# *Fundamentals of Music*

ANICIUS MANLIUS SEVERINUS BOETHIUS

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# *Preface by Series Editor*

The treatise of Boethius translated here has had a remarkable career over the fifteen centuries since he wrote it. Only recently have we come to appreciate how diversely different historical epochs exploited it. Beginning around the ninth century, *De institutione musica* became established as the foundation of Western music theory, and throughout the Middle Ages Boethius remained the authority most revered for music-theoretic matters. But soon the preoccupation with polyphony and the problems of measured music, for which Boethius was of little help, led to its neglect in the training of musicians. With the rise of universities, however, his became the basic text in music studied as one of the liberal arts of the quadrivium.

The fifteenth century was a time of transition for the *fortuna* of this work. On the one hand, Johannes Gallicus recognized it as a translation of Greek writings, hence a font of knowledge about Greek music but foreign to the Western ecclesiastical tradition. On the other hand, Franchino Gaffurio continued to uphold Boethius as the source of the only true doctrine, though he became aware of contradictory views in other ancient sources discovered by the humanists. (This will be evident in the translation and annotations of his *Theorica musice* of 1492, which are being prepared for this series by Walter Kreyszig.) Meanwhile, *De institutione musica* was one of the first musical works printed—in Boethius's *Opera* (Venice, 1491–92). A more critical text was edited in 1546 by Heinrich Glarean, who used Boethius to bolster his theory of the twelve modes in the *Dodekachordon* of 1547.

From the mid-sixteenth century Boethius was often dismissed as lacking relevance to modern musical practice—for example, by Nicola Vicentino, whose *L'Antica musica ridotta alla moderna prattica* (Ancient Music Adapted to Modern Practice) of 1555 has been translated by Maria Rika Maniates and will soon be issued in this series. At the same time, the valorization of Boethius as a transmitter of Greek music theory, to be consulted along with other ancient sources, reached a high point, and two Italian humanists completed vernacular translations that were never published—Giorgio Bartoli in 1579 and Ercole Bottrigari in 1597. The only published translation is much more recent: that of Oskar Paul (Leipzig, 1872), which, like the standard edition by Gottfried Friedlein (Leipzig, 1867), was a product of the German philological movement.

Today we value Boethius for a multiplicity of reasons. We read him to understand Western medieval theory and how it evolved. He is at the center of the theoretical quarrels of the sixteenth century. As Calvin M. Bower has shown, he appears to have handed down in a glossed translation a massive music treatise of the Hellenic period by Nicomachus that otherwise would not have survived, one of the broader windows that we have on the tonal system of the ancient world. Finally, Boethius appeals to the modern theorist, ever searching for consistent schemes and principles of tonal organization, for in the first four books he lays down such a system in great detail.

For good reasons, then, an English translation of Boethius's *De institutione musica* was the first goal of the Music Theory Translation Series when it was inaugurated by David Kraehenbuehl thirty years ago. However, it was only with the completion of Bower's dissertation "Boethius, *The Principles of Music:* An Introduction, Translation, and Commentary" at George Peabody College in 1967 that the project began its path to fulfillment. A draft by another scholar had been abandoned after receiving a careful review by Professor David Hughes of Harvard University, to whom we are indebted for his valuable counsel. My collaboration with Calvin Bower since that time has been a source of mutual instruction in Greek music theory. We have benefited also from the increasing sophistication concerning the Greek theoretical tradition in the English-speaking world, particularly as represented by the work of Thomas J. Mathiesen, whose translation of Aristides Quintilianus's *On Music* is in this series, and of Jon Solomon, whose translation of Claudius Ptolemy's *Harmonics* will soon be ready for publication in it. Professor Bower has earned our thanks for persevering through these years in investigating the sources and manuscript tradition of Boethius's music theory. The insights and information gained in this research enrich every page of his introduction and annotations.

That Boethius's famous treatise has not been published in English before is understandable to us who have worked with the text. Not that Boethius's Latin is so very difficult; major obstacles have been, rather, his terminology, his antiquated mathematics, his often ponderous explanations, and the diagrams. But most of all the challenge has been to express his thoughts in today's English. The translation we have arrived at here has benefited from the experience of my students in the history of medieval theory at Yale University, who read earlier drafts and wrote often revealing commentaries on selected chapters.





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Claude V. Palisca

# *De institutione musica*

by Boethius



# BOOK 1

## 1. *Introduction: Music forms a part of us through nature, and can ennoble or debase character* [178]

Perception through all the senses is so spontaneously and naturally present in certain living creatures that an animal without them cannot be [179]
conceived. But knowledge and clear perception of the senses themselves are not so immediately acquired through inquiry with the mind.[1] For it is indisputable that we use our senses to perceive sensible objects. But what is the nature of these very senses according to which we act? And what is the property of the sensible objects? Answers to these questions do not come easily to anyone; nor can they become clear unless appropriate inquiry has guided one in reflection concerning truth.

Sight, for example, is present in all mortal beings. Whether sight occurs by images coming to the eye or by rays sent out to sensible objects is a point of disagreement among the learned,[2] although this dispute escapes the notice of the ordinary person. Further, when someone sees a triangle

1. In this philosophical prologue, Boethius uses the word *animus* ("mind") to refer to the seat of reasoning, a function which stands in contrast to the functions of the passive senses. But *animus* is likewise used as the seat of feelings and emotions, the seat of courage and morale, and as the moral constitution of the person, all of which are affected by the senses, particularly the sense of hearing. Literary style further obscures precision of meaning, for *mens* is used as well as *animus* to give variety. Words such as "mind," "intellect," "reason," and even "soul" will be used to translate these two terms.

2. This dispute concerning sight was between the Epicureans, who held that vision was the result of images coming to the eye, and the Stoics, who held that vision took place by means of rays emitted from the eye. See Epicurus, in Diogenes Laertius *De vitis philosophorum* 10.49–50 and Lucretius *De rerum natura* 4.26–109; see also Zeno, in Diogenes Laertius 7.157; Gellius, in *Stoicorum veterum fragmenta*, ed. von Arnim, 2.871, and Cicero *Ad Atticum epistulae* 2.3.

or a square, he recognizes easily that which is observed with the eyes. But what is the nature of a triangle or a square? For this you must ask a mathematician.

Now the same can be said with respect to other sensible objects, especially concerning the witness of the ears: the sense of hearing is capable of apprehending sounds in such a way that it not only exercises judgment and identifies their differences, but very often actually finds pleasure if the modes[3] are pleasing and ordered, whereas it is vexed if they are disordered and incoherent.

From this it follows that, since there happen to be four mathematical disciplines,[4] the other three share with music the task of searching for truth; but music is associated not only with speculation but with morality as well. For nothing is more characteristic of human nature than to be soothed by pleasant modes or disturbed by their opposites. This is not peculiar to people in particular endeavors or of particular ages. Indeed, music extends
[180] to every endeavor; moreover, youths, as well as the aged are so naturally attuned to musical modes by a kind of voluntary affection that no age at all is excluded from the charm of sweet song. What Plato rightfully said can likewise be understood: the soul of the universe was joined together according to musical concord.[5] For when we hear what is properly and harmoniously united in sound[6] in conjunction with that which is harmoniously coupled and joined together within us and are attracted to it, then we recognize that we ourselves are put together in its likeness. For likeness attracts, whereas unlikeness disgusts and repels.

From this cause, radical transformations in character also arise. A lascivious disposition takes pleasure in more lascivious modes or is often made soft and corrupted upon hearing them. On the other hand, a rougher spirit finds pleasure in more exciting modes or becomes aroused when it hears them. This is the reason why musical modes were named after certain peoples, such as "Lydian" mode and "Phrygian," for in whatever a partic-

3. "Mode" here is a translation of the Latin *modus,* words with broad spectra of meaning in each language. Properly speaking, the Latin term *modus* implies measurement, and in musical contexts a musical measurement or temperament or tuning. *Modus,* as used by Boethius, does not denote "mode" as the term was used in the later Middle Ages; nor does it have the rather specific connotation of octave-species that it carries today. In this treatise the word has various meanings: it may refer to tuning and temperament (as in the present context) or to the Greek *harmoniae* (as later in this chapter in reference to Plato) or to transposed systems or *tropoi* (as in 4.15–17).

4. The four mathematical disciplines are arithmetic, music, geometry, and astronomy. For an account of the derivation of these disciplines from the kinds of quantities, see 2.3. The four mathematical disciplines are defined as the *quadrivium* in Boethius's *Arith.* 1.1 (Masi trans., p. 73); they represent a fourfold path which leads the mind away from sense perception to abstract knowledge. As such, they are propaedeutic to the study of philosophy.

5. Plato *Timaeus* 35B.

6. Friedlein ed., 180.7: *quod in senis* should read *quod in sonis*. The *e* of *senis* is an obvious typographical error, confirmed by all manuscripts.



ular people finds pleasure, by that same name the mode itself is designated. A people finds pleasure in modes because of likeness to its own character, for it is not possible for gentle things to be joined with or find pleasure in rough things, nor rough things in gentle.[7] Rather, as has been said, similitude brings about love and pleasure.[8] Thus Plato holds that the greatest care should be exercised lest something be altered in music of good character. He states that there is no greater ruin of morals in a republic than the gradual perversion of chaste and temperate music, for the minds of those listening at first acquiesce. Then they gradually submit, preserving no trace of honesty or justice—whether lascivious modes bring something immodest into the dispositions of the people or rougher ones implant something warlike and savage.[9]

Indeed no path to the mind is as open for instruction as the sense of [181]
hearing. Thus, when rhythms and modes reach an intellect through the ears, they doubtless affect and reshape that mind according to their particular character. This again can be perceived in various peoples; those who are rougher delight in the rather uncultivated modes of the Getae,[10] whereas those who are more gentle delight in more moderate modes—although in these times this hardly ever occurs.[11] Since the human race has become lascivious and impressionable, it is taken up totally by representational and theatrical modes.[12] Music was indeed chaste and modest when it was performed on simpler instruments.[13] But since it has been squandered in var-

7. The terms *mollis* and *durus* (translated from μαλακός and σκληπός) are technical terms in ancient theory. At the more general level of discussion, *mollis* describes music that is soft, tender, and effeminate in character, whereas *durus* describes music that is firm, austere, and masculine. At a more technical level, *mollis* denotes intervals, particularly semitones and quarter-tones, that are small and compact, whereas *durus* denotes intervals, particularly tones, that are broader and more expansive (see 1.21). For a broad survey of these terms in Western music, see Carl Dahlhaus, "Die Termini Dur und Moll," *Archiv für Musikwissenschaft* 12 (1955): 280–96.

8. Plato *Symposium* 187.

9. Plato *Republic* 424.

10. The Getae were the most northerly branch of the Thracian people. The province of Getae is described in Strabo *Geographica* 7.295, 304. This reference to their "uncultivated modes" is without precedent in ancient sources.

11. Boethius (or his source) may not be commenting directly on the actual music of his time in this passage; he is repeating a literary topos which was popular in musical writings of a more philosophical bent. For other examples, see Athenaeus *Deipnosophistae* 14.631E–632B and Plutarch *De musica* 1136B. This does not imply that the literary topos did not apply to music in the second century (the century of Nicomachus) or the sixth century (that of Boethius).

12. Complaints such as this are generally associated with the rise of "popular" theatrical innovations in music, such as those of Melanippides, Cinesias, Phrynis, and Timotheus, all musical innovators of the fifth century B.C. Concerning the role of audiences in determining musical tastes and values, see Athenaeus *Deipnosophistae* 14.631E–632A.

13. Idealization of an ancient simplicity is another topos of ancient theory; see Plutarch *De musica* 1135D for a statement very similar to this one. For a "history" of music in its ideal (i.e., simple) state, see 1.20.

ious, promiscuous ways, it has lost its measure of dignity and virtue; and, having almost fallen into a state of disgrace, it preserves nothing of its ancient splendor. Hence Plato prescribes that children not be trained in all modes, but only in those which are vigorous and simple.[14] This rule must be most carefully adhered to, for if henceforth anything should somehow be altered ever so slightly—albeit not noticed immediately—after some time it will make a considerable difference and will sink through the ears into one's character. Plato holds music of the highest moral character, modestly composed, to be a great guardian of the republic; thus it should be temperate, simple, and masculine, rather than effeminate, violent, or fickle.[15]

The Lacedaemonians guarded this tradition with greatest care and at considerable expense when the Cretan Thaletas of Gortyne[16] imbued children with the discipline of musical knowledge. This, in fact, was the custom
[182] among ancient peoples and persisted for a considerable time. Thus, when Timotheus of Miletus[17] added one string to those that were already established, thereby making the music more capricious, a decree was drafted to expel him from Laconica. I have inserted this decree concerning him in the original Greek; since the inscription is in the Spartan language, the letter C (sigma) is changed to P (rho).

ΕΠΕΙΔΗ ΤΙΜΟΘΕΟΡ Ο ΜΙΛΗCΙΟΡ
ΠΑΡΑΓΙΝΟΜΕΝΟΡ ΕΤΤΑΝ ΑΜΕΤΕΡΑΝ ΠΟΛΙΝ,
ΤΑΝ ΠΑΛΑΙΑΝ ΜΩΑΝ ΑΤΙΜΑCΔΕ
ΚΑΙ ΤΑΝ ΔΙΑ ΤΑΝ ΕΠΤΑ ΧΟΡΔΑΝ ΚΙΘΑΡΙΖΙΝ
ΑΠΟCΤΡΕΦΟΜΕΝΟΡ,
ΠΟΛΥΦΩΝΙΑΝ ΕΙCΑΓΩΝ ΛΥΜΑΙΝΕΤΑΙ ΤΑΡ ΑΚΟΑΡ ΤΩΝ ΝΕΩΝ
ΔΙΑ ΤΕ ΤΑΡ ΠΟΛΥΧΟΡΔΙΑΡ ΚΑΙ ΤΑΡ ΚΕΝΟΤΑΤΟΡ ΤΩ ΜΕΛΕΟΡ,
ΑΓΕΝΝΗ ΚΑΙ ΠΟΙΚΙΛΑΝ ΑΝΤΙ ΑΠΛΟΑΡ ΚΑΙ ΤΕΤΑΓΜΕΝΑΡ
ΑΜΦΙΕΝΝΥΤΑΙ ΤΑΝ ΜΩΑΝ ΕΠΙ ΧΡΩΜΑΤΟΡ CΥΝΕΙCΤΑΜΕΝΟΡ
ΤΑΝ ΤΩ ΜΕΛΙΟΡ ΔΙΑCΚΕΥΑΝ ΑΝΤΙ ΤΑΡ ΕΝΑΡΜΟΝΙΩ
ΠΟΤΤΑΝ ΑΝΤΙCΤΡΟΦΟΝ ΑΜΟΙΒΑΝ,
[183] ΠΑΡΑΚΛΗΘΕΙC ΔΕ ΚΑΙ ΕΝ ΤΟΝ ΑΓΩΝΑ ΤΑΡ ΕΛΕΥCΙΝΙΑΡ ΔΑΜΑΤΡΟΡ

14. Plato *Republic* 399C.

15. Plato *Republic* 399, 410–11.

16. In Plutarch *De musica* 1134B-C Thaletas of Gortyne on Crete (fl. seventh century B.C.) is said to have been one of the musicians responsible for the second establishment of music in Sparta. He is also said to have appeared in Lacedaemonia on the advice of a Delphic oracle (Plutarch *De musica* 1146C). Concerning the Spartan character of his music, see Plutarch *Lycurgus* 4. See also *Lyra Graeca*, a collection of fragments of Greek lyrical texts edited and translated by J. M. Edmonds, 2d, enlarged ed. (Cambridge, Mass., and London, 1928), vol. 1, pp. 35–37.

17. Timotheus of Miletus (fifth to fourth centuries B.C.) is the most infamous musician of antiquity because of his innovations; see Pausanius *Descriptio Graeciae* 3.12 and Plutarch *De musica* 1135C-D, 1141C–1142B, 1142C. Athenaeus *Deipnosophistae* 634E presents a somewhat different version of Timotheus's expulsion from Sparta, wherein he vindicates himself through rather ingenious means. See *Lyra Graeca*, vol. 3, pp. 280–333, and Timotheos Milesius *Die Perser*, ed. U. von Wilamowitz-Möllendorff (Leipzig, 1903), pp. 69–80.





ΑΠΡΕΠΗ ΔΙΕCΚΕΥΑCΑΤΟ ΤΑΝ ΤΩ ΜΥΘΩ ΔΙΑCΚΕΥΑΝ
ΤΑΝ ΤΑΡ CΕΜΕΛΑΡ ΩΔΙΝΑΡ
ΟΥΚ ΕΝΔΙΚΑ ΤΩΡ ΝΕΩΡ ΔΙΔΑΚΚΗ:
ΔΕΔΟΧΘΑΙ ΦΑ ΠΕΡΙ ΤΟΥΤΩΝ
ΤΩΡ ΒΑCΙΛΕΑΡ ΚΑΙ ΤΩΡ ΕΦΟΡΩΡ ΜΕΜΨΑΤΤΑΙ ΤΙΜΟΘΕΟΝ:
ΕΠΑΝΑΓΚΑΖΑΙ ΔΕ ΚΑΙ ΤΑΝ ΕΝΔΕΚΑ ΧΟΡΔΑΝ
ΕΚΤΑΜΩΝ ΓΑΡ ΤΑΡ ΠΕΡΙΤΤΑΡ
ΥΠΟΛΙΠΟΜΕΝΩΝ ΤΑΡ ΕΠΤΑ.
ΟΠΩΡ ΕΚΑCΤΟΡ ΤΟ ΤΑΡ ΠΟΛΙΟΡ ΒΑΡΟΡ ΟΡΩΝ
ΕΥΛΑΒΗΤΑΙ ΕΤΤΑΝ CΠΑΡΤΑΝ
ΕΠΙΦΕΡΗΝ ΤΙ ΤΩΝ ΜΗ ΚΑΛΩΝ ΕΟΝΤΩΝ
ΜΗ ΠΟΤΕ ΤΑΡΑΡΕΤΑΙ ΚΛΕΟΡ ΑΓΟΝΩΝ.[18]

This decree sets forth the following: The Spartans were indignant with [184] Timotheus of Miletus, because, by introducing a capricious music to the minds of the children, he had thwarted those whom he had accepted to teach and had steered them away from the moderation of virtue, and because he had changed the harmony,[19] which he had found temperate, into the chromatic genus, which is overrefined. Indeed, the Spartans were so attentive to music that they thought it even took possession of minds.

It is common knowledge that song has many times calmed rages, and that it has often worked great wonders on the affections of bodies or minds. Who does not know that Pythagoras, by performing a spondee, restored a [185] drunk adolescent of Taormina incited by the sound of the Phrygian mode[20] to a calmer and more composed state? One night, when a whore was closeted in the house of a rival, this frenzied youth wanted to set fire to the house. Pythagoras, being a night owl, was contemplating the courses of the heavens (as was his custom) when he learned that this youth, incited by the sound of the Phrygian mode, would not desist from his action in response to the many warnings of his friends; he ordered that the mode be changed, thereby tempering the disposition of the frenzied youth to a state of absolute calm.[21] Marcus Tullius relates the story in his book, *De consiliis*

18. The text of this decree, as printed here, is based on manuscripts *K*, *M*, *P*, and *Q*. Appendix 2 provides notes on this text, a critical apparatus, and an English translation by T. Burgess.

19. *Armonia* (ἁρμονία), like *modus* (see above, n. 3) is a term with both general and specific denotations: it can refer, like *modus*, to the general arrangement of pitches in a system (particularly an octave), or it can denote the seven octave-species which form the basis of the seven tonoi. In certain contexts (e.g., Aristoxenus *Harmonica* 1.2, 23, and Boethius 1.15 of the present treatise) the word can even refer to the enharmonic genus. In the present context Timotheus is reprimanded for changing the structure of pitches in the octave, thereby changing the genus from enharmonic to chromatic.

20. Friedlein 185.1–2: *subphrygii modi sono incitatum* should read *sub phrygii modi sono incitatum*. The appropriate reading is clear, as is the grammar, in the manuscripts and in earlier editions; several lines later the mode is again identified as "Phrygian" (185.6). To read this as a latinization of "hypophrygian" (Pizzani, "Fonti," pp. 162–63) is to legitimate a misprint in Friedlein.

21. For the story of the frenzied youth, see Quintilian *Institutio oratoria* 1.10.32, Sextus

*suis*, but somewhat differently, as follows: "But I would compare something trivial with something important, since I am drawn by a likeness between them. When drunken youths, incited by the music of auloi,[22] as happens, were about to break in the door of a chaste woman, it is said that Pythagoras admonished the aulete to perform a spondee. When this was done, the severity of the rhythms and the seriousness of the performer caused the raging fury of the youths to subside."[23]

To cite some similar examples briefly, Terpander[24] and Arion of Methymna[25] saved the citizens of Lesbos and Ionia from very serious illness through the assistance of song. Moreover, by means of modes, Ismenias the Theban[26] is said to have driven away all the distresses of many Boeotians suffering the torments of sciatica. Similarly it is said that Empedocles altered the mode of music-making when an infuriated youth attacked one of his guests with a sword because this guest had condemned the youth's father by bringing an accusation. Thereby Empedocles tempered the wrath of the youth.[27]

This capacity of the musical discipline had become so familiar in the doctrines of ancient philosophy that the Pythagoreans, when they wanted

Empiricus *Adversus musicos* 6.8, Aristotle Elias, *Prolegomena philosophiae* 2 (ed. Adolf Busse, *Commentaria in Aristotelem Graeca*, vol. 18 [Berlin, 1890], p. 31), and Ammonius *In Porphyrii isagogen sive v voces* (ed. Adolf Busse, *Commentaria in Aristotelem Graeca*, vol. 4, pt. 3 [Berlin, 1891], p. 130).

