Transverse momentum distributions in Semi-inclusive Deep Inelastic Scattering

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in collaboration with

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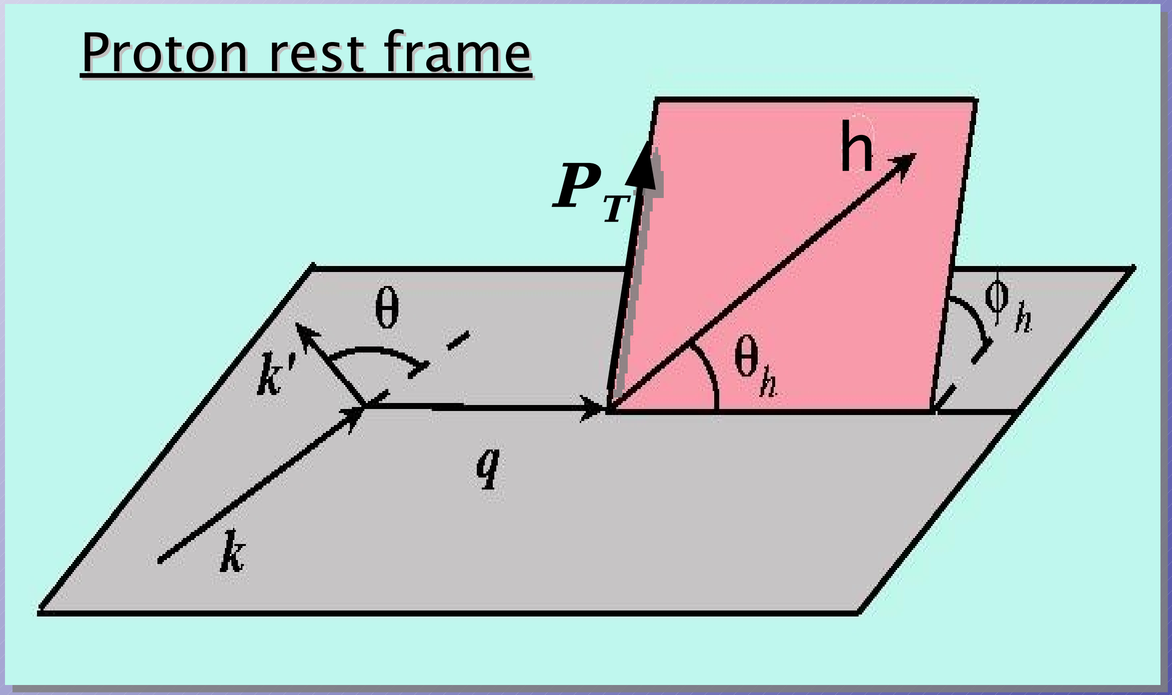
- Deep inelastic semi-inclusive reaction :

$$l(k)+H(P)\rightarrow l(k')+H'(h)+X$$

Variables set:



Hadronic scattering plane and momenta [1]

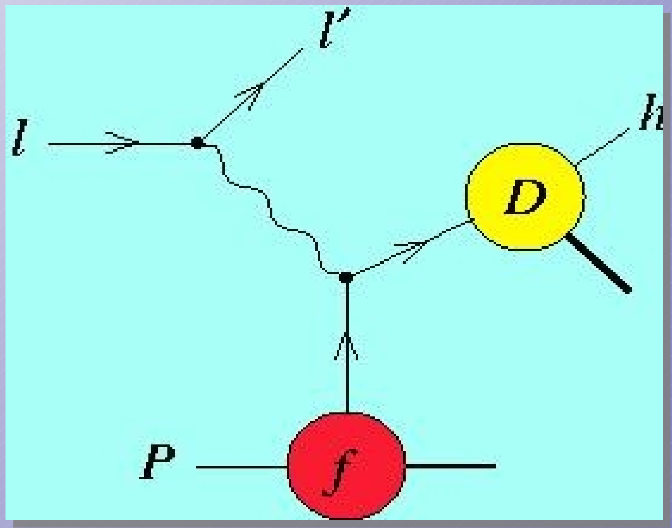
$$x_B=\frac{Q^2}{2P\cdot q}$$

$$Q^2=-q^2=-(k-k')^2$$

$$z_h=\frac{P\cdot h}{P\cdot q}$$

[1] J. Levelt, P.J. Mulders, *Phys. Rev.* **D49**, 96, (1994)

- Due to QCD-factorization [2], SIDIS cross-sections in the current fragmentation region is expressed in terms of universal:



↳ Fragmentation functions (D) from e^+e^-;

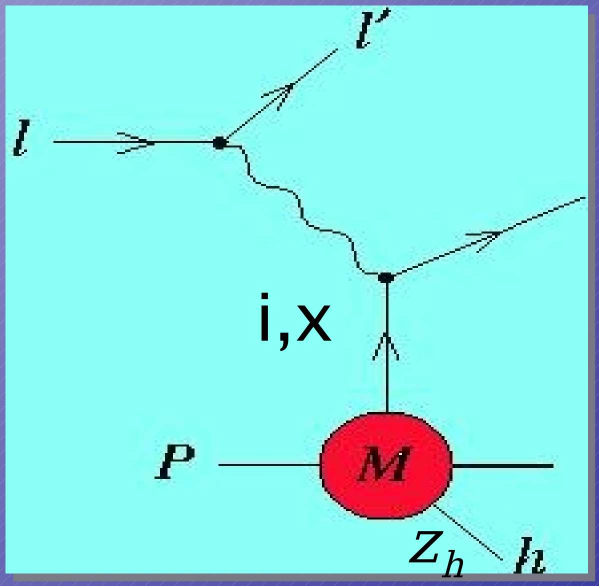
↳ Distribution functions (F) from inclusive DIS.

