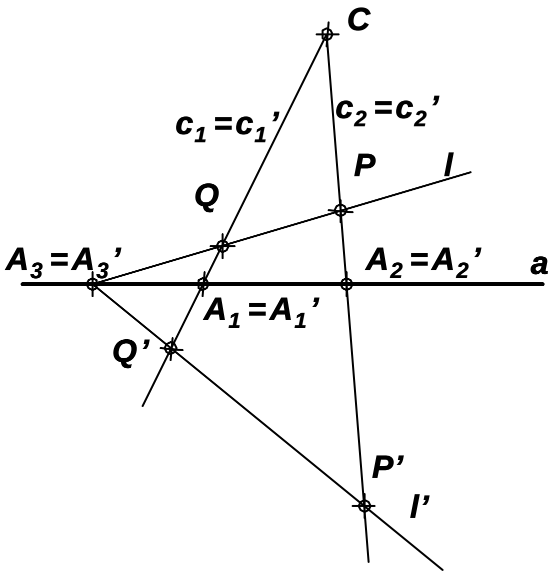
Central-axial Collineation

A *linear mapping* ***? ⇒ ?’*** of the plane onto itself is called *central-axial collineation* with center ***C*** and axis ***a***, if it leaves invariant ***C*** and ***a***. That also means, the lines passing through the center and the points lying in the axis are invariant.

The central-axial collineation is defined by means of the center, axis and a pair of points: {C, a, P → P’}.

The statement can be proved by showing that for an arbitrary point Q the Q’, for an arbitrary line l the line l’ can be found.

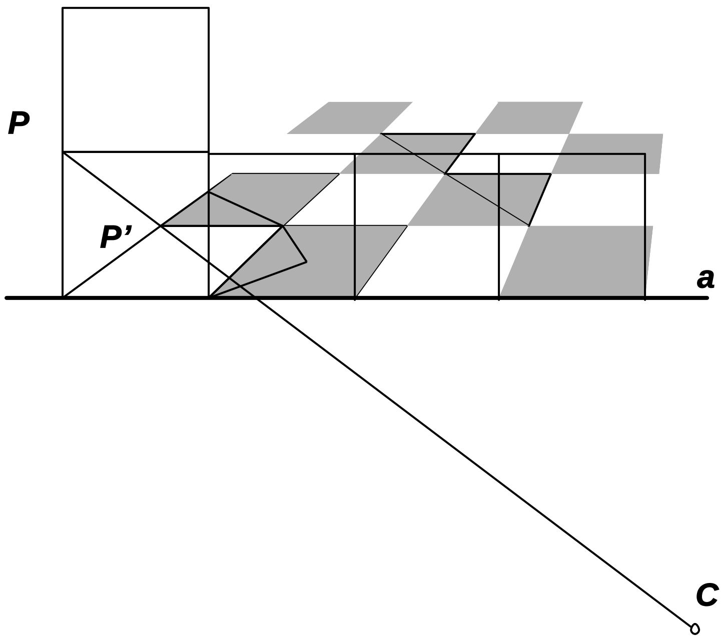
The reverse mapping {C, a, P’ → P} is also a central-axial collineation.



