Astrologic; Second-Order Logic and the *k*-order Ladder; Second-Order Axiomatized Arithmetic; Gödel's "God Theorem" & Speedup Theorem

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
3/16/2023

Astrologic; Second-Order Logic and the *k*-order Ladder; Second-Order Axiomatized Arithmetic; Gödel's "God Theorem" & Speedup Theorem

Selmer Bringsjord

Rensselaer AI & Reasoning (RAIR) Lab
Department of Cognitive Science
Department of Computer Science
Lally School of Management
Rensselaer Polytechnic Institute (RPI)
Troy, New York 12180 USA

Intro to Logic
3/16/2023

FOL

√ FOL

√ FOL

Epistemic + FOL

$\mathbf{B}_d\mathbf{B}_v\mathbf{B}_dVv$

√ FOL

Epistemic + FOL (for coverage of "killer" robots, later)

$\mathbf{B}_d\mathbf{B}_v\mathbf{B}_dVv$

√ FOL

√ Epistemic + FOL (for coverage of "killer" robots, later)

$\mathbf{B}_d\mathbf{B}_v\mathbf{B}_dVv$

√ FOL

√ Epistemic + FOL (for coverage of "killer" robots, later)

$$\mathbf{B}_d\mathbf{B}_v\mathbf{B}_d V v$$

TOL

$$\exists X[X(j) \wedge \neg X(m) \wedge S(X)]$$

√ FOL

√ Epistemic + FOL (for coverage of "killer" robots, later)

$$\mathbf{B}_d\mathbf{B}_v\mathbf{B}_dVv$$

TOL

$$\exists X[X(j) \wedge \neg X(m) \wedge S(X)]$$

√ FOL

√ Epistemic + FOL (for coverage of "killer" robots, later)

$\mathbf{B}_d\mathbf{B}_v\mathbf{B}_dVv$

TOL

? $\exists X[X(j) \wedge \neg X(m) \wedge S(X)]$

Double-Minded Man

The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161500NY

Contents

Abstract 2

1 Introduction 1

2 What is Expressivity? 3

3 *Double-Minded Man*, and Expressivity Challenges Therefrom 5
- 3.1 Gist of *Double-Minded Man* 5
- 3.2 Basic Demands of Movie Outline 3 6
- 3.3 Harriet, Joseph, and a Pair of Expressivity Challenges 6

4 The "No Way" Category and the Expressivity Challenges 7

5 The "Maybe" Catgory ACT-R, and the Expressivity Challenges 7

6 CLARION and the Expressivity Challenges 9
- 6.1 Overview of CLARION 9
  - 6.1.1 NACS — the Non-Action-Centered Subsystem 9
- 6.2 The Expressivity of CLARION 11
  - 6.2.1 Representing Structured Knowledge in General 11
  - 6.2.2 FOL-level Expressivity in CLARION 12
  - 6.2.3 Challenges (1) and (2) Met 13

7 Concerns & Objections; Replies 14
- 7.1 Concern 1: "What is the purpose?" 14
- 7.2 Concern 2: "What exactly are these 'challenges'?" 16
- 7.3 Concern 3: "Can CLARION handles *Double-Minded Man*?" 16
- 7.4 Concern 4: "What about the character-centric AI of Mateas et al.?" 16

8 Conclusion 17

References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

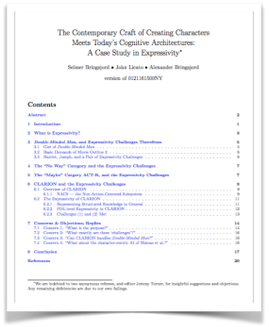
version of 0121161500NY

Contents

Abstract 2
1 Introduction 3
2 What is Expressivity? 3
3 Double-Minded Man, and Expressivity Challenges Therefrom 5
3.1 Gist of Double-Minded Man 5
3.2 Basic Demands of Movie Outline 3 6
3.3 Starlets, Jamais, and a Pair of Expressivity Challenges 8
4 The "No Way" Category and the Expressivity Challenges 7
5 The "Maybe" Category: ACT-R, and the Expressivity Challenges 7
6 CLARION and the Expressivity Challenges 9
6.1 Overview of CLARION 9
6.1.1 NACS — the Non-Action-Centered Subsystem 9
6.2 The Expressivity of CLARION 11
6.2.1 Representing Structured Knowledge in General 11
6.2.2 Hill-Climbing Rules in the CLARION 12
6.2.3 Challenges (1) and (2) Met 13
7 Concerns & Objections, Replies 14
7.1 Concern 1: "What is the purpose?" 14
7.2 Concern 2: "What exactly are these challenges?" 15
7.3 Concern 3: "Why CLARION and the Double-Minded Man?" 16
7.4 Concern 4: "What about the character-centric AI of Mateas et al.?" 16
8 Conclusion 17
References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man



The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord ♦ John Licato ♦ Alexander Bringsjord