$$\sigma_C \approx \int_{x_B}^{1}\int_{z_h}^{1}\frac{dx'}{x'}\frac{dz'}{z'} F_P^i(x',Q^2)\,\hat{\sigma}_{ij}\left(\frac{x_B}{x'},\frac{z_h}{z'},Q^2\right) D_h^j(z',Q^2).$$

[2] G.Altarelli, R.K.Ellis,G. Martinelli, So-Young Pi, ***Nucl. Phys.*** **B160**, 301, (1979).

• Hadron production in the target fragmentation region is realized through fracture functions [4] $M^i_{P,h}(x_B, z_h, Q^2)$:

$$\sigma_T = (1-x_B)\int_{\frac{x_B}{1-(1-X_B)z_h}}^{1} \frac{dx'}{x'} M^i_{P,h}\left(\frac{x_B}{x'}, (1-x_B)z_h, Q^2\right)\hat{\sigma}_i(x', Q^2)$$



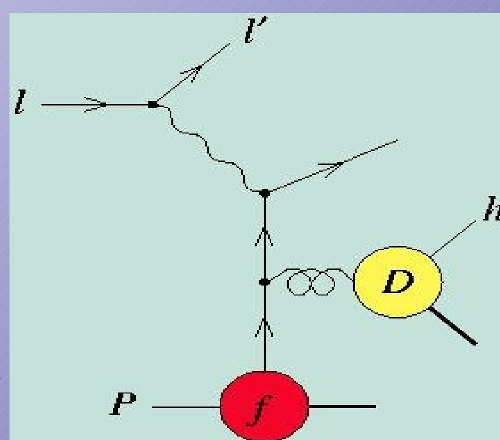
➠ If h is collinear to proton remnant, new singularities [5] arise.

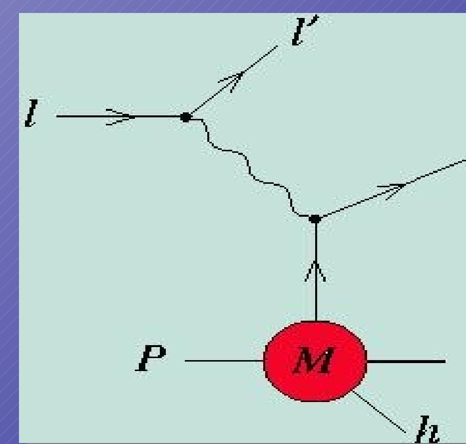
↪Factorization of collinear [6] and soft [7] singularities into M has been proved.

[4] L.Trentadue, G.Veneziano, ***Phys. Lett.*** **B323**, 201, (1994)
[5] D.Graudenz, ***Nucl. Phys.*** **B432**, 351, (1994)
[6] M.Grazzini, L. Trentadue, G. Veneziano, ***Nucl. Phys.*** **B519**, 394, (1998)
[7] J.C. Collins, ***Phys. Rev.*** **D57**, 3051, (1998)

- QCD predicts the scale dependence of M :

$$\frac{\partial}{\partial \log Q^2} M_{i,h/p}(x,z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_{x/(1-z)}^{1}\frac{du}{u} P_j^i(u)\, M_{j,h/N}\left(\frac{x}{u},z,Q^2\right) + \frac{\alpha_s(Q^2)}{2\pi}\int_{x}^{x/(x+z)} \frac{du}{x(1-u)}\hat{P}_j^{i,l}(u)\, F_{j/p}\left(\frac{x}{u},Q^2\right) D_{h/l}\left(\frac{zu}{x(1-u)},Q^2\right)$$

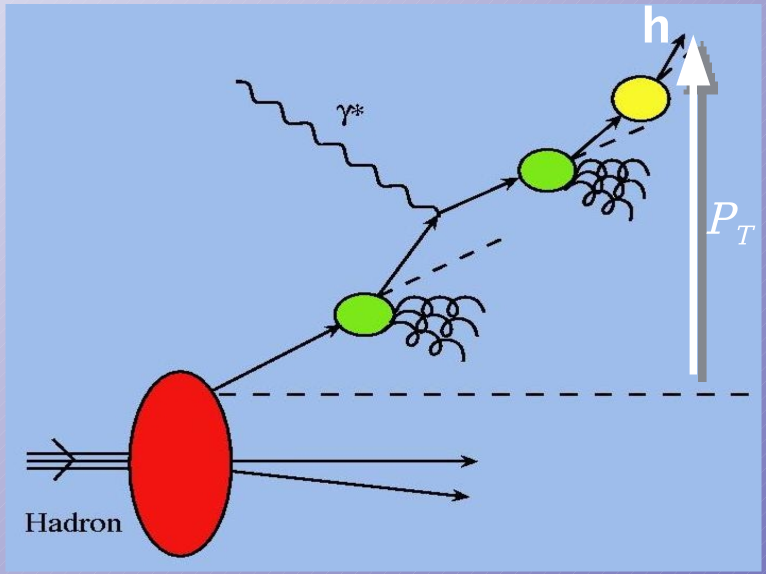




↪ Leading twist SIDIS cross section is thus:

$$\frac{d^3\sigma_p^h}{dx_B\, dQ^2\, dz_h} \propto \sum_{i=q,\bar{q}} e_i^2 \Big[\underbrace{F_{i/p}(x_B,Q^2)\, D_{h/i}(z_h,Q^2)}_{\sigma_{current}} + \underbrace{(1-x_B)\, M_{i,h/p}(x_B,(1-x_B)z_h,Q^2)}_{\sigma_{target}} \Big]$$

- Sources of transverse momentum in $l+P\rightarrow l+h+X$:



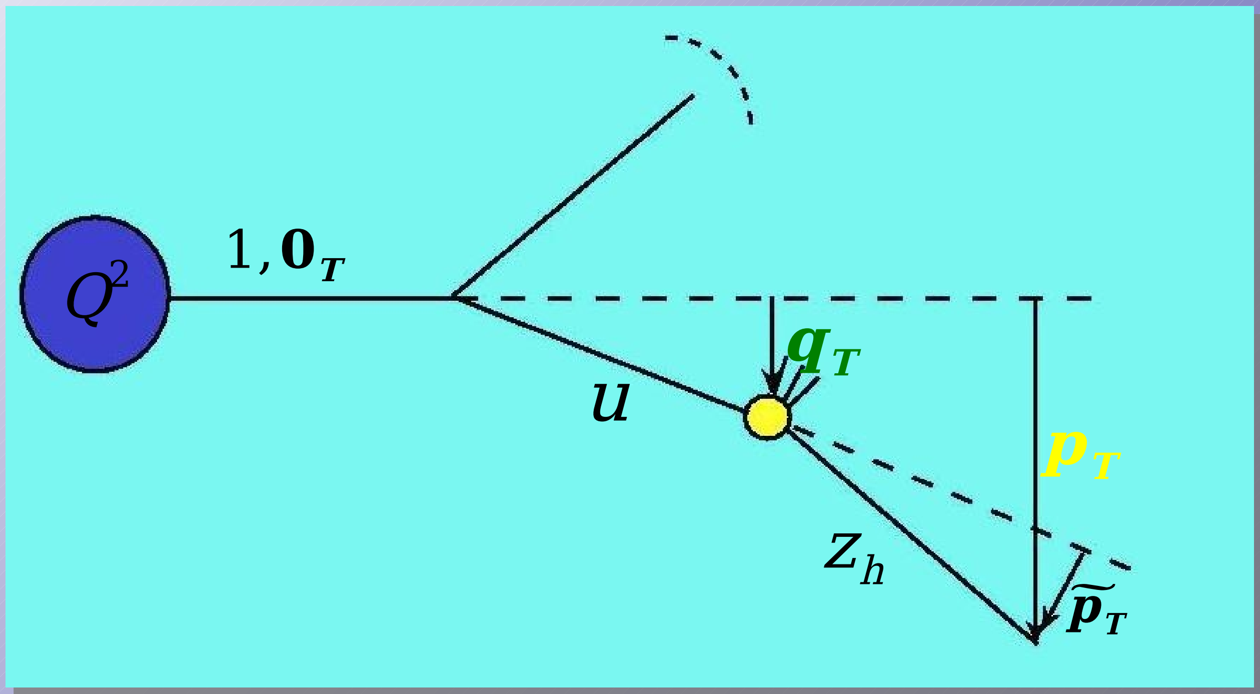
- Intrinsic distribution k_T [8] ✓
- Radiative q_T ✗
- Intrinsic fragmentation p_T [8] ✓

Detected hadron transverse momentum : $\boldsymbol{P_T}\approx \boldsymbol{p_T}+z_h\boldsymbol{k_T}$

But the cross-section $\frac{d^5\sigma^{lp\rightarrow lhX}}{dx_B\,dQ^2\,dz_h\,d^2\boldsymbol{P_T}}$ depends indirectly on q_T !

[8] M. Anselmino & al. , ***Phys. Rev.* D71**, 074006 (2005)

