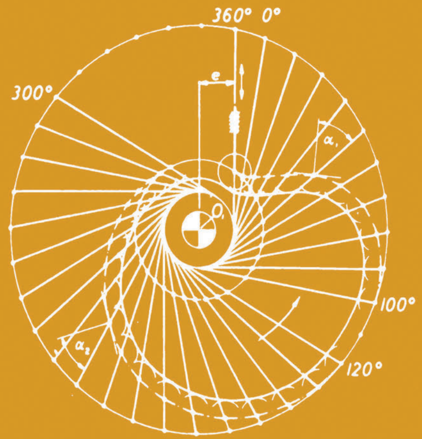
MECHANICAL ENGINEERING/55

CAM DESIGN AND MANUFACTURE

SECOND EDITION



PREBEN W. JENSEN

CAM DESIGN AND MANUFACTURE

MECHANICAL ENGINEERING

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Preface

This book was written to give the practicing engineer a sound grasp of the methods of solving the problems connected with cams—their design, application, and manufacture. The above goal is as valid for this new edition as it was for the first edition. Since that time the most important change has been the improvement of numerically controlled machine tools (NC-machines) and the availability of computers in general. Therefore the emphasis on graphical and analytical methods has shifted toward the latter; but for a design engineer who has put his creative thoughts into metal it is of the utmost importance that he can visualize the problems; therefore the graphical approach has not been neglected. Major changes occur in Chapters 5 and 12 where analytical expressions that can be programmed on a home computer have been developed.

The plan of the book is as follows: In Chapter 1 the basic types of cam and follower systems are described and illustrated. In Chapter 2 the construction and use of displacement diagrams are explained and formulas are given for the displacement, velocity, and acceleration curves for various types of cam motion.

In Chapter 3 displacement diagram synthesis is explained and methods of combining various curves to obtain a desired motion are given. Chapter 4 outlines the methods of determining cam profiles graphically when different types of followers are used and formulas for determining the cam profile using both rectangular and polar coordinates are given.

Chapter 5 takes up the importance of pressure angle, and the procedure for proportioning a cam with respect to pressure angle limitations is explained. The importance of avoiding too small a radius of curvature is also discussed.

In Chapter 6 the advantages of circular cams and methods of proportioning these are outlined. Chapter 7 continues with a discussion of circular-arc and straight-line cams, which have advantages particularly from the standpoint of ease of production.

Chapter 8 considers the important factors of forces, contact stresses, and materials. This is followed in Chapter 9 by a discussion of various methods of cam manufacture.

When cams rotate at high speed, the factors of elasticity and backlash must be taken into consideration if the desired accuracy of motion is to be obtained. One way of doing this is to use the polydyne method of cam design. In Chapter 10 this method is described in detail and the effect of various members of the cam train are determined.

In Chapter 11 the use of various types of mechanisms with cams to offset the disadvantages of the latter is illustrated with various examples.

In Chapter 12, formulas to determine velocities, accelerations, and forces in linkages have been developed. This chapter together with Chapter 8 should enable the reader to find forces in complex mechanisms other than cam mechanisms.

Chapter 13 includes six computer programs that allow the user to determine minimum cam size for given maximum pressure angles for eight different cam displacement diagrams and calculate the maximum compressive stress by both rise and return for translating as well as swinging roller followers.

My thanks to Mr. A. F. Abou-Ghaledum from Cleveland State University, who in no time linked the six programs together to make them user-friendly.

Twenty-eight nomograms are included in order to facilitate computations.

The bibliography at the end of the book lists more than 1800 titles in English and German. The list in the first edition comprised "only" approximately 500 titles. The list is in alphabetical order and is referenced in eleven groups to facilitate its use.

An author usually puts a little of his heart and convictions into his book and I am happy to express that the first edition was well accepted.

Preben W. Jensen

Contents

Using the Cam Design Software

The following should help the new user in using the cam programs included on the supplied disk. There are a number of requirements needed before these programs and the disk can be used. These requirements are concerned with the type of system you are using, and the operating system used.

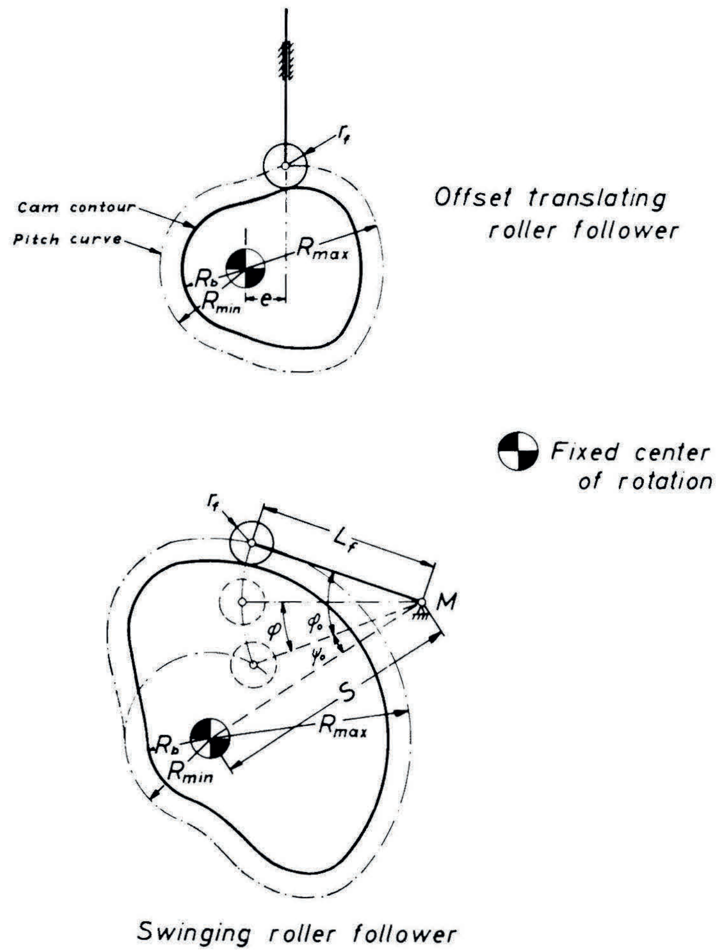
The programs were developed on an IBM-PC running on DOS 2.0, and using BASICA. However, since the programs do not use any special IBM formats and system calls they could run virtually on any compatible running MS-BASIC, GW-BASIC, or similar basic editors.

The following procedure is recommended for proper use of these programs:

1. Switch your computer ON, insert a DOS disk or a disk containing a BASIC Editor into drive A.
2. When you receive the A> prompt, type in the command to run basic (*i.e.,* A>BASICA).
3. Remove the disk from drive A and insert the cam programs disk in A.
4. Type the following command: RUN "MAIN"

This should get you into the MAIN program, which will then prompt you with the master MENU. At this point refer to Chapter 13.

Nomenclature



$a = \dfrac{dv}{dt} = \dfrac{d^2y}{dt^2}$ = acceleration of follower, in/sec^2

b = thickness of contacting cam and follower, in.
d_s = shaft diameter, in.
e = offset or eccentricity, in.
F_n = normal load, lbs.
g = gravitational constant = 386 in/sec^2 = 32.16 ft./sec^2
h = maximum displacement of follower, in.
L_f = length of oscillating follower arm, in.
N = cam speed, rpm

$p = \dfrac{da}{dt}$ = pulse of follower, in/sec^3

r = radius to trace point, in.
r_f = roller radius, in.
S = distance between cam and oscillating follower centers, in.
t = time for cam to rotate angle θ, sec
T = time for cam to rotate angle β, sec
T_1 = time for cam to rotate angle β_1, sec
T_2 = time for cam to rotate angle β_2, sec

$v = \dfrac{dy}{dt}$ = velocity of follower, in/sec

y = displacement of follower, in.

$y' = \dfrac{dy}{dt}$ = follower velocity, in/sec.

$y'' = \dfrac{d^2y}{dt^2}$ = follower acceleration, in/sec^2

$y''' = \dfrac{d^3y}{dt^3}$ = follower pulse or jerk, in/sec^3

R_{max} = maximum radius of cam (to center of roller), in.
R_{min} = minimum radius of cam (to center of roller), in.
R_b = radius of base circle (to actual cam shape), in.
R_g = radius of cutter or grinder, in.
S_c = pressure, lb./in.^{2}
α = pressure angle, degrees
α_1 = pressure angle by rise, degrees
α_2 = pressure angle by return, degrees
α_{m1} = max. pressure angle by rise, degrees
α_{m2} = max. pressure angle by rise, degrees

β = cam angle rotation for total rise h, degrees
β_1 = cam angle rotation for total rise, degrees
β_2 = cam angle rotation for total return, degrees
θ = cam angle rotation for follower displacement y, degrees
μ = coefficient of friction
μ = transmission angle, degrees
μ_c, μ_f = Poisson's ratio for cam and follower, respectively
R_c = radius of curvature of cam, in.
ϕ = angle of oscillating follower movement for cam angle θ, degrees
ϕ_0 = total angle of oscillating follower movement, degrees
ω = cam angular velocity, rad/sec

List of Nomograms

CAM DESIGN AND MANUFACTURE

CHAPTER 1

Cam and Follower Systems

Cams are used in a wide variety of machines; such as packaging machines, can-making machinery, wire-forming machines, engines, computing mechanisms, and mechanical and electronic computers. One important reason why cam mechanisms are preferred over other types is that the use of cams makes it possible to obtain an unlimited variety of motions and when certain basic requirements are followed, cams perform satisfactorily year after year.

