2 Radio Astronomy Fundamentals
2.1 Introduction

The atmosphere is transparent to only two bands of the electromagnetic spectrum: optical and radio bands.

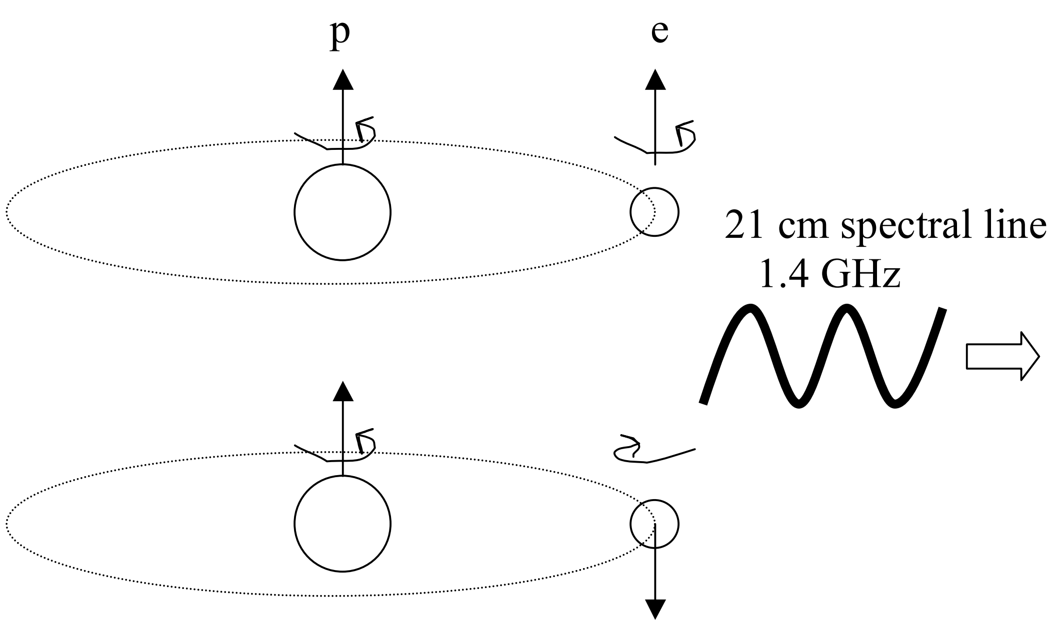
Optical band: 0.4 –0.8 μm (factor 2 in wavelength coverage)
Radio band : 1 cm – 10 m (factor 1000 in wavelength coverage)

The development of radio astronomy technology and early telephone and satellite communications have always been closely related.The science of radio astronomy started with Karl Jansky in 1931. Jansky was a radio engineer at Bell Telephone Labs. He studied thunderstorm static to improve transoceanic telephone circuits. He built an antenna operating at 20.5 MHz (14.6 m) and found static from local and distant thunderstorms. In addition he found a "steady hiss-type static of unknown origin," first seen in 1932. By 1935 he realized that the static came from outside the solar system.

Grote Reber, a radio engineer, built a parabolic reflector antenna with a 9.5 m diameter. He studied the static at 160 MHz (1.87 m) with much better angular resolution than Jansky. The beamwidth of his telescope at that frequency was $\Theta_{3dB} = 12°$. The 3-dB beamwidth is also sometimes given as the full-width at half maximum (FWHM).He identified the center of our Galaxy, the Milky Way, which is a strong emitter of radio waves. In addition he also identified other strong radio sources at the sky.

Oort at the Leiden observatory in The Netherlands realized that the static must be continuum radiation (and not spectral line radiation) extending over the whole radio spectrum from at least 1 m to many m in wavelength. He also realized that looking for a spectral line in the radio would be groundbreaking.

In 1944 van der Hulst suggested that the hyperfine transition of neutral hydrogen in the universe may be observable.



1951 Ewen and Purcell (Harvard university) detected the line in emission. A few weeks later also Muller and Oort and in 1952 Christianson and Hindman in Sydney (Australia).

With the HI line (H for hydrogen, I for neutral, by the way HII would stand for ionized hydrogen) detected, the Galaxy structure could be mapped and its spiral structure discovered.

Large radio antennas were built:

76m at Jodrell Bank, England (end of 1950's)
46m in Algonquin Park (beginning of 1960's)
64m at Parkes, Australia (beginning of 1960's)
70m for the NASA DSN in Goldstone, CA, USA, Robledo, Spain, and Tidbinbilla, Australia (1960's)
100m at Effelsberg, Germany (end of 1960's)
110m at Green Bank, VW, USA (2000's)

and then arrays of antennas:

27x25m array, VLA at Socorro, NM, USA (1970's)
several 6m antenna array at Penticton, BC, Canada

In 1963 Penzias and Wilson (Bell Labs) wanted to improve satellite communications. They built a big horn antenna to achieve a low antenna temperature. But they experienced unexplainable hiss. To find the origin of the hiss, they even looked for pigeon droppings in the antenna. However, the cause of the hiss could not be pinpointed.

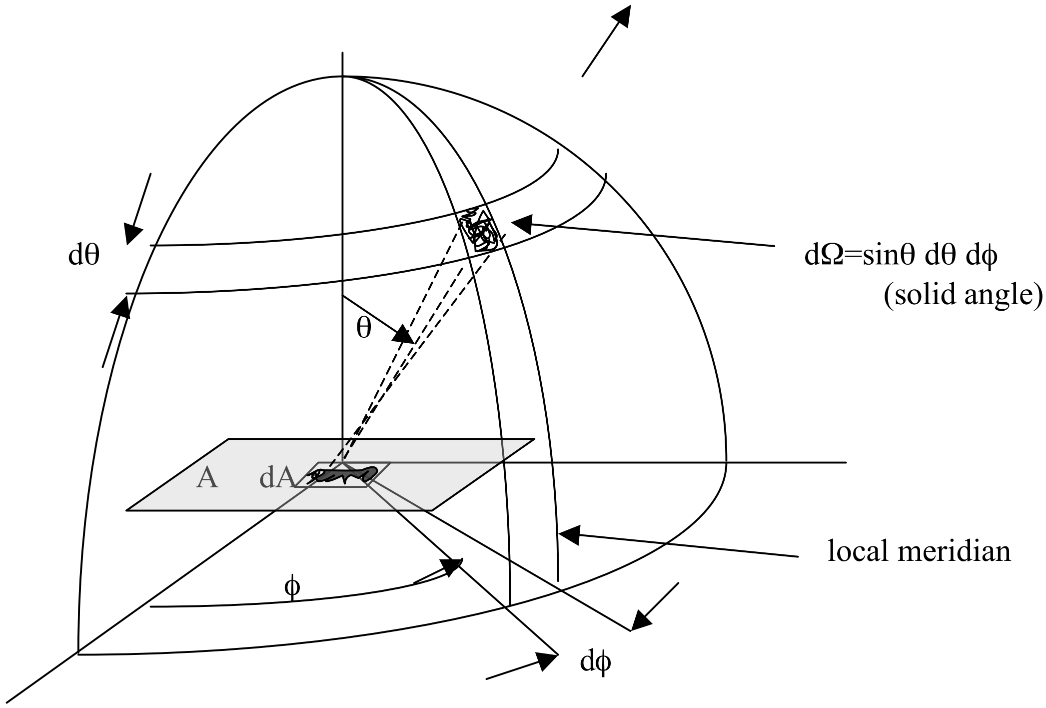
Then in 1965 it was realized that the hiss was of cosmological origin. It was caused by the 3K microwave background radiation from the early universe. Penzias and Wilson were awarded the Nobel prize for their discovery.

In 1967 A. Hewish with J. Bell discovered pulsars. M. Ryle developed the aperture synthesis technique. Hewish and Ryle were awarded the Nobel prize in 1974.

In 1974 Hulse and Taylor discovered the first binary pulsar. By carefully measuring the decay of the binary orbit they could show that the energy lost in the decay was equal to the energy expected to be radiated away from the binary system in the form of gravitational waves. This was the first (indirect) evidence for the existence of gravitational waves as predicted by Einstein's theory of general relativity. Hulse and Taylor received the Nobel prize in 1993.

2.2 Power, spectral power, brightness and flux density

Consider EM radiation from the sky falling on a flat horizontal area as sketched below.



