

# **Astrologic; Second-Order Logic and the $k$ -order Ladder; Second-Order Axiomatized Arithmetic; Gödel's “God Theorem” & Speedup Theorem**

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Intro to Logic  
3/16/2023





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**FOL**

✓ FOL

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Epistemic + FOL  
 $B_d B_v B_d V v$

✓ FOL

Epistemic + FOL (for coverage of “killer” robots, later)  
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TOL

$\exists X [X(j) \wedge \neg X(m) \wedge S(X)]$

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?  $\exists X [X(j) \wedge \neg X(m) \wedge S(X)]$



# Double-Minded Man

## The Contemporary Craft of Creating Characters Meets Today's Cognitive Architectures: A Case Study in Expressivity\*

Selmer Bringsjord • John Licato • Alexander Bringsjord

version of 0121161500NY

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\*We are indebted to two anonymous referees, and editor Jeremy Turner, for insightful suggestions and objections. Any remaining deficiencies are due to our own failings.

The Contemporary Craft of Creating Characters  
Meets Today's Cognitive Architecture:  
A Case Study in Expertise\*

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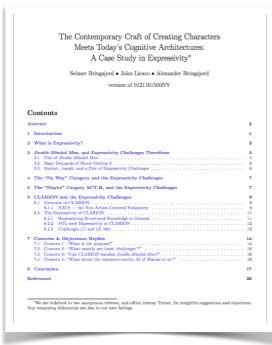
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# Double-Minded Man

# Double-Minded Man



A screenshot of a movie outline software interface. The window title is "Movie Outline - Double-Minded\_Man\_010316.mvo". The interface includes a toolbar with various editing tools, a font and size selection area (Arial, 12), and a scene heading dropdown (Scene Heading). Below the toolbar is a menu bar with options: Outline, Script, Notes, Characters, FeelFactor, Reference, Library, PowerView, Step Cards, and Story Tasks. The main content area is titled "1. TWIRL - DAY" and contains several paragraphs of script text. On the right side, there is a vertical list of scene headings: 1. TWIRL - DAY, 2. YES, THAT'S HIM - LATER, and 3. SECOND HOME - LATER. At the bottom of the window, a status bar shows "1. TWIRL - DAY" on the left and "Step 1 of 3" on the right.

## 1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.

1. TWIRL - DAY
2. YES, THAT'S HIM - LATER
3. SECOND HOME - LATER

The Contemporary Craft of Creating Characters  
Meets Today's Cognitive Architecture:  
A Case Study in Expertise\*

Silver Stripling • John Lewis • Alexander Stripling  
ISSN 1539-3062

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version of 02/16/2016

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# Double-Minded Man

Double-Minded Man  
by  
S Bringsjord & A Bringsjord

DRAFT #5  
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# Double-Minded Man

The Contemporary Craft of Creating Characters  
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The Contemporary Craft of Creating Characters  
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Silver Stripling • John Lewis • Alexander Stripling  
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# Double-Minded Man

## 1. TWIRL - DAY

68-year-old Harriet Smith sits with two wrinkled hands firmly on the wheel of her rust-eaten Subaru wagon, staring straight ahead through the top level of bifocals as she waits serenely at a red light.

Harriet is alone in the car. To her right is another vehicle, also waiting, in this case to make a right turn; it's a sleek, low-slung, black Camaro.

We are inside the cabin with Harriet. The Subaru's sound system softly plays choral music. Harriet's lips move slightly as she internally sings along, mouthing a slow aria. Her head weaves slightly side to side, in the rhythm with the music.

Things are calm as can be here inside the car with Harriet. There are a pair of well-worn Bibles on the empty passenger seat beside her, one with a gold-lettered 'Harriet' on its leather front cover, the other with a matching 'Joseph' on its front cover.

Harriet's eyes swivel up to the light: still red. We wait with her.

Suddenly there is a piercing SCREECH outside. Harriet jerks her head to the right and we follow her line of sight.

A sleek motorcycle has swerved out of its lane and is now streaking straight for the right side of the Camaro beside Harriet's car.

The bike slams with CLANG into the side of the Camaro. Its rider is flung up and forward into the air, twirling passed Harriet's windshield.

We now watch from Harriet's POV, in slow motion. The black-leather-clad motorcyclist sails by Harriet's windshield, airborne. We see a man's face, clearly: His elephant-hide skin tells us that he is well beyond middle-age. Yet thick, black curls of youthful hair emerge from under his helmet. The rider has only one half of a black, bushy, swept-out, waxed mustache. His eyes are weary and grey, and appear to lock with Harriet's for an instant.

We return to normal speed. The body is now lying on the incoming lane to the left of Harriet's Subaru, perfectly still on the blacktop, the head twisted into an impossible angle. Blood seeps from a nostril. Beside the lifeless head, a BMW medallion lies on the pavement, glinting in the sunlight.





# Climbing the $k$ -order Ladder

# Climbing the $k$ -order Ladder

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

$Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.



# Climbing the $k$ -order Ladder

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

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# Climbing the $k$ -order Ladder

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge Likes(a, b) \wedge Llama(fatherOf(a))$

$a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

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$\exists x[Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

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Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

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# Climbing the $k$ -order Ladder

$$\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$$

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**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

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# Climbing the $k$ -order Ladder

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge Likes(x, y) \wedge R(fatherOf(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge Likes(x, b) \wedge Llama(fatherOf(x))]$

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# Climbing the $k$ -order Ladder

$\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \text{Likes}(x, y) \wedge R(\text{fatherOf}(x))]$

Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \text{Likes}(x, b) \wedge Llama(\text{fatherOf}(x))]$

There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \text{Likes}(a, b) \wedge Llama(\text{fatherOf}(a))$

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# Climbing the $k$ -order Ladder

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# Climbing the $k$ -order Ladder

**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \text{Positive}(R^2) \wedge R(\text{fatherOf}(x))]$

$\mathcal{L}_3$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \text{Likes}(x, y) \wedge R(\text{fatherOf}(x))]$

$\mathcal{L}_2$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \text{Likes}(x, b) \wedge Llama(\text{fatherOf}(x))]$

$\mathcal{L}_1$  There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \text{Likes}(a, b) \wedge Llama(\text{fatherOf}(a))$

$\mathcal{L}_0$   $a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.