Cams are used to transform a rotary motion into a translating or oscillating motion. In certain cases they are also used to transform a translating or oscillating motion in to a different translating or oscillating motion.

The requirements which are imposed on cams vary from machine to machine because the requirements depend not only on the speed of the cam, but also on the kind of machine in which they are being used. In certain kinds of wrapping machines, for example, the forces imposed on the material to be wrapped should be kept as low as possible, but it doesn't matter if these forces are applied suddenly, whereas in other machines it is very important for the proper performance of the machinery that the variation of forces is smooth and gradual. The basic limiting requirements are: kind of time-displacement diagram, pressure angle, radius of curvature, and finally, the contact pressure between follower and cam. These requirements will be discussed in subsequent chapters.

The most commonly used cam is the plate cam which is cut out of a piece of flat metal or plate. Dependent on the kind of follower, various types of following systems are often employed. A *radial translating roller follower* is shown in Fig. 1-1a and is so called because the center line of the follower-stem passes through the center of the cam shaft. An *offset trans-*

lating roller follower is shown in Fig. 1-1b; here the center line of the follower-stem does not pass through the cam shaft center.

Figure 1-1c shows a *swinging roller follower* which is preferred over the translating follower because a much higher pressure angle can be allowed, and hence the overall proportions of the mechanism can be reduced. The question is often raised as to whether the rotation of the cam should be away from or toward pivot point M; in the case of Fig. 1-1c this would mean respectively CCW (counterclockwise) and CW (clockwise) rotation of the cam. There is a slight advantage in letting the cam rotate away from the pivot point, but in most cases the advantage is insignificant.

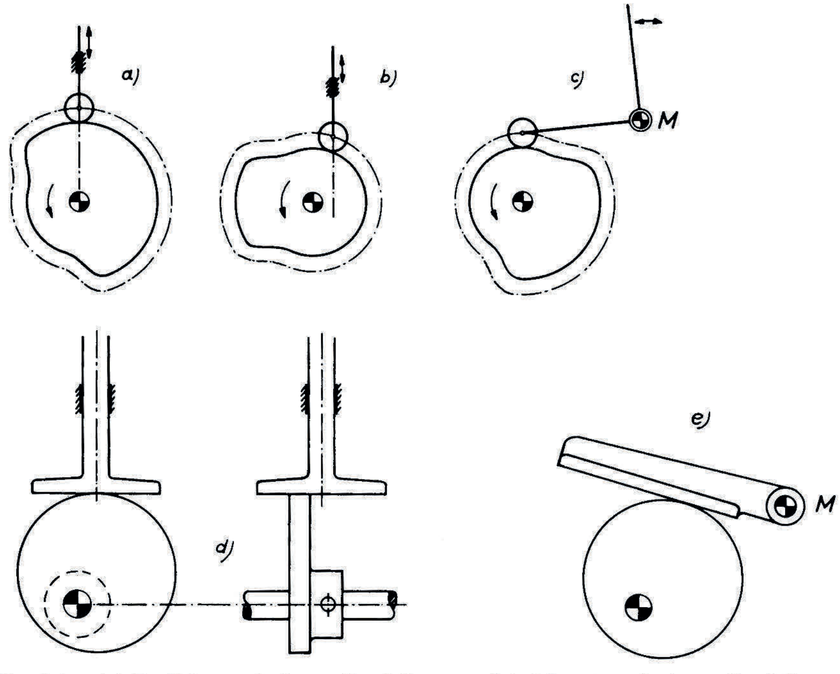


Fig. 1-1. (a) Radial translating roller follower. (b) Offset translating roller follower. (c) Swinging roller follower. (d) Flat-faced translating follower. (e) Flat-faced swinging follower.

Figure 1-1d shows a translating flat-faced follower. The flat does not necessarily have to be perpendicular to the follower-stem, although it usually is. Sometimes the center line of the follower-stem is offset as shown in the right-hand view of Fig. 1-1d. This arrangement will tend to distribute and reduce wear on the flat because the friction force created between the cam and the follower will tend to rotate the follower around its axis. It should be noticed that whether the center line of the follower-stem passes through the cam shaft center or not has no effect on the cam profile. In certain kinds of textile machines, the arrangement shown in Fig. 1-1e with a swinging flat-faced follower is used.

In the systems so far shown, the follower is kept in contact with the cam with the help of gravity forces. Obviously, this is only possible in the case of low-speed cams. For moderate- and high-speed cams other means must be employed. The use of springs is one obvious solution to keep cam and follower in contact with each other.

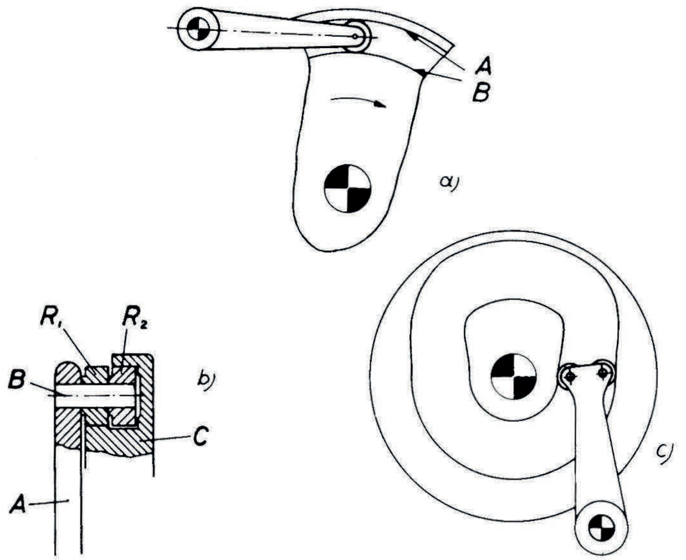


Fig. 1-2. (a) Closed-track cam. (b) Double closed-track cam. (c) Closed-track cam with two rollers.

Other possibilities are shown in Figs. 1-2a, b, and c. In Fig. 1-2a part of a closed-track cam is shown. When the roller follower is driven upward, the roller contacts B, and when driven downward the roller contacts A. A certain clearance is therefore necessary to permit the roller to roll. However, this clearance should be kept as small as possible, because the larger the clearance, the larger the impact will be when the roller changes contact from one side of the track to the other. This change takes place when the motion of the roller changes from acceleration to deceleration. If this change is made gradually, as with a cycloidal motion, the impact is greatly reduced as compared to the case of parabolic motion where the acceleration changes suddenly.

It is therefore natural to try to reduce the clearance to a minimum and it can be done with the help of the arrangement shown in Fig. 1-2b. The follower arm A carries the two rollers R_1 and R_2 on the same pin B. R_1 rolls on the inner side of the track of cam C and R_2 rolls on the outer track.

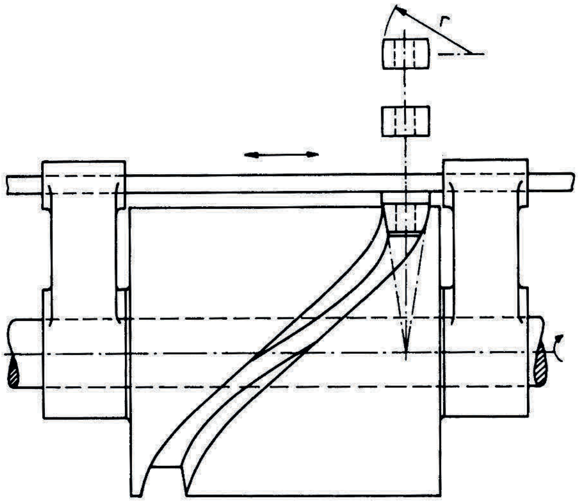


Fig. 1-3a. Cylindrical cam with translating roller follower.

Because of its cost, however, this arrangement is seldom used. In Fig. 1-2c two rollers are placed on different pins. There seems to be no advantage over that of Fig. 1-2b. The track shown in Fig. 1-2c is also more difficult to machine than that of Fig. 1-2b.

A cylindrical cam with translating roller follower, Fig. 1-3a, has the characteristic that the direction of motion of the follower is parallel to the cam shaft. Theoretically, the roller has to be cone-shaped with its apex at the cam shaft center, as shown in Fig. 1-3a, in order that the roller may roll without sliding. However, a cylindrical roller or one with a slightly spherical shape will operate satisfactorily in most cases.

