NORMS AND LOGIC

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Attitudes toward deontic logic, one of the most recent branches of logic, vary according to logical and/or philosophical presuppositions. While some wonder what its significance could be in, for example, ethics or law,[1] others see it as the product of a creative activity which gives deontic logic its formal, structural properties. These properties, according to this view, determine the proper value of deontic logic independently of the question of its use.[2]

Thus there are people who are convinced that contemporary deontic logic is useless to those jurists who wish to study the essential features of their own inferences.[3] Others, less pessimistic, even if they may assess with some severity contemporary deontic logic, admit that it can be conceived and constructed differently so as to become useful. Also they seek to formulate its conditions of applicability, for example, in juridic inferences.[4] Still others now seem to find among the systems of deontic logic elaborated up to now, systems capable, at least in part, of serving as a logical foundation for discursive thought, be it ethical, juridic, or other.[5]

This diversity of attitudes is significant in more than one regard. First of all, it permits the supposition that not all the authors of systems of deontic logic conceive of deontic logic in the same way, and consequently that their conceptions of the task (role) of the logician differ too. There is nothing surprising in this diversity of opinion with regard to the nature and the role of deontic logic. It is but a reflection of the divergent views on the nature and

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[1] The investigation of the significance of deontic logic concerning ethics and law constituted the theme of the symposium held in Vienna, September 6, 1968, during the XIV International Congress of Philosophy. The papers presented to this symposium have been published in the *Proceedings of the XIV International Congress for Philosophy,* Vienna, September 2-9, 1968, Wien, Verlad Herder, 1968, vol. II, pp. 269-311. This theme was also the basis of a symposium of December 22 and 23 in Brussels where the relationships between deontic and juridical logic were debated and the papers of which were published in *Logique et Analyse* of 1970 as well as in Volume IV of the *Etudes de Logique Juridique* published by Ch. Perelman (Brussels: Etablissements Emile Bruylant, 1970).

[2] We refer to the paper of A. R. Anderson presented to the Brussels symposium which appears in *Logique et Analyse* of 1970.

[3] See for instance the intervention of Professor Perelman at the Vienna symposium, mentioned in our communication from Brussels, "Raisonnement Juridique et Logique Juridique," *Logique et Analyse,* XIII (1970).

[4] See for example, Z. Ziembinski, "Conditions préliminaires à l'application de la logique déontique dans les raisonnements juridiques," paper presented to the Brussels symposium, *Logique et Analyse* (1970).

[5] See for example, our communication at the Vienna symposium, "La signification de la logique déontique pour la morale et le droit," *Akten des XV. internationalen Kongresses der Philosophie,* pp. 285-290.

role of logic in general. Robert Blanché recently devoted a book to a very thorough study of this subject.[6] Human thought, discursive as well as intuitive, has both an intensional and an extensional aspect: according to the emphasis placed on one or the other aspect, the corresponding conception of logic is arrived at (obtained). The choice here is determined by our inclination either towards *"l'esprit de finesse,"* leading to "reflexive logic," to use the nomenclature of M. Blanché, or towards *"l'esprit de géométrie,"* leading to the kind of logic called by Blanché "formal" or "formalistic." Without subscribing to his dualistic view of logic, for we believe on the contrary that logic is a single whole, and that the *"esprit de finesse"* and the *"esprit de géométrie"* as complements, can and must provide the basis for a harmonious synthesis of "reflexive" and "formalistic" logic, we agree completely with M. Blanché affirmation that extensionalism, when carried to the extremes of formalism, transforms logic, which is supposed to be the science of the laws regulating discursive thought, into a purely creative activity, in the limiting case, a game.[7] In this context, it is not surprising that uneasiness exists, that ethicians and jurists do not think much of contemporary deontic logic, that restricting conditions are imposed on it, when it is not condemned and eliminated once and for all. The problem is thus formulated and the debate opens. What requirements must deontic logic, as a branch of logic, satisfy in order to be what it must be, the science of the rules and laws of discursive thought, and in order not to deceive those who look to it for the theoretical knowledge, if not the practical direction, of normative discursive thought. It is to this very question that the present paper would like to make a modest contribution by analyzing from the point of view of their semantic category, the deontic functors really used by deontic logicians in their systems.

For it seems to us that deontic logic will be really useful only if it is at the same time materially adequate and formally correct. In *Pour une Conception Adequate de la Logique Deontique,*[8] we insisted on the first condition, and now propose to examine more closely the second or, more precisely, one of the numerous requirements of the second condition, that which concerns the theory of semantic categories.

I. THE NOTION OF A SEMANTIC CATEGORY

Before going further it is appropriate to consider for a moment the notion

[6] R. Blanché, *Raison et discours* (Paris: Librairie Philosophique J. Vrin, 1967). See our account of this work in *Archives de Philosophie,* XXX (1970).

[7] R. Blanché, *op. cit.,* p. 63 *in fine.*

[8] *Proceedings of the Polish Congress of Contemporary Science and Culture in Exile* (London: 1970).

of a semantic category. An interesting study of this notion could be made. Such a study would take us back to Stanislaw Lesniewski, and the Polish logicians influenced by him such as Alfred Tarski or Kazimierz Ajdukiewicz, to Bertrand Russell and Edmund Husserl, and to Aristotle. At least this is the perspective opened by Eugene C. Luschei in the following passage:

> *La grammaire des catégories sémantiques de Lesniewski rappelle formellement les théories simples des types inventées ad hoc pour empêcher les paradoxes bien connus. Mais, quant à sa conception et à son but, elle possède davantage d'affinité avec la tradition des catégories d'Aristote, les Bedeutungskategorien de Husserl et la grammaire des parties du discours dans les langues indo-européennes.*[9]

However, we shall not undertake this historical study, as interesting and enlightening as it may be. Our subject does not call for it. Let it suffice to recall that Stanislaw Lesniewski, realizing the qualities and defects of Russell's work, conceived of the ambitious project of perfecting a universal system of logic and foundations of mathematics. Mr. Luschei describes it in these terms:

> *Il n'ambitionnait pas la création de quelque nouveau calcul qui viendrait s'ajouter aux calculs déjà inventés ni même la démonstration des théorèmes généraux sur les calculs formels alternatifs au nom de la "logique comparative," mais avait pour ambition de perfectionner le système de logique et de fondements des mathématiques universalement valide de telle manière qu'il puisse formuler là rigoureusement les généralisations, se laissant exprimer uniquement en métalangage des systèmes moins riches en moyens d'expression . . . et s'yfier comme à un véritable instrument de déduction et d'investigation scientifique.*[10]

In the course of this project, Lesniewski elaborated, among other things, his theory of semantical categories (*grammar of semantical categories* in Luschei's original terminology); he used this theory to replace the hierarchy of types which he was not able to justify intuitively. The theory of semantic categories, as created by Lesniewski, is an integral part of his system, but the idea of the theory may be detached from the system and used for the rigorous analysis of any language, whether formalized or natural. In *Syntactic Connexion,*[11] by Kazimierz Ajdukiewicz, one finds a remarkable example of the

[9] E. C. Luschei, *The Logical Systems of Lesniewski* (Amsterdam: North-Holland Publishing Co., 1962), p. 35.

[10] *Ibid.*, p. 24. Lesniewski adopted the system of *Principia Mathematica* as a general model to follow its main topics. But the example of deductive exactness was provided to him by G. Frege (*Grundsatze der Arithmetik*). E. C. Luschei, *op. cit.*, p. 78.

[11] K. Ajdukiewicz, "Syntactic Connexion," *Polish Logic 1920-1939* (Oxford: Clarendon Press, 1967), pp. 207-301.

use of the generalized theory of semantical categories. K. Ajdukiewicz remarks appropriately:

> *L'échelle des catégories sémantiques ainsi détachée du système de Lesniewski et généralisée s'apparente—selon la très juste remarque de K. Ajdukiewicz—à la hiérarchie simplifiée des types logiques—bien qu'ellésoit dans une très large mesure plus ramifiée que celle-ci—et én constitue au fond le pendant de caractère grammatical et sémantique.*[12]

We are neither undertaking a historical study of Lesniewski's or Ajdukiewicz' works, nor the construction of a logical system for which we would have to guarantee not only the syntactical connectivity, but also that the meanings of all the well-formed expressions be free from antinomies. Therefore we should summarize neither Lesniewski's hierarchy of semantical categories nor the method developed by Ajdukiewicz for the verification of the syntactic connectivity of well-formed expressions. If we take into account our objective, it is sufficient merely to recall the notion of semantic category.

In Edmund Husserl's *Logical Investigations*[13] we find that, though the author does not formulate a definition of the term "semantical category," he does show what must be understood by it by means of an example. Let us take the proposition: "This tree is green." Let us replace "tree" by the symbol "S" and "green" by the symbol "P." We obtain the propositional semiotic function: "This S is P." If we want to transform this function back into a proposition, i.e., into a meaningful expression constituting a unity of meaning (every proposition constitutes a unity of meaning), we are obliged to respect a rule of substitution which requires that, in Husserl's terminology, a "nominal matter" be substituted for nominal symbols, an "adjectival matter" be substituted for adjectival symbols, a "relational matter" be substituted for relational symbols, and so on. Thus, if we substitute "gold" for "S," and "blue" for "P," we obtain the proposition: "This gold is blue." This proposition, though false, is meaningful. Moreover, it is because of the fact that this is a meaningful proposition that it can be seen as being either true or false. On the other hand, if we substitute the relational matter "is bigger than" for the nominal symbol "S," and the propositional matter "it is raining" for the symbol "P," we obtain the expression: "This is bigger than it is raining," an expression completely without meaning and therefore neither true nor false. Since it does not form a syntactically connected expression, the product

[12] *Ibid.,* p. 208.

[13] E. Husserl, *Logical Investigations,* translated into English by J. N. Findley (London: Routledge and Kegan Paul, and New York: Humanities Press, 1970), vol. 2, Investigation IV, pp. 493-529, especially the Introduction and Sections 3, 5-7, 10-14.

of the last substitution is not endowed with a unity of meaning. Each one of the words composing it possesses a meaning, but the sequence as a whole is not meaningful. Instead of being organically linked together to constitute a unity of meaning, they are only placed side by side, without forming any such unity.