22. The term *tibia* was used by Latin authors from the period of Ennius *Annales* 299 (293–169 B.C.) to translate the Greek αὐλός, the principal wind instrument of antiquity. There seems to be no transliteration of *aulos*, as the name of an instrument, into Latin. Of the six appearances of the word *tibia* in Boethius's text, two are in quotations of other authors: the present instance from Cicero and the subsequent quotation of Statius concerning funeral processions. In this translation the word *tibia* will be translated as "aulos," and the performer will be called "aulete." See below, 1.34 and n. 138.

23. This same quotation from Cicero, with minor variants, is also found in Augustine *Contra Julianum* 5.5.23. Cicero's *De consiliis suis*, or Ἀνέκδοτα, is no longer extant; see M. Schanz and C. Hosius, *Geschichte der römischer Literatur bis zum Gesetzgebungswerk des Kaisers Justinian*, vol. 1 (Munich, 1927), p. 531.

24. Terpander (of Antissa on the isle of Lesbos), who flourished between 700 and 650 B.C., is probably the most revered musician of antiquity. See *Lyra Graeca*, vol. 1, pp. 16–33. See also below, 1.20 and n. 104.

25. Arion of Methymna (Lesbos) was approximately contemporary with Terpander. This account of Terpander and Arion using music curatively seems to be unique. Concerning Arion, his music, and the dolphin legend, see Strabo *Geographica* 13.2.3, Herodotus *Historia* 1.23, and Hyginus *Fabula* 194. See also *Lyra Graeca*, vol. 1, pp. 136–39.

26. Ismenias of Thebes (fl. third century B.C.) is identified as an aulete in Plutarch *Pericles* 1.152.5 and idem, *Non posse suaviter vivi* 1095F, as well as in Lucian *Pseudologista* 5, although none of these passages treats him with much dignity or respect. Concerning the use of Phrygian, or Thracian, music to cure sciatica, see Athenaeus *Deipnosophistae* 624B. The coupling of Ismenias and the cure is found in no other source from antiquity.

27. This account of Empedocles' curative use of music is found in no other source. Concerning Empedocles' reputation as a physician, see Diogenes Laertius *De vitis philosophorum* 8.60–62, 77.





to relieve their daily concerns in sleep, employed certain melodies so that a mild and quiet slumber would fall upon them.[28] Likewise upon awakening, [186]
they purged the stupor and confusion of sleep with certain other modes, for they knew that the whole structure of our soul and body has been joined by means of musical coalescence. For just as one's physical state affects feeling, so also the pulses of the heart are increased by disturbed states of mind. Democritus is said to have related this to the physician Hippocrates, who came to treat Democritus when he was being held in custody by his fellow citizens because they thought he was mad.[29]

But to what purpose is all this? So that there can be no doubt that the order of our soul and body seems to be related somehow through those same ratios[30] by which subsequent argument will demonstrate sets of pitches,[31] suitable for melody, are joined together and united. Hence it happens that a sweet tune delights even infants, while a harsh and rough one will interrupt the pleasure of listening. Certainly people of every age

28. Concerning the Pythagoreans' use of music for sleep, see Plutarch *Isis et Osiris* 31.384, Quintilian *Institutio oratoria* 9.4.12, and Censorinus *De die natali* 12.4.

29. Democritus of Abdera and Hippocrates of Cos both flourished in the late fifth century B.C. Both were known in antiquity for their powers as physicians. Concerning the relationships among Democritus, Hippocrates, and other parties, see documents concerning Democritus in Hippocratic *Corpus*, ed. E. Littré (Paris, 1839), vol. 9, pp. 321–99; concerning authenticity of the accounts, see L. Edelstein, "Hippokrates," in A. Pauly, G. Wissowa, and W. Kroll, *Real-Encyclopädie der klassischen Altertumswissenschaft* (1893–), Supp. 6 (1935), pp. 1290–1345, esp. pp. 1303–05. A brief account of the meeting between these two is recorded by Diogenes Laertius *De vitis philosophorum* 9.42, as well as other keen insights regarding Democritus. The present account concerning the effect of feelings on the pulse seems to be unique.

30. "Ratio" is a translation of the Latin *proportio*, which Boethius translated from the Greek λόγος. Boethius uses the term frequently as technical vocabulary denoting a mathematical ratio. For definition of the term, see book 2.12 and n. 34. In the present context, Boethius is speaking in analogical terms; but ultimately, the text is Pythagorean, and Pythagoreans would argue that the reality of the relation between body and soul is a ratio of numbers.

31. The verb *modulor* ("to modulate") and its substantive form *modulatio* ("modulation") present problems similar to those associated with the word *modus* (see above, n. 3). In fact, *modulor* is derived from *modus* and means the application of measure (*modus*) to the most basic elements of music—pitch and time. Hence the classical Latin definition of music: *musica est scientia bene modulandi* (e.g., Censorinus *De die natali* 10.10). To translate the verb *modulor* as "to sing" or "to play" and the noun *modulatio* as "melody" is to miss the *quantitative* denotation of the words—viz., the application of ratios to pitch and time. To translate the verb as "measure" and the noun as "measurement" seems too abstract, for these terms lack any artistic, aesthetic connotation. Yet to use the English "modulate" and "modulation" is to bring a technical, musical notion into the translation which does not apply—viz., the notion of "changing keys." I will attempt to capture the quantitative and systematic connotation of *modulatio* with such phrases as "arrangement of pitches" or "set of pitches." The verb *modulor* is more difficult. At times it must be translated as "measure" (e.g., at 1.12); at other times it must be given a less quantitative translation. In the next paragraph, e.g., *ipsos modulantur dolentes* (literally, "they modulate their very lamentations") is rendered "they turn their very lamentations into music." See Christoph von Blumröder, "Modulatio / Modulation," in *Handwörterbuch der musikalischen Terminologie*, ed. H. Eggebrecht (1983).

and sex experience this; although they may differ in their actions, they are nevertheless united as one in the pleasure of music.

Why is it that mourners, even though in tears, turn their very lamentations into music? This is most characteristic of women, as though the cause for weeping might be made sweeter through song. Among the ancients it was even the custom that music of the aulos led the procession of mourners, as these lines of Papinius Statius testify:

> The aulos, whose practice it is to lead forth the
> youthful dead,
> Utters its mournful note from a curved horn.[32]

Someone who cannot sing well will nevertheless sing something to himself, not because the song that he sings affects him with particular satisfaction, but because those who express a kind of inborn sweetness from the soul—regardless of how it is expressed—find pleasure. Is it not equally evident that the passions of those fighting in battle are roused by the call of trumpets? If it is true that fury and wrath can be brought forth
[187] out of a peaceful state of mind, there is no doubt that a more temperate mode can calm the wrath or excessive desire of a troubled mind. How does it come about that when someone voluntarily listens to a song with ears and mind, he is also involuntarily turned toward it in such a way that his body responds with motions somehow similar to the song heard? How does it happen that the mind itself, solely by means of memory, picks out some melody previously heard?

From all these accounts it appears beyond doubt that music is so naturally united with us that we cannot be free from it even if we so desired. For this reason the power of the intellect ought to be summoned, so that this art, innate through nature, may also be mastered, comprehended through knowledge. For just as in seeing it does not suffice for the learned to perceive colors and forms without also searching out their properties, so it does not suffice for musicians[33] to find pleasure in melodies without also coming to know how they are structured internally by means of ratio of pitches.[34]

32. Statius *Thebias* 6.120–21.

33. The Latin *musicus* carries more weight than the English "musician." For a definition of "musician," see 1.34.

34. "Pitch" is a translation of the Latin *vox*, Boethius's rendering of the Greek φθόγγος (see 1.12). *Vox* is a term with a wide spectrum of meanings, even in musical contexts, for it can mean the human voice, sound in general, or musical pitch. The most common usage in the treatise is the third by far. For the Pythagorean, pitch is an expression of quantity in music (see 1.3), and knowing the ratios of pitches is the goal of the first four books of this treatise.





## 2. *There are three kinds of music, and concerning the influence of music*

Thus, at the outset, it seems proper to tell someone examining music what we shall discover about the number of kinds of music recognized by those schooled in it. There are three: the first is cosmic, whereas the second is human; the third is that which rests in certain instruments, such as the kithara or the aulos or other instruments which serve melody.

The first kind, the cosmic, is discernible especially in those things which are observed in heaven itself or in the combination of elements or the diversity of seasons.[35] For how can it happen that so swift a heavenly machine moves on a mute and silent course? Although that sound does not penetrate our ears—which necessarily happens for many reasons[36]—it is nevertheless impossible that such extremely fast motion of such large bodies should produce absolutely no sound, especially since the courses of the [188] stars are joined by such harmonious union that nothing so perfectly united, nothing so perfectly fitted together, can be realized. For some orbits are borne higher, others lower; and they all revolve with such equal energy that a fixed order of their courses is reckoned through their diverse inequalities. For that reason, a fixed sequence of modulation cannot be separated from this celestial revolution.

If a certain harmony did not join the diversities and opposing forces of the four elements, how would it be possible that they could unite in one mass and contrivance?[37] But all this diversity gives birth to variety of both seasons and fruits in such a way that it nevertheless imparts one structure to the year. Whence if you imagine one of these things which supply such diversity taken away, then all things would seem to fall apart and, so to speak, preserve none of their consonance. And just as, on the one hand, adjustment of pitch in lower strings is such that lowness does not descend into silence, while, on the other hand, adjustment of sharpness in higher strings is carefully monitored lest the excessively stretched strings break because of the tenuity of pitch, but the whole corpus of pitches is coherent and harmonious with itself, in the same way we discern in cosmic music

35. Concerning cosmic music in general, see Plato *Timaeus* 35–36; idem, *Laws* 889B-C. Concerning the harmony of the heavens, see Pliny *Naturalis historia* 2.22(20).84, Cicero *De re publica* 6.18.18; Plutarch *De musica* 1147, Nicomachus *Enchiridion* 3, Censorinus *De die natali* 12, Macrobius *In somnium Scipionis* 2.1.2 and 6.1–6, and Ptolemy *Harmonica* 3.10–16, 104–11. Concerning the harmony of the elements, see Plato *Symposium* 188A, idem, *Timaeus* 32C; and Macrobius *In somnium Scipionis* 1.5.25. Concerning harmony of the seasons, see Plato *Symposium* 188A.

36. See, e.g., Cicero *De re publica* 6.18.19 and Macrobius *In somnium Scipionis* 2.4.14.

37. There seems to be a lacuna at this point in the text. Given the opening question of the paragraph, a discussion of harmony and the diversity of the elements should follow; but the text skips abruptly to a discussion of the diversity of the seasons. Thus a development of the harmony of the elements and an introduction to the harmony of the seasons are missing. Some scribe may have jumped from one "diversity" to another.

that nothing can be so excessive that it destroys something else by its own intemperance. Everything is such that it either bears its own fruit or aids others in bearing theirs. For what winter confines, spring releases, summer heats, and autumn ripens, and the seasons in turn either bring forth their own fruit or give aid to others in bringing forth their own. But these things ought to be discussed later more studiously.[38]

Whoever penetrates into his own self perceives human music.[39] For what unites the incorporeal nature of reason with the body if not a certain harmony and, as it were, a careful tuning of low and high pitches as though
[189] producing one consonance? What other than this unites the parts of the soul,[40] which, according to Aristotle, is composed of the rational and the irrational?[41] What is it that intermingles the elements of the body or holds together the parts of the body in an established order? I shall also speak about these things later.[42]

The third kind of music is that which is said to rest in various instruments. This music is governed either by tension, as in strings, or by breath, as in the aulos or those instruments activated by water, or by a certain percussion, as in those which are cast in concave brass, and various sounds are produced from these.[43]

It seems, then, that we ought to discuss the music of instruments first in this work.[44] This is enough of a preamble; now the basic principles of music must be discussed.

38. Boethius never returns to this topic, in fact.

39. Concerning human music, see Plato *Phaedo* 86; idem, *Laws* 653B; idem, *Republic* 442–43; Cicero *Tusculanae disputationes* 1.10; Plutarch *De musica* 1140B; and Ptolemy *Harmonica* 3.5–7 (95–100).

40. This is the only place where Boethius uses the term *anima* to refer to the soul. See above, n. 1, for use of *animus*.

41. The text here refers only to Aristotle's division of the soul into a rational and an irrational part (*Nicomachean Ethics* 1.13.1102–03); it should not be understood as arguing that Aristotle thought harmony to be a principle in unifying these two parts. Significantly, Boethius does not cite *De anima* 432A-B, where Aristotle finds the division "rational-irrational" less than satisfactory.

42. In fact, Boethius never returns to this topic.

43. Concerning the division of instrumental music into "strings, winds, and percussion," see Cassiodorus *Institutiones* 2.5.6.

44. By the "music of instruments" Boethius understands the mathematical principles that determine the structure of classical systems. The remainder of book 1 presents all the basic elements of the discipline; books 2 and 3 then demonstrate abstract mathematical principles. The "music of instruments" finds its ultimate end in the division of the monochord in book 4. Book 5 (based on book 1 of Ptolemy's *Harmonica*) may be considered a continuation of the "music of instruments," for it presents and evaluates further mathematical principles for division of the system. The implication is clear, however, that "music of instruments" is not the only topic to be discussed by this text. Cosmic music and human music should be discussed for the plan set forth in this chapter to be complete (see above, nn. 38 and 42). For discussion of the complete scope of Boethius's work, see Introduction.





## 3. *Concerning pitches and concerning the basic principles of music*

Consonance, which governs all setting out of pitches, cannot be made without sound; sound is not produced without some pulsation and percussion; and pulsation and percussion cannot exist by any means unless motion precedes them. If all things were immobile, one thing could not run into another, so one thing should not be moved by another; but if all things remained still and motion was absent, it would be a necessary consequence that no sound would be made. For this reason, sound is defined as a percussion of air remaining undissolved all the way to the hearing.[45]

Some motions are faster, others slower; some motions are less frequent, others more frequent. If someone regards an uninterrupted motion, he will necessarily observe in it either speed or slowness; moreover, if someone moves his hand, he will move it in either a frequent or less frequent motion. If motion is slow and less frequent, low sounds are necessarily produced by the very slowness and infrequency of striking. But if [190]
motions are fast and more frequent, high sounds are necessarily produced. For this reason, if the same string is made tighter, it sounds high, if loosened,[46] low. For when it is tighter, it produces faster pulsation, recurs more quickly, and strikes the air more frequently and densely. The string that is looser brings about lax and slow pulsations, and, being less frequent because of this very weakness of striking, does not vibrate very long.

One should not think that when a string is struck, only one sound is produced, or that only one percussion is present in these numerous sounds; rather, as often as air is moved, the vibrating string will have struck it. But since the rapid motions of sounds are connected, no interruption is sensed by the ears, and a single sound, either low or high, impresses the sense. Yet each sound consists of many sounds, the low of slower and less frequent

45. Compare this definition of sound with Nicomachus *Enchiridion* 4 (JanS. 242–43). See also the definition of sound below, 1.8 and n. 63.

46. The verbs *intendo* and *remitto*, along with their substantive forms *intentio* and *remissio*, present problems to the translator of medieval Latin theory. *Intendo* implies increasing the tension of a string, adding to the quantity (frequency) of a pitch, and rendering a pitch higher, all at the same time. *Remitto* implies the opposite: viz., loosening the tension of a string, reducing the quantity of a pitch, and rendering a pitch lower. This complex of meaning was particularly important to Pythagoreans, for they viewed pitch as essentially quantitative, and these words functioned particularly well for them as expressions of both the quantitative and the qualitative changing of pitch. In the course of the present chapter, the identification of addition and subtraction of quantity with raising and lowering of pitch forms an important step in a logical derivation of ratios as the essential element in musical expression. Depending on the context, these verbs will be translated "to tighten" or "to loosen," "to raise" or "to lower," and so on. In all cases the quantitative implications of raising or lowering pitch should be kept in mind.

sounds, the high of faster and more frequent ones.[47] It is as if someone carefully fashions a cone—which people call a "top"—and applies one stripe of red or some other color to it and spins it as fast as possible, then the whole cone seems dyed with the red color, not because the whole thing is thus, but because the velocity of the red stripe overwhelms the clear parts, and they are not allowed to appear. But these things will be treated later.[48]

Since high pitches are incited by more frequent and faster motions, whereas low ones are incited by slower and less frequent motions, it is evident that high pitch is intensified from low through some addition of motions, while low pitch is relaxed from high through lessening of motions. For high pitch consists of more motions than low. Plurality makes the difference in these matters, and plurality necessarily consists in a kind of numerical quantity. Every smaller quantity is considered in relation to a larger quantity as number compared to number. Of those things which are compared according to number, some are equal, and others unequal.[49]
[191] Therefore, some sounds are also equal, while others stand at an interval from each other by virtue of an inequality. In those pitches which do not harmonize through any inequality, there is no consonance at all. For consonance is the concord of mutually dissimilar pitches brought together into one.[50]

## 4. *Concerning the species*[51] *of inequality*

Things which are unequal hold within themselves five criteria[52] relating to degrees of inequality. One is surpassed by another either by a multiple

47. For Pythagoreans, the theory that one sound consists of many is yet another step in the logical derivation of ratios as the proper subject of musical theory. Compare this theory of sound and the present argument with those found in Nicomachus *Enchiridion* 4 (esp. JanS. 243.17–244.11).

48. See 1.31 and 4.1.

49. In passages such as this Boethius assumes that the reader is familiar with *Arith.* Concerning these two types of related quantity (equal and unequal), see *Arith.* 1.21.

50. Compare this chapter, a Pythagorean preamble to the study of music, with 2.20 and 4.1. See also Nicomachus's theory of consonance below in 1.31.

51. Although the term *species* appears in the titles of this and the following chapter, in the text of both chapters the term *genus* is used to classify differences in inequalities. The term *genus* is here translated "class."

52. The term *momentum* challenges the translator in many places: Boethius was probably translating the Greek word ῥοπή, which denotes a weight used on a scale, but also connotes the critical moment of judgment, the turn of the scale. In the present context "criterion" conveys the notions of both standard and judgment. The three other instances of this word (1.9 [Friedlein ed., 196.8], 1.10 [Friedlein ed., 196.19 and 197.03]) present somewhat different problems. (Henceforth numbers in brackets will be references to Friedlein ed.) Concerning the term itself and its usage in antiquity, the Middle Ages, and Renaissance, see Paolo Galluzzi, *Momento, Studi galileiani* (Rome, 1979).





or by a singular part or by several parts or by a multiple and a part or by a multiple and parts.

The first class of inequality is called "multiple." The multiple is such that the larger number contains the whole smaller number within itself twice, three times, or four times, and so forth; nothing is either lacking or superfluous. It is called either "duple," "triple," or "quadruple"; and the multiple class proceeds into infinity according to this series. n+1 : n

The second class of inequality is that which is called "superparticular"; or it is such that the larger number contains within itself the whole smaller number plus some single part of it: either a half, as three to two (and this $\frac{n+1}{n}$ 3:2 is called the "sesquialter" ratio), or a third, as four to three (and this 4:3 called the "sesquitertian"). According to this manner in subsequent numbers, some single part in addition to the smaller numbers is contained by the larger numbers.

The third class of inequality is such that the larger number contains within itself the whole lesser number plus several of its parts besides. If it contains two parts more, it will be called the "superbipartient" ratio, as n+2 : n five is to three; whereas if it contains three parts more, it will be called n+3 : n "supertripartient," as seven is to four. The pattern can be the same in other numbers.

The fourth class of inequality, which is combined from the multiple and the superparticular, is such that the larger number has within itself the [192] lesser number either twice, or three times, or some other number of times, plus one other part of it. If the larger contains the smaller twice plus a half 5:2 part, the ratio will be called "duple-sesquialter,"[53] as five is to two; whereas 5=2+2+1 if the lesser number is contained twice plus a third part of it, it will be called "duple-sesquitertian," as seven is to three. But if the lesser number is contained three times plus its half part, it will be called "triple-sesquialter," as seven is to two. In this same way, the names of multiplicity and superparticularity are varied in other numbers.