Figure 1-3b shows a similar arrangement but two rollers are used instead of one. The advantage as compared with Fig. 1-3a, where one roller

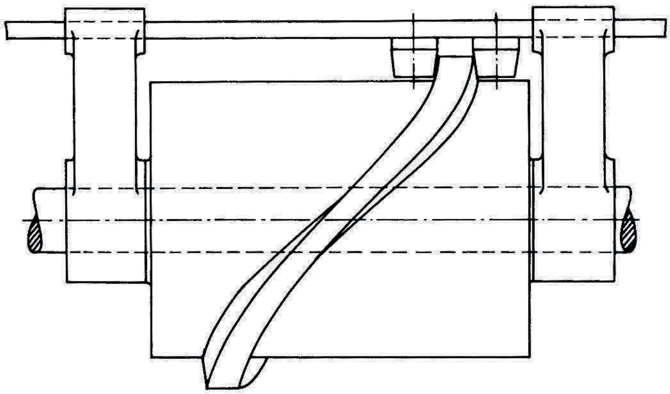


Fig. 1-3b. Double-end cam with translating roller follower.

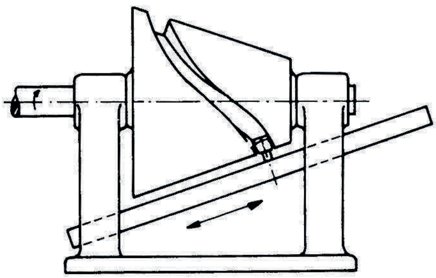


Fig. 1-4(a). Conical closed-track cam with translating roller follower

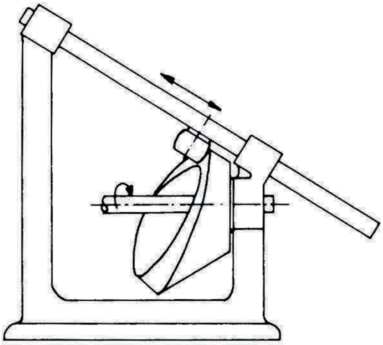


Fig. 1-4(b). Conical open-track cam with translating roller follower

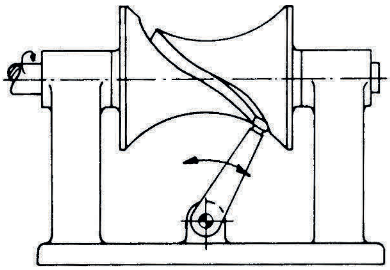


Fig. 1-5. Globoidal closed-track cam with swinging roller follower

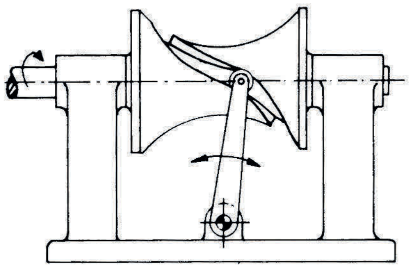


Fig. 1-6. Globoidal closed-track cam with swinging roller follower

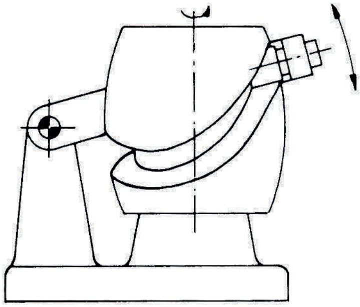


Fig. 1-7. Globoidal closed-track cam with swinging roller follower

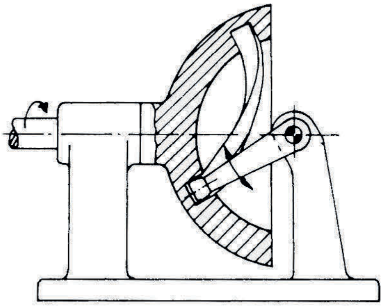


Fig. 1-8. Spherical closed-track cam with swinging roller follower

Figs. 1-4 to 1-8. Special types of cams.

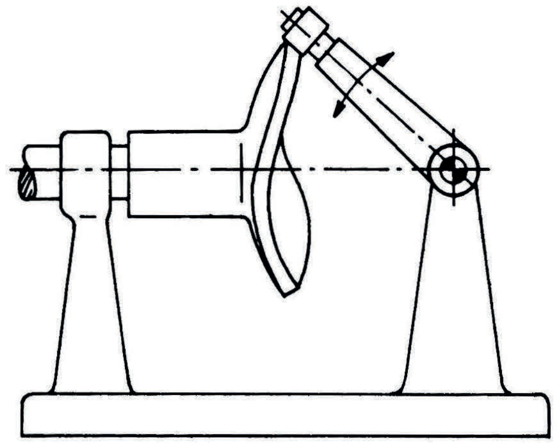


Fig. 1-9. Spherical open-track cam with swinging roller follower.

changes contact from one side of the track to the other whenever acceleration is changed in direction, is that with both rollers in contact with the track at the same time there will be no backlash.

A conical cam, Fig. 1-4a, has much the same characteristics as a cylindrical one and is used in cases when the direction of the output motion is parallel to an element of the base cone. The conical cam in Fig. 1-4a has a closed track and that in Fig. 1-4b an open track.

The cam and follower systems shown in Figs. 1-5 to 1-10 have to be cut by special devices. Because of the cost of making them, they are

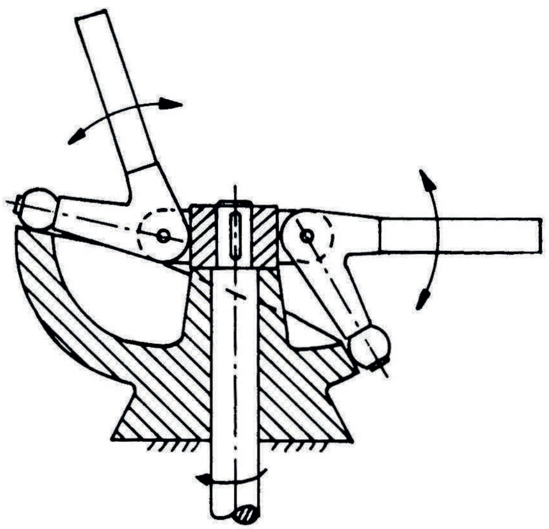


Fig. 1-10. Kinematic inversion of the spherical cam; cam is stationary and swinging roller follower rotates around cam.

seldom employed and then only for rather small mechanisms. Figures 1-5, 1-6, and 1-7 show globidal cams with swinging roller followers; the only difference is in how the roller is placed relative to the input and output shafts. Figure 1-8 shows a spherical cam with swinging roller follower and closed-track, and in Fig. 1-9 the mechanism is shown with an open track.

Figure 1-10 is a kinematic inversion of the spherical cam; the cam is stationary and the swinging roller follower rotates. This kind of mechanism is used in agricultural machinery.

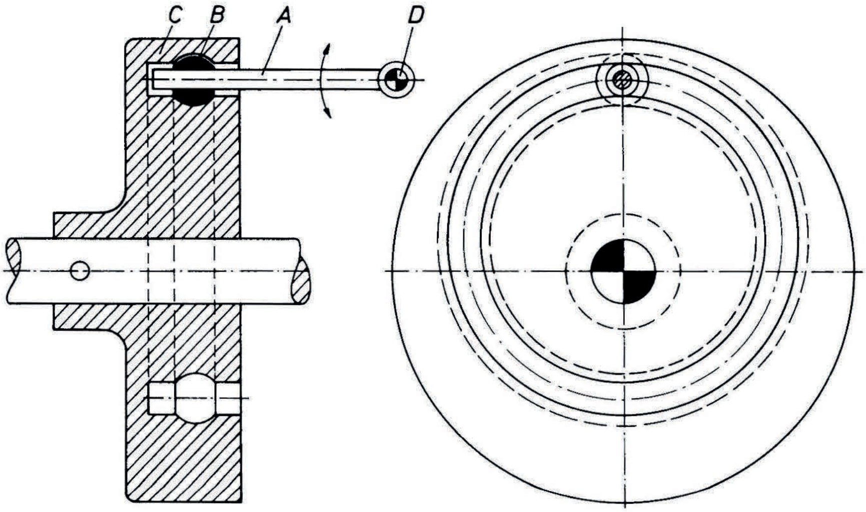


Fig. 1-11. Ball runs in groove of closed-track cam, imparting an oscillating motion to output member A.

A very special kind of cam and follower system is shown in Fig. 1-11. The cam C is a closed-track cam and the track is formed so that a ball B can be guided by either side of the track dependent upon the direction of motion of arm A. A round arm A has a sliding fit in the hole of the ball and is fastened to shaft D, which is the output shaft. This mechanism is used in a sewing machine and the cam rotates at 2000 to 3000 rpm. The advantage of this mechanism becomes clearly obvious when compared with Fig. 1-9. To cut the cam in Fig. 1-9 would require that the milling cutter be moved exactly the same way relative to the spherical cam as the roller follower, and this requires a special set-up. However, the cam in Fig. 1-11 can be cut with a milling cutter which has the form of the groove, exactly as if it were a plate cam.