A rigorous, formally correct, and materially adequate definition of the term "semantic category" can be given only for particular languages, for example, the languages of the systems of Lesniewski or Lukasiewicz. But since such a task would be beyond the scope of the present paper, we must content ourselves with a more or less imperfect definition that is satisfactory enough to suggest the general notion of semantic category. Such a definition requires resorting to the notion of a signification constituting a unity of meaning, to use Husserl's expression. We shall consider this notion as already introduced into our conceptual framework in order not to overextend our exposition. Therefore, we do not define it. Given that, we can formulate the following definition:

> The expressions A and B belong to the same semantical category if and only if there are such expressions, C and D.
> 1. D is a (not necessarily proper) part of C, and
> 2. D is equiform with A, or D is equiform with B, and there are such expressions, E and F,
> 3. E is the result of substituting F for D in C, and
> 4. If D is equiform with A, then F is equiform with B, and
> 5. If D is equiform with B, then F is equiform with A, and
> 6. C is an expression with a signification constituting a unity of meaning, and
> 7. E is an expression with a signification constituting a unit of meaning.

As can be seen, the notion of semantic category is closely tied to the rule of deductive inference known as the rule of substitution. This rule is so well known, the author of the present study is somewhat embarrassed to recall the notion of semantic category, for no rule of substitution can be formulated without either implicit or explicit recourse to this notion. If he does it anyway, it is because he is driven to it by the liberties taken by some logicians who neglect to indicate, correctly and exactly, the semantic categories of the expressions they are using.

Unfortunately this is the case with several deontic logicians, even some of the most active and famous ones, such as A. R. Anderson, A. N. Prior, and G. H. Von Wright. What is more, when we seek to fill in the gaps and want to determine the semantic categories of their expressions, we are sometimes led to reject their own partial indications because they are found to be inadequate. We realize, then, that their systems were not always conceived on the model of the real discursive thought to be explained, thought that M. Blanché calls "natural operative logic." For if these deontic logicians had desired to bring in this thought, this thought, by its easily identifiable units of meaning (since such an investigation begins in the intensional terrain of the intelligible content), would have imposed an adequate and exhaustive determination of the semantic categories of the constants, variables, and functions (other than variables) which they could have used for the construction of such and such formalism invented for the interpretation supplied initially by the real discursive thought studied. If the semantic categories of these expressions are not indicated, or if they are misrepresented, then this proves that the systems of deontic logic thus impaired have been built in a more or less mechanical and *a priori* fashion, sometimes also in a combinatorial way which abstracts partly or totally from the real discursive thought to which it returns only after the system is created, in order to give it, in addition—if it is at all possible—an interpretation and an application.[14] It is not surprising, then, that these logicians are not always successful and that finally those involved, the moralists, the jurists, etc., are hardly satisfied with the deontic logic offered to them. What was just said once more justifies the object of our analysis and shows its importance, theoretical as well as practical.

II. FUNDAMENTAL SEMANTIC CATEGORIES

If the present article were only meant for logicians, it would be superfluous to enumerate the fundamental semantic categories. But primarily, it is intended for jurists who are not supposed to know them. Thus, before examining the deontic functors used by logicians, before determining their respective semantic categories and comparing the claims of these logicians with our results, we shall review the main semantic categories to which the expressions of the usual logical languages, deontic or others, belong. Three such general categories can be distinguished: expressions having the syntactical structure of (1) propositions, (2) names, and (3) functors. Quantifiers constitute a separate group of expressions because they do not fall into the definition of semantic category as formulated above.[15]

[14] Such was not Lesniewski's attitude as he said of himself: "I am constructing my system in the manner of a radical 'formalist," because I am indeed a convinced 'intuitionist.' " (S. Lesniewski, "Grundzuge eines neuen Systems der Grundlagen der Mathematik," *Fundamenta Mathematicae,* XIV, p. 78 (1929, pp. 1-81, quoted by E. C. Luschei, *op. cit.,* p. 50). " 'Formalization as a servant of intuition'—such was his motto, if we may say so. Therefore, according to Lesniewski, interpretation was not an accessory of formalism, most certainly useful but not indispensable; it was its source and its object at the same time."

[15] Quantifiers do not fall into our definition of semantic category because this definition resorts to the notion of substitution, and one does not substitute quantifiers for each other;

This enumeration calls for some explanation. The first basic semantic category is usually the proposition (and, of course, the propositional variable, for variables and semiotic functions other than variables belong to the same semantic category as their values) or, more accurately, the logical proposition, that is, the kind of expression which has the syntactic structure of a grammatical sentence, and which can be either true or false. However, this seems unacceptable for various reasons.

First, in support of the thesis that we are disputing, the authority of tradition going back to Aristotle is invoked. To that we answer that the *argumentum ex auctoritate* is not appropriate in logic. Besides, while it is certainly true that Aristotle says in *On Interpretation*: "However, not every discourse is a proposition, but only the discourse in which the true and the false reside, which is not always the case . . . ,"[16] on the other hand he admits practical knowledge alongside theoretical knowledge, which considerably increases the number of "discourses in which the true and the false are found." Indeed for Aristotle, norms and estimates are still true or false.[17] Moreover, "discourses in which true and false are not found" are not excluded from the field of logic because according to the Stagirite, they belong to rhetoric which posterity is rightly considering as one of the branches of Aristotelian logic.

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each one constitutes, so to speak, a semantic category of its own. For even if "*every*" can be substituted for "*a certain*" or vice versa, in expressions such as: "*every poet is an artist*" or "*a certain animal is a mammal,*" without transforming a meaningful expression into a meaningless one, we do not see any possibility of introducing a symbol, in the present case a quantificatorial variable (*sit venia verbo*), for which one could, at leisure, sometimes substitute the universal quantifier, sometimes the existential quantifier, without destroying the unity of meaning of a given expression. If it were possible, we would be allowed to pass from expressions such as: "*For every x, if x is f then x is g*" to expressions such as "*There exists x such that x is f and x is g.*" But this is not allowed and should not be allowed, because the universal quantifier is a generalization of conjunctions while the existential quantifier is a generalization of disjunctions. In fact, "*for every x, fx*" means the same thing as "$fx_1 . fx_2 . fx_3 \ldots fx_n$" and "*there exists an x such that fx*" means the same things as "$fx_1 \vee fx_2 \vee fx_3 \vee \ldots fx_n$." According to the definition of the universal quantifier "*for every x*" means the same thing as "*it is not the case that there exists a non-x such that*" (in symbols "$(x)fx = \approx(\exists x)\approx fx$"), and if we also take into account De Morgan's law: "$\approx(p . q) = \approx p \vee \approx q$" then the expression: "*For every x, fx*" meaning the same thing as "$fx_1 . fx_2 \ldots fx_n$" is transformed into: "*It is not the case that there exists a non-x such that not* fx_1 *v not* $fx_2\ fx_3 \vee \ldots \vee fx_n$." This shows that in the case of simple expressions of the type "$(x)fx$" or "$(\exists x)fx$" as well as in the case of complex expressions of type "$(x)(fx \rightarrow gx)$" or "$(\exists\ x)(fx \,\&\, gx)$" the mutual substitution of universal and existential quantifiers is not possible without destroying the structure securing the unity of meaning. To that may be added the arguments given by R. Blanché in *Raison et discours,* ch. VIII, §§ 21 and 22. For this logician from Toulouse, the universal and existential quantifiers belong to two different semantic categories, because the first one, unless expressing only the conjunction of a limited number of cases, is situated on the level of the *essence,* source of the *necessity,* to which all individual cases, even if there are infinitely many, are subordinated, while the second one belongs to the field of *existence* where—aside from the Absolute—everything is *contingent.*

[16] Aristotle, *On Interpretation, 4, in fine* (17 a 2ss).

[17] *Ibid., Nicomachean Ethics,* VI, 2. See also G. Kalinowski, *Le problème de la vérité en morale et en droit* (Lyon: E. Vitte, 1967), Ch. II, § 2, Aristotle, pp. 129-134.

Let us add that Aristotle himself did not use the name "logic" to designate the entirety of the science of discursive thought as it does today. T. Kotarbinski reminds us of this in his *Leçons sur l'histoire de la logique.*[18] On the other hand, the Stagirite gave diverse parts of logic well-known names such as "analytic," "dialectic," "sophistic," etc. Only in the Byzantine epoch was the whole of his works treating the above subjects called *"Organon"* and the title of "logical writings," finally replaced by "Aristotelian logic," came even later.

In any case, if we set aside every authority, including Aristotle, we notice that discursive thought is the form of inferences which involves estimations, norms, imperatives, questions, prayers, etc., as well as judgments of reality (judgments of dispute, descriptions, explanations, in short, theoretical judgments). Therefore there is no reason to leave this part of our discursive thought outside the field of investigation of logic, if the latter must fulfill its nature of science for the laws of discursive thought as such.[19]

Second, one might argue that the functors of the theory of propositions are organically tied to statements which are either true or false (or which possess some intermediate logical value in the case of a many-valued logic), and that, consequently, they cannot have statements which, by their very nature, have no truth value as arguments. This argument is not pertinent either. For today's logic is formalized to such a degree that symbols "1" and "0" or other analogous symbols ("T" and "F" for example, or "+" and "−") used in 'truth-tables" do not necessarily represent true and false, but any mutually exclusive values capable of attribution to expressions having the grammatical syntactical structure of propositions which could be substituted for the variables "p," "q," "r," etc., from the logic of propositions.

Consequently, we do not see any valid reason not to consider propositions the first fundamental semantic category, provided that this term may be taken as broadly as it is taken in grammar. It is only because most logicians restrict its scope to the true or false propositions—without good reason in our judgment—that they reserve to them the title of "logical propositions" and exclude all the other grammatical propositions in our discourse that are just as entitled to be called logical propositions, since they equally constitute the object of logic, inasmuch as they are elements of our discursive thought. But these considerations do not exempt us from having to subdivide the generic category of propositions into various species and subspecies such as propositions endowed with truth and falsity, and propositions lacking such values. At the second level the true or false propositions can be further divided into

[18] *Op. cit.,* p. 4 s, n. 4.

[19] Concerning this see also R. Blanché, *Raison et discours,* p. 75 s.

declarative, estimative, normative propositions, while the others can be divided into imperative, interrogative, exclamative, etc., depending on whether it is possible or impossible to substitute them for a same variable symbol without destroying the given unity of meaning.