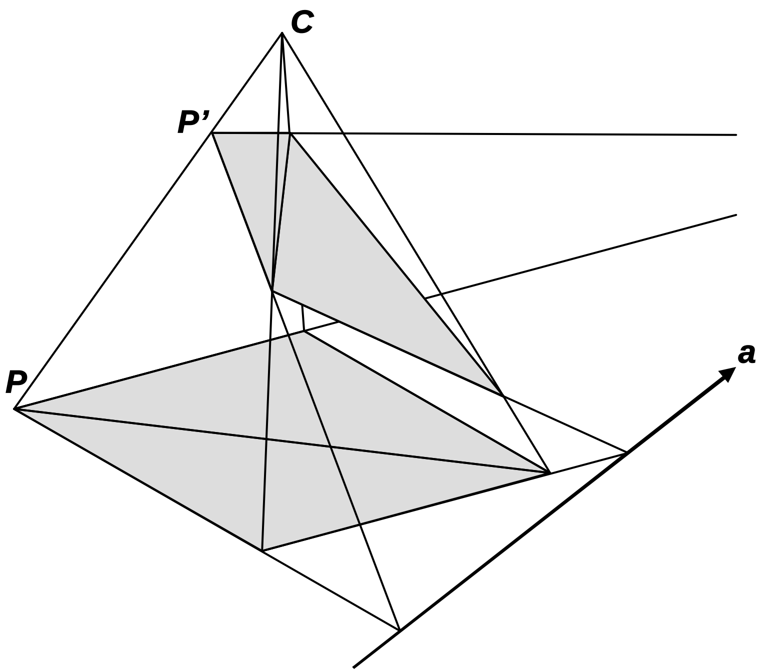
The *central-axial collineation* is also called *perspective collineation.*

Figures in Collineation

Lines parallel to the axis at a perspective collineation remain parallel.

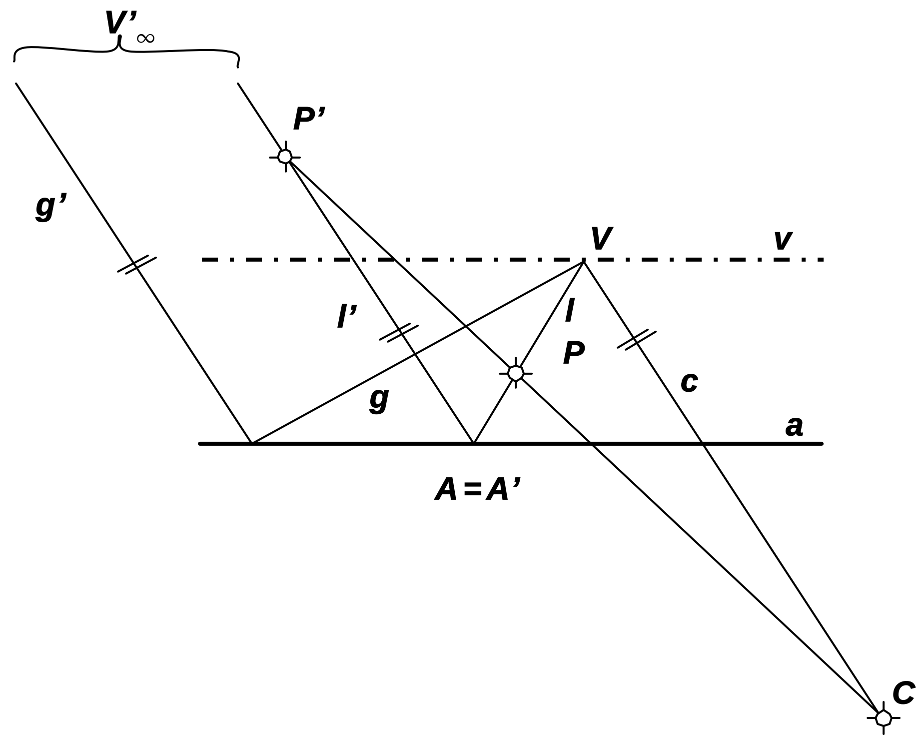


Perspective collineation will be applied at the construction of intersection of pyramid and plane.



Vanishing Line

Point ***V*** is the point of intersection of ***l*** and the line ***c*** parallel to ***l'*** through ***C***. The image of the point ***V*** will be $\boldsymbol{V'_{\infty}}$, a "*point at infinity*". If ***g'*** is parallel to ***l'*** than ***g*** and ***l*** have the same *vanishing point* ***V***.



The set of vanishing points is a line ***v*** passing through ***V***, parallel to the axis ***a***.