CHAPTER 2

Displacement Diagrams

In Chapter 1 many varieties of cams and followers are illustrated and in all of them the cam rotates at a constant angular velocity and the follower moves in a manner prescribed by the functional requirements of the machine.

The simplest follower motion is a constant velocity rise followed by a similar return with a dwell in between. A simple graph called a displacement diagram illustrates this sequence of events.

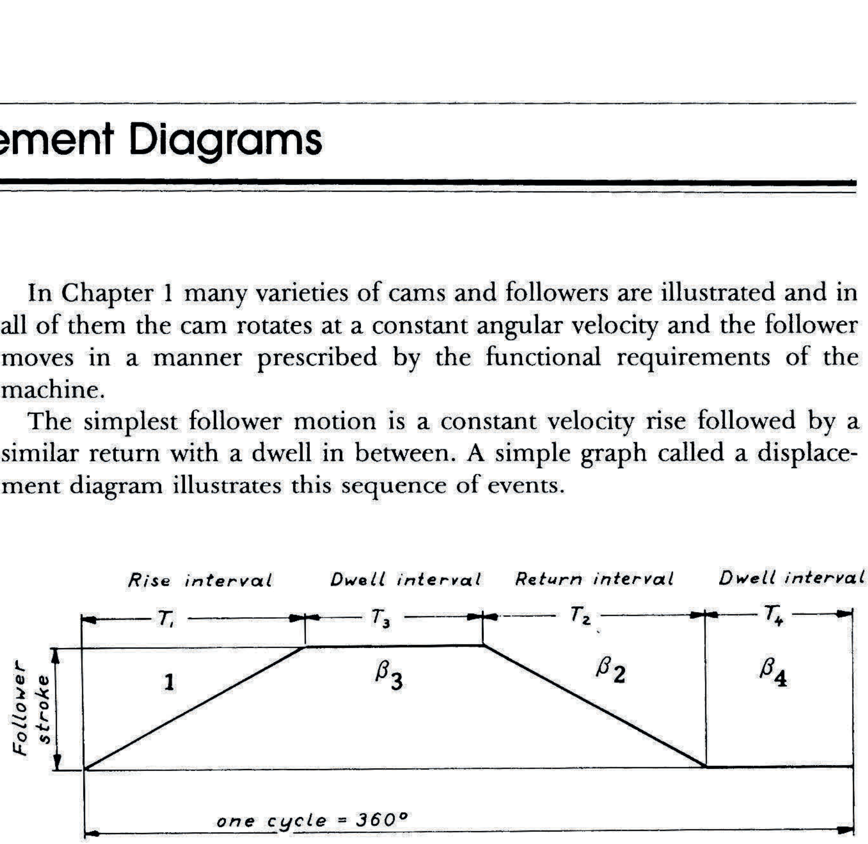


Fig. 2-1. A simple displacement diagram.

Such a diagram is shown in Fig. 2-1. Here, one cycle is taken to mean one complete revolution of the cam; that is, one cycle represents 360 degrees. From this it follows that the horizontal distances, T_1, T_2, T_3, T_4 is expressed in seconds and β_1, β_2, β_3, β_4 in degrees of rotation. Degrees are mostly used. The vertical distances represent the motion of the follower dependent on time. From Fig. 2-2 it can be seen that in place of a rotating cam it is possible to achieve the identical follower action by means of a translating

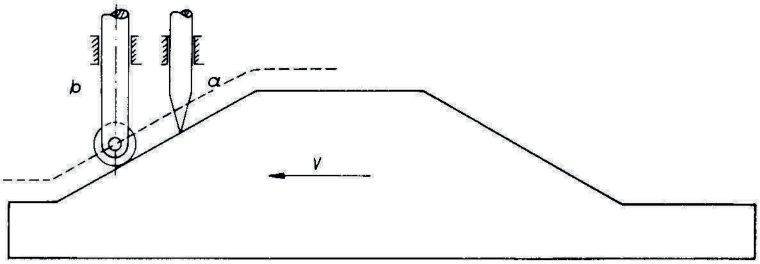


Fig. 2-2. Translating cam made from diagram in Fig. 2-1.

member, which has a profile the same as that of the displacement diagram, translating with a constant velocity v. This device is often used in machinery. To cause the follower to move as indicated by Fig. 2-1, but by means of a rotating member instead of a slider, we merely "wrap" the time displacement diagram around a circular disk as shown in Fig. 2-3. (This will be explained in more detail in Chapter 4.) Thus, the diagram in Fig. 2-1 represents the follower movement of either Figs. 2-2 or 2-3.

Cam Followers

It is important to study the effect of the kind of contact between the cam surface and the follower. For example, in Fig. 2-2, shown at (a) is a follower having a pointed end which makes line contact with the cam. It is clear that if continuous contact is maintained, the follower stem will have truly the

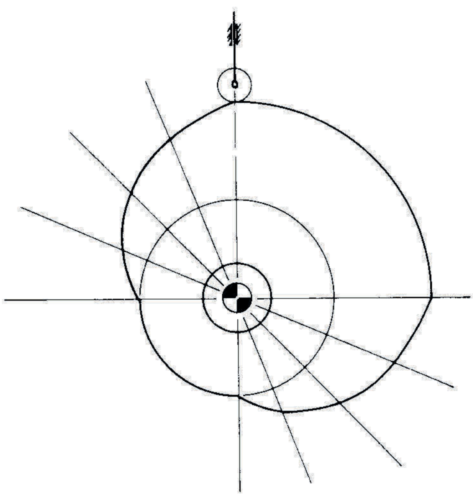


Fig. 2-3. Rotating cam made from diagram in Fig. 2-1.

motion prescribed by the time-displacement diagram. If, however, a roller is used as shown at (b), the follower stem cannot possibly have exactly the movement prescribed because of the sharp corners of the cam at the start and finish of each rise and return. The movement of the roller follower will be as shown by the dashed line in Fig. 2-2. Note the curved path at the end of the rise.

It is obvious from Fig. 2-2 that since a constant slope is used, this has the effect of moving the follower with constant velocity. Now constant velocity is a desirable form of motion for a cam follower provided that the acceleration from rest to the constant velocity value is moderate. Theoretically, an instantaneous jump from zero velocity to any value of velocity results in an infinite acceleration and since mass is always involved in machines, this theoretically results in an infinitely large force. Actually, an instantaneous change in velocity is impossible due to flexure of the machine parts and other factors. Nevertheless, any shock effect is serious and must be kept to a minimum. For this reason the rise and fall portions of a cam displacement diagram are of vital importance and need to be studied in considerable detail.

It is fortunate that in many design problems the required operation merely requires a particular machine member to be at a given point at a given time. How it gets to this point is not specified. The cam designer is therefore at liberty to choose an approach which gives the lowest shock values possible so as to reduce wear and tear on the cam assembly. It is also fortunate that what is best for the cam is almost always best for the machine and its product.

Types of Cam Displacement Curves

A wide variety of cam curves are available for moving the follower and these will be thoroughly analyzed. In the following sections only the rise portions of the total time-displacement diagram are studied. The return portions can be analyzed in a similar manner. However, formulas for the return portions are added for convenience. Complex cams are frequently employed which may involve a number of rise-dwell-return intervals in which the rise and return aspects are quite different. To analyze the action of a cam it is necessary to study its time-displacement and associated velocity and acceleration curves. The latter are based on the first and second time-derivatives of the equation describing the time-displacement curve:

$$y = \text{displacement} = f(t) = f(\theta)$$

$$\frac{dy}{dt} = \text{velocity} \qquad \frac{d^2y}{dt^2} = \text{acceleration}$$

A variety of displacement curves will now be briefly discussed. The equations for return motion are based upon the point A in Fig. 2-4 and the following figures being at the top of the diagram and point B at the bottom. The displacement curves will be more thoroughly analyzed in subsequent chapters. In the group called polynomial curves the significance of numerical prefixes is taken up in detail in Chapter 10.

y = displacement of follower, in.
h = maximum displacement of follower, in.
t = time for cam to rotate through angle θ, sec.
T_1 = time for total rise, sec
T_2 = time for total return, sec
θ = cam angle rotation for follower displacement y, degrees
β_1 = cam angle for total rise, degrees
β_2 = cam angle for total return, degrees
v = velocity of follower, in./sec
a = follower acceleration, in./sec^2
t_x = a function of t

In all the following formulas for y, θ and β_1, β_2 are used but can be replaced with t and T_1, T_2, respectively. This will facilitate calculation of velocities and accelerations when θ and β are known.

