THE DODECAHEDRON IN PLATO'S *TIMAEUS*

I

ἔτι δὲ οὔσης συστάσεως μιᾶς πέμπτης, ἐπὶ τὸ πᾶν ὁ θεὸς αὐτῇ κατεχρήσατο ἐκεῖνο διαζωγραφῶν. (*Timaeus* 55 C)

Plato's treatment of the dodecahedron in the *Timaeus* is confined to this brief statement of wide implication. Of the five regular Platonic polyhedra, the dodecahedron is the only geometric solid not assigned to one of the basic elements[1]). It is placed fifth and last in the series, although in terms of the number of faces in its structure it is fourth.

F. M. Cornford translates the passage: 'There remained one construction, the fifth; and the god used it for the whole, making a pattern of animal figures thereon.'[2]). There is an immediate and obvious difficulty in translating *διαζωγραφῶν* as *making a pattern of animal figures*. The section of the *Timaeus* (53 C–57 D) in which the statement is found is concerned only with the description of the mathematical properties of the primary bodies. Plato's attention is here devoted exclusively to abstract concepts involving mathematics and physics; a description of anything decorated with animal figures is out of place in the context. A. E. Taylor interprets the word to mean *broidering figures on it*, taking *ζῷα* as the regular word for figures in a picture or a piece of tapestry, and the *ζῷα* of this passage to be the constellations, including those of the Zodiac. Taylor's interpretation is founded upon a statement in *Timaeus Locrus* on the dodecahedron and the sphere and upon Plutarch's analysis of the pentagonal faces of the dodecahedron[3]). This is a point to which the

1) The faces of the tetrahedron of fire, the octahedron of air, and the icosahedron of water are composed of equilateral triangles; the faces of the cube of earth are composed of right-angled isosceles triangles (half-squares). The pentagonal faces of the dodecahedron cannot be used to form any of the basic elements. A detailed examination of the triangular surfaces of the solids is found in Cornford, *Plato's Cosmology* (New York, 1957), 211 ff. and is continued by Pohle, "The Mathematical Foundations of Plato's Atomic Physics", *Isis* 62 (1971), 36–46.

2) Cornford (above, note 1) 218.

3) Taylor, *A Commentary on Plato's Timaeus* (Oxford, 1928), 377: Timaeus Locrus 98 E: *Τὸ δὲ δωδεκάεδρον εἰκόνα τοῦ παντὸς ἐστάσατο, ἔγγιστα*

discussion will return; first, however, it is necessary that *διαζωγραφῶν* be examined to determine both a general range of meaning current in the fifth and fourth centuries B.C. (particularly in the Platonic Corpus) and a specific application appropriate to this passage.

The basic meaning of *ζωγραφεῖν* is *to paint from life* (*Republic* 598B); an extension of this meaning is *to adorn* (as with paint). This more general meaning is found even earlier in Aristophanes, *Ecclesiazusae* 996. In the *Philebus* Plato describes *τὰ φαντάσματα* as *ἐζωγραφημένα*, purely mental pleasures as *ἡδοναί ... ἐζωγραφημέναι*, an image a man has of himself as *ἐνεζωγραφημένον*, and images generally as *γεγραμμένα* (*Philebus* 40A–B); in each instance the meaning is very close to the English *depict*. *Philebus* 39B and 39D demonstrate a conscious transition and extension of *ζωγράφον/ζωγραφήματα* and *γραμματιστὴν/γράμματα* from concrete to conceptual depiction. The *depiction* or *portrayal* of a circle in concrete form is described as *τὸ ζωγραφούμενον* at *Ep.* VII, 342C. The usage is of interest even in light of the questionable authenticity of *Ep.* VII as a whole and of this passage (342A–244D) in particular[4]). It is significant for present considerations that *ἐνζωγραφεῖν* and *ζωγραφεῖν* are very close in meaning to *γράφειν*.

