Diagnostic/metagnostic modelling of the western Mediterranean's general circulation with a 3D primitive equations K-ε model

123908

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ABSTRACT

The Gher 3D k-ε PE Model is briefly described and its application to the study of the western Mediterranean's General Circulation is discussed. It is shown that the system-oriented model reproduces well the main trends of the general circulation and its month-to-month variations - thus providing valuable metagnostic information to assist interdisciplinary field surveys, planning and management - but also that it evidences essential synoptic processes such as deep water formation, coastal upwellings, gyres... confirming that the cogent physical mechanisms are indeed properly included in the metagnostic model. The model is thus also appropriate for process studies, the diagnostic simulations taking advantage of metagnostic nowcasts and forecasts to set realistic boundary conditions.

INTRODUCTION

Mathematical models of the marine system have considerably evolved since the early days of the hydrodynamic depth-integrated constant viscosity models of tides and storm surges on the shelf or the quasi-geostrophic models of ocean circulation in idealized basins.

The improvements include (i) the progressive confluence of shelf and ocean models with in particular the extension of the former to three dimensions and the rallying of the latter to primitive equations, (ii) the expansion of hydrodynamic models to biogeochemical and ecosystem models, (iii) the refined parameterization of sub-grid scale processes and in particular, turbulent vertical diffusion, introducing two new state variables, the turbulent kinetic energy k and the turbulent dissipation rate ε (e.g. NIHOUL *et al.,* 1989).

At this stage, one can rely on sound three-dimensional k-ε models able to describe the so-called "weather" of the sea, from mesoscale processes like tides and storm surges, inertial oscillations, diurnal cycles (time scale : from a few hours to a few days) to synoptic scale and macroscale processes like general circulation currents, seasonal cycles and associated frontal structures, upwellings, deep convection (time scales : a few days to a few months).

If there is a weak link in these models, it is, almost always, associated with the ε equation which, if written in tractable form, involves several parameters with a wide range of empirical values (e.g. NIHOUL *et al.*, 1989).

In many cases, however, using the dimensional relationship relating k, ε and the "mixing length" ℓ, and well-documented empirical formulations for ℓ, one can replace the partial differential equation for ε by an algebraic equation with fewer and better know parameters (e.g. NIHOUL *et al.*, 1989).

A robust model results, with very limited parameterization and just the right degree of sophistication to allow reliable simulations of the marine weather (NIHOUL and DJENIDI, 1987, NIHOUL *et al.*, 1989).

Such model, however, even restricted to its hydrodynamic sector, has a minimum of seven state variables, the three components of the velocity vector $\underset{\sim}{u}$, pressure p, buoyancy a (or salinity), temperature θ and the turbulent kinetic energy k, described by time-dependent three-dimensional partial differential equation.

This raises the questions of (i) the size of the data base necessary to provide the initial and boundary conditions in three dimensions, (ii) the amount of computing power required and the difficulty of designing simple,relevant, numerical tests, (iii) the volume of simulation outputs and the need to "sample" them intelligently, with the objectives of the model in mind.

Faced with all these difficulties, one is naturally tempted to simplify the model and consider reducing (i) its ambition (subject objective, purview), (ii) its support (from 3D to 2D, 1D...), (iii) its scope (leaving out, or frozen-in, state variables like buoyancy, turbulent kinetic energy...) (iv) its definition (the "inner scale" of the numerical grid), (v) its precision (the range of error tolerated on the state variables) or (vi) its realism (the ability of the model to truly represent the physical system with real bathymetry, coastal and open-sea boundaries, atmospheric forcings...).

A question naturally arises here : Is the "reduced" model a simpler version of the complete model (or does it belong to another class) ? Does it provide a simplified view of the marine system which can be used in a first approximation prognosis (or does it describe a different system?).

DIAGNOSTIC AND METAGNOSTIC MODELLING

One can easily burn one's fingers over drastic simplifications, made too lightly for the purpose of mathematical or numerical tests.