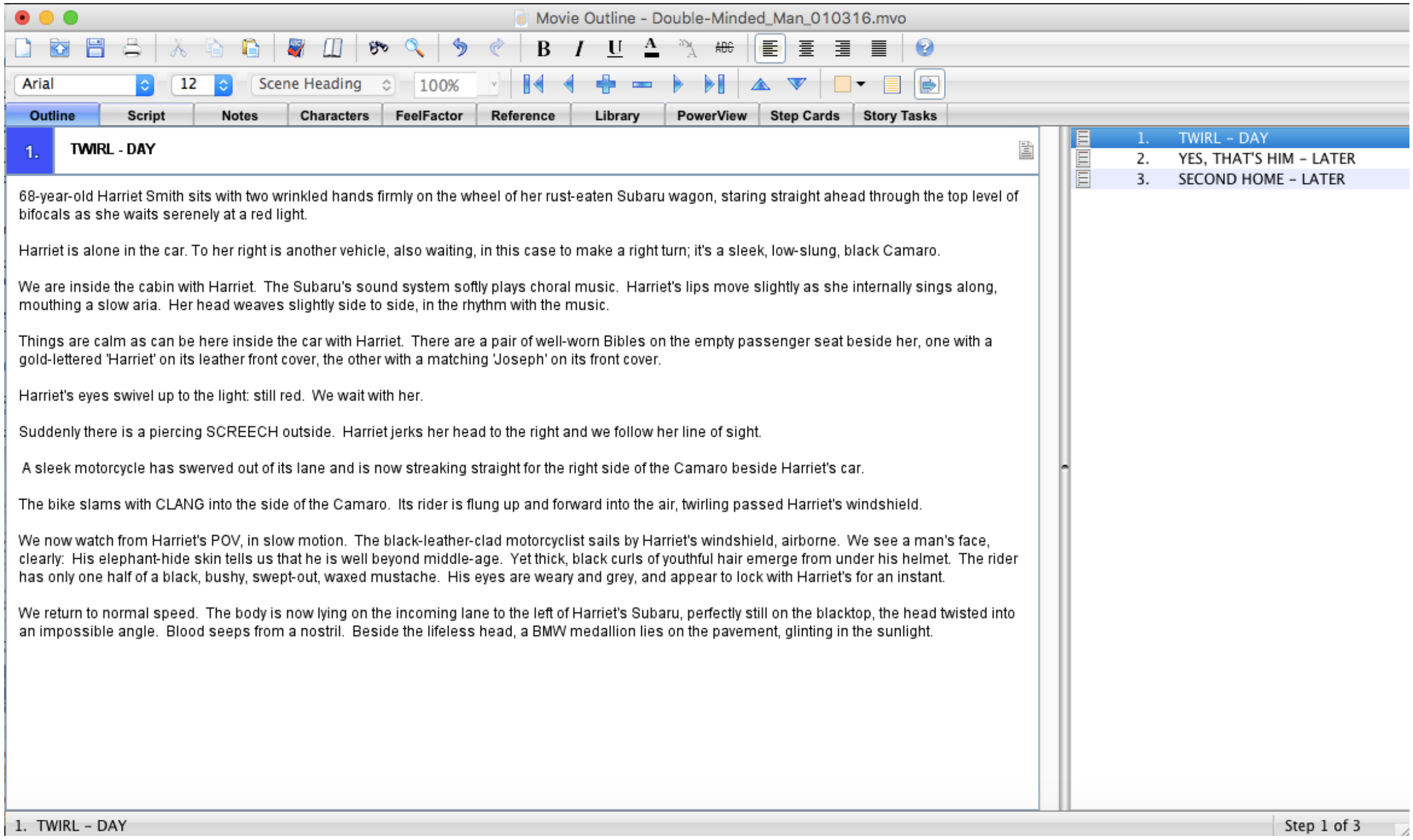
version of 0121815SSNY

Contents

Abstract 2
1 Introduction 3
2 What Is Expressivity? 5
3 Double-Minded Man, and Expressivity Challenges Therefrom 6
3.1 Gist of Double-Minded Man 6
3.2 Basic Demands of Movie Outline 3 6
3.3 Stories, Annals, and a Pair of Expressivity Challenges 8
4 The "Via Way" Category and the Expressivity Challenges 7
5 The "Digital" Category ACT-R, and the Expressivity Challenges 7
6 CLARION and the Expressivity Challenges 9
6.1 Overview of CLARION 9
6.1.1 NACS — the Non-Action-Centered Subsystem 9
6.2 The Expressivity of CLARION 11
6.2.1 Representing Structured Knowledge in General 12
6.2.2 Representing Double-Minded Man 13
6.2.3 Challenges (1) and (2) Met 13
7 Concerns & Objections; Replies 14
7.1 Concern 1: "What is the purpose?" 14
7.2 Concern 2: "What exactly are these 'challenges'?" 16
7.3 Concern 3: "Isn't this just a case of cherry-picking?Hold Shot" 16
7.4 Concern 4: "What about the character-movie AI of Mueue et al.?" 18
8 Conclusion 17
References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man



The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161500NY

Contents

Abstract 2

1 Introduction 3

2 What is Expressivity? 3

3 Double-Minded Man, and Expressivity Challenges Therefrom 5
- 3.1 Gist of Double-Minded Man 5
- 3.2 Basic Demands of Movie Outline 3 6
- 3.3 Starlets, Jamais, and a Pair of Expressivity Challenges 8

4 The "No Way" Category and the Expressivity Challenges 7

5 The "Maybe" Category: ACT-R, and the Expressivity Challenges 7

6 CLARION and the Expressivity Challenges 9
- 6.1 Overview of CLARION 9
  - 6.1.1 NACS — the Non-Action-Centered Subsystem 9
- 6.2 The Expressivity of CLARION 11
  - 6.2.1 Representing Structured Knowledge in General 11
  - 6.2.2 Hill-Climbing Rules in the CLARION 12
  - 6.2.3 Challenges (1) and (2) Met 13

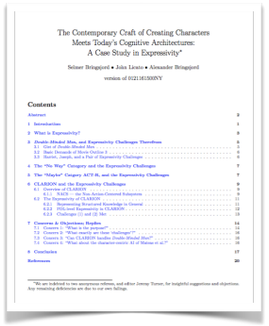
7 Concerns & Objections, Replies 14
- 7.1 Concern 1: "What is the purpose?" 14
- 7.2 Concern 2: "What exactly are these challenges?" 16
- 7.3 Concern 3: "Why CLARION and the Double-Minded Man?" 16
- 7.4 Concern 4: "What about the character-centric AI of Mateas et al.?" 16

8 Conclusion 17

References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man



The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161500NY

Contents

Abstract 2

1 Introduction 3

2 What is Expressivity? 3

3 Double-Minded Man, and Expressivity Challenges Therefrom 5
3.1 Gist of Double-Minded Man 5
3.2 Basic Demands of Movie Outline 3 6
3.3 Stories, Jumps, and a Pair of Expressivity Challenges 6

4 The "No Way" Category and the Expressivity Challenges 7

5 The "Maybe" Category AUT-R, and the Expressivity Challenges 7

6 CLARION and the Expressivity Challenges 9
6.1 Overview of CLARION 9
6.1.1 NACS — the Non-Action-Centered Subsystem 9
6.2 The Expressivity of CLARION 11
6.2.1 Representing Structured Knowledge in General 11
6.2.2 Hill-Climbing Rules in the Action Subsystem 12
6.2.3 Challenges (1) and (2) Met 13

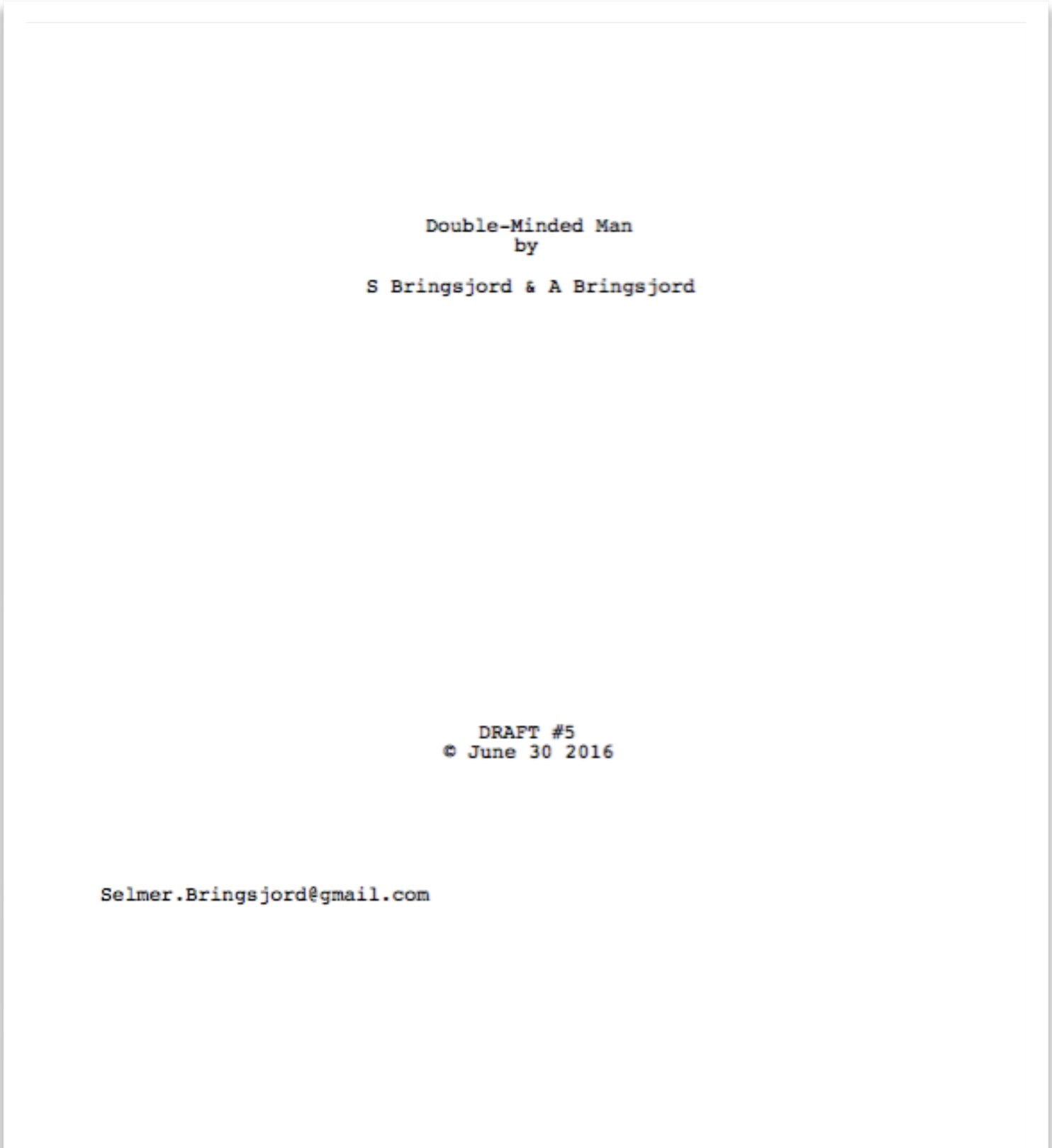
7 Concerns & Objections, Replies 14
7.1 Concern 1: "What is the purpose?" 14
7.2 Concern 2: "What exactly are these challenges?" 16
7.3 Concern 3: "Can CLARION handle Double-Minded Man?" 16
7.4 Concern 4: "What about the character-metric AI of Marino et al.?" 16