In a similar fashion *γραφή* is used by Plato to signify an outline or *delineation*[5]), although the verb *γράφειν* in uncompounded form appears not to have been used in the mathematical sense of *describe* prior to Euclid[6]). Similarly, *διαγραφή* means *delineation* (*Republic* 501A), and *διάγραμμα* means *geometrical figure* (*Phaedrus* 73B), but the verb *διαγράφειν*, while used by Plato to signify *to*

σφαίρας ἐόν (ed. Marg [Leiden, 1972], 136). Plutarch, *Platonicae Quaestiones* 5.1 (*Moralia* VI. 1, ed. Hubert, B. T. [Leipzig, 1959]), discusses the geometry of the dodecahedron and its proximity to the sphere with direct reference to *Timaeus* 55 C, concluding with the remark *διὸ καὶ δοκεῖ τὸν ζωδιακὸν ἅμα καὶ τὸν ἐνιαυτὸν ἀπομιμεῖσθαι ταῖς διανομαῖς τῶν μοιρῶν ἰσαρίθμοις οὔσαις*. The notion is predicated upon the division of each of the twelve pentagonal faces of the dodecahedron (months) into thirty scalene triangles (days). Cf. below, note 17.

4) The "Doctrine of the Fifth" in *Ep.* VII has little or no bearing upon the interpretation of *Tim.* 55 C. Cf. Edelstein, *Plato's Seventh Letter* (Leiden, 1966), 87 n. 37.

5) The form *καταγραφὴν* is used at *Smp.* 193 A, or as conjectured by Ruhnken, *κατὰ γραφήν*. The meaning is probably the same in either case. Cf. *The Symposium of Plato*[2], ed. Bury (Cambridge, 1932), 67.

6) *Tht.* 147 D: *περὶ δυνάμεών τι ἡμῖν Θεόδωρος ὅδε ἔγραφε* refers to the composition of a treatise on roots.

delineate, does not appear to have extensive application to the construction of mathematical figures[7]).

In the light of these considerations, it is not necessary that *διαζωγραφῶν* in the *Timaeus* mean specifically *painting*, or *making a pattern of animal figures*, or *embroidering*, or have any reference to astronomy. The context requires, in fact demands, only the meaning *delineating*[8]), and when taken with *ἐκεῖνο* referring to

---

7) A. J. Festugière, *Proclus, Commentaire sur le Timée* (Paris, 1966) I, 95.2 (60.27) is quite definite in his interpretation of Proclus' understanding of the word as used by Plato: "'Représentation graphique' est tout ce que Proclus a vu ici dans le mot *διαζωγραφεῖν*, et, même chez Platon, je doute qu'il y ait à chercher une allusion soit aux animaux du zodiaque (Cornford, 219) soit même à la balle formée de douze pièces de cuir aux couleurs variée dont parle le *Phédon* (110 B), bien que Proclus déjà ait fait le rapprochement, 1. IV (t. III) 141.22 ss. ..." Further, at IV, 180.2 (141.23) his observation is that "*γραφή* [in Proclus] est dessin ou peinture, *ζωγραφεῖν* dessiner ou peindre d'après la nature vivante, d'où généralement dessiner ou peindre. *διαζωγραφεῖν* (*Tim.* 55 C 6) ajoute sans doute la nuance qu'il s'agit là d'un 'arrangement final' (*δια-*), comme l'entend Rivaud. Ni chez Proclus en tout cas (*ὥσπερ ὁ οὐρανὸς τῷ δωδεκαέδρῳ παρὰ τοῦ δημιουργοῦ διεζωγράφηται* 141.23 s.) ni chez le Ps. Olympiodore, in *Phaed.* 199.4 Norvin (*πῶς δωδεκασκύτῳ σφαίρᾳ ἔοικεν; ἢ ὅτι μέχρι αὐτῆς πρόεισι τὸ δωδεκάεδρον, ᾧ διαζωγραφεῖ τὸ πᾶν ὁ Τίμαιος, ὃν ἐκεῖ λέγεται τρόπον*), il n'est question de 'figures brodées sur le Ciel' ... ou 'de figures animales peintes au Ciel' ..." Proclus has a consistent view of the relation of the dodecahedron to the sphere and to *τὸ πᾶν*. Cf. Festugière I, 31 (7.1); I, 98 (63.12); III, 255 (208.20); III, 280 (234.19); and III, 325 (281.22).