It is important to distinguish two different kinds of names, namely, empty names which do signify concepts, but do not designate (denote) any real object, and non-empty names. The names in both of these groups can be either singular or general (predicates). In turn, singular names subdivide into proper names ("Julius Caesar," for example), ostensive names (let us say, "this tree"), and descriptions ("the author of 'De bello gallico' "). Singular non-empty names designate (denote) real objects (entities). (The semantic function of proper names is to name. Thus one prefers usually to say that they name rather than to say that they designate.) Singular empty names, such as "Juno," do not designate (denote)—in this very case do not name—anything, for they are not (mediate) signs of *real* objects (entities), but of *intentional* objects (entities) which could be designated (denoted) or, more exactly, named.[20]

Any expression which is neither a quantifier nor a proposition nor a name is a *functor*. Functors subdivide into three kinds according to the category of the expressions they create. Thus we have *proposition-forming functors, name-forming functors,* and *functor-forming functors*. In turn, each of these three categories subdivides into several categories and subcategories according to the number of arguments required for the given functor in order to create the corresponding expression and the semantic category of its arguments. Thus we have proposition-forming functors with a proposition as an argument (the propositional negation, for example), or with two propositions as arguments (conjunction, for example). We have proposition-forming functors with a singular name as an argument ("Venus[21] exists"), with two singular names as arguments ("Romulus is the brother of Remus"), a singular name and a general name ("Robert Kennedy is a senator"), or two general names ("The prefect is a civil servant"). We have name-forming functors with a singular name, two singular names, a general name, two general names, etc., as arguments ("non-payment of 100 F by Peter to Paul, on January 10, 1970, at 10:30 a.m., at window 5 in the Post Office of Orsay," "Peter or Paul," "non-square," "truth or falsity," etc.). We finally have functor-forming functors with another functor of such or such semantic category as an argument ("does not . . ." is an example of such a functor-forming functor—

[20] Concerning intentional entities (intentional objects), see G. Kalinowski, *Querelle de la science normative* (Paris: Librairie Générale de Droit et de Jurisprudence, 1969), ch. III § 5.

[21] "Venus" is here the name of a planet.

together with a proposition-forming functor with a singular name as an argument, such as "exists," the functor "does not . . ." creates a new functor of the same category, namely "does not exist,") etc.

This enumeration of basic semantic categories is sufficient, but without pretending to be exhaustive. Nevertheless the two following remarks have to be added: in order to determine the semantic category of an expression in a thorough and precise manner, one should also take into account (1) the degree of the language (object-language, metalanguage, etc.) to which the given expression, as well as each one of its components, belongs, if need be; (2) the symbols of constants or variables belong to the same semantic category as their values, i.e., the expressions they are representing.

III. DEONTIC FUNCTORS AND THEIR SEMANTIC CATEGORY

We are now ready to start the analysis of those deontic functors really used by deontic logicians. They are evidently too numerous for all of them to be examined. Therefore, we shall choose only a few of the most representative ones by dividing them into two groups. The first group of logicians to be considered will be those using deontic functors which can be assigned to semantic categories precisely, without problems, even if this has not been done by those logicians. On the other hand, the second group will consist of those who are using deontic functors indeed raising problems.

A. *Unproblematic Deontic Functors*

In reality, deontic logicians use quite varied functors, functors belonging to the object-language (Castaneda, for example, or Tammelo) or, in the contrary, to the metalanguage (Von Wright, in his logic of *normative statements*, or Ziemba), deontic predicate-forming functors (general names), norm-forming functors, and proposition-forming functors on norms with the most diverse expressions as arguments: singular names, general names (predicates), and propositions. (About the functors of this last category, we shall have to ask later on whether their argument is a proposition or, indeed, the (metalinguistic) name of a proposition.) But before we come to that, let us note the limiting case with which we are going to start and in which we shall deal with a deontic logic without deontic functors.

This is the case with E. García Máynez' calculus.[22] In fact, the Mexican professor utilizes no functors, but deontic predicates such as "regulated,"

[22] E. Garcia Maynes, *Los principios de la ontologia formal del derecho y de la logica juridica* (Mexico: Imprenta Universitaria, 1953).

"licit," "illicit," "orderly," etc., which he symbolizes respectively by "J," "L," 'I," L₁," etc. These predicates are used as arguments, on one hand, to the proposition-forming functor "Σ" being interpreted "is" in the sense of belonging of an element to a set. This functor combines with, as arguments, a singular name, in the present case a name of an action, and a general name, in the present case a deontic predicate. And, on the other hand, the above predicates are used as arguments to the negation of a predicate, that which results into functions like "L" being interpreted "non-licit," or to one of the interpredicative functors (predicate-forming functors with two predicates as arguments) such as the interpredicative conjunction or the interpredicative disjunction, etc. From these elements, Mr. García Máynez sets up two, even three categories of deontic functions: propositional functions such as "xΣL" being interpreted "x is licit" where "x" is an individual name variable representing the name of an action, and predicative functions such as "L+I" = J" being interpreted "licit or illicit if and only if regulated," corresponding to which one has propositional functions, for example, "(xΣL) + (xΣI) = (xΣJ)." To these, quantifiers are added later on, the universal quantifier in the present case. Such an addition transforms the function cited as an example into the universal proposition (one of the theses of Mr. García Máynez' deontic logic) "(x){[(xΣL) + (xΣI)] = (xJ)}."

A related conception of deontic functions is found in MM. Tammelo and Klinger's works.[23] In fact, Ilmar Tammelo, and following him, his disciple Ron Klinger, build the theses of their deontic logic by means of the deontic logical functions "ba," "bo," "la," "lo," "na," "no," "pa," "po," "wa," "wo," where the letters "b," "l," "n," "p," and "w" are respectively symbolizing the deontic predicates "obligatory," "permissible" (in the sense of bilateral permission—that is permission to do and not to do), "non-regulated," "permissible" (in the sense of unilateral permission), and "lawful," where "a" and "o" stand for two other deontic predicates of "action" and "omission." Therefore, expressions "ba," 'bo," etc., really are general-name (predicative) functions representing conjunction deontic predicates "obligatory action," "obligatory omission," etc. These conjunctive predicates (like the atomic predicates of which they are composed) are deontic constants and not variables, as Mr. Klinger wrongly calls them.[24] He is equally wrong to hold their components "b," "l," etc., as functors.[25] As we have seen, any expression other than a proposition, a name, or a quantifier, is a functor.

[23] I. Tammelo, *Law, Logic and Human Communication* (Archiv für Rechts- und Sozialphilosophie L, 1964), pp. 331-366; R. Klinger, *Basic Deontic Structure of Legal Systems* (Sydney: Univ. of Sydney, Faculty of Law, 1969), mimeographed thesis.

[24] R. Klinger, *op. cit.*, p. c and p. 17.

[25] *Ibid.*, p. 6.

Now, "b," "l," etc., are general names (predicates). From these elements, MM. Tammelo and Klinger build theses such as "NKbabo" interpreted as "not simultaneously obligatory action and obligatory omission." [Implied: (the latter is) opposed to the given action.] The way in which Mr. Klinger himself is interpreting this thesis, which is different from ours, shows that in the theses of MM. Tammelo and Klinger's deontic logic, an individual name variable, representing a concrete instance of conduct, is implicit. Indeed, Mr. Klinger interprets the thesis "NKbabo" as follows: "It is not the case that an instance of conduct will be at the same time an obligatory action and an obligatory omission." (The expression "an instance of conduct," although it is borrowed from everyday language, is indeed the variable "x" written in nonsymbolic notation, so that the expression containing it remains a logical function, and would only be transformed into a proposition if the variable in question would be linked to a quantifier, or if one of its values were substituted to it, the ostensive singular name (*"this very instance of conduct,"* for example). The result is that MM. Tammelo and Klinger's theses also include another category of implied expressions: the functor "is" (in the meaning of belonging of an element to a set), a functor which, as we have seen, Mr. García Máynez made explicit and wrote by means of the symbol "Σ." If we adopt the same symbol and introduce the variable "x," we can confer to the thesis taken as an example, its exhaustive form by writing it "NKΣxbaΣxbo" and interpret it: *"it is not the case that simultaneously x will be an obligatory action and an obligatory omission."* As we can see, MM. Tammelo and Klinger's deontic logic, as well as Mr. García Máynez', is a calculus of deontic predicates, without deontic functors strictly speaking. [If our authors had conceived a system where a general name variable, "a" let us say, would have represented as does variable "A" from G. H. von Wright in his 1951 system mentioned later on, just any general name of action (the name of a positive action as well as that of a negative action, in other words, of an omission), names such as "theft," "purchase," "tax payment," "respect for other people's property," etc., then signs "b," "l," etc., could have been held as symbols for constants representing deontic functors "is obligatory," "is permissible," etc. But we have seen that this is not the case, in reality.]

Other logicians use proposition-forming functors in the case of propositions meaning norms. This is the case with H. N. Castenada's works where we find functions like "K(A[x])" which is interpreted as *"x must do A."*[26] However, this logician also utilizes functions which call to mind MM. Tam-

[26] Following MM. Tammelo and Klinger, we adopt Jan Lukasiewicz's system of symbolic notation without parentheses, known under the name of Polish notation.

melo and Klinger's functions, *"x is an obligatory action"* for instance, but which are not identical because the term "action" has, under Mr. Castenada's pen, a broader meaning than it does for MM. Tammelo and Klinger: Castenada's action designates negative actions (omissions) as well as positive ones, whereas "action" for MM. Tammelo and Klinger is opposed to "omission."

Functions similar to Mr. Castenada's "K(A[x])" are also found in our system K_2.[27] But there they form only one of all the four functions used in our two systems K_1 and K_2, the second being the prolongation and complement of the first. Deontic functors used in functions found in both Mr. Castenada's system and ours K_1 and K_2 are *proposition (norm)-forming functors*—with a singular name of action as subject argument, and a general name (predicate) of action as predicate argument. Deontic functors appearing in the functions of the tree categories of our systems K_1 and K_2 are the following: proposition (norm)-forming functors with two singular names of action as argument, analogous functors with a general name (predicate) of action and a singular name of action as arguments, and also proposition (norm)-forming functors with a general name of action and a general name of action as arguments.