- Time-like TMD DGLAP evolution equation



Branching kinematics:

$$\tilde{\boldsymbol{p}}_T = \boldsymbol{p}_T - \frac{z_h}{u}\boldsymbol{q}_T$$

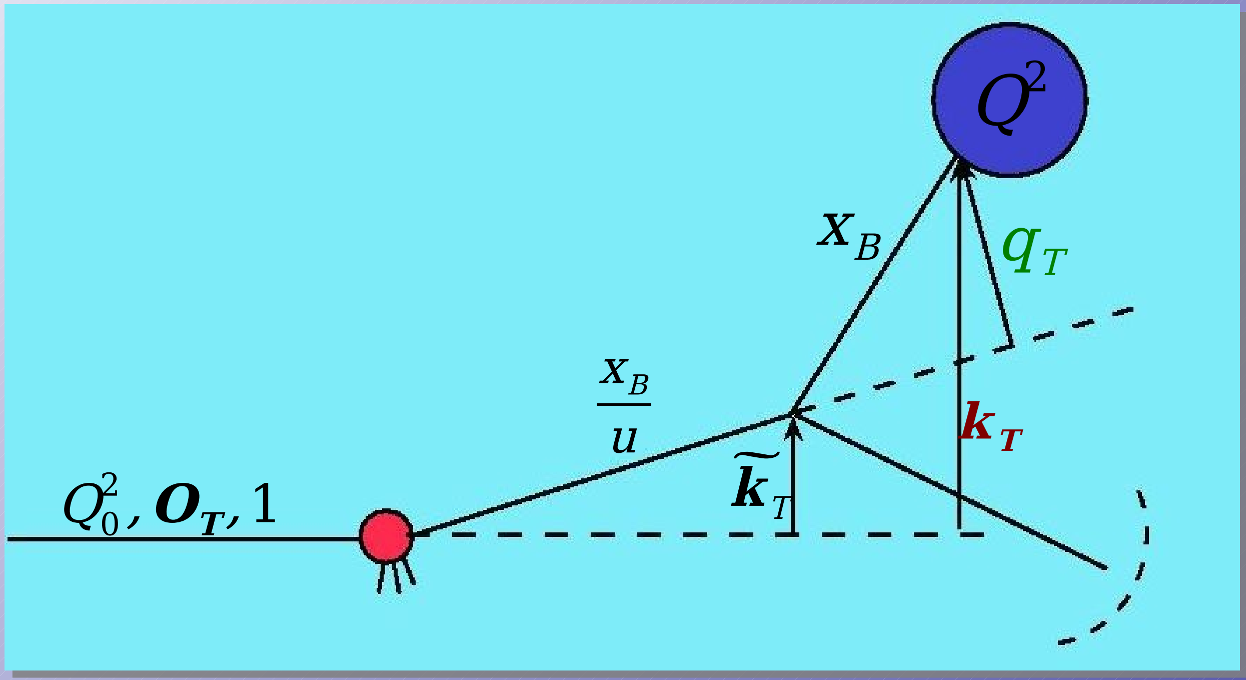
$$u(1-u)Q^2 = \boldsymbol{P}_T^2$$

$$Q^2 \frac{\partial D_a^b(Q^2, z_h, \boldsymbol{p}_T)}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{du}{u} \int \frac{d^2 \boldsymbol{q}_T}{\pi} \delta[u(1-u)Q^2 - \boldsymbol{q}_T^2] P_a^c(w) D_c^b\left(Q^2, \frac{z_h}{u}, \boldsymbol{p}_T - \frac{z_h}{u}\boldsymbol{q}_T\right)$$

A.Bassetto, M. Ciafaloni, G.Marchesini, ***Nucl. Phys.*** **B163**, (1980)

- Space-like TMD DGLAP evolution equation



Branching kinematics:

$$\tilde{\mathbf{k}}_T = \frac{K_T - q_T}{u}$$

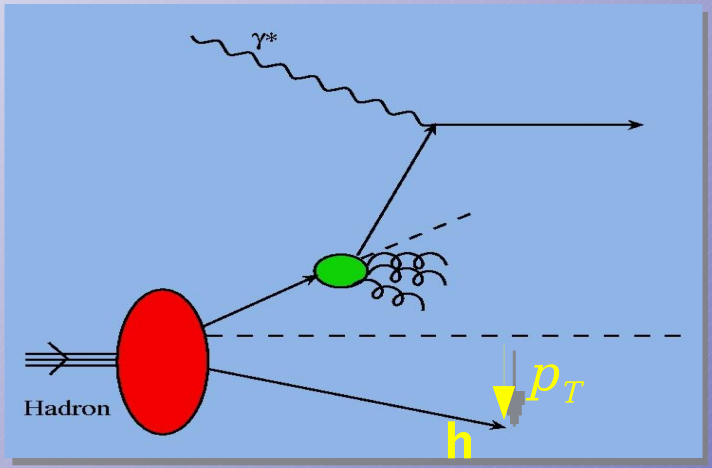
$$(1-u)\,Q^2 = \mathbf{q}_T^2$$

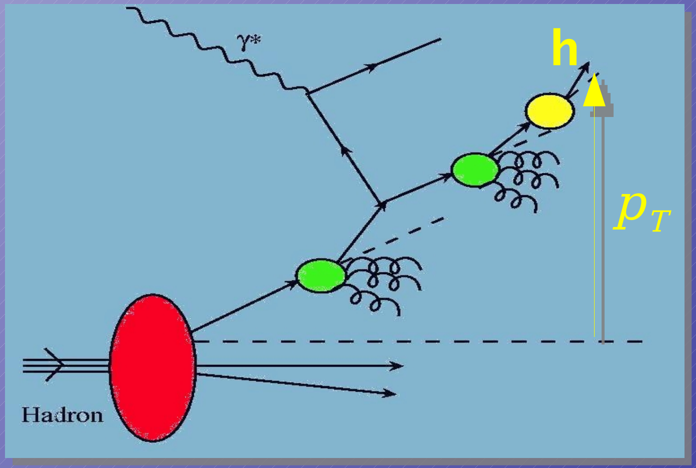
$$Q^2 \frac{\partial F_a^b(Q^2, x_B, \mathbf{k}_T)}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int_{x_B}^1 \frac{du}{u^3} \int \frac{d^2 \mathbf{q}_T}{\pi} \delta[(1-u)Q^2 - \mathbf{q}_T^2]\, P_a^c(u)\, F_c^b\left(Q^2, \frac{x_B}{u}, \frac{\mathbf{k}_T - \mathbf{q}_T}{u}\right)$$