8) Eva Sachs in *Die Fünf Platonischen Körper* (Berlin, 1917), 47, in considering this passage presents a similar interpretation: "Ich übersetze die Worte: 'Da noch eine körperliche Figur, die fünfte, übrig war, so verwandte sie Gott für das All, indem er dessen Grundriß entwarf.' *διαζῳγραφεῖν* ist schwer zu erklären, *ζῳγραφεῖν* heißt es nicht, weil es sich um Malen mit Farben handelte, sondern weil der Kosmos ein *ζῷον* ist, *διαγράφειν* aber ist 'Linien durchziehen'. Wie das gemeint ist, zeigt Platons Staat 500e, wo die Philosophen sagen: 'Der Staat wird auf keine andere Weise jemals die Glückseligkeit erreichen, als wenn die Grundlinien zu seinem Entwurf Maler zeichnen, die das göttliche Modell benutzen'." *Ἀλλ' ἐὰν δὴ αἴσθωνται οἱ πολλοὶ ὅτι ἀληθῆ περὶ αὐτοῦ λέγομεν, χαλεπανοῦσι δὴ τοῖς φιλοσόφοις καὶ ἀπιστήσουσιν ἡμῖν λέγουσιν ὡς οὐκ ἄν ποτε ἄλλως εὐδαιμονήσειε πόλις, εἰ μὴ αὐτὴν διαγράψειαν οἱ τῷ θείῳ παραδείγματι χρώμενοι ζωγράφοι;* (*R.* VI, 500 D-E). Cf. also Gadamer, *Idee und Wirklichkeit in Platos Timaios* (Heidelberg, 1974), 25, n. 11: "Daß dagegen der Werker einen Vorentwurf, sozusagen einen Umriß des All anfertigt und dafür den dem Volumen des Kreises am nächsten kommenden regelmäßigen Körper verwendet, ist plausibel. Dafür, daß in *δια* die schematisierende Tätigkeit der Umrißkonstruktion wie in *διαγραφεῖν* vorliegt, vgl. Rep. 501 A, Arist. Top. 105 b13. Hier will Platon durch das Wort wohl daran erinnern, dass es ein *ζῷον* ist, das Universum, um dessen Grundriß es sich handelt."

τὸ πᾶν, it means that the Demiurge *delineates the whole* in that fifth solid. Conversely, the geometrical delineation of the dodecahedron and its surfaces provides an insight into the integrated physical structure of the whole.

II

In the *Timaeus* matter is described in terms of the four Empedoclean elements Fire, Air, Water, and Earth. Fire and Earth are the two extremes linked by the two intermediate elements Air and Water (*Timaeus* 31B). All of the four regular solids that correspond to the elements can be inscribed in a sphere. The remaining regular solid is the dodecahedron, and while it is not identified with an element, and while it too can be inscribed in a sphere, it occupies an intermediate position between the four polyhedra of the elements and the figure of the sphere. In his commentary Cornford correctly observes of the dodecahedron that "... the Demiurge 'uses it for the whole', i.e. for the sphere, to which the figure approaches most nearly in volume...", citing *Timaeus Locrus* and Wyttenbach's note on *Phaedo* 110B where the spherical earth is compared to a ball made of twelve pentagonal pieces of leather[9]).