Constant-velocity Motion (Fig. 2-4)

$$\left.\begin{aligned} y &= h\frac{\theta}{\beta_1} \\ v &= \frac{h}{T_1} \\ a &= 0 \end{aligned}\right\} \begin{array}{c} 0 < \theta < \beta_1 \\ \text{(rise)} \end{array} \tag{2.1a}$$

except at $\theta = 0$ and $\theta = \beta_1$ where the acceleration is theoretically infinite

$$\left.\begin{aligned} y &= h\left(1 - \frac{\theta}{\beta_2}\right) \\ v &= -\frac{h}{T_2} \\ a &= 0 \end{aligned}\right\} \begin{array}{c} 0 < \theta < \beta_2 \\ \text{(return)} \end{array} \tag{2.1b}$$

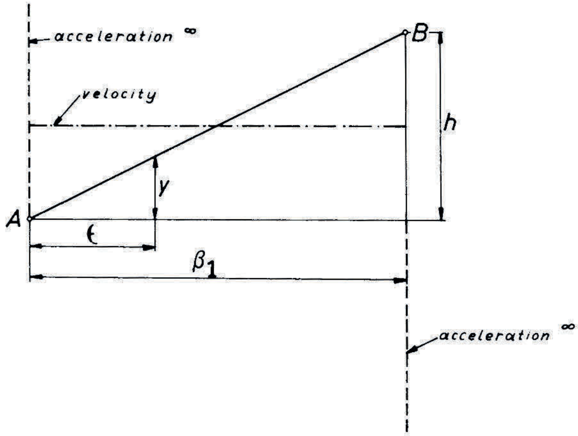


Fig. 2-4. Cam displacement, velocity, and acceleration curves for constant velocity motion.

This motion and its disadvantages were discussed earlier in the chapter. This curve is in general only to be used as a composite curve.

The angle for rise β_1 and the corresponding time T_1 are related by the formula

$$T_1 = \frac{60}{N}\frac{\beta_1}{360}$$

For return the formula becomes

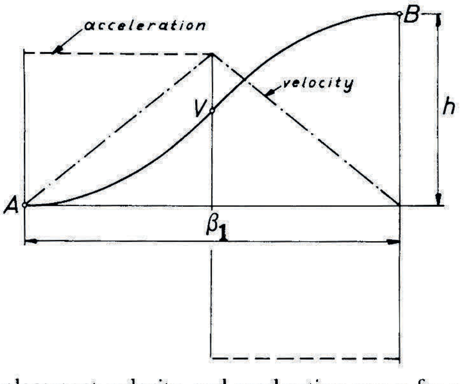


Fig. 2-5. Cam displacement, velocity, and acceleration curves for parabolic motion.

$$T_2 = \frac{60}{N}\frac{\beta_2}{360}$$

where N is the rotational speed of the cam in RPM (revolutions pr. minute).

In the unaltered form shown it is rarely used except in very crude devices, nevertheless the advantage of uniform velocity is an important one and by modifying the start and finish of the follower stroke this form of cam motion can be utilized. The modification is explained in Chapter 3.

Parabolic Motion (Fig. 2-5)

$$\left.\begin{aligned} y &= 2h\left(\frac{\theta}{\beta_1}\right)^2 \\ v &= 4\frac{h}{T_1}\frac{\theta}{\beta_1} \\ a &= 4\frac{h}{T_1^2} \end{aligned}\right\} \begin{matrix} 0 \leqslant \theta \leqslant \dfrac{\beta_1}{2} \\ \text{(rise)} \end{matrix} \qquad (2.2a)$$

$$\left.\begin{aligned} y &= h\left[1 - 2\left(\frac{\beta_1 - \theta}{\beta_1}\right)^2\right] \\ v &= 4\frac{h}{T_1}\frac{\beta_1 - \theta}{\beta_1} \\ a &= -4\frac{h}{T_1^2} \end{aligned}\right\} \begin{matrix} \dfrac{\beta_1}{2} \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{matrix} \qquad (2.2b)$$

$$\left.\begin{aligned} y &= h\left[1 - 2\left(\frac{\theta}{\beta_2}\right)^2\right] \\ v &= -4\frac{h}{T_2}\frac{\theta}{\beta_2} \\ a &= -4\frac{h}{T_2^2} \end{aligned}\right\} \begin{matrix} 0 \leqslant \theta \leqslant \dfrac{\beta_2}{2} \\ \text{(return)} \end{matrix} \qquad (2.2c)$$

$$\left.\begin{aligned} y &= 2h\left(\frac{\beta_2 - \theta}{\beta_2}\right)^2 \\ v &= -4\frac{h}{T_2}\frac{\beta_2 - \theta}{\beta_2} \\ a &= 4\frac{h}{T_2^2} \end{aligned}\right\} \begin{matrix} \dfrac{\beta_2}{2} \leqslant \theta \leqslant \beta_2 \\ \text{(return)} \end{matrix} \qquad (2.2d)$$

The most important advantage of this curve is that for a given angle of rotation and rise it produces the smallest possible acceleration. However, because of the sudden changes in acceleration at the beginning, middle, and end of the stroke, shocks are produced. If the follower system were perfectly rigid with no backlash or flexibility, this would be of little significance. But such systems are mechanically impossible to build and a certain amount of impact is caused at each of these change-over points.

Therefore this curve is not recommended for high speed.

Simple Harmonic Motion (Fig. 2-6)

$$\left.\begin{aligned} y &= \frac{h}{2}\left[1 - \cos\left(\pi \frac{\theta}{\beta_1}\right)\right] \\ v &= \frac{h}{2}\cdot\frac{\pi}{T_1}\sin\left(\pi\frac{\theta}{\beta_1}\right) \\ a &= \frac{h}{2}\cdot\frac{\pi^2}{T_1^2}\cos\left(\pi\frac{\theta}{\beta_1}\right) \end{aligned}\right\} \quad \begin{gathered} 0 \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{gathered} \tag{2.3a}$$

$$\left.\begin{aligned} y &= \frac{h}{2}\left[1 + \cos\left(\pi \frac{\theta}{\beta_2}\right)\right] \\ v &= -\frac{h}{2}\frac{\pi}{T_2}\sin\left(\pi\frac{\theta}{\beta_2}\right) \\ a &= -\frac{h}{2}\frac{\pi^2}{T_2^2}\cos\left(\pi\frac{\theta}{\beta_2}\right) \end{aligned}\right\} \quad \begin{gathered} 0 \leqslant \theta \leqslant \beta_2 \\ \text{(return)} \end{gathered} \tag{2.3b}$$

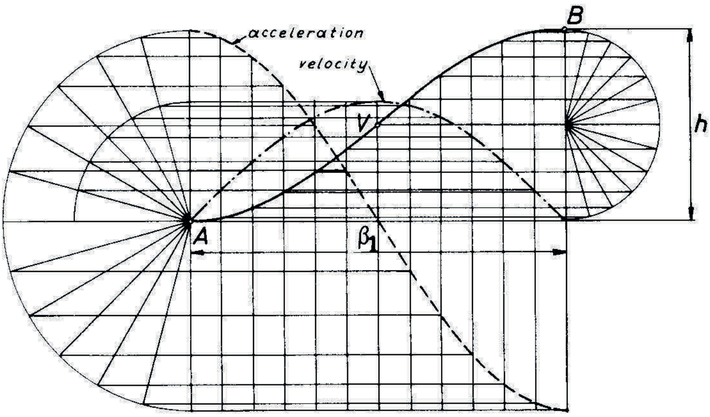


Fig. 2-6. Cam displacement, velocity, and acceleration curves for simple harmonic motion.

Smoothness in velocity and acceleration during the stroke is the advantage inherent in this curve. However, the instantaneous changes in acceleration at the beginning and end of the stroke tend to cause vibration, noise, and wear. As can be seen from Fig. 2-6, the maximum acceleration values occur at the ends of the stroke. Thus, if inertia loads are to be overcome by the follower, the resulting forces cause severe stresses in the members. These forces are in many cases much larger than the externally applied loads. This curve is not recommended for high speed.