# Climbing the $k$ -order Ladder

⋮

**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_3$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_2$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

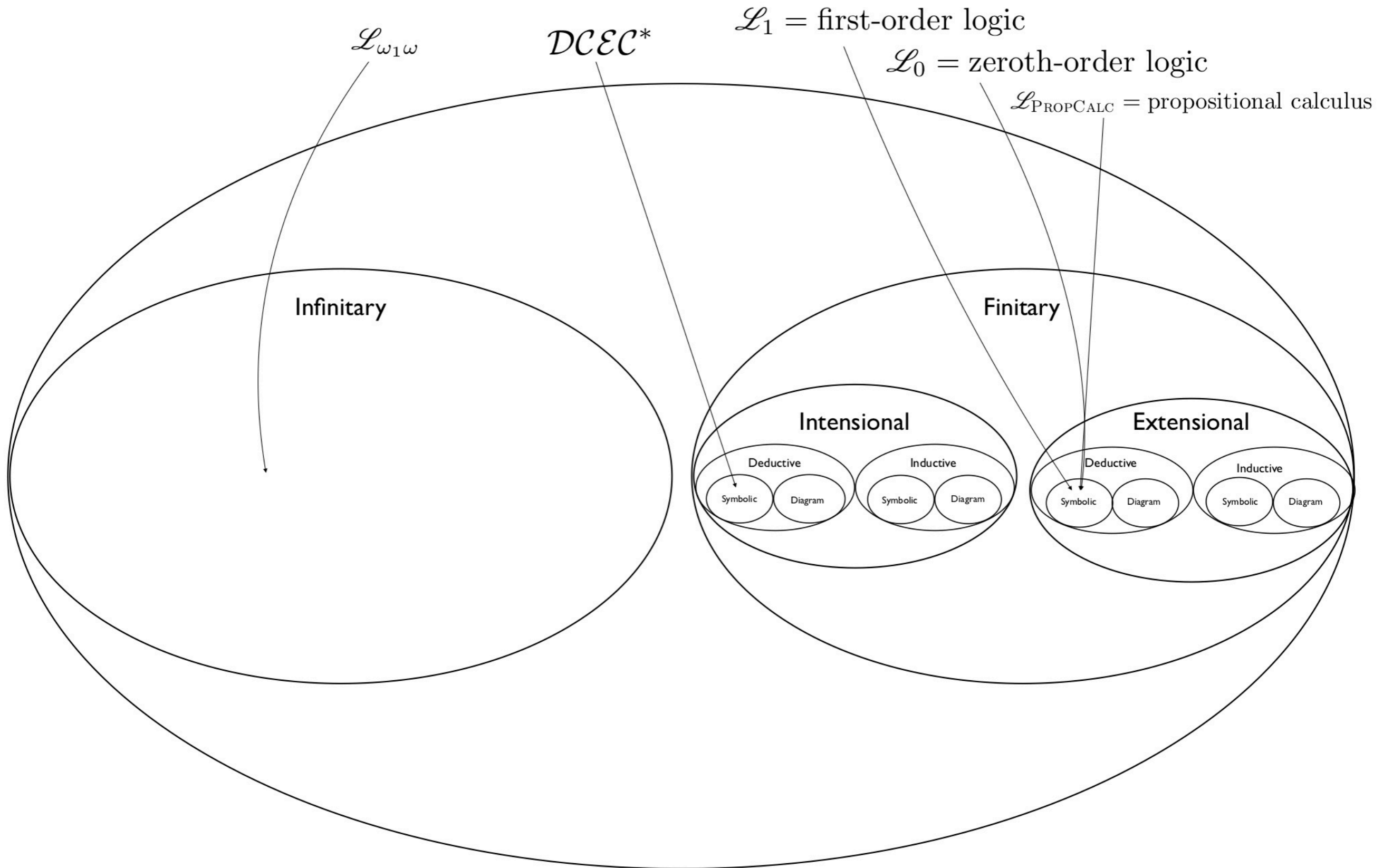
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**ZOL**  $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