F.A.C. and L.Trentadue, ***Phys. Lett.*** **B636**, (2006)

- Fracture functions TMD evolution equation





$$Q^2 \frac{\partial M^j_{p,h}(Q^2, x_B, \boldsymbol{k_T}, z_h, \boldsymbol{p_T})}{\partial Q^2} =$$

$$= \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u^3} \int \frac{d^2 \boldsymbol{q_T}}{\pi} \delta[(1-u)\, Q^2 - \boldsymbol{q_T^2}]\, P^j_i(u)\, M^i_{p,h}\left(Q^2, \frac{x_B}{u}, \frac{\boldsymbol{k_T} - \boldsymbol{q_T}}{u}, z_h, \boldsymbol{p_T}\right) +$$

$$+ \sum_c \frac{\alpha_s(Q^2)}{2\pi} \int \frac{du}{u} \int \frac{d^2 \boldsymbol{q_T}}{\pi} \delta[(1-u)\, Q^2 - \boldsymbol{q_T^2}]\, \frac{u}{x_B(1-u)} \hat{P}^{j,l}_i(u) \cdot$$

$$\cdot F^i_P\left(Q^2, \frac{x_B}{u}, \frac{\boldsymbol{k_T} - \boldsymbol{q_T}}{u}\right) D^h_l\left(Q^2, \frac{z_h u}{x_B(1-u)}, \boldsymbol{p_T} - \frac{z_h u}{x_B(1-u)} \boldsymbol{q_T}\right)$$

F.A.C. and L.Trentadue, ***Phys. Lett.* B636**, (2006) Increasing phase space

Factorization:

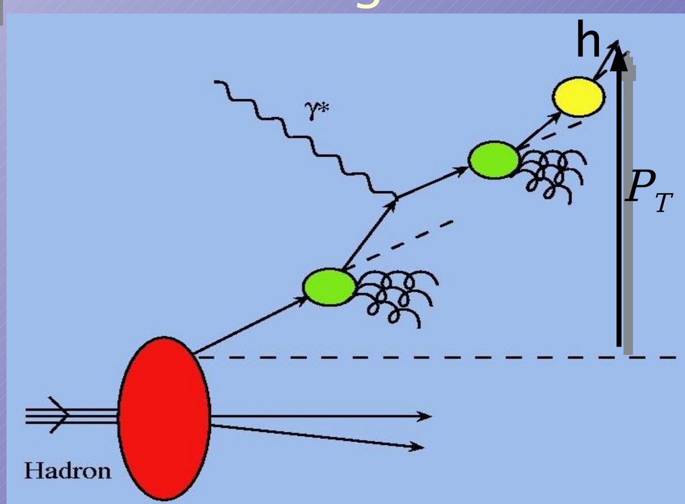
$$d\sigma \propto f(x) \circ D(z)$$ ✓

$$d\sigma \propto f(x) \circ D(z) + M(x,z)$$ ✓

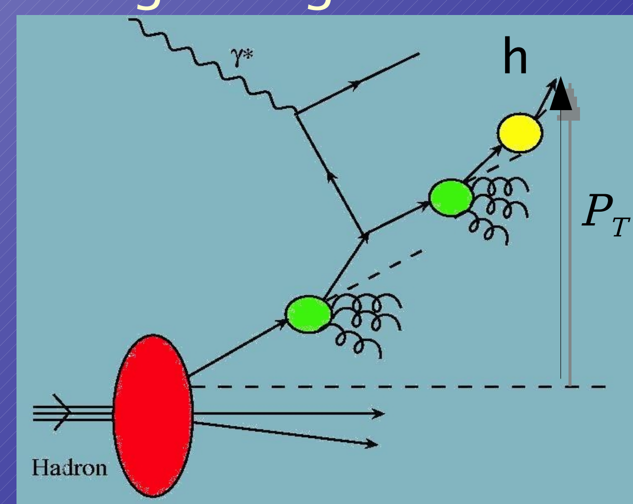
$$\frac{d\sigma}{d\boldsymbol{P_T}} \propto f(x,\boldsymbol{k_T}) \circ D(z,\boldsymbol{p_T})$$ ✓ [9]

$$\frac{d\sigma}{d\boldsymbol{P_T}} \propto f(x,\boldsymbol{k_T}) \circ D(z,\boldsymbol{p_T}) + M(x,\boldsymbol{k_T},z,\boldsymbol{p_T})$$ ✗