For instance, one has learnt that a two-dimensional sensible equivalent of a three dimensional problem cannot be obtained by simply dropping vertical derivatives. One must average over depth, with cardinal differences such as the transformation of the continuity equation from incompressible flow form ($\underset{\sim}{\nabla} . \underset{\sim}{u} = 0$ in three dimensions) to "compressible flow" form

$$(\frac{\partial H}{\partial t} + \underset{\sim}{\nabla} . (H \bar{\underset{\sim}{u}}) = 0$$

where H is the total depth and $\underset{\sim}{u}$ the depth-averaged two-dimensional horizontal velocity vector) and the appearance, beside classical turbulent and other sub-grid scale diffusion terms, of the dominating "shear effect" diffusion produced by the vertical velocity shear.

The only correct "rigid lid approximation" requires a "σ-coordinate" transformation which again modifies the nature of the continuity equation from incompressible to compressible form.

Neglecting molecular/turbulent diffusion generates singular perturbation problems which do not always reduce to the existence of unresolved boundary layers where adjustement to boundary conditions takes place (NIHOUL, 1990).

A more sensible approach to the problem of reducing the size of the model is the restriction to the diagnostic study of one or a few, isolated and idealized, processes.

Devised to investigate, in details, particular mechanisms, scrutinize the behaviour of specific state variables, elucidate fundamental questions..., often very refined in their representation of, sometimes, rather subtle processes, diagnostic models are frequently content with very crude approximations of the physical world (constant depths, rectilinear coasts, infinite ocean, steady two-dimensional fronts, rigid sea surface...). In brief, the reduction, in this case, bears on the model's ambition and realism, trying to conserve the "physical insight" of the process study.

The advantages of diagnotics models are readily listed : (i) reduced dependence on initial and boundary conditions, (ii) reduced computing cost of numerical tests and simulations, (ii) reduced complexity of simulation's outputs, easier interpretation of results and better understanding of the process (at least, in its idealized form).

The disadvantages are quite as easily pointed out : (i) risk of misunderstanding the real process if it actually results from several interacting effects, (ii) risk, if the realism is prohibitively reduced, of applying the model's equations - which have been calibrated with real field data - to an idealized system to which they are not, actually, applicable, (iii) loss of accuracy and loss of pertinence to the questions posed to the physicists by co-workers in other disciplines and managers which were, probably, the initial motivations of the modelling enterprise.

Answer to these questions must be realistic and require the operation, on a routine basis, of a system-oriented (as opposed to process-oriented) metagnostic model.

Called upon to tackle a pratical situation, a metagnostic model may not ignore the real field conditions (depths, coastlines, actual atmospheric forcing...). Its aim however is to assess the consequences of particular events and provide the marine nowcasts and forecasts which will assist interdisciplinary field studies, planning and management. The model must be sound, expeditious and efficient but it is not required to provide detailed information on the delicate machinery subtending its parameterization scheme.

The advantages of a metagnostic model are obvious but its disadvantage should not be ignored : the critical need of many - many more than for process studies - reliable data to calibrate, initiate and operate the model and, accordingly, its unavoidable sensitivity to the quality as well as the quantity of the available data. The realism of a metagnostic model may be paid - as a result of shortage of data and limitations of computing facilities - by a reduction of the model's definition and precision.

The potency and limitations of metagnostic modelling are illustrated in the following on the example of the Gher 3D General Circulation Model of the western Mediterranean.

THE GHER 3D MEDITERRANEAN MODEL

The Gher Model is primitive equation, fully 3D (multi-level), time dependent, non-linear, baroclinic model with a k-ε turbulence closure.The state variables are the three components of the velocity vector, the temperature, the buoyancy (or salinity), the pressure (or the surface elevation), the turbulent kinetic energy, the energy dissipation rate (or the mixing length) and the concentrations of passive constituents.

The model has been described in earlier publications (e.g. NIHOUL and DJENIDI, 1987, NIHOUL *et al.*, 1989, NIHOUL and BECKERS, 1989).

The model is calibrated to the study of marine weatherlike processes from mesoscale tides and storm surges to synoptic frontal and eddy structures and macroscale slowly warying currents characteristic of the so-called "general circulation".

The Gher General Circulation Model is derived from the general "marine weather" model by averaging over a time of several weeks.

In the new equations and associated boundary conditions, mean products of mesoscale fluctuations are differentiated from turbulent fluxes and treated separately (thus avoiding overplaying turbulence closure concepts).

The equation for ε - which, at general circulation scale, contains too many unknown parameters - is dropped and replaced by an algebraic mixing length closure.

