scales'; a 'quanto-qualitatively' 'metafinite' scales-regress formation of quanto-qualitatively self-similar structure which we term a 'meta-fractal' -- F.E.D.]. The totality [or 'meta-system' -- F.E.D.] is constituted by the scale of its internal forms, and each level [$\overset{n}{\underset{\vee}{Q}}_r$] is in some sense self-contained and all-pervasive; yet each gives rise to the next above it [$\overset{n}{\underset{\vee}{Q}}_{r+1}$] by virtue of the *potentiality* within it infused by the *immanent principle* of the totality [or of subject/object 'intra-duality' -- F.E.D.] in which it is no more than a *phase* [$\overset{n}{\underset{\vee}{Q}}_0 \rightarrow \overset{n}{\underset{\vee}{Q}}_1 \rightarrow \dots \overset{n}{\underset{\vee}{Q}}_r \rightarrow \overset{n}{\underset{\vee}{Q}}_{r+1} \rightarrow \dots$]. This is an idea of nature, not merely as an all-embracing living animal, but as a dynamic organismic system [or systems-progression 'meta-system', made up out of a diachronic succession of many systems -- F.E.D.], comprising a continuous range of wholes, on levels of progressively increasing complexity and integration. They are wholes mutually in dialectical relation, so that the entire system [or 'meta-system', since this "system" is composed of a diachronic sequence of sub-wholes also termed "systems" -- F.E.D.] manifests itself as an evolutionary [or 'meta-evolutionary', since each "system" in the diachronic sequence also "evolves" internally, before, and after, and leading to, its "revolutionary", ontology-net-expanding self-transformation into the next system in this diachronic systems-sequence or 'meta-system' -- F.E.D.] progression."

[Errol E. Harris, *Formal, Transcendental, and Dialectical Thinking: Logic and Reality*, State University of New York Press [Albany, NY: 1987], pp. 255-256, bold italics commentary and emphasis added by F.E.D.]

[Some examples -- (1) consider the «Physis»/Cosmos as a whole, in its diachronic 'self-meta-evolution' from a universe 'super-system' whose organization stops at the level/scale/degree of sub-nuclear particles [$\overset{1}{\underset{\vee}{n}}$], to one whose organization stops at the level/scale/degree of sub-atomic particles [$\overset{1}{\underset{\vee}{s}}$], to one whose organization stops at the level/scale/degree of atoms [$\overset{1}{\underset{\vee}{a}}$], to one whose organization stops at the level/scale/degree of molecules [$\overset{1}{\underset{\vee}{m}}$], to one whose organization stops at the level/scale/degree of prokaryotic "living" cells [$\overset{1}{\underset{\vee}{p}}$], . . . We render that natural-historical «autokinesis», or self-movement, and the 'metafractal cumula' of its successive systems, in ideographic intensional, mnemonic-heuristic symbols, as:

$$\begin{aligned} & [\overset{1}{\underset{\vee}{n}}] \rightarrow [\overset{1}{\underset{\vee}{n}} + \overset{1}{\underset{\vee}{s}}] \rightarrow [\overset{1}{\underset{\vee}{n}} + \overset{1}{\underset{\vee}{s}} + \overset{1}{\underset{\vee}{q_{sn}}} + \overset{1}{\underset{\vee}{a}}] \rightarrow [\overset{1}{\underset{\vee}{n}} + \overset{1}{\underset{\vee}{s}} + \overset{1}{\underset{\vee}{q_{sn}}} + \overset{1}{\underset{\vee}{a}} + \overset{1}{\underset{\vee}{q_{an}}} + \overset{1}{\underset{\vee}{q_{as}}} + \overset{1}{\underset{\vee}{q_{asn}}} + \overset{1}{\underset{\vee}{m}}] \rightarrow \\ & [\overset{1}{\underset{\vee}{n}} + \overset{1}{\underset{\vee}{s}} + \overset{1}{\underset{\vee}{q_{sn}}} + \overset{1}{\underset{\vee}{a}} + \overset{1}{\underset{\vee}{q_{an}}} + \overset{1}{\underset{\vee}{q_{as}}} + \overset{1}{\underset{\vee}{q_{asn}}} + \overset{1}{\underset{\vee}{m}} + \overset{1}{\underset{\vee}{q_{mn}}} + \overset{1}{\underset{\vee}{q_{ms}}} + \overset{1}{\underset{\vee}{q_{msn}}} + \overset{1}{\underset{\vee}{q_{ma}}} + \overset{1}{\underset{\vee}{q_{man}}} + \overset{1}{\underset{\vee}{q_{mas}}} + \overset{1}{\underset{\vee}{q_{masn}}} + \overset{1}{\underset{\vee}{p}}], \\ \text{or} \\ & \overset{1}{\underset{\vee}{Q}}_0 \rightarrow \overset{1}{\underset{\vee}{Q}}_1 \rightarrow \overset{1}{\underset{\vee}{Q}}_2 \rightarrow \overset{1}{\underset{\vee}{Q}}_3 \rightarrow \overset{1}{\underset{\vee}{Q}}_4, \text{ i.e., } \overset{1}{\underset{\vee}{Q}}_0^{2^0} \rightarrow \overset{1}{\underset{\vee}{Q}}_0^{2^1} \rightarrow \overset{1}{\underset{\vee}{Q}}_0^{2^2} \rightarrow \overset{1}{\underset{\vee}{Q}}_0^{2^3} \rightarrow \overset{1}{\underset{\vee}{Q}}_0^{2^4}. \end{aligned}$$