Current fragmentation



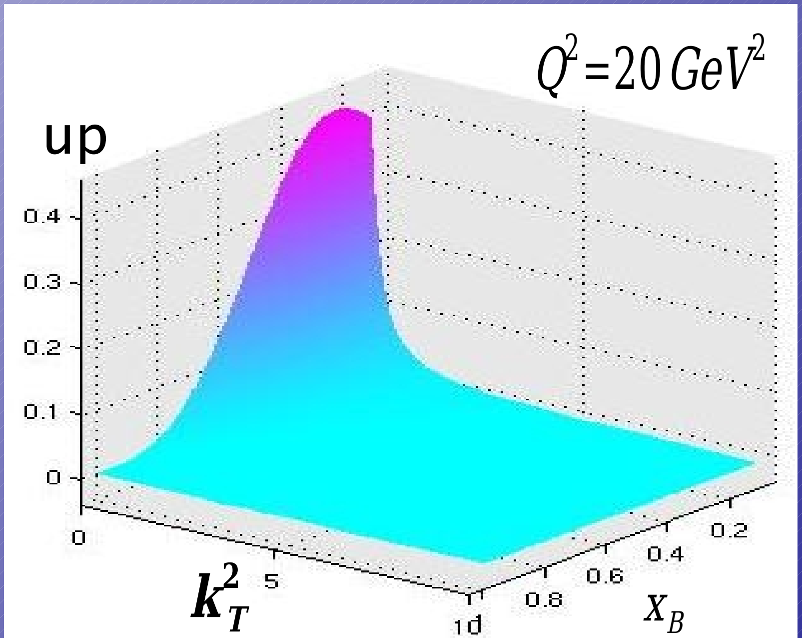
Target fragmentation

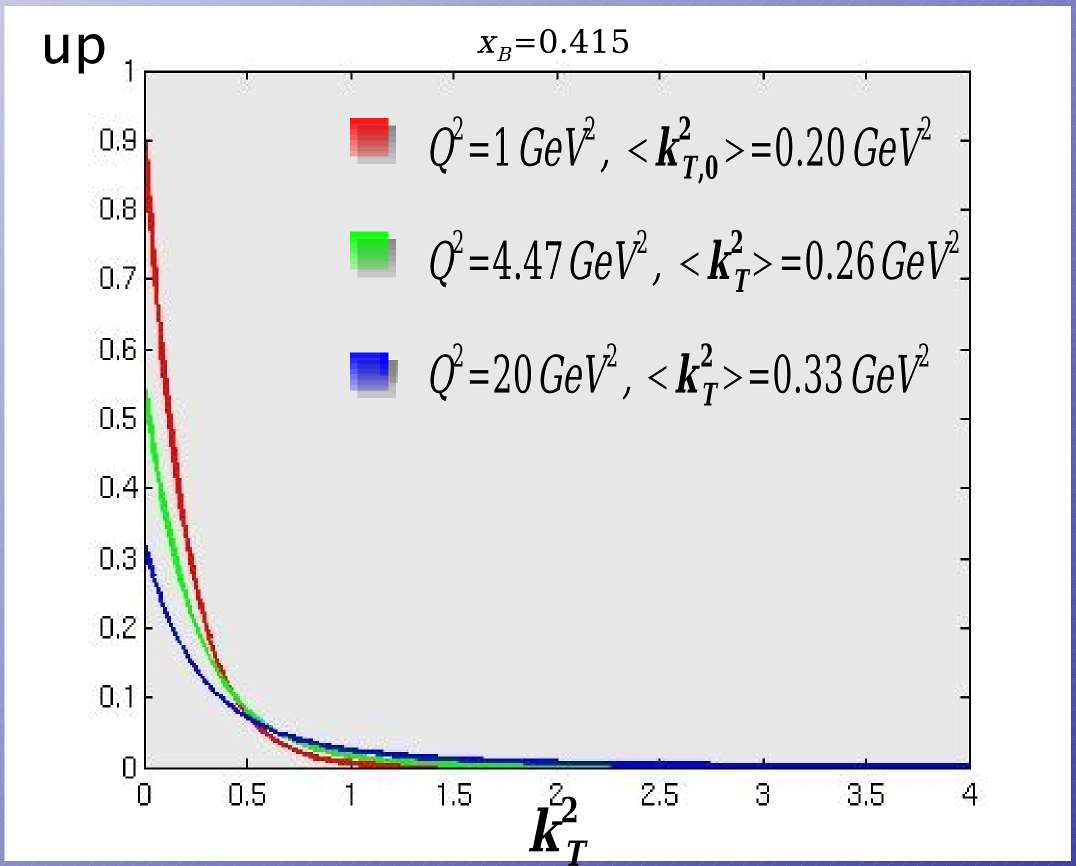


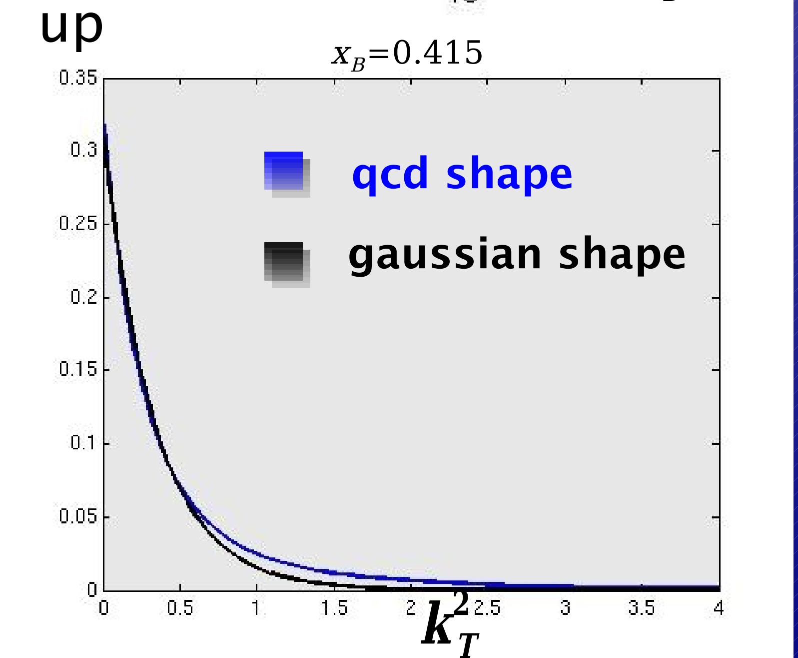
[9] X. Ji, J. Ma, F. Yuan ***Phys. Rev.* D71**, 034005 (2005)

- Numerical solution of TMD evolution equation: quark up