In the frame of the quasi-hydrostatic approximation, the vertical component of the momentum equation reduces to a simple balance between buoyancy and vertical gradient of $q = P/\rho_0 + gx_3$.

The mesocale energy production rate is proportional to the mean of the 3/2 power of the wind stress (NIHOUL *et al.*, 1989). The coefficient of proportionality is a function of depth and the flux Richardson number.

The computer code translation of the mathematical model is greatly facilitated by the introduction of new coordinates : the hat-coordinates (The hat coordinates are quite similar to the so-called "σ-coordinates" used in Meteorology and are simply rechristened here to avoid confusion with isopycnal or "iso-σ coordinates used in Oceanography).

The advantage of the new coordinates and associated variables is that horizontal and vertical length and velocity scales are then of the same order, the water column has everywhere the same height while the top and bottom boundary conditions reduce to zero vertical (hat) velocity.

Furthermore, the comparison of the vertical velocities in real and hat coordinates provides a more complete understanding of vertical motions, separating what is due to the bathymetry of the basin (upsloping and downsloping) and what is due to Ekman circulation (upwelling and downwelling).

The model is summarized in Tables I and II.

In writing the buoyancy equation, it has been assumed that production/destruction of buoyancy is essentially due to heat and mass exchanges occurring at the air-sea interface and at river outputs and taken into account in the boundary conditions. The equation for buoyancy is then the paradigm of all "passive" scalars' evolution equations and a similar equation holds for temperature, salinity, turbidity... For simplicity, these have not been explicitly written in Table I.

Considering the important variations of the western Mediterranean's bath metry, the three-dimensional model is applied conjointly to two superposed interconnected layers, the surface layer's depth following the shelf's bathymetry on the shelf and remaining at maximum shelf's depth in the deep-sea.

The introduction of two different hat-coordinate systems has also the advantage of allowing a better representation of the vertical stratification. Indeed with a single hat-coordinate transformation for the whole basin, lines of equal x_3 when retransformed to physical space cut the isopycnals and there is a risk that diffusion in hat-space destroys the vertical stratification.

The equations of Table I are applied separately in the two layers.

The numerical model uses a mode-splitting technique based on the simultaneous resolution of a depth-integrated model to compute the surface elevation. The advection term is represented by either a centered-difference scheme, a Lax-Wendorff scheme or an upwind-difference scheme, according to the numerical requirements. The numerical method is implicit in the vertical but, to avoid numerical erosion of the pycnocline, the "implicity factors" of the advection and diffusion terms are modified at each mesh

TABLE I

The GHER 3D mathematical model

Hat coordinates

$$\hat{t} = t \;, \quad \hat{x}_1 = x_1 \;, \quad \hat{x}_2 = x_2 \;, \quad \hat{x}_3 = \frac{L}{H}(x_3 + h)$$

$$\hat{u}_1 = u_1 \;, \quad \hat{u}_2 = u_2 \;, \quad \hat{u}_3 = \frac{d\hat{x}_3}{dt}$$

L is a characteristic length scale of horizontal variations, $H = h + \zeta$, h is the depth and ζ is the surface elevation.

Basic equations

$$\frac{\partial H}{\partial t} + \hat{\nabla} \cdot (H\hat{\mathbf{u}}) + \frac{\partial}{\partial \hat{x}_3}(H\hat{u}_3) = 0$$

$$\frac{\partial}{\partial t}(H\hat{\mathbf{u}}) + \hat{\nabla} \cdot (H\hat{\mathbf{u}}\hat{\mathbf{u}}) + \frac{\partial}{\partial \hat{x}_3}(H\hat{u}_3\hat{\mathbf{u}}) + f\mathbf{e}_3 \wedge H\hat{\mathbf{u}}$$
$$= H\hat{Q} + \frac{\partial}{\partial \hat{x}_3}\left[\hat{\nu}\frac{\partial (H\hat{\mathbf{u}})}{\partial \hat{x}_3}\right] + \hat{\mu}\hat{\nabla}^2(H\hat{\mathbf{u}})$$