That brings us to the subject of the deontic syllogistic with which Mr. Ziemba is trying to replace our systems K_1 and K_2.[28] In his system our Polish colleague introduces functions such as "x ob_n X" which is interpreted *"x is forced, the set of orders n taken into account, to be X."* These functors are proposition-forming functors or, more exactly, meta-proposition-forming functors which signify judgments of a determined type, on the orders with three names as arguments: the singular name of an agent (represented by the individual name variable "x"); the general name (predicate) of an agent, symbol "X,"; a name which characterizes him as a man having adopted such and such attitude, or, in other words, having behaved in such and such way, or having acted in such and such manner ("thief," "murderer," "delivery man of the sold object," "debtor paying off his debt," etc., and the name of a set of orders, "n").

The last two names, the general name of an agent and the name of a set of orders, call for explanations. As to the first one, Mr. Ziemba may,

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[27] H. N. Casteñada, "On the Logic of Norms," *Methodos,* IX (1957), pp. 209-216.

[28] G. Kalinowski, "Théorie des propositions normatives" (*Studia logica,* I (1953), pp. 147-183). This study is the summary of our typewritten Polish thesis *Logika zdan praktycznych—Logique des propositions pratiques,* presented in June, 1951, to the Faculty of Philosophy of the Catholic University of Lublin, unpublished because of life conditions in the Stalin era. Its summary was published as soon as the edition of a new magazine, in *Formal Logic,* was finally authorized in 1953, after the suppression of the Polish philosophical magazines (*Przeglad Filozoficzny, Kwartalnik Filozoficzny,* and *Ruch Filozoficzny*).

consciously or unconsciously, have been under the influence of Tadeusz Kotarbinski's *reism* (pansomatism or concretism)—Mr. Ziemba is the disciple of Mrs. Janina Kotarbinska, professor of logic at the University of Warsaw and thus finds himself in the zone of influence of her husband's philosophy. According to this system, only objects exist: (Aristotle would have said "primary substances") the other Aristotelian categories, quantity, quality, action, relation, etc., do not exist; they are nothing but fiction.[29] Whatever may be the case, by adopting only the individual name variables "x," "y," etc., and the general (predicative) name variables "X," "Y," etc., respectively representing singular names and general names of agents as such, Mr. Ziemba makes his system simpler, and therefore more elegant than ours, since he thus eliminates our variables "k" and "A" representing respectively singular names and general names of actions. But by this approach, our colleague omits from his deontic syllogistic the concrete rules that prescribe, forbid, or permit a concrete action to a concrete agent. It seems that this gap may be filled through a suitable adaptation of the substitution rule in Mr. Ziemba's system. This adaptation authorizes the substitution for the variables "X," "Y," etc., not only predicates, but also descriptions such as *"The man carrying out hic et nunc this action," "The man behaving hic et nunc in this way."* But one realizes upon reflection that there is nothing to be gained by this expedient, because one eventually runs up against new difficulties which turn out to be insurmountable.[30]

As to the name of a set or orders, to which category of names does it belong? For there are two kinds of names of sets: collective names and distributive names. The "classe 4 B_1" is a collective name (4th grade); it is the singular noun (not empty) of a set of students in *Lycee Blaise Pascal* in Orsay. The name "student in 4th grade B_1" is a distributive name: it is the affirmable name of each element of the said set of students. Mr. Ziemba does not make precise, *expressis verbis,* in which sense he uses the expression "the name of a set of orders." The names of the first kind may also be called "mereological names," for they are names of totalities whose parts can be numbered, should such be the case. Thus the name "4th grade B_1" is the name of the student community composed of the parts constituted by the students individually taken and whose proper names we write with the sym-

[29] Z. Ziemba, *Logika deontyczna jako formalizacja rozumowan normatywnych—Logique deontique, as formalization of normative reasonings* (Warszawa: Panstwowe Wydawnictwo Naukowe, 1969), Rozprawy Uniwersytetu Warszawskiego 37.

[30] T. Kotarbinski, *Gnosiology* (Oxford: Pergamon Press, s.d. As to the analysis of reism, see K. Ajdukiewicz's account, published in the original Polish edition of Mr. Kotarbinski's (1929) in *Przeglad Filozoficzny,* XXXIII (1930), and reproduced in English version, at the end of *Gnosiology,* pp. 515-536.

bols $x_1, x_2, x_3 \ldots x_k$. Mr. Ziemba's definition DII leads one to suppose that "n" is a mereological name.[31] Therefore we have a right to recognize that "n" is equivalent to the expression *"set of orders composed* of ρ, χ, $\Psi \ldots$ and ω" where "ρ," "χ," "Ψ," ... and "ω" are the names of the respective orders, constituent parts of the set in question. "n" is consequently a metalinguistic name in the sense that it is a nominal compound expression, having meta-names. From this, it follows that Mr. Ziemba's deontic functors, despite the fact that, theoretically, they have a structure analogous to the functors used in Mr. Castenada's system as well as in ours K_1 and K_2, belong to an entirely different semantic category from the latter ones. The latter are functors figuring in the vocabulary of the object language, while the former are indeed part of a metalanguage.

B. *Problematical Deontic Functors*

The deontic functions already examined up to now sometimes brought up problems. This was particularly the case with the functions used by MM. Tammelo and Klinger—but these problems were of secondary importance. On the other hand, the deontic functions which we are going to study raise much more important problems. Thus, the functors which they contain deserve to be called problematic. We find them in the works of several deontic logicians, particularly A. R. Anderson, M. Fisher, J. K. K. Hinti, A. N. Prior, N. Rescher, A. Ross, G. H. von Wright, and so many others.[32] Not all of the functors we are thinking about are used by each one of the above-mentioned logicians: each of these logicians theoretically uses nothing but one kind of functor. One of them, however, G. H. von Wright, endowed with an extra-

[31] Concerning this, we have added the following note to our article, *A New Branch of Logic: Deontic Logic:* "At first, one could think that in order to make Mr. Ziemba's deontic syllogistic no less rich in this respect than our systems K_1 and K_2, it would be sufficient to modify its rule of substitution and to authorize the introduction of descriptions through this expedient. But this solution is really closed off. For, in order to reach concrete norms which prescribe to, forbid, or allow a concrete action to a concrete subject of action, it is necessary to begin with expressions where the variable "x" is connected by the universal quantifier, as it is taking place in Mr. Ziemba's thesis T21b for instance, if we cite one of the most simple cases.

T21b $\quad x\ ob_n\ Y \rightarrow x\ doz_n\ Y$

(This thesis in interpreted: *"For each and every x, given the set of orders n, if x is required to be Y, then x, given the set of orders n, is authorized to be Y."*) "Y" would represent a description such as *"The man effecting hic et nunc this payment of 1,000 F to Paul"* for instance. Now such an expression does not encompass our ethical and juridical intuitions. For, given the set of orders n, thesis T21b is not valid for *all* men, but for a particular one, let us say Peter, if we substitute for "Y" the description in question. Therefore we do not see how to reintroduce into deontic syllogistic the concrete names expelled from it by Mr. Ziemba and which keep in systems K_1 and K_2 their right place because of their great importance in ethics and law.

[32] Z. Ziemba, *op. cit.*, p. 106.

ordinary power of invention, uses all of them, whether he has created them himself, or he has borrowed them from other logicians, in this case from A. N. Prior or N. Rescher, in his many systems of deontic logic. Consequently, we shall only speak of him. The other logicians of interest will be mentioned when need be.

We will be concerned with functions such as "OA," "P(A/B)," "Pp," "O(p/q)," "OPp," "O(d(pTp))," "P(f(pTp/qTq))," "P($s_iT(s_jIs_k)$)," "P($tTs_j/s_iT(tIs_k)$)." We are going to examine them in order.

But beforehand, let us make clear that all functors appearing in these functions do not raise the same problems and thus are not problematic in the same sense. As was the case with the previously studied functors, some of them only raise the problem of their rigorous definition, since they are not rigorously defined by these authors. These functors could, even should, be studied in the first paragraph of this section. However, for practical reasons, we think it better to keep all discussion of the functors used by G. H. von Wright together in the second paragraph. The other functors which we shall have to deal with raise much graver problems, such as the problem of their linguistic or metalinguistic nature, as well as the problem of their appropriateness in a logic which is supposed to be deontic; that is, in a logic whose name, by its etymology and its commonly accepted meaning, suggests that it captures the intuitions produced by the world of norms—ethical, juridical, technical norms in the first place—the intuitions that procure the main interpretation of this logic. What is at stake here is the fitness or, more exactly, the material adequacy of reiterated deontic functors, as we are going to see. Therefore we are going to tackle them in four sections devoted successively to the deontic functors "O," "P," and others of the same type with nominal (predicative) or propositional arguments, functions such as (1) "A," "B," etc., or "A/B," etc., (2) "p," "q," etc., or "p/q," etc., (3) "Op," "Pp," "OPp," etc., and (4) "d(pTp)," "f(pTp)," "d(pTp/qTq)," etc., as well as analogous functions "$s_iT(s_jIs_k)$," "$tTs_j/s_iT(tIs_k)$," as arguments; functions which have been borrowed from von Wright's logic of action which contains, as basic elements, functions from von Wrightean logic of change ("pTp," "~ pTp," etc.).[33]

Let us also add that deontic logic functions "OA" etc., and "O(A/B)," etc., as well as their derivatives having functions from the von Wrightean logic of action as arguments, may be compared as it will be shown below, to the functions of modal alethic logic being expressed as *de re* modal proposi-

[33] We shall omit here to indicate the works from the logicians mentioned in the text. An exhaustive list of them will be found in a bibliography added by G. H. von Wright to his book *An Essay on Deontic Logic and the General Theory of Action* (*Acta Philosophica Fennica,* XXI, 1968), Amsterdam, North-Holland Publishing Co., pp. 97-107.

tions. On the other hand, deontic functions "Op," etc., and "O(p/q)," etc., as well as their possible derivatives using the functions from the von Wrightean logic of action (if these could be considered as non-nominal (predicative) but propositional functions), as arguments, may be compared in turn to the functions of modal alethic logic being interpreted as *de dicto* modal propositions.

