AN INVESTIGATION OF COLLECTIVE OSCILLATIONS IN A PLASMA SYNTHESIZED FROM POSITIVE AND NEGATIVE ION BEAMS

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It is shown that oscillation amplification occurs in a synthesized plasma consisting of interpenetrating beams of positive and negative ions moving in the same direction with different velocities. The experimental results agree qualitatively with the linear theory.

THE theoretically predicted beam instabilities of a nonequilibrium plasma[1] have been investigated experimentally in different systems: an electron beam plus plasma,[2] an ion beam plus plasma,[3,4] a synthesized plasma consisting of ion and electron beams[5] etc. In all the previously investigated systems the negatively charged particles of the plasma or beam were electrons. It is of interest to study collective oscillations in a two-component system consisting of particles (positive and negative ions) that have equal masses and move in the same direction with close velocities. The fundamental simplicity of this plasma system, which facilitates its theoretical study, the possibility of using it as a model for other equal-mass systems (such as electron-positron beams), and its possible applications[6] arouse interest in the study of its properties. We can also point to the interesting properties of a plasma whose particles have equal Larmor radii in a magnetic field.[7]

In the present work we first synthesized a plasma consisting of interpenetrating positive and negative ion beams and investigated the conditions governing the amplification of oscillations that depends upon the relative motion of the plasma components.

CONDITIONS FOR AMPLIFICATION OF OSCILLATIONS IN A PLASMA CONSISTING OF POSITIVE AND NEGATIVE ION BEAMS

We shall consider a system that consists of interpenetrating beams of positive and negative ions having mass M and moving in the z direction with the respective velocities $v_1 = v_0 + \Delta v$ and $v_2 = v_0 - \Delta v$. For a quasi-neutral system we assume $n_+ = n_- = n$; the thermal spread of the ion velocities will be neglected ($v_{Tc} \ll \Delta v$).

Assuming that the beams are spatially homogeneous and unbounded, to describe small-amplitude oscillations in the system we make use of the dispersion equation

$$\varepsilon(\omega, k) \equiv 1 - \frac{\omega_p^2}{(\omega - kv_1)^2} - \frac{\omega_p^2}{(\omega - kv_2)^2} = 0, \qquad (1)$$

where $\omega_p \equiv (4\pi ne^2/M)^{1/2}$ is the plasma frequency and k is the z component of the wave vector. For $\Delta v \ll v_0$ the solution of this equation is

$$k = \omega / v_0 + \Delta k, \qquad (2)$$

where

$$(\Delta k)^2 = \left(\frac{\omega_p}{v_0}\right)^2 \left[1 + \left(\frac{\omega \Delta v}{v_0 \omega_p}\right)^2 \pm \sqrt{1 + 4\left(\frac{\omega \Delta v}{v_0 \omega_p}\right)^2}\right].$$

It follows from (2) that when the condition

$$\Delta v < \Delta v_{cr} \equiv \sqrt{2} v_0 \omega_p / \omega \qquad (3)$$

is fulfilled two real ($\pm \Delta k_+$) and two imaginary ($\pm \Delta k \equiv i\chi$) values exist for Δk, so that the expression for the variable potential of the electric field is

$$\varphi(t, z) = C_1 \cos\left[\omega\left(t - \frac{z}{v_0}\right) - \Delta k_+ z\right] + C_2 \cos\left[\omega\left(t - \frac{z}{v_0}\right) + \Delta k_+ z\right] + C_3 \cos\left[\omega\left(t - \frac{z}{v_0}\right)\right] e^{-\chi z} + C_4 \cos\left[\omega\left(t - \frac{z}{v_0}\right)\right] e^{\chi z}, \qquad (4)$$

where the constants C_i (i = 1, 2, 3, 4) are determined by the boundary conditions. The last term in (4) corresponds to an exponentially growing wave along the beams and describes the amplification of the oscillations in space. The inequality (3) represents the existence of a critical relative velocity that determines the threshold for amplification of oscillations at a given frequency.