$$\frac{\partial}{\partial t}(Ha) + \hat{\nabla} \cdot (Ha\hat{\mathbf{u}}) + \frac{\partial}{\partial \hat{x}_3}(Ha\hat{u}_3) = \frac{\partial}{\partial \hat{x}_3}\left[\hat{\nu}^a\frac{\partial (Ha)}{\partial \hat{x}_3}\right] + \hat{\nabla} \cdot (\hat{\mu}^a H\, \hat{\nabla} a)$$

$$\frac{\partial}{\partial t}(Hk) + \hat{\nabla} \cdot (Hk\hat{\mathbf{u}}) + \frac{\partial}{\partial \hat{x}_3}(Hk\hat{u}_3)$$
$$= HQ^k + \frac{\partial}{\partial \hat{x}_3}\left[\hat{\nu}^k\frac{\partial (Hk)}{\partial \hat{x}_3}\right] + \hat{\nabla} \cdot (\hat{\mu}^k H\, \hat{\nabla} k)$$

$$\hat{Q} = -\hat{\nabla} q_L - a\big(\hat{\nabla} h - \frac{\hat{x}_3}{L}\hat{\nabla} H\big) - \hat{\nabla}\left[\frac{1}{L}\int_L^{\hat{x}_3} Ha\, d\hat{x}_3\right]$$

$$Q^k = \tilde{\nu}\,\frac{\partial \mathbf{u}}{\partial x_3} \cdot \frac{\partial \mathbf{u}}{\partial x_3} + \pi - \tilde{\nu}^a\,\frac{\partial a}{\partial x_3} - \varepsilon \qquad \pi = \beta\big[\tau_w^{3/2}\big]_0$$

$$\hat{\mathbf{u}} = \hat{u}_1\mathbf{e}_1 + \hat{u}_2\mathbf{e}_2 \qquad \tilde{\nu}^k = \psi^k\tilde{\nu}$$

$$\hat{\nabla} = \mathbf{e}_1\frac{\partial}{\partial \hat{x}_1} + \mathbf{e}_2\frac{\partial}{\partial \hat{x}_2} \qquad \tilde{\nu}^a = \psi^a(1 - \mathrm{R}_f)^{\frac{1}{2}}\,\tilde{\nu}$$

$$\tilde{\nu} = 0.5\,\tilde{\alpha}^{\frac{1}{4}}\ell_m^0 k^{\frac{1}{2}}(1 - \mathrm{R}_f)$$

$$(\hat{\nu}, \hat{\nu}^a, \hat{\nu}^k) = \frac{L^2}{H^2}(\tilde{\nu}, \tilde{\nu}^a, \tilde{\nu}^k) \qquad \varepsilon = \tilde{\alpha}\,k^2\,(16\,\tilde{\nu})^{-1}$$

$\tilde{\alpha}$, ψ^a and ψ^k are taken as constants of order 1 ($\psi^a \sim 1.4$) and R_f is the flux Richardson number

$$\mathrm{R}_f = \frac{\tilde{\nu}^a\left|\dfrac{\partial a}{\partial x_3}\right|}{\tilde{\nu}\left\|\dfrac{\partial \mathbf{u}}{\partial x_3}\right\|^2 + \pi}$$

TABLE II.

The GHER 3D mathematical model

h : depth

ζ : surface elevation

$H = h + \zeta$

$\hat{\mathbf{u}}$: horizontal velocity vector (hat coordinates)

$\hat{u}_3$: vertical velocity (hat coordinates)

f : Coriolis frequency

$\mathbf{e}_3$: unit vector along the vertical, pointing upwards

$\tilde{\nu}$: vertical turbulent viscosity ; $\hat{\nu} = \frac{L^2}{H^2} \tilde{\nu}$

$\hat{\mu}$: horizontal sub-grid scale "turbulent" viscosity (hat coordinates)

a : buoyancy

$\tilde{\nu}^a$: vertical turbulent diffusivity of buoyancy $\hat{\nu}^a = \frac{L^2}{H^2} \tilde{\nu}_a$

$\hat{\mu}^a$: horizontal sub-grid scale "turbulent" diffusivity of buoyancy

k : turbulent kinetic energy

$\tilde{\nu}^k$: vertical turbulent diffusivity of turbulent energy, $\hat{\nu}^k = \frac{L^2}{H^2} \tilde{\nu}^k$

$\hat{\mu}^h$: horizontal sub-grid scale "turbulent" diffusivity of turbulent energy

$q = \frac{p}{\rho_0} + g x_3$; $q_L = \frac{p_a}{\rho_0} + g\zeta$

p : pressure p_a : atmospheric pressure

g : acceleration of gravity

ρ_0 : reference Boussinesq density

π : average mesoscale production rate of turbulent energy

ε : turbulent energy dissipation rate and at each iteration. Hydrostatic instabilities, as in cases of deep water formation, are taken into account using an original parameterization of the diffusion coefficients, generalized from Nihoul's formula for the stably stratified case (BECKERS, 1990).