Cycloidal Motion (Fig. 2-7)

$$\left.\begin{aligned} y &= h\left[\frac{\theta}{\beta_1} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{\beta_1}\right)\right] \\ v &= \frac{h}{T_1}\left[1 - \cos\left(2\pi\frac{\theta}{\beta_1}\right)\right] \\ a &= \frac{2\pi h}{T_1^2}\sin\left(2\pi\frac{\theta}{\beta_1}\right) \end{aligned}\right\} \quad \begin{gathered} 0 \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{gathered} \qquad (2.4a)$$

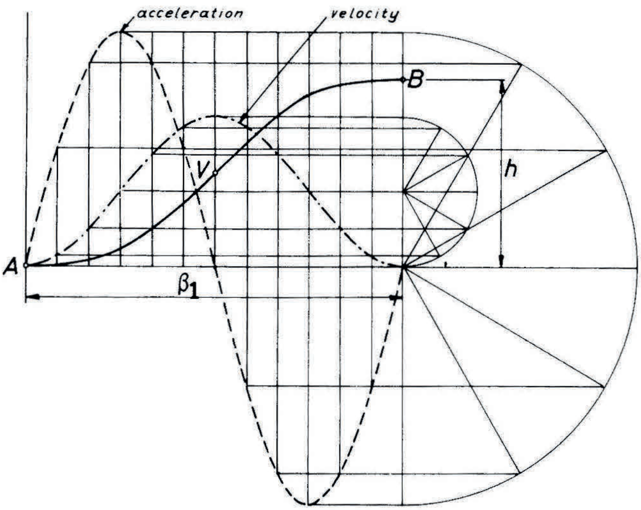


Fig. 2-7. Cam displacement, velocity, and acceleration curves for cycloidal motion.

$$\left.\begin{aligned} y &= h\left[1 - \frac{\theta}{\beta_2} + \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{\beta_2}\right)\right] \\ v &= \frac{h}{T_2}\left[\cos\left(2\pi\frac{\theta}{\beta_2}\right) - 1\right] \\ a &= -\frac{2\pi h}{T_2^2}\sin\left(2\pi\frac{\theta}{\beta_2}\right) \end{aligned}\right\} \begin{gathered} 0 \leqslant \theta \leqslant \beta_2 \\ \text{(return)} \end{gathered} \qquad (2.4b)$$

This time-displacement curve has excellent acceleration characteristics; there are no abrupt changes in its associated acceleration curve. The maximum value of the acceleration of the follower for a given rise and time is somewhat higher than that of the simple harmonic motion curve. In spite of this the cycloidal curve is used often as a basis for designing cams for high-speed machinery because it results in low noise, vibration, and wear.

The cycloidal motion displacement curve is so called because it can be generated from a cycloid which is the locus of a point of a circle rolling on a straight line. Thus, in Fig. 2-8 P is a point on the circle which rolls on the straight line BC. The radius OP of the circle is made equal to $h/2\pi$ so that the circumference of the circle equals the distance BC. The circle starts its rolling when P is at C. When the center of the circle is in the position shown

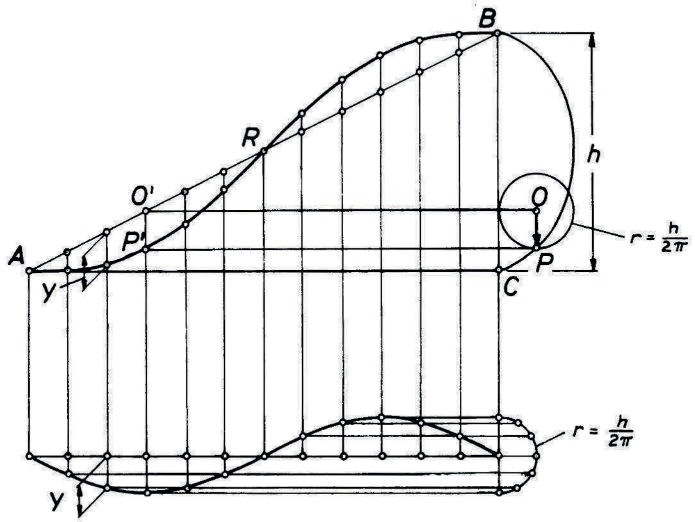


Fig. 2-8. Geometric properties of cycloidal motion.

point P has moved to its present position. A horizontal line is drawn from O to O' which is on line AB and a vertical line is drawn from O' to intersect the horizontal line through P; the point of intersection is P' which is a point on the displacement curve.

Cycloidal motion can also be considered to be composed of a straight line AB on which is superposed a sine wave the amplitude of which is $r = h/2\pi$ and the amplitude being perpendicular to the base line AC.

In Chapter 3 more is said about this and a family of curves—the modified cycloids—is developed.

Double Harmonic Motion (Fig. 2-9)

$$\left.\begin{aligned} y &= \frac{h}{2}\left[1 - \cos\left(\pi\frac{\theta}{\beta_1}\right) - \frac{1}{4}\left(1 - \cos\left(2\pi\frac{\theta}{\beta_1}\right)\right)\right] \\ v &= \frac{h}{2}\frac{\pi}{T_1^2}\left[\sin\left(\pi\frac{\theta}{\beta_1}\right) - \frac{1}{2}\sin\left(2\pi\frac{\theta}{\beta_1}\right)\right] \\ a &= \frac{h}{2}\frac{\pi^2}{T_1^2}\left[\cos\left(\pi\frac{\theta}{\beta_1}\right) - \cos\left(2\pi\frac{\theta}{\beta_1}\right)\right] \end{aligned}\right\} \begin{gathered} 0 \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{gathered} \qquad (2.5a)$$

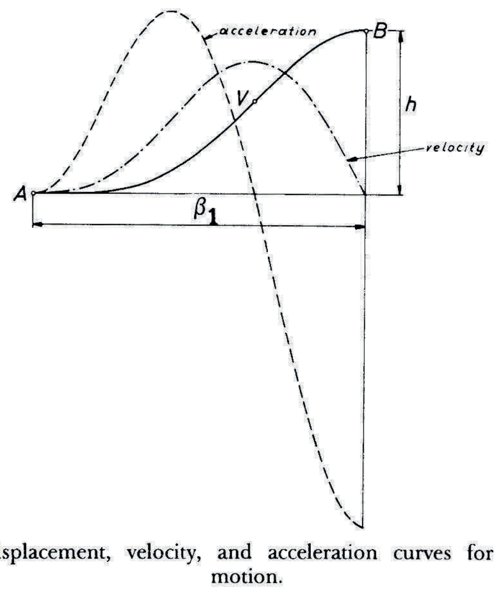


Fig. 2-9. Cam displacement, velocity, and acceleration curves for double harmonic motion.

$$\left.\begin{aligned} y &= \frac{h}{2}\left[1+\cos\left(\pi\frac{\theta}{\beta_2}\right)+\frac{1}{4}\left(1-\cos\left(2\pi\frac{\theta}{\beta_2}\right)\right)\right] \\ v &= -\frac{h}{2}\frac{\pi}{T_2}\left[\sin\left(\pi\frac{\theta}{\beta_2}\right)-\frac{1}{2}\sin\left(2\pi\frac{\theta}{\beta_2}\right)\right] \\ a &= -\frac{h}{2}\frac{\pi^2}{T_2^2}\left[\cos\left(\pi\frac{\theta}{\beta_2}\right)-\cos\left(2\pi\frac{\theta}{\beta_2}\right)\right] \end{aligned}\right\} \begin{array}{c} 0 \leqslant \theta \leqslant \beta_2 \\ \text{(return)} \end{array} \tag{2.5b}$$

The negative acceleration is about double that of the positive and there is a sudden change in acceleration at the end of the rise. This curve is only good when used as part of a compound curve, that is, a curve made up of different or similar basic curves (parabolic, simple harmonic, etc.)

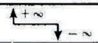
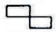
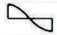
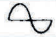
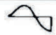
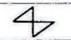
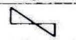
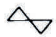
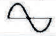
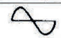
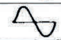
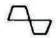
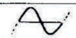
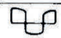
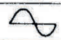
Cubic Curve No. 1 (Table 2-1)

$$\left.\begin{aligned} y &= 4h\left(\frac{\theta}{\beta_1}\right)^3 \\ v &= \frac{12h}{T_1}\left(\frac{\theta}{\beta_1}\right)^2 \\ a &= \frac{24h}{T_1^2}\frac{\theta}{\beta_1} \end{aligned}\right\} \begin{array}{c} 0 \leqslant \theta \leqslant \frac{\beta_1}{2} \\ \text{(rise)} \end{array} \tag{2.6a}$$

$$\left.\begin{aligned} y &= h\left[1-4h\left(\frac{\beta_1-\theta}{\beta_1}\right)^3\right] \\ v &= \frac{12h}{T_1}(\beta_1-\theta)^2 \\ a &= -\frac{24h}{T_1^2}(\beta_1-\theta) \end{aligned}\right\} \begin{array}{c} \frac{\beta_1}{2} \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{array} \tag{2.6b}$$

$$\left.\begin{aligned} y &= h\left[1-4\left(\frac{\theta}{\beta_2}\right)^3\right] \\ v &= -\frac{12h}{T_2}\left(\frac{\theta}{\beta_2}\right)^2 \\ a &= -\frac{24h}{T_2^2}\frac{\theta}{\beta_2} \end{aligned}\right\} \begin{array}{c} 0 \leqslant \theta \leqslant \frac{\beta_2}{2} \\ \text{(return)} \end{array} \tag{2.6c}$$