Figure 1 shows the dependence of $\chi v_0/\omega_p$ on Δv for several frequencies. We observe that for fixed values of v_0 and ω_p there exists an optimum value of the relative velocity, Δv_{opt}, at which the buildup factor ("increment") χ attains its maximum value χ_{max}. With decrease of the frequency both the amplification threshold and the location of the maximum buildup factor are shifted toward higher values, but χ_{max} is independent of the frequency. It follows from (2) that

$$\omega \Delta v_{opt} = 0.5\sqrt{3} v_0 \omega_p, \qquad \chi_{max} = 0.5\, \omega_p / v_0.$$

We note that similar relations for a system of interpenetrating electron beams were obtained in[8,9].

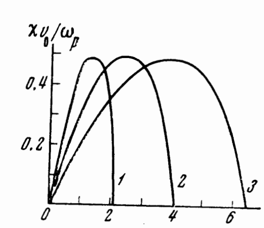

FIG. 1. Calculation space buildup factor versus relative velocity of the beam for different frequencies f: 1–104 MHz, 2–55 MHz, and 3–35 MHz; f_p = 1 MHz; v_0 = 1.6×10^8 cm/sec.

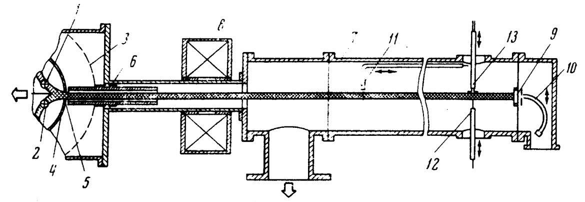

FIG. 2. Diagram of experimental apparatus.

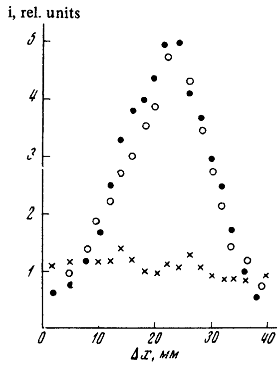

FIG. 3. Radial distribution of current density: O—of positive ions in the synthesized beam, ●—of negative ions in the synthesized beam, X—of positive ions in the absence of negative ions.

EXPERIMENTAL RESULTS AND DISCUSSIONS

The apparatus represented in Fig. 2 was used to obtain and investigate a plasma consisting of interpenetrating beams of positive and negative ions. Beams 1 and 2 of positive and negative ~13-keV hydrogen ions in currents up to 5 mA were obtained from sources[10] that were immersed in a homogeneous 2000-Oe magnetic field generated between the poles of an electromagnet 3. In the equipotential space bounded by the liner 4 and the ion beam extractors the beams were combined through the action of the magnetic field. The beams then passed through an aperture 5 in the liner, traversed a channel in the magnetic shield 6, and entered the interaction chamber 7, which was 130 cm long with a 20-cm diameter. The beams were focused by the magnetic lens 8. Despite the intense hydrogen flux that came from the negative ion source while the latter was in operation, a pressure $p \leq 10^{-5}$ mm Hg was maintained in the interaction chamber; collisions between ions and neutral particles could be neglected at this pressure. Each component of the beam synthesized in the interaction chamber was a ~ 2-mA current with a ~ 1-cm radius.

The potential of the positive ion source was higher, and that of the negative ion source was lower, by an amount U_0 than the potential of the liner; the magnetic shield and the interaction chamber were grounded. The potential difference ΔU between the liner and the magnetic shield accelerated one of the beams and retarded the other. In this way relative motion of the synthesized plasma components was induced without changing the average velocity v_0 of the plasma. Since $\Delta U \ll U_0$ in our measurements, we had the approximate equality $2\Delta v/v_0 \approx \Delta U/U_0$.

The ion beams entered the collector 9, to which a customary small positive potential was applied for the purpose of inhibiting ion-electronic emission. The ions passed through a slit in the collector into the electrostatic analyzer 10, which was moved along the slit. This analyzer enabled us to measure the current densities of the separate beam components within the beam cross section and to monitor their energies.

The single probes 11 and 12, which were matched with their cables, and the dipole antenna 13 were used to investigate oscillations generated in the system. Probe 11 was movable in a region 20—90 cm from the collector. Oscillations were registered conventionally with an IP-26 detector, either directly or with the aid of a broad-band amplifier. The range of investigated oscillations was 20—150 MHz.