At the air-sea interface, all fluxes (momentum, heat...) are imposed and calculated from the available atmospheric data.

At the bottom, hat and salt fluxes are assumed to be zero but the flux of momentum (bottom stress) and the turbulent kinetic energy production rate are calculated using an analytical model of the bottom Ekman layer. (This solution gives the classical quadratic friction law for small bottom layer meshes but has the advantage on the logarithmic approximation to allow a veering of the velocity vector with height when the grid spacing near the bottom increases).

At the coasts, heat and salt fluxes are set to zero except at the mouths of main rivers like the Rhône.

The velocity perpendicular to the coast is zero and a quadratic friction law is imposed for the tangential velocity.

At the straits, observed transports (integrated inflowing or outflowing velocities) are imposed. The boundary conditions for the other variables are basically of the Orlanski type but they are adapted to the numerical scheme and modified to make it possible to take a measured value into account when such a value is available.

The model has been tested in the following - still rather crude - conditions :

(i) 22 km horizontal mesh size

(ii) 14 vertical meshes

(iii) time step of 80 s for the subsidiary 2D Model,
800 s for the 3D model

(iv) maximum depth of the top layer : 200 m (corresponding roughly to the shelf-break in the Gulf of Lion).

The model was initialized with early September typical climatological conditions compiled from Levitus's hydrological data (e.g. LEVITUS S., 1982), preliminary results from reduced gravity models and exploratory simulations of the current field with the Gher Model (NIHOUL and BECKERS, 1989).

Figs 1, 2 and 3 show three typical examples of the climatological atmospheric mechanical and thermodynamical forcings drawn from the data of May (1982) for September, December and March... One can see clearly the prevailing Mistral wind blowing from the continent to the south-east over the Gulf of Lion and the associated heat loss from the ocean to the atmosphere.

Wind stresses generate turbulence and mixing in the upper layer which deepens by entrainment of quieter fluid from below. On the other hand, cooling and evaporation produce cold, salty, - i.e. heavier -, water which will ultimately sinks. Observations show that heavy water sinking tends to occur in rather well-defined chimneys and are associated with intensive convective vertical mixing.

Theses two kinds of vertical mixing are well illustrated by Figs 4 and 5. Fig. 4 shows the results of a process study (starting from low turbulence well-stratified water), viz the turbulent kinetic energy field at 1 m depth,

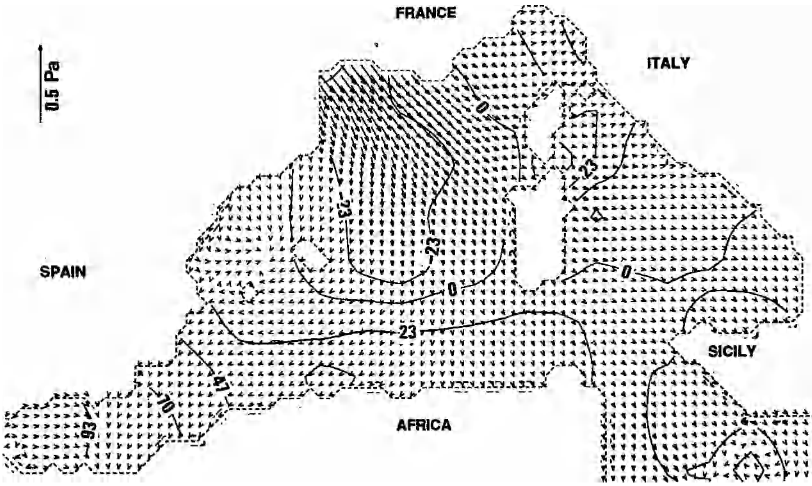

FIGURE 1. - Climatological surface heat flux and wind stress (from MAY 1982), mean value for September (wind stress reference 0.5 Pa, heat flux in w m^2 positive downwards).