The fifth class of inequality, which is called "multiple-superpartient," is such that the larger number has the whole lesser number within itself more than once, plus more than one single part of it. If the larger number contains the smaller number twice plus two parts of it besides, it will be called "duple-superbipartient," as three is to eight; or again a ratio of this class may be called "triple-superbipartient."[54]

53. Friedlein, following *I*, gives *duplex supersesqualter* [192.4], *duplex supersesquitertius* [192.6], and *triplex supersesqualter* [192.8] as names of proportions in this class; the reading of *I* is supported by *K, M, P,* and *V. Q,* however, does not contain *super* in the names of the ratios, and *super* is erased or underlined for omission in *R, S,* and *T.* A major branch of the manuscript tradition does not contain *super* (see Friedlein's apparatus, 192.4, 6, 8). When the multiple-superparticular class of inequality is discussed in *Arith.* (1.29), the prefix *super* does not appear in conjunction with names of ratios [61.19–21]. I thus lean toward the reading of *Q*, as supported by *Arith.*

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We explain these things cursorily and briefly now, since we elucidated them carefully in the books we wrote concerning the fundamentals of arithmetic.[55]

## 5. *What species of inequality pertain to consonance*

Of these classes of inequality, the last two may be set aside, since they are a mixture of others; theorizing ought to be carried out within the first three. The multiple seems to hold the greater authority for consonances, whereas the superparticular seems to occupy the next place. The super-
[193] partient is excluded from consonance of harmony[56]—as acknowledged by various theorists, with the exception of Ptolemy.[57]

## 6. *Why multiplicity and superparticularity are assigned to consonances*

Now those things which are simpler by nature are demonstrated to be harmonious when brought into relationship. Since lowness and highness consist of quantity, those things which can retain the property of discrete quantity will be discovered best to preserve the nature of consonance. One kind of quantity is discrete, while another is continuous. Quantity that is discrete is finite at its smallest point, but proceeds through larger quantity to infinity. For in this kind of quantity, unity[58] is the smallest element, and it is finite, whereas the measurement of plurality extends to infinity, in the manner that number, which begins from finite unity, increases with no limit. Quantity that is continuous, on the other hand, is finite as a whole, but is

54. Friedlein's *ut sunt tres et .XI* [192.17] is a gloss which became a part of the textual tradition in the late ninth and tenth centuries. It is not found at all in *K, P,* or *T,* whereas it (or *ut sunt .III. ad .XI.*) is found as a gloss in *I, M, Q, R,* and *V.* The only control manuscript which takes the phrase into the text is *S.* But no concrete numbers (viz., 3 and 11) are given in the original text for this proposition.

55. See *Arith.* 1.21–31, and below, 2.4.

56. "Consonance of harmony," my translation of *armoniae concinentia,* could be construed as redundant. But one must recall that *armonia* in its general meaning implies a well-ordered arrangement of pitches in a system (see above, n. 19). The text should thus be interpreted to mean that the superpartient class of proportion is separated from consonance in a well-ordered, rational system of musical pitch.

57. Ptolemy *Harmonica* 1.7.15; see below, 5.9.

58. Number is defined in *Arith.* 1.3 as a "collection of unities, or a flowing forth of quantity from unities" (*Numerus est unitatum collectio, vel quantitatis acervus ex unitatibus profusus* [13.11–12]). Unity is described as "mother of all number" (*Arith.* 1.14 [30.28], 1.17 [37.18], 2.8 [98.7], and, as implied in the present passage, unity is indivisible (*Arith.* 1.9 [17.12], 1.10 [23.8]. Boethius's *unitas* is a translation of the Greek μονάς. In ancient arithmetic, unity (the monad) had a certain ontological force, as well as a function in calculations. Often the number 1 is not used in arithmetical texts, and the word μονάς, or *unitas,* occurs in its place. In order to preserve the philosophical character of Pythagorean arithmetic, "unity" will be used in the translation where *unitas* designates the number 1 in calculations.



infinitely divisible. For a line—which represents continuous quantity—can be divided at any time into an infinite number of segments (since its length is a foot or some other finite measure). Therefore, number always increases to infinity, whereas continuous quantity decreases to infinity.[59]

Multiplicity, then, since there is no end to its increasing, preserves most of all the nature of number. The superparticular class, on the other hand, since it divides the smaller part into infinity, possesses the property of continuous quantity; it divides the smaller part because it always contains it plus a part of it—whether a half, third, fourth, or fifth. Indeed the very part named by the larger number always decreases (since a third is named by three, a fourth by four, and since four surpasses three, the fourth rather [194]
than the third is found to be smaller).[60]

The superpartient class, in a certain sense, departs from singleness of nature; for it contains two, three, or four parts over and above, and—departing from simplicity—abounds in a certain plurality of parts.

Every multiple ratio, in addition, contains the whole of itself, for a duple takes the total lesser quantity twice, the triple contains the total lesser three times, and so on in the same manner. Superparticularity, on the other hand, preserves nothing undivided, but yields an excess of a half, a third, a fourth, or a fifth; nevertheless the superparticular class achieves division into singular and simple parts. The superpartient ratio, however, neither retains undivided parts nor admits singular parts, and thus, according to Pythagoras, this class of ratios is not brought to bear on musical consonances. Ptolemy, nevertheless, placed even this class of ratios among consonances, as I will show later.[61]

## 7. *Which ratios should be fitted to which musical consonances*

Meanwhile this should be known: all musical consonances consist of a duple, triple, quadruple, sesquialter, or sesquitertian ratio. Moreover, that which is sesquitertian in number will be called "diatessaron" in sounds; that which is sesquialter in number is called "diapente" in pitches; that which is duple in ratios, "diapason" among consonances; the triple, "dia-

59. Concerning continuous and discrete quantity, see *Arith.* 1.1 [8.15–9.26] and below, 2.3.

60. This passage comparing ratios of the multiple class with discrete quantity and ratios of the superparticular class with continuous quantity seems unique to Boethius's text.

61. See below, 5.7, and Ptolemy *Harmonica* 1.7.15. In fact, Ptolemy does not place superpartient ratios among consonances, but only the multiple-superbipartient ratio 8:3; this is the ratio of the diapason-plus-diatessaron, an interval which Ptolemy believes to be consonant. This chapter is laden with Pythagorean values that will reappear in book 2: singularity, multiplicity, and discrete quantity are all prior to duplicity, superparticularity, and continuous quantity in order of being known and are thus of higher value. See esp. 2.20.

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pason-plus-diapente"; the quadruple, "bis-diapason."[62] For the present, let the above be stated generally and without particulars; later, the complete theory of ratios will be brought to light.

## [195] 8. *Definitions of sound, interval, and consonance*

Sound is a melodic instance of pitch; it is "melodic" in that it functions within a composition in a given tuning. (At present we do not wish to define sound in general, but only that which is called "phthongos" in Greek from the similarity to speaking, that is, φθέγγεσθαι.)[63]

Interval is the distance between high and low sound.[64]

Consonance is a mixture of high and low sound falling pleasantly and uniformly on the ears.[65]

Dissonance, on the other hand, is a harsh and unpleasant percussion of two sounds coming to the ear intermingled with each other.[66] For as long as they are unwilling to blend together and each somehow strives to be heard unimpaired, and since one interferes with the other, each is transmitted to the sense unpleasantly.

## 9. *Not all judgment ought to be given to the senses, but reason ought more to be trusted. Concerning the deception of the senses in this matter*

We propose, concerning these matters, that we should not grant all judgment to the senses—although the whole origin of this discipline is taken from the sense of hearing, for if nothing were heard, no argument what-

62. These terms served as the names of musical intervals in antiquity and the Middle Ages, and they have been retained in this translation: diatessaron for fourth, diapente for fifth, diapason for octave, diapason-plus-diatessaron for eleventh; diapason-plus-diapente for twelfth, and bis-diapason for fifteenth.

63. The term *sonus* in this definition clearly refers to sound as a musical entity and not in the most general sense (see definition of sound in 1.3, also n. 45). The term *vox* and its Greek counterpart φθόγγος are more ambiguous than "pitch," for they refer to the spoken or sung voice as well as to abstract tone; the particular usage of *sonus* and *vox* becomes clear in the complete text of the definition. "In a given tuning" (*in unam intensionem*) establishes the musical context for the melodic sound; that is, it must be at a particular level of pitch and function in a genus and system. Friedlein properly preserves the unique spelling of *intensio* (rather than *intentio*) at this point in the text, although the meaning of the change remains unexplained; the unique spelling is recorded in the control manuscripts, whereas all other occurrences of the word are written as *intentio* (*intencio* in *V*). Compare the use of *vox* in this chapter with that in 1.12. Compare, too, this definition of sound with Nicomachus *Enchiridion* 13 (JanS. 261.4–7) and Aristoxenus's definition of *phthongos* in *Harmonica* 15. Concerning *intentio*, see above, n. 46.

64. See Nicomachus *Enchiridion* 12 (JanS. 261.8).

65. See Nicomachus *Enchiridion* 12 (JanS. 262.1–2); also below, 1.28, 1.30, and 5.1.

66. See Nicomachus *Enchiridion* 12 (JanS. 262.5–6).



soever concerning pitches would exist. Yet the sense of hearing holds the origin in a particular way, and, as it were, serves as an exhortation; the ultimate perfection and the faculty of recognition consists of reason, which, holding itself to fixed rules,[67] does not falter by any error.

But what need is there to speak at length concerning the error of the senses, when this same faculty of perceiving is neither equal in all persons nor equal in the same person at all times? Anyone who aspires to search for truth would to no purpose trust wavering judgment. For this reason the Pythagoreans follow a certain middle path. They do not yield the whole of [196]
judgment to the ears, yet certain things are not investigated by them except through the ears.[68] The Pythagoreans estimate consonances themselves with the ear, but they do not entrust the distances by which consonances differ among themselves to the ears, whose judgments are indecisive. They delegate the determination of distances to rules and reason—as though the sense were something submissive and a servant, while reason is a judge and carries authority.

impt.

Although basic elements of almost every discipline—and of life itself—are introduced through the impression of the senses, nevertheless there is no certain judgment, no comprehension of truth, in these if the arbitration of reason is lacking. For sensation itself is impaired by excess in greatness and smallness alike. It is possible not to perceive very small things because of the minuteness of the sensible objects themselves, and sense perception is frequently confused by very large objects. In pitches, for example, the hearing grasps with difficulty those that are very soft; but if pitches are very loud, the hearing is deafened by the intensity of the sound itself.

## 10. *In what manner Pythagoras investigated the ratios of consonances*

This, then, was primarily the reason why Pythagoras, having abandoned the judgment of hearing, had turned to the weights of rules.[69] He put no credence in human ears, which are subject to change, in part through

67. The term *regula* ("rule"), inconspicuously inserted in the opening sentence of this chapter, carries increasing weight as the treatise progresses. The ultimate *regula* (κανών in Greek) for the musical discipline is that of the monochord, the instrument which makes audible the principles which rule reason in musical thinking.

68. The moderation of the Pythagoreans—especially the earliest ones—may be somewhat overstated here; see below, 5.3, and Plutarch *De musica* 1144F. The moderation expressed in this chapter is clarified in 3.10.

69. A subtle play with meanings occurs in this chapter and the next: *momentum* (ῥοπή) implies a standard weight, a weight used in measurement (see above, n. 52). Pythagoras is about to discover his rule, his measure of consonances, in the weight (*pondus*) of the hammers used in the smithy. In the next chapter he will "weigh" (*perpendo*) his theory of consonance. I suggest that subsequent uses of "weigh" in book 1 (1.28 [220.3] and 1.34 [224.19 and 225.8]) reiterate the concept of *momentum* and *perpendo* from these chapters.

nature, in part by external circumstance, and undergo changes caused by age. Nor did he devote himself to instruments, in conjunction with which much inconstancy and uncertainty often arise. When you wish to examine strings, for example, more humid air may deaden the pulsation, or drier air may excite it, or the thickness of a string may render a sound lower, or thinness may make it higher, or, by some other means, one alters a state of previous stability. Moreover, the same would be true of other instruments.

[197] Assessing all these instruments as unreliable and granting them a minimum of trust, yet remaining curious for some time, Pythagoras was seeking a way to acquire through reason, unfalteringly and consistently, a full knowledge of the criteria for consonances. In the meantime, by a kind of divine will, while passing the workshop of blacksmiths, he overheard the beating of hammers somehow emit a single consonance from differing sounds. Thus in the presence of what he had long sought, he approached the activity spellbound. Reflecting for a time, he decided that the strength of the men hammering caused the diversity of sounds, and in order to prove this more clearly, he commanded them to exchange hammers among themselves. But the property of sounds did not rest in the muscles of the men; rather, it followed the exchanged hammers. When he had observed this, he examined the weight of the hammers. There happened to be five hammers, and those which sounded together the consonance of the diapason were found to be double in weight. Pythagoras determined further that the same one, the one that was the double of the second, was the sesquitertian of another, with which it sounded a diatessaron. Then he found that this same one, the duple of the above pair, formed the sesquialter ratio of still another, and that it joined with it in the consonance of the diapente. These two, to which the first double proved to be sesquitertian and sesquialter, were discovered in turn to hold the sesquioctave ratio between themselves.[70] The fifth hammer, which was discordant[71] with all, was discarded.

Although some musical consonances were called "diapason," some "diapente," and some "diatessaron" (which is the smallest consonance)

70. The above narrative seems construed to make it difficult for the reader to follow the weights and sounds of the respective hammers. The emphasis is on the resounding consonances and the proportions discovered between hammers producing consonances. The Pythagorean tetractys is not introduced until the very end of the chapter, and then "for the sake of illustration." Some manuscripts—e.g., *Q*—contain the numbers 12, 9, 8, and 6 as interlinear glosses over words in the text, as a means of keeping track of the hammers. Many later sources—e.g., Munich, Clm 18,480—present a diagram at this point, illustrating the possible ratios contained in the four numbers 12, 9, 8, 6. The present translation seeks to keep the character of gradual revelation of Boethius's original text.

71. Boethius uses the term *inconsonans,* which I have translated "discordant." He uses this term in only one other place—in the description of music in its ideal state, which consists of only four notes, which again are related as 12:9:8:6 (1.20 [206.5]). The presence of this term in these two passages further demonstrates the close relationship between them.





before Pythagoras, Pythagoras was the first to ascertain through this means by what ratio the concord of sounds was joined together. So that what has been said might be clearer, for sake of illustration, let the weights of the four hammers be contained in the numbers written below.

$$12 : 9 : 8 : 6.$$

Thus the hammers which bring together 12 with 6 pounds sounded the [198] consonance of the diapason in duple ratio. The hammer of 12 pounds with that of 9 (and the hammer of 8 with that of 6) joined in the consonance of the diatessaron according to the epitrita[72] ratio. The one of 9 pounds with that of 6 (as well as those of 12 and 8) commingled the consonance of the diapente. The one of 9 with that of 8 sounded the tone according to the sesquioctave ratio.

## 11. *By what differing means Pythagoras weighed the ratios of consonances*

Upon returning home, Pythagoras weighed carefully by means of different observations whether the complete theory of consonances[73] might consist of these ratios. First, he attached corresponding weights to strings and discerned by ear their consonances; then, he applied the double and mean and fitted other ratios to lengths of pipes. He came to enjoy a most complete assurance through the various experiments. By way of measurement, he poured ladles of corresponding weights into glasses, and he struck these glasses—set in order according to various weights—with a rod of copper or iron, and he was glad to have found nothing at variance. Thus led, he turned to length and thickness of strings, that he might test further. And in this way he found the rule, about which we shall speak later.[74] It came to be called by that name, not because the rule with which we measure the sizes of strings is wooden, but because this kind of rule is tantamount to such fixed and enduring inquiry that no researcher would be misled by dubious evidence.[75]

72. Epitritos (ἐπίτριτος), the Greek term for one and a third—i.e., 4/3, or sesquitertian ratio.

73. At this point Boethius introduces the word *symphonia,* a transliteration of the Greek συμφωνία, fully equivalent to the Latin *consonantia.* The word had been introduced into Latin vocabulary by Vitruvius *De architectura* 1.1.9, and other writers before Boethius had used the term as equivalent to *consonantia.* Boethius seems to use it for the sake of variety, but always in a context where *consonantia* could just as well have been used. *Symphonia* is here translated, like *consonantia,* as "consonance."

74. Boethius is playing on the double meaning of the Latin word *regula,* a word similar to the English "rule." *Regula* can mean a calibrated stick or a procedural guide and standard of judgment. The musical "rule," which will be revealed in 4.5–12, is so named from the latter meaning, despite the fact that the actual tool, the monochord, is made of wood.

75. Concerning Pythagoras's discovery of these musical ratios and his experiments, see

## [199] 12. *Concerning the classification of voices*[76] *and an explanation thereof*

Enough concerning these things. Now we should consider the different kinds of voices. Every voice is either συνεχής, which is continuous, or διαστεματική, which it is named when it is sustained by means of interval.[77]

A voice is continuous when, as in speaking or reciting a prose oration, we hurry over words: the voice hastens not to get caught up in high and low sounds, but to run through the words very quickly, and the impulse of continuous voice is occupied with pronouncing and giving meanings to the words.

Διαστεματική, on the other hand, is that voice which we sustain in singing, wherein we submit less to words than to a sequence of intervals forming a tune.[78] This particular voice is more deliberate, and by measuring out differences of pitch it produces a certain interval, not of silence, but of sustained and drawn out song.

To these, as Albinus[79] asserts, is added a third, different kind, which can incorporate intermediate voices, such as when we recite heroic poems

Nicomachus *Enchiridion* 6 (JanS. 245–48), Gaudentius *Eisagoge* 9 (JanS. 340–41), Iamblicus *Vita de Pythagora* 1.24, Aristotle Elias *Prolegomena philosophiae* 2 (ed. Adolf Busse, p. 29), and Macrobius *Commentarii in somnium Scipionis* 2.1.9–14. As widespread as this story is in antiquity, most of the observations reported are physically impossible; the ratios supposedly tested by Pythagoras are valid if applied to lengths of pipes or strings, but not to weights of hammers or weights attached to strings. See Mersenne, *Questions harmoniques* (Paris, 1634), p. 166; Claude Palisca, "Scientific Empiricism in Musical Thought," pp. 127–29; and Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, pp. 374–77.

76. In this chapter and the following, the term *vox* is used unequivocally to denote the human voice (see above, n. 34). The distinction between a continuum of pitch built with discrete pitches and a continuum with no discrete steps is something of a standard topic in ancient theory; the same distinction is made in 5.5, there derived from Ptolemy and using a more abstract (pure pitch) notion of *vox*.

77. See Nicomachus *Enchiridion* 2 (JanS. 238) concerning these classifications and descriptions. Boethius chooses the word *suspensa* (*suspendo*) to contrast with *continua* (*continuo*); *suspendo* implies both interrupting and suspending, supporting, or hovering, as in the case of a melody which is both sustained and interrupted by intervals. *Suspensa* has been translated as "sustained."

78. "Sequence of intervals forming a tune" is a translation of *modulis* [199.11] (dative plural of *modulus*), a term which Boethius uses only once. It is obviously related to the *modus*/*modulatio* complex of meanings (see above, nn. 3 and 31). As a singular noun, *modulus* would imply an interval, but in the plural as in the present context, the term suggests an indefinite number of intervals forming a musical entity. See *OLD* 1124.

79. The musical writings of Albinus, also cited in Cassiodorus *Institutiones* 2.5.10, are not extant. Boethius, in *In librum Aristotelis de interpretatione editio secunda, seu maiora commentaria*, ed. Meiser, pp. 3–4, also cites writings on geometry and logic by Albinus. Concerning this third type of voice, see Martianus Capella *De nuptiis* 9.937.





not in continuous flow as in prose or in a sustained and slower moving manner as in song.

## 13. *That human nature limits the boundlessness of voices*

The voice which is continuous and that with which we run through song are inherently boundless. For by consideration generally agreed upon, no limit is placed either on flowing through words or on rising to high pitches or sinking to low ones. But human nature imposes its own limitation on both of these kinds of voice. The human breath places a limit on the continuous voice, which it cannot exceed for any reason, for every person speaks continuously as long as his natural breath permits. Human nature [200] likewise places a limit on the diastematic voice, which puts bounds on a person's high and low pitch, for a person can ascend just so high and descend just so low as the range of his natural voice allows.[80]

## 14. *How we hear*

At this time we should discuss how we hear. The same thing happens in sounds that happens when a stone, thrown from above, falls into a puddle or into quiet water. First it causes a wave in a very small circle; then it disperses clusters of waves into larger circles, and so on until the motion, exhausted by the spreading out of waves, dies away. The latter, wider wave is always diffused by a weaker impulse. Now if something should impede the spreading waves, the same motion rebounds immediately, and it makes new circles by the same undulations as at the center whence it originated.

In the same way, then, when air that is struck creates sound, it affects other air nearby and in this way sets in motion a circular wave of air; and so it is diffused and reaches the hearing of all standing around at the same time. The sound is fainter to someone standing at a distance, since the wave of activated air approaches him more weakly.

## 15. *Concerning the sequence of subjects, that is, of speculations*

Having set forth these matters, it seems that we should discuss the number of genera within which all song is composed and which the disci-

80. See Nicomachus *Enchiridion* 2 (JanS. 239–40).

pline of harmonic theory[81] contemplates. They are these: diatonic, chromatic, and enharmonic.[82] These must be explained, but first we must discuss tetrachords and in what manner the augmented number of strings came
[201] into existence (to which more are added now). This will be done after we have recalled in what ratios musical consonances are combined.[83]

## 16. *Concerning the consonances and the tone and the semitone*[84]

The consonance of the diapason is that which is made in the duple

81. Boethius here uses a word from the discipline of rhetoric—viz., *inventio*—to describe systematic musical thought. In rhetorical discourse *inventio* is used to denote the defining of subject matter and the devising of arguments; so I have translated *armonicae inventionis disciplina* as "discipline of harmonic theory." See also 5.2 and n. 4.

82. In this chapter Boethius uses the terms *diatonum, chroma* and *armonia* to designate the genera; these are substantive forms transliterated from the Greek διάτονον, χρῶμα, and ἁρμονία. See Cleonides *Isagoge* 3 (JanS. 181.12), Bellerman's Anonymous II, 14 (originally in F. Bellermann, *Anonymi scriptio de musica* [Berlin, 1841], newly edited in Dietmar Najock, *Drei anonyme griechische Traktate über die Musik* [Göttingen, 1972], 76.10–11), and Anonymous III, 52 (Najock 104.8). This is the only place where Boethius uses *armonia* for enharmonic. *Diatonum* and *chroma* are used again together in 1.21, and *chroma* also appears in 5.16 and 18. Boethius generally uses the adjectival forms of these terms: *genus diatonicum, chromaticum,* and *enarmonium*.