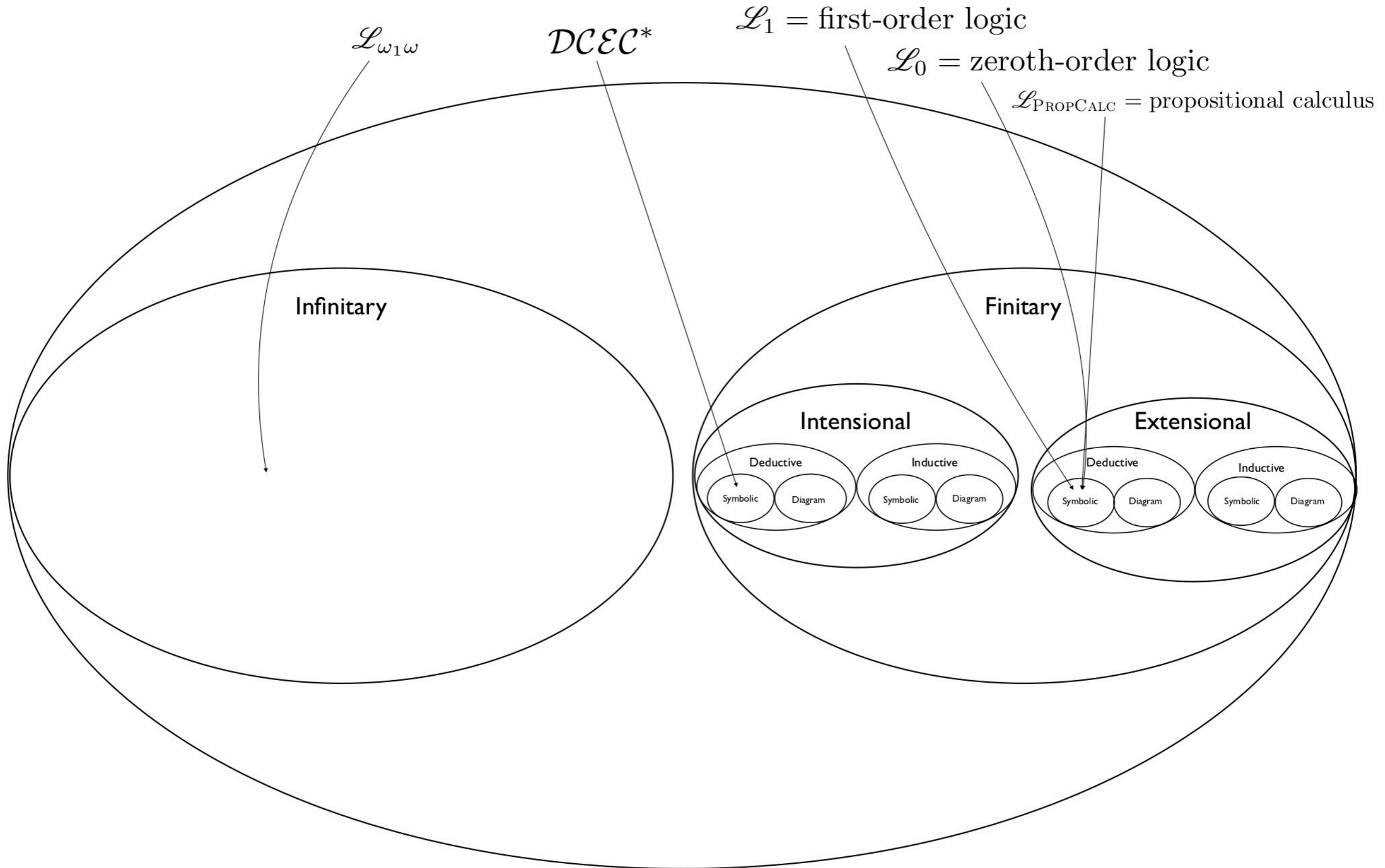
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# The Universe of Logics



$\mathcal{L}_3$   
 $\mathcal{L}_2$

# The Universe of Logics



# The Universe of Logics

$\mathcal{L}_3$

$\mathcal{L}_2$

$\mathcal{L}_{\omega_1\omega}$

$\mathcal{DCEC}^*$

$\mathcal{L}_1 =$  first-order logic

$\mathcal{L}_0 =$  zeroth-order logic

$\mathcal{L}_{\text{PROPCALC}} =$  propositional calculus

Infinitary

Finitary

Intensional

Extensional

Deductive

Inductive

Deductive

Inductive

Symbolic

Diagram

Symbolic

Diagram

Symbolic

Diagram

Symbolic

Diagram

# Climbing the $k$ -order Ladder

⋮

**TOL**  $\exists x, y \exists R, R^2 [R(x) \wedge R(y) \wedge R^2(x, y) \wedge \textit{Positive}(R^2) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_3$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property; and,  $x$   $R^2$ s  $y$ , where  $R^2$  is a positive property.

**SOL**  $\exists x \exists y \exists R [R(x) \wedge R(y) \wedge \textit{Likes}(x, y) \wedge R(\textit{fatherOf}(x))]$

$\mathcal{L}_2$  Things  $x$  and  $y$ , along with the father of  $x$ , share a certain property (and  $x$  likes  $y$ ).

**FOL**  $\exists x [Llama(x) \wedge Llama(b) \wedge \textit{Likes}(x, b) \wedge Llama(\textit{fatherOf}(x))]$

$\mathcal{L}_1$  There's some thing which is a llama and likes  $b$  (which is also a llama), and whose father is a llama too.

**ZOL**  $Llama(a) \wedge Llama(b) \wedge \textit{Likes}(a, b) \wedge Llama(\textit{fatherOf}(a))$

$\mathcal{L}_0$   $a$  is a llama, as is  $b$ ,  $a$  likes  $b$ , and the father of  $a$  is a llama as well.