The synthesized plasma is characterized in Fig. 3 by the radial distribution of current density for the separate components; for comparison, the distribution of the positive component is shown in the absence of the negative ion beam. The observed reduction of current density when one of the beams is excluded results naturally from the spreading of the remaining beam because the neutralizing space charge of the other beam is not present. Hence we conclude that the space charges of the two beams essentially neutralize each other.

It should be noted that in practice the synthesized beam will also contain electrons and ions produced in collisions between beam ions and gas molecules. Allowance for these electrons and slow ions is represented in the dispersion equation by the additional terms $\sim 1/(kd_e)^2 = (\omega_{pe}/\omega)^2(v_0/v_e)^2$ and $\sim(\omega_{pi}/\omega)^2$, where ω_{pe} and ω_{pi} are the plasma frequencies of the electrons and slow ions. In view of the absence of uncompensated space charge in the beam, we determine the concentrations of electrons and slow ions from their balance equation

$$2n\sigma_i v_0 n_0 \pi r_0^2 \approx {}^1/_2 n_e v_e \pi r_0,$$
$$n\sigma_{ch} v_0 n_0 \pi r_0^2 = {}^1/_2 n_i v_i \pi r_0,$$

where n_0 is the concentration of hydrogen molecules; σ_i and σ_{ch} are the ionization and charge exchange cross sections; v_e and v_i are the thermal velocities of the electrons and slow ions. Under our experimental conditions, in the frequency range 150—15 MHz the aforementioned added terms of the dispersion equation are calculated to have the values $\sim(10^{-5}$--$10^{-3}) \ll 1$ and $\sim(10^{-6}-10^{-4}) \ll 1$ and may therefore be neglected.

Before we proceed to the analysis of the system's oscillatory properties, it should be noted that our investigation involves initial modulation of the beam in two ways. When the negative ion beam leaves the source it is found to be modulated in the frequency range that is of present interest. We were also able to introduce a perturbation into the synthesized beam by means of a probe to which a voltage of suitable frequency was applied from a generator. Since identical results were obtained in the two cases, we shall present data obtained with the "natural" modulation of the beam.

Figure 4 shows how the amplitude of probe-registered oscillations depends on the relative velocity of the beam components. The results did not, of course, depend on the sign of the relative velocity. The concentration of the synthesized plasma, calculated from the currents and radii of the beam components, and the average ion velocity were close to the values used in calculating the curves of Fig. 1. A comparison of Figs.

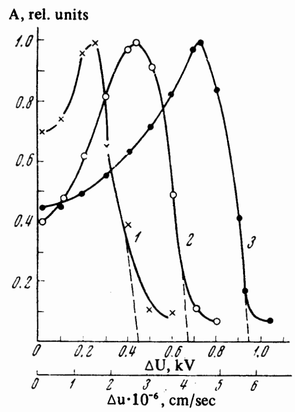

FIG. 4. Dependence of oscillation amplitude on relative velocity of the beams for different frequencies f: 1–104 MHz, 2–55 MHz, 3–35 MHz. The average concentration is $n_+ = n_- = 2.5 \times 10^7$ cm^{-3} ($f_p = 1.1$ MHz); $v_0 = 1.6 \times 10^8$ cm/sec.

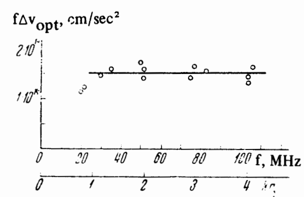

FIG. 5. Frequency dependence of the product $f\Delta v_{opt}$, shown by experimental points and a solid line representing the theoretical dependence based on Eq. (5).

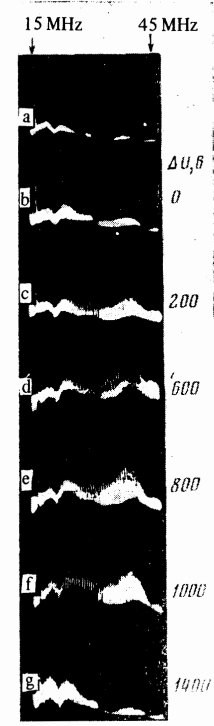

FIG. 6. Spectra of probe-detected oscillations: a–for a beam of negative ions; b–g-for a synthesized beam at different values of ΔU.