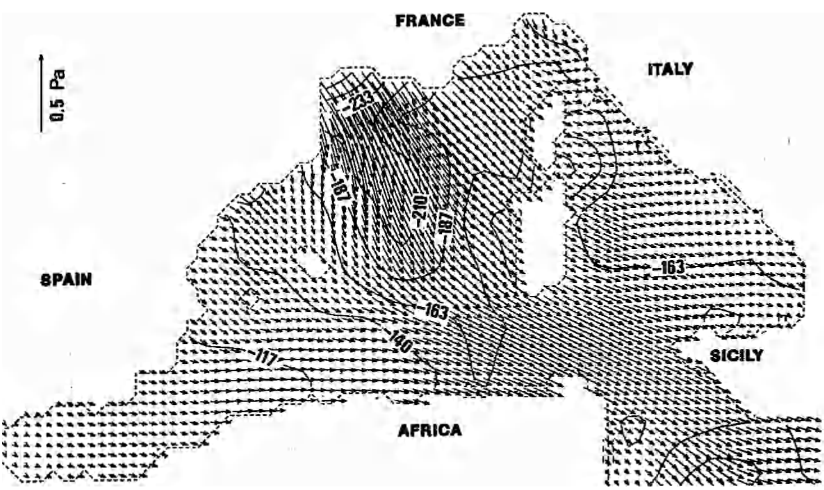

FIGURE 2. - Climatological surface heat flux and wind stress (from MAY 1982), mean value for December (wind stress reference 0.5 Pa, heat flux in w m^2 positive downwards).

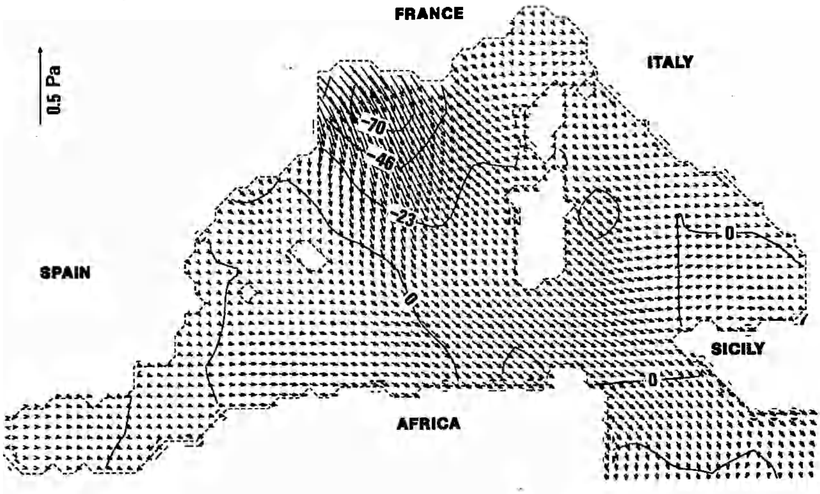

FIGURE 3. - Climatological surface heat flux and wind stress (from MAY 1982), mean value for March (wind stress reference 0.5 Pa, heat flux in w m^2 positive downwards.

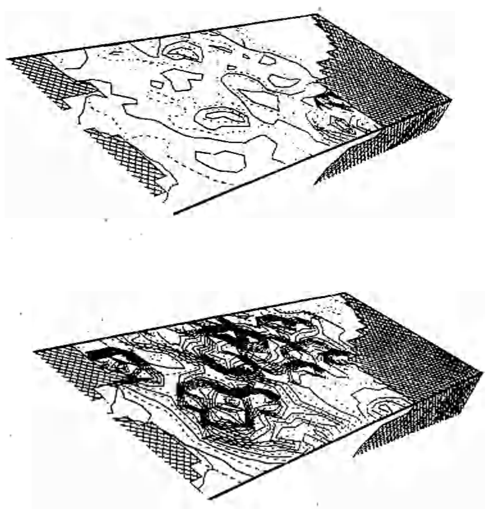

FIGURE 4. - Turbulent kinetic energy field at the surface (-1m) of the Gulf of Lion, respectively after two weeks (above) and four weeks (below) of typical winter forcing.

after respectively two weeks (above) and four weeks (below) of typical winter atmospheric forcing. One can see wind mixing appearing first near the coast in the Gulf and progressively extending off-shore in a typical patchy pattern indicative of thermodynamical modulation and the onset of deep penetrating convective mixing. Fig. 5 shows the turbulent kinetic energy field at 30m depth at the end of January after five months of real climatological atmospheric forcing (May 1982). The synoptic variability of deep water formation and convective mixing is well reproduced by the model.