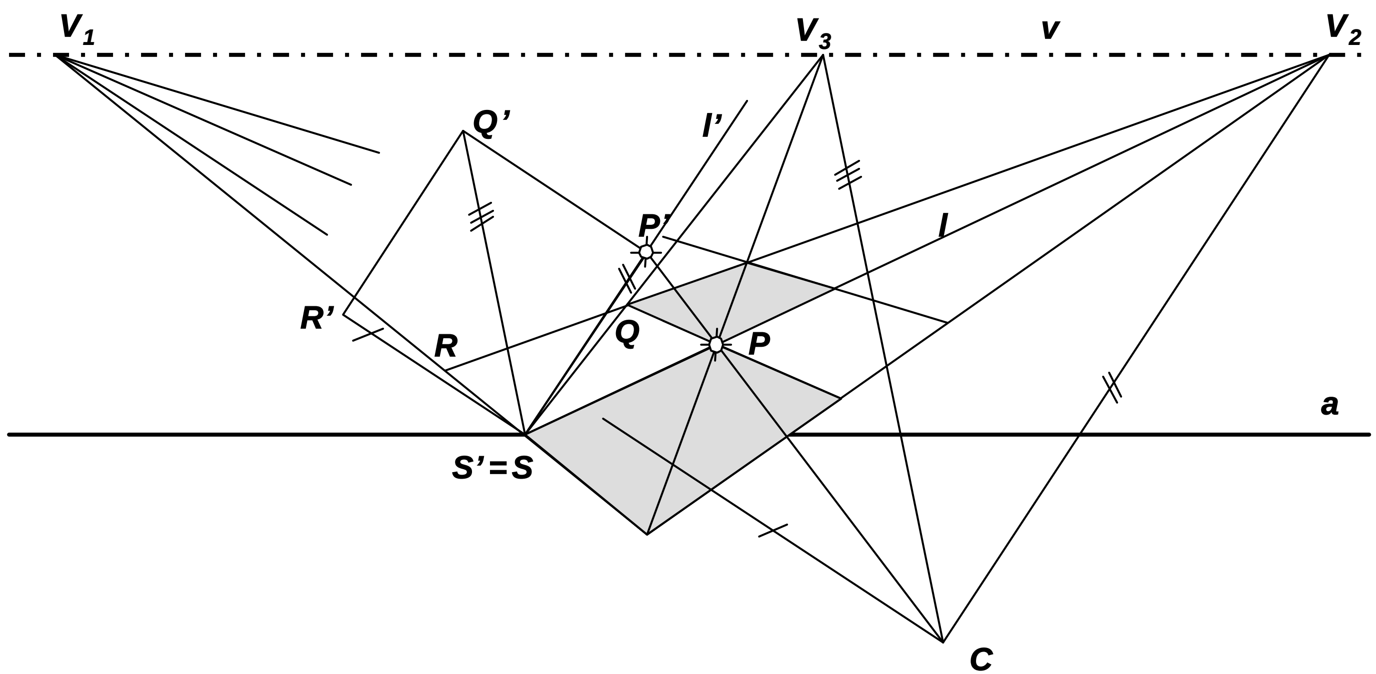
The mapping ***P*** ⇒ ***P'*** of the plane onto itself is complete if it is extended with the image of the line ***v***, an imaginary line i. e. the "*line at infinity*" $\boldsymbol{v'_{\infty}}$.

The line $\boldsymbol{v'_{\infty}}$ is also called "*ideal line*".

Construction by Means of the Vanishing Line

A perspective collineation is determined by the center ***C***, axis ***a*** and the vanishing line ***v***.