The infinitesimal power dW from a solid angle dΩ of the sky incident on a surface of area dA is:

$dW = B \cos\theta \, d\Omega \, dA \, d\nu$

dW : infinitesimal power [W]
B : brightness of sky at position of dΩ (also called surface brightness [$Wm^{-2}Hz^{-1}rad^{-2}$]
dΩ : infinitesimal solid angle of sky [rad^2]
θ : angle between dΩ and zenith [rad]
ϕ : angle in the plane of the surface from reference direction to local meridian.
dA : infinitesimal area of surface [m^2]
dν : infinitesimal element of bandwidth [Hz]

The power received on a surface of effective area, A, from a solid angle, Ω, over a bandwidth, Δν, is:

$$W = A_{eff} \iint_{\Omega} \int_{\nu}^{\nu+\Delta\nu} B \cos\theta \, d\Omega \, d\nu$$

$$w = A_{eff} \iint_{\Omega} B \cos\theta \, d\Omega$$

W is measured in [W]
W is measured in [$W \, Hz^{-1}$]

Example:
$B(\theta, \phi, \nu)$ is uniform over the bandwidth $\Delta\nu$ and uniform over the sky. Find the total power and spectral power received by a horizontal surface of 5 m^2 effective area at frequency ν with $\Delta\nu$ = 1 MHz. B = 10^{-22} [$Wm^{-2}Hz^{-1}rad^{-2}$]

$$W = A_{eff} \iint_{\Omega} \int_{\nu}^{\nu+\Delta\nu} B \cos\theta d\Omega d\nu$$

$$= A_{eff} B \Delta\nu \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \cos\theta \sin\theta d\theta d\phi$$

And with

$$\int_0^{\frac{\pi}{2}} \cos\theta \sin\theta d\theta = \frac{1}{2} \sin^2\theta \Big|_0^{\pi/2} = \frac{1}{2}$$

we get:

$$W = A_{eff} B \pi \Delta\nu$$

$$w = A_{eff} B \pi$$

W = 15.7 10^{-16} W
W = 15.7 10^{-22} W Hz^{-1}

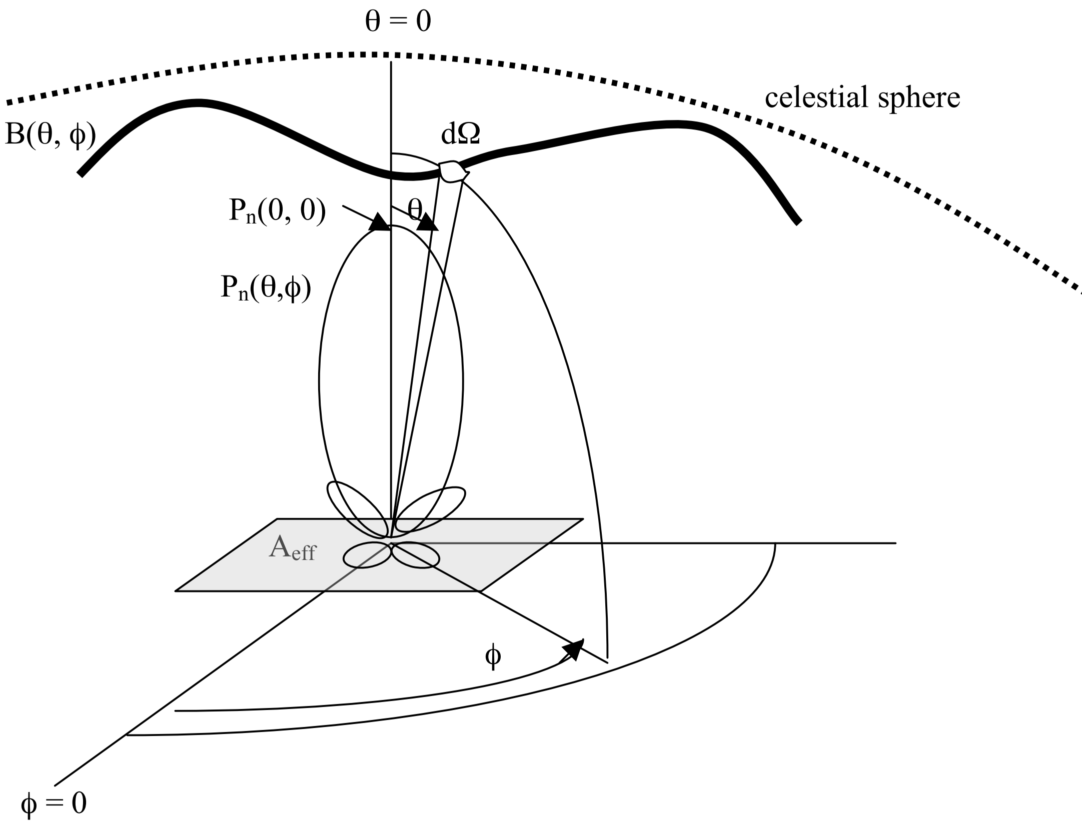
Brightness distribution

In general the brightness depends on θ and ϕ, and we call $B(\theta, \phi)$ the brightness distribution. In addition we want to introduce two more parameters:

A_{eff} : effective aperture. With η as the efficiency of an antenna (recall PHYS 3250), the Effective aperture is simply the geometric aperture multiplied with the efficiency which is typically 0.5 to 0.8 for paraboloidal antennas.

$P_n(\theta,\phi)$: power pattern of the beam of the antenna, normalized so that $P_n(0, 0) = 1$. It replaces the factor cos θ. For an isotropic antenna: $P_n(\theta,\phi) = 1$. The power pattern depends on ν, so $P_n(\theta, \phi) = P_n(\theta,\phi,\nu)$. We will get to the frequency dependence later. For now we will not deal with the frequency dependence.

We will now adjust the orientation of our surface with area A and effective area A_{eff} so that it is normal to the vector B. In other words, we are pointing our surface with its flat side toward the source of radiation.



In general, the surface brightness, or simply the brightness, is both a function of the position in the sky and the frequency:

$B(\theta, \phi) = B(\theta, \phi, \nu)$

Discrete sources:

A discrete radio source is one which is distinct or separate. We have:

1) extended sources
2) point sources

for any discrete source, integrating B over the extent of the source gives the **flux density, S**

$$S = \iint_{source} B(\theta,\phi)\, d\Omega$$

Example: source: 1 deg²
$B(\theta,\phi) = B = 2.3 \cdot 10^{-18}\ W m^{-2} Hz^{-1} rad^{-2}$
→ $S = 2.3 \cdot 10^{-18} \cdot \left(\frac{2\pi}{360}\right)^2$
$= 7.0 \cdot 10^{-22}\ W m^{-2} Hz^{-1}$
$= 70000\ Jy$

The unit of flux density is Jy (Jansky).
$1 Jy = 10^{-26}\ W\ m^{-2}\ Hz^{-1}$

If the source is observed with an antenna with a beam pattern $P_n(\theta,\phi)$, a flux density, S_o, for $S_{observed}$, is measured.

$$S_o = \iint_{source} B(\theta,\phi) P_n(\theta,\phi) d\Omega$$

$P_n(\theta,\phi)$ is simply a weighting function. In other words, if the source is much larger than the beam pattern, then the brightness along boresight (where $P_n(0,0) = 1$) will contribute most to the observed flux density and the brightness further away from boresight will contribute less. The brightness far away from boresight will not be "seen" by the antenna and therefore will contribute almost nothing, apart from radiation that gets into the antenna through the side lobes. In general:

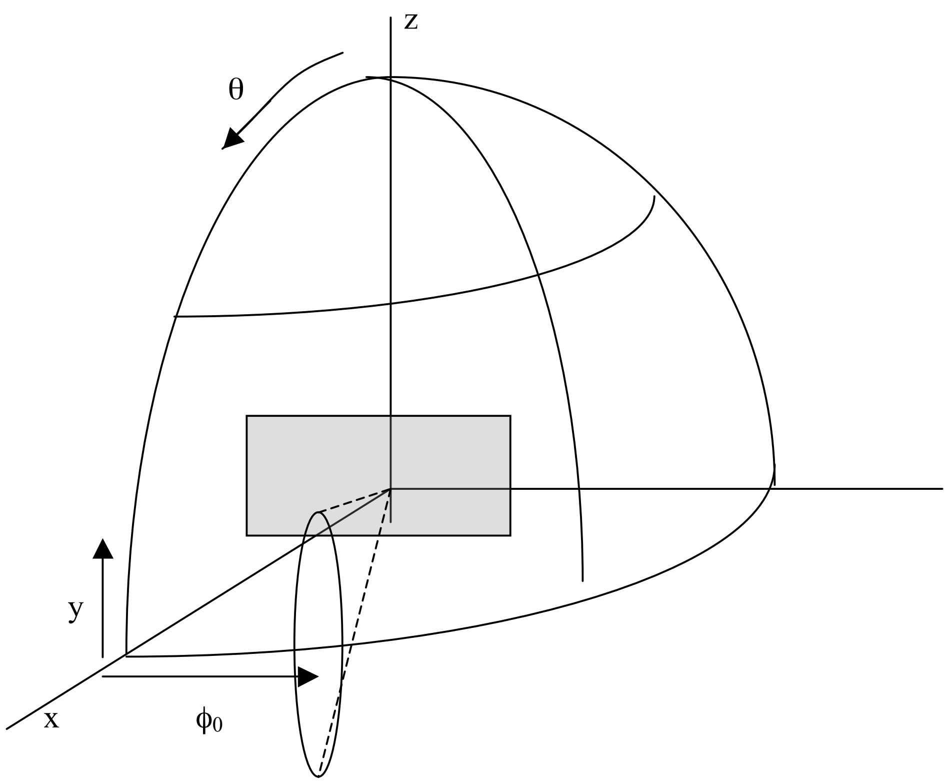
$S_0 = S$ for a point source if the antenna is pointed toward source .

