AD 742723

AD

SWERR-TR-72-30

INVESTIGATION OF THE INTERACTION OF WEAPON-AMMUNITION SUBSYSTEMS

TECHNICAL REPORT

May 1972

RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

RESEARCH, DEVELOPMENT AND ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

DISPOSITION INSTRUCTIONS:

Destroy this report when it is no longer needed. Do not return it to the originator.

DISCLAIMER:

The findings of this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Wisconsin Dept. of Mechanical Engineering and Dept. of Statistics Madison, Wisconsin	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

INVESTIGATION OF THE INTERACTION OF WEAPON-AMMUNITION SUBSYSTEMS (U)

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)

S. M. Wu

6. REPORT DATE May 1972	7a. TOTAL NO. OF PAGES 204	7b. NO. OF REFS

8a. CONTRACT OR GRANT NO. DAAF0370C0073	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO. DA 1W562604A607	
c. AMS Code 552D.11.80700.02	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) SWERR-TR-72-30
d.	

10. DISTRIBUTION STATEMENT

Approved for public release, distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY U. S. Army Weapons Command Research & Engineering Directorate Rock Island, Illinois 61201

13. ABSTRACT

The initial phase in a systematic analysis of weapon-ammunition interaction has been accomplished ~~under the guidance of the Research Directorate, Weapons Laboratory at Rock Island~~. Acceptance-test data for five manufacturers' production of 5.56mm ammunition were analyzed through time-series modeling, an empirical cumulative distribution function was formulated, and a bivariate histogram of chamber pressure and port pressure was developed for use in the selection of weapon-test ammunition. Statistical experimental design procedures based on factorial or fractional factorial approaches are included for use in tests to identify the controlling parameters in weapon-ammunition interactions and to determine whether these parameters can be identified from ammunition acceptance-test data. Some preliminary correlation analyses are included for comparison of pressure measurements by means of crusher gages and piezoelectric gages. (U) (Wu, S. M.)

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

Unclassified

Security Classification

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Weapon-Ammunition Interaction						
2. Small Arms						
3. Ammunition						
4. Weapon Dynamics						
5. Mathematical Synthesis						
6. Statistical Analysis						

Unclassified

Security Classification

i

AD

RESEARCH DIRECTORATE

WEAPONS LABORATORY AT ROCK ISLAND

RESEARCH, DEVELOPMENT AND ENGINEERING DIRECTORATE

U. S. ARMY WEAPONS COMMAND

TECHNICAL REPORT

SWERR-TR-72-30

INVESTIGATION OF THE INTERACTION OF WEAPON-AMMUNITION SUBSYSTEMS

May 1972

DA 1W562604A607 AMS Code 552D.11.80700.02

ABSTRACT

The initial phase in a systematic analysis of weapon-ammunition interaction has been accomplished under the guidance of the Research Directorate, Weapons Laboratory at Rock Island. Acceptance-test data for five manufacturers' production of 5.56mm ammunition were analyzed through time-series modeling, an empirical cumulative distribution function was formulated, and a bivariate histogram of chamber pressure and port pressure was developed for use in the selection of weapon-test ammunition.

Statistical experimental design procedures based on factorial or fractional factorial approaches are included for use in tests to identify the controlling parameters in weapon-ammunition interactions and to determine whether these parameters can be identified from ammunition acceptance-test data. Some preliminary correlation analyses are included for comparison of pressure measurements by means of crusher gages and piezoelectric gages.

FOREWORD

This report was prepared under Contract DAAF37060073 by Professor S. M. Wu, Department of Mechanical Engineering and Department of Statistics, University of Wisconsin, Madison, under guidance of the Research Directorate, Weapons Laboratory at Rock Island, with W. L. Dahl as Project Scientist.

The U. S. Army Small Arms Systems Agency supported the work as part of the Army Small Arms Program task entitled "Define Weapon Factors in the Broad Spectrum of Ammunition."

The testing of developmental weapons has often been limited to the firing of available quantities of ammunition for which little information may be available regarding either the specific characteristics of the test ammunition or its relationship within the spectrum of production ammunition.

Definition of the ammunition characteristics that are important in the operation of differing gun mechanisms and identification of the ranges of the variables will permit more meaningful tests of the weapons. These tests will, in turn, enable the establishment of more realistic boundary conditions for analysis by parameter variation with computerized mathematical models. Such improved analytical techniques will provide better predictions of weapon-ammunition interaction and better estimates of system reliability during the earliest stages of the development process.

Results of the first phase in an analysis of 5.56mm weapon-ammunition interaction are described in this report. This portion of the work has been focused on the development of methods for the identification of groups of sequentially manufactured ammunition-production-lots that apparently fall within single distributions of statistically normal production. The unique control limits, associated with these distributions, are being explored as a basis for the selection of gun-test ammunition.

In addition, some statistical experimental design techniques are outlined for use in the next phase of the work. The forthcoming effort will determine (through coordinated tests at Frankford Arsenal and Rock Island) whether the ammunition acceptance-test crusher-pressure data on which this report is based are sufficiently difinitive for determination of the "gun-powering" characteristics of the ammunition. Toward the latter objective, some preliminary correlation analyses of pressure data acquired by means of piezoelectric and crusher gages are included in this report.

One of the appendices contains a description of a preliminary mathematical model, "Empirical=Mechanistic Model for Interior Ballistics of Guns," formulated through cooperation between the University of Wisconsin and the Badger Army Ammunition Plant. Although that model was not developed under the Research Directorate contract, it was contributed by the University as a possible means by which the technology may be advanced.

CONTENTS

ACKNOWLEDGEMENTS

We are grateful to Mr. W. L. Dahl and Mr. R. Coberly of the Research Directorate, Weapons Laboratory at Rock Island for consultation and guidance during the course of this project.

We wish also to acknowledge the help of Mr. E. G. Johnson of Badger Army Ammunition Plant at Baraboo, Wisconsin, and Mr. J. Ramnarance of Olin Corporation for providing us with ammunition data and for familiarizing us with the process of production and testing of Ball Powder.

1. INTRODUCTION

Ammunition and weapon need to be compatible. Ammunition tests are performed to ascertain that the ballistic characteristics of ammunition are consistent with the gun design requirements. The tests are useful only if the responses provide meaningful information toward this end. The present methods of testing, namely copper crusher and piezo methods, need to be compared to determine the extent of useful information obtained.

Copper crusher forms the present method of acceptance testing. The control limits used for acceptance are based upon specifications which may or may not represent the real capabilities of the present production process. Additionally, the control limits may be only indirectly related to the functional requirements of automatic weapons. It is therefore possible, that unnecessarily severe requirements for versatility might have been imposed on the weapon. Analysis of acceptance testing data is required to determine more realistic control limits.

Just as ammunition tests are performed to evaluate ammunition characteristics, similarly weapon tests are needed to evaluate weapon performance, which must lie within specified limits. Since the ammunition lots are not identical in their ballistic properties, a criterion has to be established to select ammunition lots for weapon testing.

The first step toward the establishment of such a criterion is to determine the ammunition characteristics that influence weapon performance.

The capability of ammunition manufacturing process can then be analyzed in terms of these characteristics to evaluate the differences between ammunition lots. Lots that would give a large variation in weapon performance would be selected for weapon testing.

Related to the analysis of ammunition manufacturing process and weapon testing are the questions of determining proper number of tests to be conducted at each stage of data generation and detection of changes in ammunition characteristics at different stages of manufacture. The latter would help explain the final ballistic characteristics attained by the ammunition. It is felt that sufficient attention has not yet been give to these questions, satisfactory solutions of which are likely to lead to considerable savings to the government.

It is felt that a theoretical understanding of the internal ballistic mechanism involved would be very useful in the interpretation of the results obtained.

SUMMARY

The analyses in this report are based upon acceptance test data for 5.56 mm. Ball M-193 and 5.56 mm. Tracer M-196 ammunition from five manufacturers (Lake City, Twin City, Remington, Federal and Winchester) covering a period from July 1968 to March 1971. Additionally, special ammunition test data from Frankford and ammunition data from B.A.A.P. have been analyzed. Acceptance test data from B.A.A.P. covers a production period of the past three years. Copper crusher and piezo data for the same propellant lot are also available.

The purposes of these analyses are many fold. They include a comparison of copper crusher and piezo methods of testing, the determination of more realistic control limits for ammunition production and selection of ammunition lots for weapon testing. Chapter 1 brings out the importance of such analyses.

Piezo transducer gives more information regarding pressures inside the barrel. Maximum pressure at the gage location as well as ignition delay and slope can be determined. These are indicators of propellant characteristics. Crusher deformation can be considered as a weighted integral of piezo pressure-time curve and could be a good estimate of impact energy. Analysis in Chapter 2 points out that copper crusher indicates similar pressure-velocity relationship as piezo does (Figs. 7 and 10), but it has larger variability.

Coating date against copper crusher chamber pressure plot does not show any pattern (Fig. 8), whereas plot of piezo peak chamber pressure versus coating date reveals a time trend (Fig. 11). The time trend would be lost if only copper crusher data are examined.

Chapter 3 contains a detailed analysis of standard test data. Chamber pressure data from standard tests are found to exhibit marked trends. Three methods (semi averages, cumulative sum and moving averages) have been employed to estimate the underlying process behavior. In particular, the point of shift after which the trend is less predominant is determined. The method of cumulative sum is found to give the best visual indication of the point of shift.

The chamber pressure data are found to be nonstationary (Lake City data) even after the point of shift. The data are also found to be autocorrelated. Therefore, time series models have been obtained to explain the nature of correlations. Analysis of chamber pressure data from different manufacturers shows the mean chamber pressures to be quite close to each other. The chamber pressure variance is found to vary considerably (118×10^4 psi^2 for Remington to 629×10^4 psi^2 for Winchester) from manufacturer to manufacturer. The mean chamber pressure of Ball ammunition (46600 psi) is lower than that of Tracer ammunition (49200 psi).

Analysis of chamber pressure variance shows a larger value of within lot variance (29×10^5 psi^2) for ammunition from Twin City as compared to (17.6×10^5 psi^2) ammunition from Lake City. In both cases the trend is towards a reduction in variance, indicating continued improvement in production and testing processes.

In Chapter 4, a method based upon empirical cumulative distribution function has been developed to obtain control limits for ammunition production processes. The control limits are based upon 99 percentile point of the empirical cumulative distribution function. To narrow the control limits, data after 'cutoff date' alone have been considered. The conditions under which cutoff date can be taken as the date corresponding to the point of shift, are given in Section 4.2. Cutoff date and control limits have been calculated for Ball and Tracer chamber pressure data from the five manufacturers. On the average the control limits have been reduced by 2000 psi from the existing ones.

In Chapter 5, a criterion has been developed for selecting ammunition lots for weapon testing. It is based upon the bivariate histogram of chamber pressure and port pressure from standard tests. The inadequacy of basing the selection on chamber pressure or port pressure alone has been discussed. The selected lots have been classified into High, Medium and Low. Medium lots are based upon the mode and the High and the Low lots are based upon the approximate 90% confidence limits of the bivariate histogram. An experimental procedure, using fractional factorial or factorial designs, has been suggested to determine ammunition parameters that control weapon performance.

Chapter 6 contains a suggested procedure to determine the number of tests necessary to estimate ammunition parameters. The procedure is based upon the desired precision of estimates and the experimental error involved. The sequential method for estimation of experimental error shows that twenty readings do not give a proper estimate of experimental error in copper crusher testing and ten readings are not sufficient to obtain a good estimate of experimental error in piezo testing. The current practice in standard testing is to obtain an estimate of standard deviation based upon 20 tests. The implication of this analysis is that the estimate so obtained is likely to be modified considerably if it is based on sufficiently large number of tests.

Chapter 7 deals with the analysis of data at different stages of manufacture. Analysis of propellant lot characteristics from B.A.A.P. indicates the lot characteristics (charge weight and chamber pressure) to be serially correlated. Comparison of acceptance test results, for the same propellant lots from B.A.A.P., Lake City, and Twin City show the test results to be almost identical, indicating no change in propellant properties during the time period between the tests.

During the course of the project, need was felt for a model for the interior ballistic system of guns. Appendix I contains an initial attempt toward building an empirical-mechanistic model, using a Lagrangean formulation of the hydrodynamic system. Piezo pressure-time curve and velocity have been used as responses. The model is shown capable of iterative improvements. Several possible uses for the model have been outlined.

The data used in different analysis in this report are given in Appendices II, III and IV. Computer programs necessary for the analyses are included in Appendix V.

2. COMPARISON OF COPPER CRUSHER AND PIEZO METHODS OF TESTING

Copper crusher and piezo methods are currently used to conduct ammunition tests. The crusher gage has been in vogue for almost a century and is in use even today as the sole standard method of pressure measurement. The piezo gage, even though known for a considerable period of time, is still not adopted as a standard method of pressure measurement. As a result, relatively small amounts of piezo data is available compared to the large amount of copper crusher data accumulated over past years. In this section, the two methods are compared regarding their relative usefulness.

2.1. Purpose of Testing

Purpose of testing is two fold: as a means of acceptance testing and as a tool for process control. Comparison is primarily based upon the former function of the testing method. Where an ammunition lot is tested for acceptance, it is necessary to determine whether the powder can impart desired velocity to the bullet and whether the gun can withstand the pressures generated.

2.2 Description of Piezo Responses

Piezo response is a continuous pressure-time curve for the particular section along the barrel (usually mid chamber position for chamber pressure measurement) where it is located. One typical curve is shown in Fig. 1. There is an initial portion of 'ignition delay' during which the burning rate is small. Next there is a rapid rate of rise of pressure due to increased burning rate. The expansion of gases behind the bullet has a tendency to reduce the pressure. Eventually the powder burns off and as a result of these interacting causes, the pressure reduces. The pressure curve, therefore, exhibits an unimodal maximum.

Piezo transducer has a time constant of the order of 10^{-9} seconds and is quite widely used to obtain the pressure-time history.

2.3 Copper Crusher Response and its Relationship with Piezo Response

In standard acceptance testing for measurement of chamber pressure, the pressure inside the gun chamber is transmitted to the copper cylinder through a rigid steel piston. The pressure acting on the copper cylinder is the same as in Fig. 1. If the elastic and plastic behavior of the copper cylinder is known, the crusher deformation can be expressed as a weighted integral of the piezo pressure-time curve.

Piezo Response

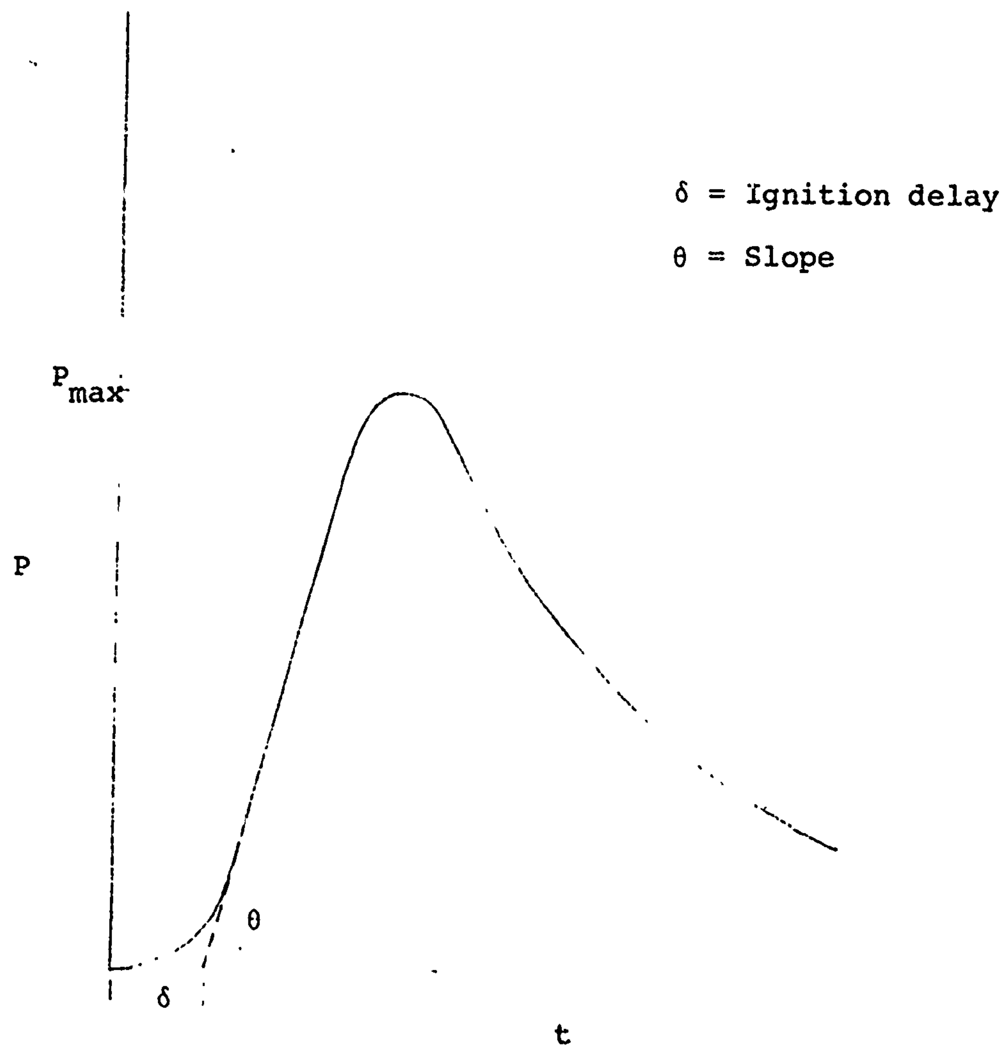


Figure 1

2.4 Methods of Choosing the Acceptance Test Criteria

Gun body may fail if the burning conditions are severe. One way to estimate the limiting condition is by means of the maximum stress produced in the gun walls which is a function of the maximum pressure developed. The peak piezo chamber pressure is perhaps a suitable indicator for this type of failure. Another possibility is impact failure, which appears more realistic for the present situation. Deformation of copper cylinder is one of the ways of measuring impact energy. However, the best indicator of impact energy can only be determined by a detailed solid mechanical analysis of the gun body.

2.5 Correlation Analysis

Correlation analysis was conducted to determine the type of information that can be obtained from the data. Three major sets of data were used, namely, standard acceptance test data, special ammunition test data supplied by Badger Army Ammunition Plant (B.A.A.P.). These are listed in Appendix II.

2.5.1 Standard Acceptance Test Data

Pertinent results in this category are summarized below:

(1) Chamber pressure has no correlation with velocity from velocity barrel. (Fig. 2)

(2) Chamber pressure and port pressure have a slight negative correlation (Fig. 3).

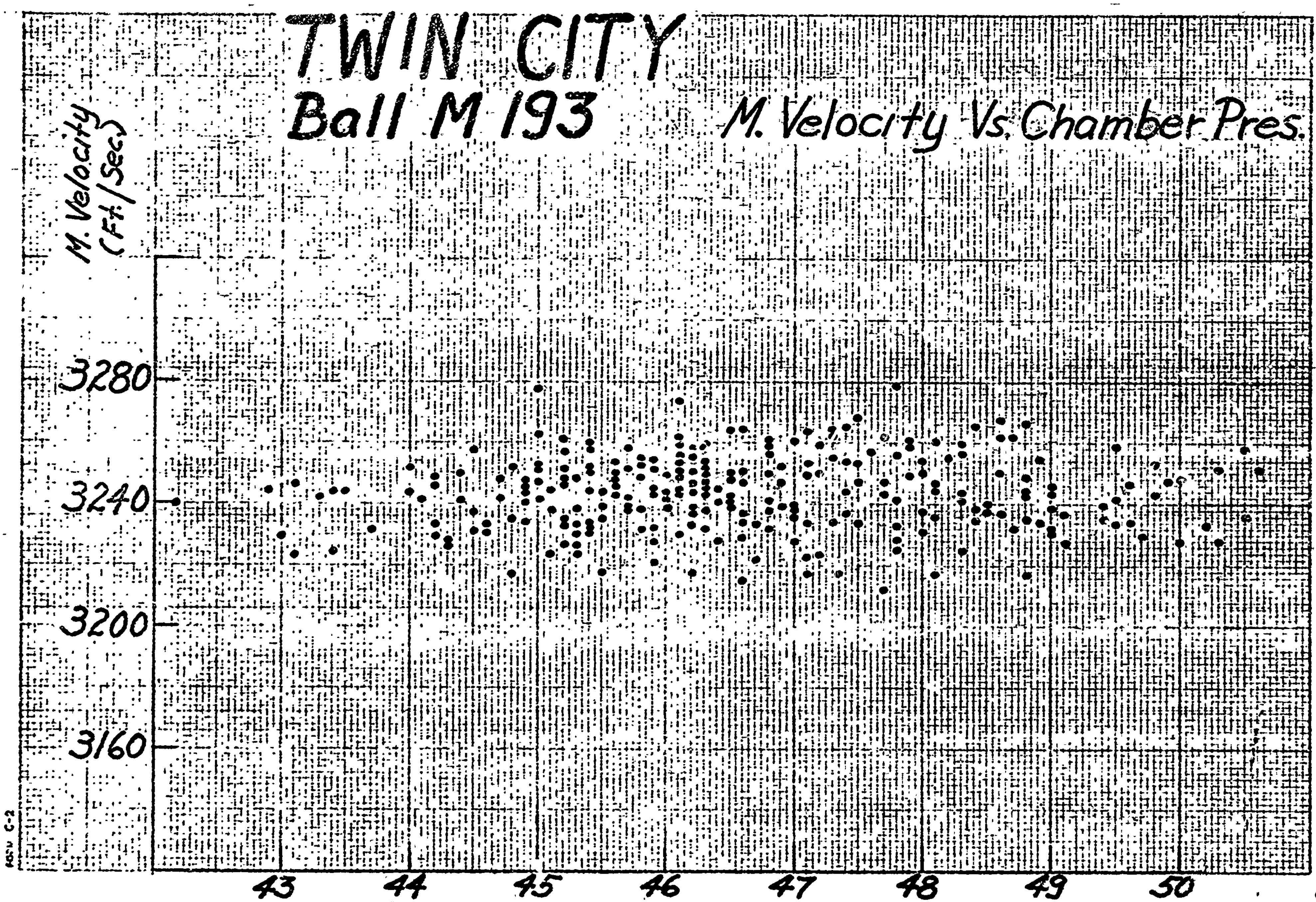


FIGURE 2 Chamber Pressure (1000 P.S.I.)

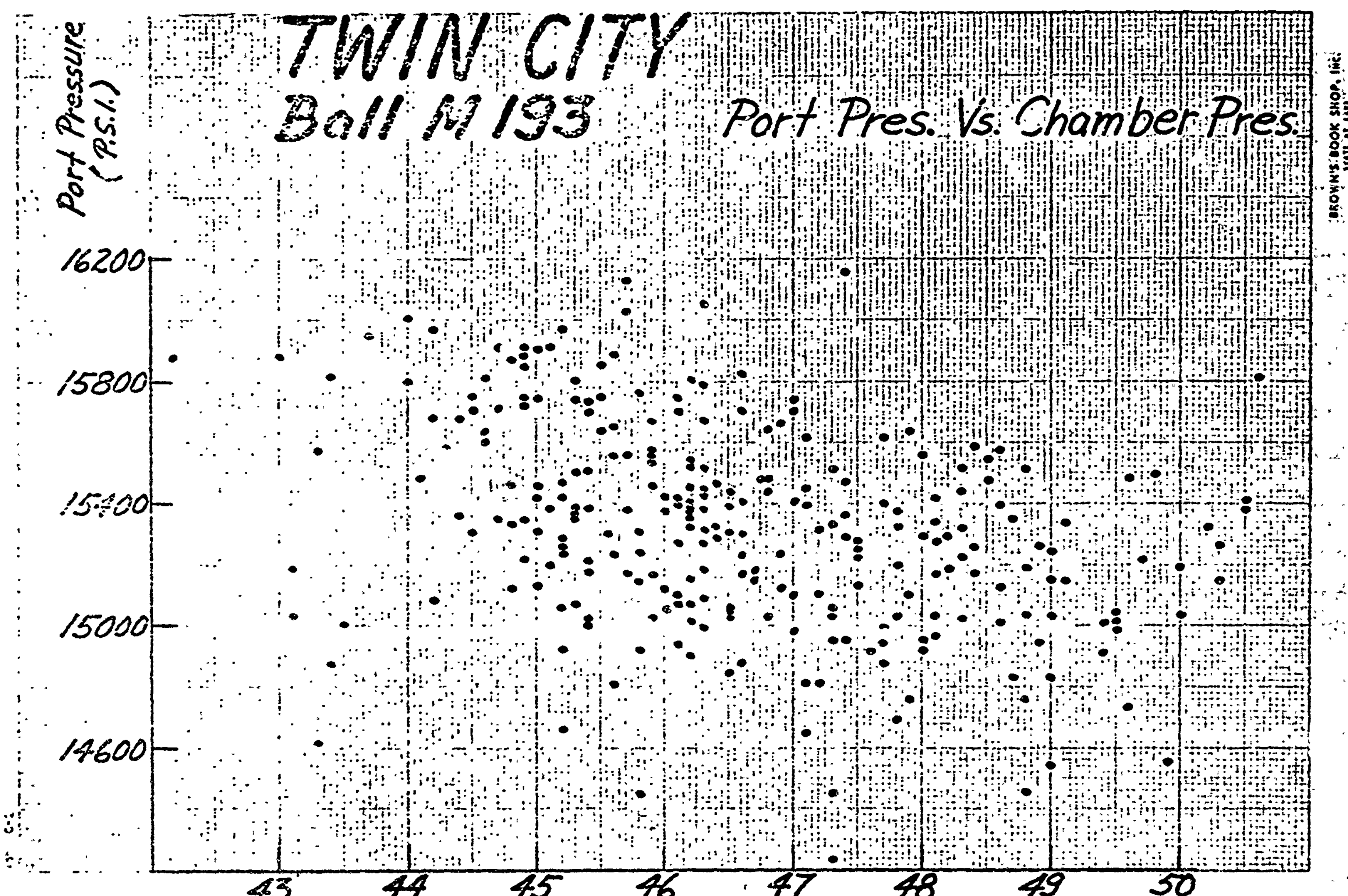

TWIN CITY
Ball M 193
Port Pres. Vs. Chamber Pres.
Port Pressure (P.S.I.)
16200
15800
15400
15000
14600
43
44
45
46
47
48
49
50
Chamber Pressure (1000 P.S.I.)
BROWN'S BOOK SHOP, INC.

FIGURE 3

2.5.2 Special Ammunition Test Data

Twenty pieces of special ammunition test data were obtained from Frankford Arsenal. Limited information available is as follows:

(1) There is a positive correlation between peak chamber pressure and velocity. (Fig. 4)

(2) Peak chamber pressure is uncorrelated with peak port pressure. (Fig. 5)

(3) Pressure-time integral is uncorrelated with velocity. (Figs. 6(a), 6(b))

2.5.3 B.A.A.P. Data

Copper crusher and piezo transducer data for composites and hand-blends were supplied by B.A.A.P. Results are summarized below:

(a) Copper Crusher Method

(1) Chamber pressure and velocity from pressure barrel are correlated. (Fig. 7)

(2) Plot of coating date against chamber pressure reveals no trend. (Fig. 8)

(3) Plot of coating date against velocity from pressure barrel has a slight trend. (Fig. 9)

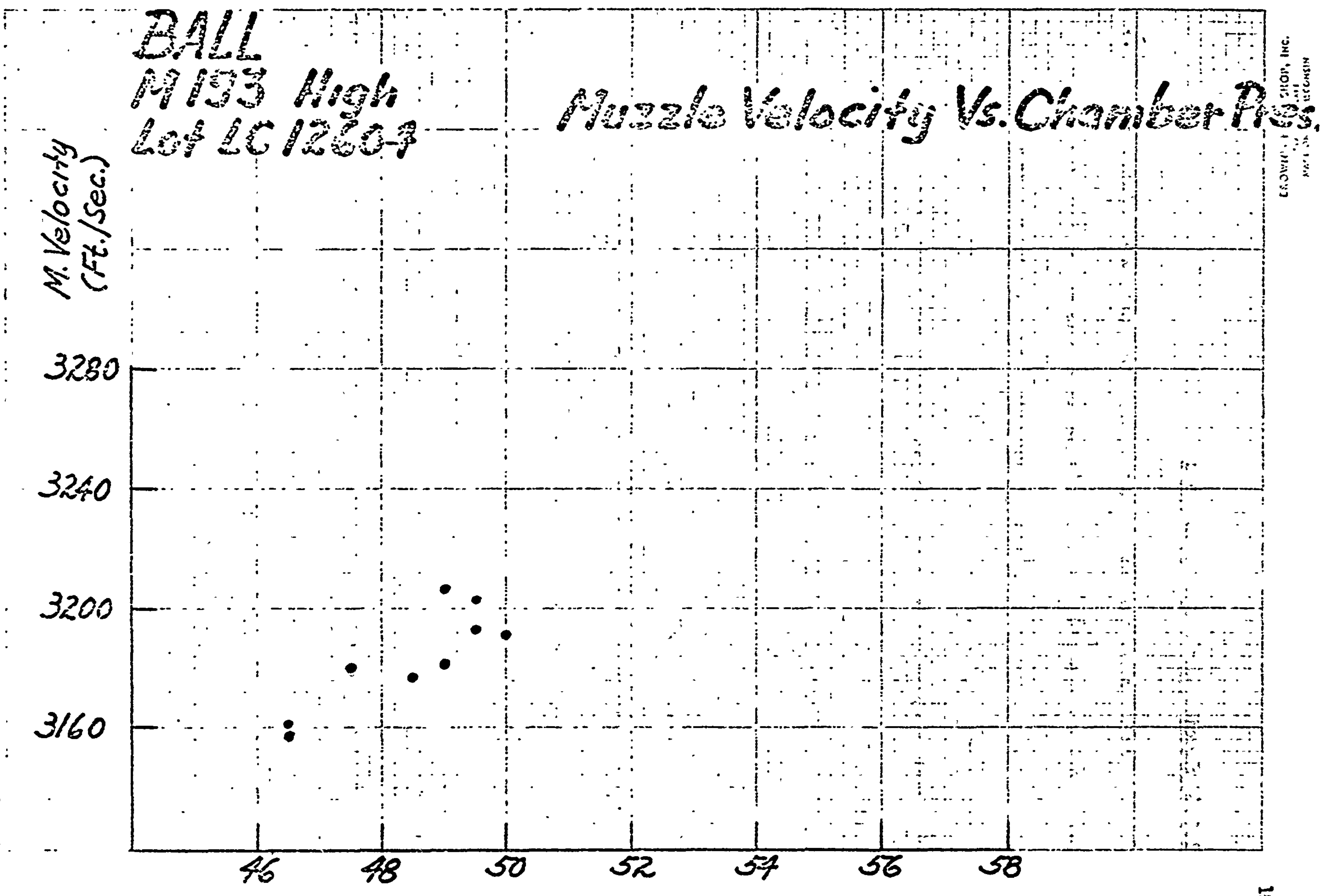

BALL
M193 High
Lot LC 12607
Muzzle Velocity Vs. Chamber Pres.
M. Velocity (Ft./Sec.)
3280
3240
3200
3160
46
48
50
52
54
56
58
Chamber Pressure (1000 P.S.I.)

FIGURE 4

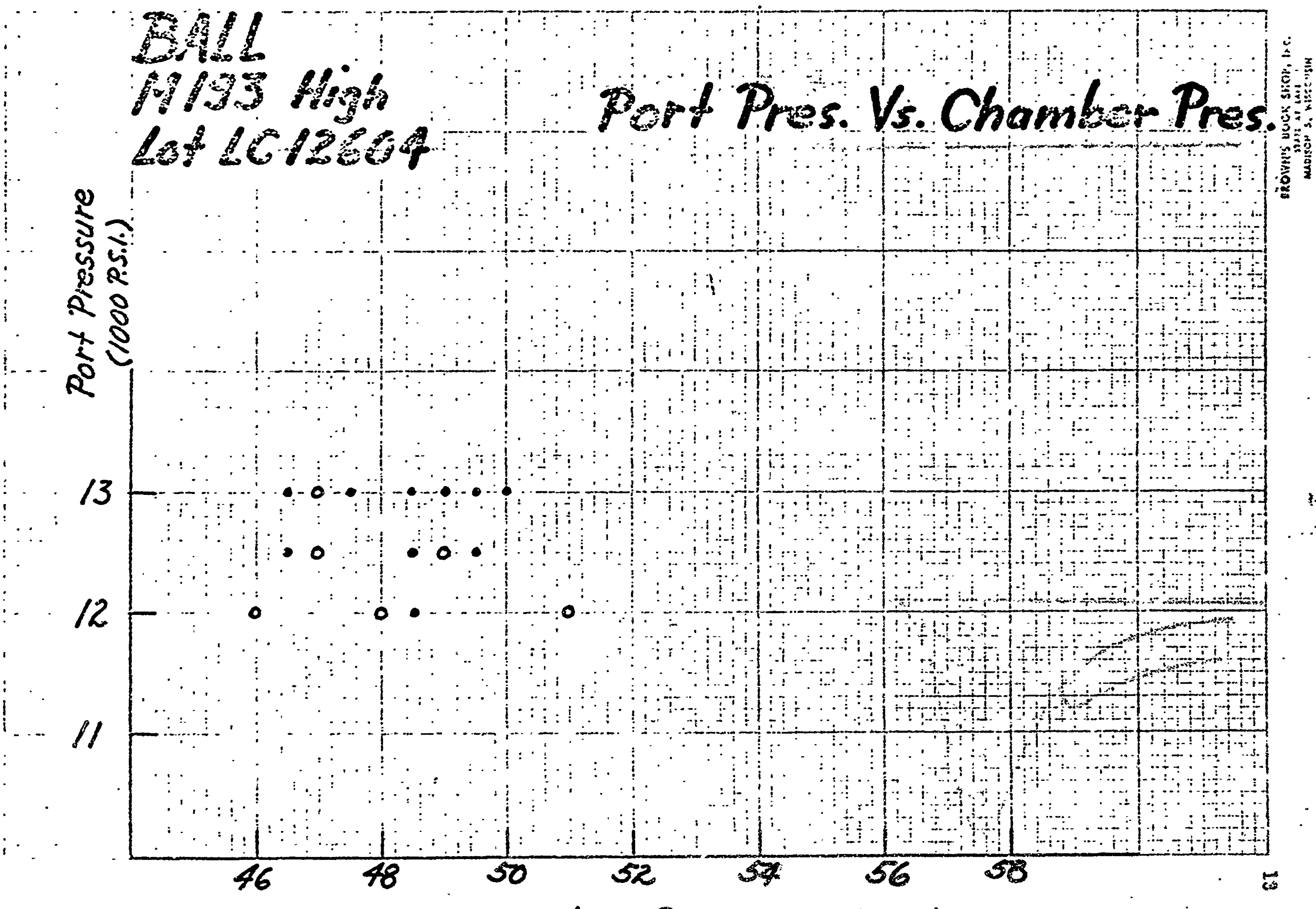

BALL
M193 High
Lot LC12604
Port Pres. Vs. Chamber Pres.
Port Pressure (1000 P.S.I.)
13
12
11
46
48
50
52
54
56
58
Chamber Pressure (1000 P.S.I.)
BROWN'S BOOK SHOP, INC.
STATE AT LAKE
MADISON 5, WISCONSIN