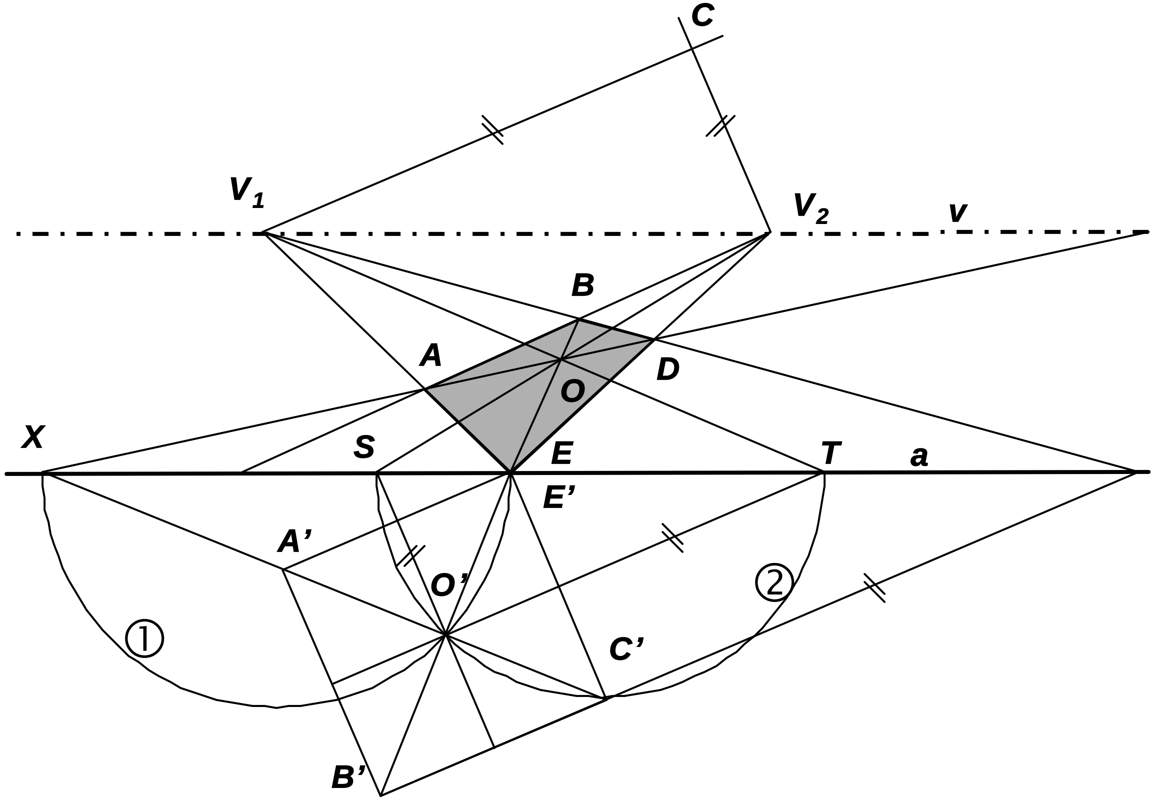
(Hint: use an auxiliary line ***l*** passing through ***P*** and its image ***l'*** that contains the point ***P'***.)



To the square ***P', Q', R', S'=S***, we can find the quadrilateral ***PQRS*** at the ***P'*** ⇒ ***P*** reverse mapping.

Quadrilateral and Square

Let a convex quadrilateral {***A***, ***B***, ***D***, ***E***} be given. Find a perspective collineation such that the image of the quadrilateral should be a square {***A’***, ***B’***, ***D’***, ***E’***} . (Hint: in a square, both the diagonals and the bimedians form right angles.)



$V_1 = |AE| \cap |BD|$,
$V_2 = |AB| \cap |DE|$,
$v = |V_1 V_2|$

$a ? v$, $E \in a$, $E = E'$

$O = |AD| \cap |BE|$,
$S = |OV_2| \cap a$,
$T = |OV_1| \cap a$,
$X = |AD| \cap a$,

Semicircles ① and ②
O'= ① ∩ ②

O’E’: semi-diagonal of the square

Construction of ***C***:

$|V_1C| ? |O'T|$
$|V_2C| ? |O'S|$