The language used to describe the formation of the sphere reflects a construction proceeding from the elements to the whole incorporating each of the elements in the structure of the whole:

Τῶν δὲ δὴ τεττάρων ἓν ὅλον ἕκαστον εἴληφεν ἡ τοῦ κόσμου σύστασις. ἐκ γὰρ πυρὸς παντὸς ὕδατός τε καὶ ἀέρος καὶ γῆς συνέστησεν αὐτὸν ὁ συνιστάς, μέρος οὐδὲν οὐδενὸς οὐδὲ δύναμιν ἔξωθεν ὑπολιπών, κτλ. (*Timaeus* 32C)

Now the frame of the world took up the whole of each of these four; he who put it together made it consist of all the fire and water and air and earth, leaving no part or power of any one of them outside. ... (tr. Cornford, p. 52)

Σχῆμα δὲ ἔδωκεν αὐτῷ τὸ πρέπον καὶ τὸ συγγενές. τῷ δὲ τὰ πάντα ἐν αὐτῷ ζῷα περιέχειν μέλλοντι ζῴῳ πρέπον ἂν εἴη σχῆμα τὸ περιειληφὸς ἐν αὐτῷ πάντα ὁπόσα σχήματα. διὸ καὶ σφαιροειδές, ἐκ μέσου πάντῃ πρὸς τὰς τελευτὰς ἴσον ἀπέχον, κυκλοτερὲς αὐτὸ

9) Cornford (above, note 1), 219.

ἐτορνεύσατο, πάντων τελεώτατον ὁμοιότατόν τε αὐτὸ ἑαυτῷ σχημάτων, κτλ. (*Timaeus* 33 B)

And for shape he gave it that which is fitting and akin to its nature. For the living creature that was to embrace all living creatures within itself, the fitting shape would be the figure that comprehends in itself all the figures there are; accordingly, he turned its shape rounded and spherical, equidistant every way from centre to extremity – a figure the most perfect and uniform of all; .... (tr. Cornford, p. 54)

... *λεῖον καὶ ὁμαλὸν πανταχῇ τε ἐκ μέσου ἴσον καὶ ὅλον καὶ τέλεον ἐκ τελέων σωμάτων σῶμα ἐποίησεν· κτλ.* (*Timaeus* 34 B)

... he made it smooth and uniform, everywhere equidistant from its centre, a body whole and complete, with complete bodies for its parts. (tr. Cornford, p. 58)

The all-embracing figure, *τὸ τοῦ παντὸς σῶμα* (*Tim.* 31 B), is given the most perfect shape comprehending in itself all the figures. The words *σχῆμα τὸ περιειληφὸς ἐν αὐτῷ πάντα ὁπόσα σχήματα* (*Timaeus* 33 B) must refer to all five of the regular solids[10]) inasmuch as they are the primary shapes of creation. If this be the case, then the dodecahedron has a place in the construction of the whole. The question of what utility, if any, it has in the formation and continuing function of the material universe is perhaps not wholly amenable to solution[11]). However, within

---

10) The dodecahedron has an intermediate position between the sphere and the four elements which is perhaps implied in the discussion of intermediate existence (*Tim.* 35 A), although no explicit mention is made of it. Similarly, the extreme and mean ratios are employed in the line used to form the pentagon in Pythagorean mathematics (below, note 21), but Plato makes no specific reference to the ratios in his discussion of the dodecahedron or in the passage on the harmonics of the World Soul (*Tim.* 35 B –36 B).