8 Conclusion 17

References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man



Double-Minded Man
by

S Bringsjord & A Bringsjord

DRAFT #5
© June 30 2016

Selmer.Bringsjord@gmail.com

The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

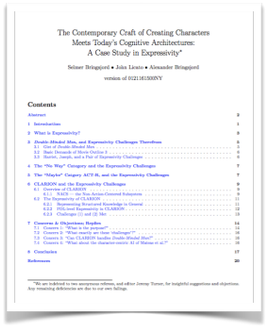
version of 0121161500NY

Contents

Abstract 2
1 Introduction 3
2 What is Expressivity? 3
3 Double-Minded Man, and Expressivity Challenges Therefrom 5
3.1 Gist of Double-Minded Man 5
3.2 Basic Demands of Movie Outline 3 6
3.3 Starlets, Jamais, and a Pair of Expressivity Challenges 6
4 The "No Way" Category and the Expressivity Challenges 7
5 The "Maybe" Category: ACT-R, and the Expressivity Challenges 7
6 CLARION and the Expressivity Challenges 9
6.1 Overview of CLARION 9
6.1.1 NACS — the Non-Action-Centered Subsystem 9
6.2 The Expressivity of CLARION 11
6.2.1 Representing Structured Knowledge in General 11
6.2.2 Hill-Climbing Rules in the Action Subsystem 12
6.2.3 Challenges (1) and (2) Met 13
7 Concerns & Objections, Replies 14
7.1 Concern 1: "What is the purpose?" 14
7.2 Concern 2: "What exactly are these challenges?" 15
7.3 Concern 3: "Why CLARION and the Double-Minded Man?" 16
7.4 Concern 4: "What about the character-centric AI of Mateas et al.?" 16
8 Conclusion 17
References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man



The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord ♦ John Licato ♦ Alexander Bringsjord

version of 0121815S9NY

Contents

Abstract 2
1 Introduction 1
2 What Is Expressivity? 3
3 Double-Minded Man, and Expressivity Challenge Therefore 5
3.1 Gist of Double-Minded Man . . . 5
3.2 Basic Demands of Movie Outline 2 . . . 6
3.3 Stories, Annals, and a Pair of Expressivity Challenges . . . 6
4 The "On Way" Category and the Expressivity Challenge 7
5 The "Digital" Category ACT-R, and the Expressivity Challenge 7
6 CLARION and the Expressivity Challenge 9
6.1 Overview of CLARION . . . 9
6.1.1 NACS — the Non-Action-Centered Subsystem . . . 9
6.2 The Expressivity of CLARION . . . 11
6.2.1 Representing Structural Knowledge in General . . . 12
6.2.2 Representing Stories in CLARION . . . 13
6.2.3 Challenges (1) and (2) Met . . . 13
7 Concerns & Objections; Replies 14
7.1 Concern 1: "What is the purpose?" . . . 14
7.2 Concern 2: "That wasn't an true 'challenge'." . . . 16
7.3 Concern 3: "You're Using a Moving Pinhole Short" . . . 16
7.4 Concern 4: "What about the character-movie AI of Murray et al.?" . . . 16
8 Conclusion 17
References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man

1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

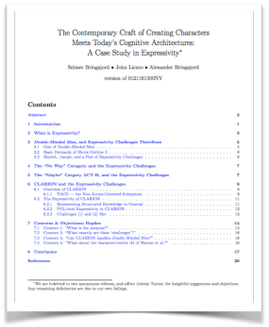
Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.



The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord ◆ John Licato ◆ Alexander Bringsjord

version of 0121161500NY

Contents

Abstract 2

1 Introduction 3

2 What is Expressivity? 3

3 Double-Minded Man, and Expressivity Challenges Therefrom 5
- 3.1 Gist of Double-Minded Man 5
- 3.2 Basic Demands of Movie Outline 2 6
- 3.3 Stories, Jumps, and a Pair of Expressivity Challenges 6

4 The "No Way" Category and the Expressivity Challenges 7

5 The "Maybe" Category ACT-R, and the Expressivity Challenges 7

6 CLARION and the Expressivity Challenges 9
- 6.1 Overview of CLARION 9
  - 6.1.1 NACS — the Non-Action-Centered Subsystem 9
- 6.2 The Expressivity of CLARION 11
  - 6.2.1 Representing Structured Knowledge in General 11
  - 6.2.2 Hill-Climbing Rules in the Action Subsystem 12
  - 6.2.3 Challenges (1) and (2) Met 13

7 Concerns & Objections, Replies 14
- 7.1 Concern 1: "What is the purpose?" 14
- 7.2 Concern 2: "What exactly are these challenges?" 16
- 7.3 Concern 3: "Can CLARION handle Double-Minded Man?" 16
- 7.4 Concern 4: "What about the character-metric AI of Mateas et al.?" 16

8 Conclusion 17

References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man

1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

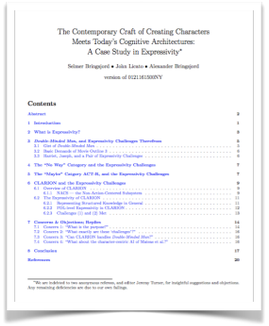
Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.



The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord ♦ John Licato ♦ Alexander Bringsjord

version of 0121815S0NY

Contents

Abstract 2

1 Introduction 3

2 What Is Expressivity? 3

3 Double-Minded Man, and Expressivity Challenge Theorems 5
3.1 Gist of Double-Minded Man 5
3.2 Basic Demands of Movie Outline 2 6
3.3 Stories, Annals, and a Pair of Expressivity Challenges 6

4 The "No Way" Category and the Expressivity Challenges 7

5 The "Maybe" Category ACT-R, and the Expressivity Challenges 7

6 CLARION and the Expressivity Challenges 9
6.1 Overview of CLARION 9
6.1.1 NACS — the Non-Action-Centered Subsystem 9
6.2 The Expressivity of CLARION 11
6.2.1 Representing Structural Knowledge in General 12
6.2.2 Representing Double-Minded Man 13
6.2.3 Challenges (1) and (2) Met 13

7 Concerns & Objections; Replies 14
7.1 Concern 1: "What is the purpose?" 14
7.2 Concern 2: "What exactly are these 'challenges'?" 16
7.3 Concern 3: "Isn't this just a case of 'Hello World' Blues?" 16
7.4 Concern 4: "What about the character-movie AI of Murray et al.?" 18

8 Conclusion 17

References 20

*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man

$$\exists X[X(joseph) \wedge \neg X(m(harriet, joseph)) \wedge Sleazy(X)]$$

1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

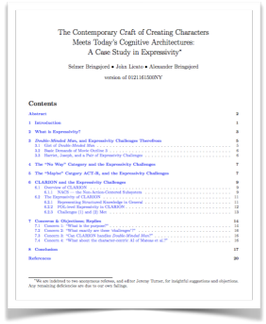
Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.



The Contemporary Craft of Creating Characters
Meets Today's Cognitive Architectures:
A Case Study in Expressivity*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161XHNY

Contents

Abstract 2

1 Introduction 3

2 What is Expressivity? 3

3 Double-Minded Man, and Expressivity Challenges Therefrom 5
3.1 Tale of Double-Minded Man 5
3.2 Basic Demands of Movie-Outline 2 6
3.3 Harriet, Joseph, and a Pair of Expressivity Challenges 6

4 The "Sea Way" Category and the Expressivity Challenges 7

5 The "Digital" Category ACT-R, and the Expressivity Challenges 7

6 CLARION and the Expressivity Challenges 9
6.1 Overview of CLARION 9
6.1.1 ToAST — the Non-Action-Centered Subsystem 9
6.2 The Expressivity of CLARION 11
6.2.1 Representing Structural Knowledge in General 12
6.2.2 Representing Double-Minded Man 13
6.2.3 Challenges (1) and (2) Met 13

7 Concerns & Objections; Replies 14
7.1 Concern 1: "What is the purpose?" 14
7.2 Concern 2: "That wasn't my idea of a challenge!" 16
7.3 Concern 3: "What about the 'Winding Path' Story?" 16
7.4 Concern 4: "What about the character-story AI of Mateas et al.?" 16

8 Conclusion 17

References 20

*We are indebted to two anonymous referees, and editor Joscep Turiel, for helpful suggestions and objections. Any remaining deficiencies are due to our own failings.