83. The purpose of this chapter is to prepare the reader for the chapters which follow: 16–19 consider the structure of musical consonances; 20 discusses basic tetrachords and the addition of strings; and 21 arrives at the genera of song.

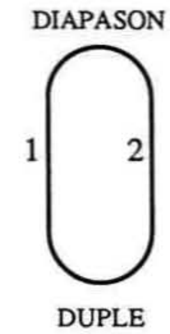
84. This chapter begins by stating seven basic tenets concerning ratios of consonances and the tone, accompanied by diagrams for each. These declarations are found in all the control manuscripts and in most later sources, and their character is emphasized through the use of majuscule script in most sources (*I, K, M, Q, T,* and many later sources). Friedlein judged these prefatory elements to be unessential accretions, and he printed them (according to *I*) and a version of the diagrams only in his apparatus. But the text which follows the declarations and diagrams builds on them, and both textual history and context argue that they are an integral part of Boethius's text. I have based my translation on the text as it appears in *K, M, P, Q,* and *T:*

> DIAPASON SYMPHONIA EST QUAE FIT IN DUPLO, UT EST HOC.
> DIAPENTE VERO EST QUAE CONSTAT HIS NUMERIS.
> DIATESSARON VERO EST QUAE IN HAC PROPORTIONE CONSISTIT.
> TONUS VERO SESQUIOCTAVA PROPORTIONE CONCLUDITUR, SED IN HOC NONDUM EST CONSONANTIA.
> DIAPASON VERO ET DIAPENTE TRIPLA COMPARATIONE COLLIGITUR, HOC MODO.
> BISDIAPASON QUADRUPLA COLLATIONE PERFICITUR.
> DIATESSARON AC DIAPENTE UNUM PERFICIUNT DIAPASON, HOC MODO.

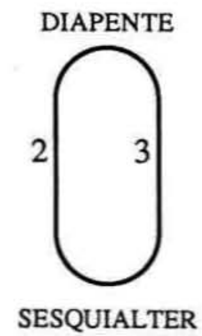




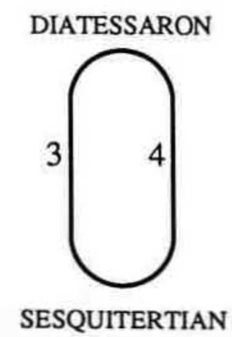
ratio, such as this [Fig. A.1]:



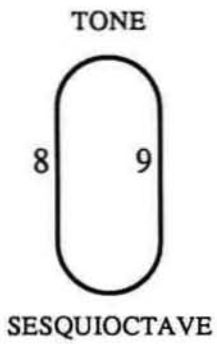

The diapente is that which consists of these numbers [Fig. A.2]:



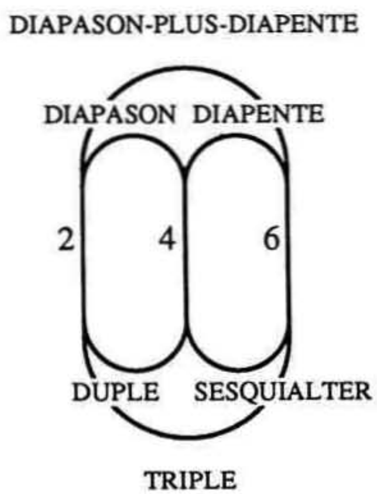

The diatessaron is that which occurs in this ratio [Fig. A.3]:



The tone is comprised of the sesquioctave ratio, but there is no consonance in this [Fig. A.4]:



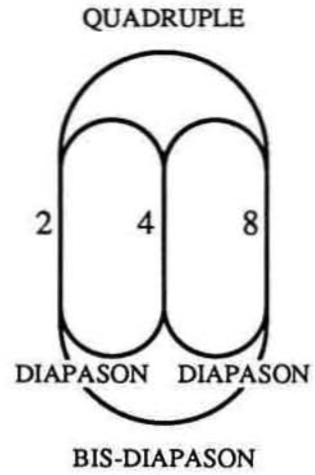

The diapason-plus-diapente is brought together through the triple ratio in this manner [Fig. A.5]:



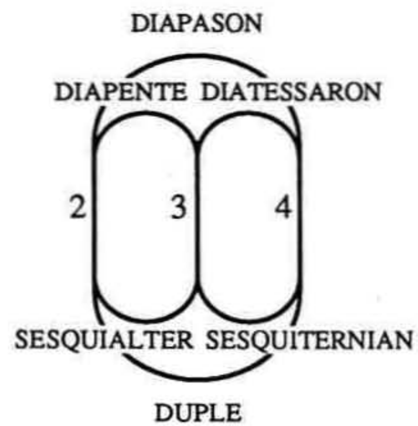

The bis-diapason is brought about through the quadruple comparison [Fig. A.6]:








The diapente plus the diatessaron produce one diapason, in this manner [Fig. A.7]:




If a pitch is higher or lower than another pitch by a duple, a consonance of the diapason will be made. If a pitch is higher or lower than another pitch by a sesquialter, sesquitertian, or sesquioctave ratio, then it [202]
will yield consonances of the diapente or of the diatessaron or a tone respectively. Likewise, if a diapason, such as 4:2, and a diapente, such as 6:4, are joined together, they will make the triple consonance, which is the

diapason-plus-diapente. If two diapasons are combined, such as 2:4 and 4:8, then the quadruple consonance will be made, which is the bis-diapason. If a sesquialter and a sesquitertian—that is, a diapente and a diatessaron, such as 2:3 and 3:4—are joined, the duple consonance, obviously the diapason, is formed; 4:3 brings about a sesquitertian ratio, while 3:2 is joined by a sesquialter relation,[85] and the same 4 related to the 2 unities within a duple comparison. But the sesquialter ratio creates a consonance of the diapente, and the sesquitertian that of the diatessaron, whereas the duple ratio produces a consonance of the diapason. Therefore, a diatessaron plus a diapente forms a consonance of the diapason.

Moreover, a tone cannot be divided into equal parts; why, however, will be explained later.[86] For now it is sufficient to know only that a tone is never divided into two equal parts. So that this might be very easily demonstrated, let 8 and 9 represent the sesquioctave ratio. No mediating number falls naturally between these, so let us multiply them by 2: twice 8 makes 16; twice 9, 18. However, a number naturally falls between 16 and 18—namely, 17. Let these numbers be set out in order: 16, 17, 18. Now when 16 and 18 are compared, they yield the sesquioctave ratio and thus
[203] a tone. But the middle number, 17, does not divide this ratio equally. For compared to 16, 17 contains in itself the total 16, plus 1/16 part of it—that is, unity. Now if the third number—that is, 18—is compared to this number—that is, to 17—it contains the total 17 and 1/17 part of it. Therefore 17 does not surpass the smaller number and is not surpassed by the larger number by the same parts; the smaller part is 1/17, the larger 1/16. Both of these are called "semitones," not because these intermediate semitones are equal at all, but because something that does not come to a whole is usually called "semi." In the case of these, one semitone is called "major," and the other "minor."[87]

## 17. *In what smallest integers*[88] *the semitone is ascertained*

At this time I shall explain more clearly what an integral[89] semitone is, or in what smallest integers it is ascertained. For what has been said

85. This is the only time in the treatise that Boethius uses the word *collatio*, a term which functions as a less technical equivalent of ratio, much like *habitudo*. See above, n. 30, and book 2, n. 3.

86. See 3.1–2.

87. Although these ratios are called "semitones" and one is named "major" and the other "minor," they are abstract elements in an argument, rather than concrete entities in a musical system. Boethius further qualifies these ratios as designating "intermediate semitones" (*semitonia media*), thereby associating the ratios with the theory of means developed in books 2 and 3. The same argument—with the same numbers and vocabulary—is developed further in 3.1. See also 3.2 and n. 7.

88. Boethius uses the expression *primi numeri* on three occasions in this treatise: twice in the present chapter and once in 3.14. In *Arith.* 1.13–18, he defines and discusses "prime



about the division of the tone has nothing to do with our wanting to show the measurements of semitones; it applies rather to the fact that we said a tone cannot be divided into twin equal parts.[90]

The diatessaron, which is a consonance of four pitches and of three intervals, consists of two tones and an integral semitone.[91] A diagram[92] of this [Fig. A.8] is set out below:

| | | | |
|---|---|---|---|
| 192 | 216 | 243 | 256 |

If the number 192 is compared to 256, a sesquitertian ratio will result, and it will sound a consonance of the diatessaron. But if 216 is compared with 192, the ratio is a sesquioctave, for the difference between them is 24, which is an eighth part of 192. Therefore it is a tone. Moreover, if 243 is compared to 216, the ratio will be another sesquioctave, for the difference between them, 27 represents an eighth part of 216. The comparison of 256 [204]
with 243 remains: the difference between these is 13, which, multiplied by

number" (also *primi numeri*); but in the present context he means something other than "prime numbers," for neither 243 nor 256 is prime. Yet 256:243 is the primary expression of that ratio in which the minor semitone is found; thus I have translated *primi numeri* as "smallest integers."

89. Boethius's use of the adjective *integer* presents problems, for he uses it in two ways, each of which has the potential of contradicting the other. At the close of the previous chapter Boethius said that a thing is usually called "semi" when it does not "come to a whole" (*ad integritatem usque non perveniat*). Yet, in this opening sentence Boethius modifies the noun "semitone" with the adjective "integral," thereby applying a qualification to the semitone, the lack of which is precisely what defines it as "semi." Thus, the remainder after two tones have been subtracted from the interval of a diatessaron is an "integral semitone"; that is, it is a constituent, functioning part of the diatessaron, even though it is not an "integral half" of a tone. When Boethius uses the term according to its first meaning, I have translated it as "whole," "complete," or "full"; when he employs it according to its second meaning, as "integral." This confusion in terminology led to textual problems in the present chapter during the Middle Ages (see below, n. 91).

90. See above, 1.16; 18:17:16 do not present the proportion of semitones as constituent parts of a tonal system.

91. Friedlein 203.20: *non integro semitonio* should read *integro semitonio*. The opening sentence of this chapter clearly states that the first possible numbers containing an "integral semitone" will be explained; those numbers are 256:243. The reading *integro semitonio* is found in all the control manuscripts except *I* and *R*, where *non* is added above the line by later hands. The *non* becomes part of the text in one branch of the manuscript tradition during the tenth century, but it was never universally adopted. The introduction of *non* is an example of hypercorrection. The semitone of the present argument, although not a half-tone, is self-contained and complete, a constituent part of the diatessaron.

92. This diagram and the one that follows qualify as "diagrams" and are numbered as figures chiefly because the word *descriptio* occurs in the text. Boethius uses this term consistently to refer to diagrams. In the control manuscripts the numbers occur on the textual line, with no lines or boxes around them. They may have been more elaborate drawings in an earlier tradition, and in later traditions they take on the character of diagrams (see, e.g., Cambridge, Trinity College, R.15.22, ff. 17r and 18r).

8, does not seem to arrive at a mean of 243. It is therefore not a semitone, but less than a semitone. For it might be reckoned to be a whole semitone, rigorously speaking, if the difference of these numbers, which is 13, multiplied by 8, could have equaled a mean of the number 243.[93] Thus the comparison of 243 with 256 yields less than a true semitone.

## 18. *The distance between a diatessaron and a diapente is a tone*

The consonance of the diapente has five pitches and four intervals, three tones and a minor semitone. Again take the number 192 and compute its sesquialter, which would make a consonance of the diapente with it. This number should therefore be 288. Then let the numbers which were related to 192 above be placed among these numbers: 216, 243, and 256. Let a diagram be formed in this manner [Fig. A.9]:

| | | | | |
|---|---|---|---|---|
| 192 | 216 | 243 | 256 | 288 |

Now in the above diagram 192 and 256 were shown to contain two tones and a semitone. Therefore, the comparison of 256[94] and 288 remains, which is a sesquioctave—that is, a tone. Their difference is 32, which is an eighth part of 256. Thus the consonance of the diapente has been demonstrated to consist of three tones and a semitone. Yet a short while ago the consonance of the diatessaron was derived from the number 192 related to 256, whereas now a diapente is extended from this same 192 to 288. Therefore
[205] the consonance of the diatessaron is surpassed by the diapente by that ratio which is found between the numbers 256 and 288, and this is a tone. Thus the consonance of the diapente surpasses that of the diatessaron by a tone.

93. The *iure* of this sentence, which I have translated as "rigorously speaking," refers to the kind of Pythagorean mathematical consistency found at the close of the previous chapter, where the ratios 18:17 and 17:16 were called "semitones." If 13 multiplied by 8 had equaled half of 243, then 256:243 would have been a "semitone" in the sense that 18:17 and 17:16 were both semitones—i.e., 256 would be the arithmetic mean between some number x and 243 such that the difference between x and 243 would be an eighth part of 243. Since 13 multiplied by 8 (i.e., 104) is less than the half of 243 (i.e., 121½), 256:243 is less than 18:17—the final demonstration of which is found in 3.13—and, "rigorously speaking," 256:243 is not a semitone. Significantly, Boethius uses the term *medietas,* rather than *dimidium,* in this context, "mean" rather than "half" (concerning means, see 2.12–14). It is clear from this chapter and the previous one that Boethius considers the tone indivisible by 2, and that he is not mistakenly considering any of these ratios a "half-tone" as Pizzani, "Fonti," pp. 55–56, would have us believe. The argument is nevertheless obtuse, which led some frustrated medieval reader to add a *non* where none was necessary.

94. Friedlein 204.21: *comparatio ducentorum ad .CCLXXXVIII.* should read *comparatio ducentorum quinquaginta .vi. ad .CCLXXXVIII.* The latter reading is found in manuscripts and is dictated by the argument.





## 19. *The diapason consists of five tones and two semitones*

The consonance of the diapason consists of five tones and two semitones, which nevertheless do not make up one tone. For it has been demonstrated that a diapason consists of a diatessaron and a diapente. Furthermore, it has been proved that a diatessaron consists of two tones and a semitone, a diapente of three tones and a semitone. These joined together, therefore, produce five tones (and two semitones)—but since the two semitones were not full halves, their conjunction does not add up to a whole tone. Their sum surpasses a half tone but falls short of a complete tone. According to this reckoning, the diapason consists of five tones and two semitones, which, just as they do not fill a complete tone, so they go beyond a full semitone. But the theory behind these things and how one comes to know these musical consonances will be explained more clearly later.[95]

Meanwhile, belief must be summoned to the present argument to make up for modest knowledge; indeed, a firm credence in all should be summoned, since each thing will be made clear by proper demonstration. Having set these matters in order, we will discuss for a while the strings of the kithara and their names, and also how they were added, since this determines their names. After first coming to an understanding of these matters, knowledge of what will be discussed subsequently will be easy.

## 20. *Concerning the additions of strings and their names*

In the beginning, Nicomachus reports,[96] music was truly simple, since [206]
it was composed of four strings. It continued in this state until the time of

95. The present chapter is pedagogical in purpose, consisting of a review of the doctrines that have been presented up to this point. The reference to later explanations is not specific, but general. Compare this chapter with 1.33, which also asks for credence and reviews the basics of Pythagorean dogma.

96. The account given in this chapter is not found in any extant work of Nicomachus; the development of the musical system presented here is the most thorough of any that have been handed down from antiquity and, as such, is unique. In his *Enchiridion* 11 (JanS. 260.4–12), Nicomachus promised a full account of the growth of the musical system, starting with the tetrachord, an account of each note, and the inventor thereof. I take the present chapter of Boethius to be the history promised by Nicomachus.

The naming of inventors is an important topos in ancient literature. By assigning an agent to a specific invention, an author characterized both the object invented and the person associated with the object, thereby giving an object, profession, or action a sanction that it would otherwise have lacked. In the naming of inventors, the author's interest extended beyond giving a certain historical chronology; it established a cultural context, a function, a value for the object and the inventor. (For a discussion of the naming of inventors in ancient literature, see Adolf Kleingünther, "Πρῶτος Εὑρετής. Untersuchungen zur Geschichte einer Fragestellung," *Philologus,* Suppl. 26, vol. 1 [1934], pp. 1–155.)

The attribution of inventions in ancient musical lore is a subject of great inconsistency and complexity. More than one inventor can usually be found for any given instrument, string, Orpheus.[97] In this period the disposition of strings was such that the first and fourth strings sounded the consonance of the diapason, while the middle strings each in turn sounded the diapente and the diatessaron with the strings nearest them and those most distant.[98] Indeed, there was nothing discordant in these, in imitation of cosmic music, which consists of the four elements. The inventor of the quadrichord is said to have been Mercury.[99]

At a later time,[100] Toroebus,[101] son of Atys and king of the Lydians,[102]

or nomos (see ibid., pp. 22–25, 135–43). The present account is characterized as much by the inventors it leaves out as by those it names; it omits, e.g., Phrynis, Simonides, Thamyris, and Melanippides, all famous inventors associated with "theatrical" music and music of questionable repute. Embedded in the present text is a "tonal genesis" with a consistent point of view: it presents the growth of the musical system as determined by the principle of species of consonances. These species determine the fundamental ethos of ancient music: the modes (see 4.14–15). The indication of these species is revealed through the geographical origins of the inventors named. In almost every case, the attributions can be corroborated by other sources. At each level of development of the tonal system, a Dorian, or native Greek, disposition is expanded into systems which accommodate Phrygian and Lydian elements.

97. Making Orpheus the *terminus ante quem* for music of four strings, or simple music, assigns the "state of grace" of music to the period of the gods and demigods; for Orpheus was the son of Apollo and a muse. Nicomachus *Excerpta* 1 (JanS. 266) reports that Mercury taught Orpheus to play the lyre, but in that account the instrument had seven strings, not four.

98. Concerning the antiquity of the four-stringed lyre, see Terpander *Fragmenta* 5 (*Lyra Graeca*, vol. 1, p. 32), Strabo *Geographica* 13.3–4, Pliny *Naturalis historia* 7.56(57).204, Censorinus *Fragmenta* 12. The disposition of the present instrument is manifestly Pythagorean; the strings are tuned to the same set of pitches discovered by Pythagoras in the weights of the hammers and represented in the ratios 12:9:8:6. The noted absence of any "discordant" (*inconsonans*) element and the tuning of the four pitches resonate strongly with 1.10 (see above, n. 71).

99. In several sources, Mercury is credited with inventing the lyre, but none of the sources calls it a "quadrichord"; see Homer *Hymnus ad Mercurium* 15–63 (seven strings), Nicomachus *Excerpta* 1 (JanS. 266) (seven strings), Diodorus Siculus *Bibliotheca* 1.16.2 (three strings), Apollodorus *Bibliotheca* 3.10.2 (no number given), and Horace *Carmina* 1.10 (no number given).

100. Entering the period of human inventions, here innocently indicated by *post* ("at a later time"), the tuning of the lyre underwent a fundamental change: whereas the four original strings were given no names and were tuned in an octave filled with a fourth and a fifth, the following narrative tells us that the first four strings were the hypate (H), parhypate (pH), lichanos (L), and mese (M). These are the fundamental pitches around which the Greek system was built, the tetrachord with the semitone in the lowest position, a species of fourth with Dorian quality.

1 2 3 4
H _ pH ___ L ___ M
*s T T*

(I use the diatonic genus in tracing the species since the same genus is used when tracing species in 4.14.)

In describing species of fourth as having a certain "quality," I am following Winnington-Ingram, except that he uses the term *flavor* rather than *quality* (see *Mode in Ancient Greek*





added a fifth string. Hyagnis the Phrygian[103] added a sixth string to these. Then a seventh string was added by Terpander of Lesbia,[104] obviously in likeness to the seven planets. The lowest of these seven was the one called "hypate," the larger and more honorable, as it were; for this reason they name Jove "Hypatos." They also call a consul by the same name because of the loftiness of his rank. This string was attributed to Saturn because of its slow motion and low sound.[105] The second string was the parhypate,

*Music* [Cambridge, 1936], p. 15); but it is important to observe Winnington-Ingram's caution concerning naming tetrachords "Lydian," "Phrygian," or "Dorian" (*Mode*, p. 12).

101. Friedlein 206.8: *post Coroebus Atyis filius* should read *post Toroebus Aetyis filius*. The combination of an adverbial *post* followed by an unfamiliar Greek name gave scribes considerable difficulty. *K*, e.g., reads *posteriobus atyis filius*, and *I* read *postroebus* before it was corrected to *post coroebus*. *R* also gives *coroebus*, but *M, P, Q, S, and V* all read *post Toroebus Atyis filius*.

102. Toroebus, the Lydian, is named as the inventor of the fifth string; Plutarch *De musica* 1136C, citing Dionysius Iambus and others, names Toroebus as the first to use the Lydian *harmonia*. The fifth string, the trite (T), makes possible the species of diatessaron of Lydian quality, with the semitone in the highest position.

2 3 4 5
pH ___ L ___ M _ T
*T T s*

103. According to Athenaeus *Deipnosophistae* 14.624, Aristoxenus named Hyagnis as inventor of the Phrygian *harmonia*. Nicomachus attributes to Hyagnis, the Phrygian, the invention of the sixth string; this string, the paranete (pN), makes possible the species of diatessaron of Phrygian quality, with the semitone in the middle position.