(1) *Functors of de Re Deontic Functions*

According to the plan agreed upon above, we begin this analysis with functions such as "OA," or "PA" used by G. H. von Wright in *Deontic Logic* and *An Essay in Modal Logic.*[34] The eminent Finnish logician interprets them as *"A is obligatory"* and *"A is permissible"* where "A" is a nominal general variable standing for the general noun of any kind of action ("parking," "sale," etc.). "O" and "P" being interpreted as *"is obligatory"* and *"is permissible"* respectively, are deontic functors forming expressions having the syntactical structure of proposition (clause). But of which proposition are we talking? It would seem that we are dealing with propositions signifying norms. However, these propositions (clauses) are considered as being either true or false by Mr. von Wright.

Also, if we take into account that norms, according to the view stated in his foreword of *Logical Studies,* are neither true nor false because they do not fall under the categories of truth or falsity, von Wright insists that he considers the deontic functions in question to represent propositions of a set category or norms, propositions such as *"There exists the juridical norm N allowing parking in front of this house."* Thus there is no longer any doubt, since the appearance of *Logical Studies,* about the semantic category of the examined functions or of the functors they contain. Indeed, Mr. von Wright adds in *Norm and Action* that the equiform functions can correspond to statements signifying the norms themselves and not just judgments on norms. But he does not want to be concerned with such functions because of the problem he thinks they raise—the problem of correct or incorrect usage, of sentence-forming functors of classical (bivalent) theory of propositions from the perspective of the metalogical rules determining the correctness of logical systems. May these functors have, as arguments, statements signifying not only theoretical judgments (of ascertainment, of description, of explanation, in short of reality) but also practical judgments, norms among others? Mr. von Wright

[34] If need be, say again that variables, nominal ones such as "A," "B," etc., propositional ones as "p," "q," etc., and also functorial ones (*sit venia verbo!*), if they are used, are the simplest logical functions from the point of view of syntactical structure.

thinks that such a usage of the functors of the theory of propositions is relevant, but he concedes an ignorance as to its correctness. Therefore, he prefers to circumvent the problem by applying himself to the constructon of the only deontic logic of propositions on norms of the aforementioned type.[35] Consequently, the deontic functors "O" and "P," as well as other analogous ones ("F" for example, being interpreted as *"is prohibited"* or "I" as *"is indifferent"*) manifest themselves as belonging to the metalanguage having as object, the language in which are stated the norms expressed by the propositions created by means of the functors in question.

Let us notice as an aside that the functions we have just examined present the double drawback of not containing nominal variables, whatever these may be: individual or general, standing for names of action subjects, or comprising only nominal variables representing general names (predicates) of actions. Consequently, the theses of von Wrightean deontic logic reveal nothing about the concrete behavior of men: they are kept at the level of universals.[36]

Mr. Hintikka noticed this drawback and proposed to overcome it by replacing the functions such as "PA" by corresponding quantified expressions "(x)PAx" to be interpreted as *"all acts of type A are permissible acts."*[37] The first formula enhances the similarity existing between Mr. Hintikka's functions (comprising an individual variable representing a singular noun of action and linked by a quantifier and a deontic molecular predicate, in this case a conjunctive predicate *"permissible action,"*) and Mr. García Máynez' functions on one hand, and those of MM. Tammelo and Klinger on the other. The second formula brings to mind the functions which would render

[35] G. H. von Wright, "Deontic Logic," *Mind,* LX (1951), pp. 1-15, reprinted in *Logical Studies*—(see the following note), *An Essay in Modal Logic* (Amsterdam: North-Holland Publishing Co., 1951). The Finnish logician introduces, further on, functions such as "O(A v B)", "O(A & B)", "O(A → B)", etc., and "P(A v B)", "P(A & B)", "P(A → B)", etc. (He also writes them without parentheses—we shall follow his example herein in this regard.) May we add that these functions engender various paradoxes (about them see G. H. von Wright, "Deontic Logics," *American Philosophical Quarterly,* IV (1967), pp. 1-8; and *ibid.,* "An Essay in Deontic Logic and the General Theory of Action," I, p. 6.) The general molecular nouns of action (alternatives, conjunctives, implicatives, etc.) replace therein atomic nouns of the same type and, as the latter, in functions of the type "OA" or "PA," play the role of the unique predicative argument of the deontic functors "O" and "P" as well as "F" (forbidden) and "I" (indifferent), that may be introduced by appropriate definitions.

[36] G. H. von Wright, *Norm and Action* (London: Routledge and Kegan Paul, 1963), VIII, 2, pp. 130-134.

[37] *Op. cit.,* "An Essay in Deontic Logic and the General Theory of Action," *Acta Philosophica Fennica,* XXI (1968), pp. 1-110. This is the criticism addressed to G. H. von Wright by K. J. J. Hintikka, "Quantifiers in Deontic Logic," *Societas Scientarum Fennica,* Commentationes Humanarum Litterarum, XXIII (1958), 4, pp. 1-23; and Mr. Z. Ziemba, *Logika deontyczna jako formalizacja rozumowan normatywnych,* p. 9, text and note).

explicit, in a more exhaustive manner, the content of the propositions on the norms in question, functions such as "(x)CAxPx" read *"for all x, if x is A, then x is P,"* and to be interpreted, at the deontic level: *"For all x if x is an action of type A, then x is permissible."*

Under N. Rescher's influence,[38] G. H. von Wright abandons in *Norm and Action* and in *A New System of Deontic Logic,* as well as in his other later writings, notably in *Deontic Logic* and in *An Essay in Deontic Logic and the General Theory of Action,* the deontic functions used in 1951 and qualified as monadic to the advantage of dyadic functions of various kinds.[39]

In *A New System of Deontic Logic,* the original monadic functions "OA," "PA," etc., are, for the first time, replaced by the new dyadic functions "O(A/B)," "P(A/B)," etc., to be interpreted as: *"Given B, A is mandatory," "Given B, A is permissible,"* etc., respectively. *"The rain starting to fall, the closing of the window is mandatory,"* for instance, or, in less rigorous terms, *"The window must be closed in case of rain,"* or even *"If it rains, one should close the window,"* the two last expressions being used here, just like the first one, not as norms but as judgments on the norms, judgments signified by the propositions on the norms of Mr. von Wright, his *normative statements* in his original English terminology. Each one of these functions is composed of four types of expressions, two of which are general nominal (predicative) variables, in this case "A" and "B," and the other two (in our examples "O" and "P" on the one hand, "/" on the other) being functors. As to the latter, the question arises precisely as to which category they belong. "O," "P," etc., are sentence-forming functors, in this case, functors forming sentences on the norms, thus really metapropositions with, as single argument, a molecular predicate built with the help of the functor "/." "/" is a predicate-forming functor, with two general nominal arguments. These arguments are atomic or molecular predicates; the first one designates a state of affairs considered as the condition for a mandatory realization, permissible or other of the same type as the state of affairs designated by the second one.

(2) *Functors of the Deontic Functions de Dicto*

In 1955, A. N. Prior adopted the deontic functions "Oa," "Pa," etc. (another "spelling" of the functions "Op," "Pp," etc.), in place of von Wright's

[38] K. J. J. Hintikka, *op. cit.,* analogous expressions are used in A. N. Prior's *Formal Logic* (Oxford: Clarendon Press, 1956), Part III, I, § 6. *Deontic Logic,* pp. 220-229. The late English logician interprets therein "α x" as "The individual action x possesses the characteristic 'α' and 'o α x'—the individual action x must leave the characteristic α."

[39] N. Rescher, "An Axion System for Deontic Logic," *Philosophical Studies,* IX (1958), pp. 24-30.

functions examined above, "OA," etc.[40] These functions lead us to consider in greater detail a particular aspect, already indicated above, of the analogy between the deontic and the alethic modalities. The analogy in question has already been acknowledged by several deontic logicians, such as Alois Höfler, Jan Nuckowski, Jean Ray, Georg Henrik von Wright, Oskar Becker, and Robert Blanché, all of whom, it seems, noticed this independently.[41] This analogy consists in the correspondence, now well known, between obligation (positive), permission (bilateral), and prohibition (negative obligation) on the one hand, and necessity, possibility (bilateral,) and impossibility on the other. As one might imagine, this analogy naturally extends further and also appears in the field of the syntactical structure of the modal alethic propositions, as they are called since the appearance, in 1951, of von Wright's *An Essay in Modal Logic.* The analogy holds in two ways, according as to whether the (alethic) *modus* bears upon the *dictum,* or whether it is incorporated within the functor forming the given modal proposition. In the Middle Ages, the first kind of modal proposition was called in Latin *"propositiones modales de dicto,"* and the second kind *"propositiones modales de re."* They can be expressed respectively by the following examples: *"It is necessary that man be mortal"* and *"Man is necessarily mortal."* G. H. von Wright's functions ("OA," etc.) and those of Z. Ziemba ("x ob_n X") for example, are analogous to the modal propositions *de re*: they correspond respectively to the *normative statements: "The window is mandatorily closed"* (pro futuro and potentially of course) and *"Peter is necessarily* (taking into account the set of orders n) the seller delivering the merchandise sold without any hidden flaw." A. N. Prior's functions "Op," "Pp," etc., and the corresponding functions of G. H. von Wright, A. R. Anderson, and of several other deontic logicians, directly or indirectly influenced by the late author of *Formal Logic,* are analogous to the modal propositions *de dicto.* Indeed, these functions are to be interpreted as *"It is mandatory that p"* and *"It is permissible that p"* respectively. We may ask here whether they belong to the object-language or to the metalanguage.