FIGURE 5

Figure 6(A) PRESSURE-TIME INTEGRAL VS. VELOCITY

Special Ammunition Test -- Frankford

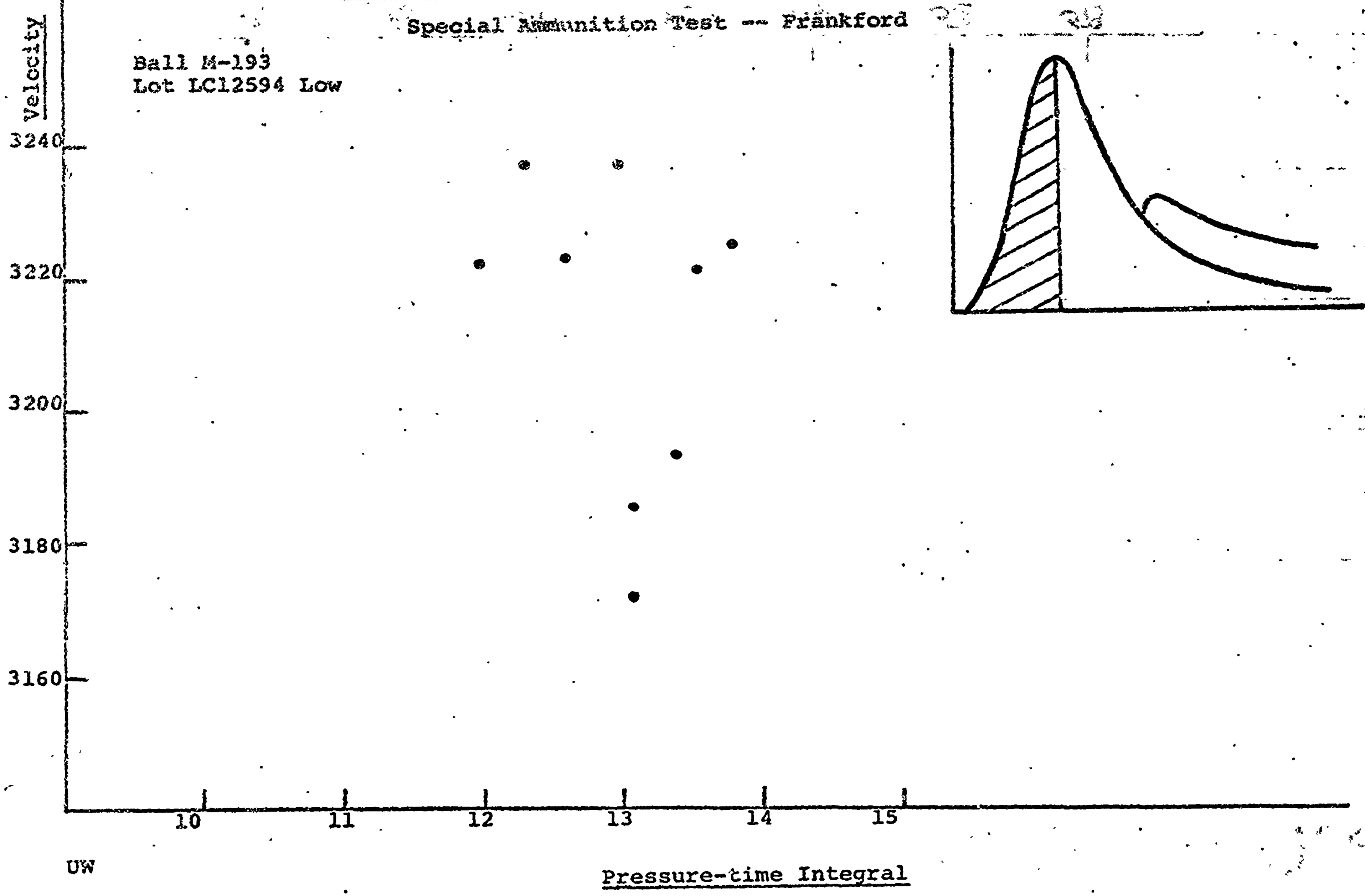


Figure 6(B) PRESSURE-TIME INTEGRAL VS. VELOCITY

Special Ammunition Test -- Frankford

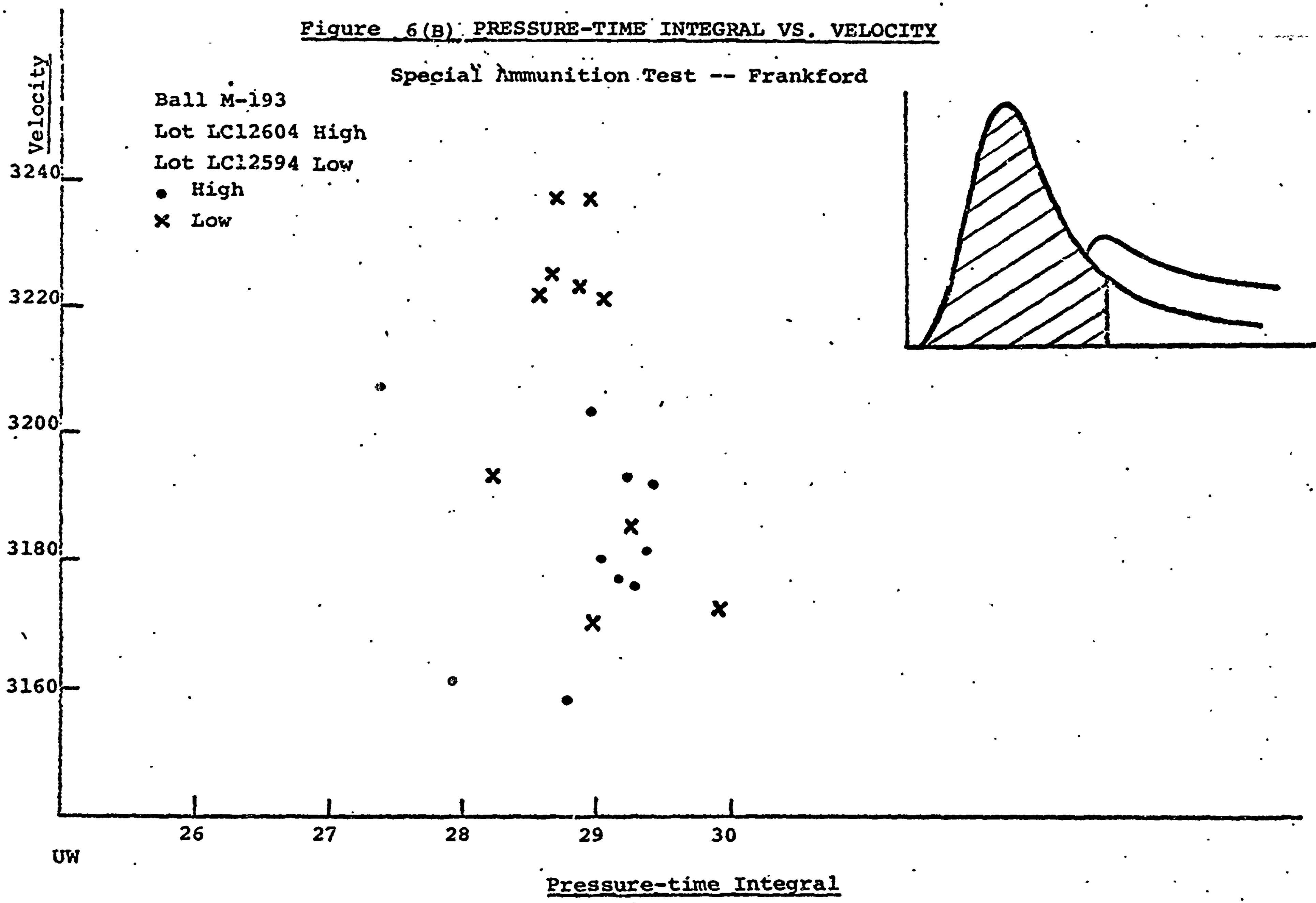


Figure 7 CHAMBER PRESSURE VS. VELOCITY (PRESSURE BARREL) AT 70°F (COPPER CRUSHER-BADGER)

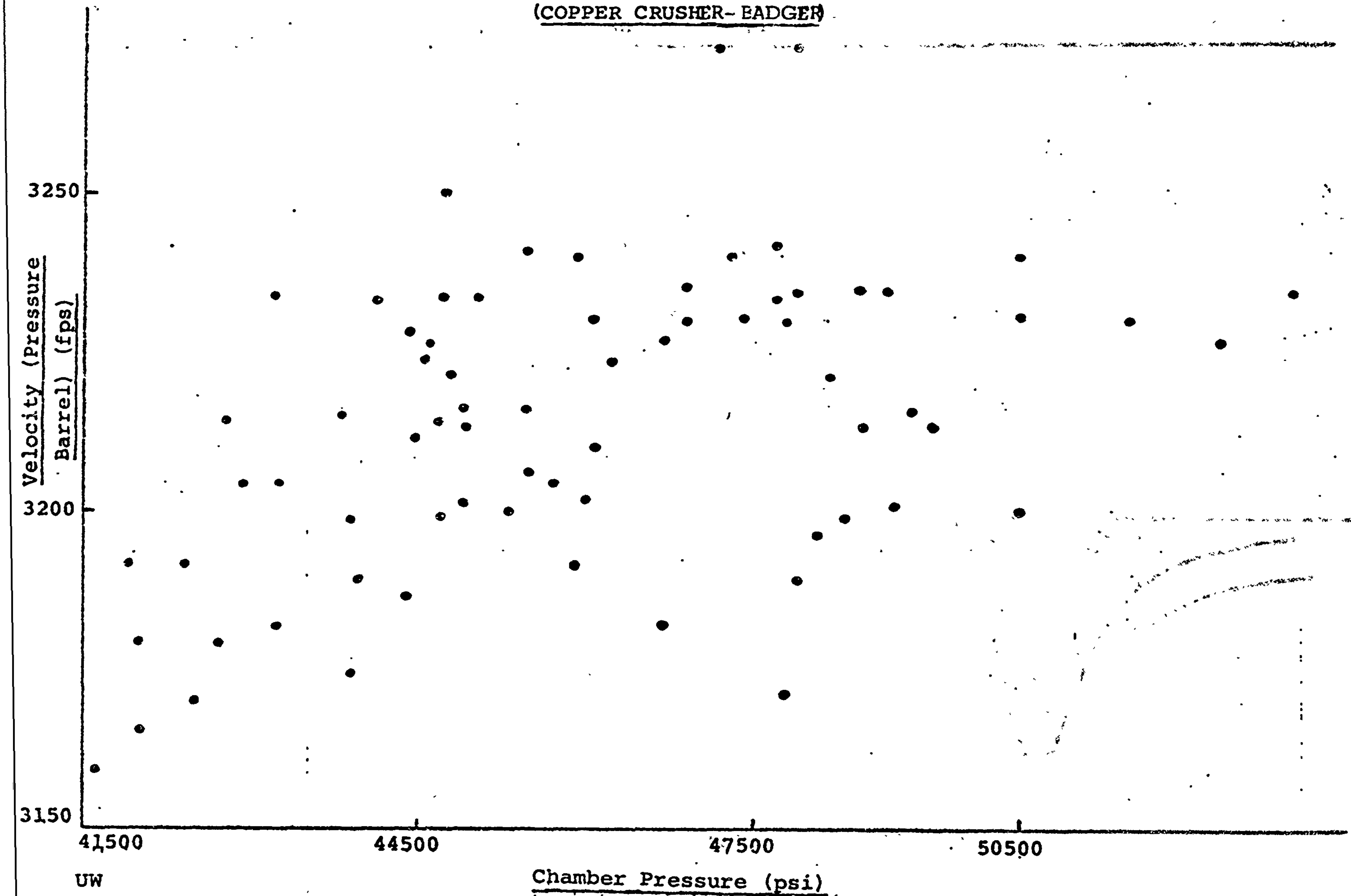
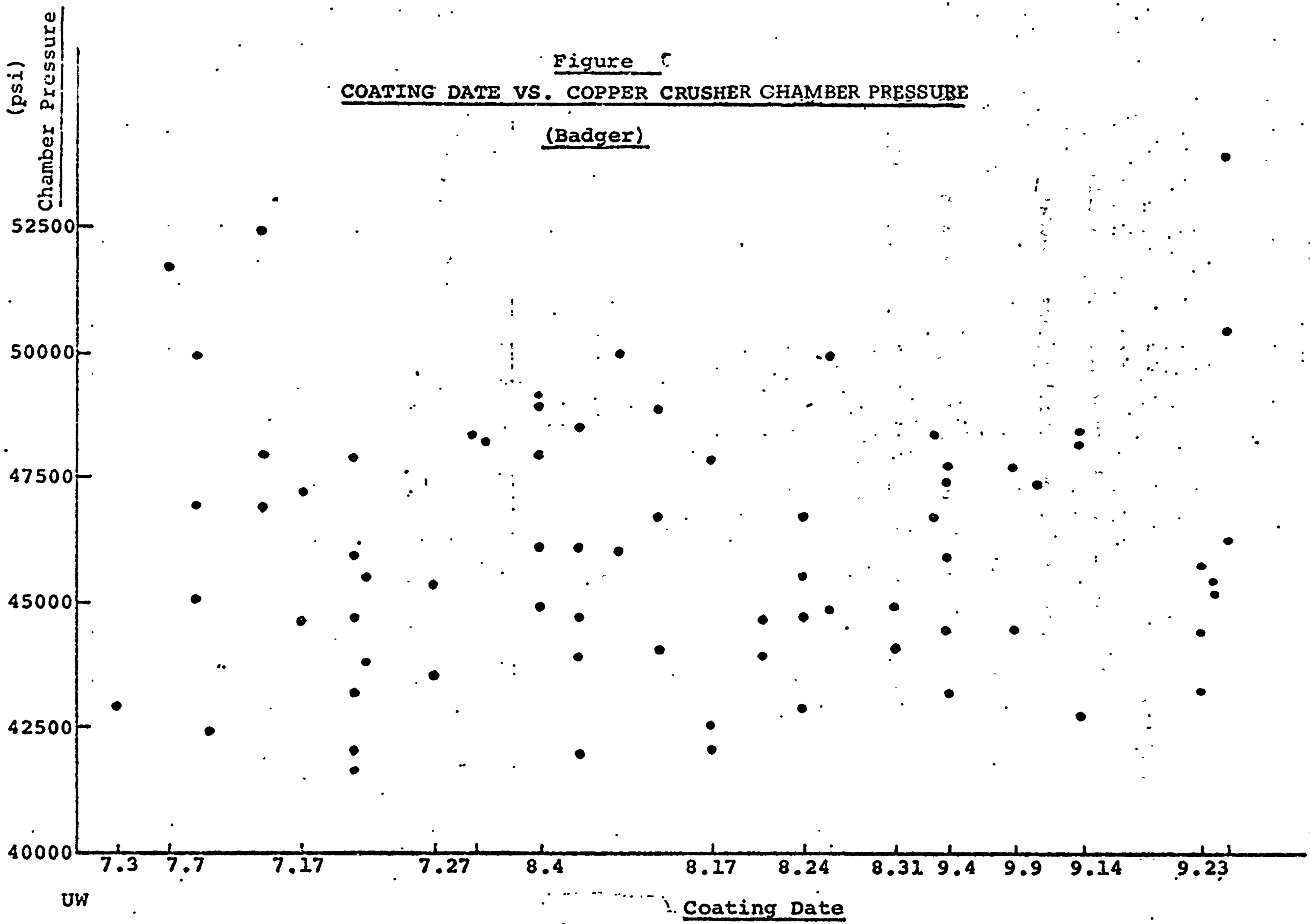


Figure C

COATING DATE VS. COPPER CRUSHER CHAMBER PRESSURE

(Badger)



(b) Piezo Method

(1) Peak chamber pressure and velocity are correlated. (Fig. 10)

(2) Plot of coating date against peak chamber pressure reveals a time trend. (Fig. 11)

(3) Slope of pressure-time curve is negatively correlated with ignition delay (Fig. 12) and is positively correlated with velocity (Fig. 13) and peak pressure. (Fig. 14)

Similar correlative structure is observed in the case of Hand Blends.

2.6 Evaluation of Copper Crusher

Copper crusher has been found to be a reliable method of comparitive measurement. It is possible that the crusher deformation is a good estimate of impact energy. From Figures 7 and 10 it can be seen that the crusher indicates the same trend as piezo does, but indicates a larger variability. Coating date against copper crusher chamber pressure plot does not show any pattern, whereas plot of coating date versus piezo peak chamber pressure reveals a time trend. This time trend could be a significant factor in production process control and would be lost if only copper crusher data are examined.

Figure 9

COATING DATE VS. COPPER CRUSHER VELOCITY (PRESSURE-BARREL)

(Badger)

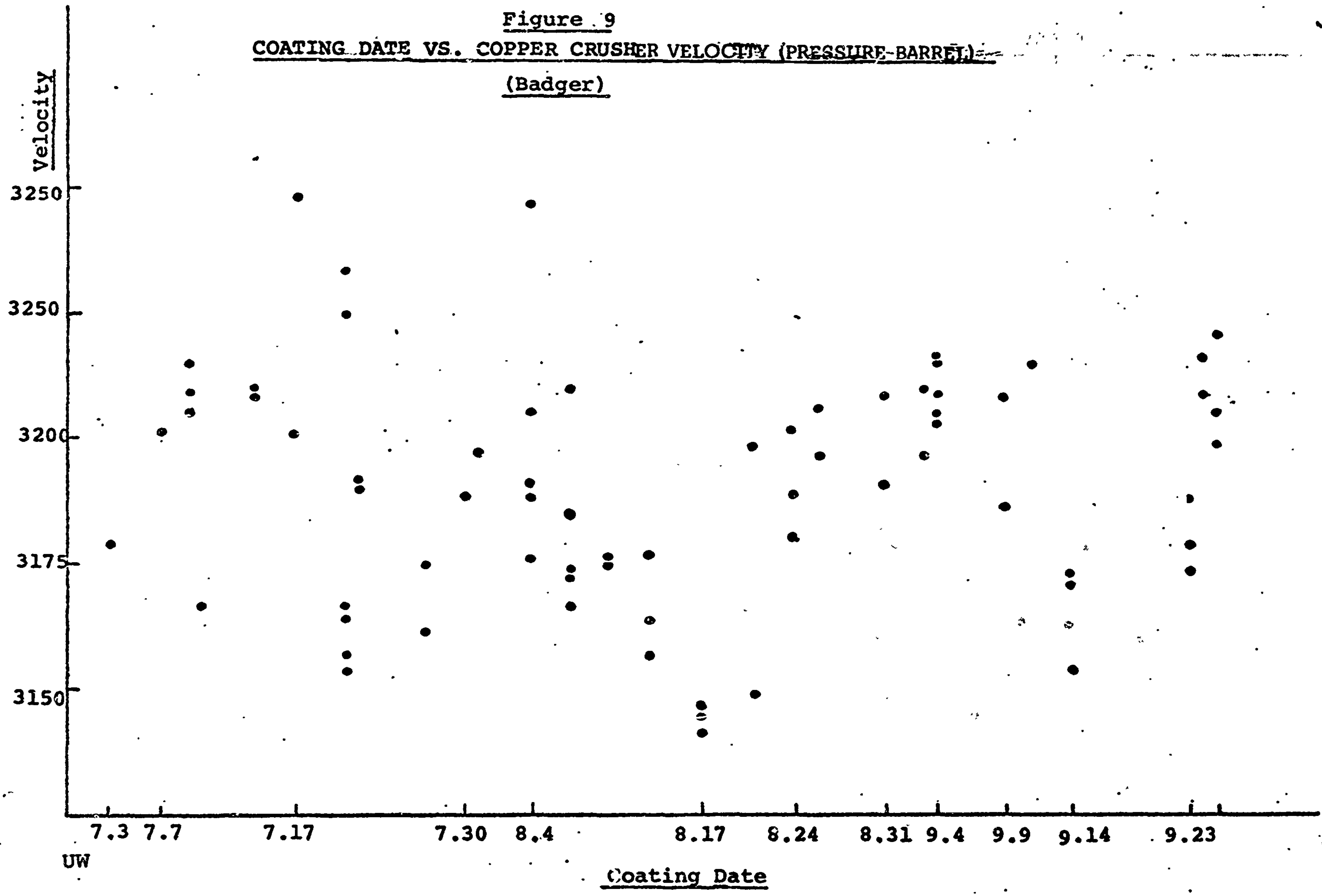


Figure 10

PEAK CHAMBER PRESSURE VS. VELOCITY AT 70°F

(Piezo Transducer - Badger)

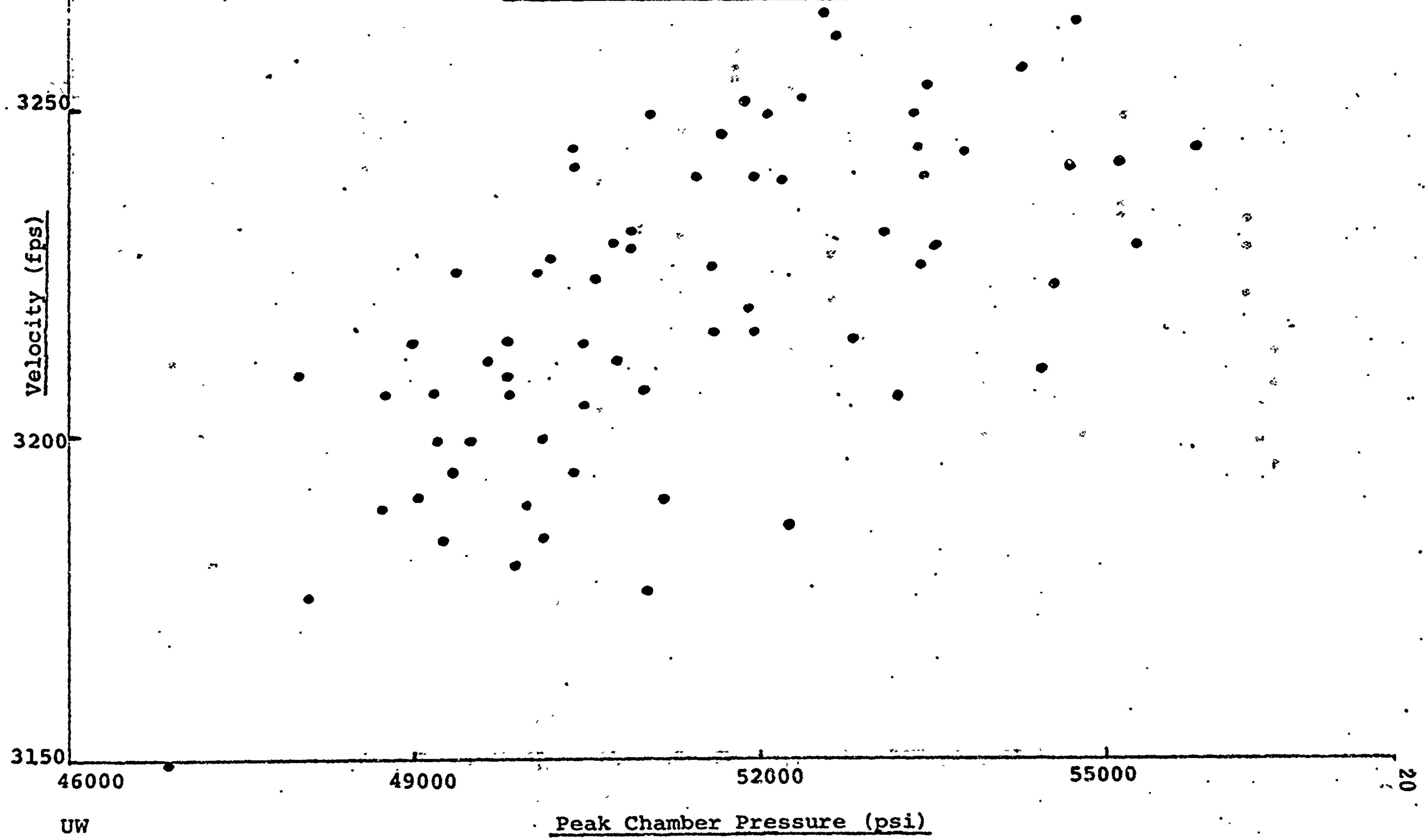
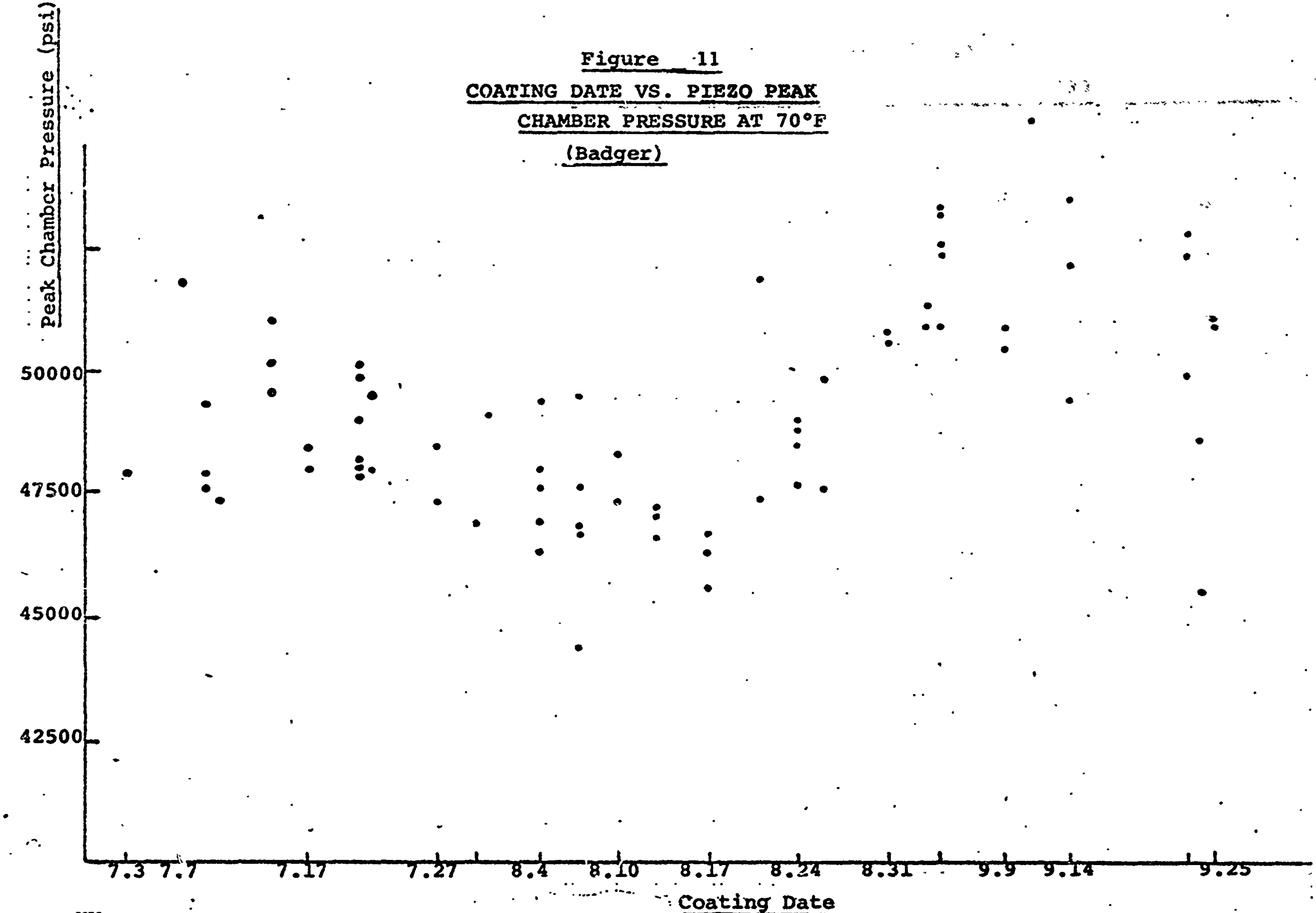


Figure 11

COATING DATE VS. PIEZO PEAK CHAMBER PRESSURE AT 70°F

(Badger)



UW

Figure 12

IGNITION DELAY VS. SLOPE AT 70°F.

(Piezo Transducer- Badger)

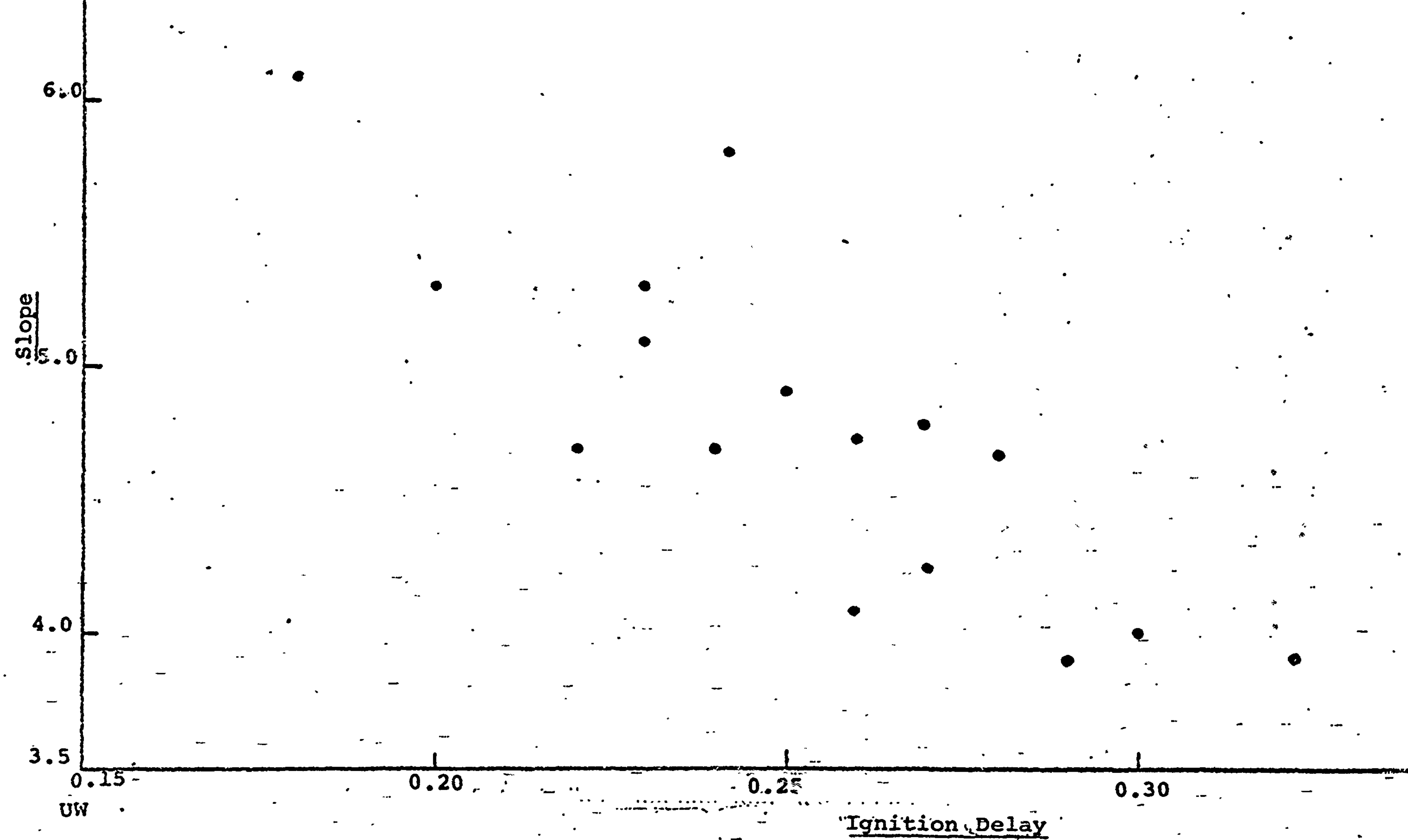


Figure . 13

VELOCITY VS. SLOPE AT 70°F

(Piezo Transducer- Badger)

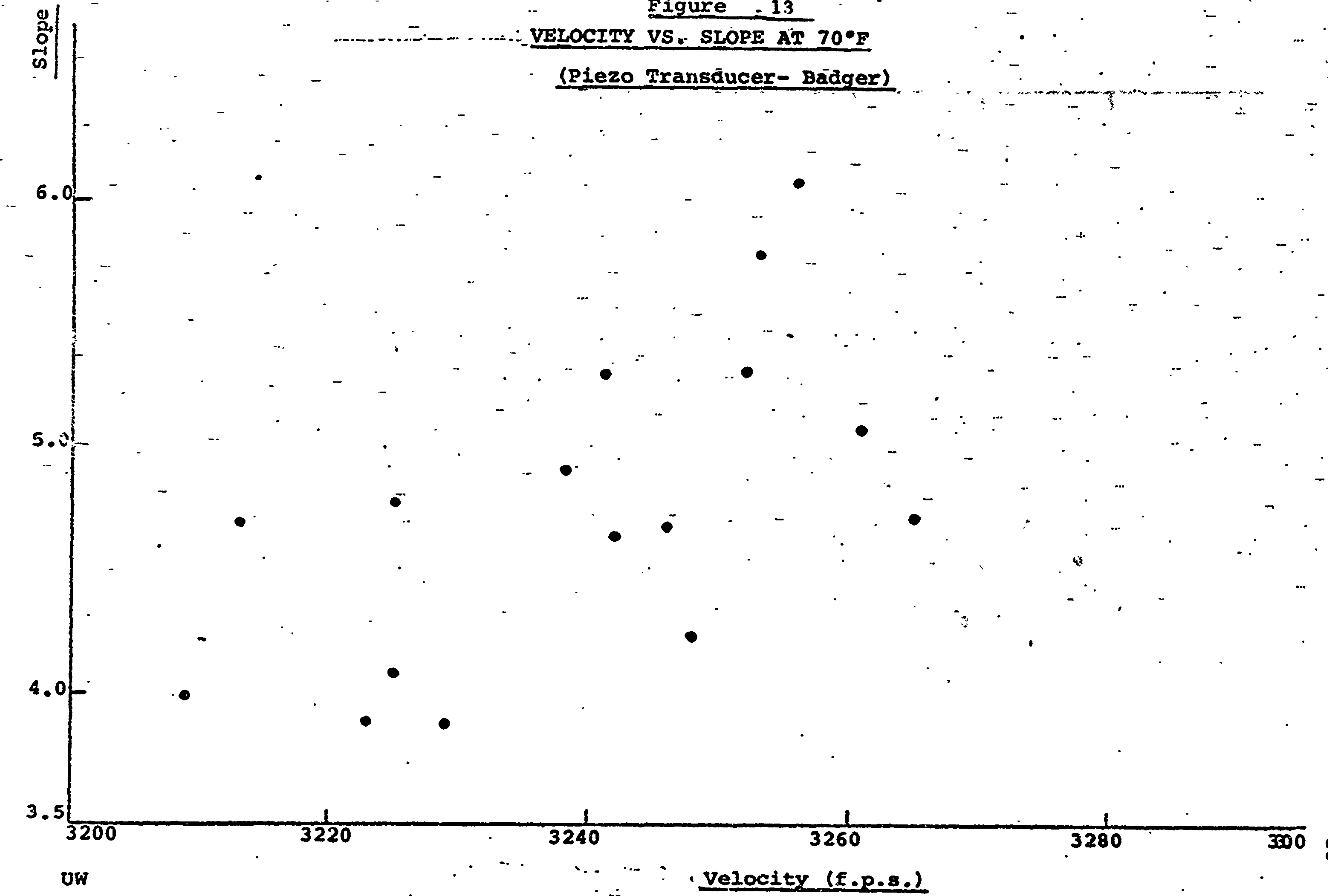
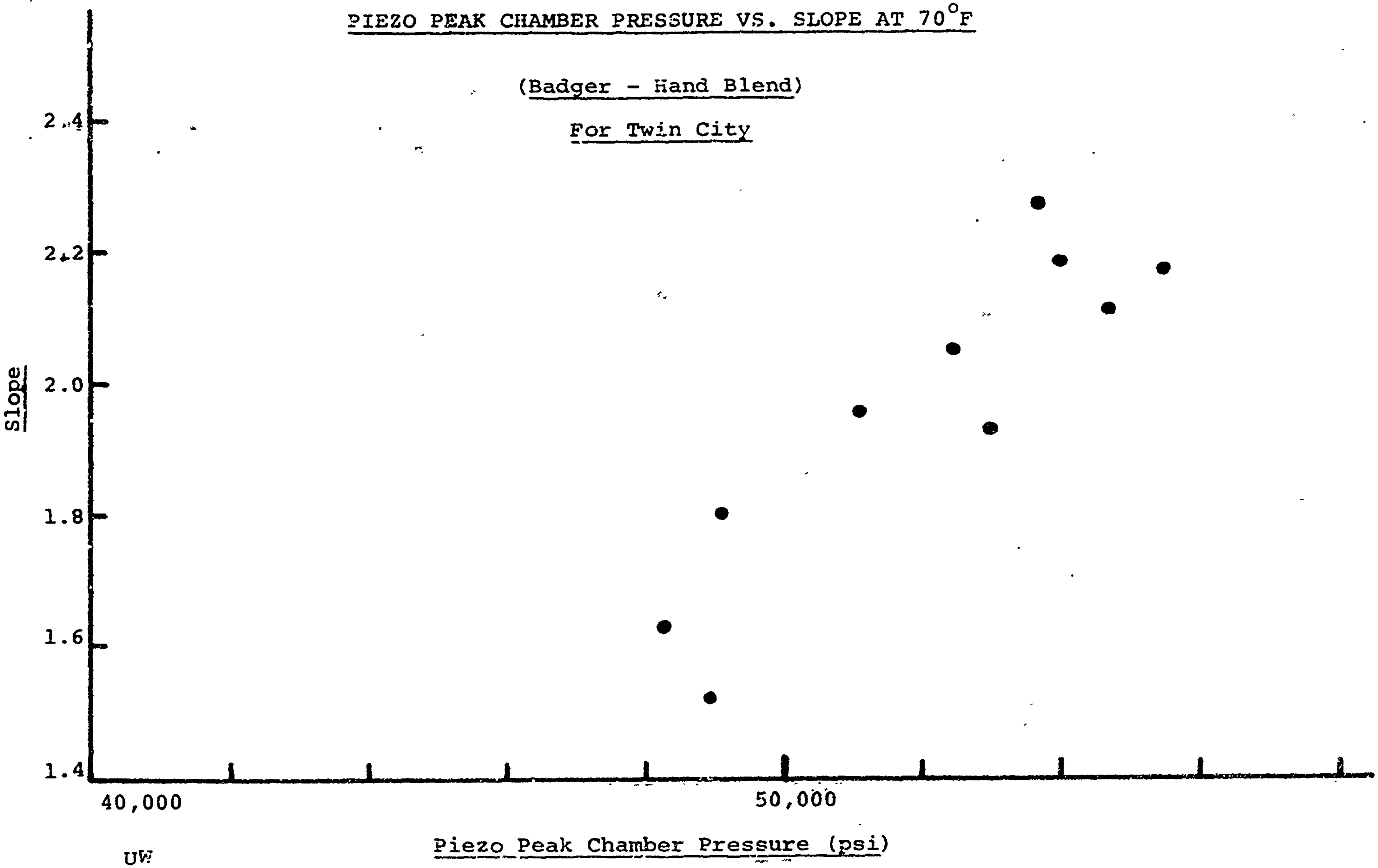


Figure 14

PIEZO PEAK CHAMBER PRESSURE VS. SLOPE AT 70°F

(Badger - Hand Blend)

For Twin City



UW

2.7 Additional Information from Piezo Transducer

Piezo transducer supplies more information as it yields the entire pressure-time curve rather than a single deformation value. An exact weighted integral can be calculated to compute impact energy. The maximum pressure is also indicated. It is also possible to compute the slope and pressure-time integrals to obtain a better understanding of ammunition properties.

3. ANALYSIS OF ACCEPTANCE TEST DATA

Acceptance test data for 5-56mm. Ball M-193 ammunition from five manufacturers (Lake City, Twin City, Remington, Federal and Winchester) as well as for 5.56mm. Tracer M-196 ammunition from three manufacturers (Lake City, Twin City and Winchester) were made available by Rock Island Arsenal for investigation. The data were subsequently up-dated to cover a production period from July 1968 to March 1971. The acceptance test data contain the following information:

(1) Ammunition lot number and date testing

(2) Propellant lot number

(3) Average charge weight used for the ammunition lot

(4) Chamber pressure - which is the average of 20 chamber pressure readings per ammunition lot

(5) Maximum value of these 20 readings

(6) Port pressure - which is the average of 20 port pressure readings per ammunition lot

(7) Muzzle Velocity - which is the average of 20 velocity readings per ammunition lot

(8) Standard deviation for chamber pressure, port pressure and velocity. This is computed from the corresponding 20 individual readings.

(9) Correction for chamber pressure and velocity

(10) Accuracy - which is a measure of how accurate the flight of the bullet is

(11) The data also contain information regarding the reference lot used, number of bullets fired using the test pressure barrel and velocity barrel, reference velocity and reference chamber pressure as well as the pressure barrel number and velocity barrel number

3.1 Preliminary Study of the Data

Graphical method was used to obtain a visual picture of the data. Figures 15 to 22 show the plots of chamber pressure, port pressure and muzzle velocity against the date of testing. The plots include Ball ammunition data from five manufacturers and Tracer ammunition data from three main Chamber factories. Pressure, port pressure, and muzzle velocity were also plotted against the corresponding ammunition lot numbers and the resulting plots are given in Figures 22 to 27.