(2) consider the second-taxonomic-level sub-universe-of-discourse of the 'meta-system' of the human species' 'meta-social meta-dynamics', i.e., going on "'inside'" the first taxonomic level 'onto, $\overset{1}{\underset{\vee}{h}}$, in terms of its 'self-meta-evolution' from a 'meta-society' whose economic organization, or "social relations of production-inducing-circulation", stop at the level/scale/degree of 'predation' -- direct Appropriation, without improvement, of the raw products of nature, [$\overset{2}{\underset{h}{A}}$] -- to one whose organization stops at the level/scale/degree of 'raw appropriation squared', 'second[-degree] appropriation', 'appropriation of the appropriation', or 'appropriation with improvement', or refinement, of raw products of pre-/extra-human nature, i.e., 'Goods'/'Gifts'-production, [$\overset{2}{\underset{h}{G}}$], to one whose organization stops at the level/scale/degree of Commodity barter, where goods acquire, beside their direct-consumption utility, an indirect, exchange-utility, [$\overset{2}{\underset{h}{C}}$], to one whose organization stops at the level/scale/degree of Money, or of the Money-mediated exchange/"circulation" of Commodities, [$\overset{2}{\underset{h}{M}}$], to one whose organization stops at the level/scale/degree of 'money-making-money', or «Kapital», [$\overset{2}{\underset{h}{K}}$], . . . We render this human-social, psycho-historical movement of that 'meta-system', and the 'meta-fractal cumula' of its successive 'human-social' systems -- ideographically, in terms, again, of intensional, mnemonic-heuristic symbols -- as:

$$\begin{aligned}
& \left[\frac{2}{h} \underline{A} \right] \rightarrow \left[\frac{2}{h} \underline{A} + \frac{2}{h} \underline{G} \right] \rightarrow \left[\frac{2}{h} \underline{A} + \frac{2}{h} \underline{G} + \frac{2}{h} \underline{q}_{GA} + \frac{2}{h} \underline{C} \right] \rightarrow \left[\frac{2}{h} \underline{A} + \frac{2}{h} \underline{G} + \frac{2}{h} \underline{q}_{GA} + \frac{2}{h} \underline{C} + \frac{2}{h} \underline{q}_{CA} + \frac{2}{h} \underline{q}_{CG} + \frac{2}{h} \underline{q}_{CGA} + \frac{2}{h} \underline{M} \right] \rightarrow \\
& \left[\frac{2}{h} \underline{A} + \frac{2}{h} \underline{G} + \frac{2}{h} \underline{q}_{GA} + \frac{2}{h} \underline{C} + \frac{2}{h} \underline{q}_{CA} + \frac{2}{h} \underline{q}_{CG} + \frac{2}{h} \underline{q}_{CGA} + \frac{2}{h} \underline{M} + \frac{2}{h} \underline{q}_{MA} + \frac{2}{h} \underline{q}_{MG} + \frac{2}{h} \underline{q}_{MGA} + \frac{2}{h} \underline{q}_{MC} + \frac{2}{h} \underline{q}_{MCA} + \frac{2}{h} \underline{q}_{MCG} + \frac{2}{h} \underline{q}_{MCGA} + \frac{2}{h} \underline{K} \right], \\
\text{OR} \\
& \frac{2}{h} \underline{Q}_0 \rightarrow \frac{2}{h} \underline{Q}_1 \rightarrow \frac{2}{h} \underline{Q}_2 \rightarrow \frac{2}{h} \underline{Q}_3 \rightarrow \frac{2}{h} \underline{Q}_4, \text{ i.e., } \frac{2}{h} \underline{Q}_0^{2^0} \rightarrow \frac{2}{h} \underline{Q}_0^{2^1} \rightarrow \frac{2}{h} \underline{Q}_0^{2^2} \rightarrow \frac{2}{h} \underline{Q}_0^{2^3} \rightarrow \frac{2}{h} \underline{Q}_0^{2^4}.
\end{aligned}$$