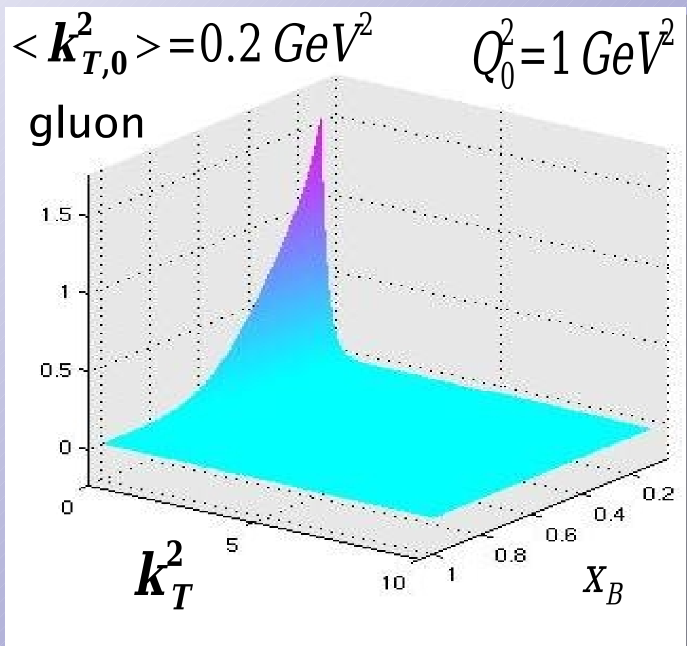
  - ↪ $<\boldsymbol{k}_{T,0}^2> = 0.2\ GeV^2 \quad Q_0^2 = 1\ GeV^2$
  - ↪ $u(x_B, Q_0^2, \boldsymbol{K}_T) = u(x_B, Q_0^2)\,\dfrac{e^{-k_T^2/<k_{T,0}^2>}}{\pi <\boldsymbol{k}_{T,0}^2>}$
  - ↪ $\int d^2\boldsymbol{k}_T\, u(x_B, Q^2, \boldsymbol{k}_T) = u(x_B, Q^2),\ \ \forall\, Q^2$