A visual examination of these graphs reveals the following:

(1) Testing is not done at regular time intervals. It appears, therefore, that plotting based upon lot numbers provides a better representation of the time sequence of the production process.

(2) The data show production trends, in particular, the chamber pressure data from Lake City and from Twin City show a marked shift.

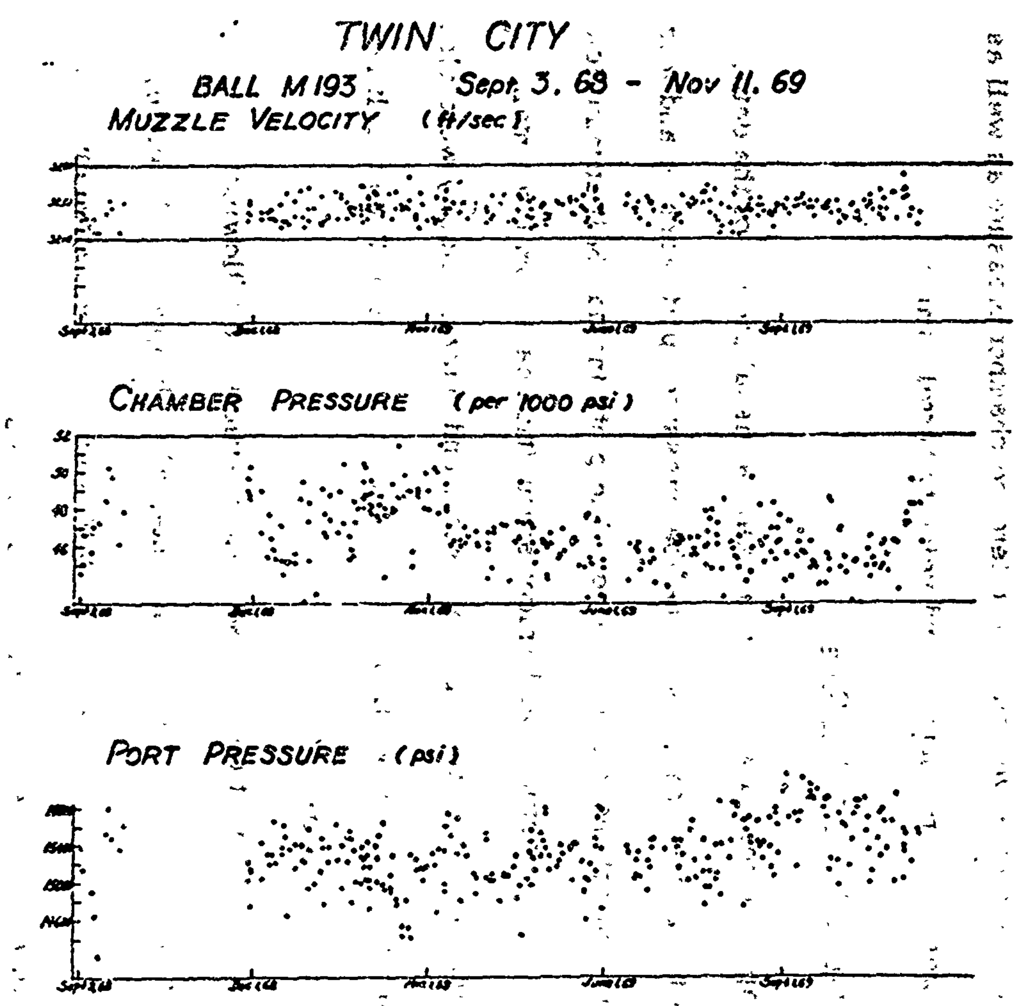

TWIN CITY
BALL M193 Sept. 3. 68 - Nov 11. 69
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 15

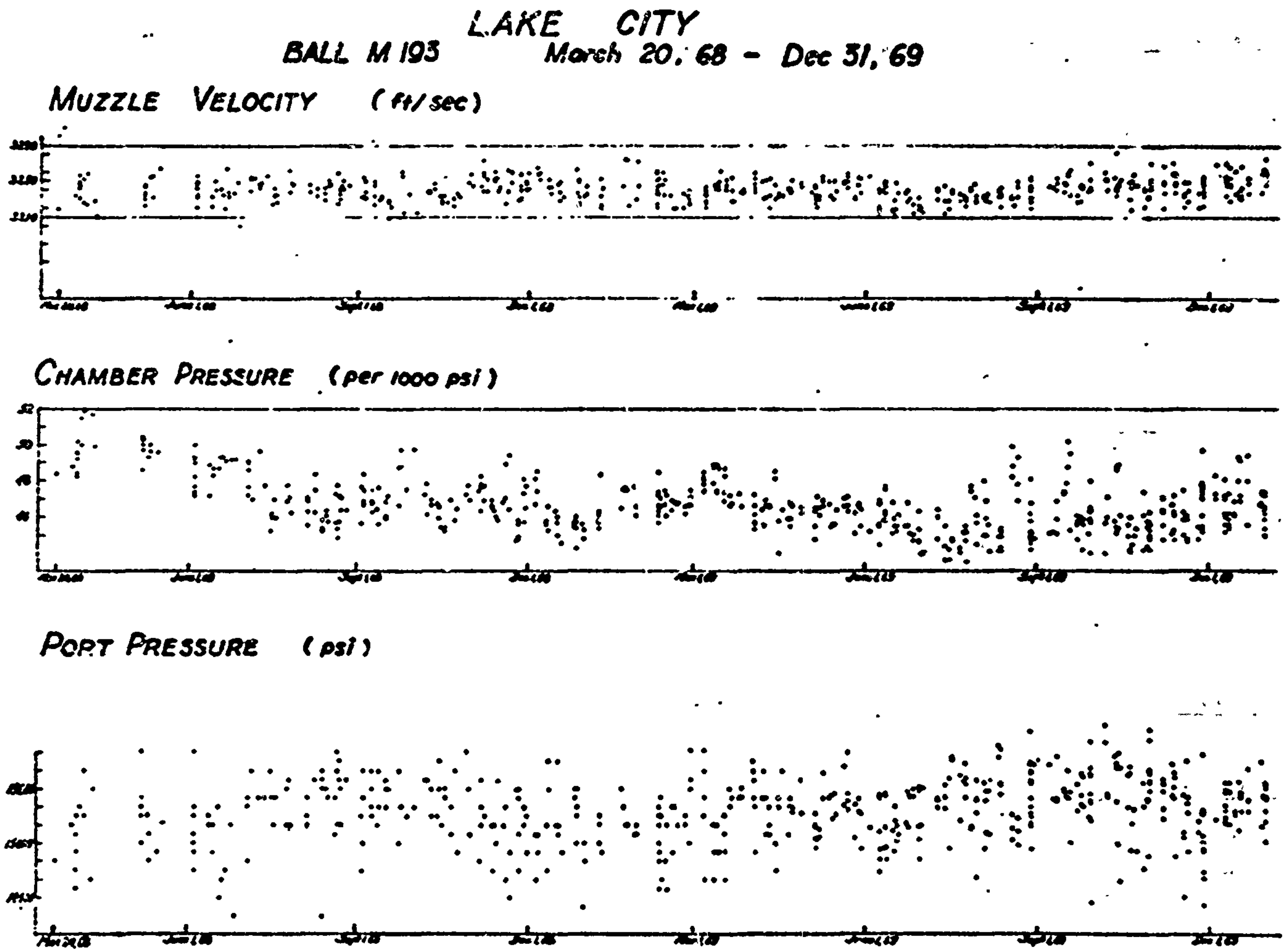

LAKE CITY
BALL M 193
March 20, 68 - Dec 31, 69
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 16

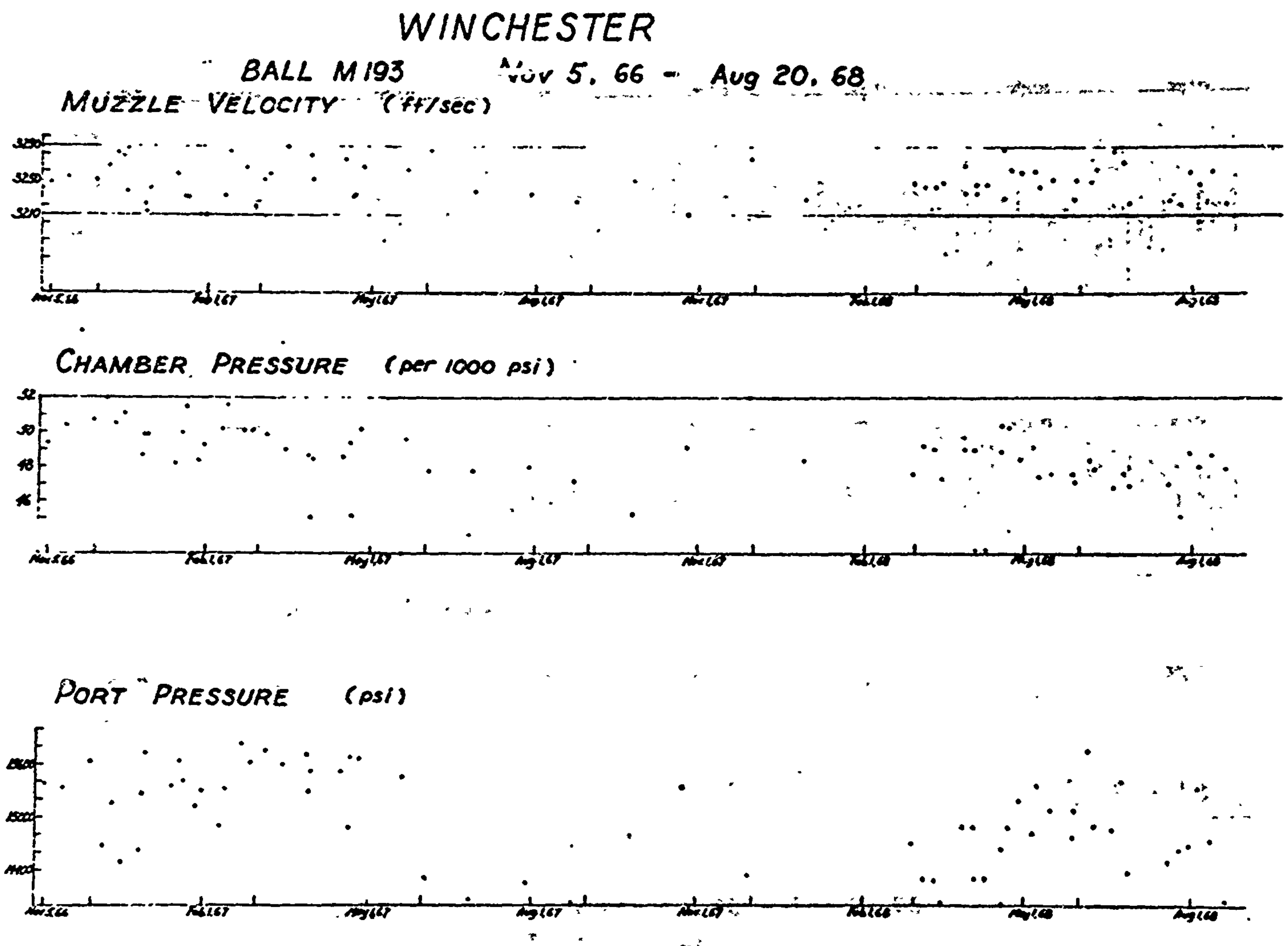

WINCHESTER
BALL M193 Nov 5, 66 - Aug 20, 68
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 17

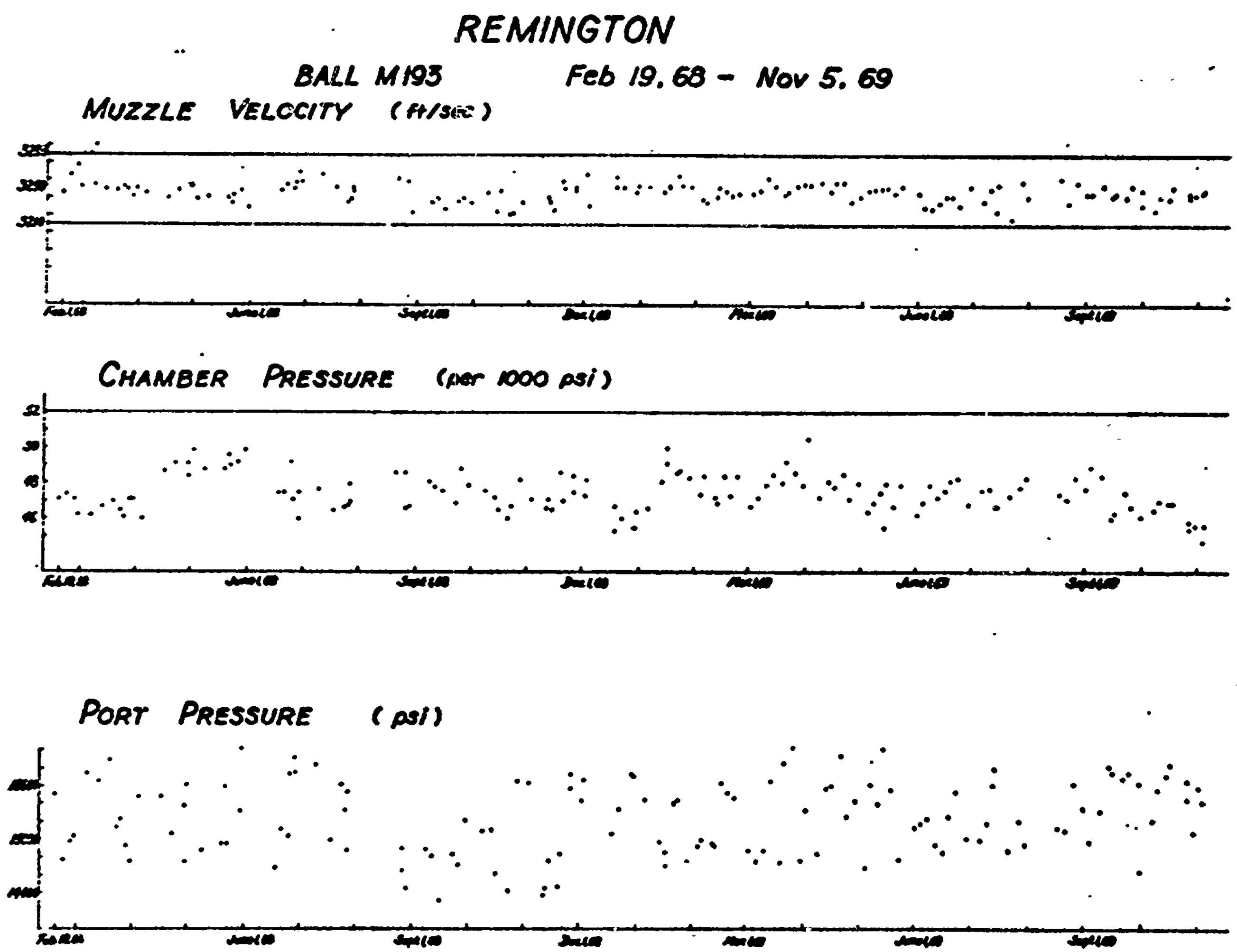

REMINGTON
BALL M193
Feb 19, 68 - Nov 5, 69
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 18

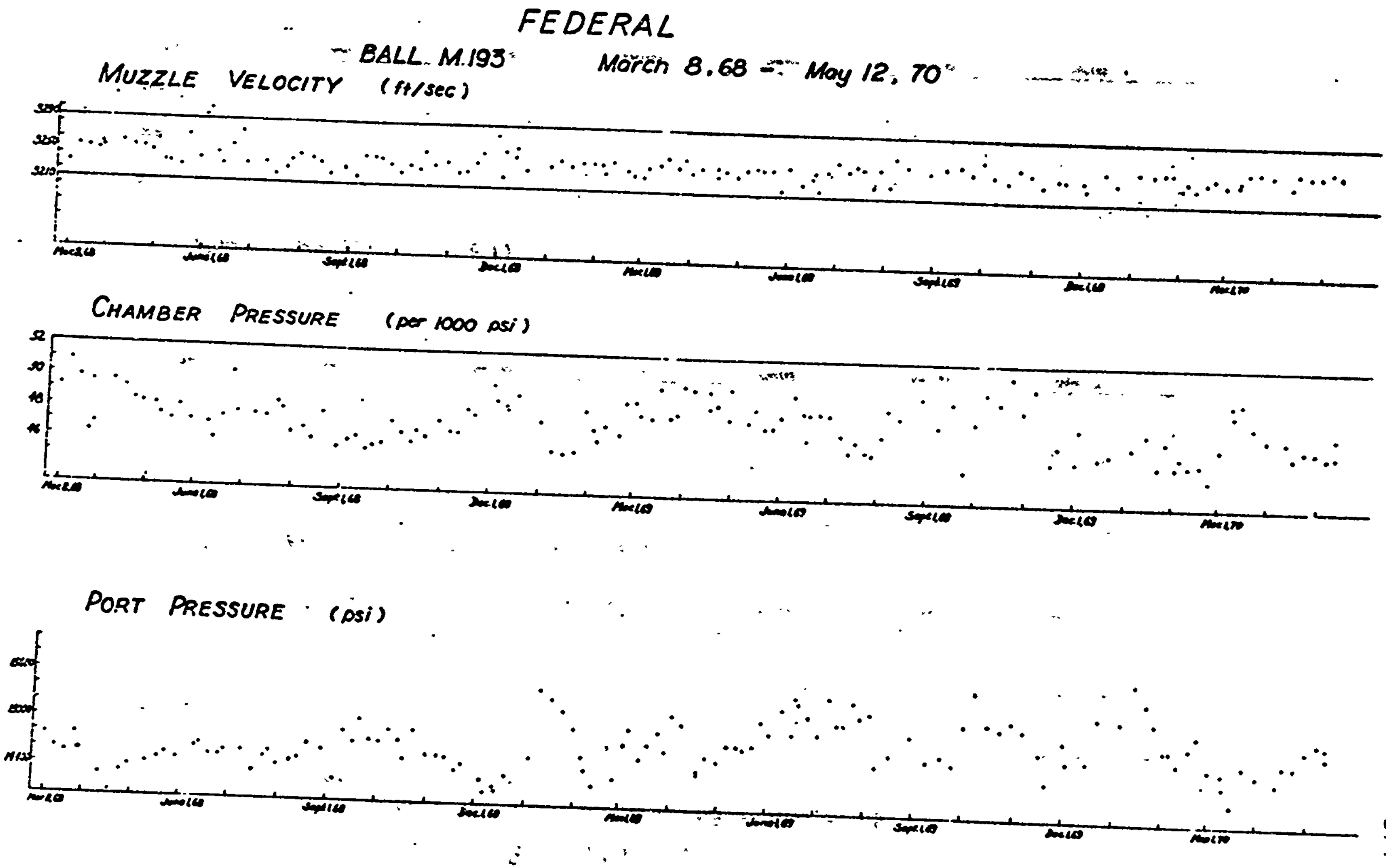

FEDERAL
BALL M.193
March 8.68 — May 12, 70
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
52
50
48
46
PORT PRESSURE (psi)

FIGURE 19

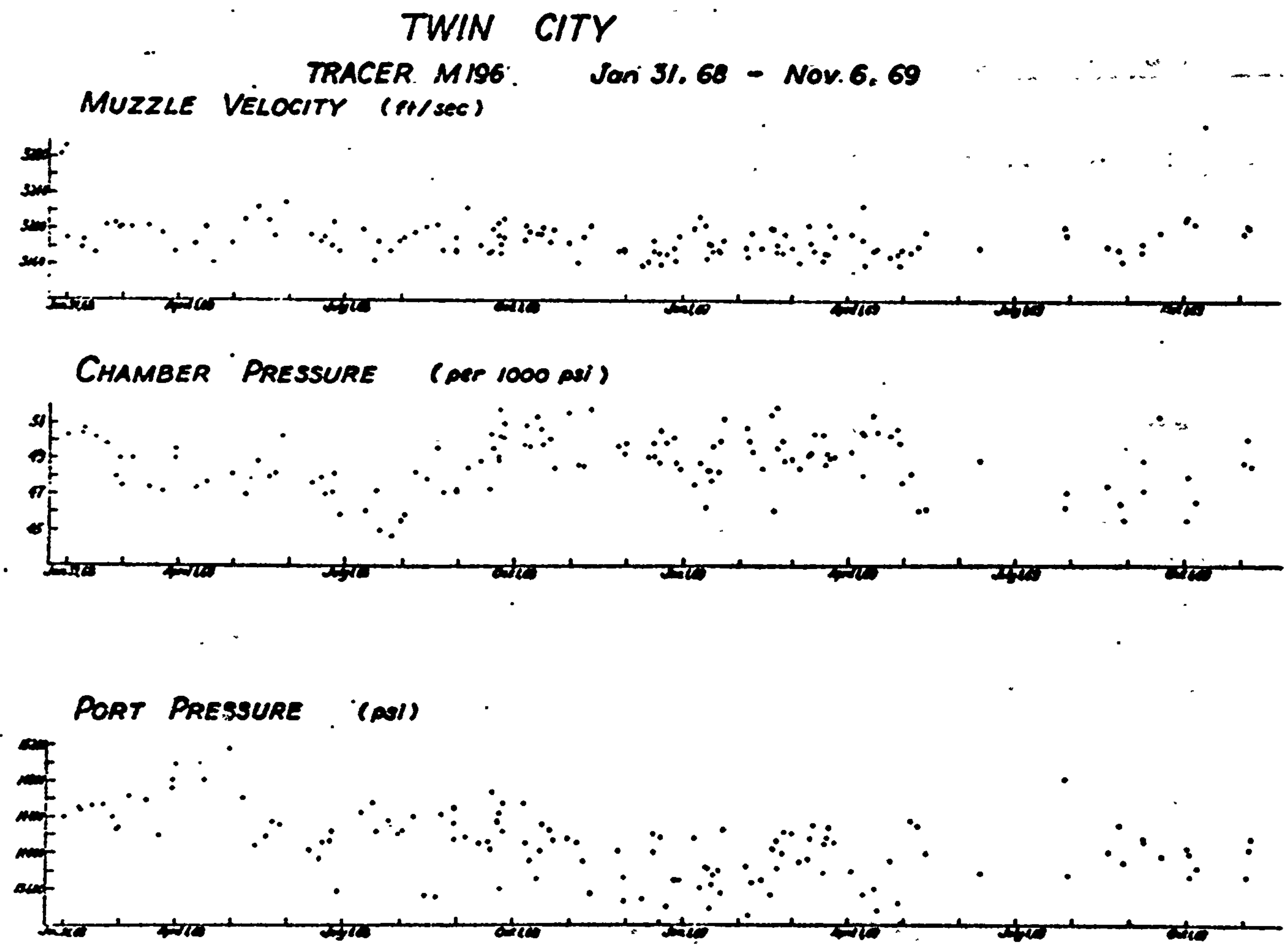

TWIN CITY
TRACER M196 Jan 31, 68 - Nov. 6, 69
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 20

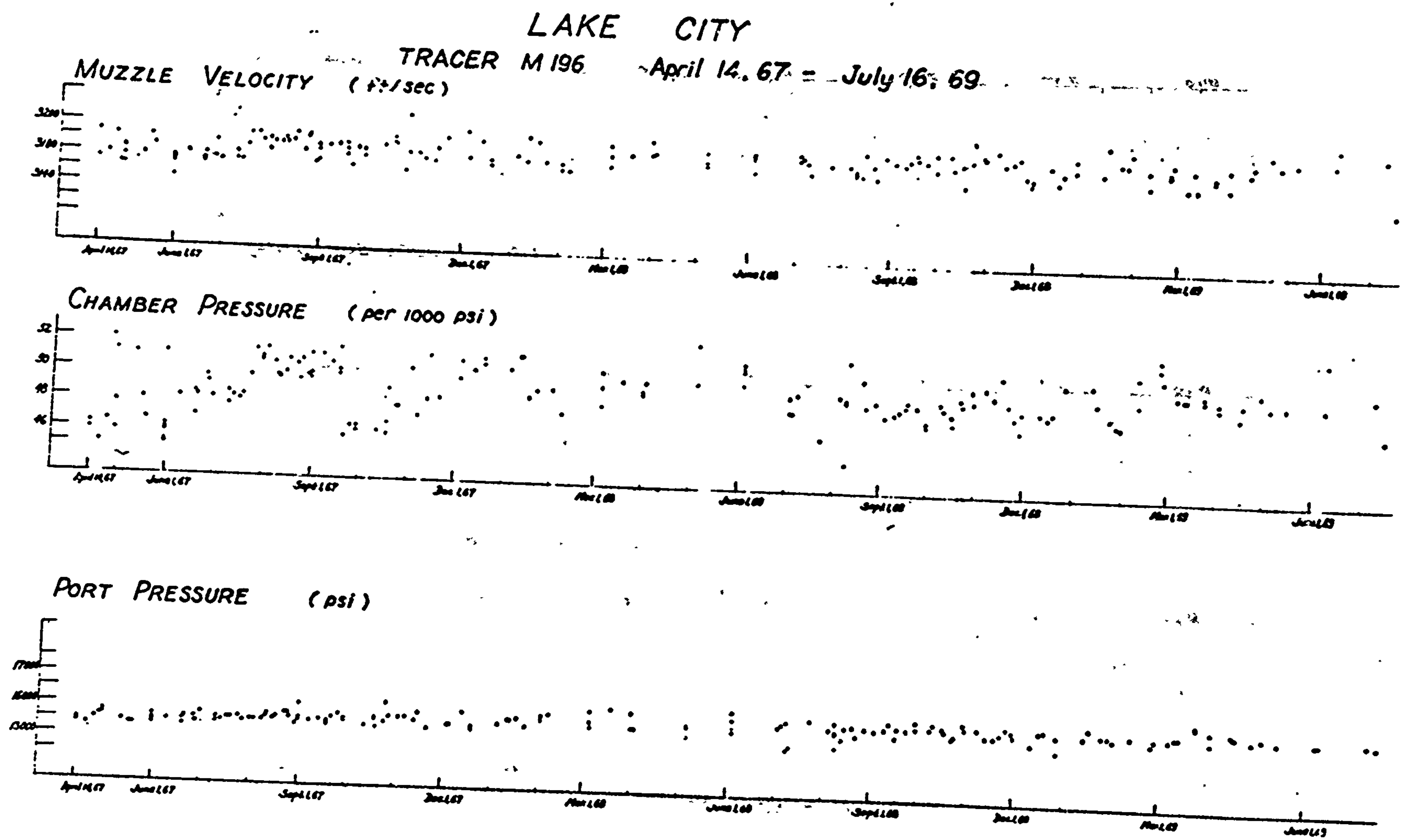

LAKE CITY
TRACER M 196 April 14, 67 — July 16, 69
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 21

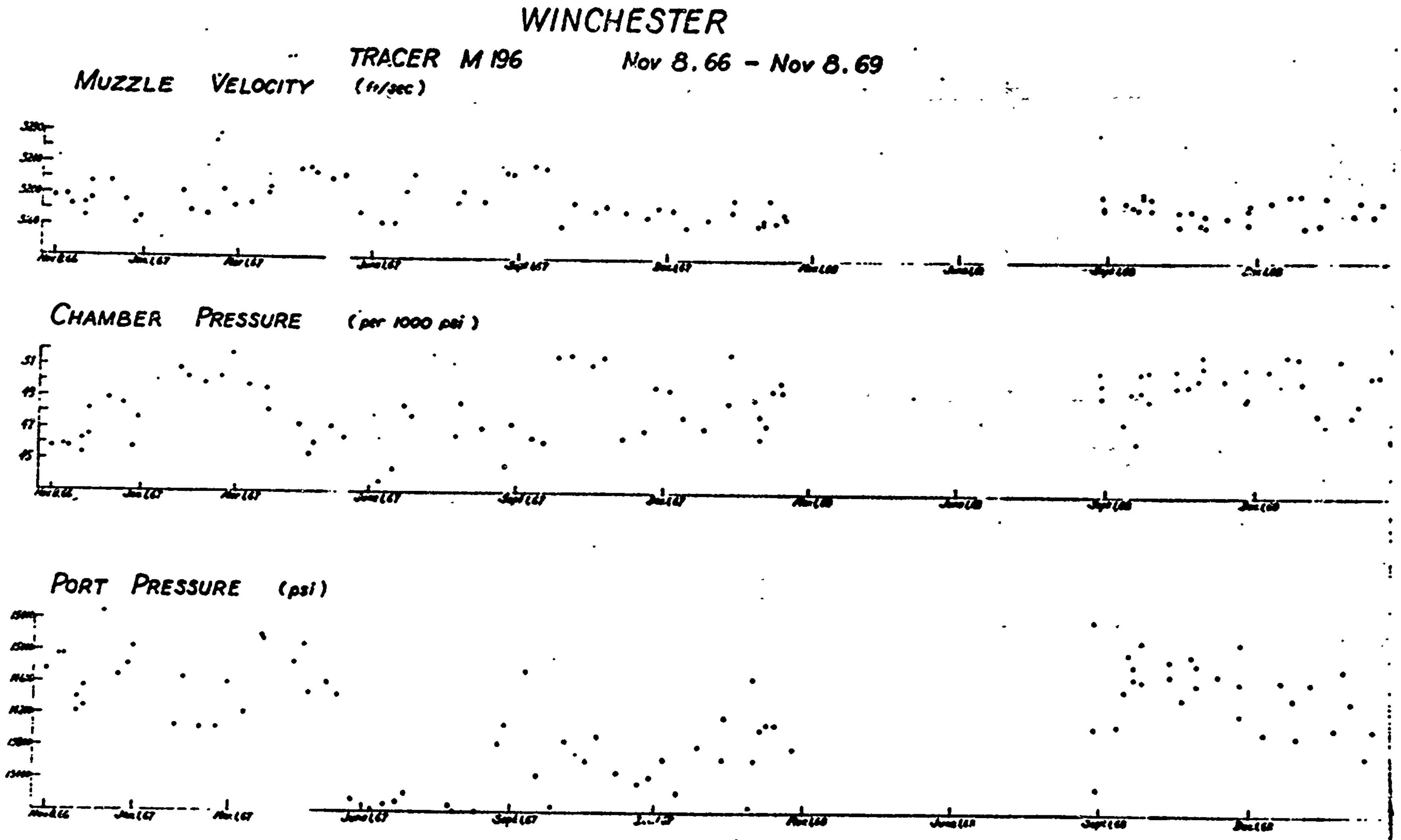

WINCHESTER
TRACER M 196
Nov 8.66 - Nov 8.69
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 22

TWIN CITY

BALL M193 LOT 18402 - 18692

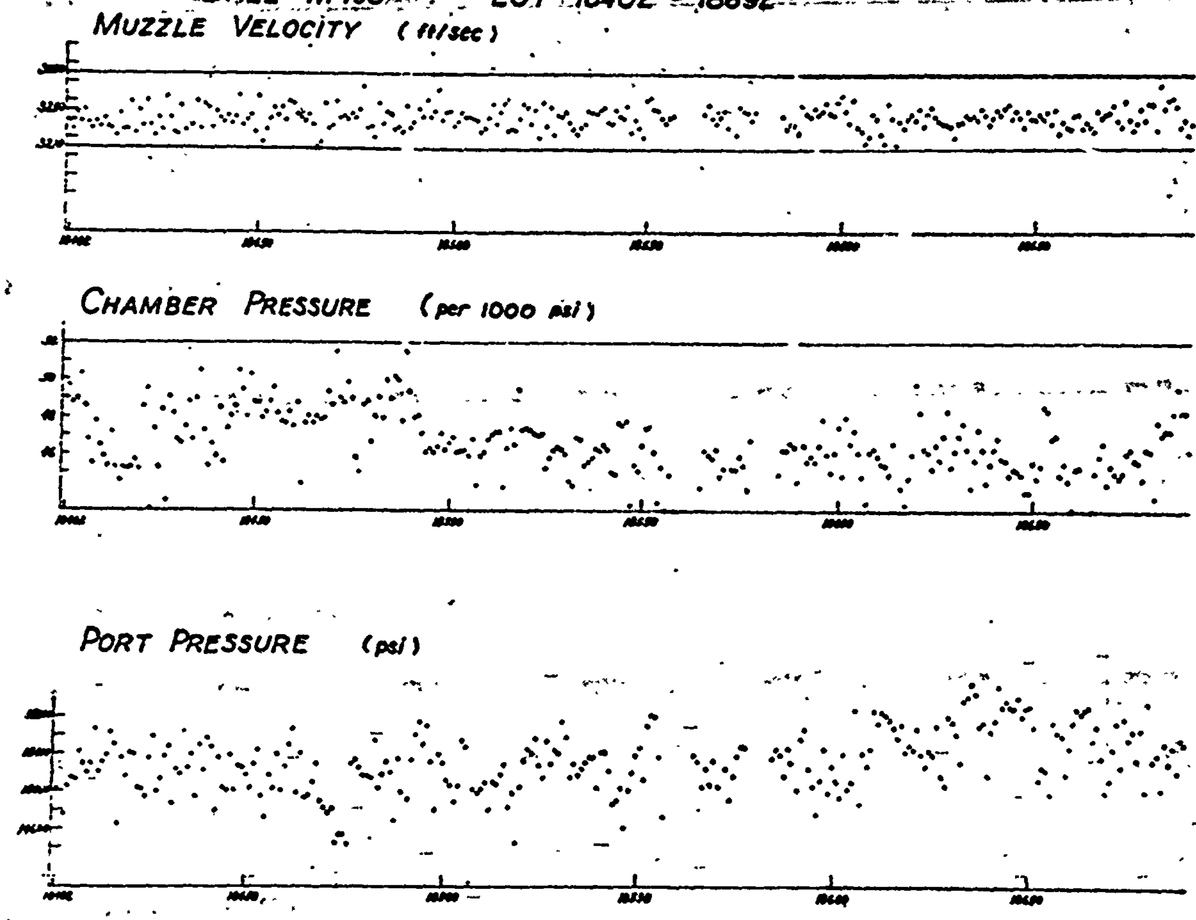


FIGURE 23

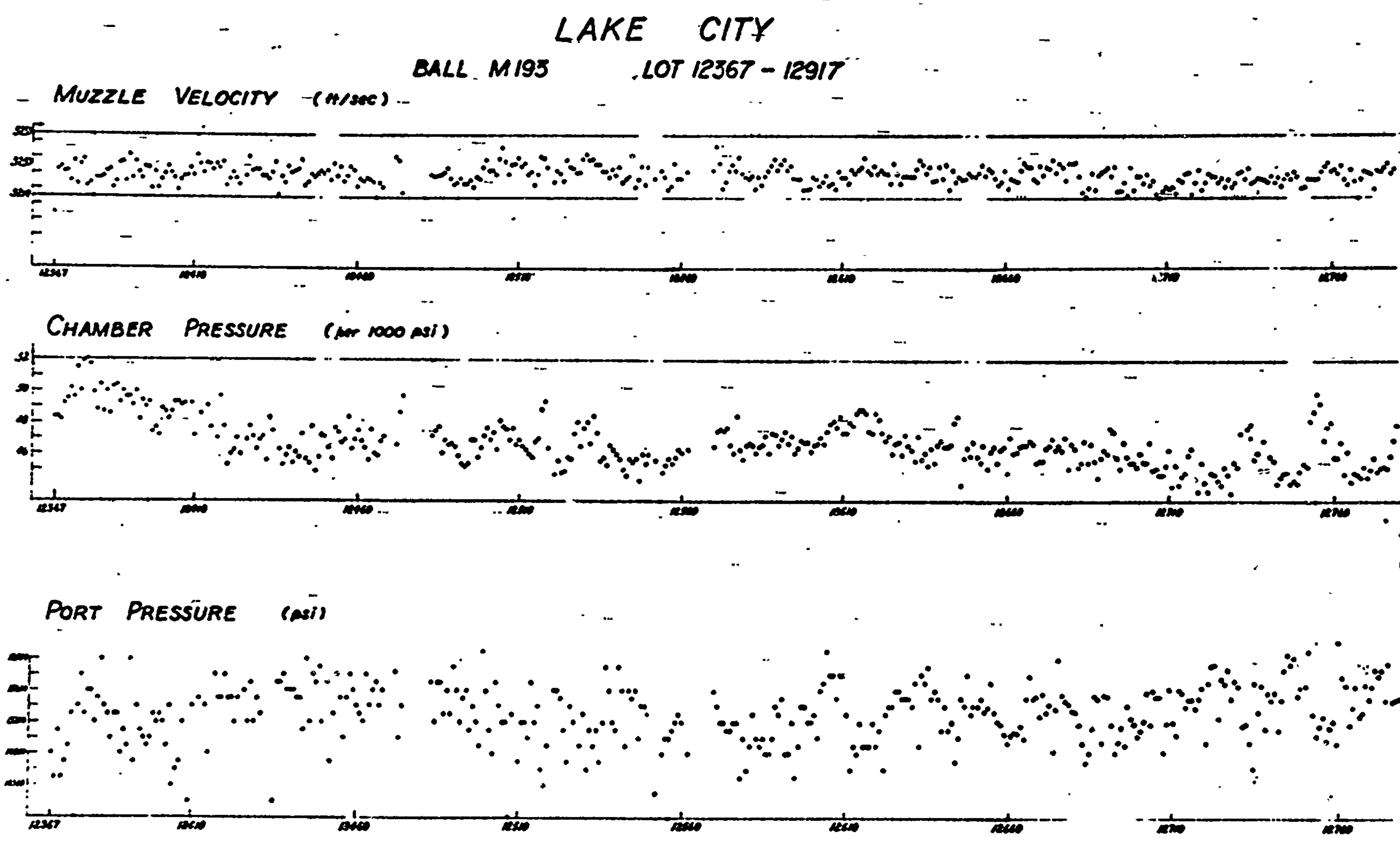

LAKE CITY
BALL M193
LOT 12367 - 12917
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 24