**“Leibniz was right that Descartes was right that ... : God exists, necessarily.”**

# Gödel's "God Theorem"

		<b>Part I</b>	
	(1')	The absence of a positive property is not positive.	premise
	(1)	The absence of a positive property is not positive; and if a property isn't positive, the absence of that property is.	premise
	(2)	Any property entailed by a positive property is itself positive.	premise
∴	(3)	Every positive property $P$ is possibly possessed by something.	(1), (2)
		<b>Part II</b>	
	(4)	Anything that is God has all positive properties.	definition
	(5)	The property of being God is itself a positive property.	premise
∴	(6)	It's possible that God exists.	(3), (5)
		<b>Part III</b>	
	(7)	Positive properties are necessarily positive.	premise
	(8)	A thing $x$ has an essence $E$ if and only if (i) $E$ is a property $x$ has; and (ii) for any property $P$ that $x$ has, $x$ 's having this property $P$ is necessarily implied by $x$ 's having essence $E$ .	definition
∴	(9)	The property of being God is an essence of any thing that has this property.	(8), (7), (4), (1)
∴	(9)	The property of <i>being God</i> ( $= G$ ) is an essential property of any thing that has $G$ .	(8), (7), (4), (1)
		<b>Part IV</b>	
	(10)	A thing has necessary existence if and only if all the essences that thing has imply that something exists and has all those essences.	definition
	(10)	A thing has necessary existence if and only if all the essential properties that thing has imply that something exists and has all those essential properties.	definition
	(11)	Necessary existence is a positive property.	premise
∴	(12)	Necessarily, God exists.	(6), (9), (10), (11)
			<b>QED</b>

# Gödel's "God Theorem" (formalized, machine verified)

	(1)	$\forall P [Pos(\neg P) \leftrightarrow \neg Pos(P)]$	premise
	(2)	$\forall P_1 \forall P_2 \{Pos(P_1) \wedge \Box \forall x [P_1(x) \rightarrow P_2(x)] \rightarrow Pos(P_2)\}$	premise
$\therefore$	(3)	$\forall P [Pos(P) \rightarrow \Diamond \exists x P(x)]$	theorem
	(4)	$\forall x [G(x) \leftrightarrow \forall P [Pos(P) \rightarrow P(x)]]$	definition
	(5)	$Pos(G)$	premise
$\therefore$	(6)	$\Diamond \exists x G(x)$	corollary
	(7)	$\forall P [Pos(P) \rightarrow \Box Pos(P)]$	premise
	(8)	$\forall x \forall P \{Ess(P, x) \leftrightarrow [P(x) \wedge \forall P' (P'(x) \rightarrow \Box \forall y (P(y) \rightarrow P'(y)))]\}$	definition
$\therefore$	(9)	$\forall x [G(x) \rightarrow Ess(G, x)]$	theorem
	(10)	$\forall x \{NE(x) \leftrightarrow \forall P [Ess(P, x) \rightarrow \Box \exists y P(y)]\}$	definition
	(11)	$Pos(NE)$	premise
$\therefore$	(12)	$\Box \exists x G(x)$ (a.k.a. "Necessarily, God exists.")	theorem

# Gödel's "God Theorem" (formalized, machine verified)

	(1)	$\forall P [Pos(\neg P) \leftrightarrow \neg Pos(P)]$	premise
	(2)	$\forall P_1 \forall P_2 \{Pos(P_1) \wedge \Box \forall x [P_1(x) \rightarrow P_2(x)] \rightarrow Pos(P_2)\}$	premise
$\therefore$	(3)	$\forall P [Pos(P) \rightarrow \Diamond \exists x P(x)]$	theorem
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	(5)	$Pos(G)$	premise
$\therefore$	(6)	$\Diamond \exists x G(x)$	corollary
	(7)	$\forall P [Pos(P) \rightarrow \Box Pos(P)]$	premise
	(8)	$\forall x \forall P \{Ess(P, x) \leftrightarrow [P(x) \wedge \forall P' (P'(x) \rightarrow \Box \forall y (P(y) \rightarrow P'(y)))]\}$	definition
$\therefore$	(9)	$\forall x [G(x) \rightarrow Ess(G, x)]$	theorem
	(10)	$\forall x \{NE(x) \leftrightarrow \forall P [Ess(P, x) \rightarrow \Box \exists y P(y)]\}$	definition
	(11)	$Pos(NE)$	premise
$\therefore$	(12)	$\Box \exists x G(x)$ (a.k.a. "Necessarily, God exists.")	theorem

$$\text{PA}_1 = \text{Z}_1$$

$$\text{A1} \quad \forall x(0 \neq s(x))$$

$$\text{A2} \quad \forall x \forall y (s(x) = s(y) \rightarrow x = y)$$

$$\text{A3} \quad \forall x (x \neq 0 \rightarrow \exists y (x = s(y)))$$

$$\text{A4} \quad \forall x (x + 0 = x)$$

$$\text{A5} \quad \forall x \forall y (x + s(y) = s(x + y))$$

$$\text{A6} \quad \forall x (x \times 0 = 0)$$

$$\text{A7} \quad \forall x \forall y (x \times s(y) = (x \times y) + x)$$

And, every sentence that is the universal closure of an instance of

$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x)))] \rightarrow \forall x \phi(x))$$

where  $\phi(x)$  is open wff with variable  $x$ , and perhaps others, free.

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$$([\phi(0) \wedge \forall x(\phi(x) \rightarrow \phi(s(x)))] \rightarrow \forall x \phi(x)) \quad \infty$$

where  $\phi(x)$  is open wff with variable  $x$ , and perhaps others, free.

$$\mathbf{PA}_2 = \mathbf{Z}_2$$

- A1  $\forall x(0 \neq s(x))$
- A2  $\forall x \forall y(s(x) = s(y) \rightarrow x = y)$
- A3  $\forall x(x \neq 0 \rightarrow \exists y(x = s(y)))$
- A4  $\forall x(x + 0 = x)$
- A5  $\forall x \forall y(x + s(y) = s(x + y))$
- A6  $\forall x(x \times 0 = 0)$
- A7  $\forall x \forall y(x \times s(y) = (x \times y) + x)$

(if we drop any restriction C, we have full second-order arithmetic.)



# $PA_2 = Z_2$

$$A1 \quad \forall x(0 \neq s(x))$$

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$$\text{Induction Axiom} \quad \forall X ([X(0) \wedge \forall x (X(x) \rightarrow X(s(x)))] \rightarrow \forall x X(x))$$

(if we drop any restriction C, we have full second-order arithmetic.)