Let us begin by examining the analogous question raised about the modal propositions *de dicto.* R. Blanché, already quoted several times in this article, thinks that the modalities, whatever they may be—alethic, deontic or whatever—originally belong to the object-language, for in modal propositions and in non-modal ones, reality is ordinarily dealt with and not propositions about reality. "When I say that 2 and 2 makes 4, my judgment bears on the

[40] G. H. von Wright, *Norm and Action, op. cit.,* "A New System of Deontic Logic," *Danish Yearbook of Philosophy,* I (1964), pp. 173-182; *op. cit., Deontic Logics.*

[41] A. N. Prior, *Formal Logic* (Oxford: Clarendon Press, 1955), pp. 220-229.

numbers 2 and 4 in such a way as to affirm a certain necessary connection between them, and does not bear on the (non-modalized) statement '2 and 2 makes 4,' in such a way as to affirm, as a matter of fact, that this statement is analytic. It is presumed that the second manner of thinking is restricted to a few logicians," writes the professor from Toulouse.[42] And he adds a bit further: ". . . according to the order of reasons, it is necessity which is the *ratio essendi* of the universitality."[43]

We often agree with Blanché. But this time, our assent is not total. We do not take exception to his last remark. We also grant him that 2 and 2 necessarily makes 4, which explains why we get 4 each time we add two occurrences of the number 2. But are we always required to express both the fact in question *and* its necessity? Can we not mentally separate one from the other and sometimes state the fact, apart from its necessity? And then what prevents us from considering the analytic character of the non-modalized statement "*2 and 2 makes 4*" and to enhance it by stating, this time in metalanguage, the proposition or more exactly the metaproposition: "*The proposition '2 and 2 makes 4' is a necessary proposition*"? And what also prevents us from adopting, out of an inappropriate concern for simplicity, an explicit or implicit convention (very much to be regretted becouse it would obscure the true metalinguistical character of our statement)—convention according to which "*It is necessary that 2 and 2 makes 4*" is semantically equivalent to "*The proposition '2 and 2 makes 4' is a necessary proposition*"? Likewise, supposing that we would want to adjudicate simultaneously upon the given fact *and* upon its necessity, are we required always to speak about it *directly?* Can we not bring to bear the same distinction as that of a while ago between the non-modalized statement about the fact, and the modalized statement about the preceding statement and directly make a decision upon the necessity of the latter and only indirectly upon the necessity of the fact enunciated by the first statement? But then, our modal propositions *de dicto* are really *de dicto* and not *de re,* that is to say propositions about propositions, in other words metapropositions. The ancient logicians seem to have seen and understood this very distinction and were precisely calling these propositions "*propositiones de dicto,*" but, alas, many modern logicians seem neither to perceive nor to respect this distinction.

[42] Just like the logicians quoted herein, the present author founded his own *Logika zdan praktycznych* (1955) and his summary of it, *Théorie des propositions normatives* in 1953 upon the same idea of the analogy between modal (alethic) logic and deontic logic. But, in contrast to these logicians, he did not discover it, but found it in the textbook of J. Nuckowski, *Poczatki logiki dla szkol srednich—Elements of logic for secondary schools* (Krakow: 1920), 3rd ed., p. 59. On this subject see G. Kalinowski, "Une nouvelle branche de logique: la logique déontique," *Archives de Philosophie* (1970).

[43] R. Blanché, *Raison et discours,* p. 266.

The metalinguistic character of *de dicto* modal propositions is not respected when "p," in expressions such as "Np," is given the interpretation *"It is necessary that p,"* or "Op" the interpretation *"It is mandatory that p,"* for a symbol of variables representing, in the first case, a proposition and, in the second, a norm (we do mean "a norm" and not "a proposition on a norm" —we explain why further on). For if this character were respected, we would know that functors such as "N" or "O" have, as unique argument, not a proposition or a norm or the corresponding variable, but either the name of a proposition (the case of "N") or the name of a norm (the case of "O") or the name of the corresponding variable, even if we should have recourse implicitly or explicitly, to the linguistic convention indicated above, which allows us to write "Np" or "Op" instead of "N'p' " or "O'p' " (although this notation is really to be deplored because it is incorrect and misleading).

Indeed, the introduction, whether explicit or implicit, of the expression " 'p' " raises problems, the first one being of the syntactico-semantic status of quotation marks. In spite of Alfred Tarski's opinion this problem does not appear to us insoluble.[44] It would take too long, however, to discuss it here. Therefore we plan to do it in a separate study devoted to the definition of the true proposition given by the eminent Polish logician. But we can say now that we do not see any imperative reason to justify the refusal to quotation marks, of the semantic category of the metanoun-forming functor with, as sole argument, just any expression. The latter can be a well-formed expression, a badly formed one, or whatever part of one or the other (a simple letter, a syllable, etc.), even any sort of written sign (a punctuation mark, for example). Looking at the matter from another point of view, it must be admitted that the expression put between quotation marks can be a constant or a variable and, as to its eventual semantic category, a noun, a proposition or a functor, or it could fail to belong to any category, since it could be a letter, a syllable, a punctuation mark, or a quantifier. This expression between quotation marks could even be a grouping or a series of propositions. Consequently —and we concede it spontaneously—there are as many species of quotation marks as species of expressions to be put between quotation marks. Consequently also, and in that A. Tarski is exactly right, the expressions formed with quotation marks are sometimes equivocal (the expression " 'a' " can be the metaname of the concrete inscription found enclosed between simple quotation marks just as it can be the metaname of every equiform expression with "a" or the metaname of the propositional variable as used by a few logicians, such as A. N. Prior, for instance). But this equivocation does not seem to us

[44] *Ibid.*, l. c.

to constitute an insurmountable difficulty, because it is only virtual and it is furthermore known. Such equivocations are frequent since they are often inevitable, practically if not theoretically. Only equivocations which are both present and hidden are truly harmful and in need of elimination. Nevertheless, in order to to reduce as much as possible virtual and known equivocation of the quotation marks, we can agree that every metaname of whatever expression, formed with the help of quotation marks, designates not only the concrete inscription put between quotation marks, but also every expression equiform with it. Thus, of the three cases of the equivocation of " 'a' " mentioned above, the first is eliminated.

Another problem is that of interpreting the *de dicto* deontic functors such as "O'p' ". The *de dicto* alethic functions such as "N'p' " are interpreted as *"Proposition 'p' is a necessary proposition"* etc. The expression: *"Proposition 'p' is a necessary proposition"* means the same thing as does: *"Proposition 'p' is an analytic proposition,"* which, in turn, means the same thing as the expression: *"Proposition 'p' is a proposition whose truth, i.e., the conformity with reality, if it is supposed that proposition 'p' is the result of a cognitive act, or another logical value which can be attributed to the propositions resulting not from a cognitive act, but from a creative act, is determined by the relations existing among the terms which compose it, relations appearing with evidence upon analysis of this proposition."* Functions such as "O'p' " or "P'p' " interpret in a similar manner as: *"The norm 'p' is a mandatory norm"* or *"The norm 'p' is a (unilaterally) permissible norm."* What is the significance of such expressions? In our opinion it is a double one: weak or strong. It is only weak —and then without interest for us—if these expressions only mean the same as the following expressions: *"The norm 'p' is a norm whose own syntactical structure and therefore its content, are the structure and content of a positive and prescriptive norm, i.e., of a norm constructed with the help of the deontic functor 'should do' or of one of its synonyms,"* etc. But the significance is strong—and therefore interesting, and even of major interest, when it signifies: *"The norm 'p' is a truly mandatory norm, i.e., not only a norm whose syntactical structure and content are those of a positive prescriptive norm, but also a norm which—according to the corresponding, objective, rationally justified criterion—is a positive prescriptive norm in force, in other words, having a mandatory force in the community* (of whatever kind this mandatory force may be: ethical, juridical, technical and other—if they are any—mandatory force on the one hand, and mandatory force of the community in question: humanity, such-and-such political community, or, also, the community of all men devoting themselves to such-and-such creative or productive activity which is submitted to given technical rules, etc., on the other hand)." These, then, are the propositions corresponding to the *de dicto* deontic functions presently examined and which are, in fact, the propositions on norms (*normative statements*) of which von Wright speaks in the preface of his *Logical Studies* and more particularly in his *Norm and Action.* In other words, if we desire to distinguish between the logic of norms and the logic of propositions on norms (normative statements), and if we want to form the logic of these propositions, then it is these *de dicto* deontic functions such as "O'p' ", "P'p' " etc., interpreted in the manner described above, which it is proper to select as constituent elements of the theses of the planned deontic logic. On the other hand, the *de re* deontic functions are the only possible elements of deontic logic, when it is conceived as the logic of norms. Finally, let us add that one of these two conceptions of deontic logic is theoretically as acceptable as the other. But we hardly see the practical usefulness of the logic of propositions on norms (normative statements). Indeed, on the one hand, the logic of norms which establishes the constant relations among states of things (in the sense of *devoir être*) designated by the norms, establishes *ipso facto* the analogous relations, sending us back to the norms in force, for these relations are always the same, whether the norms in question are presently in force, whether they no longer are in force, whether they are not yet in force, or whether they are purely imagined, for one reason or another. And, on the other hand, if we want to obtain norms in force as conclusions, it suffices to infer according to the rules of inference founded on the theses of the logic of norms, by using only norms in force as premises.

Before proceeding to examine the class of "problematic" deonic functors, it remains for us to discuss, in connection with what has just been said, the recent study, devoted to deontic and alethic *de dicto* functions, by Mr. Kazimierz Opalek, professor of the theory of law at Jagellonne University (Cracow). This study concerns his interesting communication to the Brussels Symposium.[45]

Justly impressed by the fact that the verb of the clauses subordinate to the modal alethic or deontic functors such as *"it is necessary that"* or *"it is mandatory that"* has to be in the subjunctive mood, according to the grammar rules of several languages, our Polish colleague shows that the expressions corresponding to "p" in functions such as "Np" or "Op" do not belong to the

[45] A. Tarski, *The Concept of Truth,* p. 161 ss. A. Tarski, *Logic, Semantics, Mathematics* (Oxford: Clarendon Press), pp. 152-278. Everything we say in the text of this article, in favor of the thesis which confers on quotation marks the status of metanoun-forming functors, cannot change anything to the fact that such a conception of quotation marks was unacceptable to A. Tarski, in view of the objective which he set for himself in *The Concept of Truth.* We do not discuss here either whether quotation marks are non-extensional functors, should such be the case, and to what extent their eventual intension-ability requires the rejection of our thesis.

same semantic category as the expressions replaceable by propositional variables "p," "q," "r," etc., of the theory of propositions. These expressions are called by him "*ut*-propositions" and the variables which represent them are called also by him "*ut*-propositional variables." Finally Mr. Opalek eliminates functions such as "Np" or "Op" in favor of *ut*-functions such as "N ut-p" or "O ut-p."