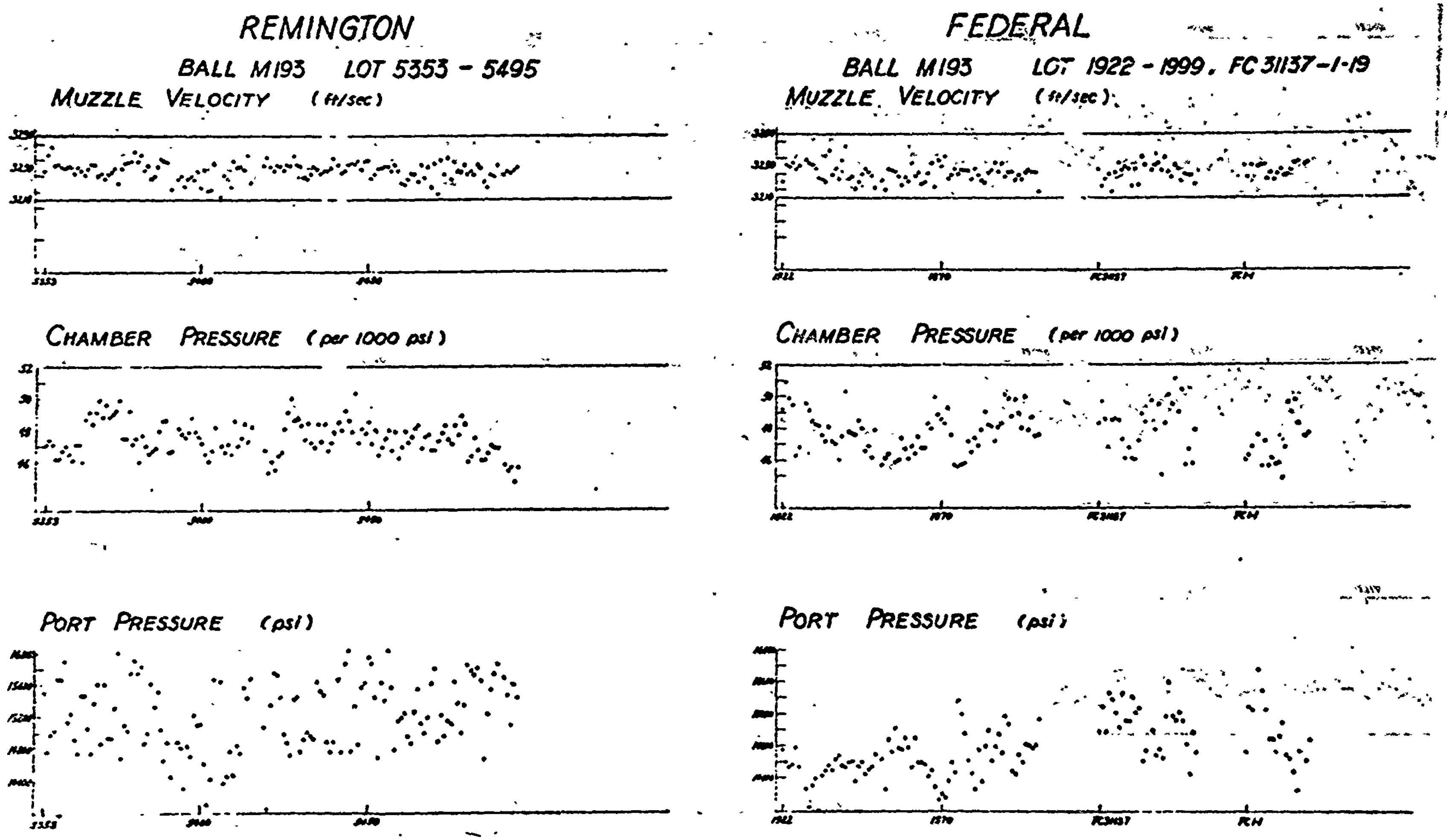

REMINGTON
BALL M193 LOT 5353 - 5495
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)
FEDERAL
BALL M193 LOT 1922 - 1999, FC 31137-1-19
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 25

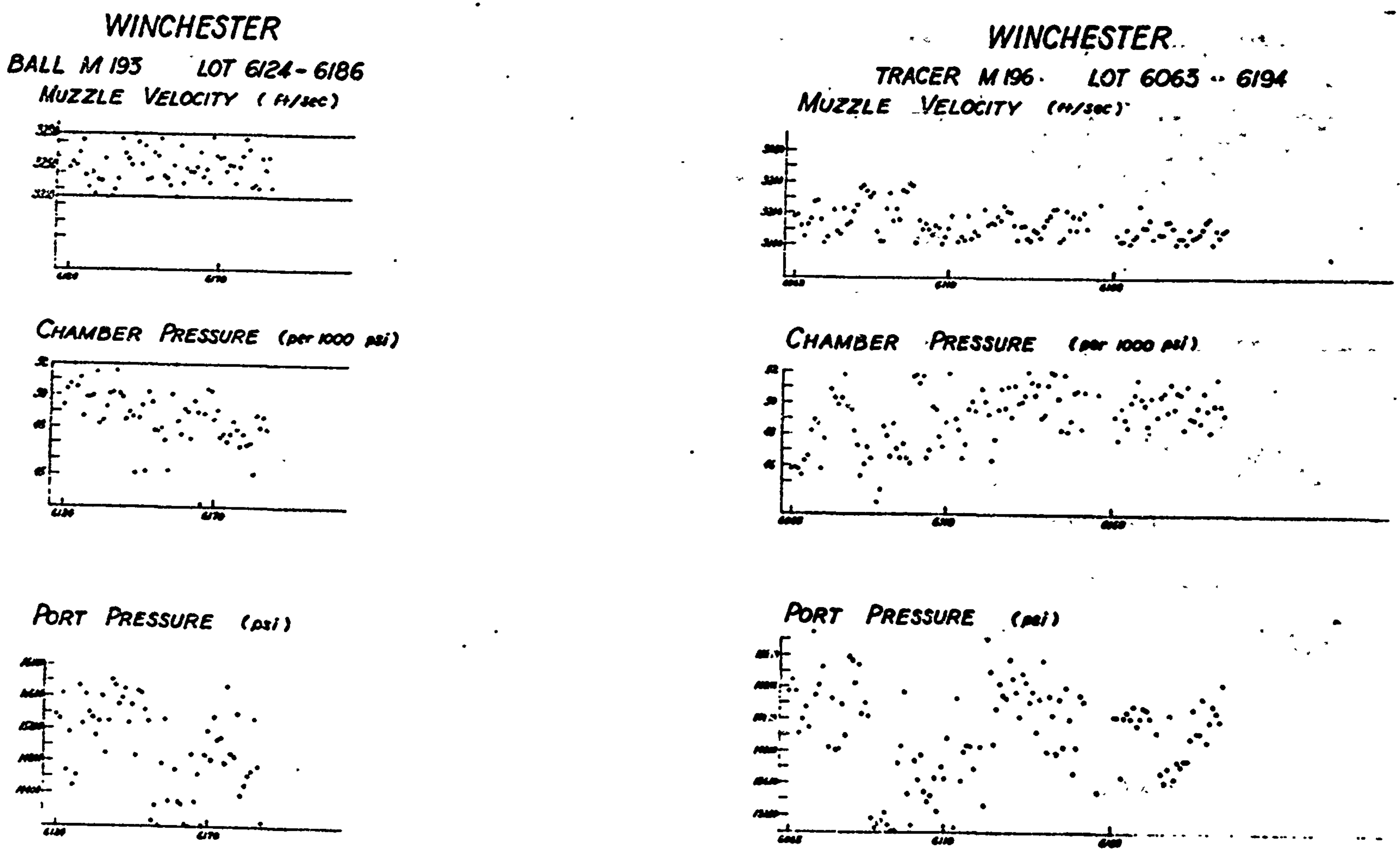

WINCHESTER
BALL M 193 LOT 6124 - 6186
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)
WINCHESTER
TRACER M 196 LOT 6063 - 6194
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 26

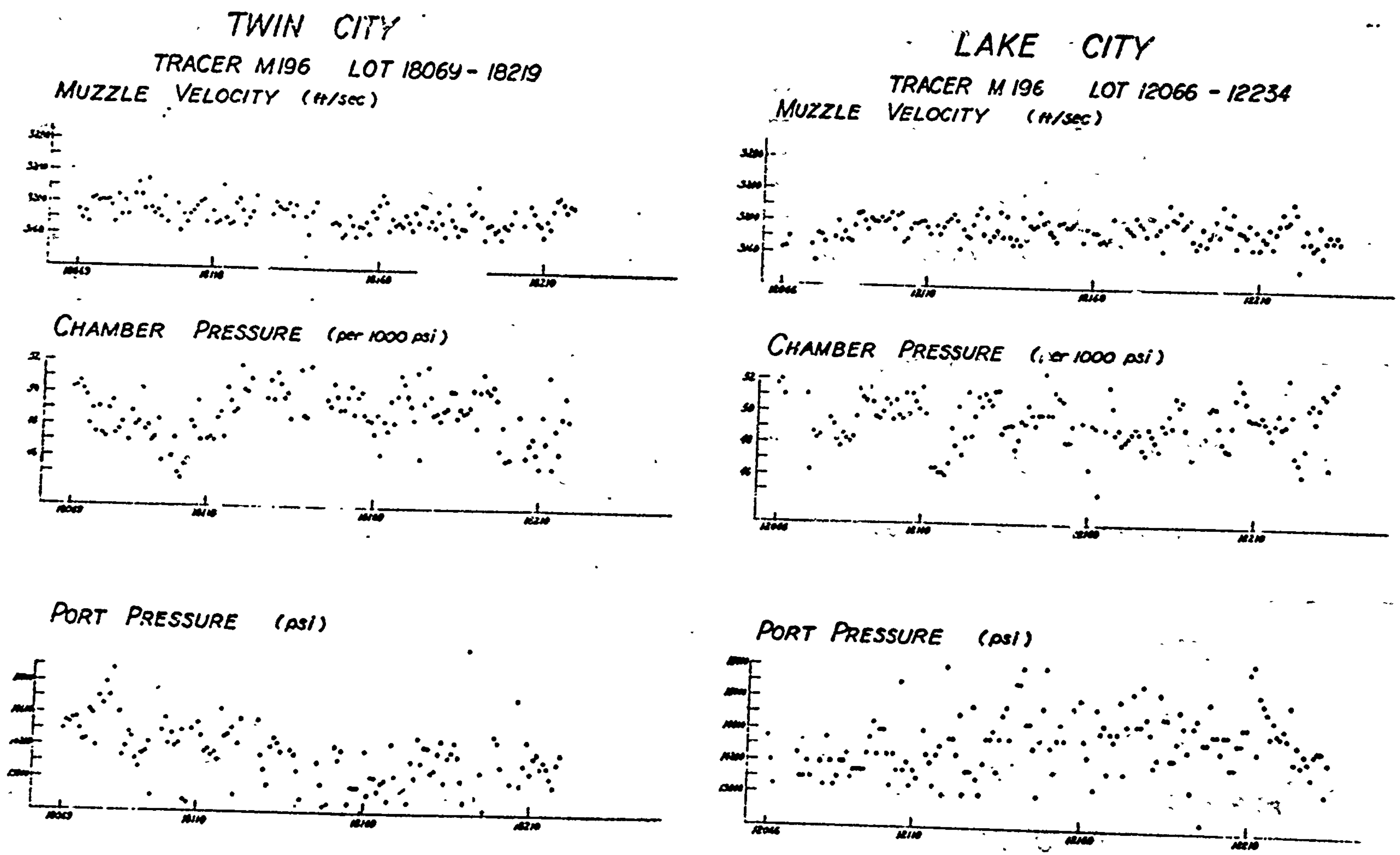

TWIN CITY
TRACER M196 LOT 18069 - 18219
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)
LAKE CITY
TRACER M 196 LOT 12066 - 12234
MUZZLE VELOCITY (ft/sec)
CHAMBER PRESSURE (per 1000 psi)
PORT PRESSURE (psi)

FIGURE 27

(3) In addition to the differences in the trend, there appear to be differences in the range (range = maximum-minimum) of the data from the five manufacturers.

(4) Mean chamber pressure for Ball ammunition is lower than that for Tracer ammunition while muzzle velocity for Ball ammunition is higher than that of the Tracer ammunition.

A quantitative analysis of the standard testing data is now conducted.

3.2 Determination of the Point of Shift

It has been noted that the chamber pressure data reveal the existence of a shift, after which the data assume a relatively stationary pattern. The large variability in the observed data makes it difficult to determine the point where the underlying process shifts. Methods are therefore needed to produce a visual picture reflecting the underlying process and minimize the random fluctuations about it. Any such method would depend upon certain assumptions regarding the process and would be only good within the assumptions made. Three methods of estimating the underlying process would now be considered. These methods would also serve to determine a unique answer for the point of shift rather than differing answers that would result by eyeball estimation.

The assumptions made are:

(1) Successive observations on chamber pressure are assumed to be independent.

(2) The observations are assumed to have the following distribution:

$$\bar{X}_t = N(\mu_t, \sigma^2)$$

where

$\bar{X}_t$ = Mean chamber pressure

For the t^{th} lot

μ_t = Expected value of $\bar{X}_t$

σ^2 = Variance of $\bar{X}_t$

It will be observed that for the data under consideration, second assumption is fairly well justified; but the first one is not exactly true since the successive observations are found to be correlated.

3.2.1 Method of Semi-Averages

The entire set of data is subdivided into equal groups each containing K_1 observations. If μ_t can be assumed constant over the K_1 observations, then the average of each group is an estimate of chamber pressure for those K_1 lots and is distributed as $N(\mu_t, \sigma^2/K_1)$. This group average is plotted against the serial number of that group. The effect of this grouping is to smoothen out the original series of observations as is evident from the reduced variance of the group average. The resulting plot presents a better visual picture of the underlying process. From Figures 28 and 29 it can be seen that large value of K_1 results in greater smoothening and a better visual picture. However, excessive smoothening might lead to a loss of information. An optimum value of K_1 is, therefore, necessary. Point of shift is the lot number after which the group averages stabilize.

3.2.2 The Method of Cumulative Sum

This is a plot of successive partial sums

$$\sum_{i=1}^{m} (X_i - K_2) \; ; \; m = 1, 2 \text{---} n$$

plotted against 'm'. Here

X_i = chamber pressure of the ith lot

n = total number of lots considered

and K_2 = reference constant

The shape of the plot changes with the different values of constant K_2. Two plots of the cumulative sum (cusum) are shown in Figures 30 and 31.

The mean level over any portion of the cusum plot is given by

$$\text{Mean Level} = K_2 + \frac{\text{Change in the cumulative sum}}{\text{Change in M}}$$

In particular, an estimate of the mean chamber pressure for a particular lot

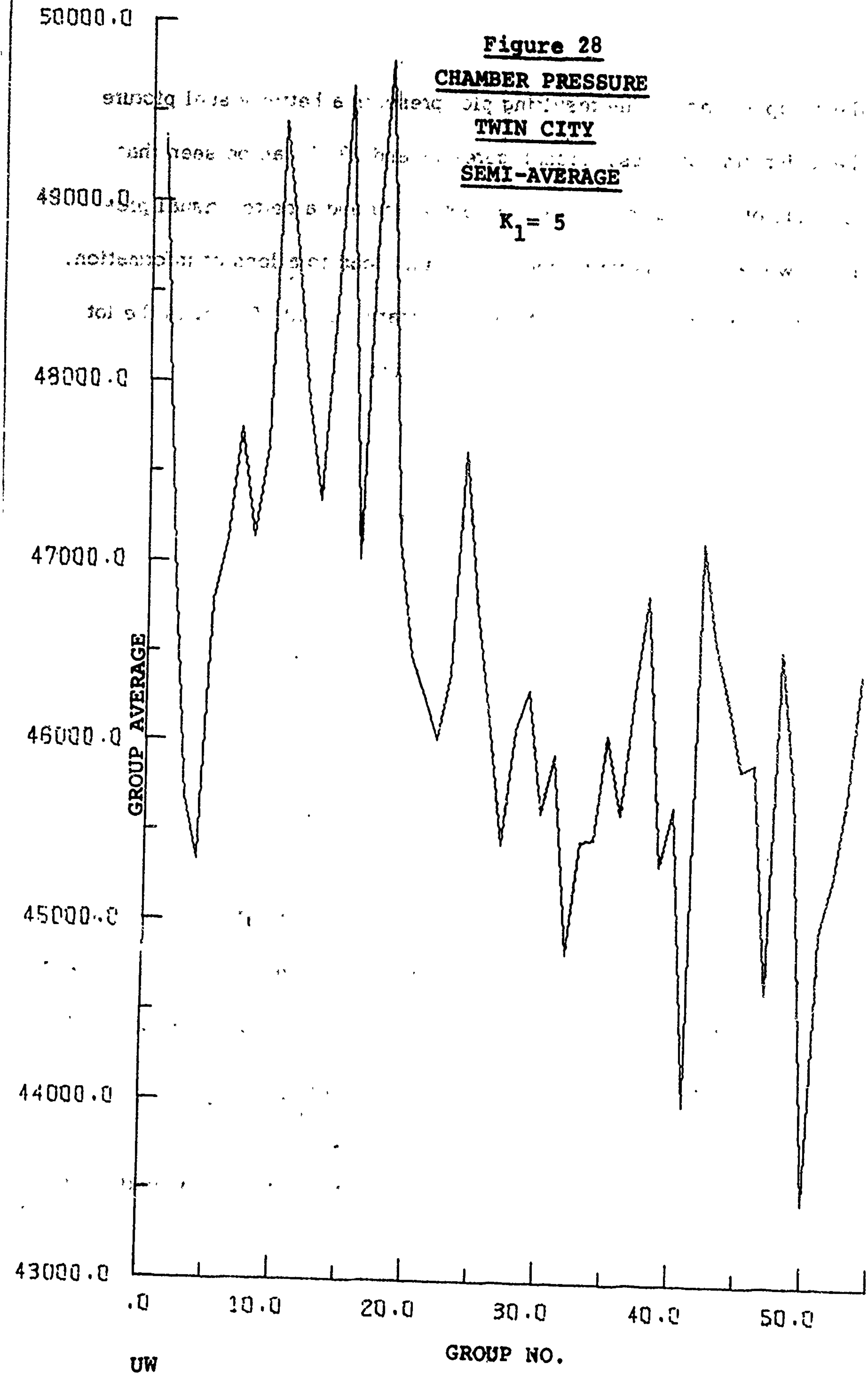
Figure 28

CHAMBER PRESSURE

TWIN CITY

SEMI-AVERAGE

$K_1 = 5$



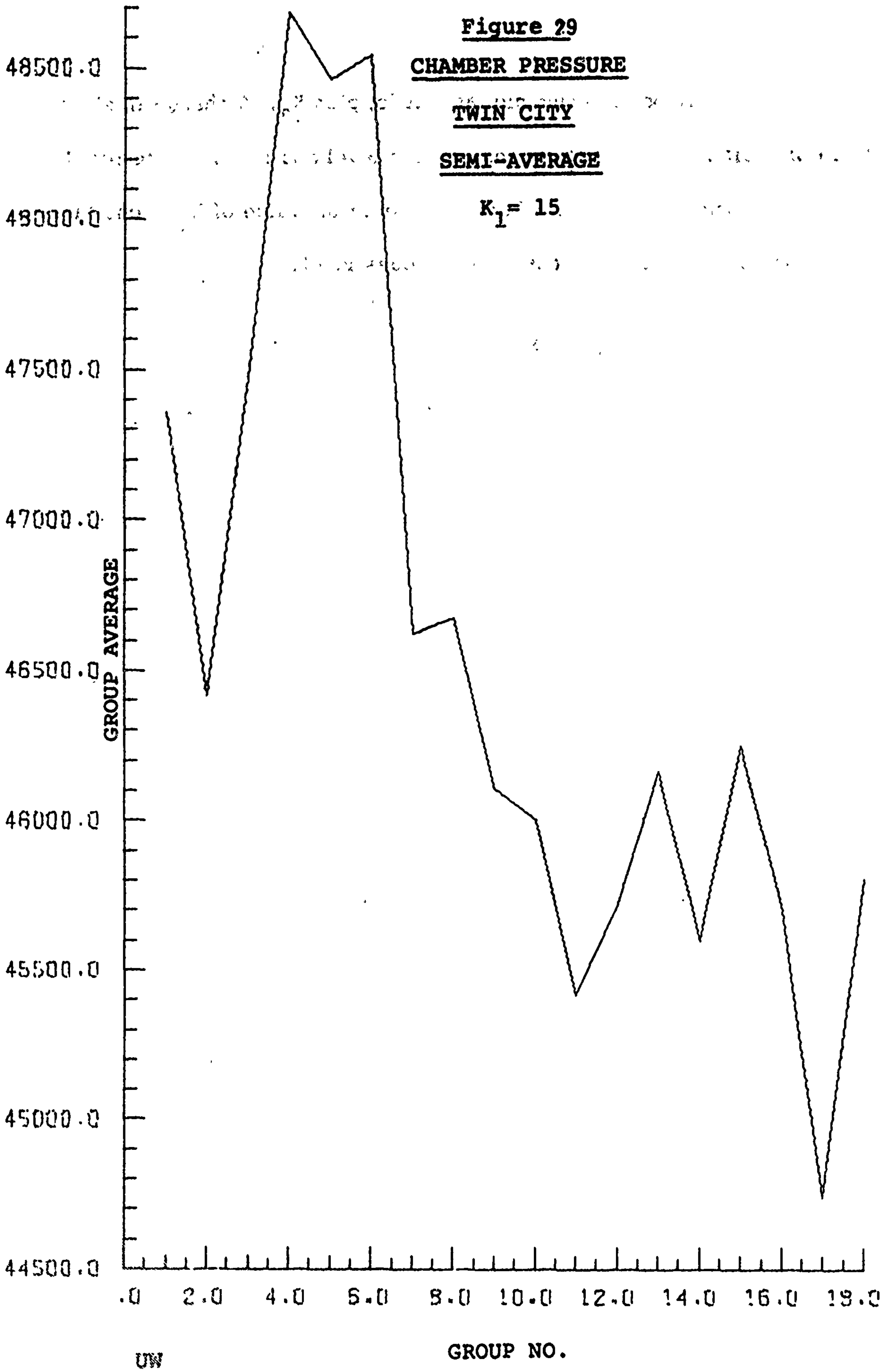


Figure 29
CHAMBER PRESSURE
TWIN CITY
SEMI-AVERAGE
K_1= 15
48500.0
48000.0
47500.0
47000.0
46500.0
46000.0
45500.0
45000.0
44500.0
GROUP AVERAGE
.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
GROUP NO.
UW

is given by the slope of cusum plot at that lot plus K_2. A change in slope, therefore, signifies a change in the process level and forms a criterion for the determination of the point of shift. An optimum value of K_2 is one that gives the best indication of change in process level.

3.2.3 Method of Moving Averages

Moving average is another technique used to produce an approximation to the underlying process. The plot is obtained as follows: An average of first K_3 observations is computed (K_3 = period) and is plotted at the mid-period position. Now the first observation is deleted and (K_3 + 1) th observation is added to get a new average. This is plotted at the mid point of this period and the process is repeated till all available observations are exhausted.

Each point on the curve is an estimate of chamber pressure for that lot. Since plotting is done at the mid-point of a period, K_3 should be an odd number so that the plotted point would correspond to an actual lot. Two such plots, with K_3 equal to 5 and 15 are shown in Figures 32 and 33. It can be observed that the longer the period, the more reduction in fluctuation. As the length of the period increases, there is a tendency for the moving average to 'iron out' the underlying process. It is prudent, therefore, to use as short a period as possible consistent with a reasonably smooth moving average.

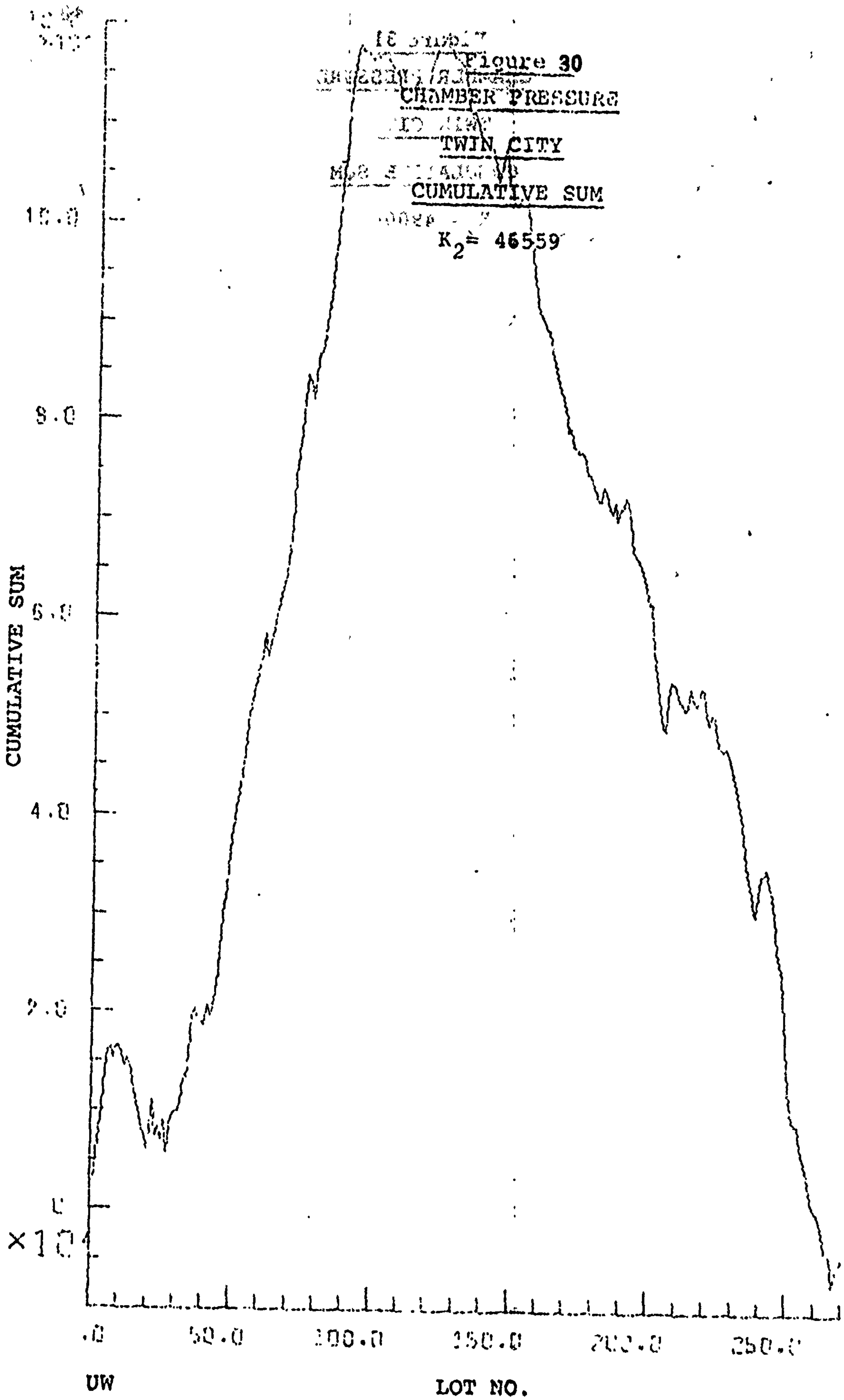


Figure 30
CHAMBER PRESSURE
TWIN CITY
CUMULATIVE SUM
K_2= 46559
CUMULATIVE SUM
×10
0
2.0
4.0
6.0
8.0
10.0
.0
50.0
100.0
150.0
200.0
250.0
UW
LOT NO.

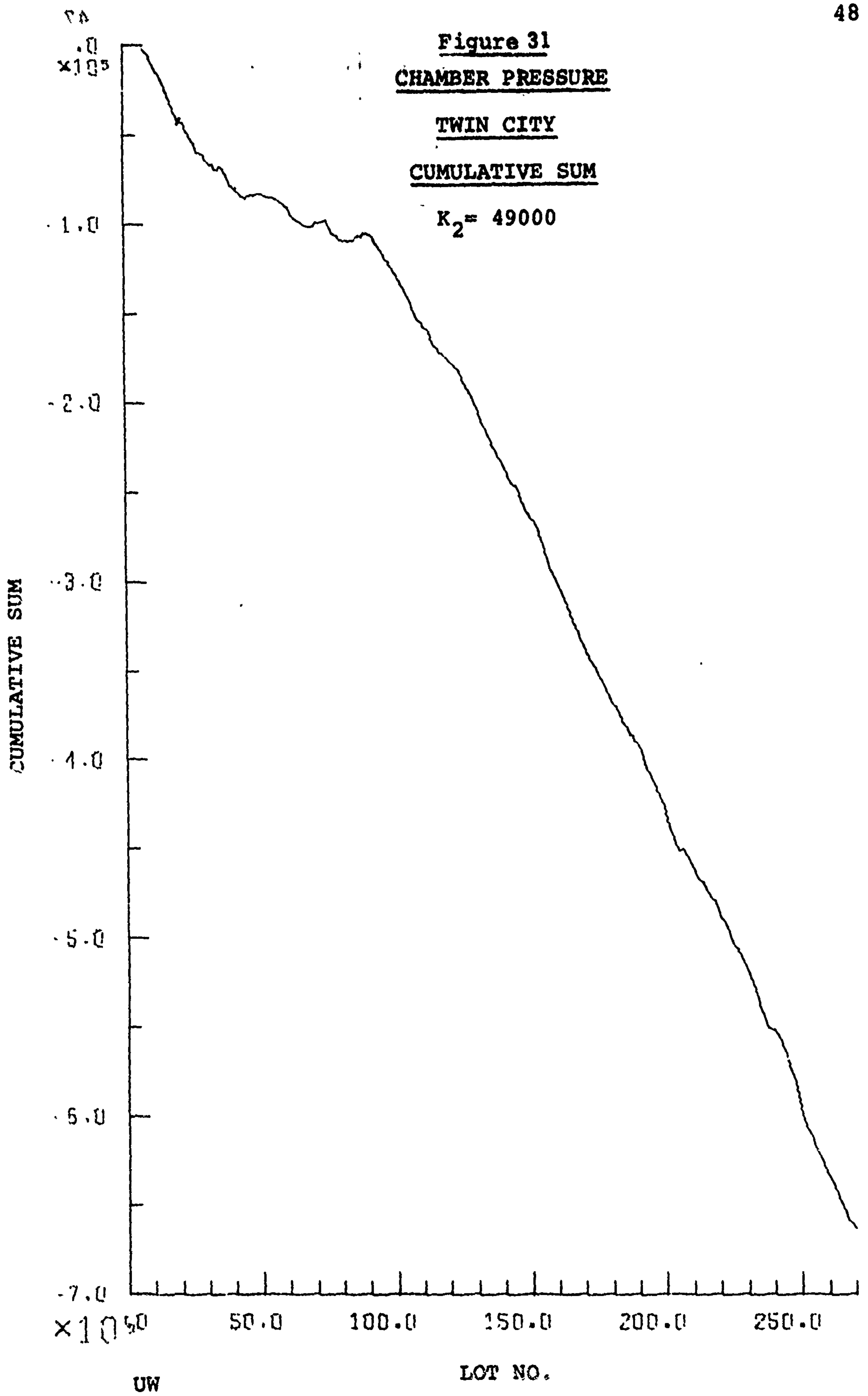


Figure 31
CHAMBER PRESSURE
TWIN CITY
CUMULATIVE SUM
K_2= 49000
.0
×10⁵
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
CUMULATIVE SUM
50.0
100.0
150.0
200.0
250.0
LOT NO.
UW

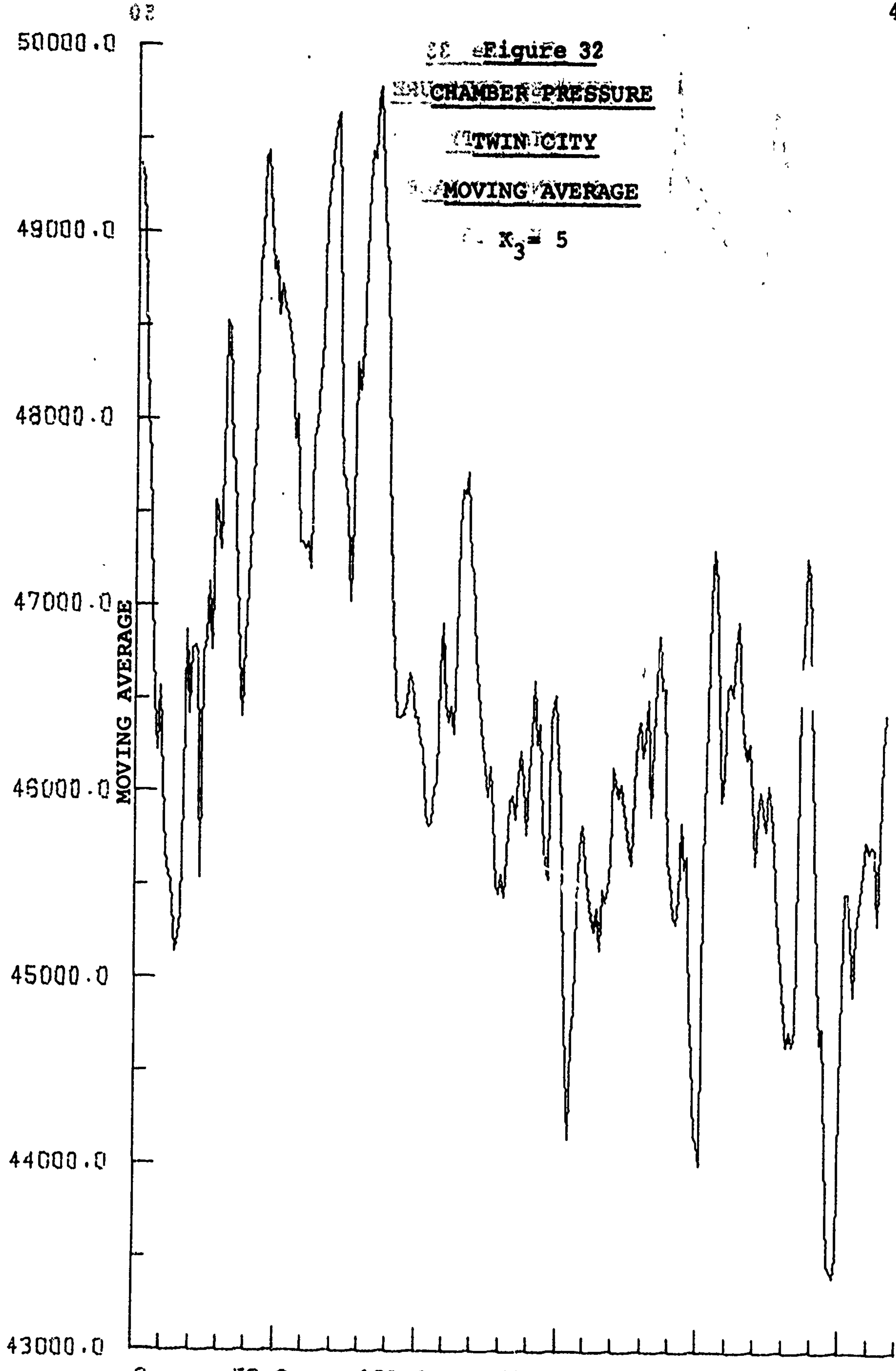


Figure 32
CHAMBER PRESSURE
TWIN CITY
MOVING AVERAGE
X_3 = 5
50000.0
49000.0
48000.0
47000.0
46000.0
45000.0
44000.0
43000.0
MOVING AVERAGE
.0
50.0
100.0
150.0
200.0
250.0
UW
LOT NO.

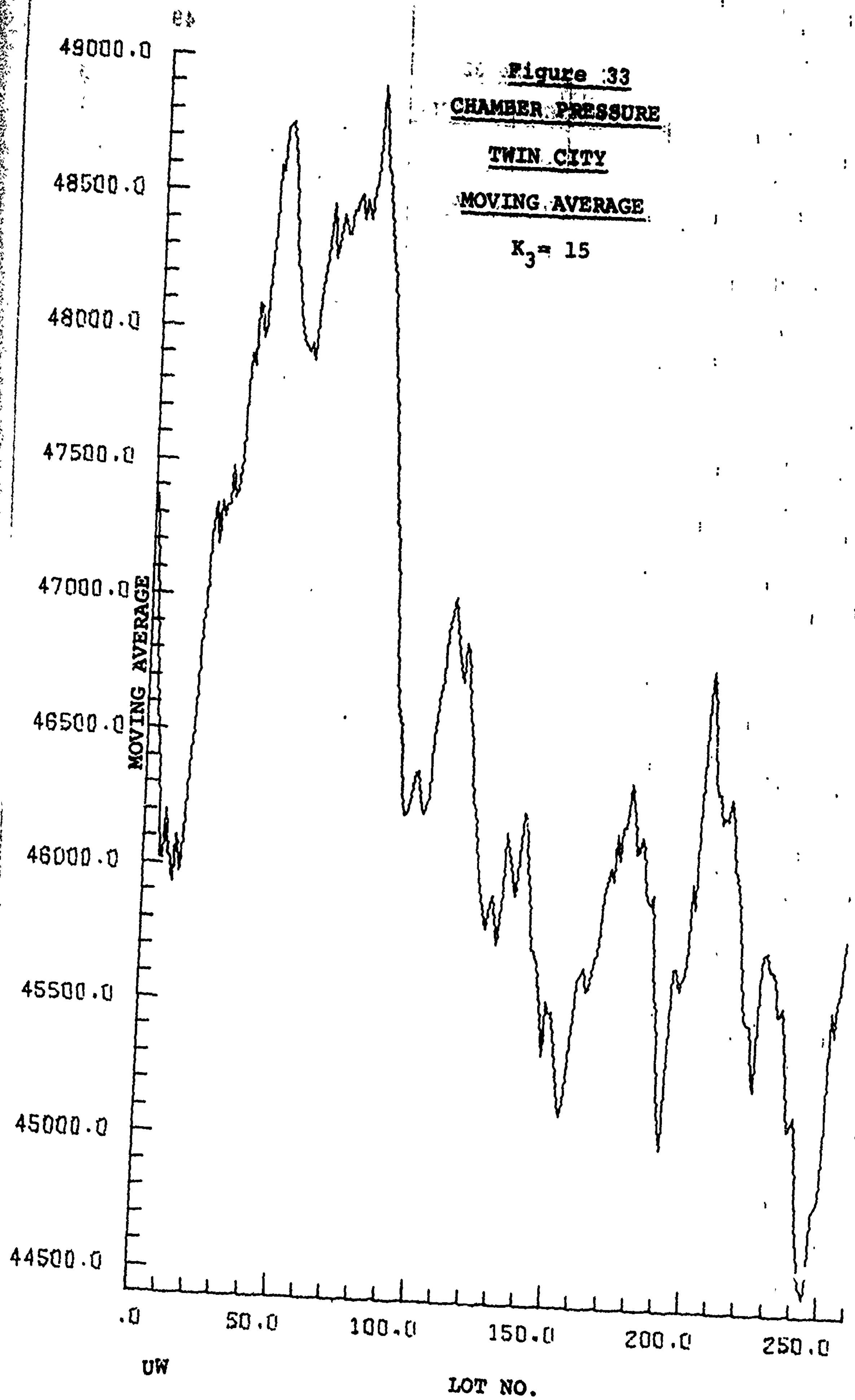


Figure 33
CHAMBER PRESSURE
TWIN CITY
MOVING AVERAGE
K3= 15
49000.0
48500.0
48000.0
47500.0
47000.0
46500.0
46000.0
45500.0
45000.0
44500.0
MOVING AVERAGE
.0
50.0
100.0
150.0
200.0
250.0
UW
LOT NO.