Table 2-1. Characteristics of Various Types of Cam Curves

TYPE OF CURVE	EQUATION NO.	ACCELERATION CURVE	VELOCITY FACTOR	ACCELERATION FACTOR	JERK FACTOR	CAM SPEED APPLICATION	PERFORMANCE AT HIGH SPEED	OTHER COMMENTS (See also chapter on cam manufacturing)
Constant Velocity Motion	**2.1a-b**		1.00	∼		Low speed and low masses	Poor	
Parabolic Motion	**2.2a-d**		2.00	4.00	3x∼	Medium speed	Good	
Simple Harmonic Motion	**2.3a-b**		1.57	4.93	2x∼	Medium speed	Good	
Cycloidal Motion	**2.4a-b**		2.00	6.28	61	High speed	Excellent	
Double Harmonic Motion	**2.5a-b**		2.00	5.5 9.9				
Cubic Curve No. 1	**2.6a-d**		3.00	12.00	1x∼	Low speed	Poor	
Cubic Curve No. 2	**2.7a-b**		1.50	6.00	2x∼	Low speed	Poor	
Cubic Curve No. 3	**2.8a-f**		2.00	8.00	32	Low speed	Poor	
3-4 Polynomial	**2.9a-d**		2.00	6.00	48	High speed	Excellent	
3-4-5 Polynomial	**2.10a-b**		1.88	5.77	60	High speed	Excellent	
4-5-6-7 Polynomial	**2.11a-b**		2.19	7.52	52.5	Medium to high speed	Good to excellent	
Trapezoidal Acceleration	**2.12a-c**		2.00	5.33	42.7	High speed	Good to excellent	
Modified Trapezoidal Acceleration	**2.13a-l**		2.00	4.89	61.4	High speed	Excellent	
Modified Sinusoidal Acceleration	**2.14a-f**		1.76	5.53	69.3	High speed	Good to excellent	
Circular Cam Profile						High speed	Best of all	Can be made on lathe with highest accuracy
Circular Arc Profile						Medium speed	Good	Small cams can be made accurately
Circular Arc and Straight Line Profile						Medium speed	Good	Small cams can be made accurately
Modified Cycloid	2.15.a 2.15.b			All values from 5.89 to ∼		Medium to high speed	Good to excellent	

$$\left.\begin{aligned} y &= 4h\left(\frac{\beta_2 - \theta}{\beta_2}\right)^3 \\ v &= -\frac{12h}{T_2}(\beta_2 - \theta)^2 \\ a &= \frac{24h}{T_2^2}(\beta_2 - \theta) \end{aligned}\right\} \begin{aligned} &\frac{\beta_2}{2} \leqslant \theta \leqslant \beta_2 \\ &\text{(return)} \end{aligned} \qquad (2.6\text{d})$$

Because of the sudden change in acceleration at the middle of the rise and because of a rather high value of the maximum deceleration, this curve is only good when used as part of a compound curve.

Cubic Curve No. 2 (Table 2-1)

$$\left.\begin{aligned} y &= h\left[3\left(\frac{\theta}{\beta_1}\right)^2 - 2\left(\frac{\theta}{\beta_1}\right)^3\right] \\ v &= 6\,\frac{h}{T_1}\left[\frac{\theta}{\beta_1} - \left(\frac{\theta}{\beta_1}\right)^2\right] \\ a &= 6\,\frac{h}{T_1^2}\left(1 - 2\,\frac{\theta}{\beta_1}\right) \end{aligned}\right\} \begin{aligned} &0 \leqslant \theta \leqslant \beta_1 \\ &\text{(rise)} \end{aligned} \qquad (2.7\text{a})$$

$$\left.\begin{aligned} y &= h\left[1 - 3\left(\frac{\theta}{\beta_2}\right)^2 + 2\left(\frac{\theta}{\beta_2}\right)^3\right] \\ v &= -6\,\frac{h}{T_2}\left[\frac{\theta}{\beta_2} - \left(\frac{\theta}{\beta_2}\right)^2\right] \\ a &= -6\,\frac{h}{T_2^2}\left(1 - 2\,\frac{\theta}{\beta_2}\right) \end{aligned}\right\} \begin{aligned} &0 \leqslant \theta \leqslant \beta_2 \\ &\text{(return)} \end{aligned} \qquad (2.7\text{b})$$

This curve is usable for low speed only.

Cubic Curve No. 3 (Table 2-1)

$$\left.\begin{aligned} y &= \frac{16}{3}\,h\left(\frac{\theta}{\beta_1}\right)^3 \\ v &= 16\,\frac{h}{T_1}\left(\frac{\theta}{\beta_1}\right)^2 \\ a &= 32\,\frac{h}{T_1^2}\left(\frac{\theta}{\beta_1}\right) \end{aligned}\right\} \begin{aligned} &0 \leqslant \theta \leqslant \frac{\beta_1}{4} \\ &\text{(rise)} \end{aligned} \qquad (2.8\text{a})$$

$$\left.\begin{aligned} y &= h\left[\frac{1}{6} - 2\frac{\theta}{\beta_1} + 8\left(\frac{\theta}{\beta_1}\right)^2 - \frac{16}{3}\left(\frac{\theta}{\beta_1}\right)^3\right] \\ v &= \frac{h}{T_1}\left[-2 + 16\frac{\theta}{\beta_1} - 16\left(\frac{\theta}{\beta_1}\right)^2\right] \\ a &= \frac{h}{T_1^2}\left(16 - 32\frac{\theta}{\beta_1}\right) \end{aligned}\right\} \begin{aligned} \frac{\beta_1}{4} \leqslant \theta \leqslant \frac{3\beta_1}{4} \\ \text{(rise)} \end{aligned} \tag{2.8b}$$

$$\left.\begin{aligned} y &= h\left[-\frac{13}{3} + 16\frac{\theta}{\beta_1} - 16\left(\frac{\theta}{\beta_1}\right)^2 + \frac{16}{3}\left(\frac{\theta}{\beta_1}\right)^3\right] \\ v &= \frac{h}{T_1}\left[16 - 32\frac{\theta}{\beta_1} + 16\left(\frac{\theta}{\beta_1}\right)^2\right] \\ a &= \frac{h}{T_1^2}\left(-32 + 32\frac{\theta}{\beta_1}\right) \end{aligned}\right\} \begin{aligned} \frac{3\beta_1}{4} \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{aligned} \tag{2.8c}$$

$$\left.\begin{aligned} y &= h\left[\frac{16}{3} - 16\frac{\theta}{\beta_2} + 16\left(\frac{\theta}{\beta_2}\right)^2 - \frac{16}{3}\left(\frac{\theta}{\beta_2}\right)^3\right] \\ v &= \frac{h}{T_2}\left[-16 + 32\frac{\theta}{\beta_2} - 16\left(\frac{\theta}{\beta_2}\right)^2\right] \\ a &= \frac{h}{T_2^2}\left(32 - 32\frac{\theta}{\beta_2}\right) \end{aligned}\right\} \begin{aligned} \frac{3\beta_2}{4} \leqslant \theta \leqslant \beta_2 \\ \text{(return)} \end{aligned} \tag{2.8d}$$

$$\left.\begin{aligned} y &= h\left[1 - \frac{16}{3}\left(\frac{\theta}{\beta_2}\right)^3\right] \\ v &= -16\frac{h}{T_2}\left(\frac{\theta}{\beta_2}\right)^2 \\ a &= -32\frac{h}{T_2^2}\frac{\theta}{\beta_2} \end{aligned}\right\} \begin{aligned} 0 \leqslant \theta \leqslant \frac{\beta_2}{4} \\ \text{(return)} \end{aligned} \tag{2.8e}$$

$$\left.\begin{aligned} y &= h\left[\frac{5}{6} + 2\frac{\theta}{\beta_2} - 8\left(\frac{\theta}{\beta_2}\right)^2 + \frac{16}{3}\left(\frac{\theta}{\beta_2}\right)^3\right] \\ v &= \frac{h}{T_2}\left[2 - 16\left(\frac{\theta}{\beta_2}\right) + 16\left(\frac{\theta}{\beta_2}\right)^2\right] \\ a &= \frac{h}{T_2^2}\left(-16 + 32\frac{\theta}{\beta_2}\right) \end{aligned}\right\} \begin{aligned} \frac{\beta_2}{4} \leqslant \theta \leqslant \frac{3\beta_2}{4} \\ \text{(return)} \end{aligned} \tag{2.8f}$$

This curve is usable only for low speed or in a compound curve.