3 4 5 6
L ___ M _ T ___ pN
*T s T*

104. Terpander is associated with the heptachord lyre through three traditions: a general association with the heptachord, as found in Aristotle *Problemata* 32, the *Suidas*, and Nicomachus *Excerpta* 1; transformation of the quadrichord into the heptachord through the addition of three strings, as in Terpander *Fragmenta* 5 (*Lyra Graeca*, vol. 1, p. 32), Strabo *Geographica* 13.3–4, and Pliny *Naturalis historia* 7.56(57).204; and the addition of one string (to six), as found in Plutarch *Instituta Laconica* 17 (238C), Plutarch *De musica* 1140F, and the present text. In Plutarch's *De musica* Terpander is credited with having added a "Dorian nete," and it is this that the present text carries forward; for Terpander's addition of a seventh string creates a second tetrachord, a tetrachord—like the first—of Dorian quality.

4 5 6 7
M _ T ___ pN ___ N
*s T T*

105. Concerning ὕπατος in reference to Jove (Zeus) and to a consul, see H. G. Liddell and R. Scott, *A Greek-English Dictionary*, 9th ed. (Oxford, 1940), p. 1854. Concerning the association of hypate with Saturn, see 1.27.

Two tetrachords are clearly set out, inasmuch as there are eight strings. But *diazeuxis*—that is, disjunction—occurs between the mese and paramese, which are separated by a whole tone.

These things will be explained more clearly in subsequent discussions, since at that time we should take up the task of explaining every single thing very carefully.[124] Meanwhile, the more attentive observer discerns five tetrachords, and no more: hypaton, meson, synemmenon, diezeugmenon, and hyperboleon.

## 26. *By what names Albinus designated the strings*

Albinus translated the names of these strings into the Latin language; thus he called the notes of the hypaton tetrachord *principales* ["principal"], those of the meson tetrachord *mediae* ["middle"], those of the synemmenon tetrachord *coniuncti* ["conjunct"], those of the diezeugmenon tetrachord
[219] *disiuncti* ["disjunct"], and those of the hyperboleon tetrachord *excellentes* ["high"].[125] But we should not linger in an extraneous work.[126]

## 27. *To what heavenly bodies the strings are compared*

At this point it would seem proper to add concerning the above tetrachords that the disposition from the hypate meson to the nete synemmenon is, as it were, a kind of exemplar of the celestial order and specification. The hypate meson is assigned to Saturn, whereas the parhypate is like the orbit of Jupiter. The lichanos meson is entrusted to Mars. The sun governs the mese. Venus holds the trite synemmenon. Mercury rules the paranete synemmenon. The nete is analogous to the orbit of the moon.[127]

Marcus Tullius draws up a different order, for in the sixth book of *De re publica* he asserts: "Nature is so disposed that low sound emanates from its one extreme part, whereas high sound emanates from its other. Therefore that high celestial orbit, that of the stars, the revolution of which is faster, moves with a high and shrill sound, whereas the weak orbit of the

124. This does not seem to point to any specific future passage in which disjunction and conjunction are treated but probably refers to the division of the monochord in 4.6–11.

125. Concerning Albinus, see above, n. 79. These Latin names for the notes were probably included because they are not quite the same as those given in Martianus Capella *De nuptiis* 9.931—a work which probably reflected the standard Latin usage—and 4.3 below. Martianus (and Boethius 4.3) translate *diezeugmenon* as *divisarum* rather than *disiunctarum*.

126. This sentence seems to imply that Boethius is working from one principal source, probably Nicomachus's lost *Eisagoge musica*, and that he has departed from the principal source to pick up Alypius's Latin terminology and is now returning to his principal text.

127. This comparison of strings to the disposition of the heavenly spheres agrees with that found in Nicomachus *Excerpta* 3 (JanS. 271–72) and idem, *Enchiridion* 3 (JanS. 241–42), although in the latter, Mercury and Venus are reversed. See R. Bragard, "L'harmonie des spheres selon Boèce," *Speculum* 4 (1921): 206–13.





moon moves with a very low sound. The earth, in ninth place, remaining immobile, is alone always fixed in place."[128] Tullius thus regards the earth as silent—that is, immobile. Next after the earth he assigns the lowest sound to the moon, which is closest to silence, so that the moon is the proslambanomenos, Mercury the hypate hypaton, Venus parhypate hypaton, the sun the lichanos hypaton, Mars the hypate meson, Jupiter the parhypate meson, Saturn the lichanos meson, and the highest heaven the mese.

The place at which I discuss the division of the monochord rule will be a more appropriate place to explain which of these strings are fixed, which are completely movable, and which stand between the fixed and the movable.[129]

## 28. *What the nature of consonance is* [220]

Although the sense of hearing recognizes consonances, reason weighs their value. When two strings, one of which is lower, are stretched and struck at the same time, and they produce, so to speak, an intermingled and sweet sound, and the two pitches coalesce into one as if linked together, then that which is called "consonance" occurs. When, on the other hand, they are struck at the same time and each desires to go its own way, and they do not bring together a sweet sound in the ear, a single sound composed of two, then this is what is called "dissonance."[130]

## 29. *Under what conditions consonances occur*

In these comparisons of low with high, it is necessary that the kind of consonances be found which are commensurable with themselves—that is, which are recognized as having a common denominator. Among multiple ratios, for example, that part which is the difference between the two terms measures the duple; between 2 and 4, for example, 2 measures both. Between 2 and 6, a triple ratio, 2 measures both. Between 9 and 8, it is unity itself which measures both. Among superparticular ratios, if the ratio is

128. Cicero *De re publica* 6.18; this "Latin" order is found also in Pliny *Naturalis historia* 2.22(20).84 and Censorinus *De die natali* 13.

129. See 4.13; the threefold division of strings into those that are immobile, those that are movable, and those in between is an important thread linking books 1 and 4; it is also found in Nicomachus *Enchiridion* 12 (JanS. 263). See Bower, "Sources," pp. 26–27.

130. Compare this definition of consonance (and dissonance) with Nicomachus *Enchiridion* 12 (JanS. 262.1–6); an important element in these definitions and those found in Nicomachus is the phrase "struck at the same time" (*simul pulsae*, a translation of the Greek ἅμα κρουσθέντες). Compare this definition with the discussion of consonance in André Barbera, "The Consonant Eleventh and the Expansion of the Musical Tetractys: A Study of Ancient Pythagoreanism," *Journal of Music Theory* 28 (1984), esp. pp. 192–93, and p. 217, n. 3.

sesquialter, such as 4:6, 2 measures both, which is also the difference between them. If the ratio is sesquitertian, such as 8:6, 2 also measures both.

This does not occur in other classes of inequalities which we discussed above, such as the superpartient, for if we couple 5 with 3, 2 (which is their difference) measures neither. For if 2 is set against 3, 2 is smaller, and
[221] doubled, it is larger. Likewise if 2 is set against 5, 2 is smaller; in fact, it is surpassed by 3. For this reason the superpartient is the first class of inequality logically separated from the nature of consonance.

Further, in the kind of terms which form consonances, many things are similar; in the other kind, very little is similar. This is proved in this manner: the duple is nothing other than a simple number twice; the triple nothing other than a simple number three times; the quadruple is nothing other than a simple number four times; the sesquialter is twice the half; and the sesquitertian is three times the third part.[131] This similarity is not found easily in other classes of inequality.



## 30. *How Plato says consonance is made*

Plato says that consonance is produced in the ear in the following manner. A higher sound, he says, is necessarily faster. Since it has thus sped ahead of the low sound, it enters into the ear swiftly, and, after encountering the innermost part of the ear, it turns around as though impelled with renewed motion; but now it moves more slowly and not as fast as when emitted by the original impulse, and, therefore, it is lower. When the lowered sound, now returning, first runs into the approaching low sound, it is similar, is blended with it, and, as Plato says, mixes in a consonance.[132]

## 31. *What Nicomachus holds against Plato's theory*

Nicomachus does not judge this to be accurately stated, for consonance is not of similar sounds, but rather of dissimilar, each coming into one and the same concord. Indeed, if a low sound is mixed with a low sound, it produces no consonance, for similitude does not produce concord of mu-
[222] sical utterance, but dissimilitude. While concord differs in individual pitches, it is united in intermingled ones.

Nicomachus holds that consonance is made in this way: it is not, he says, only one pulsation which emits a simple measure of sound; rather a string, struck only one time, makes many sounds, striking the air again and again. But since its velocity of percussion is such that one sound encom-

131. This explanation of superparticular ratios remains something of an enigma. The sesquialter, as 3:2, could be described as three halves related to two halves, and the sesquitertian, as 4:3, as four thirds related to three thirds. Such descriptions would be more consistent with 1.6.

132. See Plato *Timaeus* 80A–B.



passes the other, no interval of silence is perceived, and it comes to the ears as if one pitch. If, therefore, the percussions of the low sounds are commensurable with the percussions of the high sounds, as in the ratios which we discussed above, then there is no doubt that this very commensuration blends together and makes one consonance of pitches.[133]

## 32. *Which consonances precede others in merit*

Judgment should be exercised with respect to all these consonances which we have discussed; one ought to decide by the reason, as well as by the ear, which of them is the more pleasing. For as the ear is affected by sound or the eye by a visible form, in the same way the judgment of the mind is affected by numbers or continuous quantity.

Given a line or a number, nothing is easier to contemplate, with either the eye or the intellect, than its double. After this judgment concerning the double follows that of the half; after that of the half, that of the triple; after that of the triple, that of the third. Thus, since it is easier to represent the double, Nicomachus considers the diapason to be the optimum consonance; after this the diapente, which contains the half; then the diapason-plus-diapente, which contains the triple. The others he ranks according to the same method and plan.[134] Ptolemy, however, whose every opinion I shall explain later, does not treat them in this same manner.[135]

## 33. *How the things thus far said are to be taken*

All the things that are to be explained more fully later, we are now [223]
trying to explain cursorily and briefly, so that, for the present, they might accustom the mind of the reader to what might be called the surface of the subject; the mind will plunge into deeper knowledge in the subsequent treatment. Now this has been in the manner of the Pythagoreans: when something was said by the master Pythagoras, no one thereafter dared challenge the reasoning; rather, the explanation of the one teaching was authority for them. This continued until the time that the mind of the one learning—itself made stronger through more steadfast doctrine—came to discover the rationale of these same things, even without a teacher.

In this way we also commend what we have set forth to the belief of the reader. He should think that the diapason consists of the duple ratio,

133. This theory of consonance is not found in Nicomachus's extant works but is consistent with the theory of sound and ratio expressed in books 1–4 of the present work, esp. passages such as 1.3, 2.20, and 4.1.

134. See 2.20 for a complete treatment of the ranking of consonances according to Nicomachus.

135. See 5.7–12. The concluding sentence of this chapter is rather clear evidence that Boethius intended to translate the whole of Ptolemy's musical treatise.

the diapente of the sesquialter, the diatessaron of the sesquitertian, the diapason-plus-diapente of the triple, and the bis-diapason of the quadruple. In subsequent discussion we will explain very carefully both the theory of these consonances and the means by which musical consonances should be reckoned through the judgment of the ears. Fuller treatment will disclose all the other things which were discussed above: that the sesquioctave ratio produces the tone, and that it cannot be divided into two equal parts, any more than any other ratio of that class (the superparticular); that the consonance of the diatessaron consists of two tones and a semitone; that there are two semitones, one major and one minor; that the diapente is comprised of three tones and a minor semitone; and that the diapason is made up of five tones and two minor semitones and in no way adds up to six tones. All these things I shall prove both through mathematical reasoning and aural judgment.[136] Enough of this for the time being.



## 34. *What a musician is*

Now one should bear in mind that every art and also every discipline considers reason inherently more honorable than a skill which is practiced
[224] by the hand and the labor of an artisan. For it is much better and nobler to know about what someone else fashions than to execute that about which someone else knows; in fact, physical skill serves as a slave, while reason rules like a mistress. Unless the hand acts according to the will of reason, it acts in vain. How much nobler, then, is the study of music as a rational discipline than as composition and performance![137] It is as much nobler as the mind is superior to the body; for devoid of reason, one remains in servitude. Reason exercises authority and leads to what is right; for unless the authority is obeyed, an act, lacking a rational basis, will falter.

It follows, then, that rational speculation is not dependent on the act of making, whereas manual works are nothing unless they are guided by reason. Just how great the splendor and merit of reason are can be perceived from the fact that those people—the so-called men of physical skill—take their names not from a discipline, but rather from instruments; for instance, the kitharist is named after the kithara, the aulete after the

136. The present chapter, like 1.19, is pedagogical in purpose: it calls again for belief on the part of the reader, restates the basic tenets of Pythagorean theory, and promises theoretical treatment of each.

137. "As composition and performance" is a translation of *in opere efficiendi atque actu*—literally, "at the work of making and performance." Boethius is developing a threefold classification of people concerned with music in this chapter, in which "performer" and "composer" describe the first two classes, both of which, because of the servile nature of their work, are subservient to the third. There is no necessary correspondence between performer and composer in antiquity and musicians of these classes today.



aulos,[138] and the others after the names of their instruments. But a musician is one who has gained knowledge of making music by weighing with the reason, not through the servitude of work, but through the sovereignty of speculation.

We see this, of course, in the building of monuments and the waging of wars—that is, in the contrary ascription of titles; for monuments are inscribed and triumphs are celebrated with the names of those by whose authority and reason they were ordained, not with the names of those by whose labor and slavery they were completed.

Thus, there are three classes of those who are engaged in the musical art. The first class consists of those who perform on instruments, the second of those who compose songs, and the third of those who judge instrumental performance and song.

But those of the class which is dependent upon instruments and who spend their entire effort there—such as kitharists and those who prove their skill on the organ and other musical instruments—are excluded from com- [225]
prehension of musical knowledge, since, as was said, they act as slaves. None of them makes use of reason; rather, they are totally lacking in thought.

The second class of those practicing music is that of the poets, a class led to song not so much by thought and reason as by a certain natural instinct. For this reason this class, too, is separated from music.

The third class is that which acquires an ability for judging, so that it can carefully weigh rhythms and melodies and the composition as whole. This class, since it is totally grounded in reason and thought, will rightly be esteemed as musical. That person is a musician who exhibits the faculty of forming judgments according to speculation or reason relative and appropriate to music concerning modes and rhythms, the genera of songs, consonances,[139] and all the things which are to be explained subsequently, as well as concerning the songs of the poets.

138. The text at this point, *auloedus ex tibia,* is peculiar; the derivation of *auloedus* from *tibia* is by no means obvious, and the implication that it is strictly parallel to that of *citharoedus* from *cithara* does not help. Since there is a word in Latin for tibia-player—viz., *tibicen*—it is something of a puzzle why Boethius did not use it for the parallel construction. Perhaps he was being ostentatious with his knowledge of the Latin technical terms used by Cicero, for Cicero uses the term *auloedus* in *Pro Murena* 29.

139. *Permixtio,* a word which is clearly associated with consonance, is used by Boethius one other time—viz., in 2.20 [253.9]: *consonantia [est] duarum vocum rata permixtio* ("consonance is an appropriate mixture of two pitches").

# BOOK 2

[227] ## 1. *Introduction*

The preceding book laid out all the things which I now propose to demonstrate very carefully. But before I come to these things which should be taught in terms of their own particular properties, I should add a few comments. In this way the more enlightened mind of the student should be prepared to understand the things which are to be spoken of.[1]

## 2. *What Pythagoras established as philosophy*[2]

Pythagoras was the first person to call the study of wisdom "philosophy." He held that philosophy was the knowledge and study of whatever may properly and truly be said "to be." Moreover, he considered these things to be those that neither increase under tension nor decrease under pressure, things not changed by any chance occurrences. These things are

1. This brief chapter establishes the order of the second book: chaps. 2–17 contain the material whereby the student will become enlightened; of these, 2–5 review elements of arithmetic, while 6–17 present "axioms" (see last paragraph of 2.5). Following an excursion into the nature and merit of consonances (18–20), the classes of ratios and the specific ratios of consonances are demonstrated (21–27), along with the ratios of the tone, the major and minor semitone, and the comma (28–31). These ratios are the "things" that are to be "taught in terms of their own particular properties."

2. A fundamental dependence on the *De institutione arithmetica* is exhibited throughout the first 3 books of *Fundamentals of Music*. The dependence is ultimately on Nicomachus's *Eisagoge arithmetica*, since Boethius's treatise is a translation of Nicomachus's. In chaps. 2–5 of Book 2 Boethius rebuilds the philosophical and mathematical underpinnings of music as the discipline treating related quantity. Parallel passages are noted.





forms, magnitudes, qualities, relations,[3] and other things which, considered in themselves, are immutable, but which, joined to material substances, [228]
suffer radical change and are altered in many ways because of their relationship with a changeable thing.[4]

## 3. *Concerning different kinds of quantity, and the discipline with which each is associated*

According to Pythagoras all quantity is either continuous or discrete. That which is continuous is called "magnitude," whereas that which is discrete is called "multitude." The properties of these are different and even opposite.

Multitude, beginning from a finite quantity and increasing to an infinite quantity, proceeds in such a way that there is no limit to increasing. Multitude is limited with regard to the smallest term, but unlimited with regard to the larger; its origin is unity, and there is nothing smaller than unity. Multitude increases through numbers and is extended to infinity; there is no number that places a limit on its increasing.

Magnitude, on the other hand, likewise assumes a finite quantity as its measure, but it is infinitely divisible. For if there is a line one foot long, or any other length for that matter, it can be divided into two equal parts, and its half can be divided in half, and this half again divided into another half, so that there will never be any limit to dividing magnitude.

Magnitude is thus limited insofar as the larger measure is concerned, but it is infinite when it begins to divide. Number (that is, multitude), to the contrary, is limited with regard to the smallest measure but begins to be infinite when it multiplies. Although these things are in a sense infinite, nevertheless, philosophy investigates them as finite things; philosophy discovers something discrete in infinite things and, concerning discrete things, can rightly summon the acuity of its own system of thought.

Some magnitudes are fixed, such as squares, triangles, or circles, whereas others are movable, such as the sphere of the universe and what- [229]
ever is turned within it at a prescribed speed. Some quantities are discrete in themselves, such as three or four or other numbers, whereas others are discrete in relation to something, such as double, triple, and others that

3. This is the first occurrence of the word *habitudo* in the *Fundamentals of Music*, Boethius's translation of the Greek σχέσις. This term, which will here be translated "relation," is used in a general sense for any relation between two numbers and is often used as an equivalent of "ratio" (*proportio*, or λόγος). Nicomachus frequently uses σχεσις in his *Eisagoge arithmetica*, and consequently *habitudo* appears often in Boethius's *Arith*. That *habitudo* occurs only in Books 2 and 3 of *Fundamentals of Music* is an indication of just how dependent these two books are on the arithmetical treatise. For a definition and discussion of *habitudo*, see Nicomachus of Gerasa, *Introduction to Arithmetic*, pp. 307–08.

4. Concerning philosophy as the study of that which can truly be said "to be," see *Arith*. 1.1 [7.20–8.15] (Nicomachus *Eisagoge arithmetica* 1.1).

arise from comparison. Geometry speculates about fixed magnitude, while astronomy pursues knowledge of movable magnitude; arithmetic is the authority concerning quantity that is discrete in itself, whereas music is clearly expert concerning quantities related to other quantities.[5]

## 4. *Concerning different kinds of relative quantity*

We discussed sufficiently that quantity which is discrete in itself in the arithmetic books.[6] There are three simple classes of quantity in which one quantity is related to another: the first is multiple, the second superparticular, and the third superpartient. When the multiple is mixed with the superparticular or the superpartient, two other classes result: the multiple-superparticular and the multiple-superpartient.[7] The rule for all of these is as follows.

If you wish to compare unity with all other quantities in natural numerical series,[8] a fixed sequence of multiples should be formed. For two to one is duple, three to one is triple, four is quadruple, and so on in the same manner, as the following diagram [Fig. B.1] illustrates.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 3 | 4 | 5 | 6 | 7 |

If you should seek a superparticular ratio, compare the quantities in a natural numerical series with each other (with unity removed, of course).

5. Concerning the subdivision of quantity and the derivation and characters of the four mathematical disciplines, see *Arith.* 1.1 [8.15–10.7] (Nicomachus *Eisagoge arithmetica* 1.3).

6. Boethius makes reference to his treatise on arithmetic only once by its title, at 1.4 [192.19]. His more usual way of referring to it is simply *in arithmeticis*, "in the arithmetic [books]."

7. Concerning "quantity related to another" (*quantitas ad aliquid relata*), see *Arith.* 1.21 (Nicomachus *Eisagoge arithmetica* 1.17). The subdivision of related quantity is somewhat more developed in *Arith.*; there are two basic genera, equal and unequal related quantity. Unequal related quantity is further divided into "major" and "minor," depending on whether the larger or the smaller term appears first in the ratio: if the larger appears first—e.g., 3:2—it is major; if the smaller appears first—e.g., 2:3—it is minor. A ratio of the minor unequal related quantity species is denoted with the verbal prefix *sub*—e.g., subsuperparticular (*Arith.* 1.22); by analogy, a ratio of the major species would be denoted with the prefix *super*, which accounts for the appearance of such terminology in the musical treatise.

8. Boethius never defines the term *numerus naturalis*, although he uses it in a technical sense throughout Book 2 (and in *Arith.*), usually in conjunction with some form of *dispono* (*dispositio, dispositus*). From the context one can determine that one "natural number" is whole—that is, an integer. But in the present context, as well as several others, *naturalis numerus* refers to more than a single number; it implies a series of numbers generated from unity, and hence it has been translated as "a natural numerical series." See also 2.5 [230.23], 2.6 [231.18], 2.8 [236.24], 2.9 [239.27–28], 2.18 [250.4], and *Arith.* 1.23 [46.23–47.1].