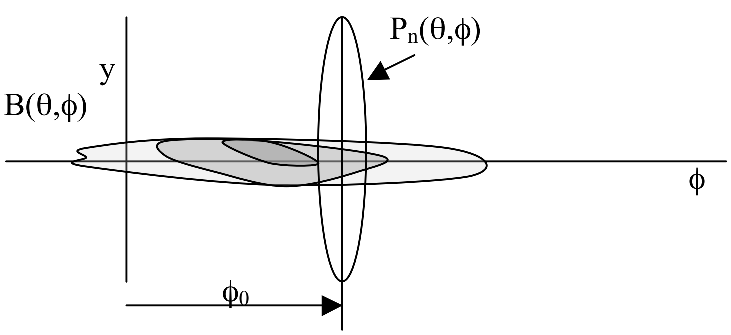
$S_0 < S$ for extended source.

If B of a source is constant over the main lobe, then

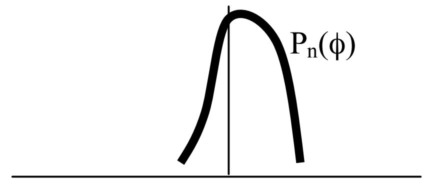
$S_0 = B(\theta,\phi)\ \Omega_M$ where Ω_M is the main lobe solid angle

So far we have assumed that the source is directed along boresight or close to it and not changing the direction in which it is pointing. We want to now look at a more general case where we change the antenna's pointing by scanning across a patch of the sky. In this case we have to consider convolution. For simplicity, let us first consider an antenna with a "fan beam" sweeping across a source in the ϕ-direction.

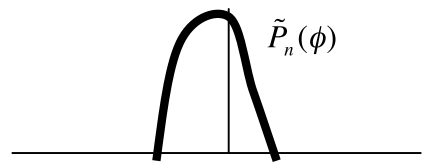




Let's just think of the 1-dim case where $B(\theta,\phi) = B(\phi)$ and $P_n(\theta,\phi) = P_n(\phi)$. Then the observed flux density, $S_0(\phi_0)$,



$$S_0(\phi_0) = \int B(\phi) P_n(\phi - \phi_0)\, d\phi$$
$$= \int B(\phi) \tilde{P}_n(\phi_0 - \phi)\, d\phi$$



with $\tilde{P}_n(\phi)$ being the mirror image of $P_n(\phi)$ with

$$\tilde{P}_n(\phi) = P_n(-\phi)$$

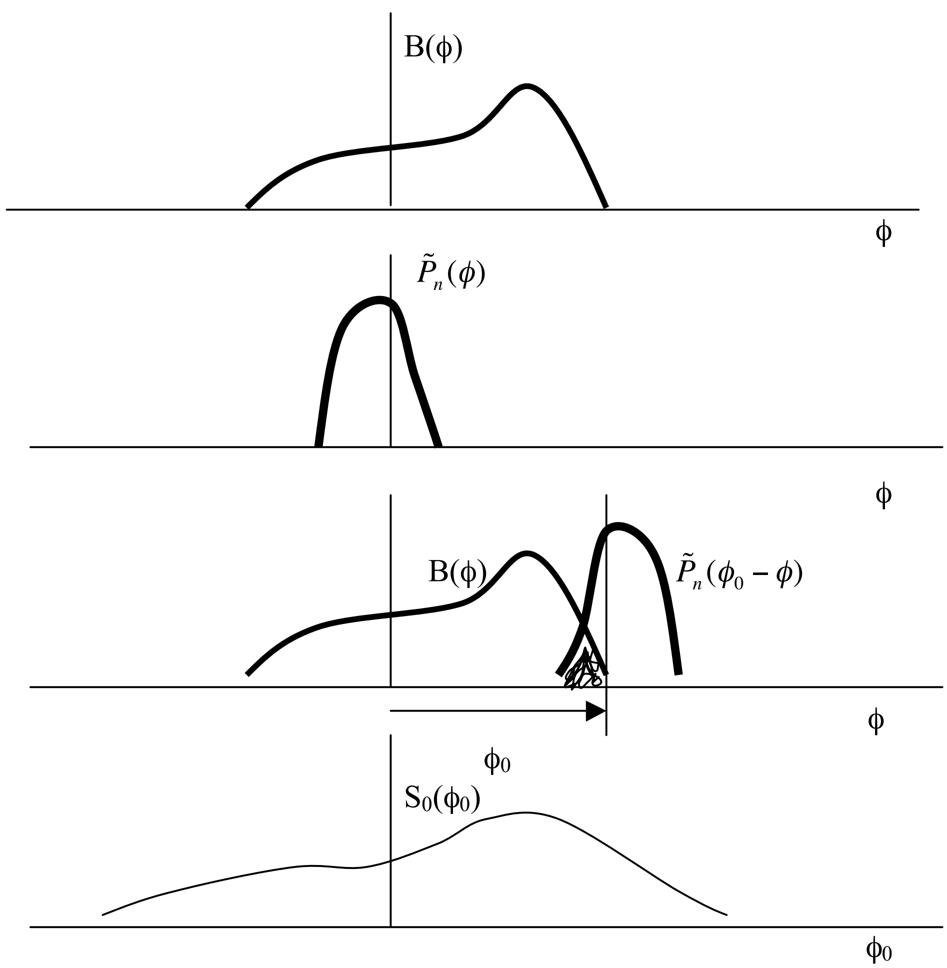
We conclude that the observed flux density is the result of a convolution of the brightness distribution with the mirror image of the beam pattern.

$$S_0 = B * \tilde{P}_n$$

If the beam pattern is symmetric, then

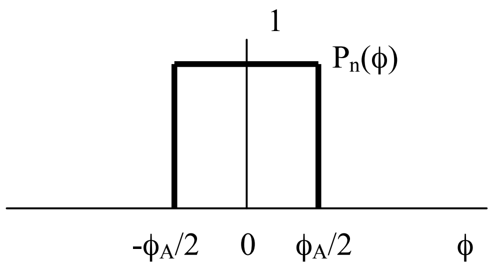
$\tilde{P}_n(\phi) = P_n(\phi)$ and
$S_0 = B * P_n$

Graphical example (with guestimated convolution function)



Antenna beam angle for 1-dim case:

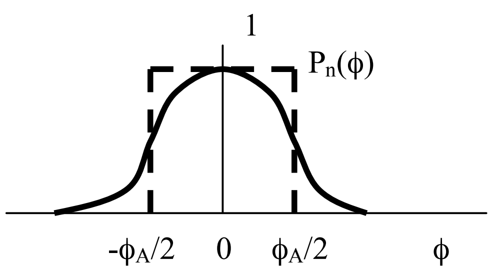
If we integrate the antenna beam pattern, $P_n(\phi)$ from $-\infty$ to $+\infty$, we get ϕ_A, the antenna beam angle. Note, that $P_n(\phi)$ is normalized and dimensionless. If $P_n(\phi)$ were a rectangular pattern with unity height,



then

$$\int_{-\infty}^{+\infty} P_n(\phi)d\phi = \phi_A$$

if $P_n(\phi)$ is a Gaussian for example



then ϕ_A gives the effective width. Note, that the same is true for $\tilde{P}_n(\phi)$ and $\tilde{P}_n(\phi_0 - \phi)$:

$$\int_{-\infty}^{+\infty} \tilde{P}_n(\phi)d\phi = \phi_A$$

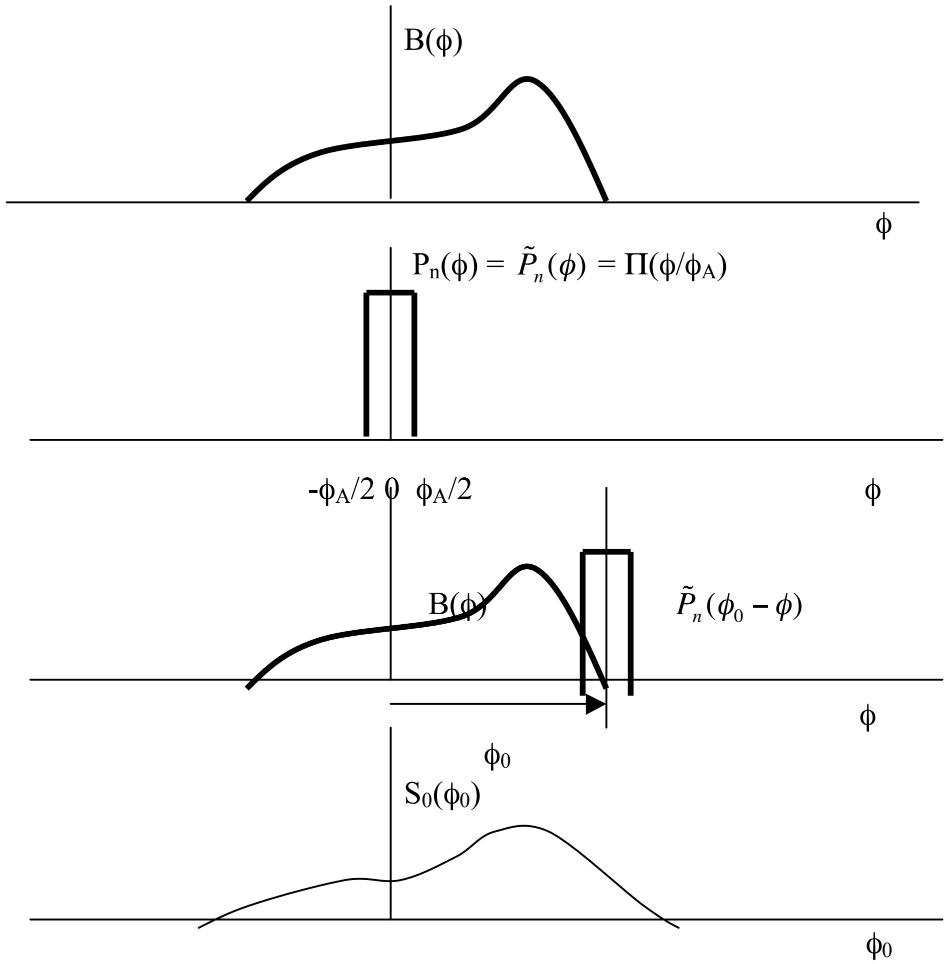
$$\int_{-\infty}^{+\infty} \tilde{P}_n(\phi_0 - \phi)d\phi = \phi_A$$

Now assume a very thin beam which in the limit can be represented by a delta function multiplied by a dimensionless constant, K_1. The constant is needed, since the integral of the delta function is equal to unity, but the integral of the beam pattern is not necessarily equal to unity. Then, with the delta function being symmetrical,

$$\int_{-\infty}^{+\infty} \tilde{P}_n(\phi_0 - \phi)d\phi = K_1 \int_{-\infty}^{+\infty} \delta(\phi_0 - \phi)d\phi = \phi_A$$

Scanning with a rectangular beam

Assuming that we have the same brightness distribution, $B(\phi)$, as before what is the observed flux density distribution S_0 if we scan the source with a rectangular beam of width ϕ_A ?

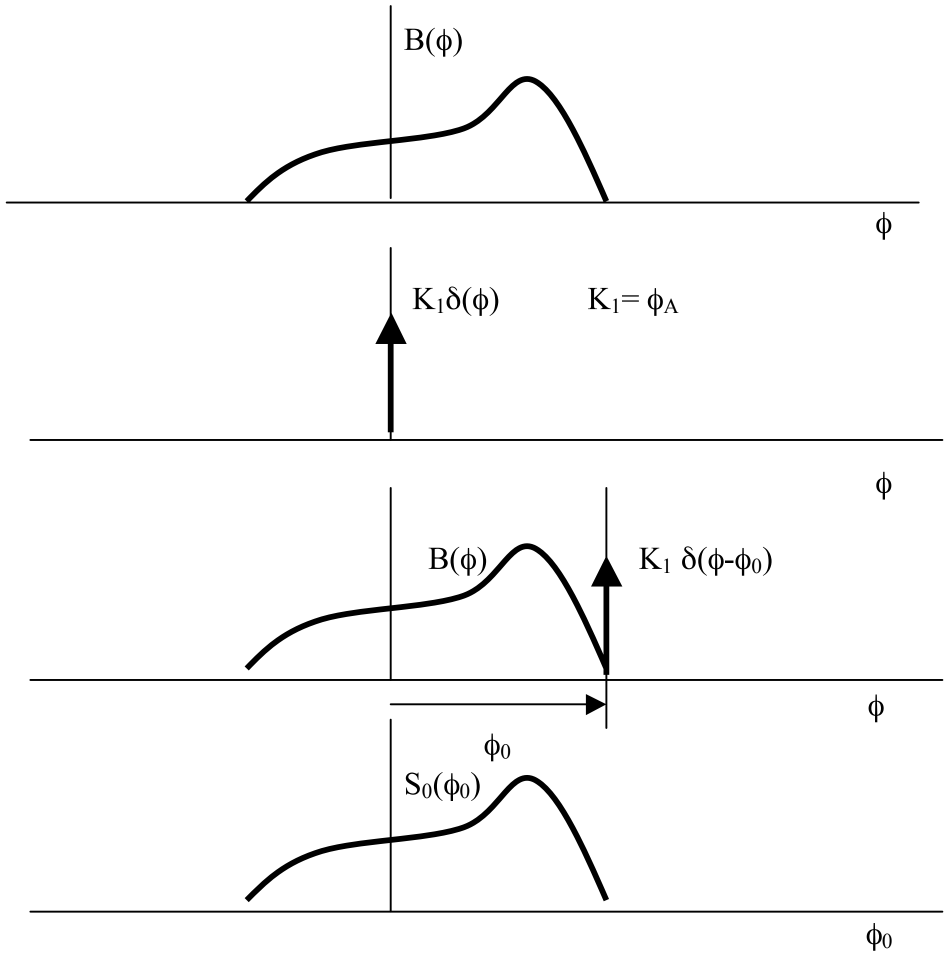


$$
\begin{aligned}
S_0(\phi_0) &= \int B(\phi)\tilde{P}_n(\phi_0 - \phi)d\phi \\
&= \int B(\phi)P_n(\phi_0 - \phi)d\phi \\
&\approx B(\phi_0)\phi_A
\end{aligned}
$$

Scanning with an infinitesimally sharp beam

Assume that we have an antenna with an infinitesimally sharp beam and want to scan a celestial source with a brightness distribution, $B(\phi)$. Then the measured or observed flux density at any sky position or *pointing* of the antenna, ϕ_0, is

$S_0(\phi_0)$ in W m^{-2} Hz^{-1}
$B(\phi_0)$ in W m^{-2} Hz^{-1} rad^{-1}
ϕ_A in rad



Thus, the observed flux density distribution, $S_0(\phi_0)$, is identical in form to the actual brightness distribution, $B(\phi)$.

Scanning a point source

Now let us assume another extreme case namely where the source is infinitesimally compact in its angular size. This could be a celestial object as big as the center of a whole galaxy, for instance a quasar, but sufficiently far away from us that it has only a very small angular diameter which appears point-like. Another example is a spacecraft sending down to us a modulated carrier signal. We call these kind of sources a point source. For the antenna beam we want to assume it to be of finite width.

Then with K_2 as another constant we can write:

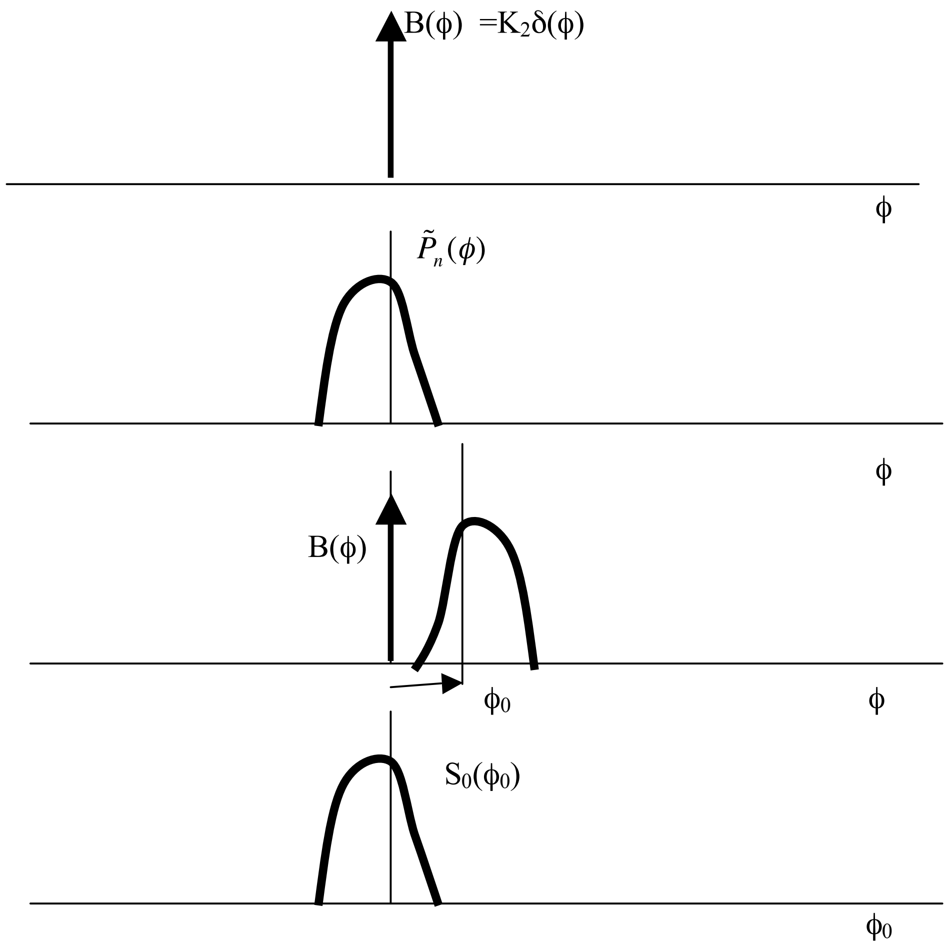
$$B(\phi) = K_2 \delta(\phi)$$

$$S_0(\phi_0) = K_2 \int_{-\infty}^{+\infty} \delta(\phi) \tilde{P}_n(\phi_0 - \phi) d\phi$$