Double-Minded Man

1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.

✓ $\exists X[X(joseph) \wedge \neg X(m(harriet, joseph)) \wedge Sleazy(X)]$

Climbing the *k*-order Ladder

Climbing the *k*-order Ladder

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

$$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

$$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

FOL $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

Things *x* and *y*, along with the father of *x*,
share a certain property (and *x* likes *y*).

FOL $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

$$\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

SOL $\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

$$\exists x, y\ \exists R, R^2[R(x) \wedge R(y) \wedge R^2(x, y) \wedge Positive(R^2) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

TOL $\exists x, y\ \exists R, R^2[R(x) \wedge R(y) \wedge R^2(x, y) \wedge Positive(R^2) \wedge R(fatherOf(x))]$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

TOL $\mathscr{L}_3$

$$\exists x, y\ \exists R, R^2[R(x) \wedge R(y) \wedge R^2(x, y) \wedge Positive(R^2) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\mathscr{L}_2$

$$\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\mathscr{L}_1$

$$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $\mathscr{L}_0$

$$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

Climbing the *k*-order Ladder

⋮

TOL $\mathscr{L}_3$

$$\exists x, y\ \exists R, R^2[R(x) \wedge R(y) \wedge R^2(x, y) \wedge Positive(R^2) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\mathscr{L}_2$

$$\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\mathscr{L}_1$

$$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$$

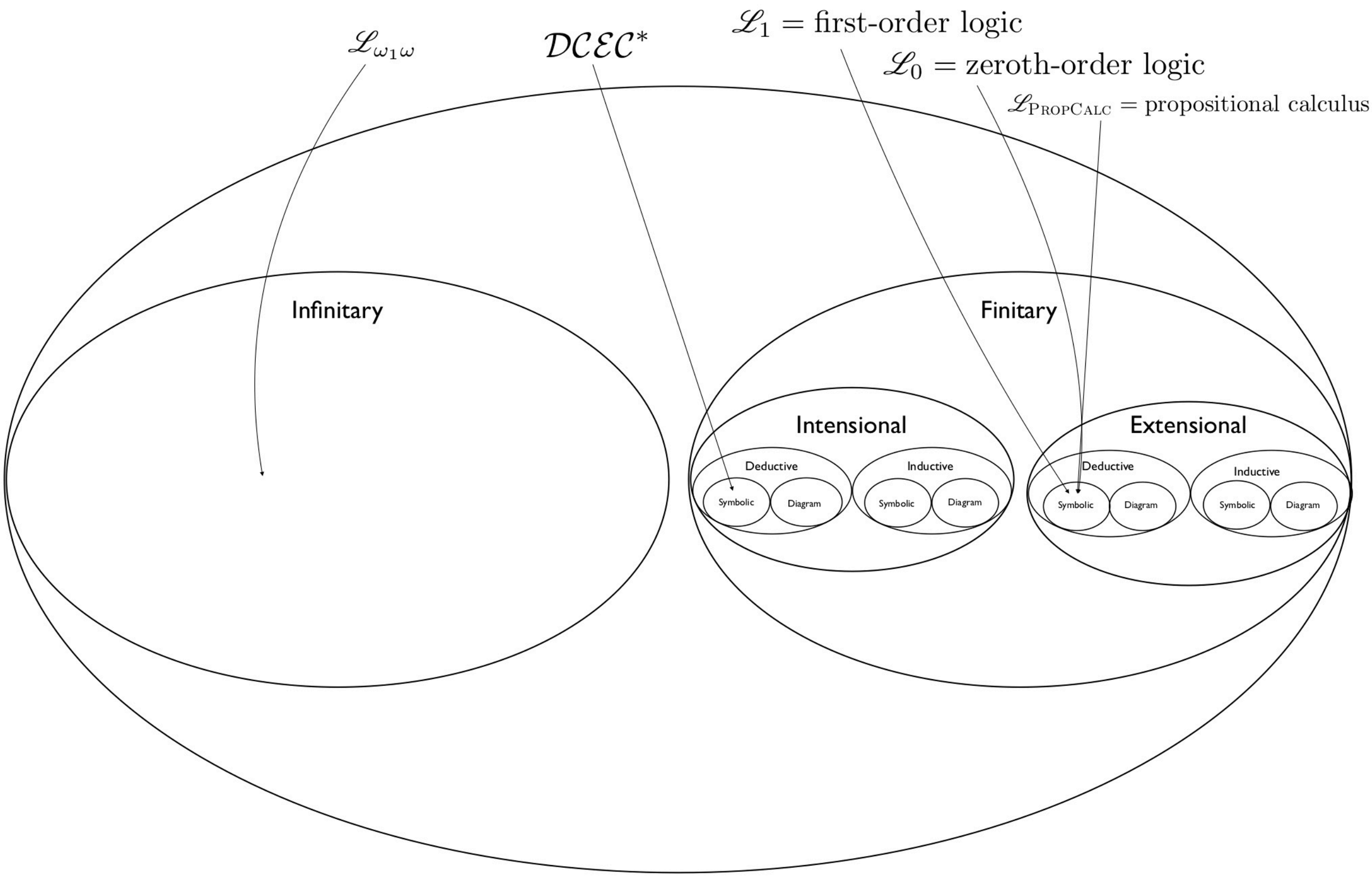
There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $\mathscr{L}_0$

$$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$$

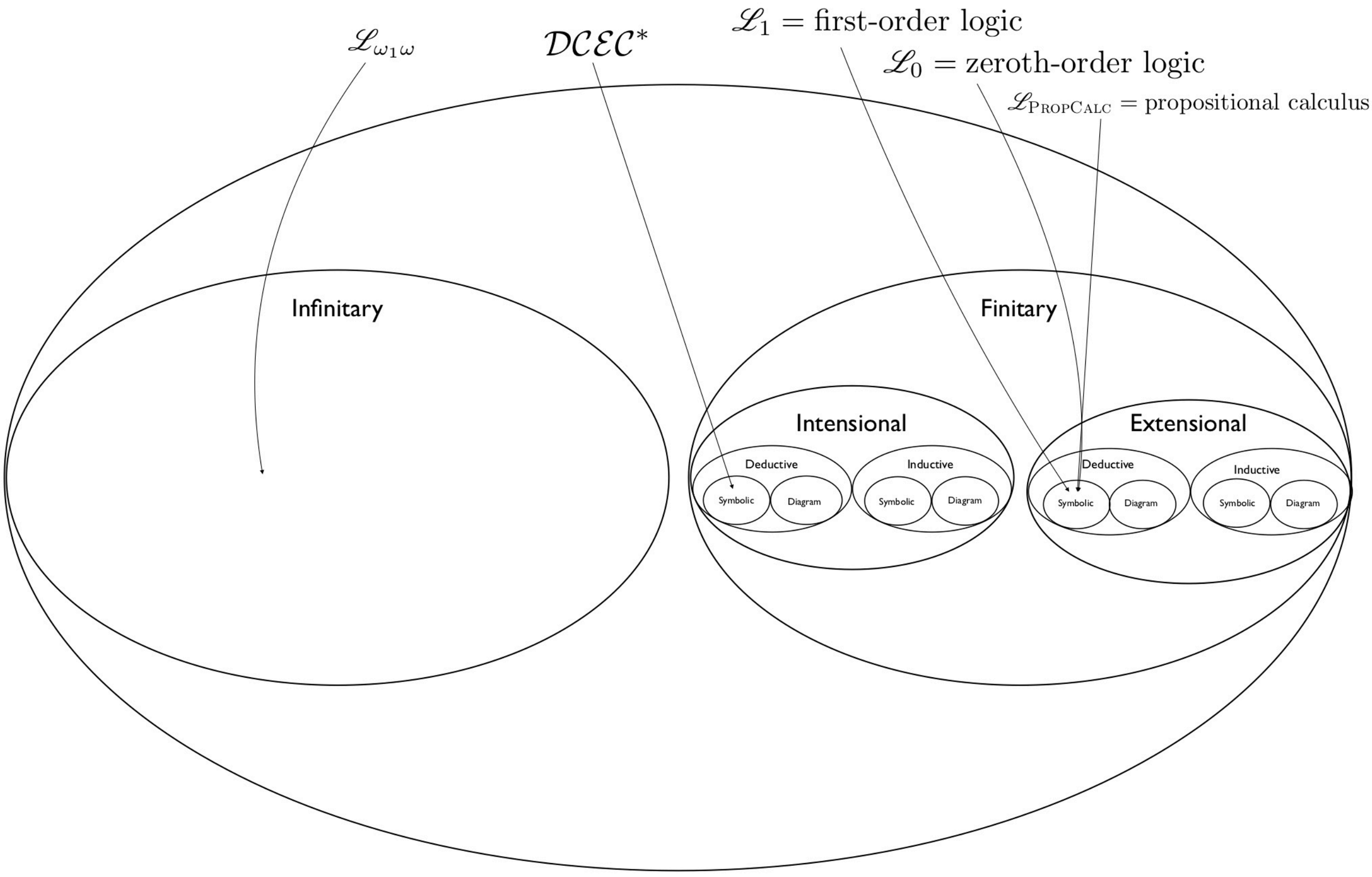
a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