3.2.4 Discussion of the Three Methods

All the three methods serve the purpose of estimating the underlying process by reducing the random fluctuations about it. If the successive values of chamber pressure can be assumed to be uncorrelated, then optimum values of constants K_1, K_2, and K_3 can be determined. For example, in the method of moving averages, the underlying process can be estimated for any particular value of K_3. The deviation of the observations from this underlying process can be computed. The value of K_3 should be so chosen that the variance of these deviations is equal to the average internal (within lot) variance of the mean chamber pressure. This has not been actually carried out since the chamber pressure readings are found to constitute a time series of correlated observations.

The purpose of determing the point of shift (the date on which the shift occured is termed the 'cutoff date') is to be able to consider only the stationary part of the data. Control limits based upon data after the cutoff date are expected to be narrower than the present control limits. If the data are plotted a rough estimate of the point of shift can be obtained visually. Referring to Figures 28 through 33, it is observed that the point of shift is best determined by the cumulative sum plot. A good value for the reference constant K_2 seems to be the mean of the entire series of data because this value of reference constant gives a large change in slope at the point of shift. This procedure will be used in Chapter 4 to obtain the cutoff date for the chamber pressure data for Ball and Tracer ammunition.

3.3 Stationarity of Lake City Chamber Pressure Data After Cutoff

It is to be determined if the chamber pressure series is stationary after cutoff. The chamber pressure data consist of a time series of observations which are means of 20 chamber pressure readings. Strict stationarity demands that all these readings come from the same distribution, in particular, they have the same mean and the same variance.

Lake City chamber pressure variance can be considered to be approximately constant (it is not exactly constant, as will be seen later) and has a mean value of 1760806 $(psi)^2$. The variance of mean chamber pressure is, therefore, equal to 88040 $(psi)^2$ (= 1760806/20). This is a measure of the average internal (within a lot) variance of the mean chamber pressure. A measure of variance between the mean chamber pressures of different lots can be obtained by calculating the variance of the chamber pressure series after cutoff. This is found to be 452000 $(psi)^2$. If all the chamber pressure readings had the same mean, then the variance between lots would be approximately the same as variance within the lot. In reality this ratio (452000/88040) is of the order of 50, hence chamber pressure data after cutoff is still nonstationary.

3.4 NonNormality of Lake City Chamber Pressure Data

It is of interest to see if the chamber pressure readings constitute observations from a normal distribution. Figure 34 shows the histogram for Lake City chamber pressure data. The best possible normal distribution to fit the data is found to be $N(462, 12.7^2)$. Table 1 shows the Goodness of Fit test to determine if the data fit this normal distribution. Since the calculated chi-square value of 23.76 is greater than $\chi^2_{12-3,0.05}$ (=16.9), normality assumption is not justified at 95% level of confidence. Figures 35 to 38 show histograms of chamber pressure data (after cutoff) from Twin City, Winchester, Remington, and Federal. The histograms are clearly seen to be nonnormal. Because of the presence of trend even after cutoff, the underlying normal distribution of each observation is distorted. The result is, therefore, not unexpected.

3.5 Normality of Lake City Port Pressure Data

Histogram of port pressure readings after cutoff is shown in Fig. 39. Table 2 shows the Goodness of Fit test to determine if the data fit the estimated normal distribution. The calculated chi-square value is 113.4 and is greater than the critical value 18.3 ($\chi^2_{13-3,\ 005}$ = 18.3). Normality assumption is not justified at 95% level of confidence.

Figure 34

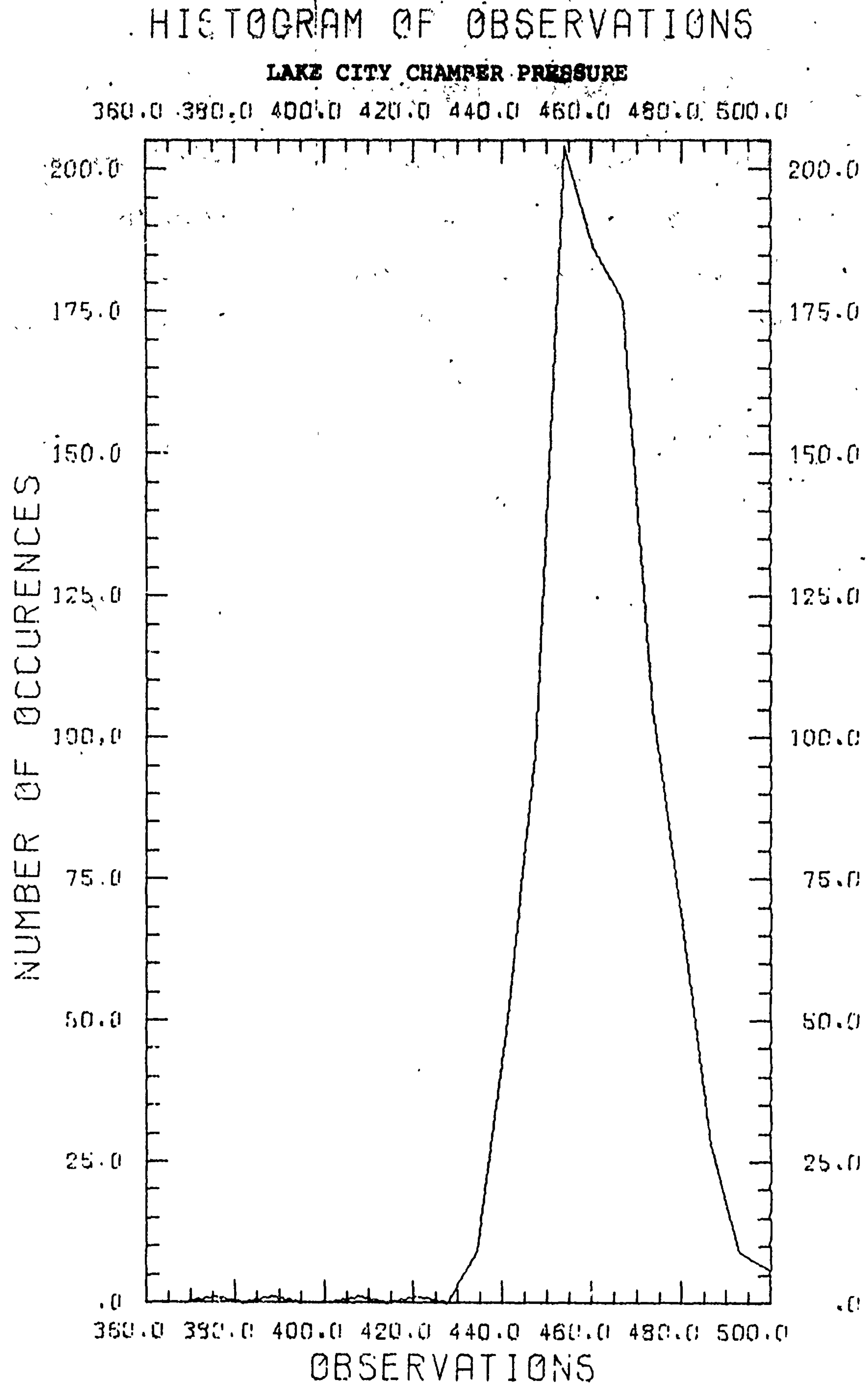

HISTOGRAM OF OBSERVATIONS
LAKE CITY CHAMBER PRESSURE
360.0 380.0 400.0 420.0 440.0 460.0 480.0 500.0
200.0
175.0
150.0
125.0
100.0
75.0
50.0
25.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

TABLE 1

Goodness of Fit Test for Lake City Chamber Pressure Data

(After Some Regrouping)

Interval x 100 psi	observed no. of occurances f_i	Expected no. of occurances $n\theta_i$ Fitted Normal∿ $N(462,12.7)^2$	$\frac{(f_i-n\theta_i)^2}{n\theta_i}$
372.0 to 430.95	4	6.6	1.00
430.95 to 437.5	9	17.9	4.35
437.5 to 444.05	48	51.0	0.18
444.05 to 450.6	97	99.0	0.04
450.6 to 457.15	204	162.0	10.90
457.15 to 463.7	186	188.5	0.03
463.7 to 470.25	177	177.0	0.00
470.25 to 476.8	105	127.0	3.80
476.8 to 483.35	69	70.0	0.16
483.35 to 489.9	28	30.0	0.13
489.9 to 496.45	9	10.3	0.17
496.45 to 503.0	6	3.0	3.00
	Total=942	Total=942	Total= 23.76

Figure 35

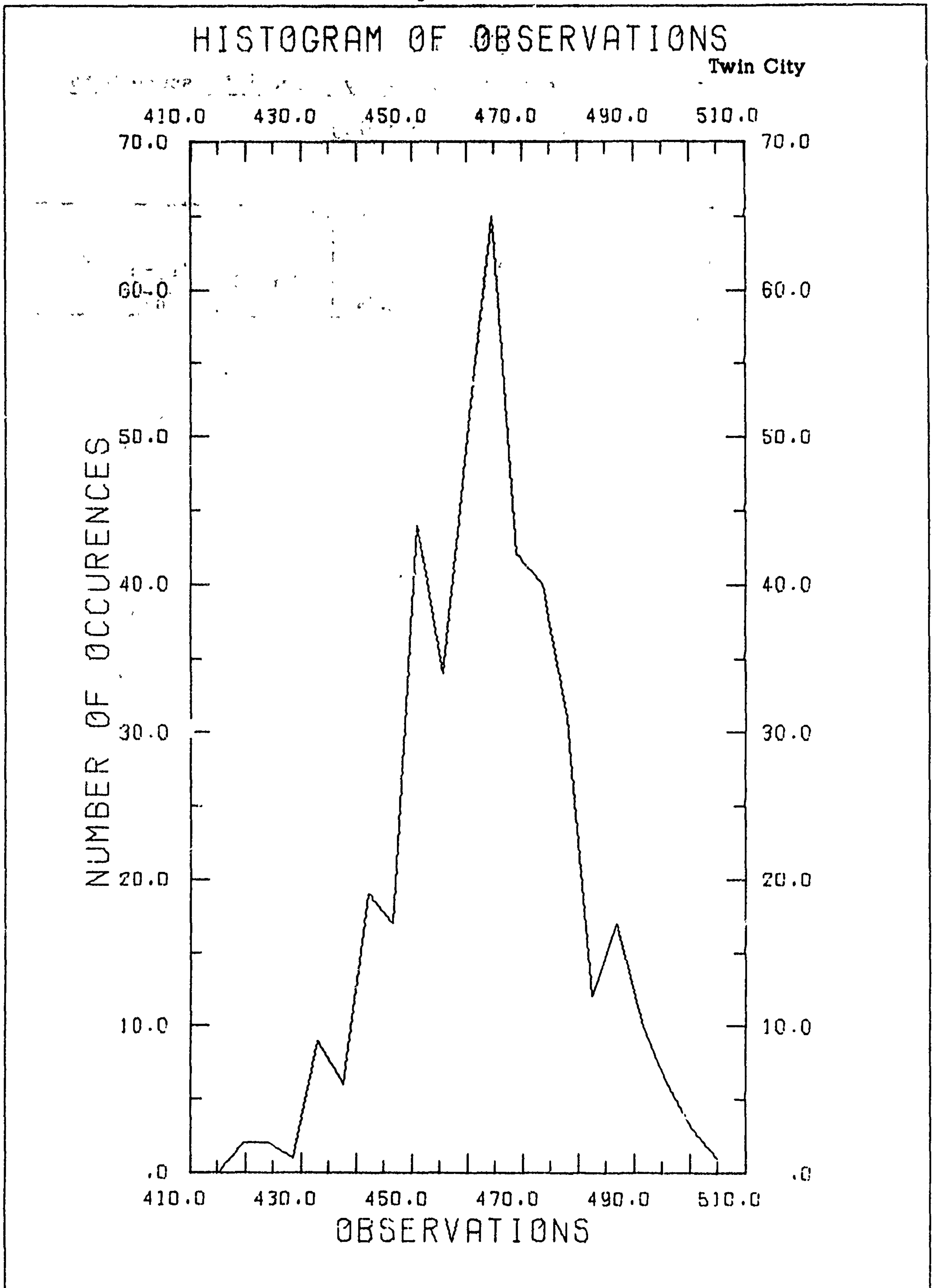

HISTOGRAM OF OBSERVATIONS
Twin City
410.0
430.0
450.0
470.0
490.0
510.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

Figure 36

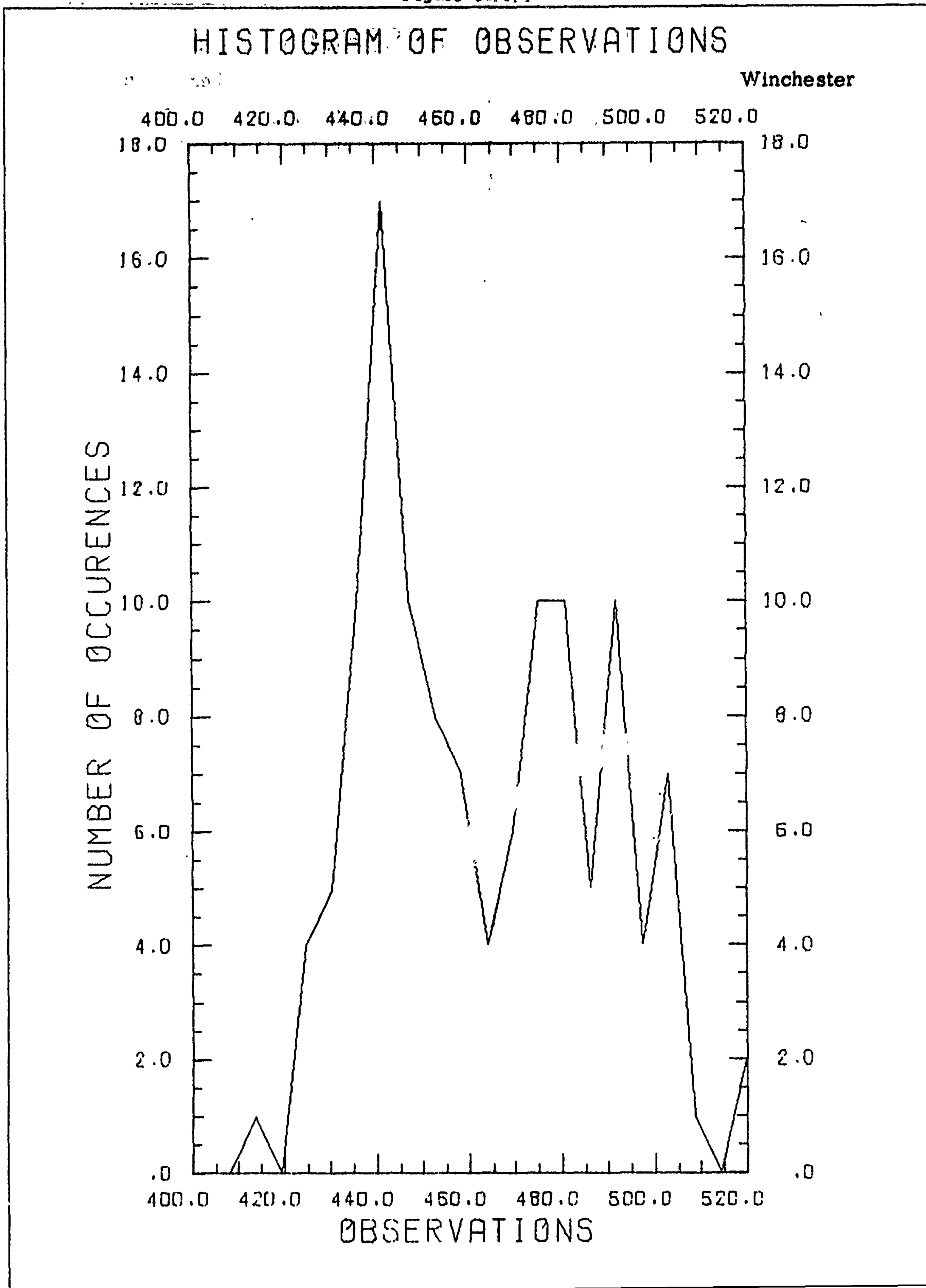

HISTOGRAM OF OBSERVATIONS
Winchester
400.0 420.0 440.0 460.0 480.0 500.0 520.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

Figure 37

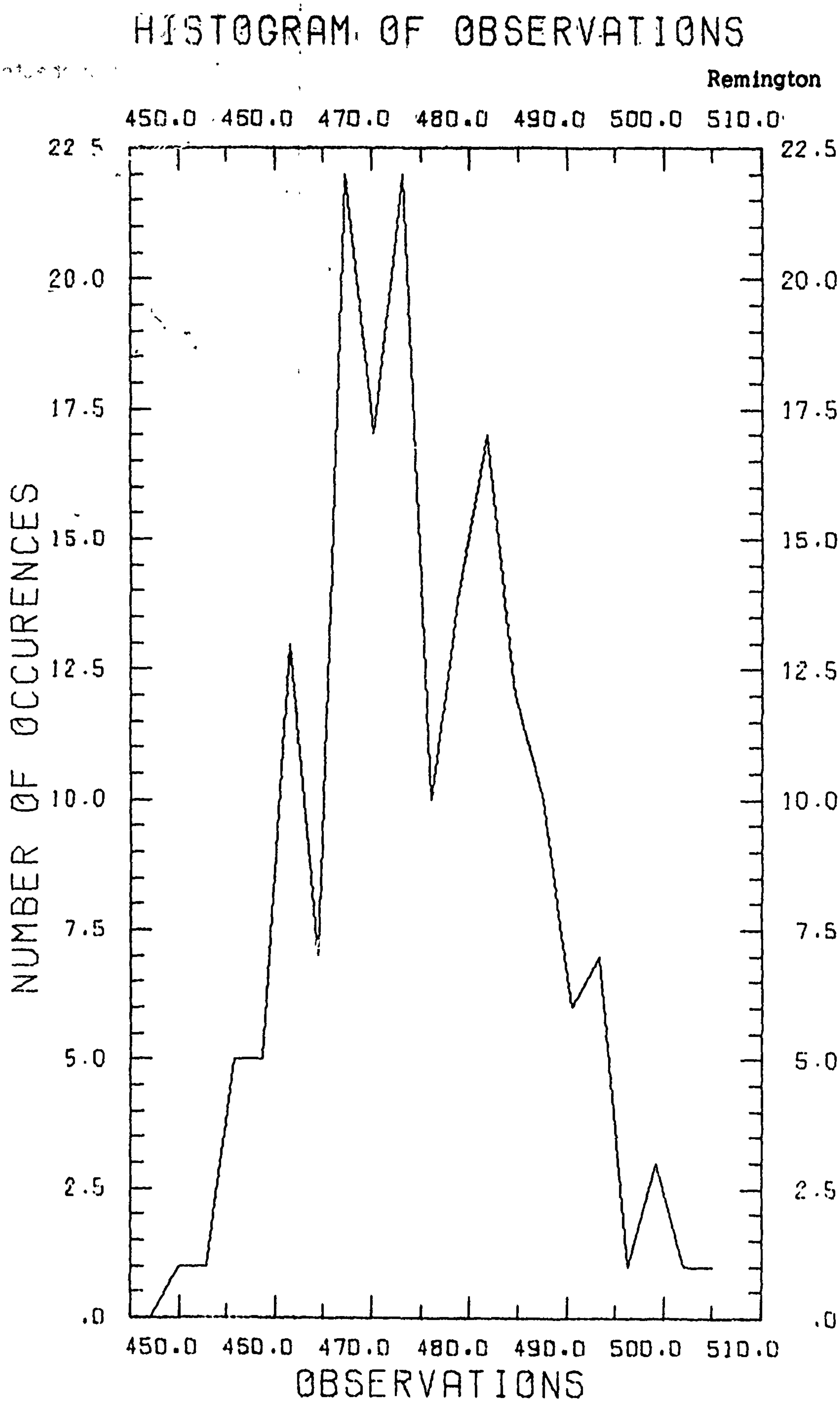
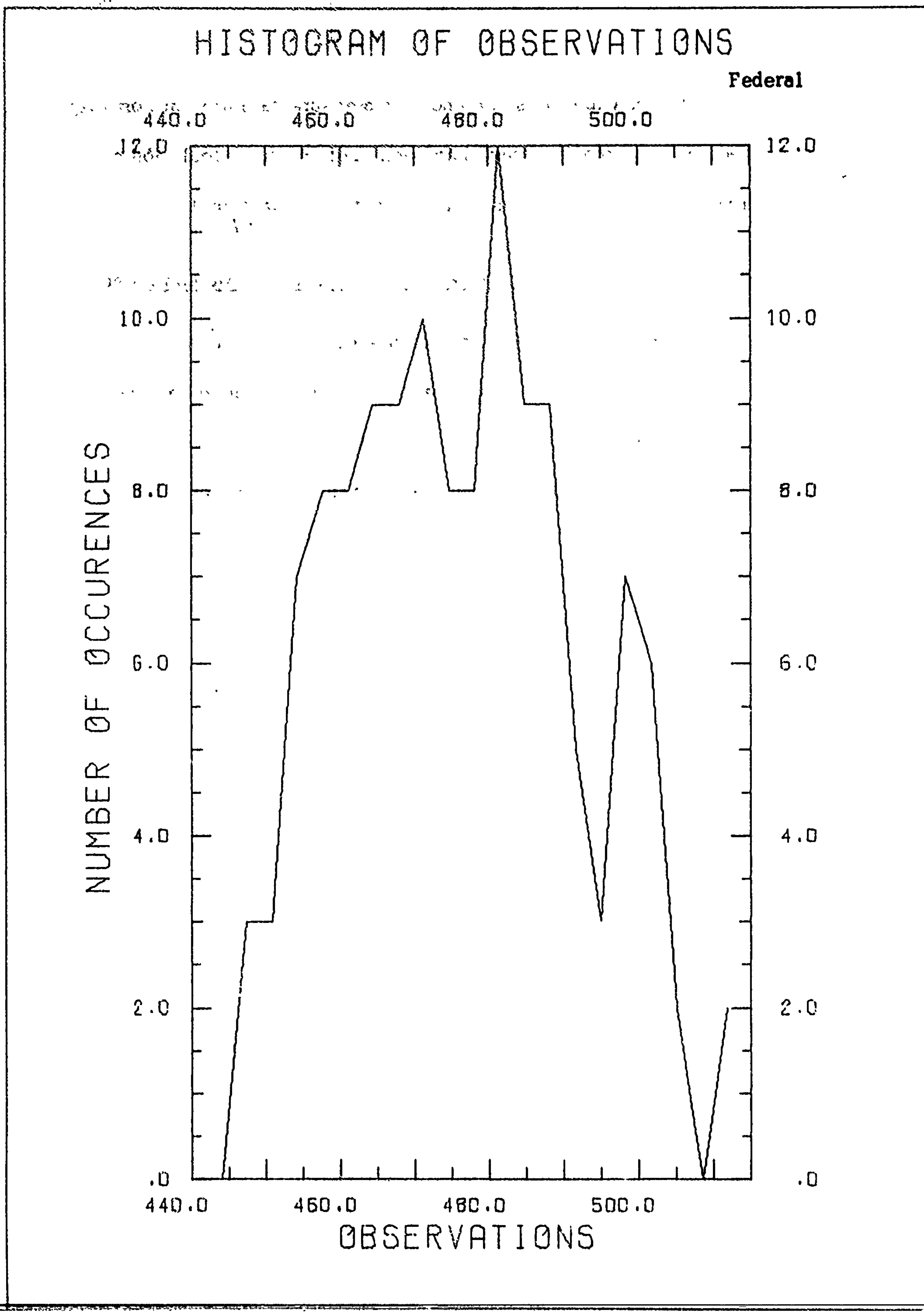

HISTOGRAM OF OBSERVATIONS
Remington
450.0 460.0 470.0 480.0 490.0 500.0 510.0
22.5
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
.0
NUMBER OF OCCURENCES
OBSERVATIONS

Figure 38


HISTOGRAM OF OBSERVATIONS
Federal
440.0
460.0
480.0
500.0
12.0
10.0
8.0
6.0
4.0
2.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

Since the chamber pressure and port pressure data are not normally distributed, an empirical approach has been used for the calculation of control limits as well as for ammunition selection for weapon tests.

3.6 Comparison of Chamber Pressure Data from Different Manufacturers

Table 3 shows the mean and variance of chamber pressure data from different manufacturers. Data after the cutoff date alone have been used for this comparison. The variance (between lots) of chamber pressure is seen to vary considerably from manufacturer to manufacturer. Means are observed to be quite close to each other. The mean chamber pressure for Ball ammunition is lower than that for Tracer ammunition.

Figure 39

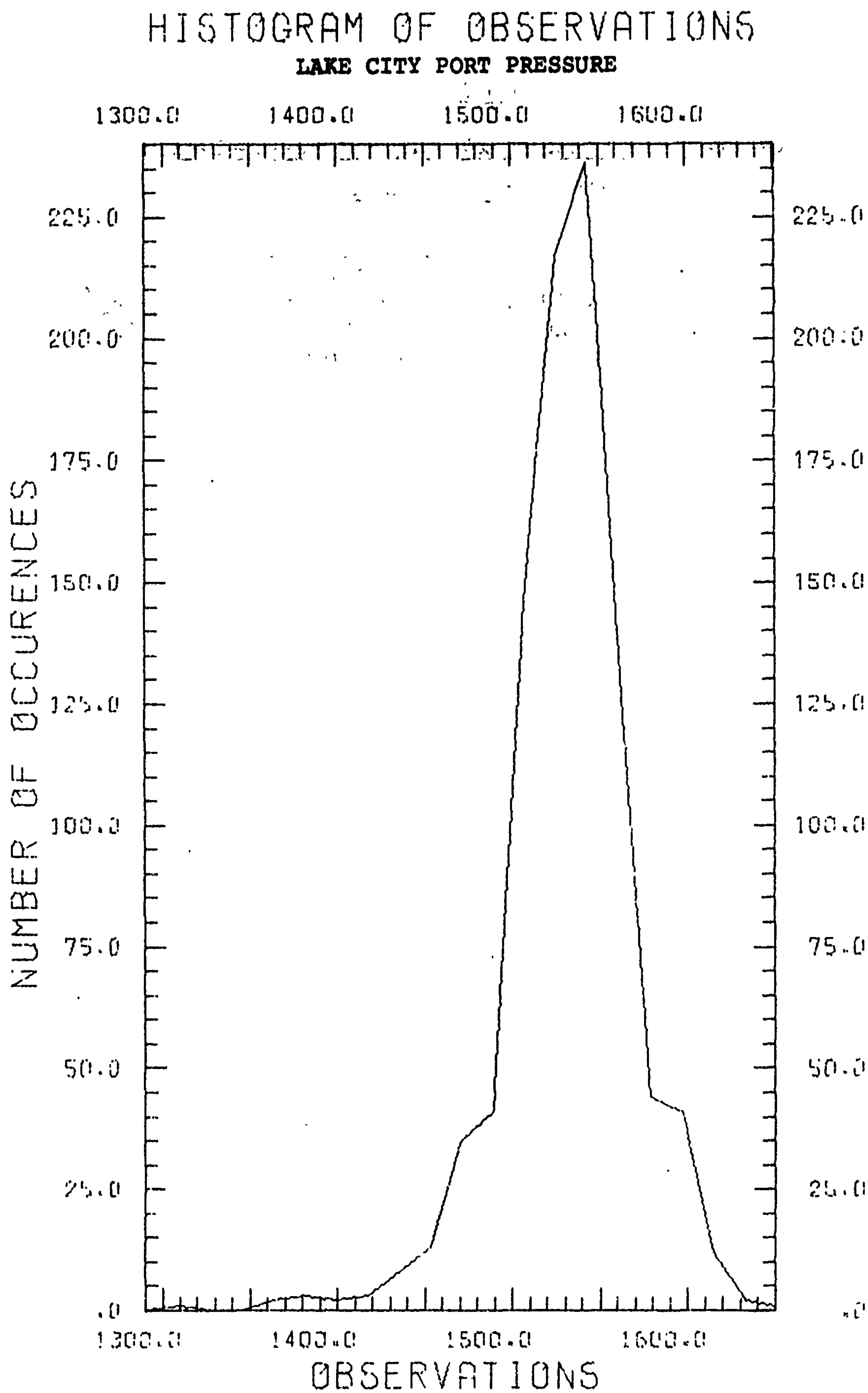

HISTOGRAM OF OBSERVATIONS
LAKE CITY PORT PRESSURE
1300.0
1400.0
1500.0
1600.0
225.0
200.0
175.0
150.0
125.0
100.0
75.0
50.0
25.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

TABLE 2

Goodness of Fit Test for Lake City Port Pressure Data

(After Some Regrouping)

Interval x 10 psi	Observed No. of Occurances fi	Expected No. of Occurances nθi Fitted Normal, $N(1533, 36.6^2)$	$\frac{(Fi-n\theta i)^2}{n\theta i}$
1300 to 1390	6	1	25
1390 to 1426	5	2	4.5
1426 to 1444	8	5.6	1.0
1444 to 1462	13	17	1.0
1462 to 1480	35	45	2.2
1480 to 1498	41	91.4	27.8
1498 to 1516	146	146	0
1516 to 1534	217	164	17.1
1534 to 1552	236	185	14.0
1552 to 1570	135	139	0.1
1570 to 1588	44	84	19.0
1588 to 1606	41	41	0
1606 to 1660	15	21	1.7
	Total = 942	Total = 942	Total = 113.4

TABLE 3

Comparison of Mean and Variance of Chamber Pressure from Five Manufacturers (Data after Cutoff Date)

Manufacturers	Chamber Pressure (Ball) Mean x 100 psi	Chamber Pressure (Ball) Variance x 10,000 psi^2	Chamber Pressure (Tracer) Mean x 100 psi	Chamber Pressure (Tracer) Variance x 10,000 psi^2
Lake City	462	462	488	321
Twin City	461	217	490	236
Winchester	461	629	499	140
Remington	474	118		
Federal	474	238		

3.7 Time Series Analysis of Lake City Data after Cutoff

Time Series Analysis was carried out on Lake City Chamber pressure data after cutoff. A total of 942 observations were considered. The model was found to exhibit a nonstationary seasonal behavior. The fitted model is

$$(1 - \phi_{22}B^{22} - \phi_{23}B^{23})(1 - B)Z_t = (1 - \theta_1 B)a_t$$

where Z_t is the chamber pressure at t^{th} lot.

The estimated parameter values are

$$\hat{\phi}_{22} = -0.05613$$
$$\hat{\phi}_{23} = 0.1307$$
$$\hat{\theta}_1 = 0.6651$$

The parameter confidence intervals are

$$-0.1169 \leq \phi_{22} \leq 0.0046$$

$$0.0704 \leq \phi_{23} \leq 0.1910$$

$$0.6161 \leq \theta_1 \leq 0.7142$$

Even though the confidence interval on ϕ_{22} includes zero, it was decided to retain it because a much better fit is obtained by its inclusion.

The correlation matrix is

$$\begin{array}{ccc} \phi_{22} & \phi_{23} & \theta_1 \end{array}$$
$$\begin{bmatrix} 1.0 & & \\ 0.0319 & 1.0 & \\ 0.0309 & 0.0312 & 1.0 \end{bmatrix}$$

The error autocorrelations and χ^2 test indicate the model to be adequate.

3.8 Analysis of Chamber Pressure Variance (Lake City and Twin City)

Chamber pressure variance within each lot for Lake City and for Twin City is plotted in Figures 42 and 43 respectively. The average value of within lot variance for Lake City is 1760806 $(psi)^2$ and for Twin City is 2905842 $(psi)^2$. Lake City, therefore, has a smaller within lot chamber pressure variance as compared to Twin City. If it were possible to assume that identical ball powder is supplied to both the plants, then this difference in variance should be attributed to variation in components and/or standard testing procedures.

The method of cumulative sum is now employed to determine the underlying process. The results are shown in Figures 44 and 45. In both cases, the trend is found to be toward a reduction of within-lot variance. This indicates the overall improvement in production and testing processes. Twin City shows a larger reduction as compared to Lake City.