3-4 Polynomial (Table 2-1)

$$\left.\begin{aligned} y &= h\left[8\left(\frac{\theta}{\beta_1}\right)^3 - 8\left(\frac{\theta}{\beta_1}\right)^4\right] \\ v &= \frac{h}{T_1}\left[24\left(\frac{\theta}{\beta_1}\right)^2 - 32\left(\frac{\theta}{\beta_1}\right)^3\right] \\ a &= \frac{h}{T_1^2}\left[48\frac{\theta}{\beta_1} - 96\left(\frac{\theta}{\beta_1}\right)^2\right] \end{aligned}\right\} \begin{gathered} 0 \leqslant \theta \leqslant \frac{\beta_1}{2} \\ \text{(rise)} \end{gathered} \qquad (2.9a)$$

$$\left.\begin{aligned} y &= h\left[1 - 8\frac{\theta}{\beta_1} + 24\left(\frac{\theta}{\beta_1}\right)^2 - 24\left(\frac{\theta}{\beta_1}\right)^3 + 8\left(\frac{\theta}{\beta_1}\right)^4\right] \\ v &= \frac{h}{T_1}\left[-8 + 48\frac{\theta}{\beta_1} - 72\left(\frac{\theta}{\beta_1}\right)^2 + 32\left(\frac{\theta}{\beta_1}\right)^3\right] \\ a &= \frac{h}{T_1^2}\left[48 - 144\frac{\theta}{\beta_1} + 96\left(\frac{\theta}{\beta_1}\right)^2\right] \end{aligned}\right\} \begin{gathered} \frac{\beta_1}{2} \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{gathered}$$

$$(2.9b)$$

$$\left.\begin{aligned} y &= h\left[1 - 8\left(\frac{\theta}{\beta_2}\right)^3 + 8\left(\frac{\theta}{\beta_2}\right)^4\right] \\ v &= \frac{h}{T_2}\left[-24\left(\frac{\theta}{\beta_2}\right)^2 + 32\left(\frac{\theta}{\beta_2}\right)^3\right] \\ a &= \frac{h}{T_2^2}\left[-48\frac{\theta}{\beta_2} + 96\left(\frac{\theta}{\beta_2}\right)^2\right] \end{aligned}\right\} \begin{gathered} 0 \leqslant \theta \leqslant \frac{\beta_2}{2} \\ \text{(return)} \end{gathered} \qquad (2.9c)$$

$$\left.\begin{aligned} y &= h\left[8\frac{\theta}{\beta_2} - 24\left(\frac{\theta}{\beta_2}\right)^2 + 24\left(\frac{\theta}{\beta_2}\right)^3 - 8\left(\frac{\theta}{\beta_2}\right)^4\right] \\ v &= \frac{h}{T_2}\left[-48\frac{\theta}{\beta_2} + 72\left(\frac{\theta}{\beta_2}\right)^2 - 32\left(\frac{\theta}{\beta_2}\right)^3\right] \\ a &= \frac{h}{T_2^2}\left[-48 + 144\frac{\theta}{\beta_2} - 96\left(\frac{\theta}{\beta_2}\right)^2\right] \end{aligned}\right\} \beta_2 \leqslant \theta \leqslant \beta_2 \qquad (2.9d)$$

This curve has characteristics very similar to that of cycloidal motion; it is a simple polynomial and in Chapter 10 more advanced polynomials are discussed.

3-4-5 Polynomial (Fig. 2-10)

$$\left.\begin{aligned} y &= h\left[10\left(\frac{\theta}{\beta_1}\right)^3 - 15\left(\frac{\theta}{\beta_1}\right)^4 + 6\left(\frac{\theta}{\beta_1}\right)^5\right] \\ v &= \frac{h}{T_1}\left[30\left(\frac{\theta}{\beta_1}\right)^2 - 60\left(\frac{\theta}{\beta_1}\right)^3 + 30\left(\frac{\theta}{\beta_1}\right)^4\right] \\ a &= \frac{h}{T_1^2}\left[60\frac{\theta}{\beta_1} - 180\left(\frac{\theta}{\beta_1}\right)^2 + 120\left(\frac{\theta}{\beta_1}\right)^3\right] \end{aligned}\right\} \begin{matrix} 0 \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{matrix} \qquad (2.10a)$$

$$\left.\begin{aligned} y &= h\left[1 - 10\left(\frac{\theta}{\beta_2}\right)^3 + 15\left(\frac{\theta}{\beta_2}\right)^4 - 6\left(\frac{\theta}{\beta_2}\right)^5\right] \\ v &= \frac{h}{T_2}\left[-30\left(\frac{\theta}{\beta_2}\right)^2 + 60\left(\frac{\theta}{\beta_2}\right)^3 - 30\left(\frac{\theta}{\beta_2}\right)^4\right] \\ a &= \frac{h}{T_2^2}\left[-60\frac{\theta}{\beta_2} + 180\left(\frac{\theta}{\beta_2}\right)^2 - 120\left(\frac{\theta}{\beta_2}\right)^3\right] \end{aligned}\right\} \begin{matrix} 0 \leqslant \theta \leqslant \beta_2 \\ \text{(return)} \end{matrix} \qquad (2.10b)$$

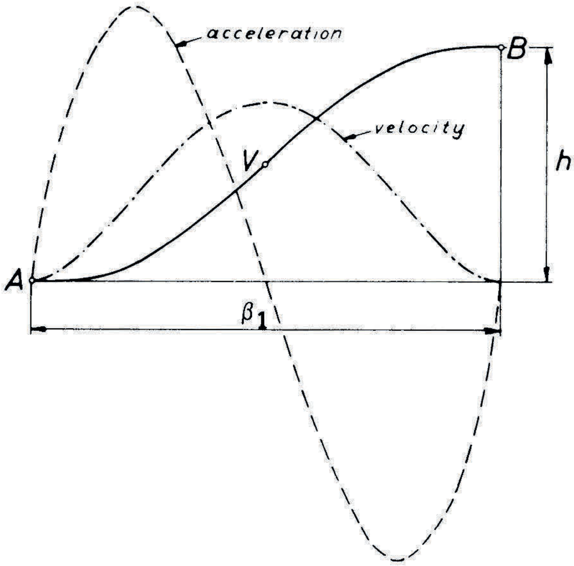


Fig. 2-10. Cam displacement, velocity, and acceleration curves for 3-4-5 polynomial motion.

This curve has good acceleration characteristics.

4-5-6-7 Polynomial (Table 2-1)

$$\left.\begin{aligned} y &= h\left[35\left(\frac{\theta}{\beta_1}\right)^4 - 84\left(\frac{\theta}{\beta_1}\right)^5 + 70\left(\frac{\theta}{\beta_1}\right)^6 - 20\left(\frac{\theta}{\beta_1}\right)^7\right] \\ v &= \frac{h}{T_1}\left[140\left(\frac{\theta}{\beta_1}\right)^3 - 420\left(\frac{\theta}{\beta_1}\right)^4 + 420\left(\frac{\theta}{\beta_1}\right)^5 - 140\left(\frac{\theta}{\beta_1}\right)^6\right] \\ a &= \frac{h}{T_1^2}\left[420\left(\frac{\theta}{\beta_1}\right)^2 - 1680\left(\frac{\theta}{\beta_1}\right)^3 + 2100\left(\frac{\theta}{\beta_1}\right)^4 - 840\left(\frac{\theta}{\beta_1}\right)^5\right] \end{aligned}\right\} \begin{gathered} 0 \leqslant \theta \leqslant \beta_1 \\ \text{(rise)} \end{gathered} \qquad (2.11a)$$

$$\left.\begin{aligned} y &= h\left[1 - 35\left(\frac{\theta}{\beta_2}\right)^4 + 84\left(\frac{\theta}{\beta_2}\right)^5 - 70\left(\frac{\theta}{\beta_2}\right)^6 + 20\left(\frac{\theta}{\beta_2}\right)^7\right] \\ v &= \frac{h}{T_2}\left[-140\left(\frac{\theta}{\beta_2}\right)^3 + 420\left(\frac{\theta}{\beta_2}\right)^4 - 420\left(\frac{\theta}{\beta_2}\right)^5 + 140\left(\frac{\theta}{\beta_2}\right)^6\right] \\ a &= \frac{h}{T_2^2}\left[-420\left(\frac{\theta}{\beta_2}\right)^2 + 1680\left(\frac{\theta}{\beta_2}\right)^3 - 2100\left(\frac{\theta}{\beta_2}\right)^4 + 840\left(\frac{\theta}{\beta_2}\right)^5\right] \end{aligned}\right\} \begin{gathered} 0 \leqslant \theta \leqslant \beta_2 \\ \text{(return)} \end{gathered} \qquad (2.11b)$$

This curve is sometimes used for high speed cams because of good acceleration characteristics.

Trapezoidal Acceleration; $C = 0.25\,\frac{\beta_1}{2}$ by rise and $0.25\,\frac{\beta_2}{2}$ by return (Table 2-1)

$$\left.\begin{aligned} y &= \frac{64}{9}h\left(\frac{\theta}{\beta_1}\right)^3 \\ v &= \frac{64}{3}\frac{h}{T_1}\left(\frac{\theta}{\beta_1}\right)^2 \\ a &= \frac{128}{3}\frac{h}{T_1^2}\frac{\theta}{\beta_1} \end{aligned}\right\} \quad 0 \leqslant \theta \leqslant \frac{\beta_1}{8} \qquad (2.12a)$$