Inter IISER-NISER Mathematics Meet (IINMM) 2021
12 - 14 July, 2021

12 July, 2021 (Monday)

Plenary talk

Cancelation of Tensor Products for Vector Bundles on Projective space
Kapil Paranjape, IISER Mohali

In a discussion on MathOverflow initiated by Hailong Dao, a method to cancel a vector bundle from a tensor product on projective space was provided by Will Savin using a result of Peter O'Sullivan. This resulted in a collaboration between the speaker, N. Mohan Kumar, Madhav Nori and V. Srinivas to understand the proof given by O'Sullivan of the existence of a "pro-Reductive Group Scheme attached to a smooth projective variety" whose group of connected components is the fundamental group scheme as defined by Nori. This expository talk will present the key ideas of the above.

Research / Invited talks

Schurs Exponent Conjecture and Related Problems
Viji Z Thomas, IISER Thiruvananthapuram

We will discuss Schurs exponent conjecture, and discuss the progress made towards the conjecture in recent years. Finally we will describe our contribution towards this conjecture. This is joint work with my students Dr. A. E Antony and P. Komma.

Contragredient representations of $p-$adic groups
Amiya Kumar Mondal, IISER Berhampur

In this talk we will present an overview on an explicit realization of the contragredient of irreducible smooth representations of $p-$adic groups, known as the duality theorems. In particular, we will present a proof of the duality theorem for $p-$adic general spin groups.

On Stably Free Modules over Smooth Real Affine Algebras

Md. Ali Zinna, IISER Kolkata

Let R be a smooth affine algebra over reals of dimension n. I will talk about the structure of isomorphism classes of stably free R-modules of rank n.

Cut locus, Morse-Bott Functions and their relation

Sachchidanand Prasad, IISER Kolkata

Associated to every closed, embedded submanifold N in a connected Riemannian manifold M, there is the distance function d_N which measures the distance of a point in M from N. We analyze the square of this function and talks about the regularity of this function. Finally, we will prove that the complement of the cut locus $Cu(N)$ of N, deformation retracts to N.

Secant Bundles on Symmetric Power of Curves

Krishanu Dan, NISER, Bhubaneswar

Let C be a smooth, connected, projective curve over $\mathbb{C}$. The symmetric group S_n of n elements acts on the product $C^n := C \times \cdots \times C(n-\text{times})$, and the resulting quotient space, $S^n(C)$, is a smooth, connected, projective variety of dimension n, called $n-$th symmetric power of C. If E is a vector bundle on C of rank r, then the action of S_n on C^n lifts to an action on $\oplus_i p_i^* E$ where $p_i : C^n \to C$ is the $i-$th coordinate projection. This induces a rank nr vector bundle on $S^n(C)$. In this talk we will discuss (semi-)stability of this bundle.

$\mathbb{A}^1-$connectedness of standard norm varieties

Chetan Balwe, IISER Mohali

Norm varieties are splitting varieties for Milnor symbols, which are used in Voevodsky's proof of the Bloch-Kato conjecture. A construction for such varieties was given by Rost. We will prove if k is a field of characteristic 0, the standard norm varieties over k are $\mathbb{A}^1-$connected over the algebraic closure of k. This talk is based on joint work with Amit Hogadi and Anand Sawant.

Initial-boundary value problem for pressureless Euler system

Abhrojyoti Sen, NISER, Bhubaneswar

In this talk, we will discuss the question of solvability of the initial boundary value problem for the pressureless Euler system, in one space dimension. The study of the initial value problem for the pressureless Euler system in one and higher space dimensions has been extensively pursued in the literature in the past decades. To the best of our knowledge, no attempts have been made in the literature so far to solve the initial-boundary value problem. The reason for this neglect may be that it is not clear in what sense boundary data can be prescribed. Using the method of generalized potentials and characteristic triangles, extended to the boundary value case, an explicit way of constructing measure-valued solutions will be presented. Specifically, we extend the idea of Huang and Wang [1] by introducing a new type of potential- boundary potential. The prescription of boundary data will be shown to depend on the behavior of the generalized potentials at the boundary. The constructed solution satisfies an entropy condition and it conserves mass, whereby mass may accumulate at the boundary. Conservation of momentum again depends on the behavior of the generalized boundary potentials.

References

[**1**] Huang, Feimin; Wang, Zhen Well-posedness for pressureless flow. Comm. Math. Phys. 222 (2001).

[**2**] Lukas Neumann, Michael Oberguggenberger, Manas R Sahoo, and Abhrojyoti Sen, Initial-boundary value problem for 1d pressureless gas dynamics. arXiv:2104.10537, 2021.