The overall cyclonic transport in the northen part of the basin is also reasonably well apprehended by the model (fig. 6) as well as the most significant secondary gyres such as the well-known Alboran Sea gyres (fig. 7).

Here however the performances of the metagnostic model are severely limited by the incertitude on initial conditions. Small modifications of these conditions can be shown to produce sometime quite dramatic changes in the general circulation pattern : while the main features remain qualitatively the same, small quantitative changes of these persistent features occur, associated with drastic modifications of secondary traits of the circulation, the reversal of subsidiary gyres... (NIHOUL and BECKERS 1989) and, in this respect, Figs 6 and 7 must be regarded simply as illustrative examples without real prognostic value.

The Gher metagnostic model is presently being applied to the simulation of a complete annual cycle, using the climatological forcings provided by May (1982) and assimilating new data supplied by the joint application of objective analysis and inverse modelling (BRASSEUR P. and ROUSSENOV V., 1990).

A complete annual cycle in climatological conditions will hopefully provide a much better set of initial conditions, on which realistic metagnostic and prognostic modelling can be based. The model will then be run, on a routine basis, with boundary inputs supplied by a Meteorological Forecasting Model operated at the French Meteorological Office, with a view of exploi-

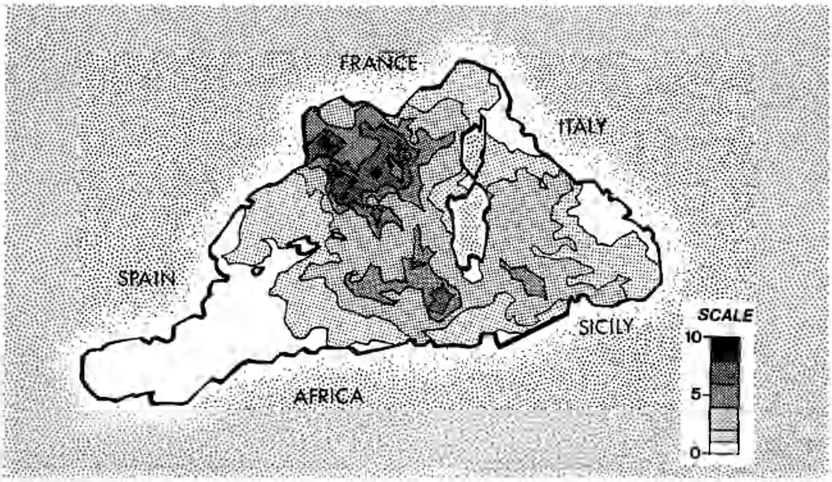

FIGURE 5. - Turbulent kinetic energy field at 30 m depth at the end of January after 5 months of real forcing (MAY 1982) (10^{-3} m^2 s^{-2}).