Resuming the passage by Errol Harris --

"Let me once more recapitulate *the dialectical relation* in its full complexity. The wholes which it *relates* are each, in one aspect, *self-contained* and *self-dependent* [or, initially, *expandedly self-reproducing* -- F.E.D.], and, in another, *mutually implicated* and *inseparably interrelated* [for each predecessor system evolves from a state of *expanding self-reproductivity* to one of *partial self-dis-reproductivity* in the form of a *self-revolutionizing* or '*self-meta-evolutionizing*' production of its *qualitatively, ontologically different* -- *ontologically net-expanded* -- successor system -- F.E.D.]. Essentially the *relation* is *serial*, each *successor whole* being a fuller and more adequate realization [or a *fuller self-development* and *self-out-working* of its proto-subject/object '*intra-duality*' or '*internal ontological self-contradiction*' -- F.E.D.] of the systematic principle governing the entire *series*. So each is related to its predecessors as their fulfillment [or as the '*explicitization*' of what they held implicit -- F.E.D.], *requiring* and *incorporating the prior forms*, while *actualizing potentialities* of which they were incapable. For this reason, while *the subsequent involves the antecedent*, it also *supercedes* and, in some sense, *negates* its forebears. Each whole, then, is a *grade* [or '*metafinite*', '*meta-fractal scale/level* -- F.E.D.], a *developmental stage*, within the total *series*, but also a distinct *relatively self-subsistent* [or *transitorily self-reproducing* -- F.E.D.] *phase* standing in *qualitative contrast* and *opposition* [a relation herein denoted via the *picto-ideographic signs* '⊂' for the '*successor* ⊂ '*predecessor*' ordering, and '⊃' for the '*predecessor* ⊃ '*successor*' ordering -- F.E.D.] to its neighbors [⊂_{Q_{n+1}} ⊂_{Q_n} ⊂_{Q_{n-1}}]. Yet because this *opposition* is *resolved* in the *higher phase* (which *preserves* the contrast while it *supersedes* it [i.e., as the result of a '*self-aufheben*' operation -- F.E.D.]), the entire *series* remains continuous and coherent."

[Errol E. Harris, *ibid.*, p. 256, *bold italics commentary and emphasis* added by F.E.D.].

[The generic historical-dialectical *sequence/series* is, to the fifth stage, phase, or epoch, per the Q arithmetic/algebra for dialectics --

$$\frac{n}{u} \underline{Q}_0 \rightarrow \frac{n}{u} \underline{Q}_1 \rightarrow \frac{n}{u} \underline{Q}_2 \rightarrow \frac{n}{u} \underline{Q}_3 \rightarrow \frac{n}{u} \underline{Q}_4 \rightarrow \dots, \text{ or } \frac{n}{u} \underline{Q}_0^{2^0} \rightarrow \frac{n}{u} \underline{Q}_0^{2^1} \rightarrow \frac{n}{u} \underline{Q}_0^{2^2} \rightarrow \frac{n}{u} \underline{Q}_0^{2^3} \rightarrow \frac{n}{u} \underline{Q}_0^{2^4} \rightarrow \dots$$

Again, note that, because of the «*aufheben*» principle and the *additive idempotency* embedded in the Q Rules-System, a $\sum \frac{n}{u} \underline{Q}_k$ *series* or '*qualitative sum*' [*pure-ontological sum*] equals the leading term in the $\{ \frac{n}{u} \underline{Q}_k \}$ *sequence*, which already contains all of its predecessor terms. For example --

$$\frac{n}{u} \underline{Q}_0 + \frac{n}{u} \underline{Q}_1 + \frac{n}{u} \underline{Q}_2 + \frac{n}{u} \underline{Q}_3 + \frac{n}{u} \underline{Q}_4 = \sum_{(k=0,4)} \frac{n}{u} \underline{Q}_k = \frac{n}{u} \underline{Q}_4. \text{ [sequence = series in } \underline{Q}\text{].}$$

Each system/whole in the '*meta-system*' *series/sequence* thus '*causally implies*' ['→'] its successor(s): $\frac{n}{u} \underline{Q}_0 \rightarrow \frac{n}{u} \underline{Q}_1 \rightarrow \frac{n}{u} \underline{Q}_2 \rightarrow \frac{n}{u} \underline{Q}_3 \rightarrow \frac{n}{u} \underline{Q}_4 \rightarrow \dots$

Each also *ontologically exceeds* its predecessor(s): $\frac{n}{u} \underline{Q}_0 \dashv \frac{n}{u} \underline{Q}_1 \dashv \frac{n}{u} \underline{Q}_2 \dashv \frac{n}{u} \underline{Q}_3 \dashv \frac{n}{u} \underline{Q}_4 \dashv \dots$