And finally:

remember: $S = \int_{source} B(\phi)\delta(\phi)d\phi$

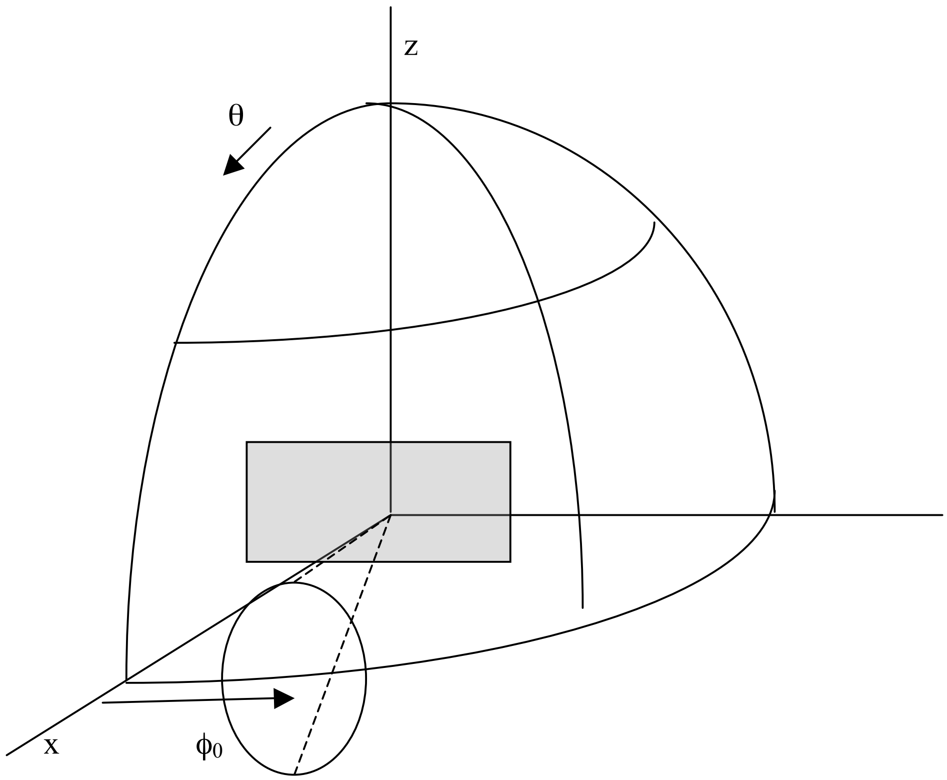
$= S$

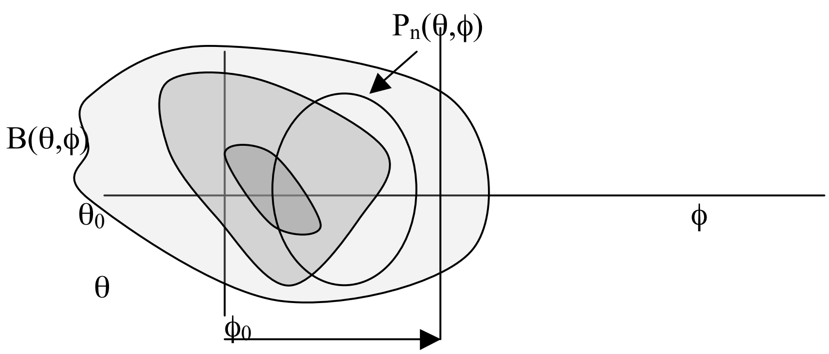


Thus, the observed flux density distribution, $S_0(\phi_0)$, is identical to the mirror image of the antenna beam pattern, $\tilde{P}_n(\phi)$. When the antenna boresight is aligned with the source ($\phi_0 = 0$), the observed flux density, $S_0(\phi_0)$, equals the true flux density of the source, S.

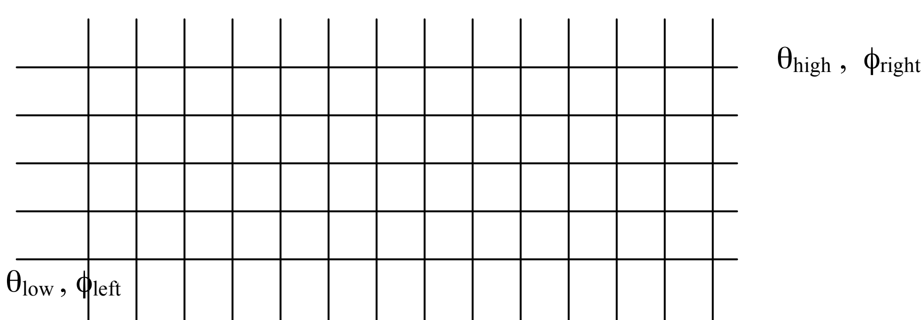
Scanning in 2-dimensions

In general the brightness distribution is 2-dimensional at the sky and the beam pattern of the antenna is also 2-dimensional.





In this case, we have to scan the source in two dimensions. We start for instance somewhere In the lower left corner at, say, θ_{low} , ϕ_{left} and sweep the antenna by increments of ϕ_0 in the ϕ - direction up to θ_{low} , ϕ_{right} Then we move the pointing of the antenna to θ_{low+1}, ϕ_{left} and sweep the antenna to θ_{low+1}, ϕ_{right}. Then we move the pointing of the antenna up again and repeat the procedure till we have reached the point θ_{high} , ϕ_{right} in the upper right corner.

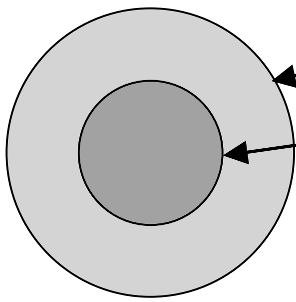


The observed flux density is then:
$S_0(\theta_0,\phi_0) = \int B(\theta,\phi)\tilde{P}_n(\theta_0-\theta,\phi_0-\phi)d\theta d\phi$

The graphical convolution is done as in the 1-dimensional case but repeated several times by shifting in the θ-direction. The result is a 2-dimensional convolution function.

A source a bit smaller than the main-beam area

If the source is smaller than the main-beam area as sketched below with the antenna being aligned with the source,



Then the observed or apparent brightness, B_0, is

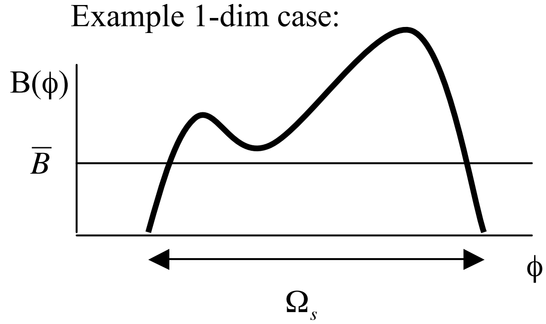
recall p57

$$B_0 = \frac{\iint_{source} B(\theta,\phi)P_n(\theta,\phi)d\Omega}{\iint_{4\pi} P_n(\theta,\phi)d\Omega} = \frac{S_o}{\Omega_A}$$

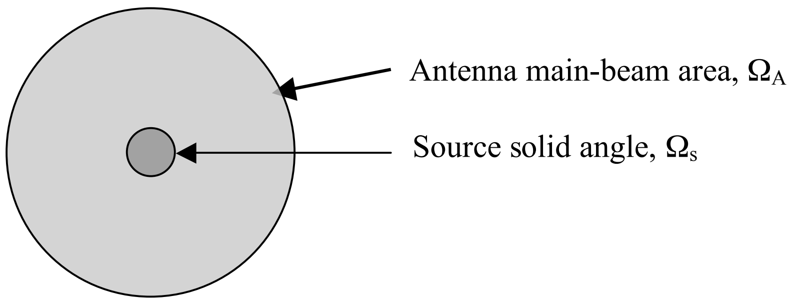
If $\Omega_s \ll \Omega_A$, then the average brightness, $\overline{B}$, of the source is:

$$\overline{B} = \frac{S}{\Omega_s} = \frac{1}{\Omega_s}\iint_{source} B(\theta,\phi)d\Omega$$