The Universe of Logics

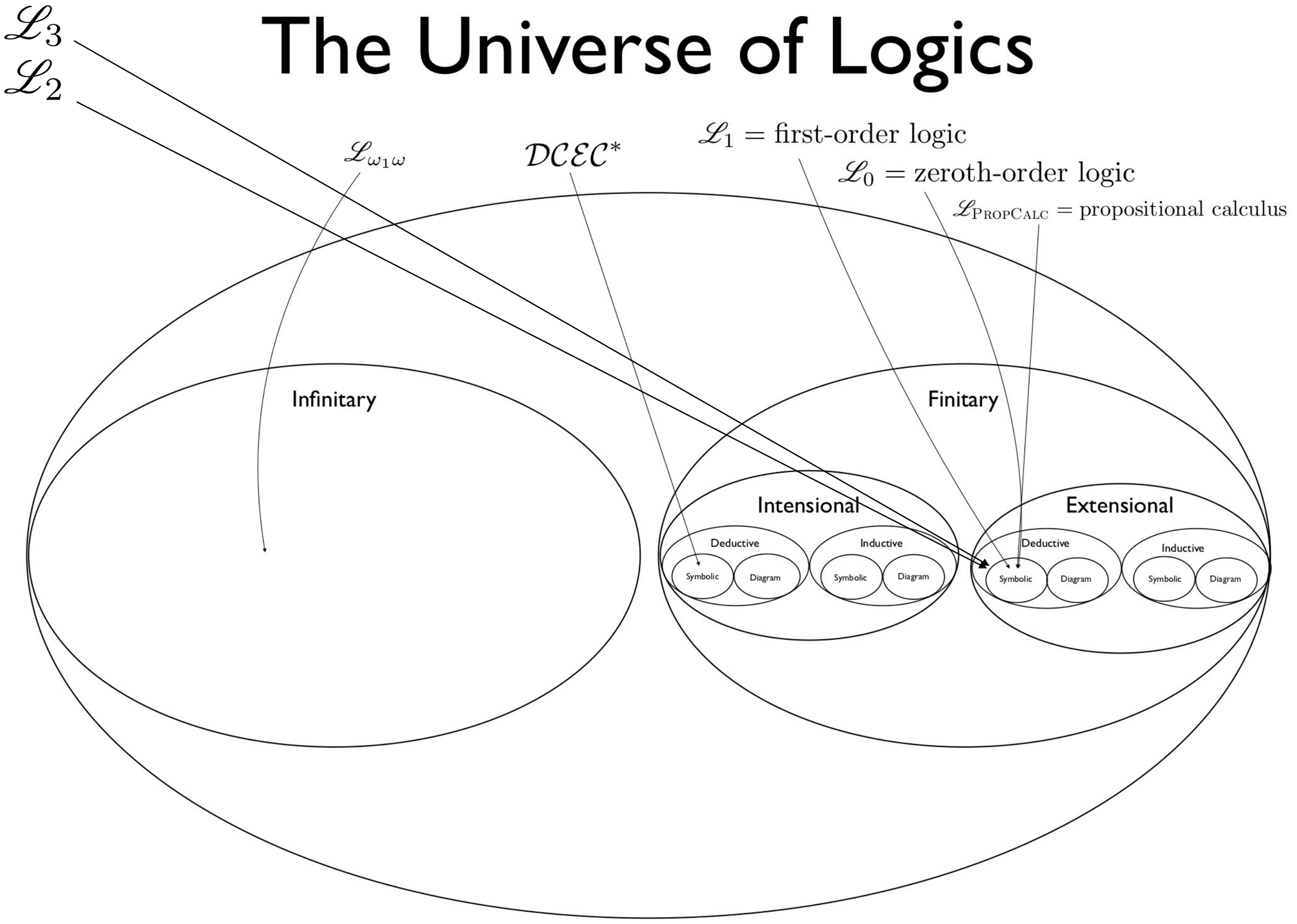


$\mathscr{L}_3$
$\mathscr{L}_2$

The Universe of Logics



The Universe of Logics



Climbing the *k*-order Ladder

⋮

TOL $\mathscr{L}_3$

$$\exists x, y\ \exists R, R^2[R(x) \wedge R(y) \wedge R^2(x, y) \wedge Positive(R^2) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property; and, *x* R^2s *y*, where R^2 is a positive property.

SOL $\mathscr{L}_2$

$$\exists x \exists y \exists R[R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

Things *x* and *y*, along with the father of *x*, share a certain property (and *x* likes *y*).

FOL $\mathscr{L}_1$

$$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$$

There's some thing which is a llama and likes *b* (which is also a llama), and whose father is a llama too.

ZOL $\mathscr{L}_0$

$$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$$

a is a llama, as is *b*, *a* likes *b*, and the father of *a* is a llama as well.

"Leibniz was right that Descartes was right that … : God exists, necessarily."

Gödel's "God Theorem"

		Part I	
	(1′)	The absence of a positive property is not positive.	premise
	(1)	The absence of a positive property is not positive; and if a property isn't positive, the absence of that property is.	premise
	(2)	Any property entailed by a positive property is itself positive.	premise
∴	(3)	Every positive property P is possibly possessed by something.	(1), (2)
		Part II	
	(4)	Anything that is God has all positive properties.	definition
	(5)	The property of being God is itself a positive property.	premise
∴	(6)	It's possible that God exists.	(3), (5)
		Part III	
	(7)	Positive properties are necessarily positive.	premise
	(8)	A thing x has an essence E if and only if (i) E is a property x has; and (ii) for any property P that x has, x's having this property P is necessarily implied by x's having essence E.	definition
∴	(9)	The property of being God is an essence of any thing that has this property.	(8), (7), (4), (1)
∴	(9)	The property of *being God* ($= G$) is an essential property of any thing that has G.	(8), (7), (4), (1)
		Part IV	
	(10)	A thing has necessary existence if and only if all the essences that thing has imply that something exists and has all those essences.	definition
	(10)	A thing has necessary existence if and only if all the essential properties that thing has imply that something exists and has all those essential properties.	definition
	(11)	Necessary existence is a positive property.	premise
∴	(12)	Necessarily, God exists.	(6), (9), (10), (11) **QED**

Gödel's "God Theorem" (formalized, machine verified)