# $PA_2 = Z_2$

$$A1 \quad \forall x(0 \neq s(x))$$

$$A2 \quad \forall x \forall y (s(x) = s(y) \rightarrow x = y)$$

$$A3 \quad \forall x (x \neq 0 \rightarrow \exists y (x = s(y)))$$

$$A4 \quad \forall x (x + 0 = x)$$

$$A5 \quad \forall x \forall y (x + s(y) = s(x + y))$$

$$A6 \quad \forall x (x \times 0 = 0)$$

$$A7 \quad \forall x \forall y (x \times s(y) = (x \times y) + x)$$

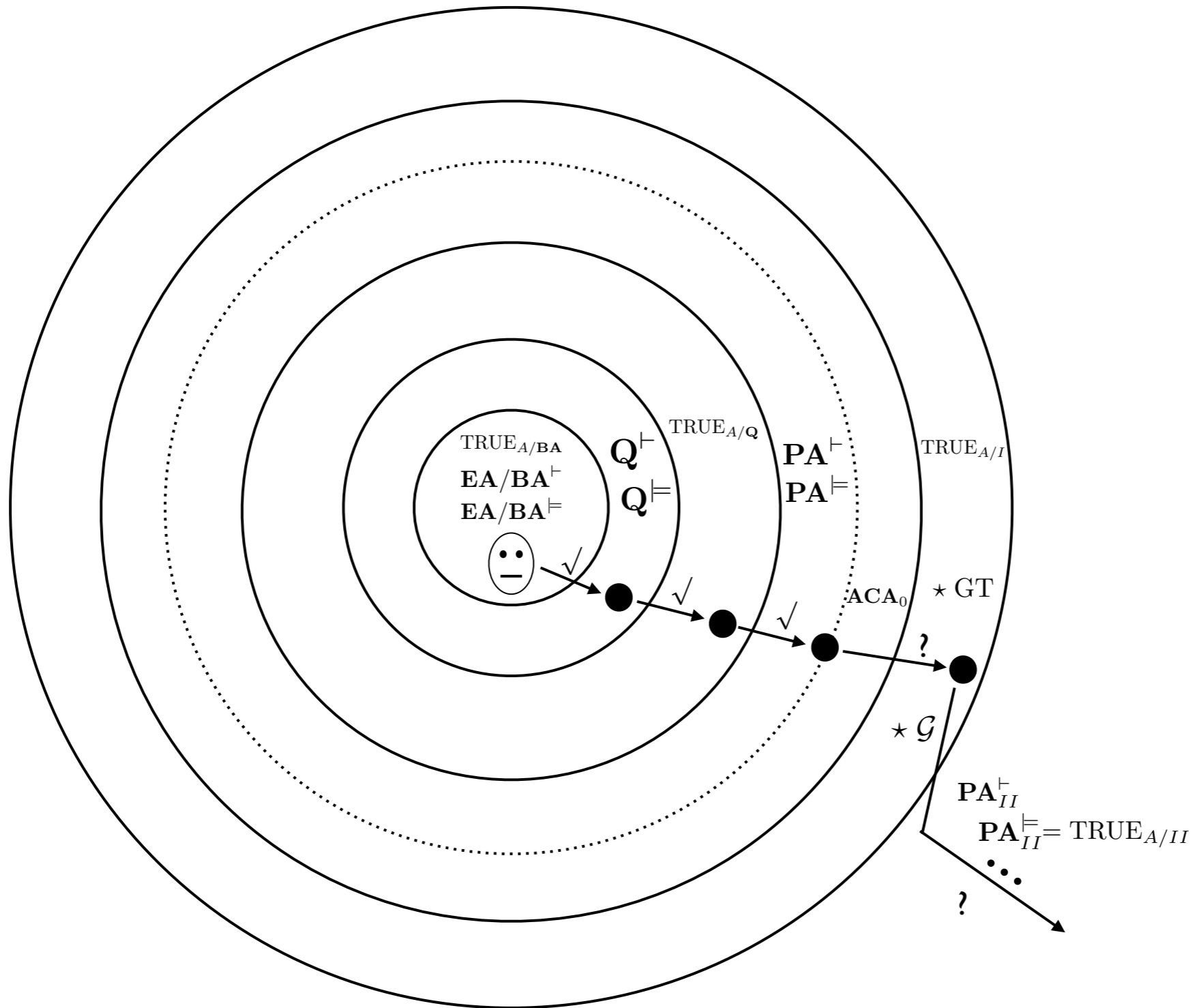
$$\text{Induction Axiom} \quad \forall X ([X(0) \wedge \forall x (X(x) \rightarrow X(s(x)))] \rightarrow \forall x X(x))$$

$$\text{Comprehension Schema} \quad \exists X (\forall x X(x) \leftrightarrow \phi(x)) \quad \text{where } \phi(x) \in \mathcal{C}$$

(if we drop any restriction  $\mathcal{C}$ , we have full second-order arithmetic.)