In our opinion, however exact his analysis, it stops too soon and does not reach the inevitable conclusion. Mr. Opalek is quite right to bring out the specific character of the expressions corresponding to the modal alethic and deontic functions represented heretofore by functions such as "Np" and "Op." This specific character is illustrated by the requirement that the verb in the expression represented by variable "p" be in the subjunctive mood. But this is not enough. We absolutely have to conclude with the following verification. There are two possibilities: either expressions like, *"It is necessary that man be mortal"* or *"It is mandatory that every seller deliver the sold merchandise without any flaw"* are enunciated in object-language because they express reality, even if this reality were considered under the aspect of *devoir-être,* or they are expressed, contrary to appearances, in metalanguage because, all things considered, they speak of propositions or of norms. In the first case, they constitute only a syntactical variant of *de re* propositions, modal, alethic, or deontic, in their ordinary form which the following examples illustrate: *"Man is necessarily mortal"* and *"The seller is mandatorily the deliverer of merchandise sold without hidden flaws,"* a variant that we have no interest to utilize. For what do we gain by eliminating "Np" or "Op" in favor of "N ut-p" or "O ut-p" interpreted in this way? We are only compelled to introduce one new semantic category of expressions: the *ut*-propositions and the *ut*-propositional variables. Since this multiplication of semantic categories is not necessary, it seem to us to be *a priori* condemned by this analogy of Occam's famous dictum *"Non sunt multiplicandae categoriae semanticae praeter necessitatem."* The only remaining possibility is the second which rejoins exactly the previously defended thesis.

The introduction of deontic functions such as "Op," "Pp," etc., has resulted in the appearance of various derivative functions, such as for example: "Op v q," or "Op → q"[46] on the one hand, and functions like "Ot," "Op/q," "Op/t," "Ot/t," etc.,[47] on the other. Bivalent logic—whether both values

[46] K. Opalek, "On the Logical Structure of Directives," *Logique et Analyse,* XIII (1970).

[47] They are mostly found in G. H. von Wright's works, notably in *Deontic Logic* and *An Essay in Deontic Logic and the General Theory of Action* and in works of logicians who follow him, such as Alf Ross, *Directives and Norms.*

in question be truth or falsity, validity or invalidity, or others—here appears to be insufficient. In fact, the well-known matrix

p	F_1	F_2	F_3	F_4
1	1	1	0	0
0	1	0	1	0

characterizes only functors (sentence-forming functors with, as unique argument, a proposition) of verum, assertium, negation and falsum; this matrix does not characterize any of the deontic functors in question. In modal logic, we have recourse to polyvalent matrices (see for example, I. M. Bochenski, *Z historii logiki zdan modalnych,* Lwów, Wydawnictwo OO. Dominkanów, 1938 or J. Lukasiewicz, "A System of Modal Logic," *The Journal of Computing Systems,* I (1953), pp. 111-149). Are there analogous polyvalent matrices for deontic functors in question? We think that we have very good reasons to doubt it. In any case, trivalent matrices for deontic functors constructed by Mr. Fisher ("A Three-Valued Calculus for Deontic Logic," *Theoria,* XXVII (1961), pp. 107-118) and by L. Aquist ("Postulate Sets and Decision Procedures for Some Systems of Deontic Logic," *Theoria,* XXIX (1963), pp. 154-175, as well as by the author of the present article in his *Logika zdan praktycznych* (1951) and in his *Théorie des Propositions normatives* (1953)—matrices identical in their structure although independent in their source—they cannot be used as an argument against our doubt because they present quite a specific character: they combine the bivalence of normative propositions (norms) with the trivalence of nouns of actions. However, let us add in order not to leave anything out, that matrices of modal alethic functors proposed by I. M. Bochenski, p. 89 of his book quoted above, have all the same directly inspired our trivalent matrices of deontic functors adopted in *Logika zdan praktycznych* and later on in *Théorie des Propositions normatives.* We shall not analyze them in any more detail in order not to repeat ourselves. For, the remarks made above regarding functions such as "Op" or "Pp" are valid, *mutatis mutandis,* for the deontic functions which derive from them and which we just pointed out. We limit ourselves only to mentioning the analogy between functions derived from "Op" and "Pp" and those derived from "OA" and "PA." Mr. von Wright and several other deontic logicians taking their inspiration from him, used functions such as "OA v B," "OA&B," "OA → B," etc., on the one hand, and functions like "Op v q," "Op&q," "Op → q," etc., on the other. Here we only note that functions of type "Op v q" and "Op → q" lead to the same paradoxes of derived obligation as the functions of type "OA v B" and "OA → B."[48]

[48] Regarding these functions, consult especially G. H. von Wright, *An Essay in Deontic Logic and the General Theory of Action.*

(3) *Reiteration of Deontic Functors*

The fundamental analogy between alethic and deontic modalities which we briefly mentioned above is a very important and fruitful analogy because it finally made possible the birth of deontic logic. Deontic logicians are reaping more and more benefits from this fruitfulness and are discovering new inferences from it. The first one has been the transposition of the distinction between modal alethic propositions *de re* and *de dicto,* in the field of deontic propositions. The fact that many alethic as well as deontic logicians do not know the true character well, syntactical and deontic, of the alethic and deontic *de dicto* propositions, does not take anything from the practical and theoretical importance of this distinction applied consecutively to modal alethic propositions and to modal deontic ones. As a second consequence, there is the reiteration of deontic functors conceived as the reiteration of modal functors. We will examine it a little more closely.

Logicians who devote themselves to modal alethic logic sometimes practice the reiteration of modal functors and introduce functions such as "NNp" (*It is necessary that it be necessary that p*"), "MLp" (*It is possible that it be impossible that p*"), "NMLp" (*It is necessary that it be possible that it be impossible that p*"), etc. Let us leave aside the question of whether these kinds of functions are built on sound material grounds. And let us limit ourselves to the examination of their formal side. According to our conception of *de dicto* modal alethic propositions presented above, the functions in question should be written, to be exact, as follows: <N "N'p' ">, <M "L'p' ">, <N M "L'p' " > etc., and should be interpreted: < *The proposition "the proposition 'p' is a necessary proposition" is a necessary proposition* > etc. If an analogous reiteration of deontic functors is practiced, the following functions are obtained: "OOp," ("*It is mandatory that it be mandatory that p*"), "IFp" ("*It is indifferent that it be forbidden that p*"), "OIFp" ("*It is mandatory that it be indifferent that it be forbidden that p*"), etc. As long as we stay with this manner of writing and interpreting the given functions, they seem as acceptable, in theory if not in practice, as the analogous alethic functions. But if we concede that they should be written, to be exact, <O "O'p' "> etc., as we tried to prove, it would be fitting to interpret them as: <The proposition "the norm 'p' is a mandatory norm" is a mandatory proposition. Right away it is noticed that in alethic propositions, " 'p' " represents the name of a proposition, the name of the proposition substituted for variable. It cannot be otherwise because *de dicto* deontic propositions, just as substituted for this variable whereas, in deontic propositions, " 'p' " represents the name of a *norm,* namely of the norm substituted for variable "p," and

<"O'p' ">, the name of a *proposition* on the norm substituted for the given variable. It cannot be otherwise because *de dicto* deontic propositions, just as *de dicto* alethic propositions, are theoretical metapropositions (of ascertainment, of reality); they are not practical propositions, in this case normative propositions, signifying norms). But is it possible to say of a proposition on a norm that this proposition is mandatory in a way compatible with the *devoir-être* which provides the intuitions having to guide us in our work of building deontic logic which seeks to be a cognition-logic and not a pure-construction logic? In our opinion, the expression "mandatory proposition" is in this context a *contradictio in adjecto* in the proper meaning of the term, for the proposition in question, truly a metaproposition, is a theoretical and not a practical proposition, whereas the epithet "mandatory" which is found here placed side by side with the name of "proposition," is exactly an epithet which can only be attributed to a norm therefore to a practical proposition, in this case proposition signifying norm.[49]

In conclusion, even if, according to the opinion of specialists to whom we are leaving the decision, the reiteration of modal alethic functors were found to be not only formally correct but also materially adequate (by these words we mean "not only consistent with the rules of syntax but also collecting our intuitions produced by the knowledge of modal reality), then still the analogy between deontic logic and alethic logic would not go that far. The analogy would stop at the distinction between *de re* and *de dicto* deontic propositions. It would not reach the level of reiteration. For the latter is neither theoretically nor practically in the field of deontic logic. In fact reiteration, if it were practiced here, would act at the level of deontic *de dicto* propositions and it would label them as mandatory, which would constitute a *contradictio in adjecto,* whether the term mandatory be taken in its strong or weak sense (see above.) Thus reiteration proves to be inadmissible if it is desired to elaborate a deontic cognition-logic and not a pure construction-logic having ultimately nothing in common with ethical, juridical, technical, and other standards experienced by men. If our preceding analysis of *de dicto* deontic propositions and consequently of the eventual reiteration of deontic functors operating in their field is exact, logicians such as G. H. von Wright are victims of an illusion

[49] We have already mentioned this in passing. For the bibliography about this matter, see not only G. H. von Wright, *An Essay in Deontic Logic and the General Theory of Action,* I, 6, but also the bibliography at the end of this book (pp. 97-107).
Regarding deontic functions such as "Op," "Pp," etc., if they were—what we don't believe for reasons explained in the present article—not only formally correct but also materially adequate (collecting our intuitions)—we leave to the specialists of the modal alethic logic, as we already mentioned it, the task of coming to a conclusion about analogous alethic functions—then the problem would be raised to find out how to characterize functors "O," "P," etc.