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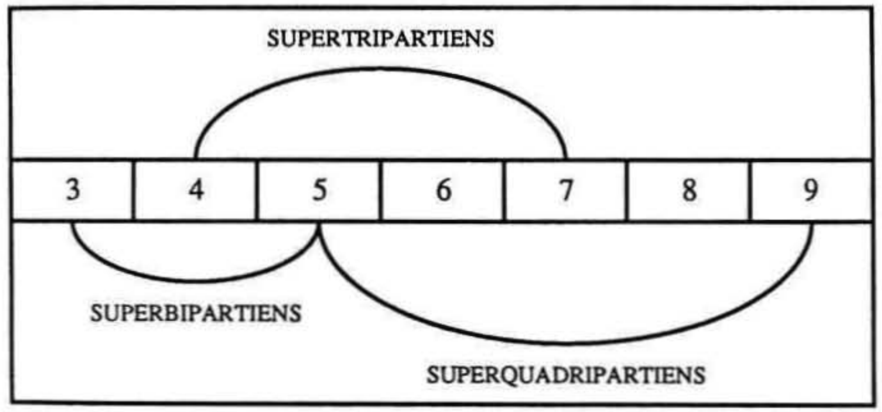
For example: three to two, which is sesquialter; four to three, which is
sesquitertian; five to four, which is sesquiquartan; and the others in the [230]
same manner, as the following diagram demonstrates [Fig. B.2].

<table>
<tr><td colspan="2">SESQUIALTER</td><td colspan="2">SESQUIQUARTAN</td><td colspan="2">SESQUISEPTIMAL</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td></td><td colspan="2">SESQUITERTIAN</td><td colspan="2">SESQUIQUINTAN</td><td></td></tr>
</table>

You will find superpartient ratios in this way: You should arrange the natural numerical series beginning from three. If you skip one number, you will see a superbipartient produced; if two, a supertripartient; if three, a superquadripartient, and likewise for subsequent numbers [Fig. B.3].




Directing attention to this same procedure, the careful reader will also see by observation the ratios put together from multiple and superparticular or multiple and superpartient. But all these things were discussed more fully in the arithmetic books.[9]

## 5. *Why multiplicity is superior to others*

In these matters it should be borne in mind that the multiple class of inequality seems far superior to the remaining two. For the formation of a natural numerical series is set out in multiples of unity, and unity is prior in the comparison.[10] Now the superparticular class is not made by compar-

9. Concerning classes of inequality, see *Arith.* 1.22 (Nicomachus *Eisagoge arithmetica* 1.17).

10. Concerning the formation and nature of multiples, see *Arith.* 1.23 (Nicomachus *Eisagoge arithmetica* 1.18).

ison with unity, but by comparison of those numbers which are set out after unity (for example, 3:2, 4:3, and so on in subsequent numbers).[11] The
[231] formation of the superpartient class is quite backwards. It is set down neither in continuous numbers—but rather by skipping over numbers—not always with an equal skip—but rather at one time over one, at another over two, at another over three, at another over four, and in this way it increases to infinity.[12] Further, multiplicity begins from unity, superparticularity from two, the superpartient from three. But this should suffice for now.

At this time it will be necessary to set out certain things which the Greeks describe as "axioms,"[13] the applicability of which will become clear when we treat individual matters through demonstration.

## 6. *What square numbers are: a reflection concerning them*[14]

A square number is one that grows by multiplying a measure by itself—for example, twice two, thrice three, four times four, five times five, six times six, of which this is a diagram [Fig. B.4]:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |

The natural numerical series set out in the upper row is thus the root of the square numbers presented in the lower; the squares, which follow one another in the lower row, are by nature continuous, for example, 4, 9, 16, and so on.

If I subtract a smaller square from a larger square that is continuous with it, what remains is the quantity that is the sum of the roots (or sides) of these same squares. For example, if I subtract 4 from 9, 5 remains, which is composed by joining 2 and 3—that is, the roots of both the
[232] squares. Likewise if I subtract 9 from 16, then 7 remains, which of course

11. Concerning the formation and nature of superparticulars, see *Arith.* 1.24 (Nichomachus *Eisagoge arithmetica* 1.19).

12. Concerning the formation and nature of superpartients, see *Arith.* 1.28 (Nicomachus *Eisagoge arithmetica* 1.20).

13. The axioms are presented in chaps. 6–17.

14. This axiom concerning square numbers seems to have no direct relevance to the subsequent axioms or to the musical proofs that follow. Perhaps the author thought it would serve as a fitting orientation to the arithmetical reasoning of the axioms and proofs. This chapter is clearly related to *Arith.* 2.12, although the latter is concerned primarily with the generation of squares from a natural numerical series, whereas the passage here seems more concerned with the "harmony" of differences between squares and their sides.





is composed by joining 3 and 4, which are the roots of the aforementioned squares. The same occurs with other square numbers.

If the square numbers are not continuous, but one between two others is omitted, then half the difference between these two will be that which is made from their roots. For example, if I subtract 4 from the square 16, 12 remains; half of this is the number that converges from the roots of both squares. The roots of the two are 2 and 4, which produce 6 when joined together. And in other squares the measure is the same.

If two squares are skipped over, a third of their difference will be the number which is made by joining together their roots. For example, if I subtract 4 from 25, two squares having been omitted, then 21 remains; the roots of these squares are 2 and 5, which produce 7, and 7 is a third part of 21.

This then is the rule: if three square numbers are skipped, then a fourth part of that which remains when the smaller square is subtracted from the larger is that which is made from the roots; if four are skipped over, then a fifth part. And so the parts will occur with the denomination of the number one degree larger than the number of squares skipped.

## 7. *All inequality proceeds from equality, and the proof thereof*

As unity is the origin of plurality and number, so equality is the origin of ratios. As we said in *Arithmetic,* once three terms have been assumed, we produce multiple ratios from equality.[15] We generate superparticular relations, on the other hand, from reversed multiples. Likewise, we make superpartient comparisons from reversed[16] superparticular ratios. For instance, let three unities be set out, or three twos or three threes or any three equal terms; then let a first term (in a subsequent row, of course) be [233]
made equal to the first, a second equal to the first plus the second, and a third equal to the first plus twice the second plus the third. The duple, the first ratio of multiplicity, is thereby made through numerical progression, as this diagram shows [Fig. B.5].

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 4 |

In this case the unity of the lower series is made equal to the unity placed

15. See *Arith.* 1.32 (Nicomachus *Eisagoge arithmetica* 1.23).

16. "Reversed" is a translation of *conversus.* Boethius means a transposition of order: the multiple (duples) 1:2:4 is reversed to produce the superparticular (sesquialters). See *Arith.* 1.32 [68.21–69.1, 70.3–8].

at first in the upper; likewise the two is equal to the first unity plus the second unity of the upper row, and similarly the four is equal to the first unity plus twice the second unity plus the third unity of the upper row, and 1, 2, 4 are in the duple ratio. If you work in the same manner from these, the triple ratio will be generated, and from the triple the quadruple, from the quadruple the quintuple, and the generation of relations continues, proceeding in this manner.[17]

If the same three numbers are taken up again, superparticular ratios may be made, as we demonstrate with one example. We now reverse them and place the larger number first: 4, 2, 1. A first term is then set down equal to the first (that is, 4), a second equal to the first plus the second (that is, 6), and a third equal to the first plus twice the second plus the third (that is, 9). When these are set out, the ratio is seen to be sesquialter [Fig. B.6].

| | | |
|---|---|---|
| 4 | 2 | 1 |
| 4 | 6 | 9 |

If this procedure is performed with triple terms, then the sesquitertian is made; if with quadruple, then the sesquiquartan. Thus with similar designations on either side, superparticular proportion[18] is born from multiplicity.

The superpartient relation is put together from reversed superparticular ratios. Let the sesquialter comparison be set out in reverse order: 9,
[234] 6, 4; then a first term should be set down equal to the first (that is, 9), a second equal to the first plus the second (that is, 15), and a third equal to the first plus twice the second plus the third (that is, 25). Let these be arranged in the lower row in this manner [Fig. B.7]:

| | | |
|---|---|---|
| 9 | 6 | 4 |
| 9 | 15 | 25 |

Thus the superbipartient relation has been produced from reversed sesquialters. If a careful student applies this process of reckoning, he produces

17. From the duple series: 1; 1 + 2 = 3; 1 + (2 × 2) + 4 = 9. Thus, from 1:2:4 the triples 1:3:9 are produced, and triples treated similarly produce quadruples, and so on.

18. This represents the first use of the word "proportion" (*proportionalitas*) in Boethius's treatise. For the distinction between "proportion" (*proportionalitas*) and "ratio" (*proportio*), see 2.12 and n. 34 below.





the supertripartient ratio from reversed sesquitertians, and he will be surprised to produce all superpartient species from superparticularity using the corresponding opposites with parallel designations.

Multiple-superparticular ratios have to be created not from reversed superparticular ratios, but rather from superparticular ratios remaining just as they were created from the multiple. From superpartient ratios remaining just as they were produced from superparticular ratios, none other than multiple-superpartient ratios will be generated. But this is enough concerning these things, for this comparison has been discussed more thoroughly in the arithmetic books.[19]

## 8. *Rules for finding any continuous superparticular ratios*

It often happens that someone discussing music might seek three, four, or some other number of equal ratios of superparticulars. But lest some error should entangle the process in difficulties through chance or through ignorance, we will produce any number of equal superparticular ratios from multiplicity by means of this rule: Any single multiple ratio, computed from unity, comes before as many superparticular relations (with comparable names in the opposite class, of course), as the multiple itself departs from unity;[20] in this way the duple ratio precedes a sesquialter, the triple a sesquitertian, the quadruple a sesquiquartan, and so on in this manner. Thus [235]
a diagram of duple terms [Fig. B.8] may be set out.

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 4 | 8 | 16 |
| | 3 | 6 | 12 | 24 |
| | | 9 | 18 | 36 |
| | | | 27 | 54 |
| | | | | 81 |

In the above diagram, the first multiple (2) has one term related to it (3) which can make a sesquialter ratio. There is nothing related to the 3, however, which could make a sesquialter with it, for it has no half. The second duple is 4; this precedes two sesquialters, 6 and 9. Nine has no half, and for this reason, nothing is related to it by a sesquialter relation. The same occurs in subsequent numbers. Triple ratios create sesquitertian ratios in the same manner. Here is a similar diagram for the triple ratio [Fig. B.9].

19. See *Arith.* 1.29 (Nicomachus *Eisagoge arithmetica* 1.22) for multiple-superparticular; *Arith.* 1.31 (Nicomachus *Eisagoge arithmetica* 1.23) for multiple-superpartient.

20. Concerning the production of continuous ratios of the superparticular class, see also *Arith.* 2.2 (Nicomachus *Eisagoge arithmetica* 2.3).

the difference went beyond them, will be a smaller ratio than 48:53, which the same difference of 5 had measured.

Indeed, larger and smaller ratios are recognized in this manner: a half is larger than a third part, a third part is larger than a fourth, a fourth part is larger than a fifth, and so on. It follows that a sesquialter ratio is larger than a sesquitertian, and a sesquitertian surpasses a sesquiquartan, and so on. From this it is evident that a ratio of superparticular numbers is always observed to be larger in smaller numbers. This is obvious in a natural numerical series. Let a natural numerical series be set out:

1 2 3 4 5

[240] Now 2 is the duple of unity, 3 the sesquialter of 2, and 4 the sesquitertian of 3. The larger numbers are 3 and 4, and the smaller 3, 2, and unity. Therefore, the smaller ratio is contained in the larger numbers, and the larger in the smaller numbers. From this it is obvious that whenever an equal plurality is added to any numbers containing a superparticular ratio, the ratio is larger before the addition of the equal plurality than after the equal plurality has been added to them.



## 10. *What is produced from multiplied multiple and superparticular ratios*[30]

It seems that something that will be demonstrated shortly should be anticipated in this place.[31] If a multiple interval is multiplied by 2, that which arises from the multiplication is multiple. But if the product of multiplication by 2 is not multiple, then that which was multiplied by 2 was not multiple.

Likewise, if a superparticular ratio is multiplied by 2, that which is produced is neither superparticular nor multiple. But if the product of a multiplication by 2 is neither multiple nor superparticular, then what was multiplied by 2 was either superparticular or from another class, but was surely not multiple.[32]

## 11. *Which superparticulars produce which multiples*[33]

To this should be added that the first two superparticular ratios make
[241] the first multiple ratio. Thus, if a sesquialter and a sesquitertian are joined together, they create a duple. Take the numbers 2, 3, and 4: 3:2 is sesquialter, 4:3 sesquitertian, and 4:2 duple. Likewise, the first multiple added to the first superparticular creates the second multiple. Take the numbers 2, 4, and 6: 4:2 is duple, the first multiple; 6:4 is sesquialter, which is the first superparticular; and 6:2 is triple, which is the second multiple. But if you add a triple to a sesquitertian, a quadruple will be produced; if you add a quadruple to a sesquiquartan, a quintuple will be produced. In this manner, by joining ratios of the multiple and superparticular classes, multiples are generated infinitely.

30. Parallels exist between this chapter and Iamblicus, *In Nicomachi arithmeticam introductionem* 77 (Pistelli ed., p. 55).

31. See 4.2, second proposition.

32. For example, the double of the sesquialter proportion 6:4 is 9:4. As 6 is to 4, so 9 is to 6, and 9:4 is neither multiple nor superparticular.

33. See also *Arith.* 2.3 (Nicomachus *Eisagoge arithmetica* 2.5).



## 12. *Concerning the arithmetic, the geometric, and the harmonic mean*

Since we have discussed the matters concerning ratios which had to be considered, we should now discuss means. A ratio is a certain comparison of two terms measured against themselves. By *terms* I mean numerical wholes. A proportion is a collection of equal ratios. A proportion consists of at least three terms, for when a first term related to a second holds the same ratio as the second to the third, then this is called a "proportion," and the "mean" among these three terms is that which is second.[34]

There is, then, a threefold classification of middle terms joining these ratios together. Either the difference between the lesser term and the mean term is equal to that of the mean and the largest, but the ratio is not equal—in the numbers 1:2:3, for example, unity alone is the difference between 1 and 2 as well as 2 and 3, but the ratio is not equal (2:1 forms a duple [242]
whereas 3:2 forms a sesquialter)—or an equal ratio is established between both pairs, but not an equal difference—in the numbers 1:2:4, for example, 2:1 is duple, as is 4:2, but the difference between 4 and 2 is 2, whereas that between 2 and unity is 1. There is a third class of mean which is characterized by neither the same ratios nor the same differences, but is derived in such a way that the largest term is related to the smallest in the same way that the difference between the larger terms is related to the difference between the lesser terms—in the numbers 3:4:6, for example, 6:3 is duple, and 2 stands between 6 and 4, while unity stands between 4 and 3, but 2 compared to unity is again duple. Therefore as the largest

34. There is a certain ambiguity in ancient arithmetic between "proportion" and the theory of means. Strictly speaking (as made clear in the definition and in the text that follows), only two or more equal ratios can form a proportion. But since two other classes of division of ratios were crucial to music theory, all three divisions (and by extension even more) came to be called "proportion," and language concerning "means" and "proportion" became equivocal. Boethius's definitions of proportion and ratio in this context seem even clearer than those in *Arith.* 2.40 [137.10–16] (Nicomachus *Eisagoge arithmetica* 2.21). A verbal relationship between the words *proportio* and *ratio* in Latin and Greek is unfortunately lost in English; *proportio* (λόγος) must be translated "ratio," and *proportionalitas* (ἀναλογία) "proportion." A clear exposition of Nicomachus's (Boethius's by extension) theory of proportion is found in d'Ooge's translation of Nicomachus *Eisagoge arithmetica*, pp. 60–65, esp. p. 264, n. 2.

term is related to the smallest, so the difference between the larger terms is related to the difference between the lesser terms.

That mean in which the differences are equal is called "arithmetic," that in which the ratios are equal "geometric," and that which we described third "harmonic."[35] We submit the following examples of them [Fig. B.15].

| ARITHMETIC | | | GEOMETRIC | | | HARMONIC | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 2 | 4 | 3 | 4 | 6 |

We are not unaware that there are also other means of ratios, which we discussed in the arithmetic books.[36] But only these three are necessary for the present discussion. Yet among these three means, only the geometric is strictly and properly called a "proportion," since it is the only one totally constructed according to equal ratios. Nevertheless, we use the same word indiscriminately, calling the others "proportions" as well.



## [243] 13. *Concerning continuous and disjunct means*[37]

Apropos of these things, some proportion is continuous, and some disjunct. Continuous proportion is that discussed above, for one and the same mean number is placed after the larger number and, at the same time, before the smaller number. On the other hand, when there are two means, then we call the proportion "disjunct," as in a geometric series of this kind: 1:2, 3:6. In this example, just as 2 is related to unity, so 6 is related to 3, and this is called "disjunct proportion." Hence it can be recognized that continuous proportion is acquired in at least three terms, disjunct, on the other hand, in four. However, proportion can be continuous in four or more terms if it occurs in this manner: 1:2:4:8:16. In this example there are not two ratios, but many, and always one fewer than there are terms given.

35. These three means—geometric, arithmetic, and harmonic—probably date back to the time of Pythagoras himself, and they occupy an important place in traditional musical speculation. Their invention and definition have been attributed to Hippasus, Philolaus, and Archytas; see Hermann Diels, *Die Fragmente der Vorsokratiker,* 11th ed., ed. Walther Kranz (Zurich and Berlin, 1964), I.18.15, p. 110 (Hippasus); I.44.A24, pp. 404–05 (Philolaus); I.47.B2, pp. 435–36 (Archytas).

36. *Arith.* 2.51–52 (Nicomachus *Eisagoge arithmetica* 2.28). See also *Arith.* 2.41 (Nicomachus *Eisagoge arithmetica* 2.22) concerning the development of ten types of proportion among the ancients.

37. Concerning continuous and disjunct means, see *Arith.* 2.40 (Nicomachus *Eisagoge arithmetica* 2.21).



## 14. *Why the means enumerated above are named as they are*

Of these means, one is named "arithmetic," because the difference between the terms is equal according to number. A second is called "geometric," because it is characterized by similarity of ratio. The "harmonic" is so named, because it is fitted together in such a way that equality of ratios is observed between differences and between extreme terms. A more thorough exposition of these things was given in the arithmetic books,[38] whereas now we run through them quickly merely to call them to mind.

## 15. *How the means discussed above arise from equality*

It is necessary to discuss briefly how these proportions are generated [244]
from equality. It has been established that just as unity governs number, so equality governs ratios; just as unity is the origin of number, so equality is the beginning of ratio.[39] Consequently the arithmetic mean arises from equality in the following manner.

Once three equal terms are given, there are two ways of producing this proportion.

A first term is set out equal to the first, a second equal to the first plus the second, and a third equal to the first plus the second plus the third. This is shown in this example: Take three unities (upper row), and let a first term of the lower row then be set down equal to the first unity (that is, 1); a second equal to the first plus the second (that is, 2); and a third equal to the first plus the second plus the third, that is, 3. This diagram [Fig. B.16] results:

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 3 |

Likewise, let three twos be set out in equality: 2 2 2 (upper row). A first term of the lower row should be made equal to the first (that is, 2), a second equal to the first plus the second (that is, 4), and a third equal to the first plus the second plus the third (that is, 6). This diagram [Fig. B.17] results:

38. The second book of *Arith.* concludes with a very thorough discussion of means and proportion (2.40–53, Masi trans., pp. 163–88; and Nicomachus *Eisagoge arithmetica* 2.21–29).

39. See above, 2.7; see also *Arith.* 1.32.

| | | |
|---|---|---|
| 2 | 2 | 2 |
| 2 | 4 | 6 |

And likewise with 3 [Fig. B.18].

| | | |
|---|---|---|
| 3 | 3 | 3 |
| 3 | 6 | 9 |

But in relation to these, it should be observed that if unity has been set down as the foundation of equality, unity will likewise be found in the differences of the numbers, and these particular numbers allow nothing to come between them. But if 2 represents equality, 2 is the difference, and one number always falls between the terms. If 3 represents equality, it is likewise the difference, and two numbers in the natural numerical succession are skipped over between the numbers, and so on in this manner.

[245] There is a second way of producing arithmetic proportion. Again take three equal terms in the upper row, and let the first in the lower row be made equal to the first plus the second, the second equal to the first plus twice the second, the third equal to the first plus twice the second plus the third. Assuming three unities, then the first of the lower row is equal to the first plus the second (that is, 2), the second equal to the first plus twice the second (that is, 3), and the third equal to the first plus twice the second plus the third (that is, 4) [Fig. B.19].

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 3 | 4 |

Here, then, unity is the difference between the terms, for unity falls between 2 and 1 and between 3 and 2; indeed, no natural number can intervene, for immediately following unity, 2 is set down in series, and after 2, 3.

The same may be accomplished with 2, so take three twos, and let the first term be equal to the first plus the second (that is, 4), the second equal to the first plus twice the second (that is 6), and the third equal to the first plus twice the second plus the third (that is, 8) [Fig. B.20].





| | | |
|---|---|---|
| 2 | 2 | 2 |
| 4 | 6 | 8 |

Here 2 likewise holds the difference between the terms, and one number naturally falls between them, for 5 naturally occurs between 4 and 6, and 7 between 6 and 8.

But if 3 is the foundation of equality, 3 will constitute the difference, with one less than this number always skipped between the terms; the same is observed with 4 and 5. The careful reader, with these same rules, will discover for himself the things about which we now remain silent for the sake of brevity.