11) The connection of the fifth element with aether made by Xenocrates: *τὰ μὲν οὖν ζῷα πάλιν οὕτω διῃρεῖτο εἰς ἰδέας τε καὶ μέρη πάντα τρόπον διαιρῶν, ἕως εἰς τὰ πάντων στοιχεῖα ἀφίκετο τῶν ζῴων, ἃ δὴ πέντε σχήματα καὶ σώματα ὠνόμαζεν, εἰς αἰθέρα καὶ πῦρ καὶ ὕδωρ καὶ γῆν καὶ ἀέρα* (*apud* Simp. *in Ph.* VIII 1 *Comm. in Arist. Graeca* X, p. 1165.33 = Brandis, *Scholia in Arist.* 427ᵃ15 in *Aristotelis Opera* ed. Gigon, v. IV [Berlin, 1961]. Cf. also Simp. *in Cael.* I 2, *Comm. in Arist. Graeca* VII, p. 12.16 and I 3, v. VII, p. 87.19 = Brandis, *Scholia* 470ᵃ24.) is not to be read into Plato's physics. Plato himself says at *Tim.* 58 C-D: *Μετὰ δὴ ταῦτα δεῖ νοεῖν ὅτι πυρός τε γένη πολλὰ γέγονεν... κατὰ ταὐτὰ δὲ ἀέρος, τὸ μὲν εὐαγέστατον ἐπίκλην αἰθὴρ καλούμενος, ὁ δὲ θολερώτατος ὁμίχλη τε καὶ σκότος, κτλ.* The discussion of aether as a fifth element is entirely appropriate in the context of Aristotle's physics (*de Caelo* 270b22); on the importance of the suspect fragments of Philolaus (*Vorsokrat.*[12] I 44 B) and of the Platonic *Epinomis* 981 C in the

the context of the dialogue and apart from its material form, the dodecahedron has a discernible geometrical significance in its relation to the other four solids.

It is notable that in the *Timaeus* the solids are described in terms of ordinary speech. The presentation very probably reflects Plato's procedure of constructing the solids by putting together triangular surfaces rather than by means of geometrical propositions[12]. If one assumes that as he wrote, Plato had in front of him solid models of the figures[13] with the triangles inscribed on the faces, then his insistence upon the interchangeability of the triangles of three of the solids (fire, air, and water) and the separate nature of the fourth (earth) (*Tim.* 54B) can be seen to stem from his method of observation in which only the surfaces of the solids are considered rather than from a conviction that the two types of triangles are incompatible in the sense that no relation pertains between them in the solids. The tetrahedron, octahedron, and icosahedron are all composed of equilateral triangles. The surfaces of the polyhedra composed of equilateral triangles are subdivided into half-equilateral triangles having the proportions 1, 2, $\sqrt{3}$[14]. Theoretically the surfaces of any one of these polyhedra could be reduced to the component triangles and then reformed into any one of the other solids in

---

interpretation of *Tim.* 55 C, cf. Martin, *Études sur la Timée de Platon* (Paris, 1841), II, 133, Note XXXVIII. iii and 247, Note LXIX. ii; Zeller, *Plato and the Older Academy* [*Philosophie der Griechen*, II,2,2 *Plato und die ältere Akademie*] (London, 1888; repr. New York, 1962), 273f.; Moraux in *R.E.* XXIV *s. quinta essentia*, col. 1181 sqq. Cf. also Claghorn, *Aristotle's Criticism of Plato's 'Timaeus'* (The Hague, 1954), 22, n. 11 where one must note a slight misinterpretation of Zeller's observations; and Cornford (above, note 1), 221.

12) Cornford (above, note 1), 212f.

13) Early Celtic and Etruscan stone dodecahedra are cited by Rivaud, *Timée-Critias* [Budé ed. vol. X] (Paris, 1925), 82; Burnet, *Early Greek Philosophy*[4] (London, 1930), 284 n. 1; and the more extensive citations of Burkert, *Lore and Science in Ancient Pythagoreanism* (Cambridge, Mass., 1972), 460, n. 65. Decorated bronze dodecahedra of the third century A.D. are illustrated by Deonna, "Les dodécaèdres gallo-romains en bronze, ajourés et bouletés", *Association Pro Aventico, Bulletin* 16 (1954), 19–89.

