

Substantival vs. relational space(-time) 1: Pre-relativistic theories

Pt. II Philosophy of Physics Lecture 1, 16 January 2015, Adam Caulton (aepw2@cam.ac.uk)

1 Newton's three laws

From Newton's *The Mathematical Principles of Natural Philosophy* (1687):

1. *Every body perseveres in its state of rest or of uniform motion in a right line unless it is compelled to change that state by forces impressed thereon.*
(\Leftrightarrow existence of *inertial frames*)
2. *The alteration of motion is ever proportional to the motive force impressed and is made in the direction of the right line in which that force is impressed.*
($\Leftrightarrow \mathbf{F}_i = m_i \mathbf{a}_i$)
3. *To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal and directed to contrary parts.*
($\Leftrightarrow \mathbf{F}_i = \sum_j \mathbf{f}_{ij}$ and $\mathbf{f}_{ij} = -\mathbf{f}_{ji} \Rightarrow$ conservation of linear momentum)

2 Absolute vs. relative; substantival vs. relational

Absolutism vs. relativism about velocity:

- *Absolutism about velocity.* The velocity of any material body (e.g. point particles) is a property of that body that makes no reference to any other material body. (Monadic property or relation to spatial points?)
- *Relativism about velocity.* Velocity is an *irreducible relation* between material bodies.

We can formulate absolutism and relativism about any other mechanical quantity: e.g. location, acceleration, etc.

- *Relativism about all motion.* All motion is a relation between material bodies.

Substantivalism vs. relationism about *space* (we'll consider space-time later):

- *Substantivalism about space.* Spatial points/regions are real objects, *which persist over time*. The geometrical properties (& relations) of material bodies are parasitic on the geometrical properties (& relations) of these spatial points/regions.
- *Relationism about space.* Spatial points do not exist/are "ideal".

Inertia is a geometrical property, so substantivalists and relationists must disagree about its origin.

What are the relationships between absolutism/relativism and substantivalism/relationism?

- Absolutism about $X \Rightarrow \neg(\text{relativism about } X)$ (converse?)
- Substantivalism about $X \Rightarrow \neg(\text{relationism about } X)$ (converse?)
- Substantivalism about space \Rightarrow absolutism about all motion (converse?)

3 Leibniz against substantivalism about space

The argument from the *Principle of Identity of Indiscernibles*:

To say that God can cause the whole universe to move forward in a right line, or in any other line, without making otherwise any alteration in it; is another chimerical supposition. For two states indiscernible from each other, are the same state; and consequently, 'tis a change without a change. (Alexander 1956: 38)

The argument from the *Principle of Sufficient Reason*:

Space is something absolutely uniform; and without the things placed in it, one point of space does not absolutely differ in any respect from another point in space. Now it hence follows, (supposing space to be something in itself, besides the order of bodies among themselves,) that 'tis impossible that there be a reason why God, preserving the same situations of bodies among themselves, should have placed them in space after one certain particular manner, and not otherwise; why every thing was not placed the quite contrary way, for instance, by changing East into West. But if space is nothing else, but that in order or relation; and is nothing at all without bodies, but the possibility of placing them; then those two states, the one such as it is now, the other supposed to be the quite contrary way, would not at all differ from one another. Their difference therefore is only to be found in our chimerical supposition of the reality of space in itself. (Alexander 1956: 26)

4 Clarke for substantivalism about space

A Newtonian-Clarkean argument (a reconstruction!):

- (i) Acceleration is absolute.
- (ii) If acceleration is absolute, then velocity is absolute.
- (iii) If velocity is absolute, then space is substantival.
- (C) Therefore (from (i)-(iii)): space is substantival.

Some (e.g. Sklar 1977: 229-32) suggest denying (iii). Later we will consider objections to (ii). Newton aimed to establish that (i) is true with the following two thought experiments.

1. *Newton's spinning bucket.*
2. *Rotating globes.*

In both cases, inertial effects (centrifugal "forces") are observable, so must be accepted even by the relationist.

5 Mach for relativism about acceleration

Mach's strategy is simply to *deny* (i) (after all, Newton had only been appealing to our intuitions about a distant possibility). This does not require him to deny any of Newton's three laws; but they must be reinterpreted. Mach argues that there is a privileged inertial frame: the frame according to which:

- (a) the centre of mass of the universe is at rest; and
- (b) the total angular momentum of the universe is zero.

The mathematical details of this idea were not fully worked out until 1977! (See Barbour & Bertotti 1982.) The result is that inertia arises from a non-local phenomena between bodies.

6 Symmetries of space-time vs. symmetries of the dynamical laws

A *symmetry* is some form-preserving transformation.

A *space-time symmetry* is a form-preserving transformation on space-time.

A *dynamical symmetry* is a form-preserving transformation on the dynamical laws.

Any space-time symmetry must also be a dynamical symmetry. Ockham's razor would recommend that any dynamical symmetry be a space-time symmetry.