	(1)	$\forall P\ [Pos(\neg P) \leftrightarrow \neg Pos(P)]$	premise
	(2)	$\forall P_1\ \forall P_2\ \{Pos(P_1) \wedge \Box\forall x\ [P_1(x) \rightarrow P_2(x)] \rightarrow Pos(P_2)\}$	premise
$\therefore$	(3)	$\forall P\ [Pos(P) \rightarrow \Diamond\exists x\ P(x)]$	theorem
	(4)	$\forall x\ [G(x) \leftrightarrow \forall P\ [Pos(P) \rightarrow P(x)]$	definition
	(5)	$Pos(G)$	premise
$\therefore$	(6)	$\Diamond\exists x\ G(x)$	corollary
	(7)	$\forall P\ [Pos(P) \rightarrow \Box Pos(P)]$	premise
	(8)	$\forall x \forall P\ \{Ess(P,x) \leftrightarrow [P(x) \wedge \forall P'\ (P'(x) \rightarrow \Box\forall y(P(y) \rightarrow P'(y))\}$	definition
$\therefore$	(9)	$\forall x\ [G(x) \rightarrow Ess(G,x)]$	theorem
	(10)	$\forall x\ \{NE(x) \leftrightarrow \forall P\ [Ess(P,x) \rightarrow \Box\exists y\ P(y)]\}$	definition
	(11)	$Pos(NE)$	premise
$\therefore$	(12)	$\Box\exists x\ G(x)$ (a.k.a. "Necessarily, God exists.)	theorem

Gödel's "God Theorem" (formalized, machine verified)

	(1)	$\forall P\ [Pos(\neg P) \leftrightarrow \neg Pos(P)]$	premise
	(2)	$\forall P_1\ \forall P_2\ \{Pos(P_1) \wedge \Box\forall x\ [P_1(x) \rightarrow P_2(x)] \rightarrow Pos(P_2)\}$	premise
$\therefore$	(3)	$\forall P\ [Pos(P) \rightarrow \Diamond\exists x\ P(x)]$	theorem
	(4)	$\forall x\ [G(x) \leftrightarrow \forall P\ [Pos(P) \rightarrow P(x)]$	definition
	(5)	$Pos(G)$	premise
$\therefore$	(6)	$\Diamond\exists x\ G(x)$	corollary
	(7)	$\forall P\ [Pos(P) \rightarrow \Box Pos(P)]$	premise
	(8)	$\forall x \forall P\ \{Ess(P,x) \leftrightarrow [P(x) \wedge \forall P'\ (P'(x) \rightarrow \Box\forall y(P(y) \rightarrow P'(y))\}$	definition
$\therefore$	(9)	$\forall x\ [G(x) \rightarrow Ess(G,x)]$	theorem
	(10)	$\forall x\ \{NE(x) \leftrightarrow \forall P\ [Ess(P,x) \rightarrow \Box\exists y\ P(y)]\}$	definition
	(11)	$Pos(NE)$	premise
$\therefore$	(12)	$\Box\exists x\ G(x)$ (a.k.a. "Necessarily, God exists.)	theorem

$PA_I = Z_I$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x))] \rightarrow \forall x \phi(x))$$

where $\phi(x)$ is open wff with variable x, and perhaps others, free.

$PA_I = Z_I$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x))] \rightarrow \forall x \phi(x)) \quad \infty$$

where $\phi(x)$ is open wff with variable x, and perhaps others, free.

$\mathsf{PA}_2 = \mathsf{Z}_2$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

(if we drop any restriction C, we have full second-order arithmetic.)

$\mathsf{PA}_2 = \mathsf{Z}_2$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

Induction Axiom $\quad \forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

(if we drop any restriction C, we have full second-order arithmetic.)

$\mathsf{PA}_2 = \mathsf{Z}_2$

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

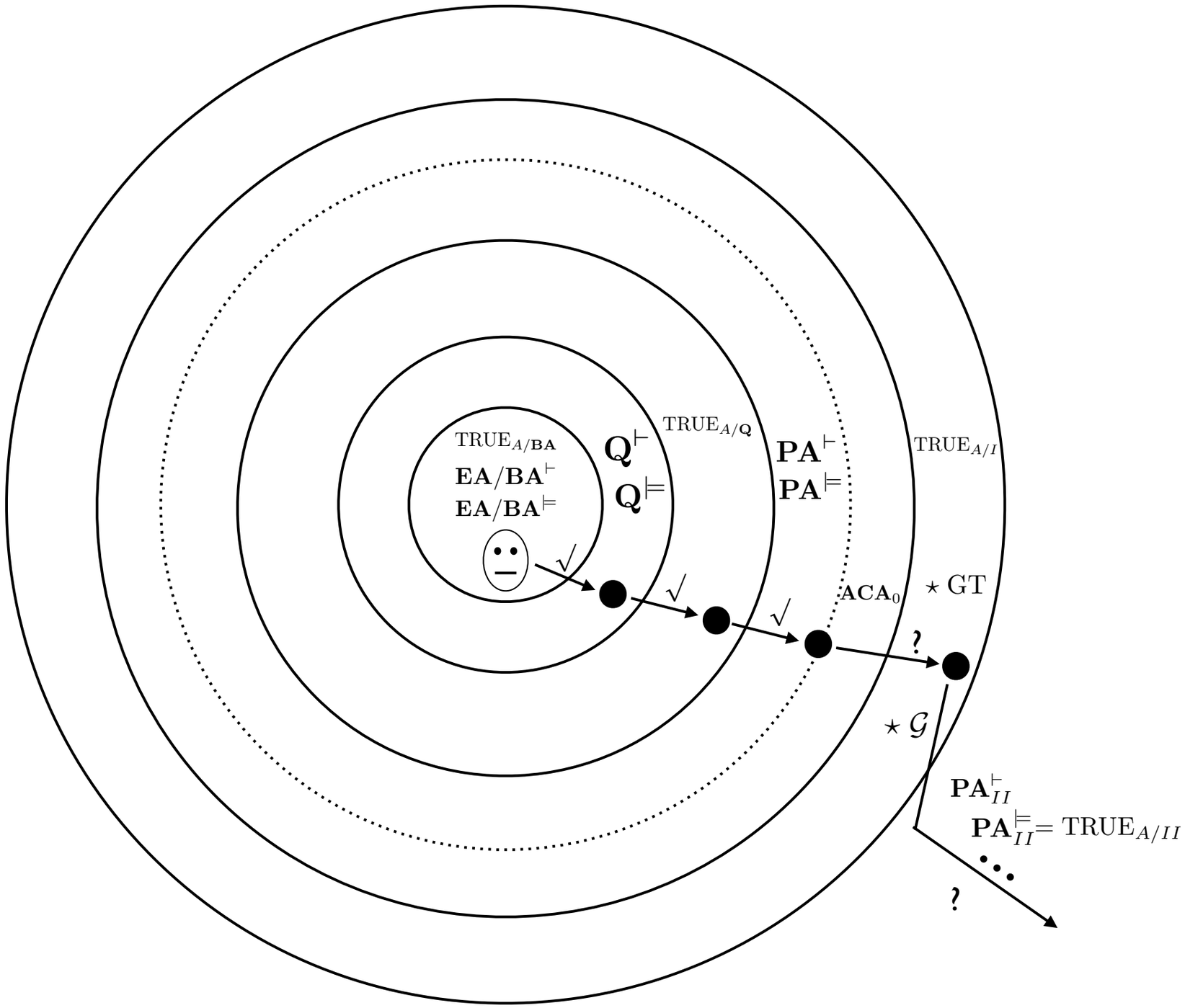
Induction Axiom $\forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

Comprehension Schema $\exists X(\forall x X(x) \leftrightarrow \phi(x)) \qquad \text{where } \phi(x) \in C$

(if we drop any restriction C, we have full second-order arithmetic.)