14) Taylor (above, note 3), 373ff.; Cornford (above, note 1), 210ff.; Friedländer, *Plato*[2] (Princeton, 1969), I, 246–60. The decomposition of the primary bodies and the interpretations of the process given by Cornford and Rivaud are discussed by Pohle, "Dimensional Concepts and the Interpretation of Plato's Physics", *Exegesis and Argument; Phronesis, Supplementary Volume I* (Assen, 1973), 306–323.

the series (*Timaeus* 53E and 54B–C)[15]). Inasmuch as the cube is formed of the half-square or right-angled isosceles triangle (1, 1, $\sqrt{2}$), the triangles will not serve to form any of the other solids, producing an apparent lapse in the otherwise consistent elaboration of the movement from point or line to a solid[16]). Quite clearly the pentagonal faces of the dodecahedron cannot be generated by the simple expedient of putting together combinations of either of the preferred triangles[17]).

The two types of surface triangles then have no apparent necessary relationship in the four material-forming solids when the solids are viewed as individual entities. Nor are the polyhedra of the *Timaeus* examined in section. If any attempt had been made to do so, one would expect Plato to have discussed, for example, the relationship of the tetrahedron to the cube: the tetrahedron can be inscribed in the cube, but not in any way that would show one of the triangular sides on the face of the cube; each edge of the tetrahedron appears as a diagonal of a square face of the cube[18]). The square, however, is present as the median base of the pyramids of equilateral triangles forming the octahedron; and two pyramids of equilateral triangles with pentagonal bases are employed in the construction of the

15) The criticism of the interchange leveled by Aristotle is detailed at length by Cherniss, *Aristotle's Criticism of Plato and the Academy* (Baltimore, 1944), I, 148ff.

16) The discussion by Taylor (above, note 3), 364–373 is of particular interest, as is the survey of the geometrical theories of the Early Academy by Philip, "The 'Pythagorean' Theory of the Derivation of Magnitudes", *Phoenix* 20 (1966), 32–50.

17) Martin (above, note 11), II, 245ff., Note LXIX. i comments critically upon the subdivision of the pentagon into triangles given by Alcinous and Plutarch and followed by Stallbaum. A diagram is given by Heath, *A History of Greek Mathematics* (Oxford, 1921), I, 296. Lasserre, *The Birth of Mathematics in the Age of Plato* (London, 1964), 76 is of the opinion that it may have been Theaetetus who first established that the straight line joining two successive angles of a pentagon intersect in extreme and mean ratio, and that the construction of the pentagon employing isosceles triangles was already known.

18) Heron in his *Definitiones* 104 (p. 66 Heiberg) reports Archimedes to have said that Plato was familiar with the semi-regular solids that may have been constructed through the sectioning of the cube (Heath, [above, note 17], I, 295). The cube and the octahedron can be sectioned for the purpose of forming semi-regular solids, and while it is possible that the principle was known at the time Plato wrote, he makes no mention of it. However, the observer of the solid models can readily see that the cross-section of the dodecahedron produces a hexagon, a figure not otherwise associated with a structure formed of pentagons.

icosahedron. The outline of the square in the octahedron and of the pentagon in the icosahedron appears as the solid model is turned[19]). It is a short step to the observation that the right-angled isosceles triangle is a component in the structure of the octahedron, a solid composed entirely of faces formed of equilateral triangles, just as the pentagon is a component in the structure of the icosahedron. None of these properties of the polygons or polyhedra is commented upon by Plato, and yet they form a body of observations which are easily made, are indeed inescapable, even when the solid models are examined independently of any formal mathematical analysis[20]).

