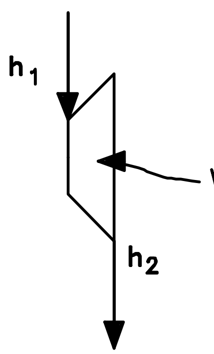
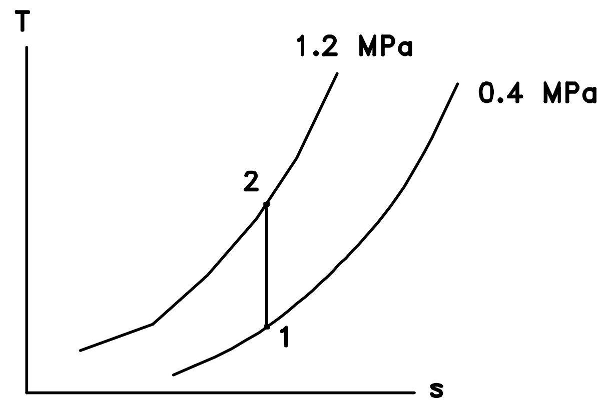
Problem: 10-10

Given: A mixture of 40% argon and 60% hydrogen by volume is compressed isentropically from 40°C at 0.4 MPa to 1.2 MPa

Find: a) Final temperature of the mixture
b) work required

Assume: Perfect gas behaviour





	$X_i \times \tilde{M}_i = (m_i/n)/\tilde{M}_i = Y_i$			
	X_i	$\tilde{M}_i$	m_i/n	Y_i
Component	$\frac{kmole_i}{kmole_{mix}}$	$\frac{kg_i}{kmole_i}$	$\frac{kg_i}{kmole_{mix}}$	$\frac{kg_i}{kmole_{mixt}}$
Ar	0.4	39.94	15.976	0.9296
H_2	0.6	2.016	1.2096	0.0704

where $\tilde{M} = 17.1856\ kg/kmole$
For isentropic compression of a perfect gas

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad ; \quad k = \frac{\hat{c}_{p_{mixt}}}{\hat{c}_{v_{mixt}}}$$

$$\hat{c}_{p_{mixt}} = \sum X_i \hat{c}_{p_i}$$

From Table B.6b

$$\begin{aligned} \hat{c}_{p_{Ar}} &= 20.89\ kJ/kmole \cdot K \\ \hat{c}_{p_{H_2}} &= 28.86\ kJ/kmole \cdot K \end{aligned}$$

$$\begin{aligned} \hat{c}_{p_{mixt}} &= (0.4)(20.89) + (0.6)(28.86) = 25.672\ kJ/kmole \cdot K \\ \hat{c}_{v_{mixt}} &= \hat{c}_{p_{mixt}} - R = 25.672 - 8.314 = 17.358\ kJ/kmole \cdot K \end{aligned}$$

Therefore

$$k_{mixt} = \frac{25.672\ kJ/kmole \cdot K}{17.358\ kJ/kmole \cdot K} = 1.479$$

Therefore

$$\begin{aligned} T_2 &= (313.16\ K)\left(\frac{1.2\ MPa}{0.4\ MPa}\right)^{(1.479-1)/1.479} \\ &= \boxed{447\ K} \\ &= 173.8\ ^\circ C \end{aligned}$$

Energy Balance: assume Δpe and Δke are negligible

$$\begin{aligned} \dot{m}_1(h_1 + pe_1 + ke_1) + \dot{W} + \cancelto{0}{\dot{Q}} &= \dot{m}_2(h_2 + pe_2 + ke_2) \\ \dot{m}_1 &= \dot{m}_2 \end{aligned}$$

$$\begin{aligned} \frac{\dot{W}}{\dot{m}} = W &= h_2 - h_1 = c_p(T_2 - T_1) = \frac{\hat{c}_p}{\tilde{M}}(T_2 - T_1) \\ &= \frac{(25.672\ kJ/kmole \cdot K)}{17.1856\ kJ/kmole \cdot K}(447.0 - 313.2)\ K \\ &= \boxed{199.9\ kJ/kg} \end{aligned}$$

From Eq. 8-15

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT - R \ln\left(\frac{P_2}{P_1}\right)$$

or on a molal basis with constant c_p

$$(\hat{s}_2 - \hat{s}_1)_1 = \hat{c}_{p_i} \ln\left(\frac{T_{2_i}}{T_{1_i}}\right) - \mathcal{R} \ln\left(\frac{P_{2_i}}{P_{1_i}}\right)$$

where the subscript i indicates for each constituent gas.
Therefore

$$(\hat{s}_2 - \hat{s}_1)_{Ar} = \hat{c}_{p_{Ar}} \ln\left(\frac{T_2}{T_1}\right) - \mathcal{R} \ln\left(\frac{P_2}{P_1}\right)$$

Note: $T_{2_{Ar}} = T_{2_{H2}} = T_2$, $T_{1_{Ar}} = T_{1_{H2}} = T_1$, and $P_{2_i} = X_{2_i} P_2$; $P_{1_i} = X_{1_i} P_1$; $X_{2_i} = X_{1_i}$.
Therefore

$$\frac{P_{2_i}}{P_{1_i}} = \frac{P_2}{P_1}$$

Therefore

$$\begin{aligned}(\hat{s}_2 - \hat{s}_1)_{Ar} &= (20.89\ kJ/kmole \cdot K) \ln\left(\frac{447\ K}{313\ K}\right) - (8.314\ kJ/kmole \cdot K) \ln\left(\frac{1.2\ MPa}{0.4\ MPa}\right) \\ &= -1.700\ kJ/kmole \cdot K\end{aligned}$$

Similarly

$$\begin{aligned}(\hat{s}_2 - \hat{s}_1)_{H2} &= (28.86\ kJ/kmole \cdot K) \ln\left(\frac{447\ K}{313\ K}\right) - (8.314\ kJ/kmole \cdot K) \ln\left(\frac{1.2\ MPa}{0.4\ MPa}\right) \\ &= 1.136\ kJ/kmole \cdot K\end{aligned}$$

Check

$$\begin{aligned}(\hat{s}_2 - \hat{s}_1)_{mixt} &= X_{Ar}(\hat{s}_2 - \hat{s}_1)_{Ar}) + X_{H2}(\hat{s}_2 - \hat{s}_1)_{H2}) \\ &= (0.4)(-1.70) + (0.6)(1.136) = 0\end{aligned}$$