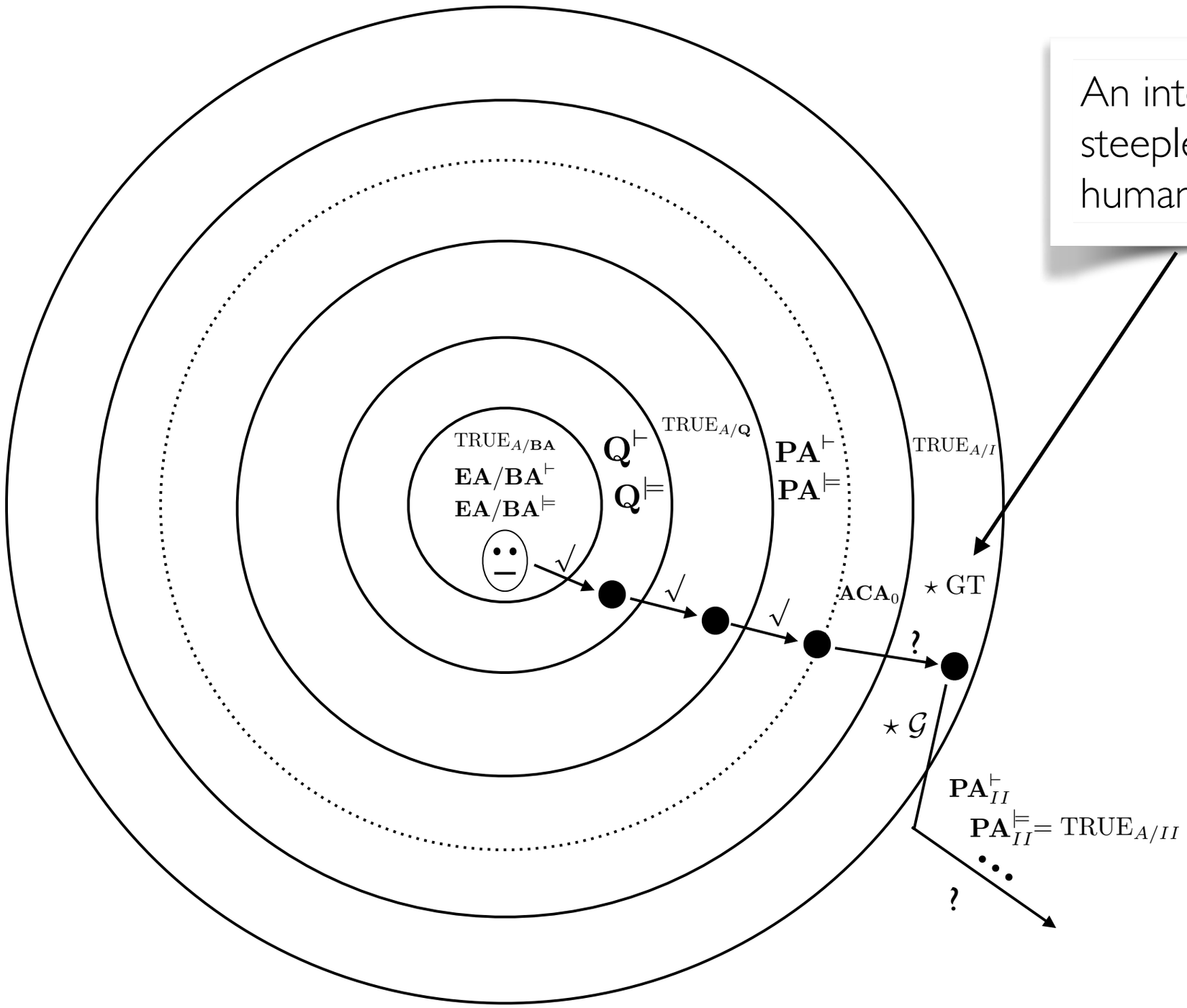
Astrologic

(Aliens & Angels on the Same "Race Track")



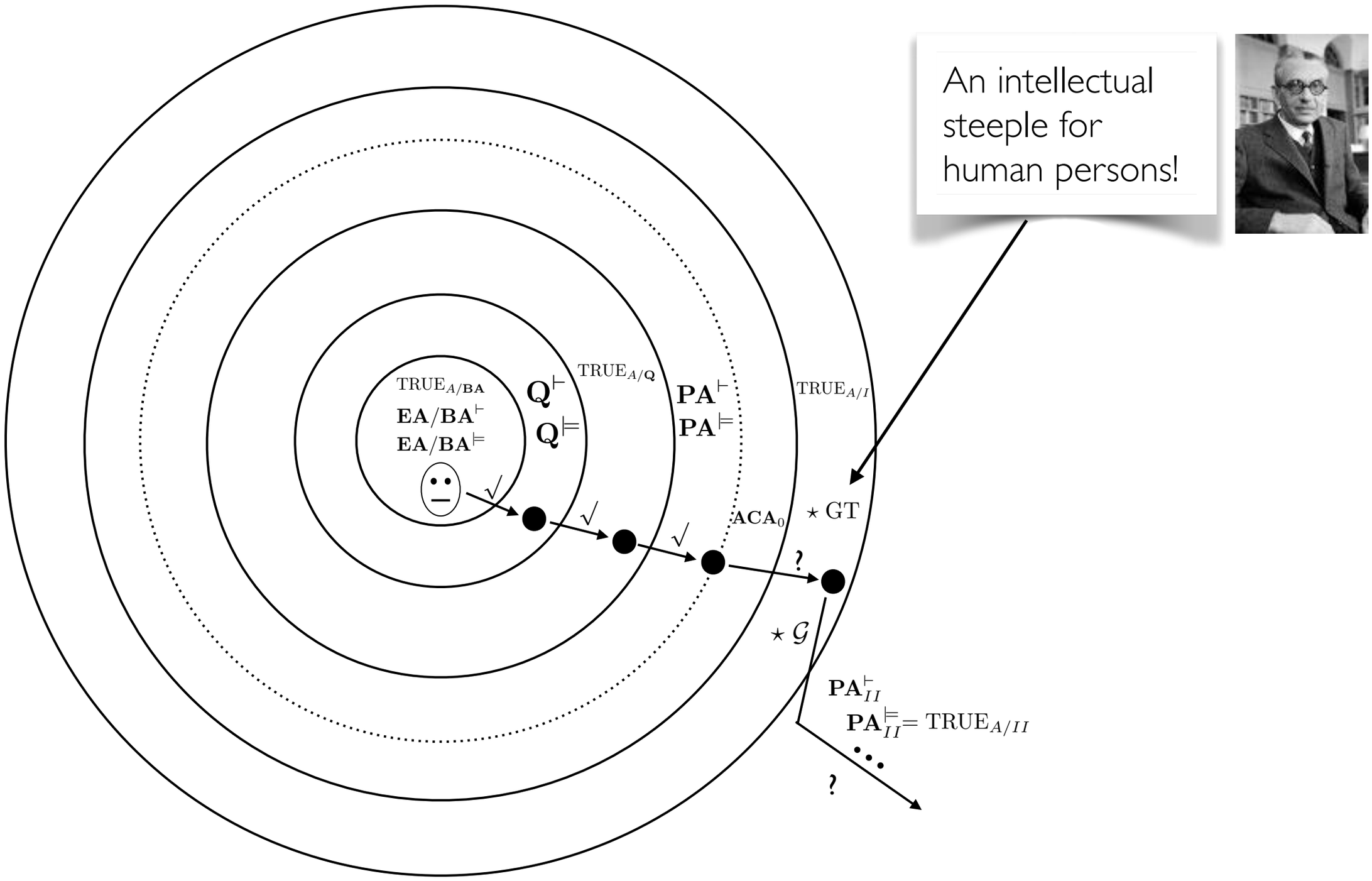
Astrologic

(Aliens & Angels on the Same "Race Track")