Enumeration of singular curves with tangencies

Anantadulal Paul, NISER, Bhubaneswar

Enumeration of nodal curves is a classically important question in enumerative geometry. A parallel question to this is the enumeration of curves with tangencies to some divisor. In this talk, we shall discuss the enumeration of singular curves with tangencies. Our objects of study are the degree d curves in $\mathbb{CP}^2$ having A_k type singularities that are tangent to a fixed-line in $\mathbb{CP}^2$. As a consequence, we shall discuss the enumeration of curves with a tacnode using tangencies.

Uniform Poincaré inequalities on measured metric spaces

Soma Maity, IISER Mohali

Consider a proper geodesic metric space (X, d) equipped with a Borel measure μ. We establish a family of uniform Poincaré inequalities on (X, d, μ) if it satisfies a local Poincaré inequality (P_{loc}), and a condition on the growth of volume. Consequently, if μ is doubling and supports (P_{loc}) then it satisfies a uniform $(\alpha, \beta, \sigma)-$Poincaré inequality. If (X, d, μ) is a Gromov- hyperbolic space, then using the volume comparison theorem introduced by Besson, Courtoise, Gallot, and Sambusetti, we obtain a uniform Poincaré inequality with the exponential growth of the Poincaré constant. Next, we relate the growth of Poincaré constants to the growth of discrete subgroups of isometries of X, which act on it properly. This is Joint work with Gautam Nilakantan.

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13 July, 2021 (Tuesday)

Plenary talk

Inverse Problems and its applications

M.P. Rajan, IISER Thiruvananthapuram

In this talk, we briefly discuss inverse problems; the ill-posed nature of the problem and the solution strategy. We also highlight some of the applicable areas.

Research / Invited Talks

Strong ill-posedness for nonlinear Schrödinger equations

Divyang Bhimani, IISER Pune

We consider nonlinear Schrödinger equations in Fourier-Lebesgue and modulation spaces involving negative regularity. The equations are posed on the whole space, and involve a smooth power nonlinearity. We prove two types of norm in ation (strong ill-posedness) results. We first establish norm in ation results below the expected critical regularities. We then prove norm in ation with infinite loss of regularity under less general assumptions. We shall also discuss similar results for fractional Hartree equation. The talk is based on a joint work with Rémi Carles and Saikatul Haque.

On Endomorphisms of Moduli space of vector bundles on Curves

Sarbeswar Pal, IISER Thiruvananthapuram

Let X be a Fano Manifold of Picard rank 1 different from the projective space. It has been conjectured that any surjective endomorphism of X is an automorphism. It is known that the moduli space of stable vector bundles over a curve is a Fano variety and has Picard rank 1.
In this talk we will verify the conjecture for such Fano varieties.

A method for computing the Perron root and eigenvectors associated with a subshift of finite type

C. Haritha, IISER Bhopal

The Perron-Frobenius theorem is one of the most celebrated results in matrix theory. It guarantees the existence of the largest real eigenvalue (known as the Perron root) and associated positive left and right eigenvectors of an irreducible matrix. In this talk, we present a combinatorial method to compute the Perron root and eigenvectors associated with an irreducible subshift of finite type. As an application, we obtain an expression for the Perron root and eigenvectors of an irreducible $0-1$ matrix. We use techniques from combinatorics and ergodic theory to compute the normalization factor of these eigenvectors. This in turn gives us an alternate definition for the Parry measure on a subshift of finite type that is invariant under the left shift map.

On Atiyah sequences of Lie $2-$*group Bundles*

Saikat Chatterjee, IISER Thiruvananthapuram

Abstract: We introduce a notion of a principal $2-$bundle over a Lie groupoid. We show that every such Lie $2-$group bundle admits a short exact sequence of VB groupoids over a Lie groupoid, namely the Atiyah sequence of VB groupoids. Two notions of connection structures viz. strict connections and semi-strict connections on them arising respectively, from a retraction of the Atiyah sequence and a retraction upto a natural isomorphism have been introduced. An existence criterion for the connections on a Lie $2-$group bundle over a proper, étale Lie groupoid have been proposed. The action of the gauge $2-$group of the gauge transformations on the category of strict and semi-strict connections have been studied. A special type of gauge symmetry of the category of semi-strict connections have been observed.

This is a joint work with A. Chawdhury and P. Koushik

Class Groups and Annihilators

Prem Prakash Pandey, IISER Berhampur

Ring of integers of a number fields need not be a UFD. To measure this deficiency one introduces the concept of class groups of number fields. Non-trivial class group makes arithmetic in the ring of integer difficult. Annihilators provide a tool to give some relief. Starting from a classical theorem in algebraic number theory we develop an idea for a very simple way to construct some annihilators.