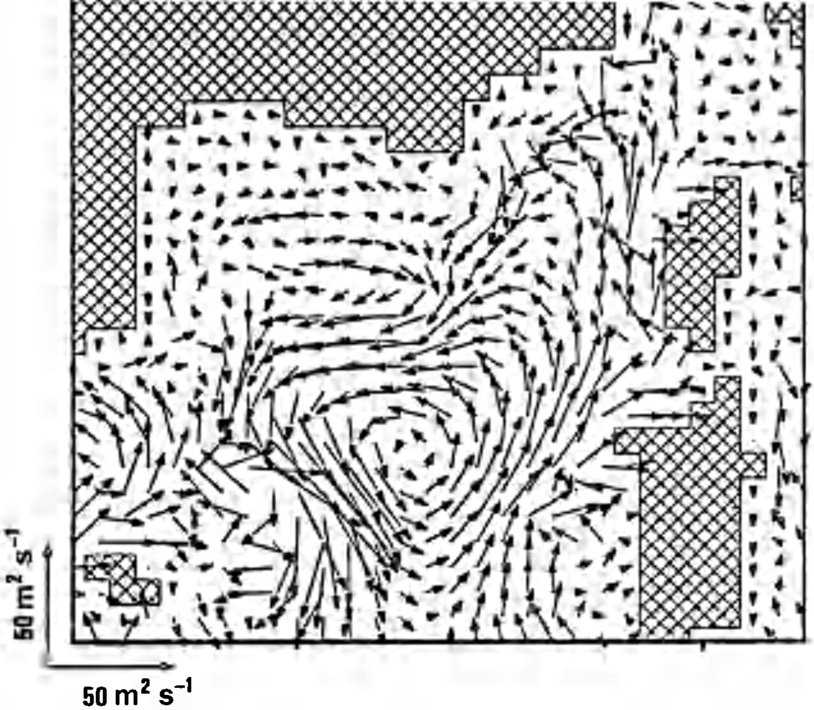

FIGURE 6. - Total transport ($M^2\ S^{-1}$) in the northern part of the basin after eight weeks of typical winter forcing.

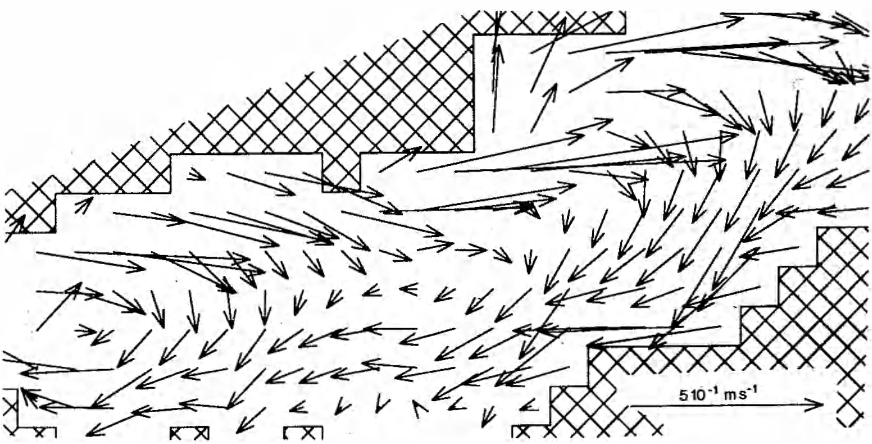

FIGURE 7. - Horizontal velocity field at 10 m depth in the Alboran Sea at the end of January after 5 months of real forcing (MAY 1982).

ting the results, as a regular service in the scope of the Eros 2000 Project, to assist interdisciplinary field surveys, planning and management.

CONCLUSIONS

On the few selected examples given above, one can see that the Gher metagnostic model reproduces well the main trends of the general circulation - thus providing valuable information for interdisciplinary surveys and management - but also that it evidences essential synoptic processes such as deep water formation, coastal upwellings, gyres...

More detailed process studies may be pursued with the model by investigating localized areas with finer grid diagnostic submodels operating in parallel with the basin metagnostic model ; diagnostic studies providing useful information to refine the mathematical representation of dominant processes, metagnostic nowcasts and forecasts supplying a realistic overview of the system in which the individual processes are immersed and improved boundary conditions for the process studies.

The handicap of the metagnostic model remains however the complexity imposed by its realism. The model is very demanding in initial and boundary conditions and the qualitative and quantitative deficiency of reliable data may severely reduce the model's resolution and predictability.

A more thorough exploitation of the data - in particular, applying variational inverse methods to reconstruct complete data fields - and the operation of the model in a preliminary simulation with climatological forcings to reshape the initial conditions will hopefully bring significant improvements in this respect, in the near future.

ACKNOWLEDGMENTS

The authors are very grateful to Dr. P.W. May for kindly providing his data on the atmospheric mechanical and thermodynamical forcings in computer usable form.

This research has been undertaken in the framework of the "Eros 2000" (European River Ocean System) project. The authors acknowledge with gratitude the support from the Commission of the European Communities 4th Environment R & D Programme under contract EV4V-0111-F.

The authors are indebted to the Ministry of Education and Research, the Ministry for Science Policy, the National Fund for Scientific Research (Belgium) and IBM for their financial and technical assistance in providing supercomputers facilities.

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