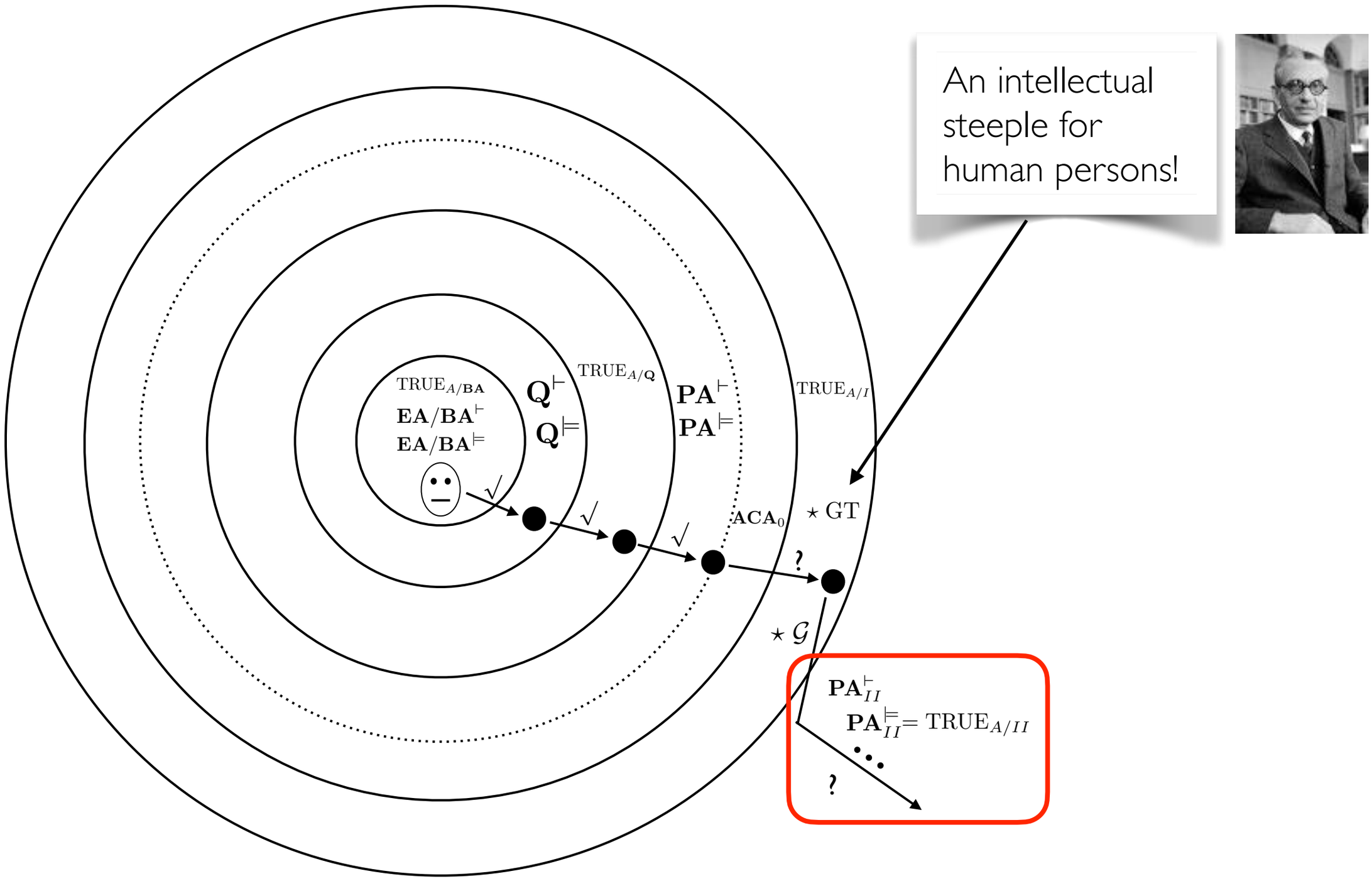
Astrologic

(Aliens & Angels on the Same "Race Track")



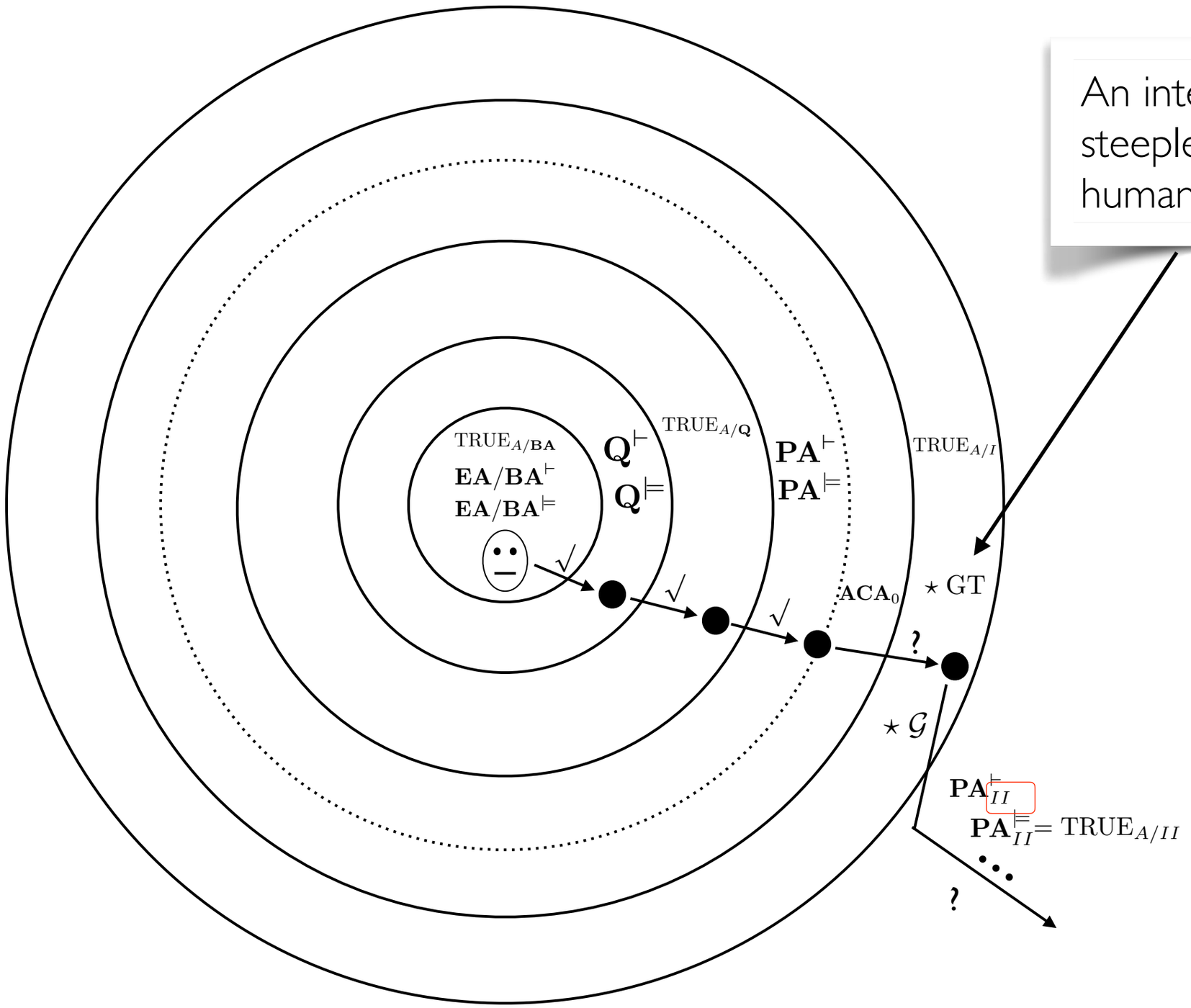
Astrologic

(Aliens & Angels on the Same "Race Track")



Astrologic

(Aliens & Angels on the Same "Race Track")



An intellectual steeple for human persons!

New!

ACA_0

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

Comprehension Schema $\exists X[\forall x\ X(x) \leftrightarrow \phi(x)]$ where $\phi(x)$ has no second-order quantifiers

New!

ACA_0

$$
\begin{array}{ll}
\text{A1} & \forall x(0 \neq s(x)) \\
\text{A2} & \forall x \forall y(s(x) = s(y) \rightarrow x = y) \\
\text{A3} & \forall x(x \neq 0 \rightarrow \exists y(x = s(y)) \\
\text{A4} & \forall x(x + 0 = x) \\
\text{A5} & \forall x \forall y(x + s(y) = s(x + y)) \\
\text{A6} & \forall x(x \times 0 = 0) \\
\text{A7} & \forall x \forall y(x \times s(y) = (x \times y) + x)
\end{array}
$$

Induction Axiom $\forall X([X(0) \wedge \forall x(X(x) \rightarrow X(s(x))] \rightarrow \forall x X(x))$

Comprehension Schema $\exists X[\forall x\ X(x) \leftrightarrow \phi(x)]$ where $\phi(x)$ has no second-order quantifiers

Slutten