On a generalisation of a theorem of Ax

Nilkantha Das, NISER, Bhubaneswar

In 1969, James Ax proved that any injective endomorphism of an algebraic variety over an algebraically closed field of characteristic zero is an automorphism. In 2005, M. Miyanishi proposed a conjecture generalizing the above result of Ax which we refer to as Miyanishi conjecture. We talk about some recent development of the Miyanishi conjecture.

Associativity, Binary Trees and Stasheff's Polyhedra

Somnath Basu, IISER Kolkata

We shall discuss associative binary structures and analyze why such a structure is not homotopy invariant. This naturally leads to higher associative structures, often called $A_\infty-$structures, which are prevalent in topology, geometry and mathematical physics. We will explain why certain polytopes which govern maps between A_∞ spaces are equivalent to Stasheff's polytopes which govern A_∞ structures. This is joint work with Sandip Samanta.

Quantum $E(2)$ groups for complex deformation parameters

Atibur Rahaman, NISER, Bhubaneswar

In this talk we will focus on the construction a family of q deformations of $E(2)$ group for nonzero complex parameters $|q| < 1$ as locally compact braided quantum groups over the circle group $\mathbb{T}$ viewed as a quasitriangular quantum group with respect to the unitary $R-$matrix $R(m,n) := (q/\overline{q})^{mn}$ for all $m, n \in \mathbb{Z}$. For real $0 < |q| < 1$, the deformation coincides with Woronowicz's $E_q(2)$ groups. Moreover, if time permits, we will go on to talk about the braided analogue of the contraction procedure between $SU_q(2)$ and $E_q(2)$ groups in the spirit of Woronowicz's quantum analogue of the classic Inönü-Wigner group contraction.

On Poincaré series of modules over local rings

Anjan Gupta, IISER Bhopal

Let R be a local ring with maximal ideal $\mathfrak{m}$ and residue field k. The Poincaré series of a finitely generated module M over R is defined as the formal power series

$$P_M^R(t) = \sum_{i \geq 0} dim_k \; Tor_i^R(M,k)t^i \in \mathbb{Z}[|t|].$$

An example due to Anick shows that modules with irrational Poincaré series exist. However, there are classes of rings over which all finitely generated modules have rational Poincaré series.
The aim of this talk is to collect some important results and discuss recent developments on this topic. We discuss new classes of rings over which modules have rational Poincaré series sharing a common denominator. If time permits, we also discuss the connection between rationality of Poincaré series and Auslander-Reiten conjecture.

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14 July, 2021 (Wednesday)

Plenary talk

Arithmeticity

A. Raghuram, IISER Pune

Suppose $\sum a_n$ is a convergent series of complex numbers, and s is a field automorphism of $\mathbb{C}$, then in general it need not be the case that $s(\sum a_n) = \sum s(a_n)$, i.e., s need not commute with the summation sign. This phenomenon, given the appellation "arithmeticity", is the source of many interesting challenges in number theory. In this talk, I will discuss arithmeticity in certain local and global problems in automorphic number theory.

Research / Invited Talks

Short $\mathbb{C}^{2\prime}s$ and their automorphism groups

Ratna Pal,IISER Berhampur

In holomorphic dynamics the class of Hénon maps is the most important class of polynomial automorphisms in $\mathbb{C}^2$. For a Hénon map in $\mathbb{C}^2$, it is known that the sub-level sets of the associated Green's function are Short $\mathbb{C}^{2\prime}$s. A Short $\mathbb{C}^2$ is a proper domain of $\mathbb{C}^2$ that can be expressed as an increasing union of unit balls such that the Kobayashi metric vanishes identically therein, but allows a bounded above pluri-subharmonic function. In this talk, we shall explore the holomorphic automorphism groups of the sub-level sets of Green's functions. We shall see that although these sets admit exhaustions by biholomorphic images of the unit ball, the automorphism groups cannot be too large. On the other hand, examples will be provided to show that these automorphism groups are non-trivial in general. This is a joint work with Sayani Bera and Kaushal Verma.

Factoriality of the von Neumann algebras associated to Yang-Baxter operators

Rajeeb Ranjan Mohanta,NISER, Bhubaneswar

Bozejko and Speicher associated a finite von Neumann algebra M_T to a self-adjoint operator T on a complex Hilbert space of the form $H \otimes H$ which satisfies the Yang-Baxter relation and $\|T\| < 1$. In this talk, we will discuss the factoriality of M_T for $dim(H) \geq 2$ when T admits an eigenvector of some special form.

On bilinear Bochner-Riesz problem

Saurabh Shrivastava, IISER Bhopal

In this talk we will discuss some recent developments on L^p boundedness properties of bilinear Bochner-Riesz means and associated operators. In particular, we will introduce the notion of Stein's square function associated with bilinear Bochner-Riesz means and study its L^p boundedness properties. This is based on a joint work with Surjeet Singh Choudhary, K. Jotsaroop and Kalachand Shuin.