Figure 41

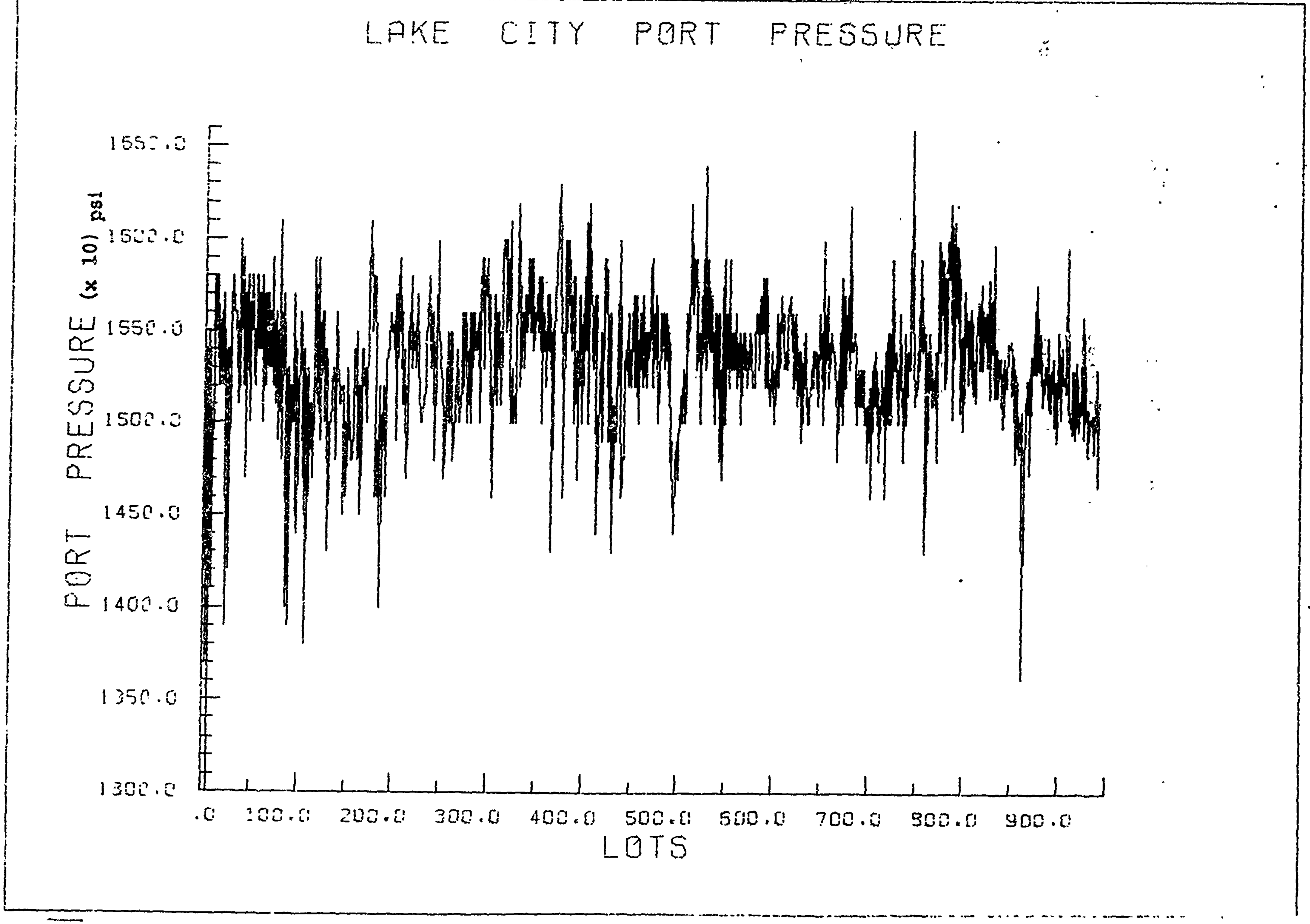

LAKE CITY PORT PRESSURE
PORT PRESSURE (x 10) psi
1600.0
1550.0
1500.0
1450.0
1400.0
1350.0
1300.0
.0
100.0
200.0
300.0
400.0
500.0
600.0
700.0
800.0
900.0
LOTS

Figure 42

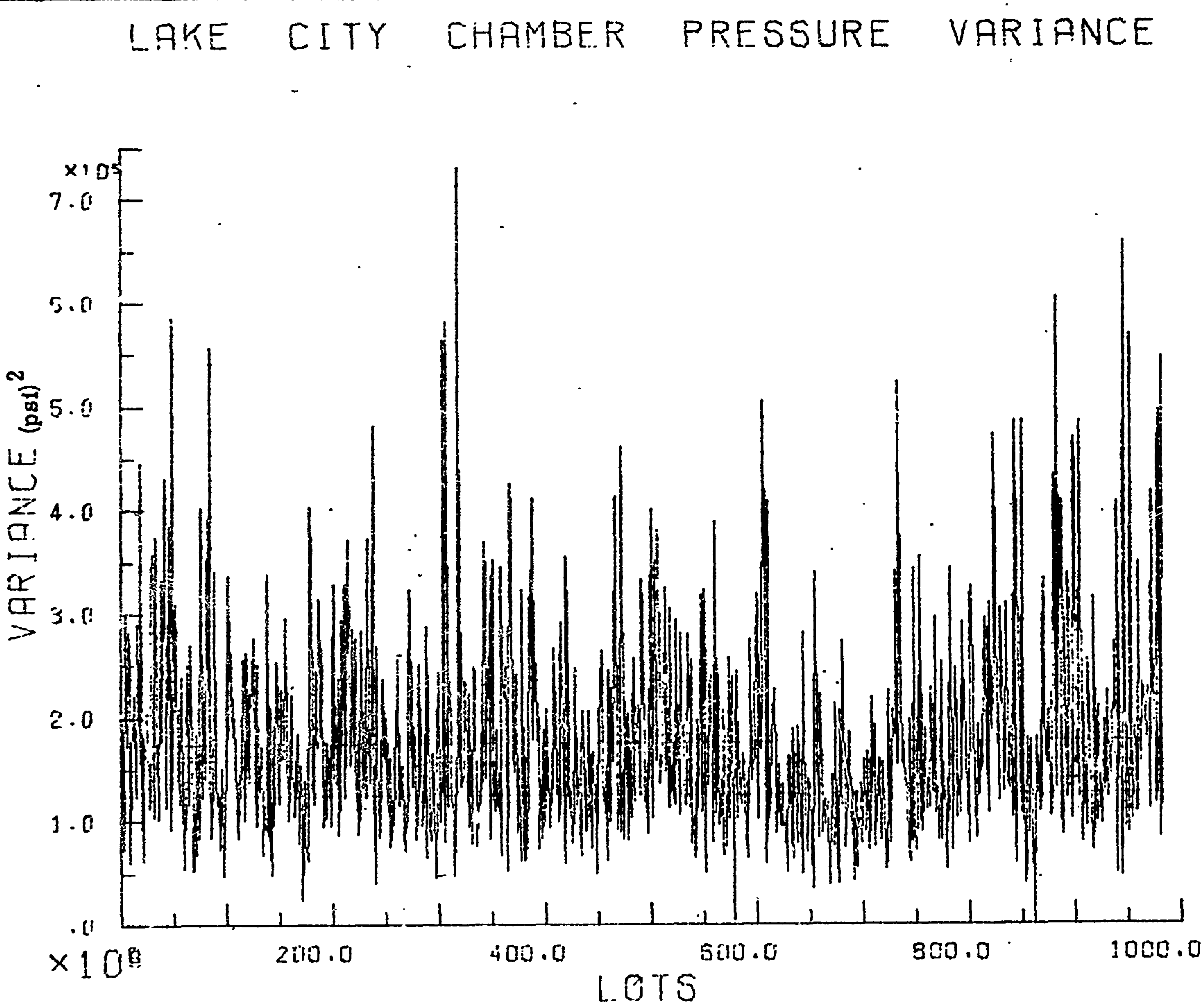
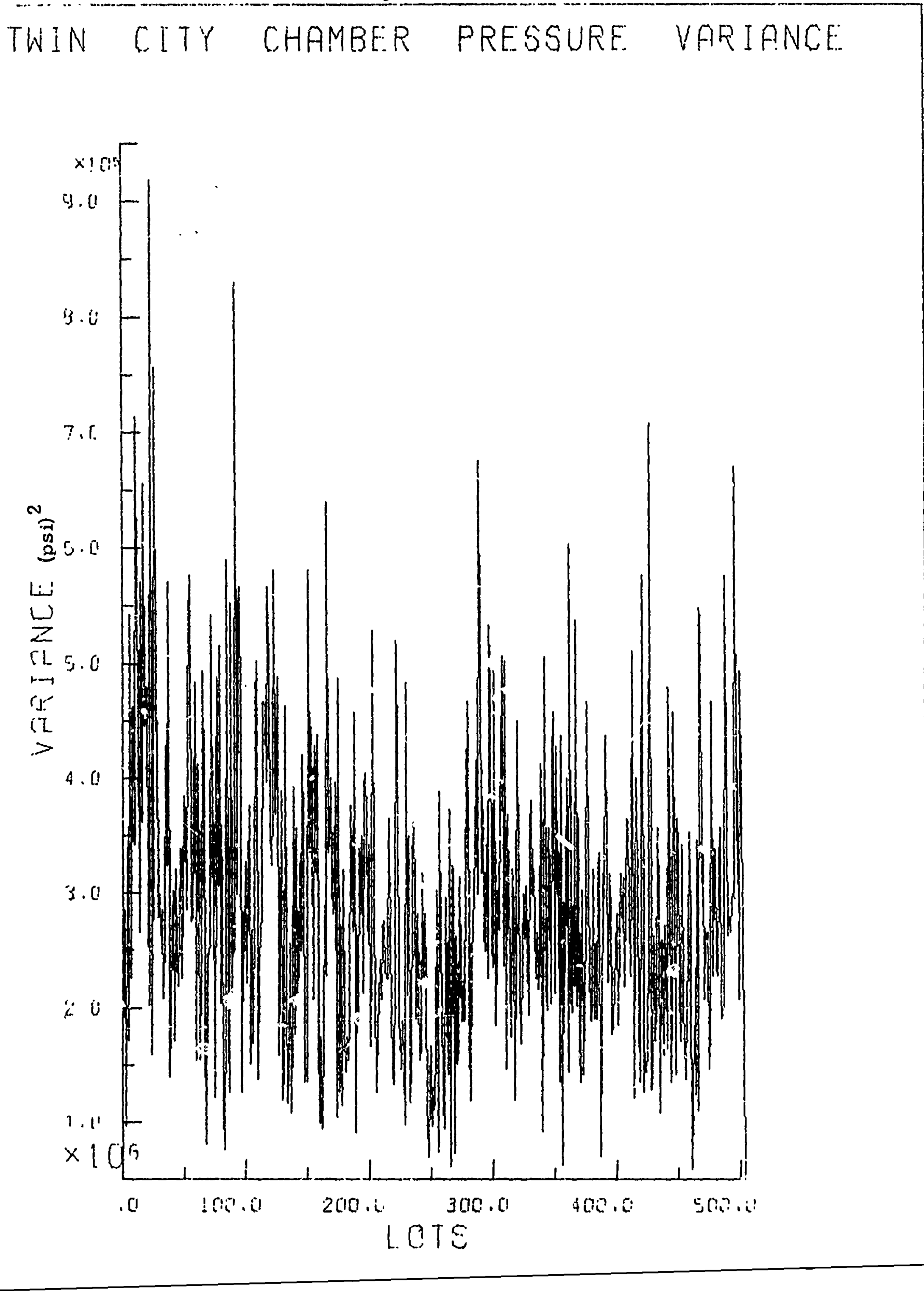

LAKE CITY CHAMBER PRESSURE VARIANCE
VARIANCE (psi)^2
x10^5
7.0
4.0
3.0
2.0
1.0
.0
x10^0
200.0
400.0
600.0
800.0
1000.0
LOTS

Figure 43


TWIN CITY CHAMBER PRESSURE VARIANCE
x10⁵
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
x10⁶
VARIANCE (psi)²
.0
100.0
200.0
300.0
400.0
500.0
LOTS

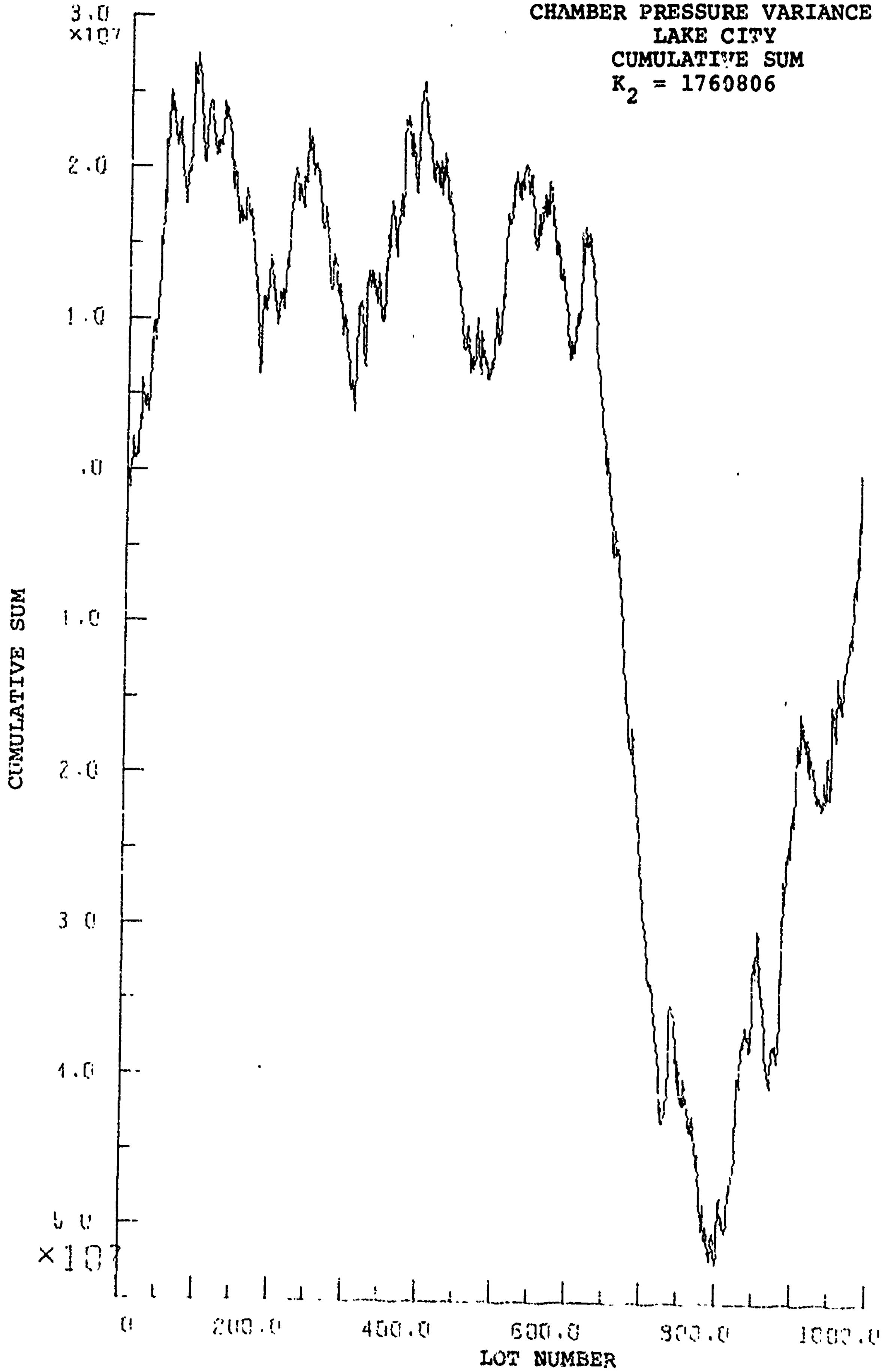
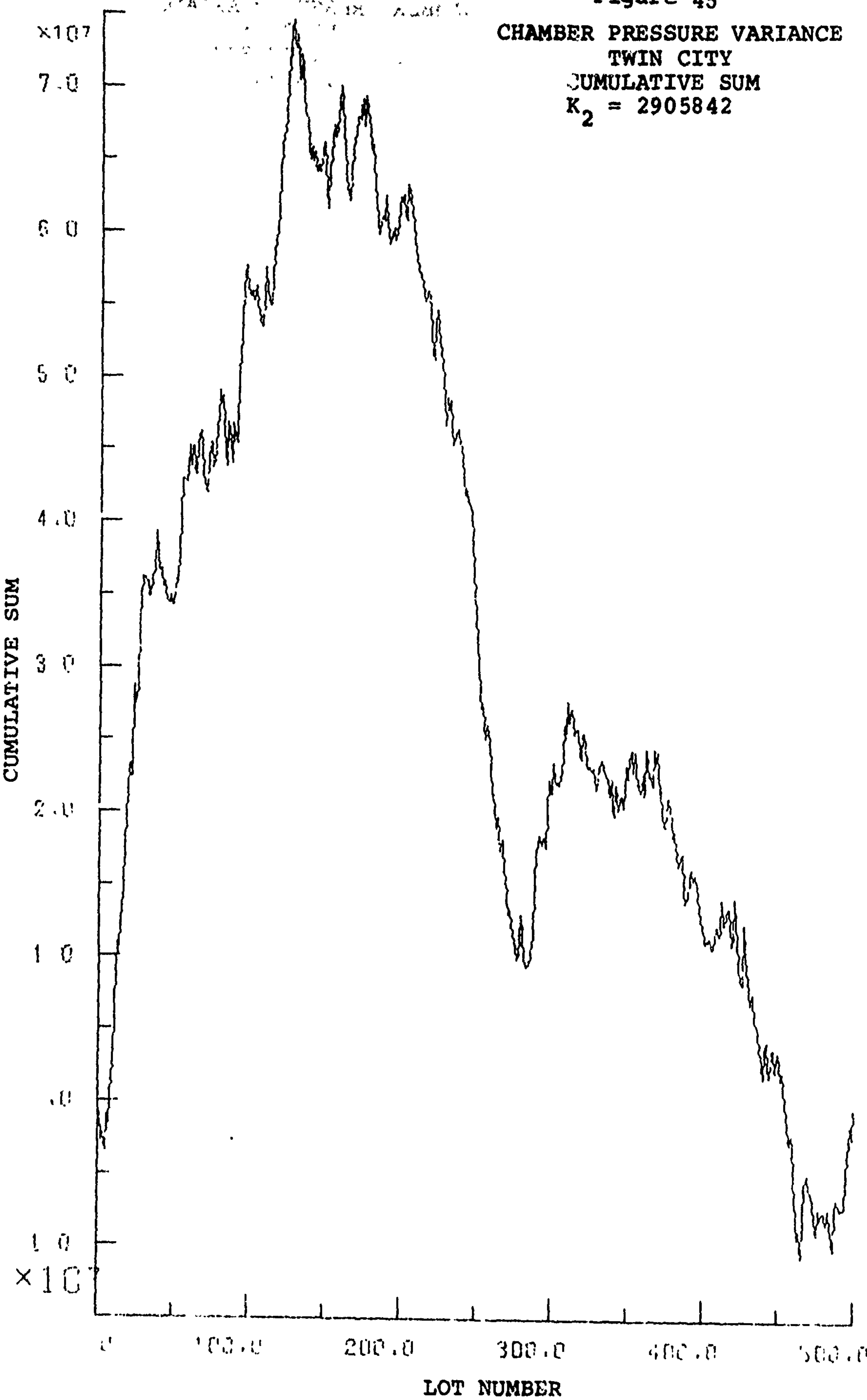


Figure 44
CHAMBER PRESSURE VARIANCE
LAKE CITY
CUMULATIVE SUM
$K_2 = 1760806$

Figure 45

CHAMBER PRESSURE VARIANCE
TWIN CITY
CUMULATIVE SUM
K_2 = 2905842



4. CUTOFF DATE AND CONTROL LIMITS

4.1 Method of Approach

Present control limits for ammunition production are wide leading to severe gun design requirements. Attempts are therefore to be made to narrow the control limits.

It is observed that the chamber pressure plots show a downward trend. This can be attributed to drastic process changes or the initial experimental stage. If the chamber pressure control limits are based upon chamber pressure values after the process changes are completed (i.e., after the cutoff date), narrower control limits can be obtained.

Cumulative sum technique with reference constant equal to the mean of the series is used to determine the cutoff date. An empirical approach using cumulative distribution function is then employed to obtain the control limits. These control limits are based upon data after the cut-off date.

4.2 Cutoff Date

The cutoff date should satisfy the following requirements:

(1) It should be possible to narrow the control limits when based upon observations after the cutoff date. This implies that the chamber pressure data after the cutoff date should be sufficiently smaller in magnitude than the data before the cutoff date.

(2) This reduction in magnitude should be maintained for a considerable past period of time so that it could be attributed to an improvement in process rather than to cyclic or chance variations.

(3) Information might be available regarding the date on which process changes intended to reduce the chamber pressure were introduced. If the observed point of shift corresponds to this prior information, then it may be taken as the cutoff date.

Method employed to determine the cutoff date is as follows: Computer plots of available chamber pressure data are obtained. The plots are updated as more data become available. Usually the visual picture indicates whether a point of shift exists; for example, Figures 46 and 47 show the tracer chamber pressure plots for Lake City and Twin City respectively. (These plots are updated over those given in Figures 20 and 21). A careful examination of these plots shows a point of shift approximately 80 lots away from the initial reference value in both cases. No such downward shift is observed in the data from Winchester (Fig. 48).

Cusum plotting is now employed to quantify the point of shift. The resulting cusum plots for Ball and Tracer ammunition from different manufacturers are shown in Figures 49 through 56. It is assumed that these points of shift satisfy the conditions necessary for cutoff date. The point of shift gives the lot number where the shift occurred. Cutoff date is the corresponding date given in the standard lot testing results. Cutoff dates are tabulated in Table 4.

Figure 46

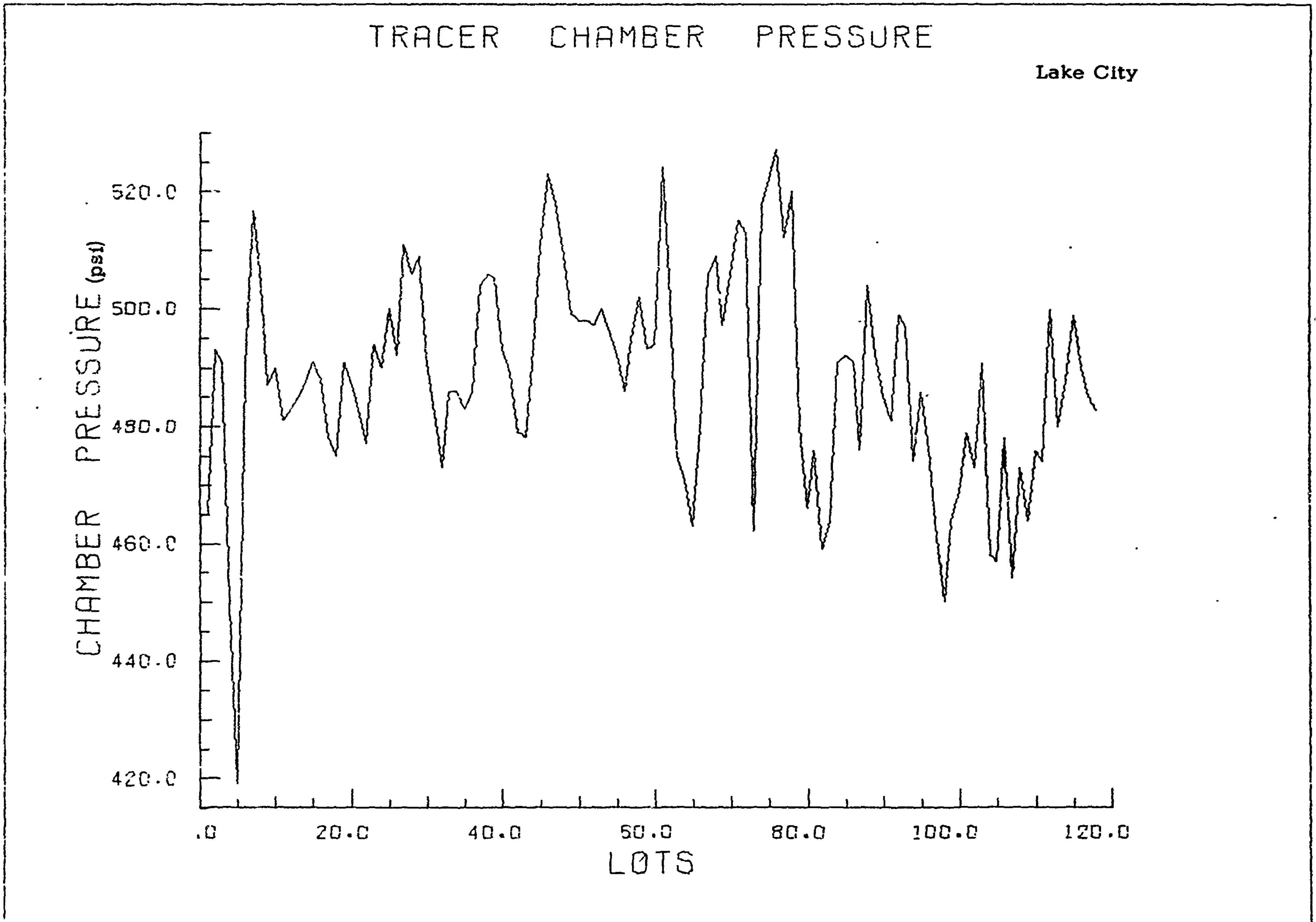

TRACER CHAMBER PRESSURE
Lake City
CHAMBER PRESSURE (psi)
520.0
500.0
480.0
460.0
440.0
420.0
.0
20.0
40.0
60.0
80.0
100.0
120.0
LOTS

Figure 47

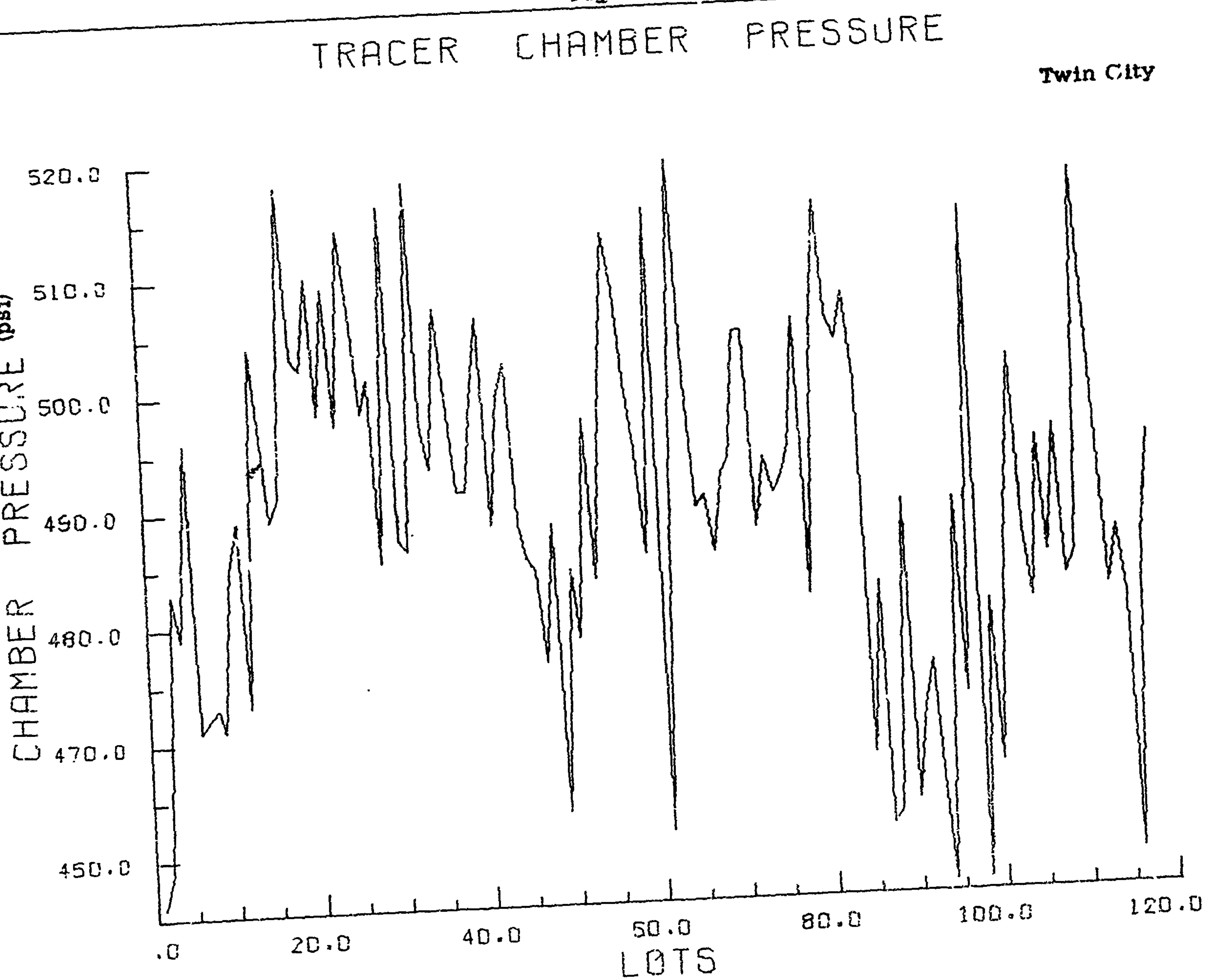
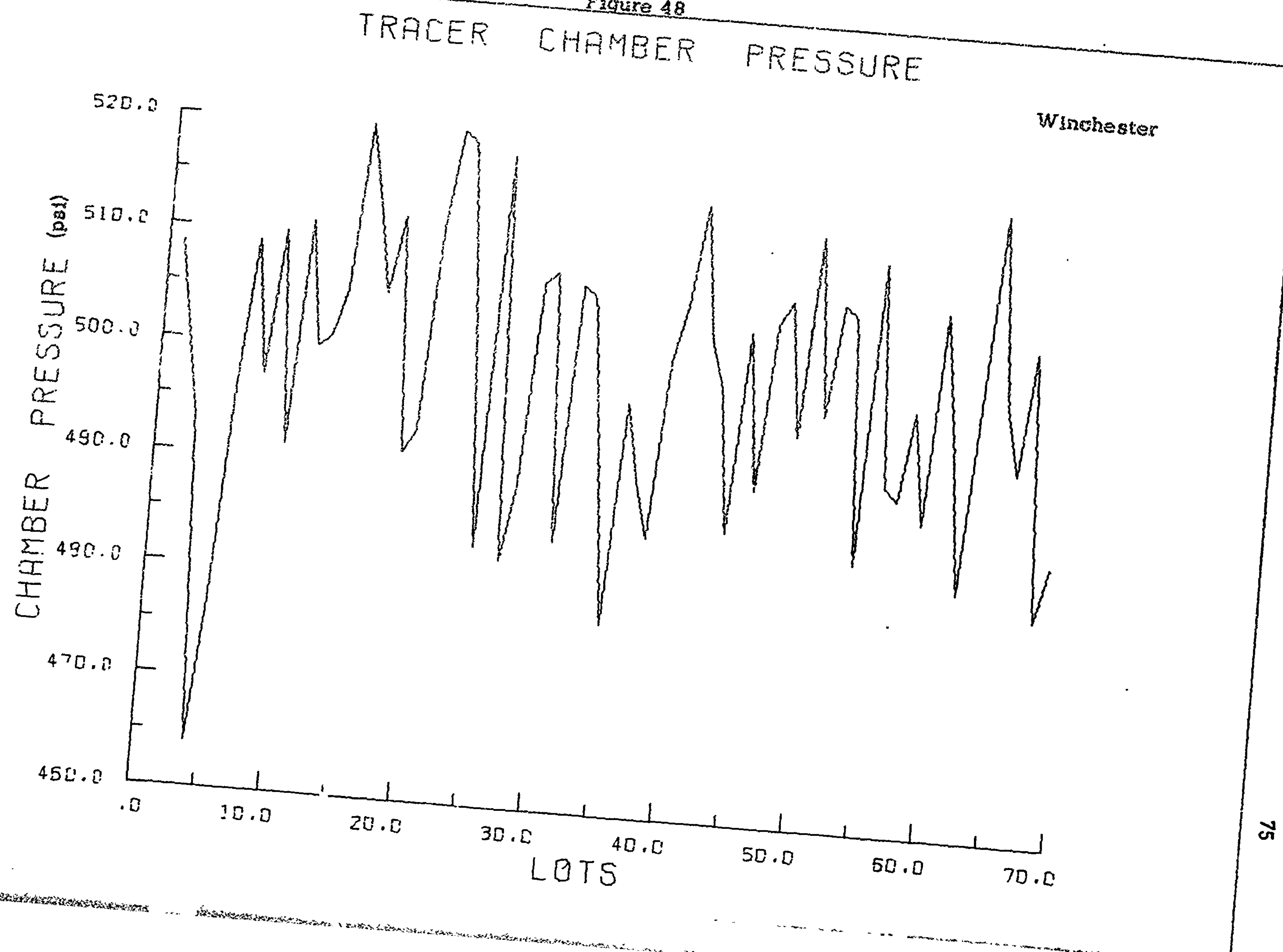

TRACER CHAMBER PRESSURE
Twin City
CHAMBER PRESSURE (psi)
520.0
510.0
500.0
490.0
480.0
470.0
450.0
.0
20.0
40.0
50.0
80.0
100.0
120.0
LOTS

Figure 48


TRACER CHAMBER PRESSURE
Winchester
CHAMBER PRESSURE (psi)
520.0
510.0
500.0
490.0
480.0
470.0
460.0
.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
LOTS

4.3 Control Limits

Control limits are based upon data after the cutoff date. The following procedure is used to determine control limits.

The range of chamber pressure readings (range = maximum chamber pressure - minimum chamber pressure) is subdivided into twenty equal parts and the number of lots belonging to each subgroup is calculated. These numbers are then divided by the total number of lots to obtain an estimate of the probability of belonging to each subgroup. Cumulative sum of these probabilities is plotted against the corresponding chamber pressure. The resulting plot is known as the empirical cumulative distribution function (empirical c.d.f.).

Only one sided (upper) control limit need to be calculated for the chamber pressure data. This is taken as the 99 percentile of the empirical c.d.f.

Plots of empirical c.d.f. for chamber pressure data (Ball and Tracer) from the different manufacturers are given in Figures 57 through 64. The calculated control limits are given in Table 5.

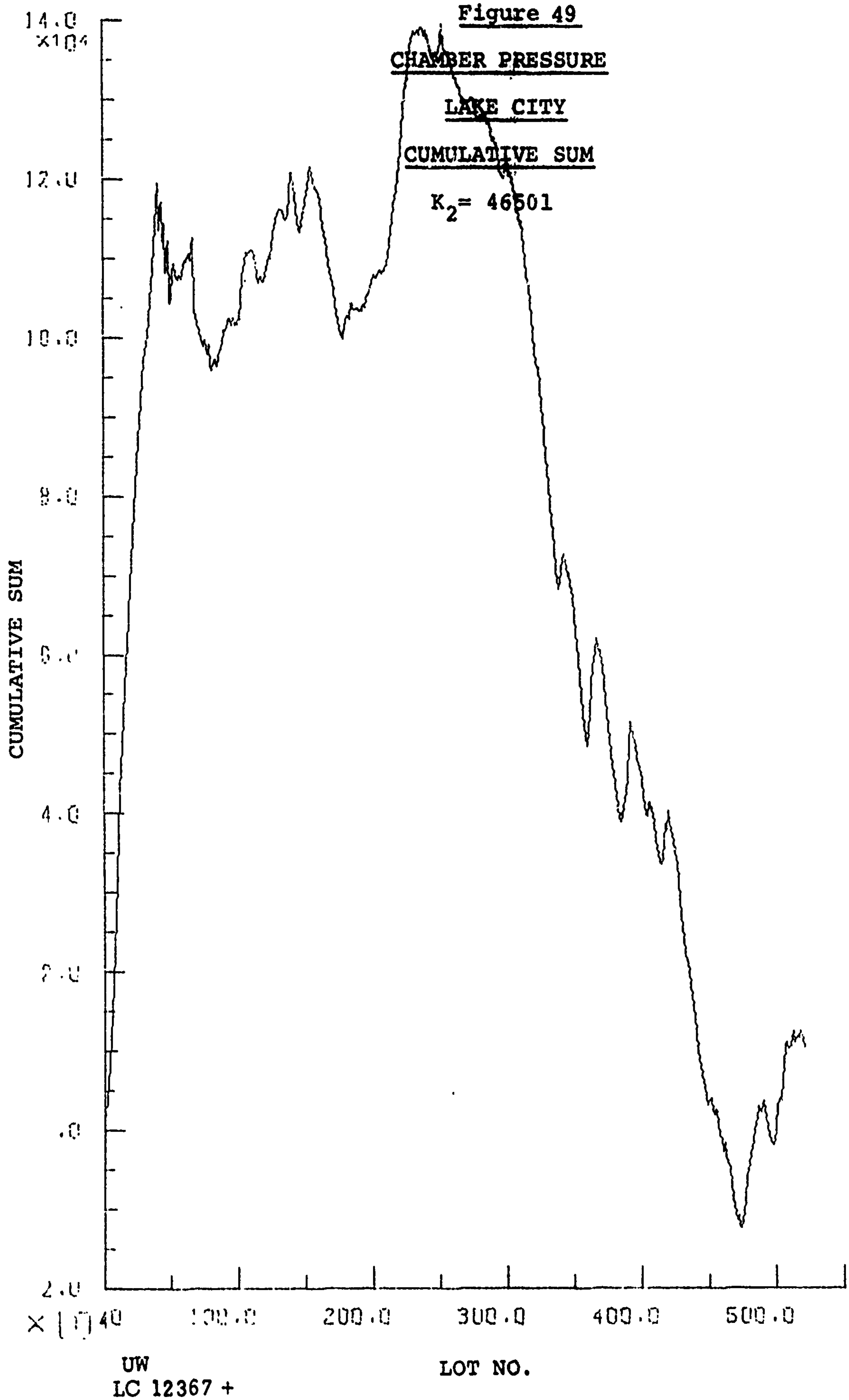


Figure 49
CHAMBER PRESSURE
LAKE CITY
CUMULATIVE SUM
K_2= 46501
CUMULATIVE SUM
14.0
×10⁴
12.0
10.0
8.0
6.0
4.0
2.0
.0
2.0
40
100.0
200.0
300.0
400.0
500.0
UW
LC 12367 +
LOT NO.

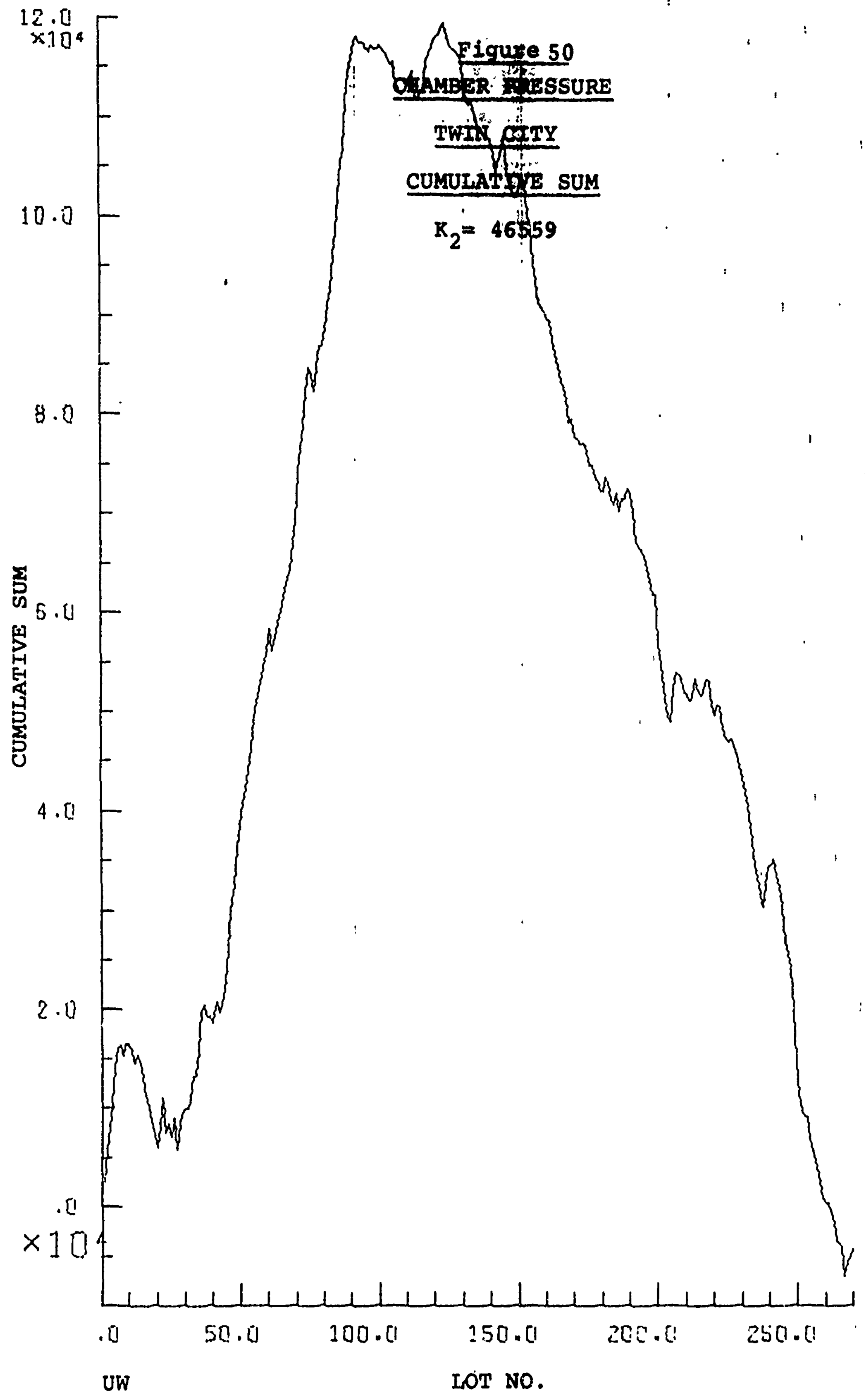


Figure 50
CHAMBER PRESSURE
TWIN CITY
CUMULATIVE SUM
K_2= 46559
12.0
×10⁴
10.0
8.0
6.0
4.0
2.0
.0
×10⁴
CUMULATIVE SUM
.0
50.0
100.0
150.0
200.0
250.0
UW
LOT NO.