Newton's First Law entails the existence of inertial frames. So Newton's laws seem to entail *absolutism about acceleration* (bracketing Mach's theory). But Newton's laws do not require a choice of a privileged inertial frame: as far as the laws are concerned, any inertial frame is as good as any other. So a change of velocity (a 'Galilean boost') is a dynamical symmetry of Newton's laws. But *absolutism about velocity* is entailed by Newton's substantivalism about space. So Galilean boosts are *not* a space-time symmetry of Newton's theory, even though they are a dynamical symmetry.

So, inspired by the above discussion, we seek to:

- (i) reform our conception of spacetime; or
- (ii) reform our interpretation of the laws

(or both!) to bring the space-time symmetries and dynamical symmetries into alignment.

7 Space-times: denying (ii)

- Absolutism about location \Rightarrow about velocity \Rightarrow about acceleration \Rightarrow ...
- ... $\not\Rightarrow$ absolutism about acceleration $\not\Rightarrow$ about velocity $\not\Rightarrow$ about location
- Substantivalism about space-time $\not\Rightarrow$ substantivalism about space

Think of a space-time as a continuum of distinct points (with zero size and duration!) with a particular structure (a particular family of properties and relations) defined on them. Different space-times are characterised by having different structures. I list them in order of strongest to weakest structure.

8 Five types of classical space-time

- *Newtonian space-time.*

- Absolute simultaneity (space-time \rightarrow space + time).
- Each ‘spatial slice’ has Euclidean geometry.
- Any two points have a defined distance and duration between them.
- *Space-time symmetries:* rotations, translations (in space and time):
 $x \mapsto x' = Rx + \text{const.}; \quad t \mapsto t' = t + \text{const.}$
- *Invariants:* velocity (\Rightarrow acceleration), inter-particle distances, durations.

- *Galilean space-time, a.k.a. neo-Newtonian space-time.*

- Absolute simultaneity.
- Each ‘spatial slice’ has Euclidean geometry.
- Any two points *on the same ‘spatial slice’* have a defined distance between them.
- Any two points have a defined duration between them.
- A standard of *straightness*, a.k.a. *affine structure*, i.e. absolute acceleration.
- *Space-time symmetries:* rotations, translations (in space and time), boosts:
 $x \mapsto x' = Rx + vt + \text{const.}; \quad t \mapsto t' = t + \text{const.}$
- *Invariants:* acceleration, inter-particle distances, durations.

- **But** Newton’s Corollary 6: *If bodies, any how moved among themselves, are urged in the direction of parallel lines by equal accelerative forces, they will all continue to move among themselves, after the same manner, as if they had been urged by no such forces.*

This suggests a *reformulation* of Newton’s laws in purely relational quantities (e.g. $\mathbf{a}_i - \mathbf{a}_j$). But still there are space-times with appropriate structure.

- *Maxwellian space-time.*

- Absolute simultaneity.
- Each ‘spatial slice’ has Euclidean geometry.
- Any two points *on the same ‘spatial slice’* have a defined distance between them.
- Any two points have a defined duration between them.
- A standard of *twist*, i.e. absolute angular velocity.
- *Space-time symmetries:* rotations, translations (in space and time), arbitrary time-dependent accelerations (hence we must revise Newton’s Laws):
 $x \mapsto x' = Rx + a(t); \quad t \mapsto t' = t + \text{const.}$
- *Invariants:* angular velocity, inter-particle distances, durations.

- *Leibnizian space-time.*
 - Absolute simultaneity.
 - Each ‘spatial slice’ has Euclidean geometry.
 - Any two points *on the same ‘spatial slice’* have a defined distance between them.
 - Any two points have a defined duration between them.
 - *Space-time symmetries: arbitrary time-dependent* rotations, translations (in space and time), arbitrary time-dependent accelerations (hence we must revise Newton’s First & Third Laws):
 $x \mapsto x' = R(t)x + a(t); \quad t \mapsto t' = t + \text{const.}$
 - *Invariants:* relative linear motions, inter-particle distances, durations.

- *Machian space-time.*
 - Absolute simultaneity.
 - Each ‘spatial slice’ has Euclidean geometry.
 - Any two points on the same ‘spatial slice’ have a defined distance between them.
 - *Space-time symmetries:* arbitrary time-dependent rotations, translations in space, arbitrary time-dependent accelerations, arbitrary reparametrisation of time (so long as it is monotonic):
 $x \mapsto x' = R(t)x + a(t); \quad t \mapsto t' = f(t) \quad \left(\frac{df}{dt} > 0\right).$
 - *Invariants:* inter-particle distances.

9 Further Reading

- Alexander, H. G. (1956), *The Leibniz-Clarke Correspondence*. Manchester: Manchester University Press.
- Barbour, J. & Bertotti, B. (1982), ‘Mach’s Principle and the Structure of Dynamical Theories’, *Proceedings of the Royal Society of London A: Mathematical and Physical Sciences* **382**, pp. 295-306.
- * Dainton, B. (2010), *Time and Space*, second edition (Acumen: Durham), Chs. 11-12.
- * Earman, J. (1989), *World Enough and Space-Time: Absolute versus Relational Theories of Space and Time* (Cambridge, MA: MIT Press), Chs. 1-3.
- Newton, I. (1729/1962), *Mathematical Principles of Natural Philosophy*, translated by A. Motte & F. Cajori. Berkeley, CA: University of California Press.