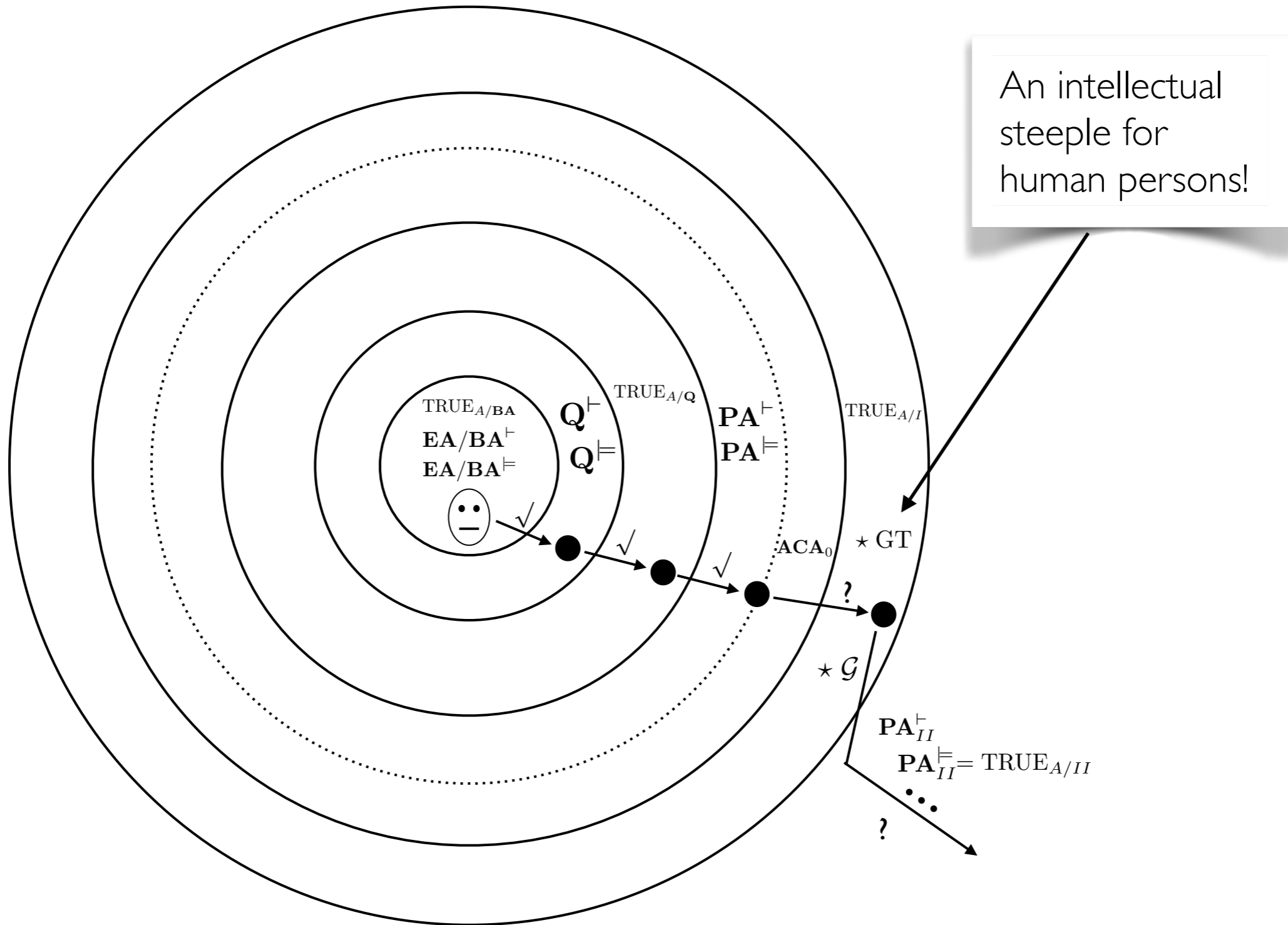
# Astrologic

(Aliens & Angels on the Same "Race Track")