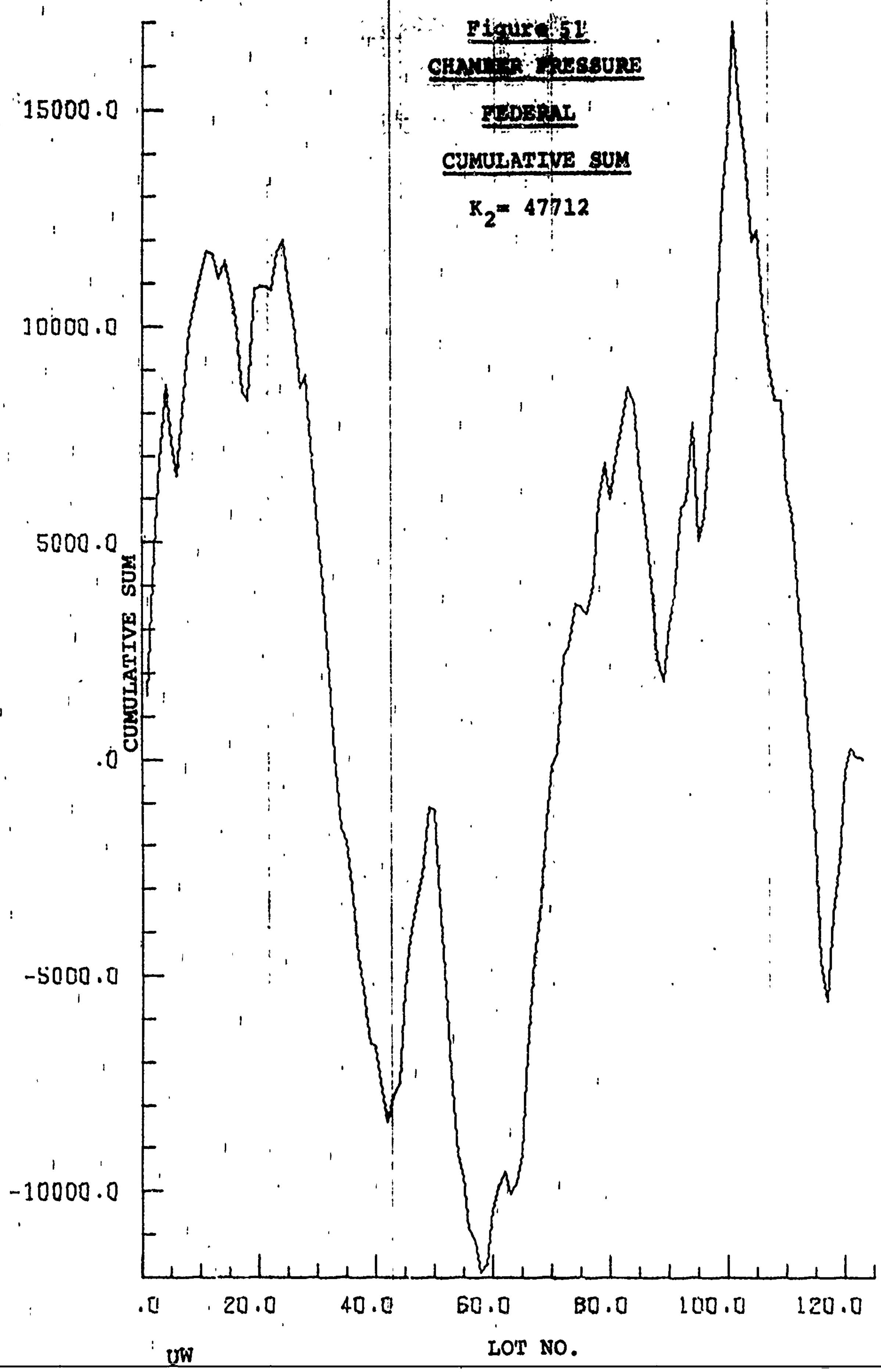


Figure 51
CHAMBER PRESSURE
FEDERAL
CUMULATIVE SUM
K2= 47712
15000.0
10000.0
5000.0
.0
-5000.0
-10000.0
CUMULATIVE SUM
.0
20.0
40.0
60.0
80.0
100.0
120.0
LOT NO.
UW

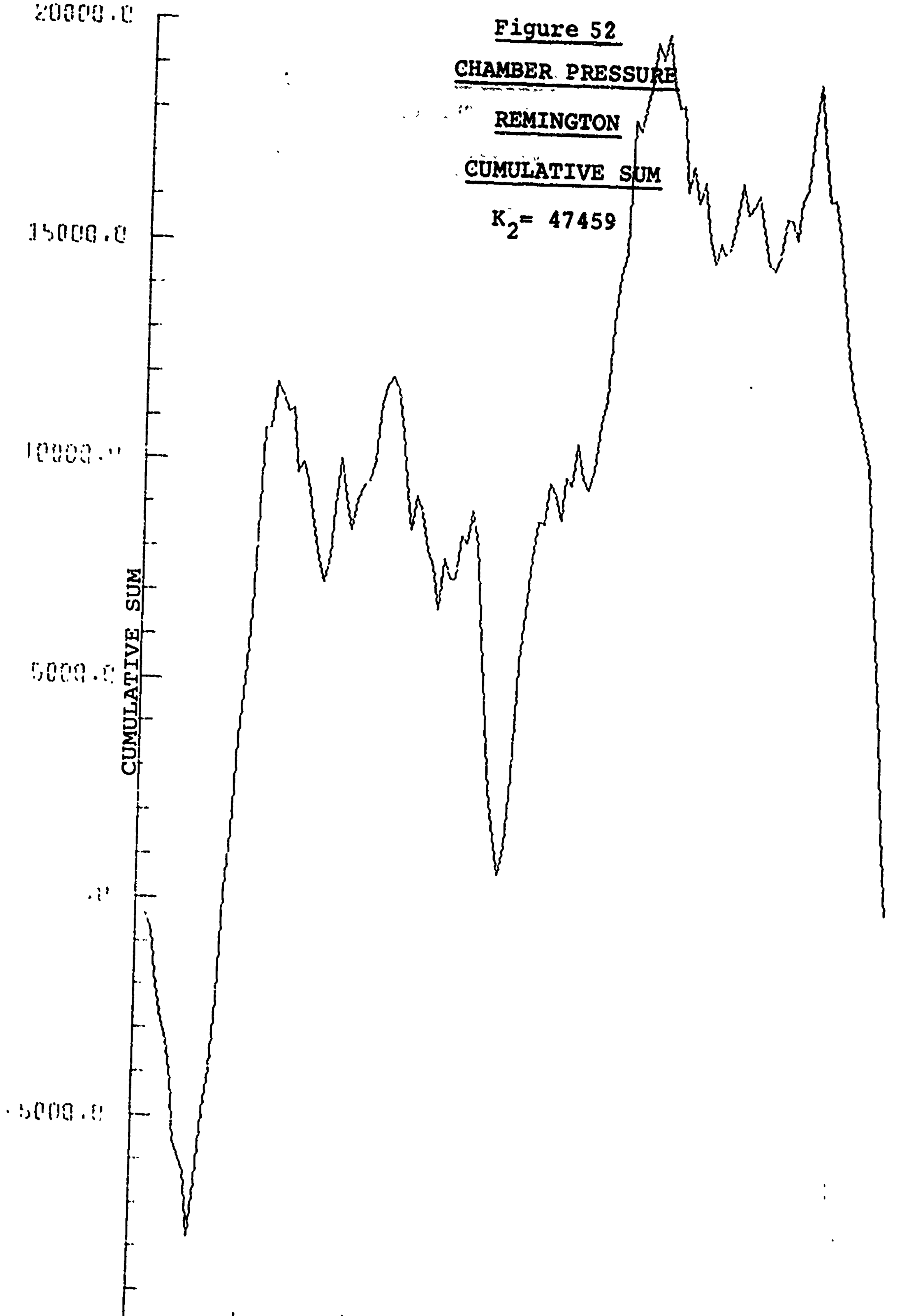


Figure 52
CHAMBER PRESSURE
REMINGTON
CUMULATIVE SUM
K2= 47459
CUMULATIVE SUM
20000.0
15000.0
10000.0
5000.0
.0
-5000.0
-10000.0
.0
20.0
40.0
60.0
80.0
100.0
120.0
140.0
LOT NO.

UW
5353 +

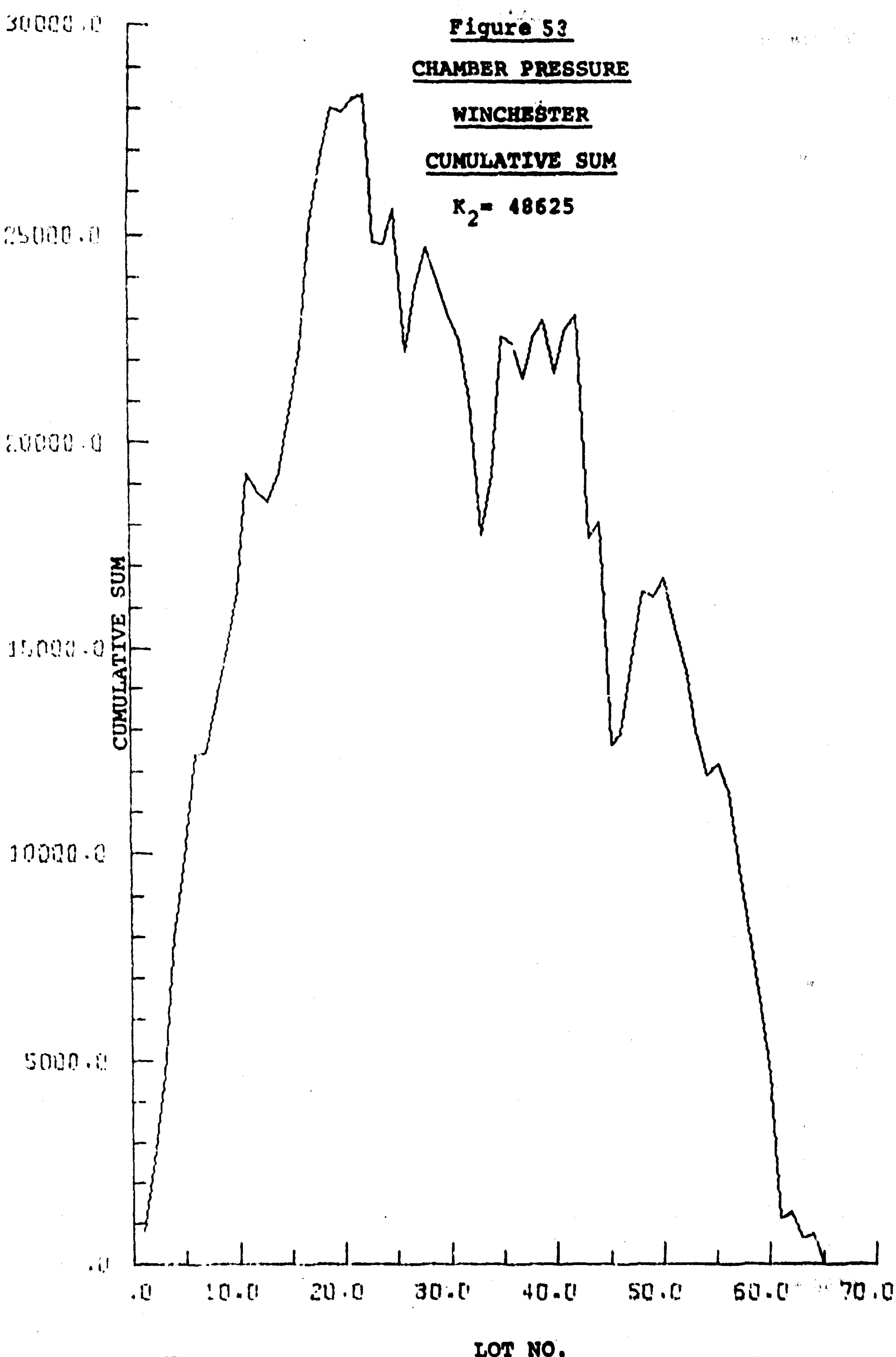


Figure 53
CHAMBER PRESSURE
WINCHESTER
CUMULATIVE SUM
K_2= 48625
30000.0
25000.0
20000.0
15000.0
10000.0
5000.0
.0
CUMULATIVE SUM
.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
LOT NO.

UW
6124 +

Figure 54

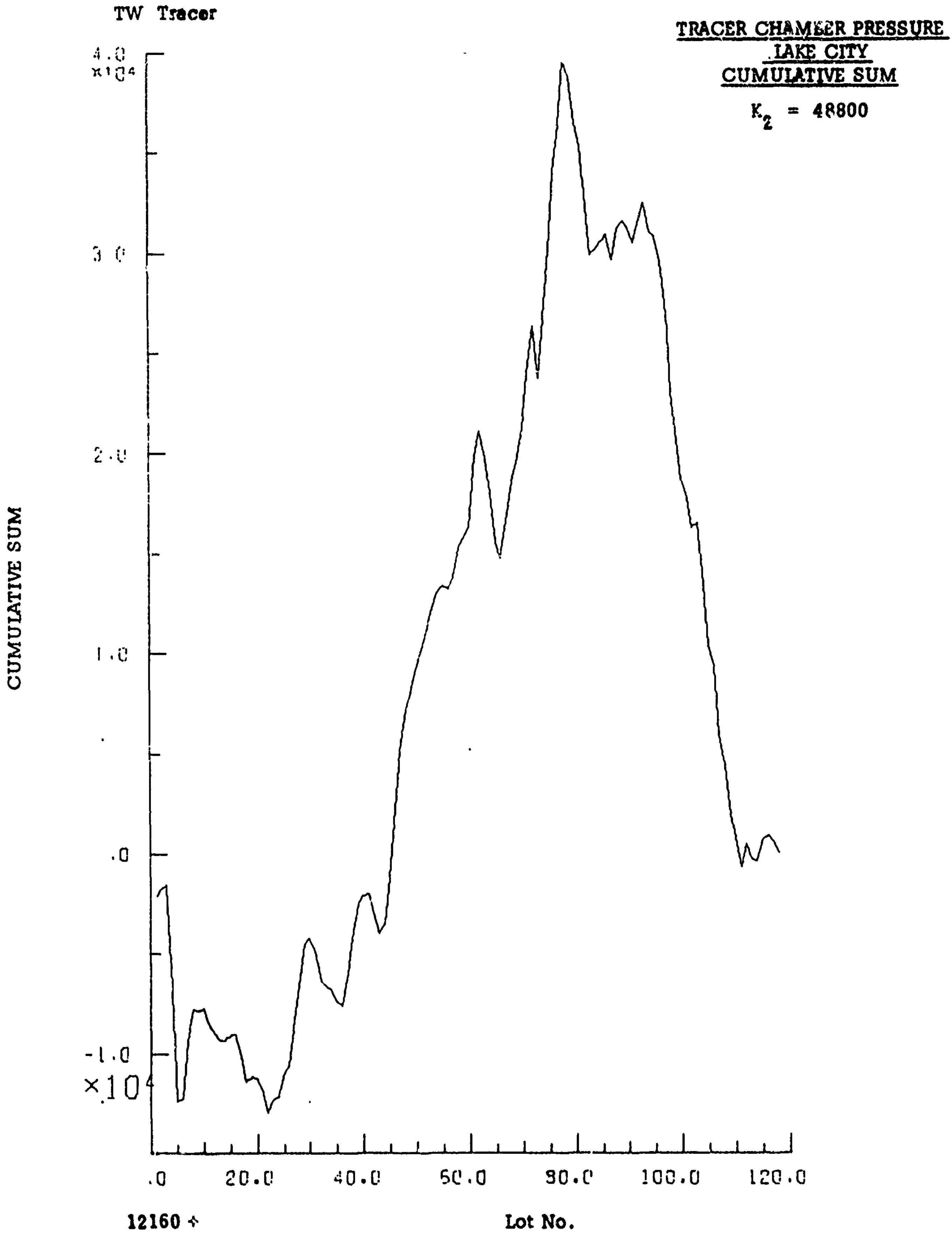

TW Tracer
TRACER CHAMBER PRESSURE
LAKE CITY
CUMULATIVE SUM
K_2 = 48800
4.0 x10⁴
3.0
2.0
1.0
.0
-1.0
×10⁴
CUMULATIVE SUM
.0
20.0
40.0
60.0
80.0
100.0
120.0
12160 +
Lot No.

Figure 55

TW Tracer

TRACER CHAMBER PRESSURE
TWIN CITY
CUMULATIVE SUM

K_2 = 49000

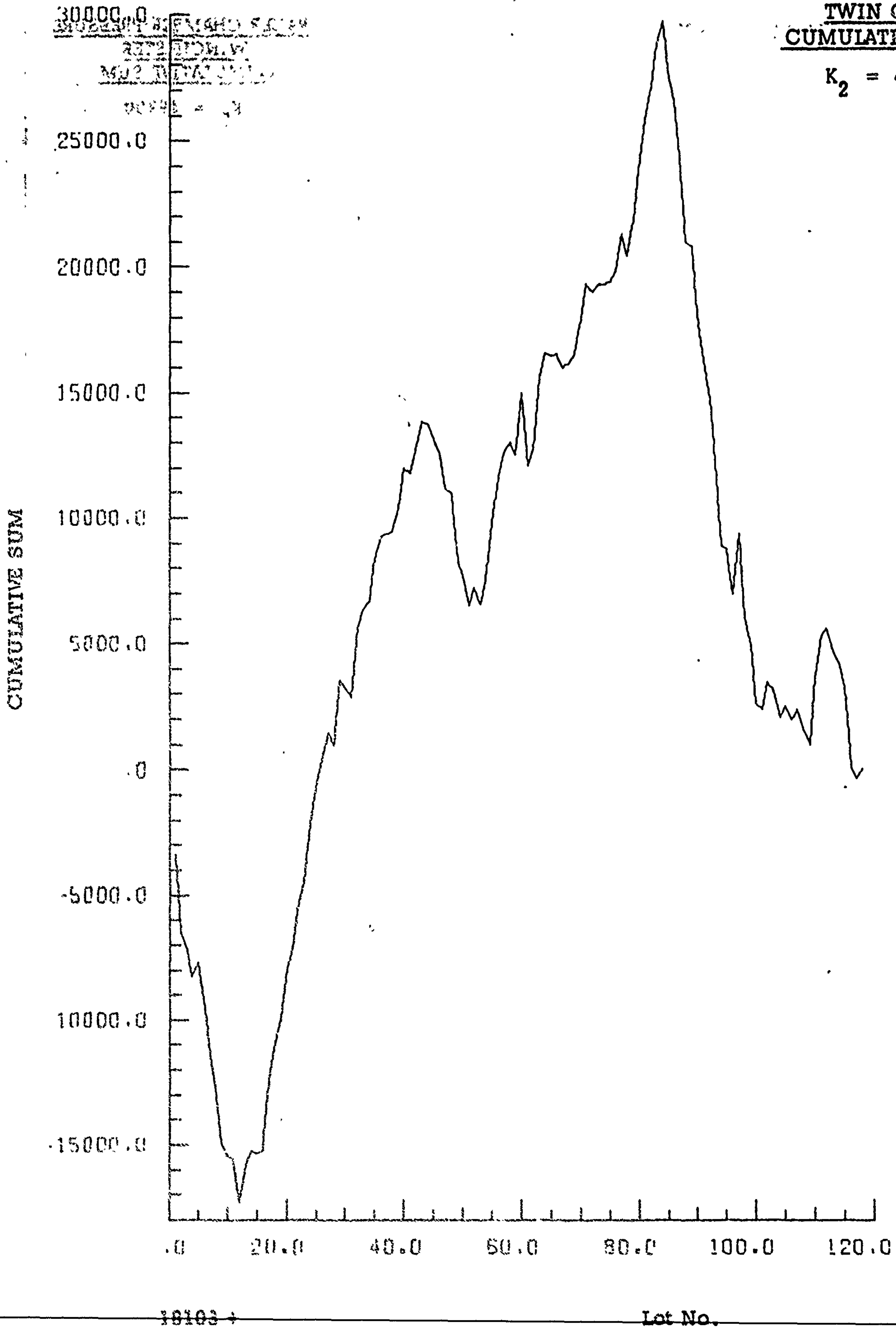


Figure 56

Winchester Tracer

TRACER CHAMBER PRESSURE
WINCHESTER
CUMULATIVE SUM

K_2 = 49900

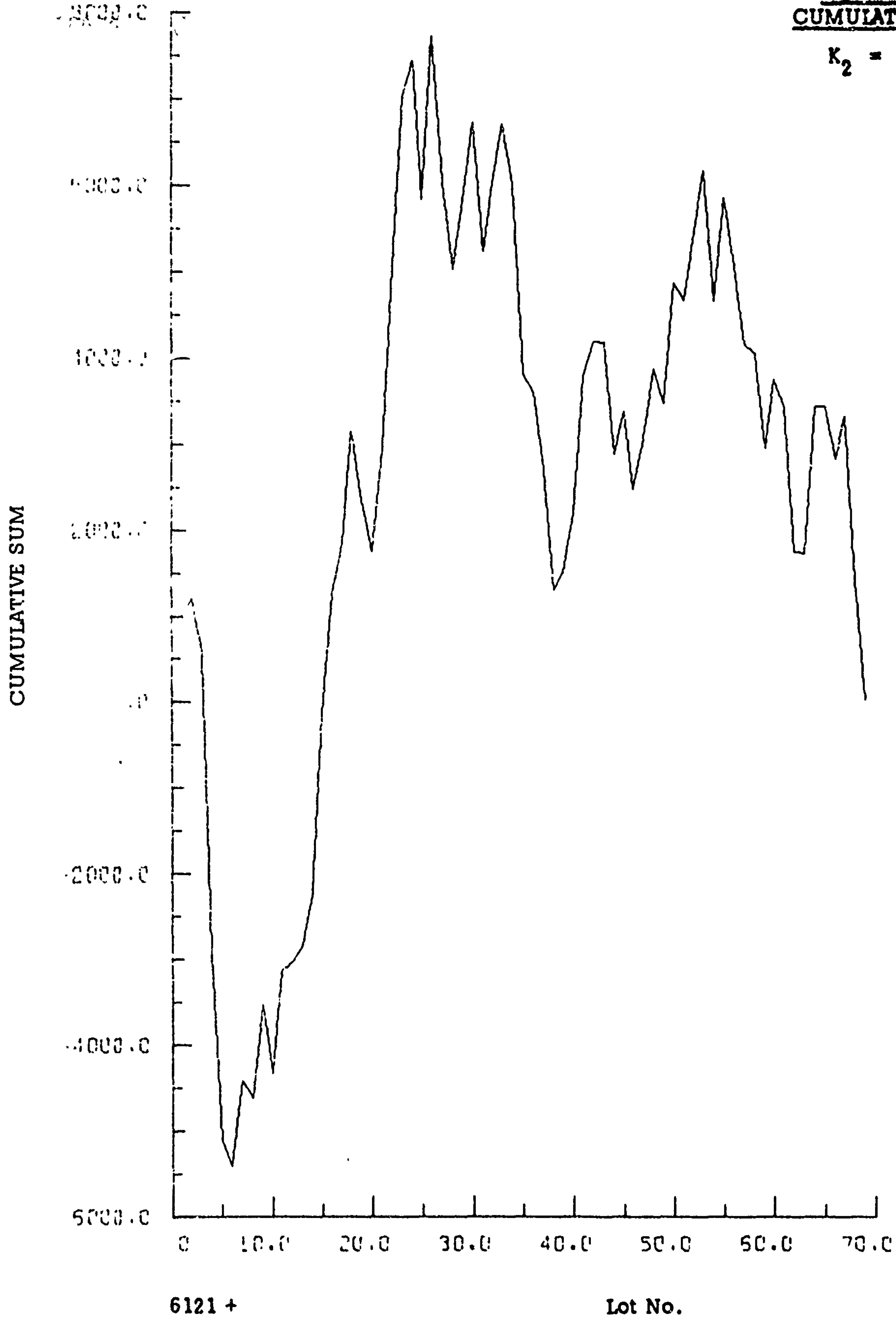


TABLE 4

Cutoff Date for Ball and Tracer Ammunition

Manufacturer	Cutoff Date	
	Ball	Tracer
Lake City	June 21, 1968	January 30, 1970
Twin City	March 10, 1969	April 15, 1969
Winchester	February 22, 1967	Nil
Remington	Nil	X
Federal	Nil	X

Figure 57

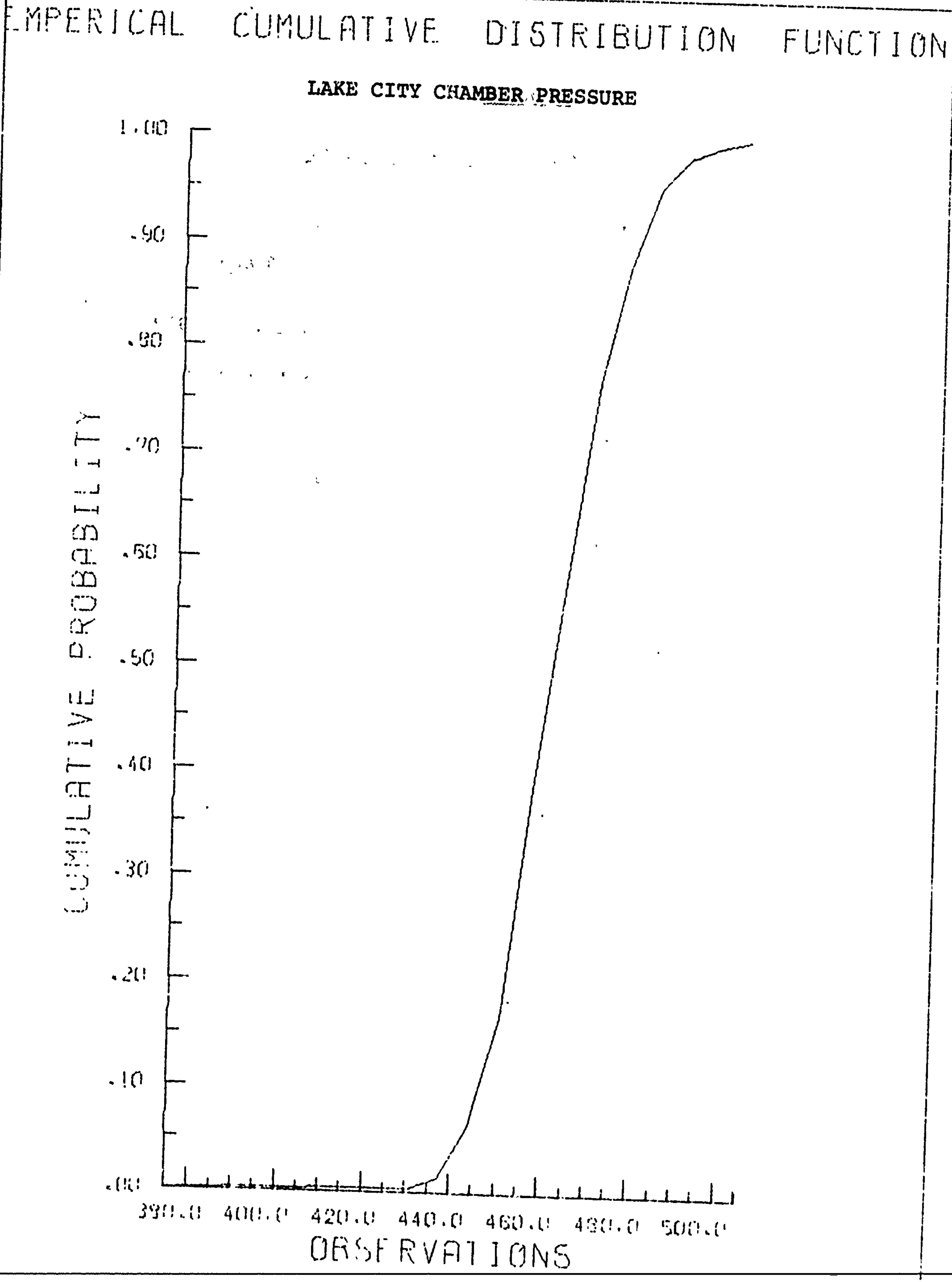

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
LAKE CITY CHAMBER PRESSURE
CUMULATIVE PROBABILITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
390.0 400.0 420.0 440.0 460.0 480.0 500.0
OBSERVATIONS

Figure 58

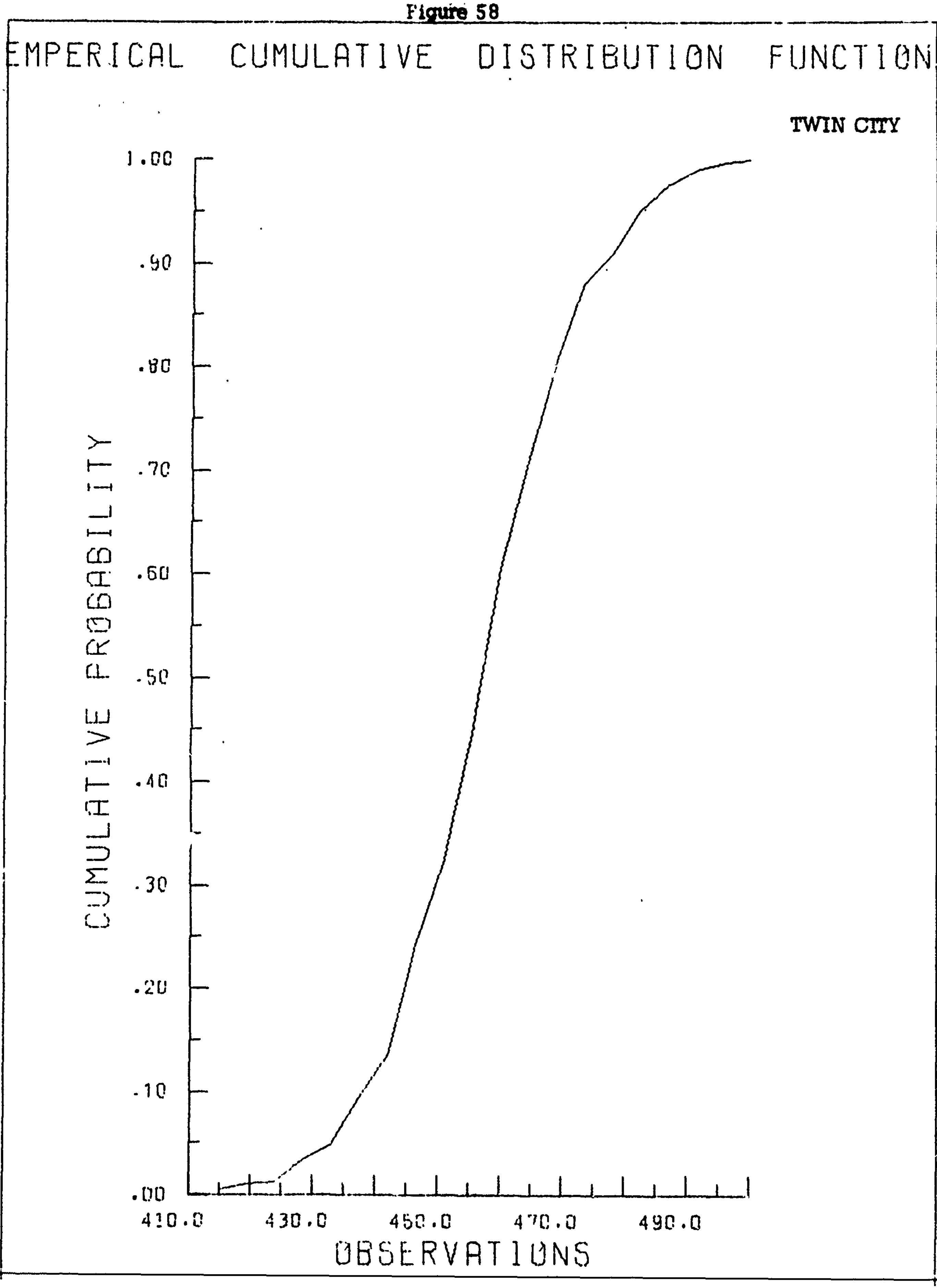

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
TWIN CITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
CUMULATIVE PROBABILITY
410.0
430.0
450.0
470.0
490.0
OBSERVATIONS

Figure 59

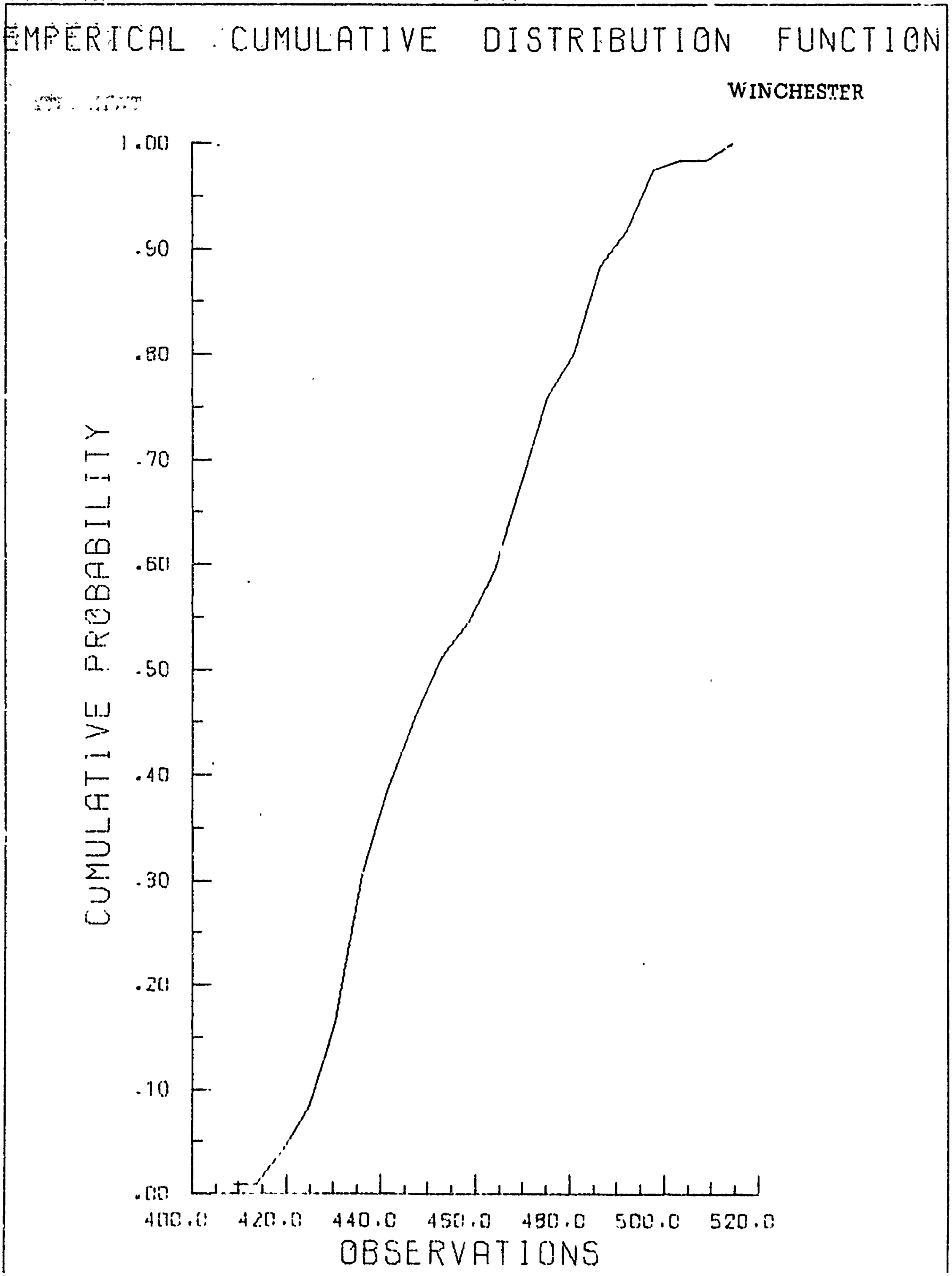

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
WINCHESTER
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
CUMULATIVE PROBABILITY
400.0
420.0
440.0
460.0
480.0
500.0
520.0
OBSERVATIONS