Whether or not the mathematics of the *Timaeus* is Pythagorean[21]), it is reasonable to assume that Plato was familiar with the early Pythagorean construction of the star pentagon and of the dodecahedron from twelve pentagons. In view of his remarks in the *Republic* (528 A–E) on the incomplete state of knowledge about the regular solids, and the likelihood that Theaetetus was the first mathematician "to construct all five [solids] theoretically and investigate fully their relations to one another and the circumscribing spheres"[22]), the absence of a

---

19) The pentagons of the icosahedron are described in Euclid XIII. 16 (Heiberg, *Euclidis Elementa* [Leipzig, 1885], IV. 305; Heath, *The Thirteen Books of Euclid's Elements*² (New York, 1956), III, 481; Lasserre (above, note 17), 75. Later mathematical analyses of the solid are discussed by Bruins, "The Icosahedron from Heron to Pappos", *Janus* 46 (1957), 173–182.

20) Although the Theory of Forms is re-emergent in the *Timaeus* and is applicable to the primary bodies (*Tim.* 51B–52C. Cf. Cornford [above, note 1], 188ff. and Morrow, "Plato's Theory of the Primary Bodies in the *Timaeus* and the Later Doctrine of Forms", *AGPh* 50 [1968], 12–28), it is not brought to bear specifically upon the present points of consideration.

21) Burkert (above, note 13), 70f., and Heath, "Excursus I, Pythagoras and the Pythagoreans", *The Thirteen Books of Euclid's Elements* (above, note 19), I, 413. The Pythagorean construction of the pentagon probably involved the juxtaposition of isosceles triangles displaying the division of a line into extreme and mean ratios as in Euclid II, 11; IV, 10 & 11; XII, 8 (Stamatis *post* Heiberg, *Euclidis Elementa* [Leipzig, 1969], I, 86 & 165–68; Heiberg, [above, note 19], IV, 269). Cf. Heath (above, note 19), II, 98f. and Stapleton, "Ancient and Modern Aspects of Pythagoreanism", *Osiris* 13 (1958), 12–45 (38f.). A connection between the very early work of Babylonian mathematicians on the pentagon and that of Pythagoras has been posited by de Vogel, *Pythagoras and Early Pythagoreanism* (Assen, 1966), 39f. Appendix A, "*On The Babylonian Origin of the Pentagram,*" 292ff. Heron's later treatment is analyzed by Bruins, "Regular Polygons in Babylonian and Greek Mathematics", *Janus* 48 (1959), 5–23 (15f.).

22) Heath (above, note 17), I, 160ff.: "It would be easy to conclude that the dodecahedron is inscribable in a sphere, and to find the centre of

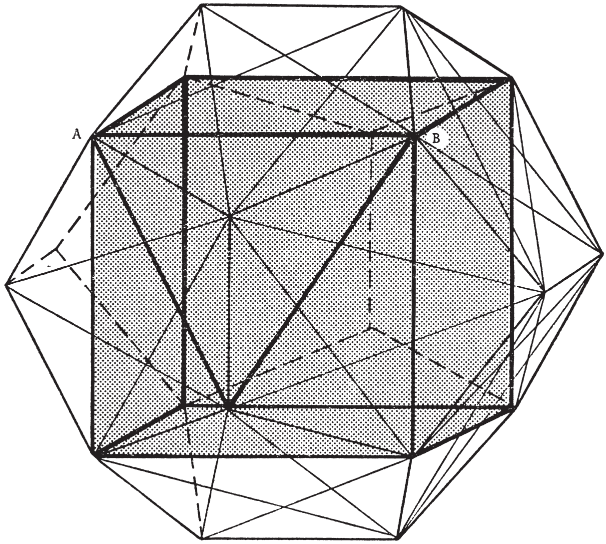

Fig. 1 (after Bruins)

detailed treatment of the dodecahedron in the *Timaeus* is understandable.