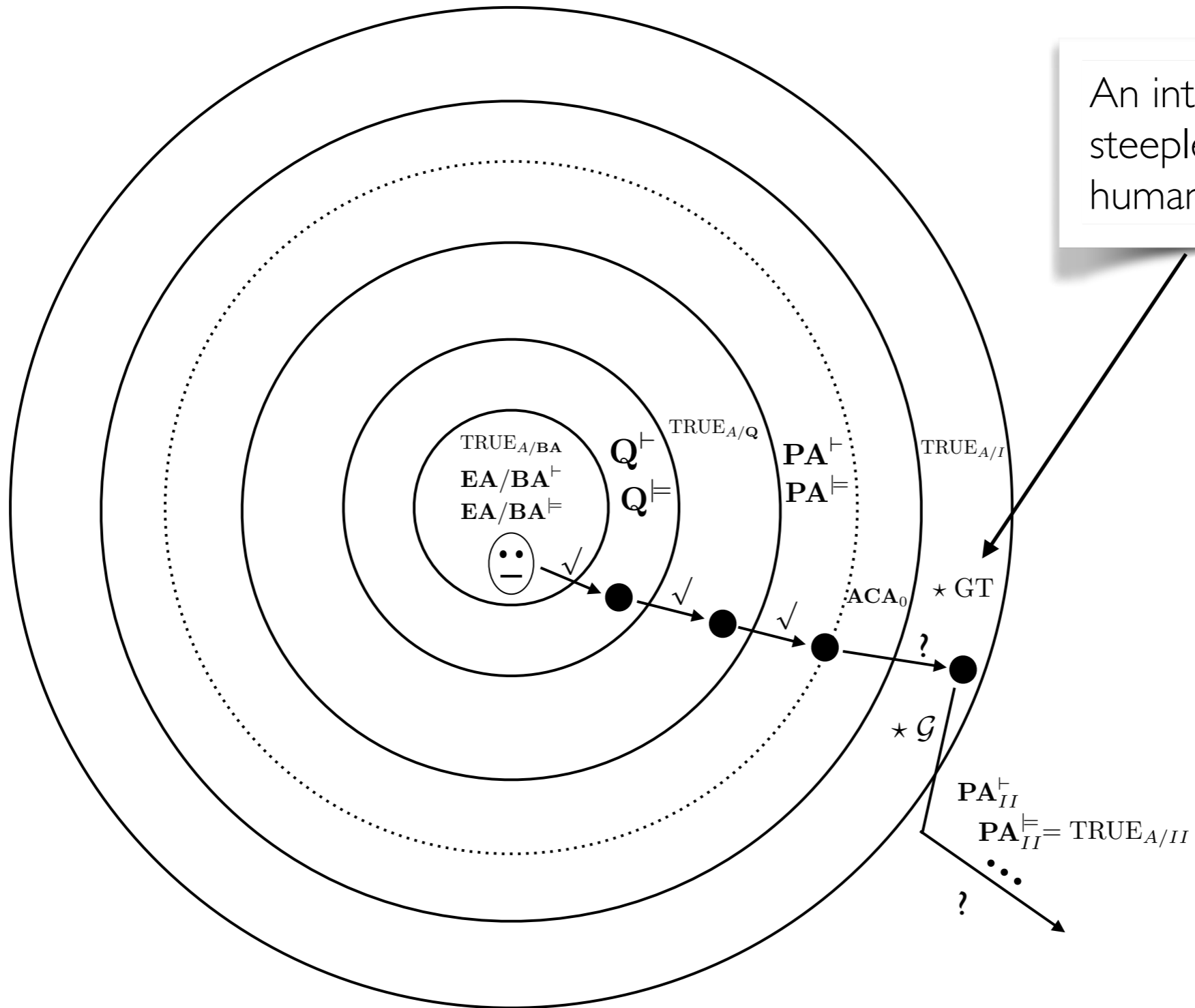
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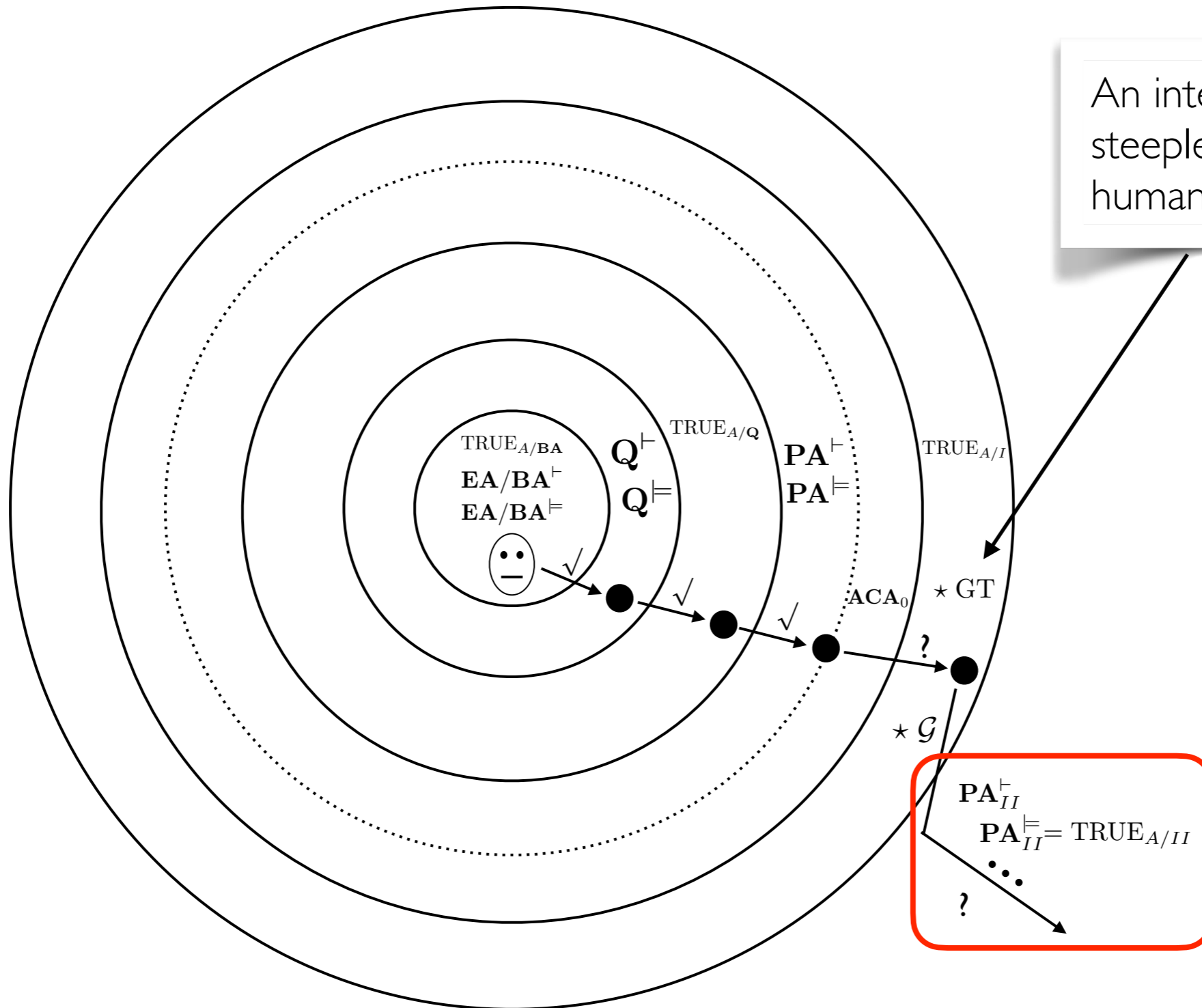


An intellectual  
steeple for  
human persons!