Example 1-dim case:



If the brightness is uniform over the source and if $\Omega_s \ll \Omega_A$,

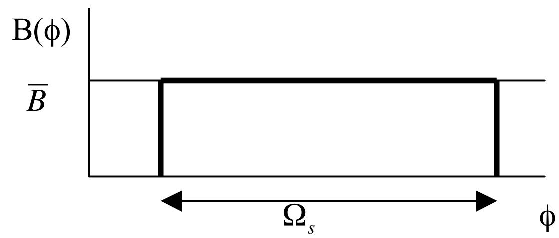


Then
$S_0 = S$

and
$\overline{B} = B$

$$B_0 = \frac{S}{\Omega_A}$$

$$= \frac{\Omega_s}{\Omega_A} B$$

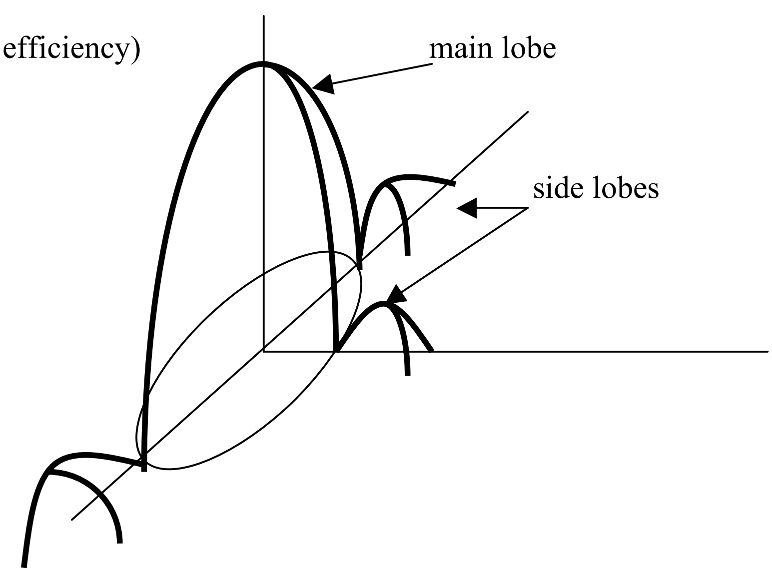


where B_o is the observed brightness and B the true brightness.

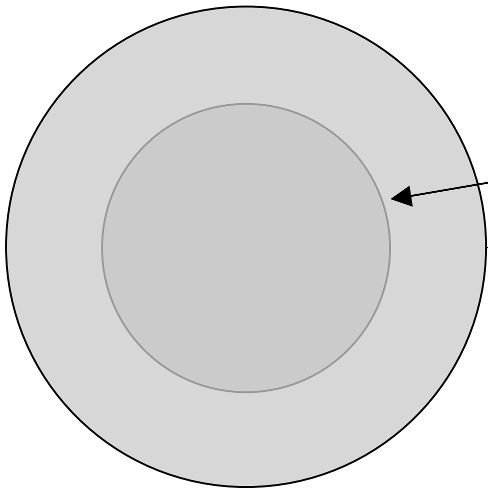
Beam efficiencies

$$\Omega_M = \iint_{\substack{main\\lobe}} P_n(\theta,\phi)\,d\Omega \qquad \text{(beam area of main lobe)}$$

$$\frac{\Omega_M}{\Omega_A} < 1 \qquad \text{(main beam efficiency)}$$



If the brightness is uniform over the source and if $\Omega_s > \Omega_A$,



Then

$$S_0 < S$$
$$S_0 = B\Omega_M$$
$$B_0 = B$$

Example:
A discrete round radio source of size 2 deg in diameter has a true average brightness of $\overline{B}$ = 10^{-20} W m^{-2} Hz^{-1} rad^{-2} at frequency ν.

1) Calculate the true flux density of the source

$$S = \overline{B}\Omega_s = 10^{-20}\pi\left(\frac{2\pi}{360}\right)^2 = 9.6 \bullet 10^{-24} Wm^{-2}Hz^{-1}$$

2)Calculate the observed flux density if the main beam of the antenna has a solid angle of 1 deg^2 and if $B = \overline{B}$.

$$S_0 = \overline{B}\Omega_M = 10^{-20}\left(\frac{2\pi}{360}\right)^2 = 3.0 \bullet 10^{-24} Wm^{-2}Hz^{-1} = 300Jy$$

The total observed power, W, received by an antenna with an effective aperture, $A_{eff,}$ sensitive to one sense of polarization only (factor ½), over a bandwidth Δν from a source of extent Ω_s is:

$$W = \frac{1}{2}A_{eff}\int_{\nu}^{\nu+\Delta\nu}\iint B(\theta,\phi)P_n(\theta,\phi)d\Omega$$

$$W = \frac{1}{2}A_{eff}\int_{\nu}^{\nu+\Delta\nu} S_0 d\nu$$

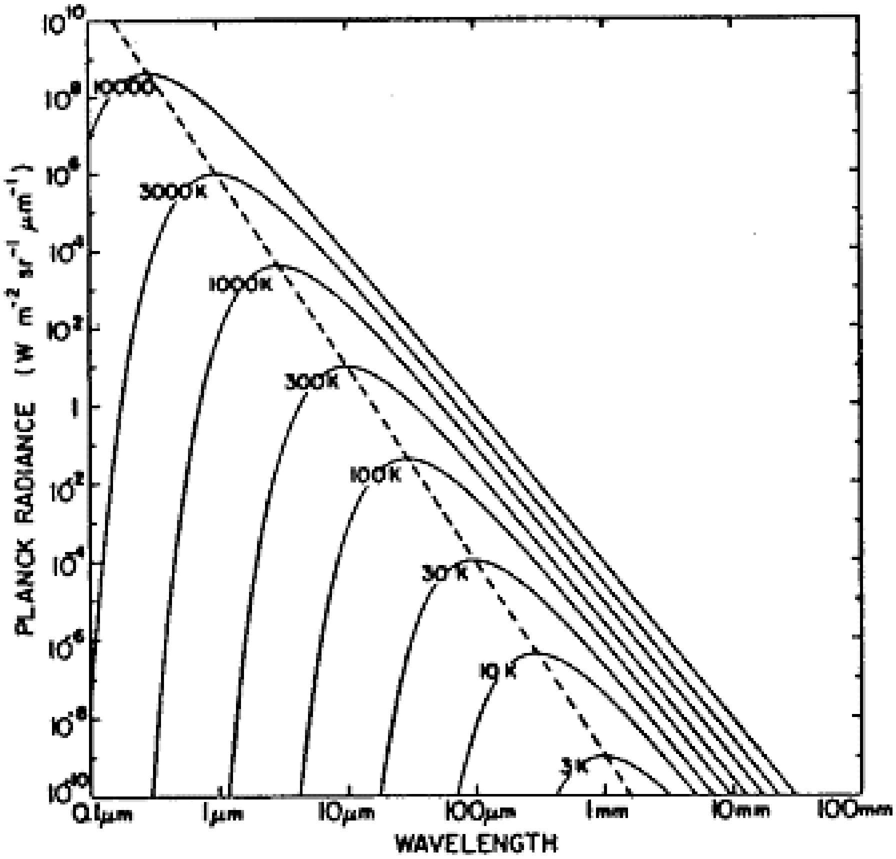
If $A_{eff} = 150m^2$ and the bandwidth Δν= 10 MHz, then in case of Example 2) we get $W=1/2 \bullet 150 \bullet 1 \bullet 10^7 \bullet 3 \bullet 10^{-24} \bullet = 2.25 \bullet 10^{-15}$ W.