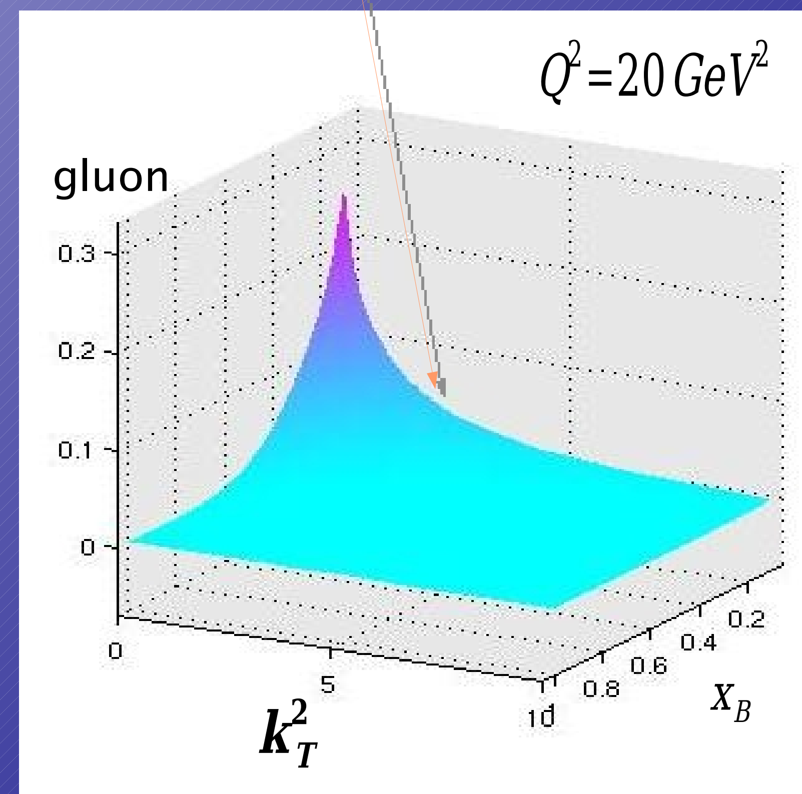


- Numerical solution of TMD evolution equation: gluon



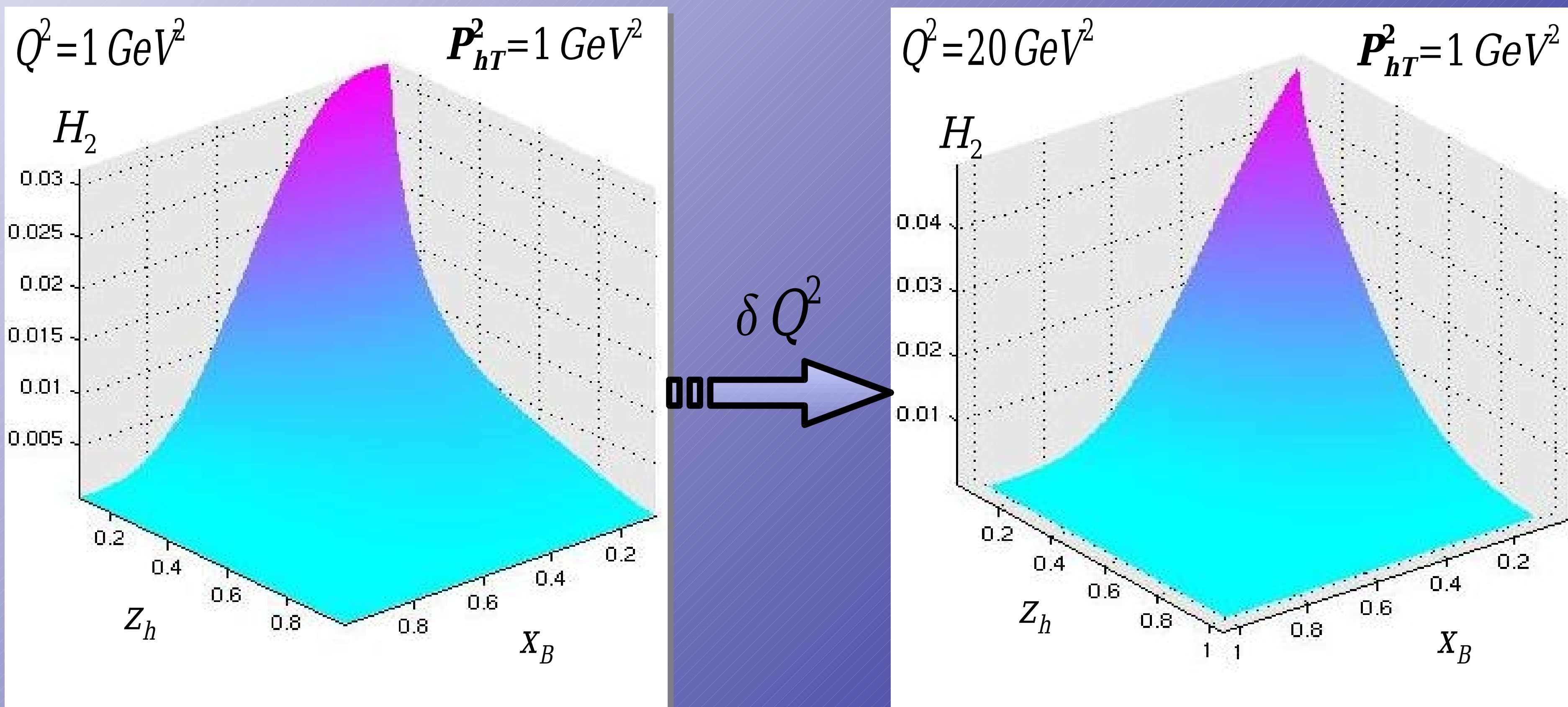
★ The spread of transverse momentum is enhanced by small-x gluon dynamics

δQ^2



☛ quark channel at large x: resum soft gluons to extract intrinsic $< k_{T,0}^2 >$ properly

- Phenomenology:
  - ↪ Current fragmentation: π^+ electroproduction



$$H_2^{\pi^+/P}(x_B, z_h, \boldsymbol{P}_{hT}, Q^2) = \sum_{i=q,\bar{q}} e_i^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}(z_h \boldsymbol{k}_T + \boldsymbol{p}_T - \boldsymbol{P}_{hT})\, F_P^i(x_B, Q^2, \boldsymbol{k}_T)\, D_i^h(z_h, Q^2, \boldsymbol{p}_T)$$

- Conclusions:

  - radiative transverse momentum can be quantitatively taken into account using TMD DGLAP evolution equations;
  - QCD dynamic drives the smearing of P_T ;
  - if factorization holds for TMD fracture functions

    ⇓

    we have a five-dimensional description of SIDIS final state both in current and in target fragmentation region.