We showed how geometric proportion can be obtained from equality when we were demonstrating how all inequality flows from equality.[40] [246] Nevertheless, unless it is bothersome, it should now be repeated again briefly. When three equal terms have been set out as an upper row, the first of the lower row is made equal to the first, the second equal to the first plus the second, and the third equal to the first plus twice the second plus the third. The same procedure may be continued. In this way, geometric proportion takes its first principle from equality. We discussed the properties of these ratios very thoroughly in the arithmetic books,[41] so if the reader instructed in these matters approaches this, he will not be disturbed by any error of doubt.

The harmonic mean, which should now be discussed a little more fully, is generated according to this reasoning.[42] It should be created, if we desire to produce duple proportion, through setting out three equal terms in the upper row, and then making the first of the lower row equal to the first plus twice the second, the second equal to twice the first plus twice the second, and the third equal to the first plus twice the second plus three times the third. Take three unities in this way [Fig. B.21].

| | | |
|---|---|---|
| 1 | 1 | 1 |

The first term should be made equal to the first plus twice the second (that is, 3), the second equal to twice the first plus twice the second (that is, 4),

40. See above, 2.7.

41. See *Arith.* 2.44 concerning the properties of geometric proportion.

42. This method of setting forth the harmonic mean shows strong parallels with a text presented in Iamblicus *In Nicomachi arithmeticam introductionem* 157 (Pistelli ed., pp. 111–12). For a discussion of the relationship between Boethius, Nicomachus, and Iamblicus (in particular with regard to 2.9, 15, and 16), see Pizzani, "Fonti," pp. 66–78.

and the third equal to the first plus twice the second plus three times the third (that is, 6). If the equality is established with 2, or with 3, the same calculation of the mean appears, with the terms and their differences being spaced according to the duple, as the following diagrams show [Fig. B.22].

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 4 | 6 |

| | | |
|---|---|---|
| 2 | 2 | 2 |
| 6 | 8 | 12 |

| | | |
|---|---|---|
| 3 | 3 | 3 |
| 9 | 12 | 18 |

But if it is required that there be a triple ratio between the extreme terms, set out three equal terms in the upper row, and make the first of the lower
[247] row from the first plus the second, the second from the first plus twice the second, and the third from the first plus twice the second plus three times the third, as the following diagram shows [Fig. B.23].

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 3 | 6 |

| | | |
|---|---|---|
| 2 | 2 | 2 |
| 4 | 6 | 12 |

| | | |
|---|---|---|
| 3 | 3 | 3 |
| 6 | 9 | 18 |

## 16. *Concerning the harmonic mean: a much fuller investigation of it*[43]

Since we have embarked upon a discussion of harmony, I do not believe we should tacitly pass over things which can be discussed more thoroughly. Thus, a harmonic proportion should be set out in a lower row, and the differences between its terms placed between them in an upper row, as in this diagram [Fig. B.24].

| DIFFERENCES | | |
|---|---|---|
| 1 | 2 | |
| 3 | 4 | 6 |
| TERMS | | |

Do you not see then that 4:3 produces a consonance of the diatessaron, 6:4 yields a diapente, 6:3 mixes a consonance of the diapason, and their

43. Concerning the harmonic mean, see *Arith.* 2.48 (Nicomachus *Eisagoge arithmetica* 2.26) and Iamblicus *In Nicomachi arithmeticam introductionem* 152–54 (Pistelli ed., pp. 108–09).





differences themselves again bring forth the same consonance? For 2:1 is duple, fixed in the consonance of the diapason. When the extreme terms are multiplied by themselves, and the mean term is increased by multiplication of itself, then the numbers compared will hold the relation and concord of a tone.[44] For 3 times 6 makes 18, 4 times 4 makes 16; the number 18, of course, surpasses the smaller number 16 by an eighth part of 16. Again, if the smallest term is multiplied by itself, it will make 9; if the larger term is increased through multiplication of itself, it will make 36. The numbers 9 and 36 set in relation with each other hold the quadruple—that is, the consonance of the bis-diapason. If we inspect these numbers carefully, everything will be seen to be either multiplication of differences or of terms by themselves. For if the smallest term is multiplied by the [248]
mean, it makes 12; likewise if the smallest term is multiplied by the largest, it makes 18. If the mean term is increased by the quantity of the largest, it makes 24; moreover if the smallest term is increased by itself, it makes 9, and if the mean is multiplied in the same manner, it makes 16. If 6, which is the largest, is multiplied by itself, it makes 36. Therefore these should be set out in series:

36 24 18 16 12 9

The terms sounding the consonance of a diatessaron are 24:18 and 12:9; the diapente 18:12, 24:16, and 36:24; the triple, which is the diapason-plus-diapente, 36:12; the quadruple, which is the bis-diapason, 36:9; the epogdous, which is a tone, is held in the comparison of 18 to 16.

## 17. *How the means discussed above are each in turn placed between two terms*

Two terms are likely to be presented and arranged in such a way that we sometimes place an arithmetic mean between them, sometimes a geometric mean, and sometimes a harmonic mean. We also discussed these things in the arithmetic books,[45] but we should nevertheless explain the same thing here briefly.

If the arithmetic mean is required, the difference of the given terms must be sought and then divided and added to the smaller term. Thus, let

44. The words in this passage, *toni habitudinem concordiamque,* could merely imply "ratio and sound of a tone," but *concordia* is a term Boethius uses as synonymous with *consonantia.* The tone is obviously not a consonance, however, despite the language.

45. See *Arith.* 2.50 (Nicomachus *Eisagoge arithmetica* 2.27), where the exact same processes are described and the same numbers are used. The passage in *Arith.* places the computation of means in *musical* terms—viz., the drilling of holes in a reed and the tuning of strings (Masi trans., p. 180). The principle of placing arithmetic means will be used in book 4 for the division of chromatic and enharmonic genera on the monochord (see book 4, nn. 58–59).

the terms 10 and, on the other side, 40 be set out, and let their mean be sought according to arithmetic proportion. I first consider the difference between the two, which is 30. This I divide, which makes 15. I add this to the smaller term, 10, which makes 25. Thus, if this mean is located between
[249] 40 and 10, an arithmetic proportion will be made in this manner [Fig. B.25].

| | | |
|---|---|---|
| 10 | 25 | 40 |

Likewise if we wish to place a geometric mean between the same terms, we multiply the extremes: 10 times 40 makes 400. We take the square root of this, which makes 20, for 20 times 20 produces 400. If we place this mean, 20, between 10 and 40, the geometric mean is made, as set out in the following diagram [Fig. B.26].

| | | |
|---|---|---|
| 10 | 20 | 40 |

Now if we seek the harmonic mean, we add the extremes: 10 plus 40 makes 50. We multiply the difference of these terms, which is 30, by the smaller term, 10, and 10 times 30 makes 300. This we divide by the 50, which makes 6; when we add this to the smaller term, 16 is produced. If we then place this number between 10 and 40, a harmonic proportion is displayed [Fig. B.27].

| | | |
|---|---|---|
| 10 | 16 | 40 |

## 18. *Concerning the merit or measure of consonances according to Nicomachus*

Enough concerning these matters. Now we should add the justification given by the Pythagoreans for associating musical consonances with the ratios discussed above. Ptolemy appears not to have agreed with them about this; we shall speak about this later.[46]

The consonance whose property the critical faculty more easily comprehends ought to be classified as the very first and most pleasing consonance. For just as every single thing is in itself, so also is it recognized by

46. For Ptolemy's criticisms of the Pythagoreans and his classification of relative pitches, see 5.8–12.





the critical faculty.[47] Thus, if that consonance which consists of the duple ratio is easier to know than all the others, then there is no doubt that the consonance of the diapason, since it precedes the others in being known, is the first of all and surpasses the others in merit. The remaining consonances, according to the Pythagoreans, necessarily hold a rank determined
by increments of multiple ratios and the reductions of superparticular re- [250]
lations. Now it has been demonstrated that multiple inequality should transcend superparticular ratios in priority of value.[48] Therefore let a natural numerical series be set out from unity to 4.

1 2 3 4

The 2 compared to 1 makes the duple ratio and produces that consonance of the diapason which is the most excellent and, because of its simplicity, the most knowable. If 3 is related to unity, it resounds the consonance of the diapason-plus-diapente. The 4 related to unity holds the quadruple, producing, of course, the consonance of the bis-diapason. But if the 3 is placed in relation to 2, it adds the consonance of the diapente; if 4 to 3, the diatessaron. This, then, is the ranking of these when each is compared to every other. One comparison now remains: if we relate 4 to 2, they fall in the duple ratio, which 2 held in relation to unity. Sounds then are at their greatest distance in the bis-diapason, since they are separated from each other by a quadruple measure of interval. The closest sounds forming a consonance between themselves seem to occur when the higher surpasses the lower by a third part of the lower. And so the measure of consonances comes to a halt: it can neither be extended beyond the quadruple nor reduced to less than a third part. According to Nicomachus,[49] then, this is the ranking of consonances: first is the diapason; second, diapason-plus-diapente; third, bis-diapason; fourth, diapente; and fifth, diatessaron.

## 19. *The opinion of Eubulides and Hippasus concerning the ranking of consonances*

Eubulides[50] and Hippasus[51] maintain a different ranking of consonances, for they say that increments of multiplicity correspond to diminu-

47. "Critical faculty" is a translation of *sensus,* a word with a broad spectrum of meaning, ranging from "perception through the senses" to "understanding." Boethius, or his Pythagorean source, is obviously not arguing that "as every single thing is in itself, so it is perceived by the sense"; to do so would blatantly contradict the basic tenet of Pythagorean thought that the senses are unreliable (see, e.g., 1.9). "Critical faculty" seems the best translation for *sensus* in this context.

48. See above, 2.5.

49. The ranking of consonances is not found in any extant work of Nicomachus.

50. No works or fragments of the early Pythagorean Eubulides survive. In fact, this is probably the only specific theory that can be definitively attributed to him; see Diels, *Vorsokratiker,* I.14.8, pp. 99, 1.

Although Nicomachus said much concerning this matter,[70] we, in bringing to light in part the very things the Pythagoreans affirm and in arguing in part the same consequences, have demonstrated—inasmuch as we have been able to do so briefly—that if a diatessaron is added to the consonance of the diapason, a consonance cannot result. I will discuss later what Ptolemy thought about this.[71] But enough of this. Now the semitone must be considered.

## 28. *Concerning the semitone: in what smallest numbers it is found*

It seems that semitones were so named not because they are truly halves of tones, but because they are not complete tones.[72] The size of the interval which we now call "semitone," but which was called "limma" or "diesis" by the ancients,[73] is determined as follows: when two sesquioctave relations (which are tones) are subtracted from a sesquitertian ratio (which is a diatessaron), an interval called a "semitone" remains. Let us try to write the two tones in a continuous arrangement. As was said, since these consist of the sesquioctave ratio, and we cannot join two continuous sesqui-
[261] octave ratios unless a multiple from which these can be derived is found, let unity be set out and also its first octuple, 8. From this I can derive one sesquioctave. But since I am seeking two, 8 should be multiplied by 8, making 64. This will be the second octuple, from which we can extract two sesquioctave ratios, for 8 (an eighth part of 64 unities), when added to the same, makes the total sum of 72. Similarly, let an eighth part of this be taken, which is 9, and it gives 81. And these two first continuous tones are written down in this arrangement:

64 : 72 : 81

70. Nothing concerning the consonance or dissonance of the diapason-plus-diatessaron is found in the extant works of Nicomachus. Concerning the history of the theory of this interval, see C. André Barbera, "The Consonant Eleventh and the Expansion of the Musical Tetractys: A Study of Ancient Pythagoreanism," *Journal of Music Theory* 28 (1964): 191–223.

71. See 5.9–10.

72. See 1.17.

73. *Limma* is the technical term traditionally used in Greek music theory for the interval discursively called "semitone." The traditional definition of this interval is "that which remains after two tones have been subtracted from the consonance of the diatessaron" (see the definition which follows in Boethius's text and that in Ptolemy *Harmonica* 1.10 (Düring ed., 23.2); *limma* thus means remainder—i.e., that which remains after the subtraction. The remainder after two tones have been subtracted from a diatessaron, the limma, is to be distinguished from the "remnant," that which remains after a limma has been subtracted from a tone, which is named *apotome* (see 2.30 and n. 79).

The term *diesis* is not used consistently: normally it is reserved for the very small interval (quarter-tone) found in the enharmonic genus (see, e.g., 1.21); but in Pythagorean circles it is used for the limma and the minor semitone as well (see Theon of Smyrna *Expositio* 12, ed. Dupuis, p. 87).





Now we should seek out the sesquitertian of 64 unities. But since 64 proves not to have a third part, then, if all these numbers are multiplied by 3, the third part forthwith comes into being, and all remain in the same ratio as they were before the multiplier 3 was applied to them. Thus, let 64 be multiplied by 3, which makes 192. A third of this (64) added to it produces 256. Then 256:192 is the sesquitertian ratio, which holds the consonance of the diatessaron. Now let us assemble in appropriate series the two sesquioctave ratios to 192, ratios that will be contained in two numbers: let 72 be multiplied by 3, making 216, and 81 by 3, making 243. Let these be arranged between the two terms cited above in this manner:

192 : 216 : 243 : 256

In this arrangement of ratios, the ratio of the first number to the last constitutes the consonance of the diatessaron, and those of the first to the second and the second to the third contain two identical tones. The interval that remains consists of the ratio 243:256, which constitutes in smallest integers the form of the semitone.

## 29. *Demonstrations that 243:256 is not half a tone* [262]

I am showing, then, that the interval of 243:256 is not the full magnitude of half a tone. The difference between 243 and 256 is contained in only 13 unities, which (13)[74] cover less than an eighteenth part of the smaller term, but more than a nineteenth part (for if you multiply 13 by 18, you will make 234, which by no means is equal to 243, and if you multiply 13 by 19, it surpasses 243). Every semitone, if it holds a full half of a tone, ought to be placed between the sixteenth part and the seventeenth—which will be demonstrated later.[75]

Now[76] it will become clear that such an interval of a semitone, if doubled, cannot complete one interval of a tone. So let us, without further

74. Friedlein's *qui .XIII. minus quidem quam minoris* [262.6] gave medieval scribes and scholars difficulty. Of the control manuscripts used for the present translation, none agree with Friedlein without qualifications. *M* and *R* read the same, but in both sources *xiii* has been corrected from *ccxliii*. *Q* and *T* read *qui minus quidem*, thus omitting any number; but this reading was changed from *quia ccxliii quidem* in both. In *Q*, the gloss *id est xiii* is written above *qui*, and *id est ccxliii* above *minoris;* given the alteration and the glosses, *Q* presents the clearest and most unequivocal reading. *V* follows the reading of *Q* and *T* with no alteration, omitting the gloss for *qui* but repeating the gloss for *minoris*. All other sources (*I, K, P, S*) give *qui ccxliii quidem*, a reading that makes no sense. Both numbers, 13 and 243, probably represent scribal attempts to clarify the antecedent of *qui*. The ambiguity arises because the relative pronoun for *unitates*, the actual noun which is antecedent, should be *quae* rather than *qui*.

75. See 3.1.

76. Friedlein's punctuation and capitalization are ambiguous at this point [262.13–14]: the comma at the end of line 13 should be a period; line 14 ("*Nunc illud* . . .") begins a new argument.

delay, arrange two such ratios continuous with each other that contain the same relation as 256:243, according to the rule presented above.[77] So let us multiply 256 by itself, and put the result as the largest term: 65,536; likewise 243 is increased by its own quantity, and the result is the smallest term: 59,049; again 256 is increased by the number 243, and this gives the number 62,208. Let the mean term be set down in this manner:

65,536 : 62,208 : 59,049

Therefore 256 and 243 are in the same ratio as 65,536 and 62,208, as well as 62,208 and 59,049. But the largest term of these (65,536) to the smallest (59,049) does not produce one whole tone. But if the ratio of the first to
[263] the second, which is equal to the ratio of the second to the third, should prove to be whole semitones, the two halves joined together would necessarily produce one tone. Since the ratio of the extreme terms is not sesquioctave, it is clear that these two intervals do not represent true halves of tones, for whatever is half of something, if it is doubled, makes that of which it is said to be the half. If it cannot fill that, then the part that is doubled is less than a half part; whereas if it exceeds it, it is more than a half part. Furthermore, it will be proved that 65,536 does not make a sesquioctave ratio with 59,049 unities if an eighth part of 59,049 is taken according to the rules that were given in the arithmetic books. Since this eighth part does not consist of a whole number, we leave the computing of the eighth part to the diligence of readers.[78] It is thus evident that the ratio consisting of 256:243 is not a whole half of a tone. That which is truly called "semitone" is, then, less than a half part of the tone.

## 30. *Concerning the larger part of the tone: in what smallest numbers it consists*

The remaining part, which is larger, is called "apotome" by the Greeks, whereas it can be called "remnant" by us.[79] For nature has so ordered things that when something is cut in such a way that it is not divided into equal parts, to the degree that the smaller part is less than half, by the same degree the larger part exceeds the half and is larger than the

77. See 2.8.

78. The reference here to "arithmetic books" is general, rather than specific. The person so instructed would have computed the difference between 65,536 and 59,049 and determined that it does not "measure" 59,049 eight times. An eighth part of 59,049 is 7,381 1/8, which added to 59,049 makes 66,430 1/8. The fraction of 1/8 is difficult to express in ancient and medieval mathematics, and one wonders if this might not be why the computation is left to the reader.

79. Boethius translates the Greek *apotome* with the Latin *decisio*—literally, "that which is cut away." No Latin treatise prior to Boethius equates the terms *apotome* and *decisio*. This "remnant" is the interval left after a *limma* (see above, n. 73) has been subtracted from a tone.





same. Therefore to the degree that the minor semitone is smaller than half a tone, to that same degree the apotome surpasses half a tone. And since we have taught that the semitone in its first instance stands in the ratio of
256 to 243, we should now prove in what smallest numbers the interval [264]
called apotome consists. If 243 could admit division by an eighth part, which would allow a sesquioctave ratio to be formed with it, then the relation of 256 to the sesquioctave of the smaller number would reveal the apotome with incontrovertible reasoning. Since, however, it is known that an eighth part of it is lacking, let both numbers be multiplied by 8. From 243 multiplied by 8 we get the number 1,944. If to this we add its eighth (243), we get 2,187. Let 256 be multiplied by 8, making 2,048. This number may now be set down in the middle of the terms cited above.

1,944 : 2,048 : 2,187

Here the third term holds the ratio of a tone with the first, while the second holds that of a minor semitone with the first; the third to the second thus forms the apotome. So it appears that the ratio of the apotome consists in these smallest integers, since the interval of a semitone is contained in the smallest numbers 243 and 256. The numbers 1,944 and 2,048 are in the same ratio as 243 and 256, since they were obtained by multiplying 256 and 243 by 8. For if one number multiplies any other two numbers, those born from that multiplication will be in the same ratio as were those numbers which the first number multiplied.

## 31. *Of what ratio the diapente and diapason consist; furthermore, the diapason does not consist of six tones*

Since we have discussed the consonance of the diatessaron at some
length, we should examine the consonance of the diapason and the diapente [265]
briefly and, one might say, with simple numbers. The diapente consists of three tones and a semitone—that is, of a diatessaron plus a tone. Let those numbers be set out which the diagram above[80] encompassed:

192 : 216 : 243 : 256

In this disposition, the first term to the second and the second to the third hold ratios of tones, but the third to the fourth that of the minor semitone, as demonstrated above.[81] If an eighth part of the 256 is added to the same (of which it is an eighth), it will make 288, which, compared to 192, produces the interval of the sesquialter ratio. There are then three tones, if the first is related to the second, the second to the third, and the fifth to

80. These numbers are found as a diagram (*descriptio*) in 1.17, Fig. A.8. They are also found set out in the text in 2.28.

81. See 2.28.

# BOOK 3

## [268] 1. *Demonstration against Aristoxenus that a superparticular ratio cannot be divided into equal parts, and for that reason neither can the tone*

In the above book it was demonstrated that the consonance of the diatessaron was made by joining two tones and a semitone, the diapente three tones and a semitone, but that these semitones, if treated through examination in and of themselves, cannot make an integral half of a tone, and therefore, that a diapason can by no measure extend to six tones. But the musician Aristoxenus, yielding all things to the judgment of the ears,[1] did not, following the Pythagoreans, consider these semitones to be smaller than the half. Rather, just as they are called "semi" tones, he considered that they were halves of tones.[2] Therefore, it must be argued and proved once again, albeit briefly, that no superparticular relation can be divided
[269] into an integral half by any known number. For between two numbers comprising a superparticular ratio, whether they are the smallest integers—between which the difference is unity—or subsequent numbers, no middle number can be placed in such a way that the smallest term holds the ratio with the middle that the middle holds with the last, as in a geometric ratio. But either the middle term can produce equal differences, so that there is equality according to an arithmetic mean, or the middle term placed between these same terms will make a harmonic mean or some other mean that we mentioned in the arithmetic books.[3] But if this can be proved, the argument that a sesquioctave ratio (which is the tone) can be separated

1. Aristoxenus *Harmonica* 2.33–34.
2. Ibid., 2.46, 56–57, et al.
3. See *Arith.* 2.56–57.





into halves will not stand, since every sesquioctave ratio is located in the superparticular class of inequality.

This will be more easily proved by induction. If among individual ratios—superparticular, of course—when they are examined carefully, none is found that is divided into equal ratios by a term positioned in the middle, then there is no doubt that the superparticular comparison cannot be divided into equal parts.