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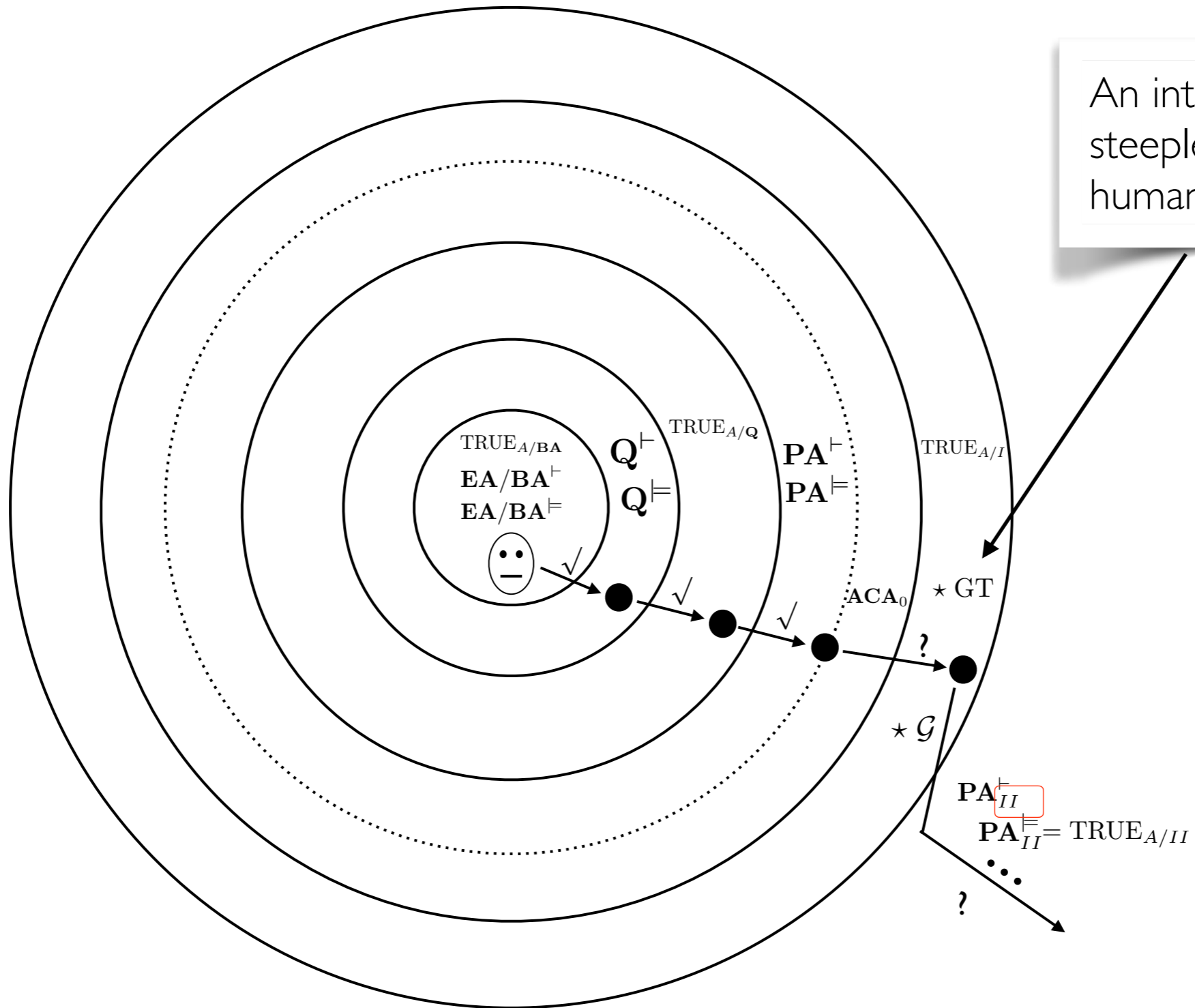


An intellectual steeple for human persons!



# Astrologic

(Aliens & Angels on the Same "Race Track")



An intellectual  
steeple for  
human persons!



New!

# ACA<sub>0</sub>

- A1  $\forall x(0 \neq s(x))$
- A2  $\forall x\forall y(s(x) = s(y) \rightarrow x = y)$
- A3  $\forall x(x \neq 0 \rightarrow \exists y(x = s(y)))$
- A4  $\forall x(x + 0 = x)$
- A5  $\forall x\forall y(x + s(y) = s(x + y))$
- A6  $\forall x(x \times 0 = 0)$
- A7  $\forall x\forall y(x \times s(y) = (x \times y) + x)$

Comprehension

Schema

$\exists X[\forall x X(x) \leftrightarrow \phi(x)]$  where  $\phi(x)$  has no second-order quantifiers



New!

# ACA<sub>0</sub>

$$A1 \quad \forall x(0 \neq s(x))$$

$$A2 \quad \forall x \forall y (s(x) = s(y) \rightarrow x = y)$$

$$A3 \quad \forall x (x \neq 0 \rightarrow \exists y (x = s(y)))$$

$$A4 \quad \forall x (x + 0 = x)$$

$$A5 \quad \forall x \forall y (x + s(y) = s(x + y))$$

$$A6 \quad \forall x (x \times 0 = 0)$$

$$A7 \quad \forall x \forall y (x \times s(y) = (x \times y) + x)$$

$$\text{Induction Axiom} \quad \forall X ([X(0) \wedge \forall x (X(x) \rightarrow X(s(x)))] \rightarrow \forall x X(x))$$

Comprehension

Schema

$$\exists X [\forall x X(x) \leftrightarrow \phi(x)] \quad \text{where } \phi(x) \text{ has no second-order quantifiers}$$



*Slutten*