If *διαζωγραφῶν* at *Timaeus* 55C is understood as *delineating* and is applied to the inscribed surfaces of the solids, and if in the case of the dodecahedron the figure is the triangular construction of the star pentagram, the relations of the cube and of the equilateral triangle to the dodecahedron are displayed upon the surface of the solid. The cube can then easily be inscribed in the dodecahedron, one line from each of the twelve pentagrams being utilized in its formation[23]). It is important to note that

---

it, without constructing both in the elaborate manner of Eucl. XIII. 17 and working out the relation between an edge of the dodecahedron and the radius of the sphere, as is there done: ..." (p. 162). Raven, *Pythagoreans and Eleatics* (Cambridge, 1948), 150–155, examines a passage in Aëtius (II, 6,5; *Vorsokrat.*[12] 44 A 15) supposedly derived from Theophrastus which attributes to Pythagoras the formation of the sphere of the whole from the dodecahedron and concludes that Aëtius (or Theophrastus) was probably mistaken.

23) This construction is analogous to the early Pythagorean construction of the pentagon and dodecahedron discussed by Heath (above,

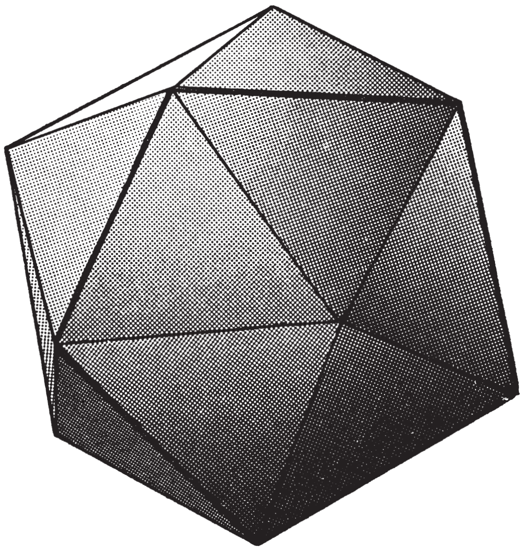

Fig. 2

none of the six surfaces of the cube lies on any of the plane surfaces of the dodecahedron in which it is inscribed (*Fig.* 1). Similarly, the equilateral triangle appears on the surface of the dodecahedron when the pentagram is drawn on each of the pentagonal faces. The triangle does not appear on a plane, but rather at the bases of three contiguous planes forming a solid angle; it has a relationship to the dodecahedron which is similar to that of the cube: one of the sides of the triangle when so inscribed can be the same line of the pentagram used to inscribe the cube (*Fig.* 1, *line* AB).

The dodecahedron and the icosahedron are said to be a dual pair: either polyhedron can be constructed by connecting points placed in the center of the faces of the other[24]). In the case

note 17), I, 159ff. Pappus' later treatment is outlined by Bruins (above, note 19), 176.

24) These relationships are well illustrated by Holden, *Shapes, Space, and Symmetry* (New York, 1971) 4–9. Ancient comparisons of the dodecahedron and the icosahedron along with the inscriptions of certain of the solids in one-another are to be found in the so-called Books XIV and XV of Euclid (Heiberg [above, note 21], V). A treatise on the comparison of the five figures by Aristaeus referred to by Hypsicles (second century B. C.) in

of the icosahedron the form of the pentagon is clearly visible in the structure of the solid, but it is not present as a plane surface (*Fig.* 2). It is a manifestation of the duality of these two solids that the surfaces of one are visible in the structure of the other; but for the purposes of the present inquiry, the important point is that when the pentagrams are drawn upon the pentagonal faces of the dodecahedron, one may clearly see both the cube (and hence the right-angled isosceles triangle) and the equilateral triangle. Thus the phrase *ἐπὶ τὸ πᾶν* has greater significance than the closeness of the dodecahedron to the sphere: the dodecahedron visibly incorporates in itself all the surfaces that combine and recombine to form the other four regular solids of Platonic physics. In so doing, it constitutes a geometrical matrix in the formation of the physical universe.

Irvine Ronald F. Kotrč

Book XIV may possibly be dated as early as the fourth century B.C. The dating of the material and a summary of the contents of the books is provided by Heath (above, note 17), I, 419ff.