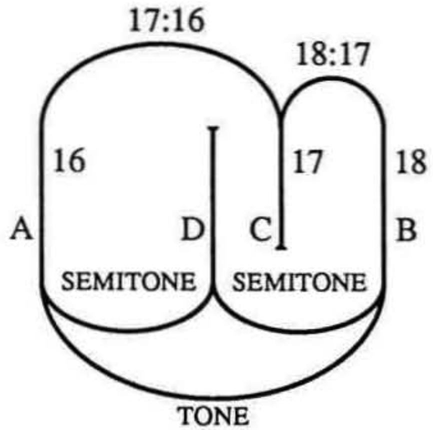
Just because something seems to sound consonant to the ears when some vague pitch is compared to another pitch at a distance of two tones and an integral semitone does not mean that it is actually consonant; for inasmuch as each sense is unable to grasp things that are very small, so the sense of hearing cannot distinguish this difference that progresses beyond consonance. But perhaps the difference may come to be perceived if such particles continually increase through these same errors. For that which is scarcely discerned in the smallest thing, when placed alongside another and joined with it, is clearly perceived, for then it begins to be of some magnitude.

So from which ratio should we begin? Will we not find a shortcut in the inquiry if we begin with the one in question? That is, with whether the tone can be divided into two equal parts. So now we should thoroughly discuss the tone and demonstrate just how it cannot be divided into two
equal parts. If this demonstration is applied to other superparticular com- [270]
parisons, it will be similarly shown that a superparticular cannot be divided into equal parts by any integral number known.

The first numbers containing the tone are 8 and 9. But since these follow each other in natural sequence in such a way that there is no mean number between them, I multiply both these numbers by two, which, of course, is the smallest I can use. This makes 16 and 18. Between these a number, 17, falls naturally. Thus 18:16 is a tone, but 18 compared to 17 contains the latter wholly plus 1/17 part of it. Now 1/17 part is naturally smaller than 1/16 part, so the ratio contained in the numbers 16 and 17 is larger than that between 17 and 18. Let these numbers be set out in this manner: let 16 be A, 17 C, and 18 B.[4] Therefore, an integral *half* of a tone will by no means fall between C and B. For the ratio CB is smaller than the ratio CA. Thus a proportional half should be placed within the larger part. Let the half be D. Then, since the ratio DB (which is half of a tone) is larger than the ratio CB (which is the minor part of a tone), but the ratio AC (which is the larger part of a tone) is larger than the ratio AD (which

4. Some manuscripts (e.g., Munich, Bayerische Staatsbibliothek, Clm 18,478, and Clm 6,361) give Fig. C.1 at this place in the text rather than in its proper place later in the chapter. Friedlein was thus led to place the letters and numbers of this passage on a separate line, thereby creating something of an additional diagram at the beginning of book 3. The control manuscripts present these letters and numbers as text, and Fig. C.1 appears at the end of this paragraph.

is half of a tone), and since the ratio AC is 17:16 and CB is 18:17, there is no doubt that an integral half falls between 17:16 and 18:17. But this
[271] will by no means be discovered in an integral number [Fig. C.1].




Since the number 17 compared with the number 16 creates the ratio 17:16,[5] let us work out a sixteenth part of the same number 17; that will be unity plus a sixteenth part of unity. If we join this together with the same number 17, it will make 18 and 1/16 part. But if 18 1/16 is compared with the number 16, it is seen to exceed the proper measure of a tone; for with 16 only the number 18 holds the sesquioctave ratio. Whence it follows that since the ratio 17:16 extended twice transcends a tone, it is not an integral half of a tone. For when something taken twice transcends another, the former evidently surpasses half the latter. For that reason the ratio 17:16 will not be half a tone. And because of this, no other ratio larger than the ratio 17:16 can be half a tone, since 17:16 itself is larger than an integral half of a tone.

But since the ratio 18:17 follows next after 17:16, we should see whether, multiplied by two, it will not fill a tone. The term 18 contains 17 plus one part of 17. So if we produce another number in relation to 18 with the same ratio that 18 has to 17, it will be 19 and 1/17 part. But if we produce a number situated in the sesquioctave ratio in relation to the term

5. Throughout this chapter and the next, Boethius names the ratios 17:16, 18:17, and 19:18 *sesquisextusdecimus, sesquiseptimusdecimus,* and *sesquioctavusdecimus* respectively. In the present paragraph the terms *sesquisextusdecimus* and *sesquiseptimusdecimus* are prefixed with the term *super*—e.g., *supersesquiseptimusdecimus.* No attempt is made to translate the *super,* and the terms are rendered as numerical ratios. The prefix is probably intended to specify that the larger term is to be given first position in the ratio. See also *Arith.* 1.22–24, where major and minor classes of quantity are discussed, the major class having the larger term in first position, the minor class the smaller term; ratios in the minor class use the prefix *sub* with the name of the ratio—e.g., *subsesquitertius*—and, by analogy, the major type might use the prefix *super.*





17, it will make 19 and 1/8 part. An eighth part is larger than a seventeenth [272]
part, so the ratio of numbers 17 and 19 1/8 is larger than that comprised of 17 and 19 1/17 (which, of course, consists of two continuous 18:17 ratios). Thus, two continuous ratios of 18:17 are seen not to complete one tone. Therefore, 18:17 is not half a tone, since these terms, when duplicated, do not fill a whole; they do not form halves, for a half, when doubled, is always equal to that of which it is half.

## 2. *Half of a tone does not remain when two tones are subtracted from a sesquitertian ratio*

Now if we set out those numbers that remain after two tones are taken away from a sesquitertian ratio, we can determine whether that ratio which remains should be reckoned as the space of an integral semitone. If it were found to be that, then the consonance of the diatessaron would also be proved to be made up of two tones and an integral semitone. A first term was given above, 192, to which 256 formed the sesquitertian ratio.[6] To this first term, 216 made a tone, and moreover, to 216, the number 243 made the space of a tone. Thus what remains from the whole diatessaron ratio is that relation which consists in 243 and 256 unities. So if this proves to be half an integral tone, there can be no doubt that the diatessaron consists of two tones and a semitone.

Since it has been demonstrated that half a tone is located between the
ratios 17:16 and 18:17, this comparison, 256:243, ought to be measured in [273]
relation to that ratio. Lest our progress be hindered, I take an eighteenth part of 243, which is 13 1/2. If I add this to 243, it makes 256 1/2. Thus the ratio 256:243 appears to be less than the relation 19:18. But if the larger "half-tone" is in the ratio 17:16, the smaller in the ratio 18:17,[7] then, since

6. See 1.22, 2.28.

7. Friedlein's reading of the first part of this sentence [273.7–8 and apparatus] cannot be justified by manuscripts from the ninth century. It clearly represents a later textual tradition and is found in only two of the five sources he used for the chapter: Clm 14,480 (*f* in the edition) and Clm 6,361 (*h* in the edition). (Of these two, only the latter gives the reading without alterations to an earlier version.) The text for the first part of this sentence, according to Friedlein, is *Quod si dimidius tonus minor quidem est sesquisextadecima, maior vero sesquiseptimadecima proportione* ("But if the half-tone is larger than the ratio 17:16, but smaller than the ratio 18:17"). This version is correct arithmetically, and it articulates clearly the argument that Boethius is making. It may, in fact, represent a "better" version of this passage. But it does not reflect the earliest manuscripts of the treatise. The text according to *M, Q,* and *V* should read *Quod si dimidius tonus maior quidem est in sesquisextadecima, minor vero in sesquiseptimadecima proportione.* The same reading is found in *B, R, S,* and *T,* but with the *in* of each clause added above the line. *I, K,* and *P* omit the prepositions, and *maior* and *minor* are expunged in *P* and altered to give the later reading found in Friedlein. If the two ablatives (the ratios) are read as ablatives of comparison, only Friedlein's version makes sense; but they can be read as ablatives of respect, particularly given the prepositions, and the

## 5. *How Philolaus divided the tone*

Philolaus, a Pythagorean, tried to divide the tone in another manner,[14] postulating that the tone had its origin in the number that constitutes the first cube of the first odd number—for that number was highly revered among the Pythagoreans. Since 3 is the first odd number, if you multiply 3 by 3, then this by 3, 27 necessarily arises, which stands at the distance of a tone from the number 24, the same 3 being the difference. For 3 is an eighth part of the quantity 24, and, added to the same, it gives the first
[277] cube of 3, 27. From this number, 27, Philolaus made two parts, one that is more than half, which he called the "apotome," and the remainder, which is less than half, which he called the "diesis." (The diesis later came to be called the "minor semitone.") The difference between these he called the "comma."

To begin with, Philolaus thought that the diesis consisted of 13 unities, because this had been discerned to be the difference between 256 and 243, and because the same number—that is, 13—consists of 9, 3, and unity, of which unity holds the place of the point, 3 the first odd line,[15] and 9 the first odd square. Because of all this, he identified 13 as the diesis, which he called the "semitone"; the remaining part of the number 27, comprised of 14 unities, he set down to be the apotome. But since unity is the difference between 13 and 14, he said that unity ought to be considered to represent the comma. So he gave the whole tone 27 unities, for 27 is the difference between 216 and 243,[16] which stand at the interval of a tone.

## 6. *The tone consists of two semitones and a comma*

From all this it is easily seen that the tone consists of two minor semitones and a comma. For if the total tone consists of an apotome and a semitone, whereas a semitone differs from an apotome by a comma, an apotome is nothing other than a minor semitone and a comma. Thus, if two minor semitones are subtracted from a tone, the remainder is a comma.

## 7. *Demonstration that there is the difference of a comma between a tone and two semitones*

The same can also be proved in this manner. If the diapason is com-
[278] prised of five tones and two minor semitones, and six tones exceed the

14. This brief chapter and also 3.8, attributed to the late fifth-century B.C. Pythagorean Philolaus, are found in no other extant source. A third fragment attributed to Philolaus is found in Nicomachus *Enchiridion* 9 (JanS. 252–53); see Diels, *Vorsokratiker*, I.44, pp. 398–415.

15. Concerning linear numbers, see *Arith.* 2.5.

16. Friedlein 277.17: *quod inter .CCXVI. ab .CCXLIII.* should read: *quod inter .CCXVI. ac .CCXLIII.*





consonance of a diapason by one comma, there is no doubt that if five tones are subtracted from the diapason, the remainder will be two minor semitones, whereas from six tones the remainder will be a tone. Moreover, this tone exceeds these remaining semitones by a comma. But if a comma is combined with the same two semitones, together they would equal a tone. Therefore one tone has as its equivalent two minor semitones and a comma; the comma is discovered to equal the first integer 7,153.

## 8. *Concerning intervals smaller than a semitone*

Philolaus incorporates these and intervals smaller than these in the following definitions.[17]

> The *diesis*, he says, is the interval by which a sesquitertian ratio is larger than two tones.
>
> The *comma* is the interval by which the sesquioctave ratio is larger than two dieses—that is, larger than two minor semitones.
>
> The *schisma* is half a *comma*.
>
> The *diaschisma* is half a *diesis*—that is, half a minor semitone.

From these definitions it can be concluded that since the tone is first divided into a semitone and an apotome, it is also divided into two semitones and a comma; whence it follows that the tone may be divided into four diaschismata and a comma. So an integral half of a tone (which is a semitone) consists of two diaschismata, which make up one minor semitone, and a schisma, which is half a comma. Since the total tone is joined together from two minor semitones and a comma, if someone wants to divide it equally, he should produce one minor semitone and half a comma. But one minor semitone is divided into two diaschismata, whereas half a comma is one schisma. Therefore it is properly said that half a tone can be properly divided into two diaschismata and one schisma; whence it follows that an integral semitone is seen to differ from a minor semitone by one schisma. An apotome, on the other hand, is different from a minor semi-
tone by two schismata, for it is different by a comma, and two schismata [279]
make one comma.

## 9. *Perceiving the parts of the tone by means of consonances*

But enough concerning these things. Now it seems that we should state how we can calculate intervals, on the one hand, to reach a higher pitch, and, on the other hand, a lower pitch, under the control of musical con-

17. See above, n. 14.

scholar should be to blend these two faculties into a concord.[15] Hence Ptolemy is quite critical of Aristoxenus and the Pythagoreans in this matter, for Aristoxenus does not trust reason at all but only the senses, while the Pythagoreans are too little concerned with the senses and too much concerned with the ratios yielded by reason.

## 4. *The basis of high and low pitch according to Aristoxenus, the Pythagoreans, and Ptolemy*[16]

All agree that sound is percussion of the air.[17] The followers of Aristoxenus and the Pythagoreans account for the difference between high and low pitch by means of contrasting theories. Aristoxenus expresses the judgment that differences between sounds with respect to highness or lowness are qualitative.[18] The Pythagoreans, on the other hand, hold that these differences are quantitative.[19] Ptolemy appears to side with the Pythagoreans insofar as he also thinks that the basis for high and low pitch does not reside in quality, but rather in quantity, because he holds that more compact, thinner bodies emit high pitch, and less compact, very large bodies emit low pitch. At this moment nothing is said concerning the measure
[356] of tension or slackness, although when something becomes slack, it becomes, as it were, less dense and thicker, whereas when it becomes tauter, it is restored to a more compact condition and is stretched thinner.[20]

## 5. *Ptolemy's opinion concerning differences between sounds*

With these matters set in order, Ptolemy divides differences between sounds in this manner. Of pitches, some are unison, while others are not. Unison pitches are those between which one sound occurs, either high or low. Pitches are not unison when one is lower, another higher. Of these non-unison pitches, some are such that their difference is not defined by a point of distinction between them, for their difference is not discrete, but

15. Boethius's general rendering of Ptolemy's "goal of harmonics" is accurate, particularly in the opening sentence of this paragraph [Düring 5.14–19]. However, this particular musical metaphor—the blending of sense and reason into a concord—is not found in Ptolemy.

16. This chapter presents a highly condensed version of Ptolemy *Harmonica* 2.3. Ptolemy does not mention Aristoxenus or the Pythagoreans in his treatment of the problem, but discusses quality of sound—e.g., tone color and loudness—and establishes that the difference between high and low sound is quantitative.

17. See, e.g., 1.3 above.

18. Although "qualitative" is the antithesis of "quantitative," it is a superficial description of Aristoxenus's theory of pitch (see Aristoxenus *Harmonica* 1.10–13). This theory of sound is ascribed to Aristoxenus by Boethius, but not by Ptolemy.

19. See, e.g., the opening of *Sectio canonis* (JanS. 148–49); compare 4.1 above.

20. This closing statement may represent a concession to Aristoxenus, who held that tension and relaxation were the "causes" of high and low "quality" of sounds (*Harmonica* 1.10–13).





is so drawn from the low to the high that it seems continuous. There are other non-unison pitches between which the difference is marked by an intervening silence.[21]

Pitches not defined by a point of distinction occur in this manner. Just as when a rainbow is observed, the colors are so close to one another that no definite line separates one color from the other—rather it changes from red to yellow, for example, in such a way that continuous mutation into the following color occurs with no clearly defined median falling between them—so also this may often occur in pitches. If someone strikes a string and—while it is sounding—tightens it, it happens that at first the pulsation is lower, whereas when it is tightened, the pitch becomes sharper, and the sounding is continuous between the low pitch and the high.[22]

## 6. *Which pitches are appropriate for harmony*[23]

Thus some non-unison pitches are continuous, others discrete. Continuous pitches are such that the difference between them is joined by a [357]
continuous line, and the high pitch—or the low—does not maintain a clearly defined position. Discrete pitches, on the other hand, have their own positions, just as unmixed colors do, between which a difference is perceived by virtue of a clearly established position.

Non-unison[24] pitches that are continuous are not considered by the faculty of harmonics, for they are dissimilar from each other and yield no single entity of sound. Discrete pitches, on the other hand, are subject to the harmonic discipline, for the difference between dissimilar pitches separated by an interval can be comprehended; such pitches, which, when joined together, can make a melody are called ἐμμελής. Those that cannot make a melody when joined are called ἐκμελής.

## 7. *What number of ratios the Pythagoreans proposed*

Those pitches are called consonant that make intermingled and pleasant sounds when joined together, while those are dissonant which do not.

21. The first paragraph of this chapter is loosely related to the first part of Ptolemy *Harmonica* 1.4 [Düring 9.29–10.5].

22. This second paragraph is a slightly expanded paraphrase of the second part of Ptolemy *Harmonica* 1.4 [Düring 10.6–9]. Although Boethius elaborates on Ptolemy's comparison of continuous sound to a rainbow, he omits Ptolemy's comparisons of low sound of the continuous kind to the bellow of an ox and high sound of this kind to the howl of a wolf [Düring 10.9–11].

23. This chapter is a paraphrase of Ptolemy *Harmonica* 1.4 [Düring 10.11–25], except for the last sentence [Düring 10.25–28], which forms the beginning of the following chapter of Boethius. Concerning continuous and disjunct pitches, see above, 1.12.

24. Boethius's text here reads *non aequisonae* [257.5]; it should read *non unisonae*.

This represents the judgment of Ptolemy concerning the different kinds of sounds.

At this point it seems that we should discuss how he disagrees with others concerning the disposition of consonances.[25]

The Pythagoreans held that the consonances of the diapente and the diatessaron were simple consonances, and from these joined together they formed one consonance of the diapason. There are, moreover, the diapente-plus-diapason and the bis-dispason, the former in triple, the latter in quadruple ratio. But they did not consider the diapason-plus-diatessaron to be a consonance, on the grounds that it does not fall among superparticular or multiple comparisons, but among the multiple-superpartient. Its ratio of pitches is 8:3, for if someone were to place a 4 in the middle of these, he would make these terms: 8:4:3. Of these 8:4 yields the consonance of a diapason, 4:3 that of a diatessaron. The ratio 8:3 is situated in the multiple-superpartient class of inequality. One should know what a multiple-super-
[358] partient comparison is from the arithmetic books and the things we discussed in the second book of this work.[26]

The Pythagoreans posit consonances in the multiple and the superparticular classes, as was discussed in the second and fourth books, but they dissociate consonances from the superpartient and from the multiple-superpartient classes. One should look in the second and fourth books of this treatise on the fundamentals of music to discover how the Pythagoreans associate the diapason with the duple, the diatessaron with the sesquitertian, and the diapente with the sesquialter.[27]

## 8. *Ptolemy's criticism of the Pythagoreans with respect to the number of ratios*[28]

Ptolemy reproves the Pythagoreans and rejects the proof that we have expounded in previous books by which, through various means, they associate the diapente with the sesquialter and the diatessaron with the sesquitertian but relate no consonances whatsoever to other superparticular ratios, even though these are of the same class.

25. The remainder of this chapter is an abridgment of Ptolemy *Harmonica* 1.5. The Pythagorean position has been presented thoroughly in the first four books.

26. See *Arith.* 1.31 and above, 2.4.

27. Concerning the disposition of consonances by the Pythagoreans (Nicomachus, Eubulides, and Hippasus), see 2.18–20. Concerning the association of certain consonances with certain ratios, see 2.21–27 and 4.2.

28. In this chapter and the two that follow, Boethius reorganizes and paraphrases Ptolemy *Harmonica* 1.6, albeit omitting many details of Ptolemy's arguments. The present chapter is a translation of a short passage from the middle of Ptolemy's chapter [Düring 13.23–14.2].



## 9. *Demonstration according to Ptolemy that the diapason-plus-diatessaron is a consonance*[29]

Ptolemy proves that a particular consonance is made from the diapason-plus-diatessaron in this manner. The consonance of the diapason produces a conjunction of pitch such that the string seems to be one and the same. Even the Pythagoreans agree on this point. For this reason, if some consonance is added to the diapason, it is preserved whole and inviolate, as though the consonance added to the consonance of the diapason were [359]
added to only one string.

Take the consonance of the diapason contained between the hypate meson and the nete diezeugmenon. These two combine and are united together in sound so that one pitch, as if produced from one string and not mixed from two, strikes the hearing. If we joined any consonance to this consonance of the diapason, its consonance would be preserved intact, because it would be joined as though it proceeded from only one sound and one string. If two upper diatessarons are joined, one to the hypate meson and one to the nete diezeugmenon—the nete hyperboleon to the nete diezeugmenon and the mese to the hypate meson—the two extremes will sound consonant with each other. Moreover, the mese will sound consonant with the nete diezeugmenon, as will the mese with the hypate meson; likewise, the nete hyperboleon will sound consonant with the nete diezeugmenon and also with the hypate meson. Furthermore, if the two lower consonances of the diatessaron are tuned, one between the hypate hypaton and the hypate meson, the other between the paramese and the nete diezeugmenon, the hypate hypaton will sound consonant with the hypate meson and the nete diezeugmenon, and the paramese will sound consonant with the nete diezeugmenon and the hypate meson, but in such a manner that the lower note maintains a consonance of the diatessaron with that closest to it, but a consonance of the diatessaron-plus-diapason with that farthest from it. The hypate hypaton forms the diatessaron with the hypate meson, a diatessaron-plus-diapason with the nete diezeugmenon. [360]
Likewise, the nete hyperboleon makes a consonance of the diatessaron above with that closest to it, the nete diezeugmenon, but a consonance of the diatessaron-plus-diapason with the hypate meson.

## 10. *The property of the consonance of the diapason*[30]

Ptolemy argues that this occurs because the diapason is almost like a single pitch, and that it forms such a consonance by creating, as it were, a

29. The basic contents of this chapter follow the opening argument of Ptolemy *Harmonica* 1.6 [Düring 13.1–23], but the references to specific musical notes, however, are not found in Ptolemy.

30. This chapter is based on a brief remark of Ptolemy in *Harmonica* 1.6 comparing the diapason with the number 10 [Düring 13.6–7]. The expansion of the comparison and the recapitulation of Ptolemy's theory that concludes this chapter are not found in *Harmonica*.