On ratios of critical values of $L-$functions

Chandrasheel Bhagwat, IISER Pune

We will discuss certain rationality results for the ratios of critical values of the degree$-2n$ $L-$functions attached to the automorphic representations of $GL_1 \times O(n, n)$ over a totally real number field for an even positive integer n. This is a joint work with A. Raghuram.

Convergence of adaptive finite element method for elliptic problems

Asha K. Dond, IISER Thiruvananthapuram

The adaptive finite element method (AFEM) is an elegant and powerful technique used to compute the numerical solution to initial and boundary value problems with a minimal computational cost. The explicit residual-based adaptive estimator allows reliable and efficient error control and motivates adaptive discretization, improving the empirical convergence rates. The quasi-optimal analysis of AFEM provides the mathematical foundation for this optimal behavior of AFEM.
In the talk, the quasi-optimal analysis of AFEM will be discussed for the lowest-order Raviart-Thomas mixed FEM for general second-order linear elliptic problems. The standard adaptive mesh-refinement strategy faces difficulty in the flux error control for these problems. Therefore, to achieve optimal convergence, a separate marking strategy: a split of a Dörfler marking for the estimator and some optimal data approximation strategy for the data approximation term has been utilized.

Estimates of automorphic forms

Anilatmaja Aryasomayajula, IISER Tirupati

Estimates of automorphic forms associated to arithmetic subgroups is an area of deep interest in number theory. Together with my graduate students, and my collaborators, we have been trying to estimate certain automorphic forms, and we give a brief survey of our progress.

Fourier integral operators and maximal operators

Ramesh Manna, NISER, Bhubaneswar

The theory of Fourier integral operators was developed by Hörmander in 1971. In this talk, we prove the local smoothing estimates for Fourier integral operators with phase function $h(x,t,\xi) = x\cdot\xi + tq(\xi)$, where q is smooth, homogeneous of degree one and amplitude function $a(x,t,\xi)$ belongs to S^m, the symbol class of order m less or equal to zero.
Local smoothing was a phenomenon originally observed in studying the circular maximal operator by C. D. Sogge. We give an overview of the regularity

results which have been proven to date. Finally, we give an application of the local smoothing estimate to the maximal operators.

Calderón-Vaillancourt-type theorems for pseudo-multipliers associated to the Grushin operator

Sayan Bagchi, IISER Kolkata

In this talk we discuss Calderón-Vaillancourt-type theorems for pseudo- multipliers associated to the Grushin operator $G = -\Delta_{x'} - |x'|^2\Delta_{x''}$ on $\mathbb{R}^{n_1+n_2}$. For appropriately defined symbol classes $S^0_{\rho,\delta}(\sqrt{G})$ with $0 \leq \delta < \rho \leq 1,\ \delta \neq 1$, we establish the L^2-boundedness result for the corresponding operators. This is a joint work with Rahul Garg.

On the existence of a non-principal Euclidean ideal class in biquadratic fields with class number two

Pasupulati Sunil Kumar, IISER Thiruvananthapuram

In 1979, H.W. Jun. Lenstra introduced the definition of the Euclidean ideal which is a generalization of Euclidean domain.

Definition 1. Let R be a Dedekind domain and $\mathbb{E}$ be the set of non zero integral ideals of R. If C is an ideal of R, then it is called Euclidean if there exists a function $\Psi : \mathbb{E} \to \mathbb{N}$, such that for every $I \in \mathbb{E}$ and $x \in I^{-1}C \setminus C$ there exist a $y \in C$ such that

$$\Psi((x-y)IC^{-1}) < \Psi(I).$$

H.Graves constructed an explicit biquadratic field $\mathbb{Q}(\sqrt{2}, \sqrt{35})$ which has a non-principal Euclidean ideal class. C Hsu generalized the result by Graves and proved Suppose $K = \mathbb{Q}(\sqrt{q}, \sqrt{kr})$. Then K has a non-principal Euclidean ideal class whenever $h_K = 2$. Here the integers q, k, r are all primes ≥ 29 and are all congruent to 1 modulo 4. The family of biquadratic field given by C. Hsu is extended by Chattopadhyay and Muthukrishnan and proved that, If $K = \mathbb{Q}(\sqrt{q}, \sqrt{kr})$ where $q \equiv 3$ and $k, r \equiv 1(mod\ 4)$ are prime numbers. Suppose that $h_K = 2$. Then K has a Euclidean ideal class.
In this talk, I will prove that a new family of biquadratic fields having non principal Euclidean ideal class whenever the class number of biquadratic field equal to two.

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