2.3 Antenna temperature and noise

The brightness, B, of the radiation from a blackbody is given by Planck's radiation law:

$$B = \frac{2h\nu}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

B: brightness in W m^{-2} Hz^{-1} rad^{-2}
h: Planck's constant 6.63 •10^{-34} Js
k: Boltzmann's constant 1.38 •10^{-23} JK^{-1}
T: temperature of blackbody in K



from http://www.heliosat3.de/e-learning/remote-sensing/Lec4.pdf

The total brightness is:

$$B_{tot} = \int_{-\infty}^{+\infty} B d\nu \qquad \text{[W m}^{-2}\text{ rad}^{-2}\text{]}$$

$$= \pi\sigma T^4$$ (Stefan-Boltzmann law) σ = 5.67·10^{-8} W m^{-2} K^{-4}

$$\int_0^{2\pi}\int_0^{\pi/2} \cos\Theta \sin\Theta \, d\Theta \, d\phi = \pi$$ see p 53

Note: B_{tot} is often given as the integral value over one hemisphere. Then we have

$$B^*_{tot} = \iint_{\pi} B_{tot} d\Omega = \pi B_{tot} = \sigma T^4 \qquad \sigma = 5.67 \bullet 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

For radio frequencies we are mostly in the Rayleigh-Jeans regime and $h\nu \ll kT$.
Then

$$e^{h\nu/kT} = 1 + h\nu/kT$$

$$B = \frac{2h\nu^3}{c^2}\frac{h\nu}{kT}$$

$$= \frac{2kT\nu^2}{c^2}$$

$$= \frac{2kT}{\lambda^2} \qquad \text{Rayleigh-Jeans law}$$

Example: T = 250K
ν = 1 GHz

Then: $h\nu = 6.63\ 10^{-25}$ J
$kT = 3.45\ 10^{-21}$ J ➔ $h\nu \ll kT$

Radiation laws applied to a discrete source

If a blackbody of temperature T subtends a solid angle, Ω_s, then the flux density of the source, S, is

$$S = \iint_{\Omega_S} B(\theta,\phi) d\Omega$$

and for $h\nu \ll kT$

$$S = \frac{2kT\nu^2}{c^2}\Omega_S$$

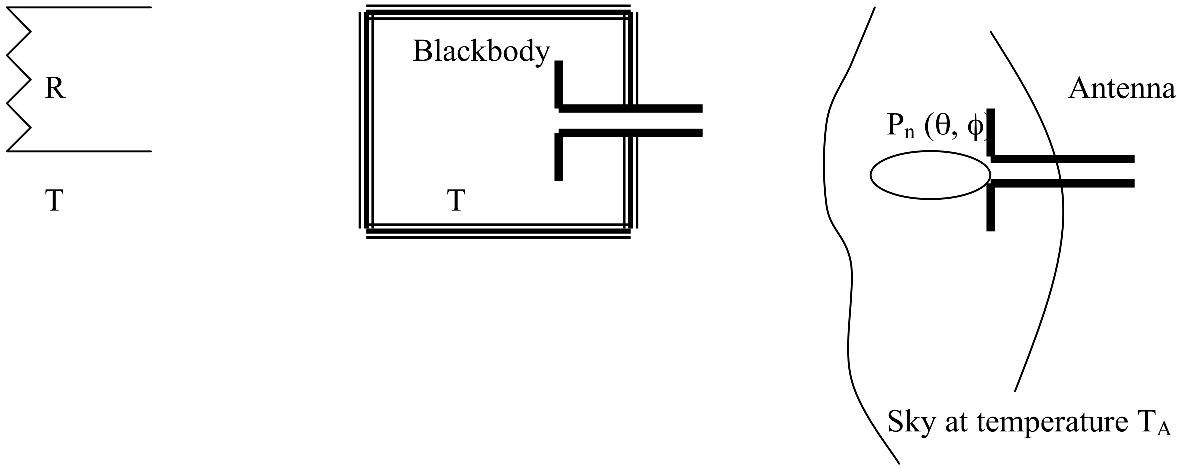
If T is not constant over the source, then we need to integrate of the the temperature distribution of the source:

$$S = \frac{2k\nu^2}{c^2}\iint_{\Omega_S} T(\theta,\phi) d\Omega$$

It was shown in 1962 that S gives not only the flux density of a source at temperature T or with a temperature distribution T(θ, φ), but also when T, or T(θ, φ), is the observed antenna temperature. When the antenna is lossless.

Relation between spectral power, w, brightness, B, and antenna temperature, T_A.

Consider the following scenario:



a) **b)** **c)**

It can be shown that the same spectral noise power is available at the terminals in all three cases.

Case a)

w = kT

Case b)

$$w = \frac{1}{2} A_{eff} \iint_{4\pi} B(\theta,\phi) P_n(\theta,\phi) d\Omega$$

And if we assume that B is constant, that is:

$$B(\theta,\phi) = \frac{2kT}{\lambda^2} = constant$$

$$\iint_{4\pi} P_n(\theta,\phi)\, d\Omega = \Omega_A$$

then:

$$w = \frac{kT}{\lambda^2} A_{eff} \Omega_A$$

Further, with

$$A_{eff} = \eta \frac{d^2}{4} \pi$$

$$\Omega_A = \frac{\theta_A^2}{4} \pi$$

$$\theta_A \approx 1.3 \frac{\lambda}{d}$$

we get:
$A_{eff}\Omega_A = \eta \frac{\pi^2}{4^2} 1.3^2 \lambda^2 \approx \lambda^2$
and finally:
$w = kT$

In other words, the spectral power received by the antenna in the box is the same as that for the resistor.

Case c)

This case is very similar to case b). The temperature the antenna "sees" through its antenna beam pattern is the temperature of the emitting body. This temperature is called the antenna temperature, T_A

We can now write:

$$w = \frac{1}{2} A_{eff} \iint_{4\pi} B(\theta,\phi) P_n(\theta,\phi) d\Omega = kT_A$$

The observed flux density of a discrete source is then:
$$S_0 = \frac{2kT_A}{A_{eff}}$$

If we express the brightness, B(θ, φ), through T(θ, φ), by using the Rayleight-jeans law, with T(θ, φ) = T_s (θ, φ) (source temperature), then
$$T_A = \frac{A_{eff}}{\lambda^2} \iint_{source} T_s(\theta,\phi) P_n(\theta,\phi) d\Omega$$
$$= \frac{1}{\Omega_A} \iint_{source} T_s(\theta,\phi) P_n(\theta,\phi) d\Omega$$

If the source is very small compared to the width of the main beam lobe, and if we are pointing our antenna right at the source, then P_n (θ, φ) is essentially equal to 1 and we get:

$$T_A = \frac{1}{\Omega_A} \iint_{source} T_s(\theta,\phi) d\Omega$$

$$= \frac{\Omega_S}{\Omega_A} \overline{T} \qquad \Omega_S << \Omega_A \quad , \overline{T} : \text{mean temperature}$$

T_S in these equations is equal to the thermal temperature of the source if the radiation is due to thermal emission. If the radiation is generated by non-thermal mechanisms, such as synchrotron radiation or emission from a spacecraft when sending a carrier wave to the earth station, then T_S may be much greater than the thermal temperature of the source. In this case, T_S is the temperature a bl;ackbody radiator would need to have to give radiation equal to that observed at wavelength λ. T_S is also called the equivalent blackbody temperature.

Example:
An antenna is pointed at Mars.
$\theta_S = 18''$ angular diameter
$\Omega_A = 0.018\ \mathrm{deg}^2$
$T_A = 0.24$ K

Then;
$\Omega_S = \pi\ \theta_s^2/4 = 0.00002\ \mathrm{deg}^2$
$T_S = T_A\ \ \Omega_A/\Omega_S$
$= 0.24 \cdot 0.018 / 0.00002$
$= 216$ K

On the other hand, one could also determine the size of a source like a planet or any other celestial body that radiates like a blackbody if there is a means to determine the temperature of the body. A few years ago that was done for one of the newly found large Kuiper belt objects. The Kuiper belt is located beyond the orbit of Neptun and contains millions of celestial objects, mostly comets 10s of km in size but also some larger moon-like objects a couple of 1000 km in size. When a particular object was found recently, the temperature was estimated from the distance to the Sun. Then the antenna temperature was measured with a radio antenna and then the size was obtained. It came out to be larger than Pluto. Should that object be called the 10th planet? Other objects just a bit smaller than Pluto were already known. The decision of the International Astronomical Union was to bump Pluto from its pedestal and consider it to be "just" another member of the Kuiper belt. How would you have voted?

2.4 Minimum detectable temperature and flux density

The minimum T_A a radio antenna can detect is limited by fluctuations in the receiver output caused by the statistical nature of noise, The noise is given by T_{sys} where

$T_{sys} = T_A + T_R$ T_R : receiver noise contributions $T_R = T_{e1} + T_{e2} / G_1 + T_{e3} / G_1G_2$

The fluctuations due to T_{sys} can be reduced by using a large bandwidth $\Delta\nu$, and/or by increasing the integration time, Δt.

$$\Delta T_{\min} = \frac{K_s T_{sys}}{\sqrt{\Delta t \Delta \nu}}$$

K_s : sensitivity constant,

$\Delta T_{\min}$: minimum detectable temperature which is equal to the rms system noise temperature fluctuations.