due to the fact that neither the determination of semantic categories of the functions in question and of their components has been carried far enough, nor have the results of such an analysis, to the extent that a partial determination was made, been confronted with reality which, although only a *devoir-être* in this case, is nevertheless a reality constituting an object of our knowledge and thus providing us the necessary intuitions to guide us. In fact, only the defect of such analysis could permit the author of *An Essay in Deontic Logic and the General Theory of Action* to write: "Let 'OPp' be read as 'one ought to see to it that *it is permitted to see to it that it is the case that p*'. *This is clumsy—but a glimpse of meaning shines through. How can one see to it that something is permitted? Evidently by giving permission, by permitting somebody to do something (in the case: to see to it that p).*"[50] We explained above the sole formally correct and materially adequate interpretation of such a function as "OPp," which is incorrectly notated and which therefore should be replaced by the correctly notated function <O "P'p' ">. However Mr. von Wright is right: *a glimpse of meaning shines through.* We understand what the eminent Finnish logician means. But it is necessary to resort to a formula different from his in order to express it in a way which is formally correct and materially adequate. It would be necessary, for example, to write —using the symbolic notation of our *Théorie des Propositions normatives* adapted to this case—"$Sx\alpha^P$" where the nominal variable "α" representing any singular name of action would be indexed with "P." The function would be interpreted: "x must make α^P" ("The action subject x must accomplish the action α^P"). The index "P" placed to the right above "α" would indicate that this variable represents in this case the name of the promulgating action of a permissive norm corresponding to the function '*y has the right to make* β.' Thus the complete and fully clarified meaning of the function "x must make α^P" would be the following: *"x must promulgate a norm corresponding to the function 'y must make* β.' " By introducing the index "(P, η, δ)" instead of "P," we could indicate that x has the duty to promulgate not only a norm corresponding to the function *"y has the right to make* β*"* but the concrete norm *"η has the right to make δ,"* let us say, *"Peter has the right not to go to his office on December 6, 1970."* In short, the idea of norms stipulating, permitting, or prohibiting the promulgation of new norms, an extremely important idea since Merkel and Kelsen have drawn the attention of legal theorists to what is called in German terminology *"die stuffbau des rechts,"*

[50] We call *"normative proposition"* a proposition such as "every man must love his neighbor" and *"norm"* what signifies such a proposition. Regarding this see our *Problème de la vérité en morale et en droit, (Problem of Truth in Ethics and Law)* (Lyon: E. Vitte, 1967), ch. III.

can find its formally correct and materially adequate expression only in the indexing of the variable which represents in the ordinary deontic functions, the names of actions, or, in Mr. Ziemba's syllogistic, the predicates attributable to subjects of action and never in the reiteration of deontic functors.

(4) *Functors of the Deontic Logic of Action*

Mr. von Wright—whose extraordinary fertile mind and ingenuity in finding more and more varied deontic functions, rich in details, and bringing into deontic logic new terrains which have remained unexplored up to now, whom we admire and envy in the best sense of the term—puts us however to some trouble by "excusing" himself from determining precisely, correctly, and adequately the semantic categories of the functions that he creates and of the elements that the latter contain. Although the task is hard, we pursue it all the more happily that in what follows as well as in what precedes, we do not have disputatious intentions but, on the contrary, we intend to collaborate by thus rendering to the great Finnish logician the homage that he deserves.

In *Norm and Action,* he uses deontic functions such as "O(d(pTp))," "P(f(pTp))," "O(f(pTp/qTq))," etc., and in *An Essay in Deontic Logic and the General Theory of Action* deontic functions like "$P(s_iT(s_jIs_k))$," "$P(tTs_j/s_iT(tIs_k))$," etc. How are they to be interpreted? What are their semantic categories, and what are the semantic categories of their component parts? Let us begin with the last question. The reply to the first will be given indirectly and gradually as we will proceed in the search for the answer to the second.

Theoretically speaking, the functions in question can derive, by way of substitution, either from functions such as "OA," "PA," etc., or such as "Op," "Pp," etc., all having been discussed previously. If all the expressions appearing to the right of the functors "O" and "P" in the functions given above as examples, are nominal functions, that is functions which represent names of actions, these expressions can be considered as substitutable for variables "A," "B," etc., appearing in functions "OA," "OB," etc. But if these functions are propositional functions, that is functions which represent any propositions (theoretical or practical, we leave the question open for the moment), they can be considered substitutable for variables "p," "q," etc., appearing in functions "Op," "Oq," etc. Either or both are possible in the view of Georg Henrik von Wright, given that "*ad esse ad posse valet illatio*." We have the impression that in fact the functions quoted above, from *Norm and Action,* come from functions such as "OA," "PB," etc., which makes it seem that it is the nominal interpretation, and not the propositional one, of expressions of action "d" and "f" read respectively, "*doing*" (positive action) and "*forebearing*" (negative action or omission) that is being used in von Wrightean logic. On the other hand, in *An Essay in Deontic Logic and the General Theory of Action,* the context of the functions in question does not leave us any doubt about the fact that the author considers them as derived from such functions as "Op," "Pq," etc. In our view, this last way of looking at things is absolutely impossible. For we are dealing with *de dicto* deontic functions which are functions representing propositions on norms. Therefore only names of norms can be substituted for "p," "q," etc., in "Op," "Pq," etc.—or, more exactly, in "O'p' ", "P'p' ", etc. Now expressions placed to the right of functors "O," "P," etc., in functions used in the deontic logic of action presented in *An Essay in Deontic Logic and the General Theory of Action,* are in no way functions representing norms. If we considered them as propositional functions, they would represent theoretical propositions of ascertainment or of description such as *"The state of the universe is at this moment* s_i *and at the next moment at the state of the universe* s_j *from before the action of the given agent comes the state of the universe* s_k*," etc.* Therefore we see other possibilities of constructing deontic logic of action only in the prolongation of functions such as "OA," etc. This simplifies our task in that we shall limit ourselves as a matter of principle to the studies of the functions in *Norm and Action* which fulfill this condition and will devote to the functions of *An Essay in Deontic Logic and the General Theory of Action* only a few complementary remarks. Taken in itself, apart from its role as a constituent part of an index, which is assigned to it inside of the expression examined, "I" is truly a functor, namely a sentence-forming functor with, as arguments, two propositions represented by variables "s_j" and "s_k" respectively and which is interpreted: "*. . . before the given action of the agent in question and after the latter . . . .*" The expression "$s_j I s_k$" taken out of its context in the *TI-calculus* would in turn be interpreted: *"The universe is in state* s_j *before the given action of the agent in question and after that action is in state* s_k*."* An analogous remark is valid for functions "pTp," etc., of the logic of change in *Norm and Action* when they are considered by themselves. In effect, "T" serves there as a sentence-forming functor with, as arguments, two propositions represented by functions such as "p," " '," etc. A function such as "pTp" is then interpreted as "*p, at the moment of time* t_1*, and, at the moment of time* t_2*, p*"; but an equiform expression appearing in one of the expressions of von Wrightean logic of action presented in *Norm and Action* or, even more so, in one of the formulas of deontic logic of action which the same book contains, no longer plays a role except as an index stipulating the field of values substitutable for general nominal variables (predicative) "d" or "f." The difficulties facing anyone who wants to express the meaning of the von Wrightean formulas in a way that is adequate to his thought as well as correct from the point of view of the theory of semantic categories are immediately evident.

From what precedes, it follows that expressions such as "d(pTp)" etc.—a similar remark is also valid for expressions such as "$s_i T(s_j I s_k)$" etc.—are nominal functions (functions representing names of actions) or more precisely nominal variables (i.e., the simplest nominal functions) indexed. Each of them includes the variable, in the proper sense of the word, symbolized—by "d," "f," or "T" and the index attached to the variable in question. The variable can be compared to the closest category of the so-called real definition *per genus proximum et differentiam specificam* and its index, to the given specific difference. The variable represents a general name of action (positive action, negative action, positive or negative action) and the index states precisely which kind of action is involved: it indicates it by stating precisely the effect that the action brings about. Thus the index "pTp" which is interpreted "*p at the moment t_1 and p at the moment t_2*" (or, as Mr. Ziemba reads it: "*p up to the moment t and p beginning at moment t*") indicates that the action whose name is represented by "d" or "f" causes the keeping at moment t_2 of the state of things existing at moment t_1 and which happens to be designated by the proposition substituted for variable "." The other possible indexes are "pTp," "pTp" "pTp," "pTp/qTq," etc. The reader himself will easily determine the meaning of it. In the case of functions such as "$s_i T(s_j I s_k)$" the nominal variable representing a general name of action is found in the middle of the index which indicates, in this very case, that the variable in question—we concede that it is symbolized this time by "T"[51]—represents the name of an action which has the effect, as the index "s_i . . . ($s_j I s_k$)" indicates it, that to the state of the universe s_i follows the passage of the latter of state s_j prior to action given to state s_k afer this action. It seems that the formal correctness and the material adequacy of von Wright's deontic logic of action, a system that constitutes an extremely precious contribution by extending deontic logic in depth, can be provided only through the determination of the syntactico-semantic rule for the deontic functions, utilized in this system, and of their parts, determinations which are self-evident by the end of the analysis that we thought we had better undertake. If they are exact, the functors "O" and "P" of deontic logic of action remain, like the corresponding functors

[51] G. H. von Wright qualifies "T" and "I" as (*connective*) functors—*An Essay in Deontic Logic* . . . , p. 71 (5). But in order to respect simultaneously the spirit of his TI-calculus and the requirements of the theory of semantic categories, we are led to deal with "T" as a general nominal variable matched with such and such indexes, in this case "s_i . . . (s_j Is_k)" and "I" as one of the constituting part of the indexes such as the preceding index.

of the von Wrightean system of 1951, sentence-forming functors with, as unique argument, a nominal variable representing a general name (predicate) of action. The only difference among the deontic functions that they create is found in the variables serving as arguments: the variables of the deontic logic of action are indexed while those of *Deontic Logic* were not. As to the rest, the new deontic functions of Mr. von Wright, just as his former functions of nominal general and individual variables representing singular names of action, lack individual variables representing singular nouns of action and quantifiers. In other words, G. H. von Wright's deontic logic, whatever be the forms borrowed from it, remains at the level of the universals without our being able to come down from them among the living and their concrete actions.

Conclusion

After this long analysis preceded by preliminary explanations necessarily as abundant, we can conclude in a few words. Several conceptions of deontic logic and the logical functions utilized in it are possible without, however, having always the same value from the point of view of the theoretical or practical interest that they present. But whatever they may be, the formal correctness and, what is more, the material adequacy of deontic logic, adequacy without which it could not be what it has to be, namely a fully successful cognition-logic and not a pure construction-logic indifferent to reality, to the intuitions which this reality provides and to the interpretation that it imposes, this correctness and adequacy cannot be provided unless the theory of semantic categories is known and respected and unless the syntactico-semantic rule for deontic functions and their components be determined as exhaustively and as rigorously as possible. Deontic logicians, even among the most eminent, are not always concerned enough about this. That is too bad. For only when deontic logic is not only formally correct but also materially adequate, will it truly serve moralists, jurists, or technicians. The present study has been undertaken, more in a spirit of collaboration than of disputation in order to render it a little more useful to them. The reader will judge if it has succeeded, and if so, to what extent.