4 and 1 shows that the character of the dependence on Δv that is observed for the oscillatory amplitudes agrees satisfactorily with the theoretical prediction for oscillation buildup. The theory predicts an optimal value Δv_{opt} and a critical relative velocity Δv_{cr} above which the oscillation amplitudes becomes very small. The experimental ratios $\Delta v_{cr}/\Delta v_{opt}$ for different frequencies lie in the range 1.4--1.7; this agrees with the value 1.64 obtained from (3) and (5).

It follows from (5) that the product $f\Delta v_{opt}$ should be independent of the frequency. In Fig. 5 the solid line represents calculations of this product as a function of frequency; the small circles represent experimental values. (The abscissa axis represents values of $kr_0 = \omega r_0/v_0$ as well as frequencies.) Good agreement is observed between experiment and theory for beams of unbounded cross section down to $kr_0 \sim 1$. For $kr_0 < 1$ the product $f\Delta v_{opt}$ tends to decrease. A detailed study of the $kr_0 < 1$ region is prevented by the pronounced decrease of oscillation buildup in this region at the lower frequencies.

At the lowest frequencies in the investigated region our amplitude measurements were accompanied by direct observation of the oscillation spectrum on the screen of a panoramic spectral analyzer. The oscillations were detected with a single probe. Figure 6 shows the oscillation spectrum in the 15–45-MHz range for (a) a negative ion beam alone and (b–g) for a synthesized beam at different values of ΔU. The upper portion of this frequency range reveals a marked variation of the oscillation spectrum, and optimal values of ΔU can be determined for different frequencies. As already mentioned, at the lowest frequencies the amplification becomes small and there is little variation of the spectrum.

Finally, Fig. 7 shows the (automatically registered) distributions of oscillation amplitudes along the beam for a certain frequency at different values of Δv. Similar distributions were obtained for other frequencies

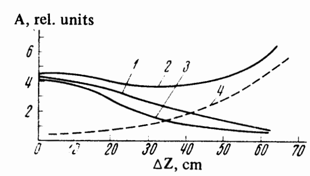

FIG. 7. Distribution of oscillation amplitudes along a synthesized beam for different relative velocities of the beam components: 1–$\Delta v = 0$, 2–$v = \Delta v_{opt}$, 3–$\Delta v = \Delta v_{cr}$, 4–difference between the ordinates of curves 2 and 1; f = 90 MHZ, $n_+ = n = 2.5 \times 10^7$ cm^{-3} ($f_p = 1.1$ MHz).

in the region of $f\Delta v_{opt}$ = const. The analysis of these distributions requires the use of (4). We note, to begin with, that when we go from (4) to the corresponding expression for beams with $\Delta v = 0$ we have $C_3 = -C_4 = C$. Therefore (4) can be put into the form

$$\varphi(t, z) = F(\Delta k_+, z) + C\,\mathrm{sh}\,\chi z. \tag{6}$$

The experimental curves 1 and 3, obtained for $\Delta v = 0$ and $\Delta v = \Delta v_{cr}$, correspond to the first term in (6), since $\chi = 0$. The manner in which Δk_+ depends on Δv leads to the assumption that values of $F(\Delta k_+, z)$ for all intermediate values of Δv will be found between the ordinates of the curves 1 and 3. Subtracting the corresponding ordinates of curves 1 and 2, we obtain somewhat reduced values for the second term at $\Delta v = \Delta v_{opt}$. The resultant curve is also shown in Fig. 7. By comparing this curve with the term $C\,\mathrm{sh}\,\chi z$ we determine the value $\chi = 2.5 \times 10^{-2}$ cm^{-1} for the buildup factor, in satisfactory agreement with $\chi = 2 \times 10^{-2}$ cm^{-1} calculated from (5).

In the present work we have shown the amplification of oscillations that results from drift instability arising in a system of interpenetrating positive and negative ion beams propagating in the same direction with different velocities. It is shown that the conditions of oscillation amplification agree with the theory of unbounded beams down to $kr_0 \sim 1$. Among the properties of the system we mention the absence of a stationary component and the inpossibility of feedback. The "simplicity" of the system permitted good agreement between experiment and the linear theory. The lengthening of the interaction region, the enhancement of beam currents, and the utilization of an external magnetic field will permit us also to investigate nonlinear effects and other interesting properties of the system that was produced and investigated in the present work.

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