Each also explicitly *incorporates / contains all* of its predecessor(s): $\frac{n}{u} \underline{Q}_0 \sqsubset \frac{n}{u} \underline{Q}_1 \sqsubset \frac{n}{u} \underline{Q}_2 \sqsubset \frac{n}{u} \underline{Q}_3 \sqsubset \frac{n}{u} \underline{Q}_4 \sqsubset \dots$

Each also '*explicitizes*' / *actualizes* what was only *implicit / potential* in its predecessor(s): $\frac{n}{u} \underline{Q}_0 \sqsupset \frac{n}{u} \underline{Q}_1 \sqsupset \frac{n}{u} \underline{Q}_2 \sqsupset \frac{n}{u} \underline{Q}_3 \sqsupset \frac{n}{u} \underline{Q}_4 \sqsupset \dots$

Each successor is also the '*ontological negation*' of its predecessor: $\frac{n}{u} \underline{Q}_{n+1} = \sim[\frac{n}{u} \underline{Q}_n] = \frac{n}{u} \underline{Q}_n [\frac{n}{u} \underline{Q}_n] = [\frac{n}{u} \underline{Q}_n]^2 = [\frac{n}{u} \underline{Q}_n + \Delta[\frac{n}{u} \underline{Q}_n]] \frac{1}{2} \frac{n}{u} \underline{Q}_n$.

Each system/whole in the '*meta-system*' *series / sequence / progression* also *opposes* its predecessors and its successor(s) in their key qualities/meaning/attributes:

$$\frac{n}{u} \underline{Q}_0 \dashv \frac{n}{u} \underline{Q}_1 \dashv \frac{n}{u} \underline{Q}_2 \dashv \frac{n}{u} \underline{Q}_3 \dashv \frac{n}{u} \underline{Q}_4 \dashv \dots, \text{ and } \frac{n}{u} \underline{Q}_0 \frac{1}{2} \frac{n}{u} \underline{Q}_1 \frac{1}{2} \frac{n}{u} \underline{Q}_2 \frac{1}{2} \frac{n}{u} \underline{Q}_3 \frac{1}{2} \frac{n}{u} \underline{Q}_4 \frac{1}{2} \dots.$$

Concluding the passage by Errol Harris --

"The relation of mankind to nature has now to be understood in the light of *this dialectic conception*. Human personality, developing within social structures peculiar to its appropriate *level* in the *scale*, is *integral to the whole*. On the other hand, as one *level* distinct from others, it *confronts the prior phases as other and opposed*. But this is only one aspect of its relation to them, for *they are also its forebears and progenitors* in which *the potentiality of its emergence is instant*. What *humanity sees as nature is its own self in becoming*; but more than this, nature is the very matrix from which its very being is contrived and the soil out of which it is nourished."

[Errol E. Harris, *ibid.*, pp. 256-257, *bold italics commentary and emphasis* added by **F.E.D.**].

Transition to the Briefing on the $\underline{U} \leftrightarrow \hat{u}_3$ Dialectical Ideography. The next *Briefing* evokes the $\underline{N}\underline{U}$ arithmetic out of the mutual and immanent [self-]critiques of both the \underline{N} and the $\underline{N}\underline{Q}$ arithmetics, as the dialectical "unification", "complex unity", "higher unity", or '*uni-thesis*' of the «arché» *thesis*, \underline{N} , and its '*contra-thesis*', $\underline{N}\underline{Q}$, a '*uni-thesis*' also denoted by \hat{u}_{QN} & by \hat{u}_U , completing the '*synthesis-sum*' denoted by $\underline{N} \oplus \underline{N}\underline{Q} \oplus \underline{N}\underline{U}$.

The foregoing briefing has set forth the apparatus of mainly the $\underline{N}\underline{Q}$ and $\underline{W}\underline{Q}$ epochs of the $\{\underline{Q}\}$ dialectical arithmetics and their dialectical algebras, without addressing their $\underline{Z}\underline{Q}$, $\underline{Q}\underline{Q}$, $\underline{R}\underline{Q}$, $\underline{C}\underline{Q}$, etc., "epochs", and at a rather generic level, largely detached from their interpretations -- i.e., rather remote from the [meta-]modeling of specific processes of *exo-human natural history* and of *human/'meta-social'* "psycho-history", or of the [psycho-]historical processes of development of particular *human conceptual systems* and *traditions of thought*.

The syntax of this *ideographical 'language of ontic meta-evolution'* more clearly reveals its semantic potential and limitations as we engage its interpretations. We so engage in the next full section, **Section II.**, where we employ the $\underline{N}\underline{Q}$ incarnation of dialectical ideography, among others, to model the observable, historical, human, "psycho-historical" phenomenon of *The Meta-Evolution Of Arithmetics* itself.