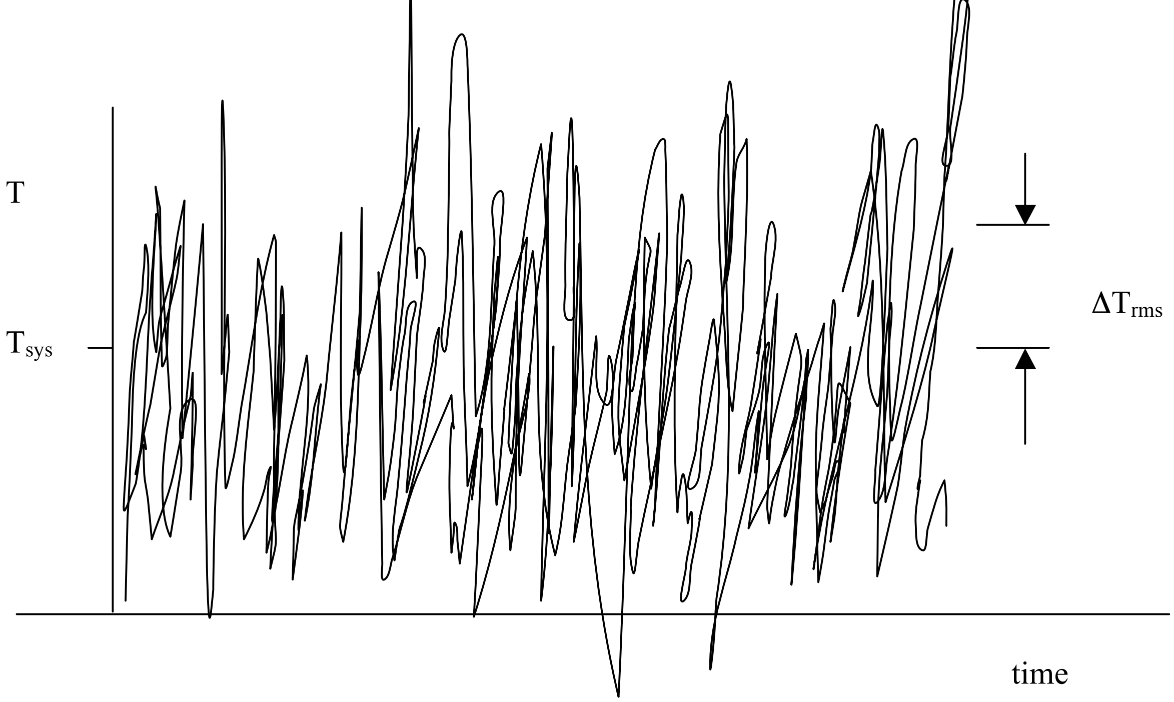
With the Rayleigh-Jeans relation we obtain the minimum detectable brightness:

$$\Delta B_{\min} = \frac{2k}{\lambda^2} \frac{K_s T_{sys}}{\sqrt{\Delta t \Delta \nu}}$$

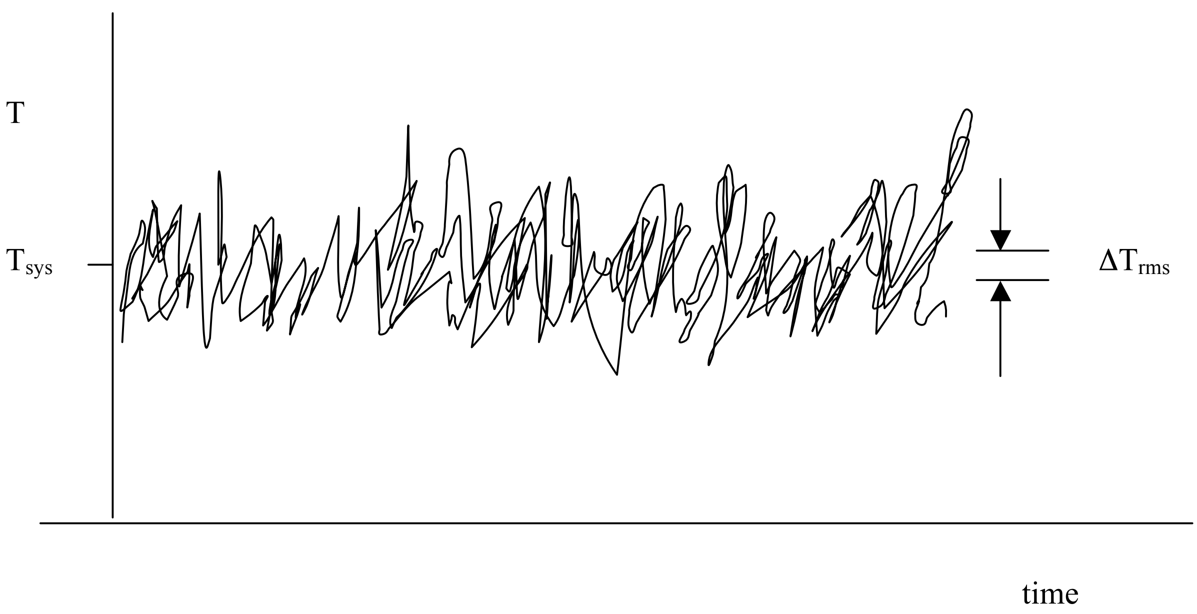
and the minimum detectable flux density:

$$\Delta S_{\min} = \frac{2k}{A_{eff}} \frac{K_s T_{sys}}{\sqrt{\Delta t \Delta \nu}}$$

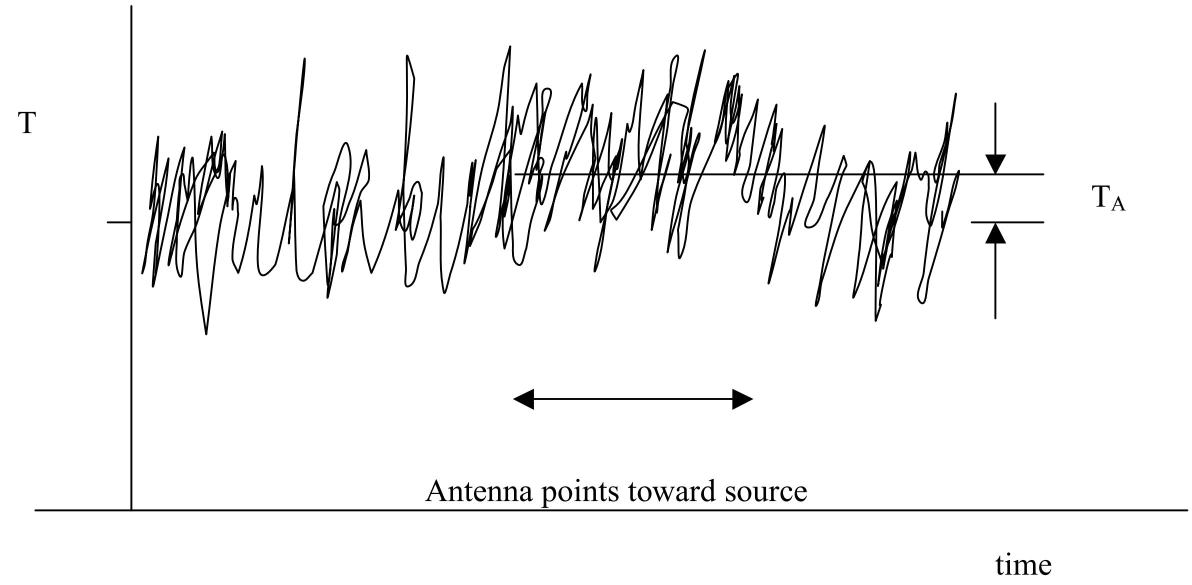
Example:
If $\Delta\nu \, \Delta t = 1$, then scanning across blank sky may give at the detector output terminals a power reading in terms of temperature that may look like this:



Now, if we increase the product $\Delta\nu \, \Delta t$, then the fluctuations, ΔT_{rms}, will decrease with the square root of the product. So, if for instance $\Delta\nu \, \Delta t = 16$, then the fluctuations will decrease by a factor 4 and the power output may look like this:



Now suppose that we are sweeping with our antenna across a celestial source like a planet. That means that the antenna temperature T_A which was nearly 0K (3K to be exact) will increase, and so will the temperature power reading. It may look like this:



The source is just barely visible as an increase of the temperature reading. The minimum temperature increase that can be seen is ΔT_{rms}. So this source causes an antenna temperature that is just a bit higher than the minimum detectable temperature increase.

Example:

$T_{sys} = 125$ K
$\lambda \ \ = 1415$ MHz
$\Delta\nu \ \ = 5$ MHz
$\Delta t \ \ = 10$s
$A_{eff} = 700\ m^2$
$K_s \ \ = \pi / \sqrt{2}$

Then: $\Delta T_{min} = 0.03$K

$\Delta S_{min} = 1.2 \bullet 10^{-27}\ W\ m^{-2}\ Hz^{-1} = 0.12$ Jy

Questions: How would you choose your parameters if you wanted to detect small fluctuations in the microwave background radiation of, say, 0.00001K?