Figure 60

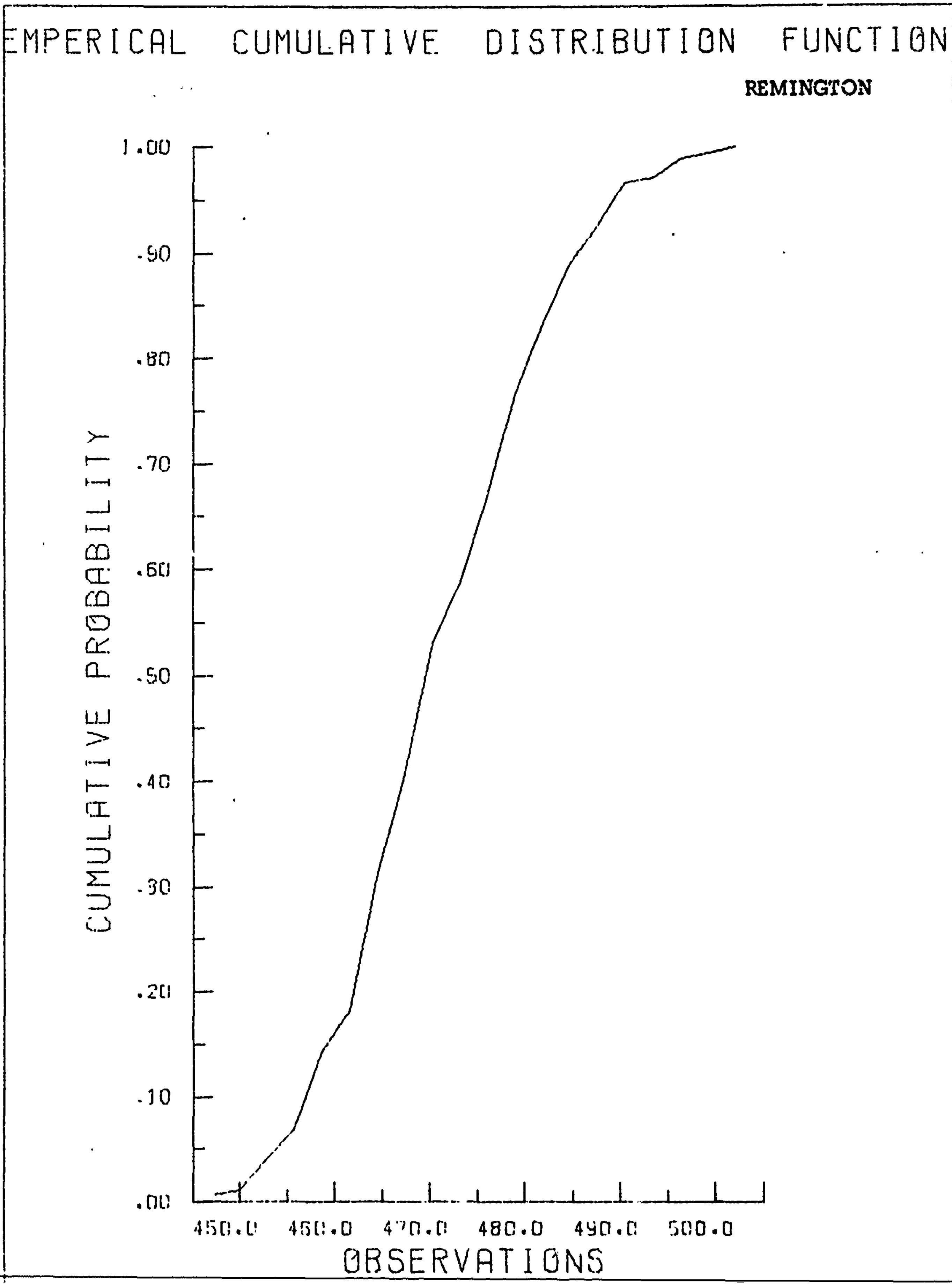
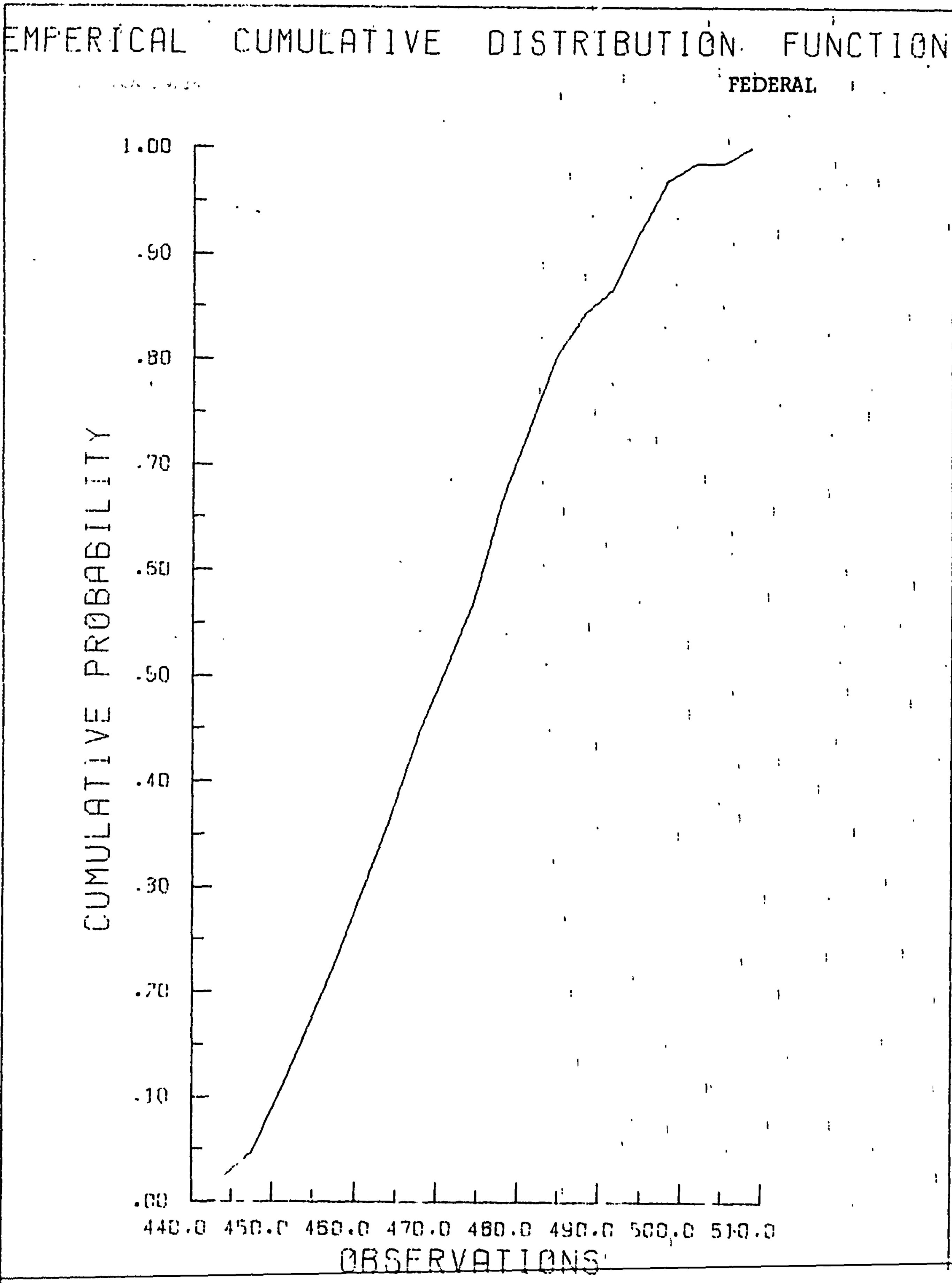

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
REMINGTON
CUMULATIVE PROBABILITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
450.0
460.0
470.0
480.0
490.0
500.0
OBSERVATIONS

Figure 61



EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
FEDERAL
CUMULATIVE PROBABILITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
440.0 450.0 460.0 470.0 480.0 490.0 500.0 510.0
OBSERVATIONS

Figure 62

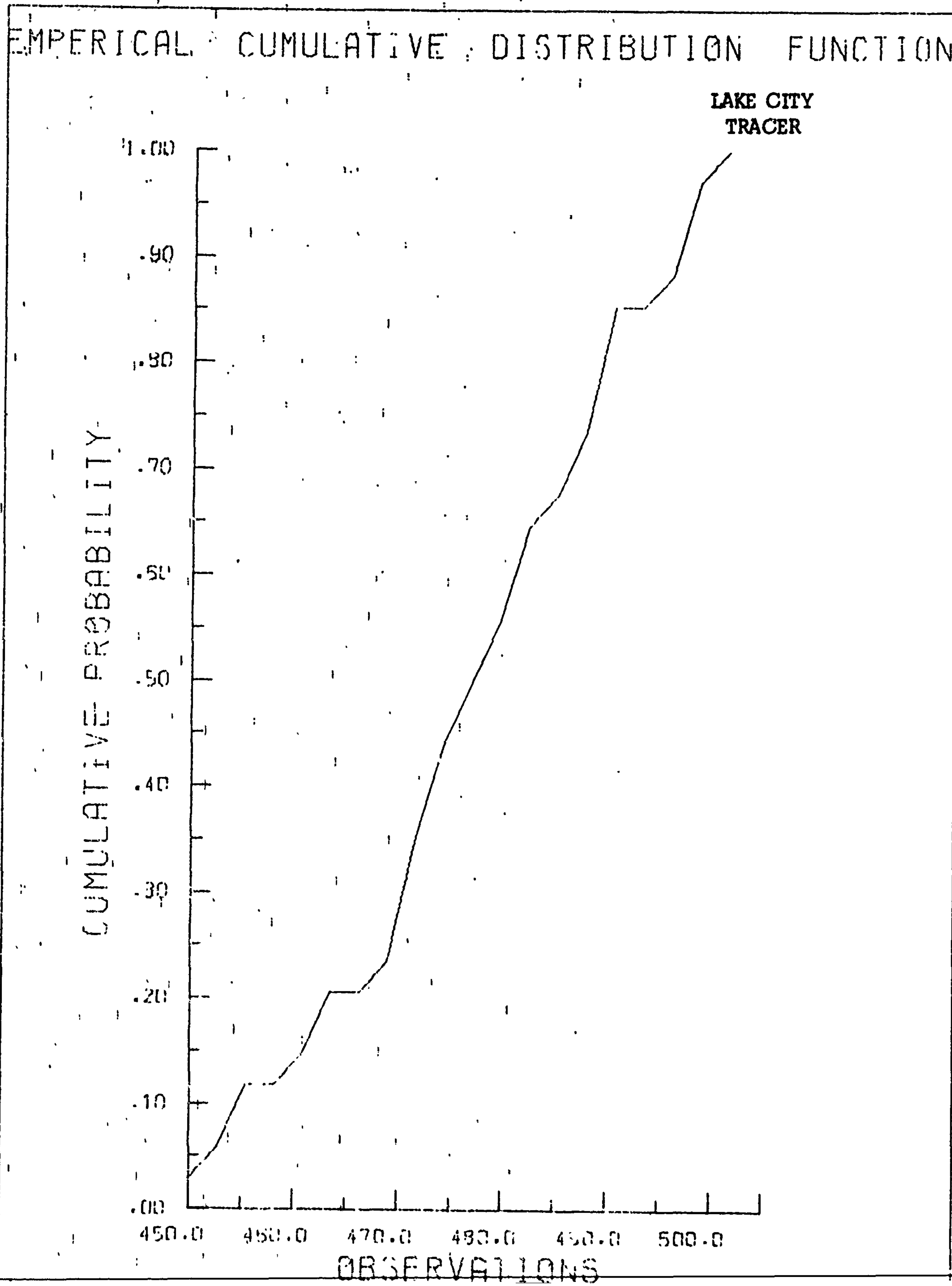

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
LAKE CITY TRACER
CUMULATIVE PROBABILITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
450.0
460.0
470.0
480.0
490.0
500.0
OBSERVATIONS

Figure 63

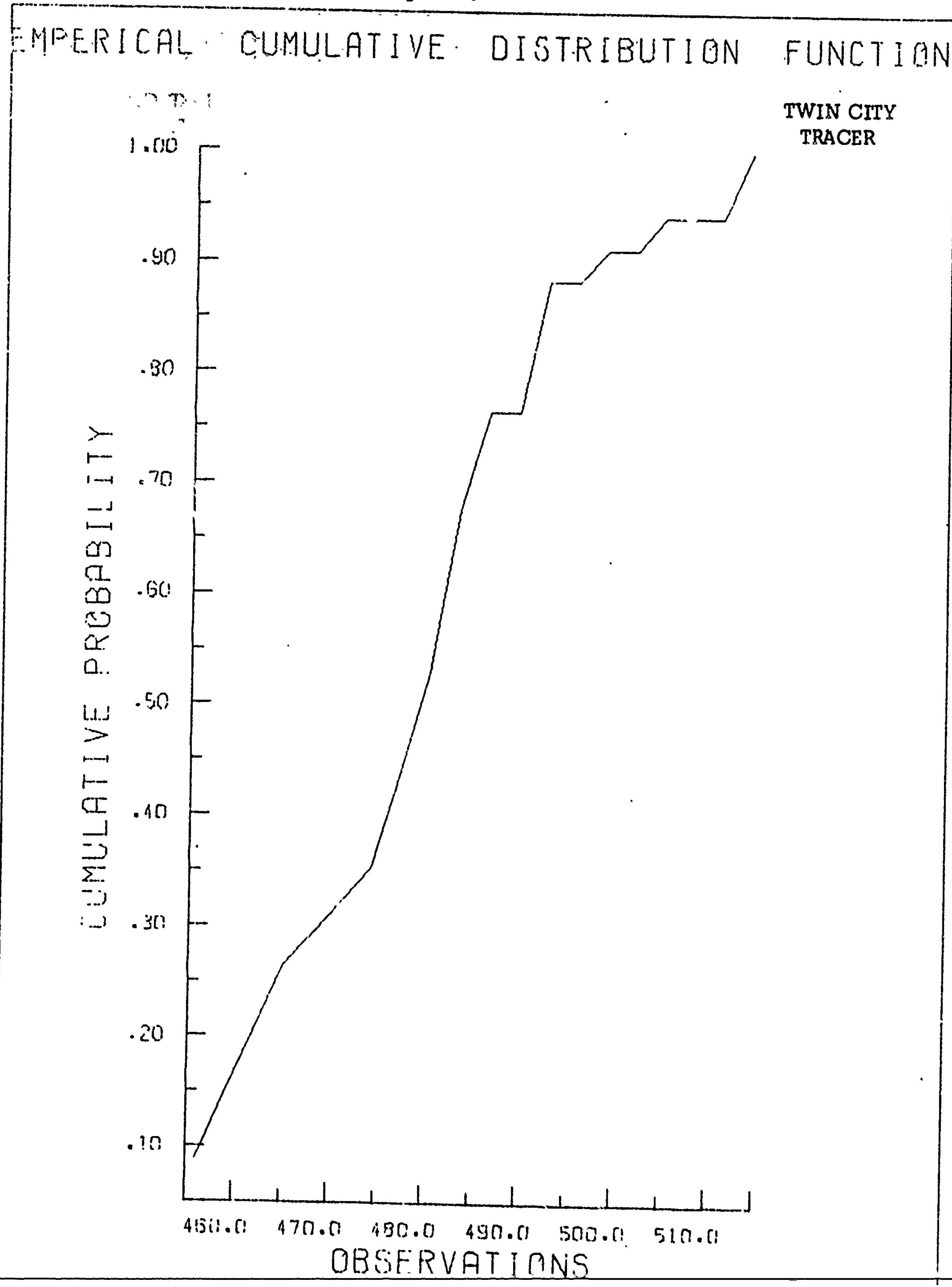

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
TWIN CITY
TRACER
CUMULATIVE PROBABILITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
460.0
470.0
480.0
490.0
500.0
510.0
OBSERVATIONS

Figure 64

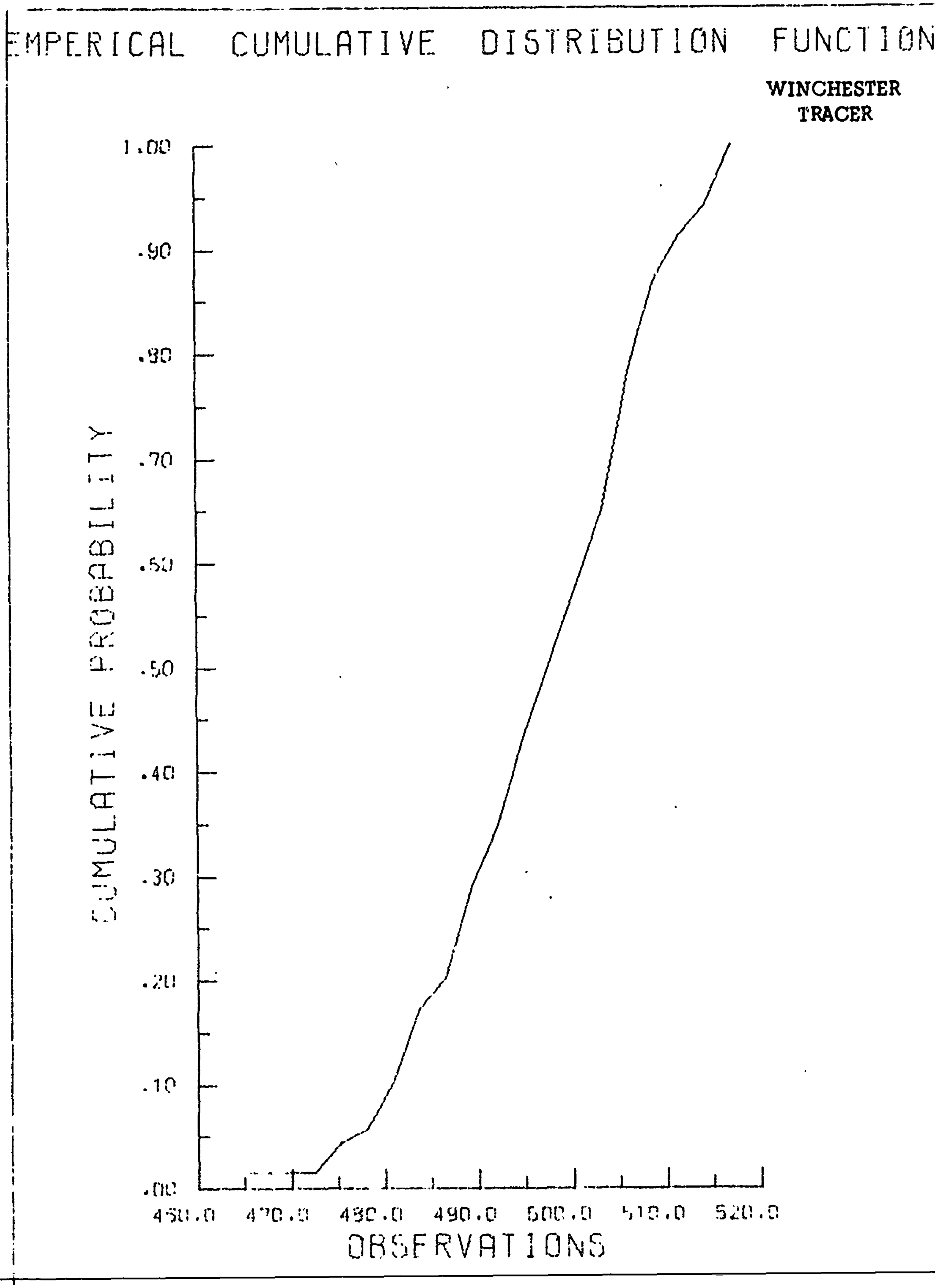

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
WINCHESTER TRACER
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
CUMULATIVE PROBABILITY
450.0
470.0
480.0
490.0
500.0
510.0
520.0
OBSERVATIONS

TABLE 5

Control Limits for Ball and Tracer Chamber Pressure

Manufacturer	Control Limits	
	Ball	Tracer
Lake City	49700 psi	50500 psi
Twin City	49200 psi	51200 psi
Winchester	51000 psi	51600 psi
Remington	49800 psi	X
Federal	50600 psi	X

5. AMMUNITION SELECTION FOR WEAPON TESTS

5.1 Method of Approach

The capabilities of contemporary ammunition production are such that differences exist between different ammunition lots. Gun performance is influenced by ballistic properties of ammunition, therefore, selection of proper ammunition lots for weapon testing is of importance.

The ammunition responses that determine gun performance are not yet known. The present analysis assumes that gun performance is governed by chamber pressure and port pressure. The analysis of ammunition production process indicates the presence of a cutoff date. Since the consideration of ammunition lots from the production period before the cutoff date leads to more severe gun requirements, ammunition lots from the production period after the cutoff date alone have been used to develop the selection criterion.

The analytical approach is to use a probabilistic viewpoint in selecting those ammunition lots from normal production that are likely to give a large variation in weapon performance. The analysis is first detailed for ammunition from Lake City Ammunition Plant and a similar analysis is then carried out for Twin City Ammunition Plant. The cutoff date for Lake City is July 21st 1968 and for Twin City is October 3rd 1969. The analysis for Lake City is based upon a total of 942 lots, covering a period from July 21st 1968 to January 1st 1971. A total of 411 lots covering a period from

October 3rd 1969 to January 1st 1971 have been used to select ammunition from Twin City.

5.2 Selection Based Upon Chamber Pressure (Lake City)

Chamber Pressure from standard tests is assumed to control the gun performance. Figure 65 is the plot of Lake City chamber pressure against the corresponding ammunition lots covering the period after the cutoff date. The range of chamber pressure readings (range = max. chamber pressure - min. chamber pressure) is subdivided into twenty equal parts and the number of lots belonging to each subgroup is calculated. Figure 66 shows the resulting histogram. Figure 67 is a plot of empirical cumulative distribution function (empirical c.d.f.). This is obtained by first calculating the probability of belonging to each subgroup. Empirical c.d.f. is a plot of the cumulative sum of these probabilities, plotted against the corresponding chamber pressure. The entire sequence of calculations is tabulated in Table 6.

If chamber pressure controls the gun performance, then a large variation in gun responses would be obtained by selecting lots with chamber pressure toward the high and the low ends of the spectrum. To these categories of High and Low lots another category of Medium lots is added to determine gun performance that will be most frequently met in practice.

Figure 65

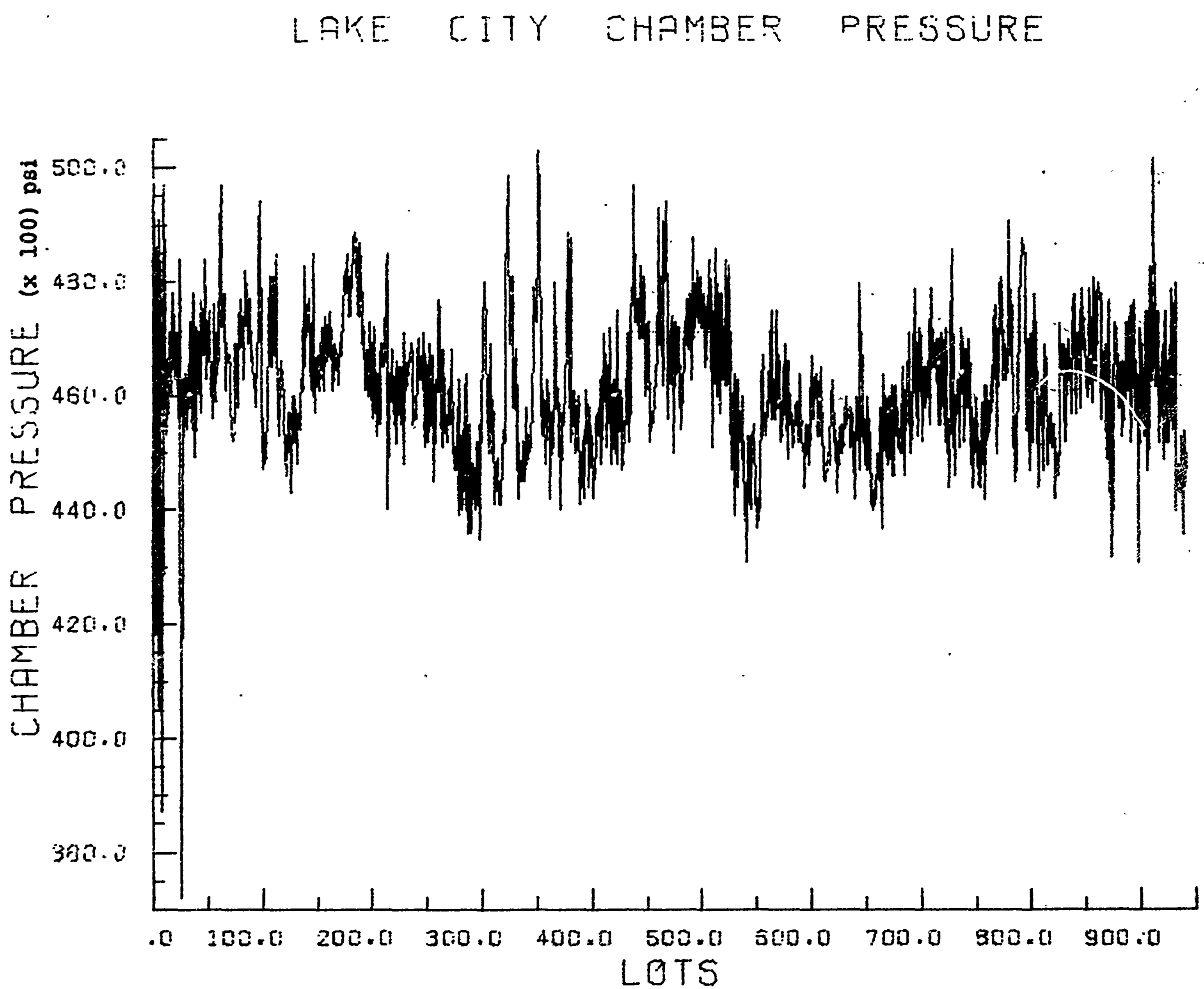
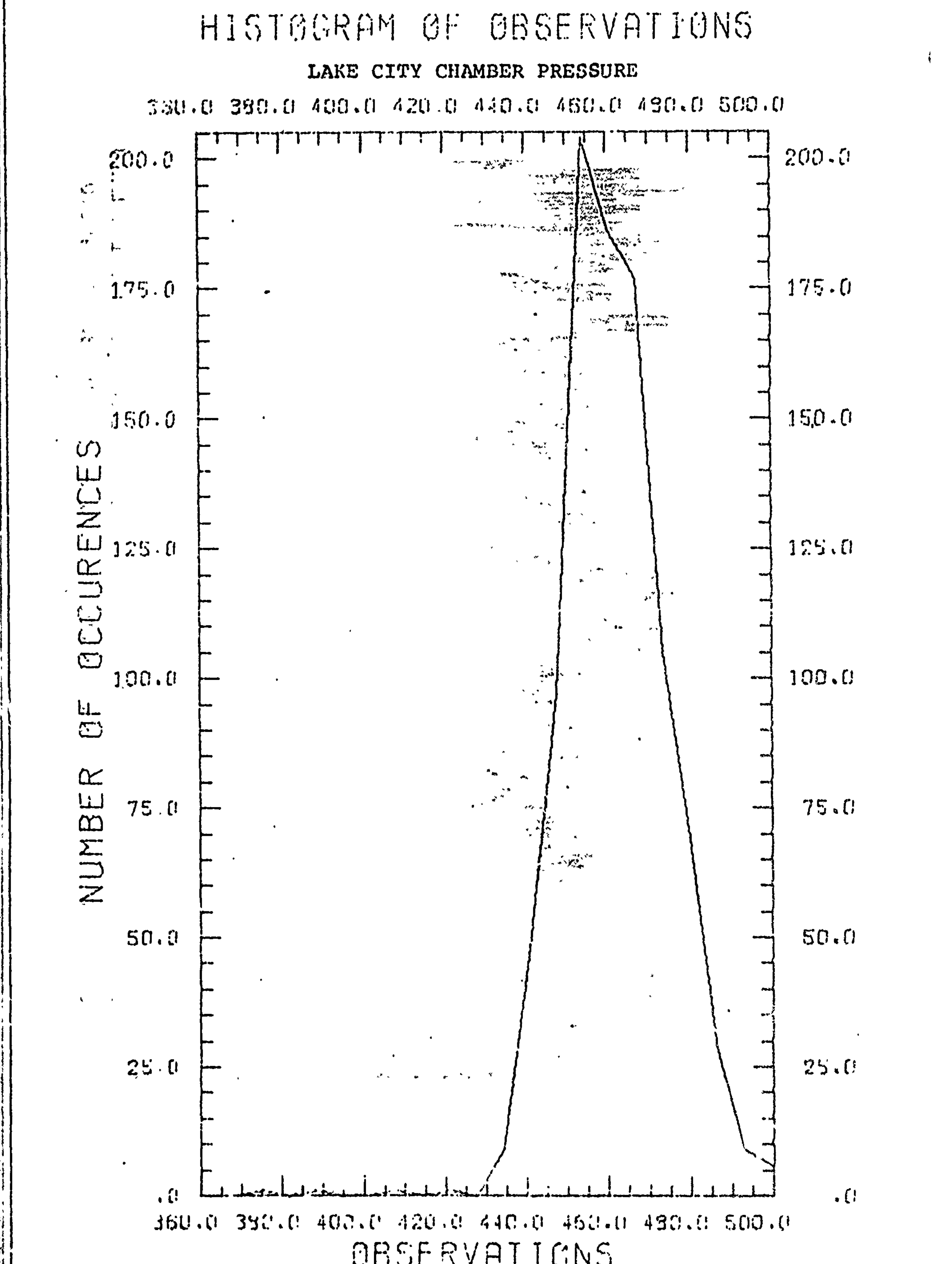

LAKE CITY CHAMBER PRESSURE
CHAMBER PRESSURE (x 100) psi
500.0
480.0
460.0
440.0
420.0
400.0
380.0
.0
100.0
200.0
300.0
400.0
500.0
600.0
700.0
800.0
900.0
LØTS

Figure 66



HISTOGRAM OF OBSERVATIONS
LAKE CITY CHAMBER PRESSURE
360.0 380.0 400.0 420.0 440.0 460.0 480.0 500.0
200.0
175.0
150.0
125.0
100.0
75.0
50.0
25.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

Figure 67

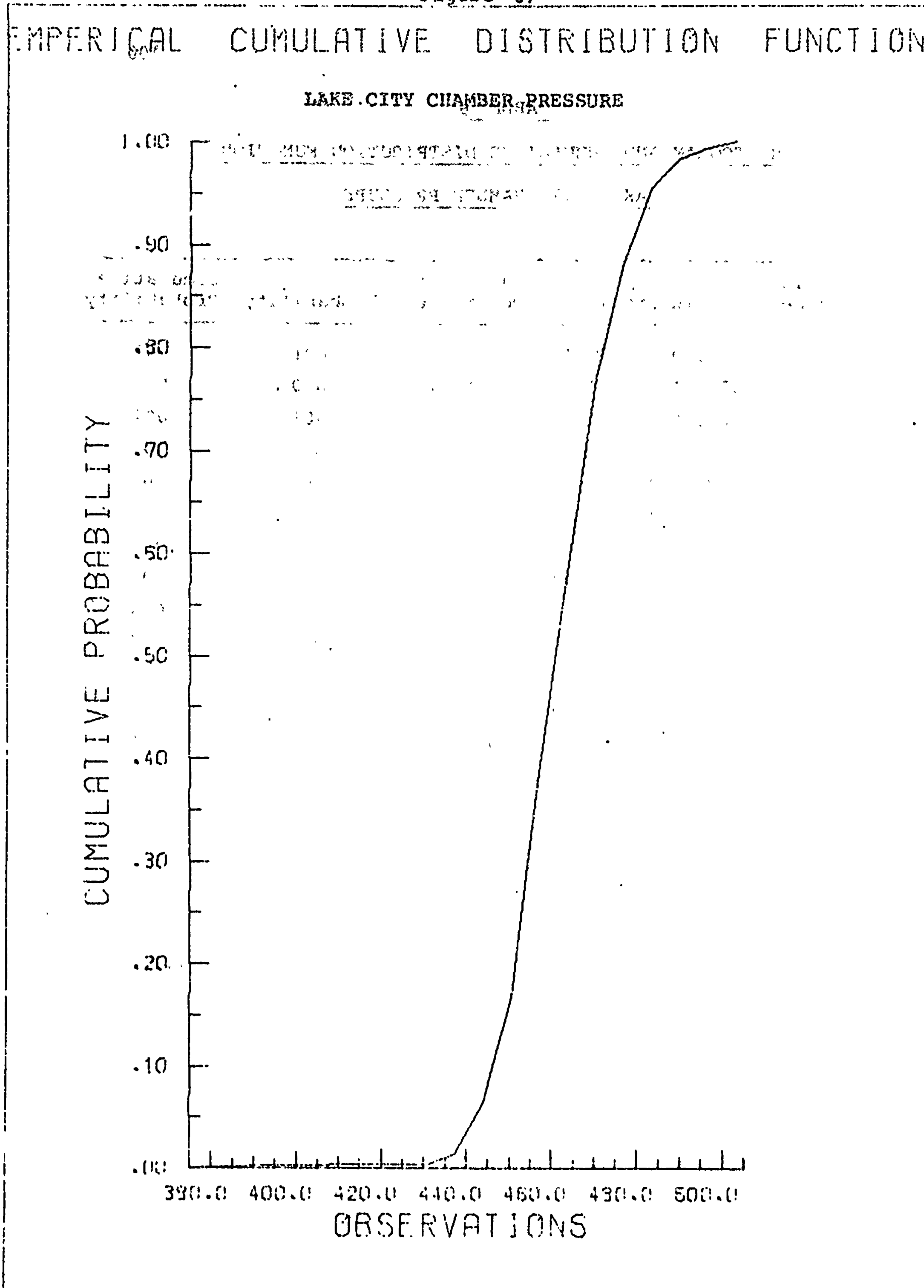

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
LAKE CITY CHAMBER PRESSURE
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
CUMULATIVE PROBABILITY
380.0 400.0 420.0 440.0 460.0 480.0 500.0
OBSERVATIONS

TABLE 6

HISTOGRAM AND CUMULATIVE DISTRIBUTION FUNCTION

LAKE CITY CHAMBER PRESSURE

Serial Number	Interval	Number of Occurences	Probability	Cumulative Probability
1	372.00 to 378.55	1.	.001	.001
2	378.55 to 385.10	0.	.000	.001
3	385.10 to 391.65	1.	.001	.002
4	391.65 to 398.20	0.	.000	.002
5	398.20 to 404.75	0.	.000	.002
6	404.75 to 411.30	1.	.001	.003
7	411.30 to 417.85	0.	.000	.003
8	417.85 to 424.40	1.	.001	.004
9	424.40 to 430.95	0.	.000	.004
10	430.95 to 437.50	9.	.009	.014
11	437.50 to 444.05	48.	.051	.065
12	444.05 to 450.60	97.	.103	.168
13	450.60 to 457.15	204.	.216	.384
14	457.15 to 463.70	186.	.197	.582
15	463.70 to 470.25	177.	.188	.769
16	470.25 to 476.80	105.	.111	.881
17	476.80 to 483.35	69.	.073	.954
18	483.35 to 489.90	28.	.030	.984
19	489.90 to 496.45	9.	.009	.994
20	496.45 to 503.00	6.	.006	1.0
		Total=942	Total = 1	

The classification of lots into High, Medium, and Low categories is done according to the following probabilistic point of view. A lot is classified as High if the probability of getting a chamber pressure higher than the chamber pressure of that particular lot is 5%. Similarly, if the probability of getting a chamber pressure lower than that of a particular lot is 5%, the lot is classified as Low. Medium lot is one that is most probable. High and Low lots, therefore, correspond to the 95 and 5 percentiles of the empirical c.d.f. Medium lot corresponds to the mode of the histogram. Another statistic that could be used to classify Medium lots is mean. However, mode appears more appealing, because it would lead to the determination of most frequent gun performance rather than the average gun performance as determined by the mean.

The calculated critical values of chamber pressure are given below:

High:	48300 psi
Medium:	45488 psi
Low:	44150 psi

Table 7 gives the selected ammunition lots. These lots have chamber pressure values close to the critical values.

5.3 Selection Based Upon Port Pressure (Lake City)

The lot selection can also be based solely on port pressure, where it is assumed that port pressure alone, governs weapon performance.

Figure 68 is the plot of port pressure against ammunition lots, covering the period of ammunition production after the cutoff date. Figure 69 is the histogram of port pressure and Fig. 70 is the empirical c.d.f.. The calculations are detailed in Table 8. The critical values of port pressure for classification into the three groups are as follows:

High:	15921 psi
Medium:	15430 psi
Low:	14738 psi

Table 9 gives the selected lots that have port pressure values close to the critical values.

5.4 Discussion of the Two Methods of Ammunition Selection

Both the methods of selection are extremely simple to use, however, they suffer from the following disadvantages:

(1) A comparison of Table 7 and Table 9 reveals that classification based upon chamber pressure alone and based upon port pressure alone leads to different lot selection. For example, lots L-1-204 and L-1-158 are classified as Medium based upon the port pressure but are classified as Medium and Low respectively, based upon the chamber pressure. It is, therefore, necessary to determine whether it is the chamber pressure or it is the port pressure that controls gun performance.

TABLE 7

Lot Selection (Based on Chamber Pressure Alone)

Lake City

High		Medium		Low	
Lot No.	Chamber Pressure psi	Lot No.	Chamber Pressure psi	Lot No.	Chamber Pressure psi
L-		L-		L-	
1-43	48300	1-288	45400	1-284	44200
1-41	48400	1-287	45300	1-79	44200
1-26	48400	1-275	45400	1-177	44100
1-15	48200	1-216	45300	1-174	44100
		1-211	45400	1-158	44200
		1-204	45300	1-142	44300

Figure 68

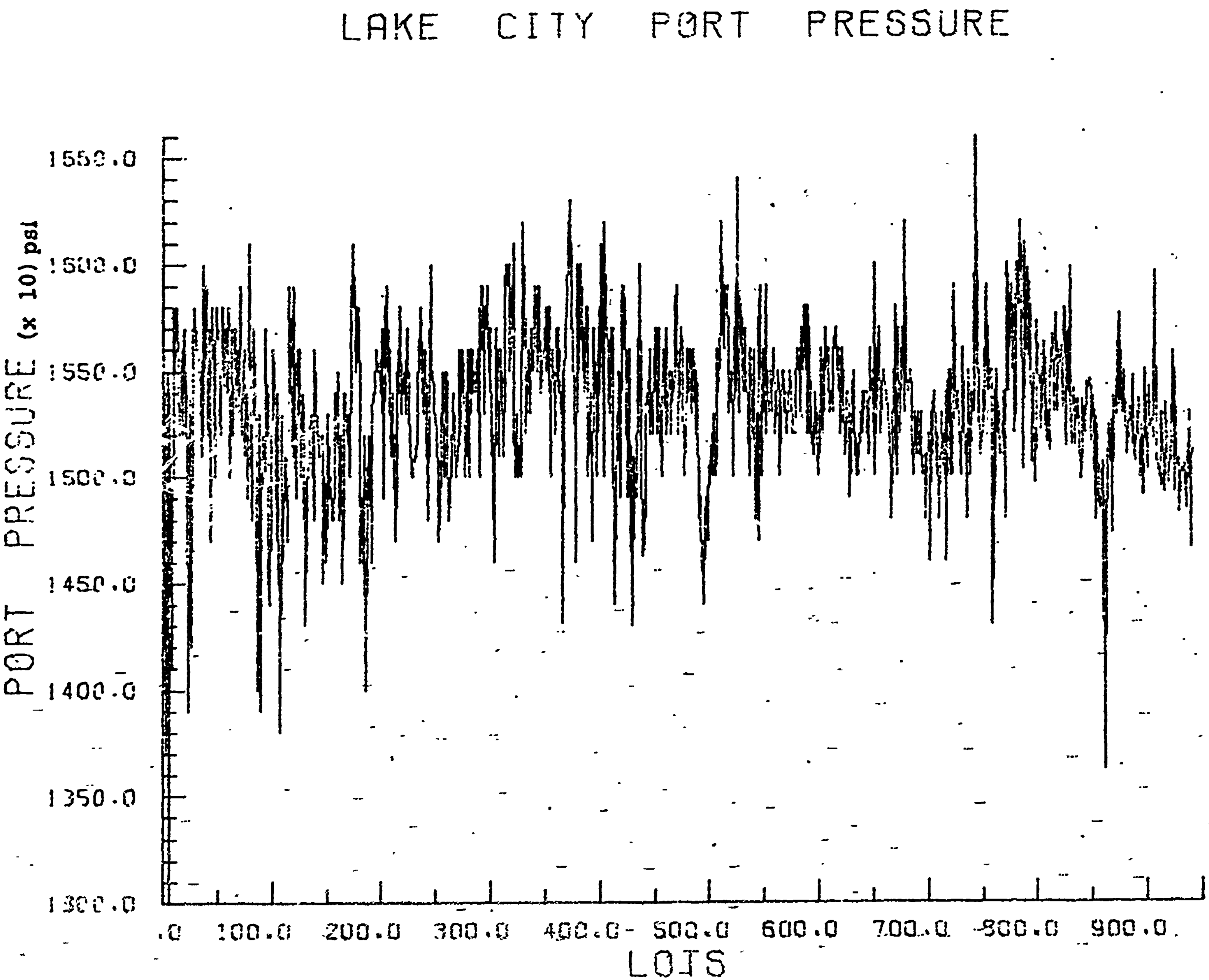

LAKE CITY PØRT PRESSURE
PØRT PRESSURE (x 10) psi
1550.0
1500.0
1550.0
1500.0
1450.0
1400.0
1350.0
1300.0
.0
100.0
200.0
300.0
400.0
500.0
600.0
700.0
800.0
900.0
LOTS

Figure 69

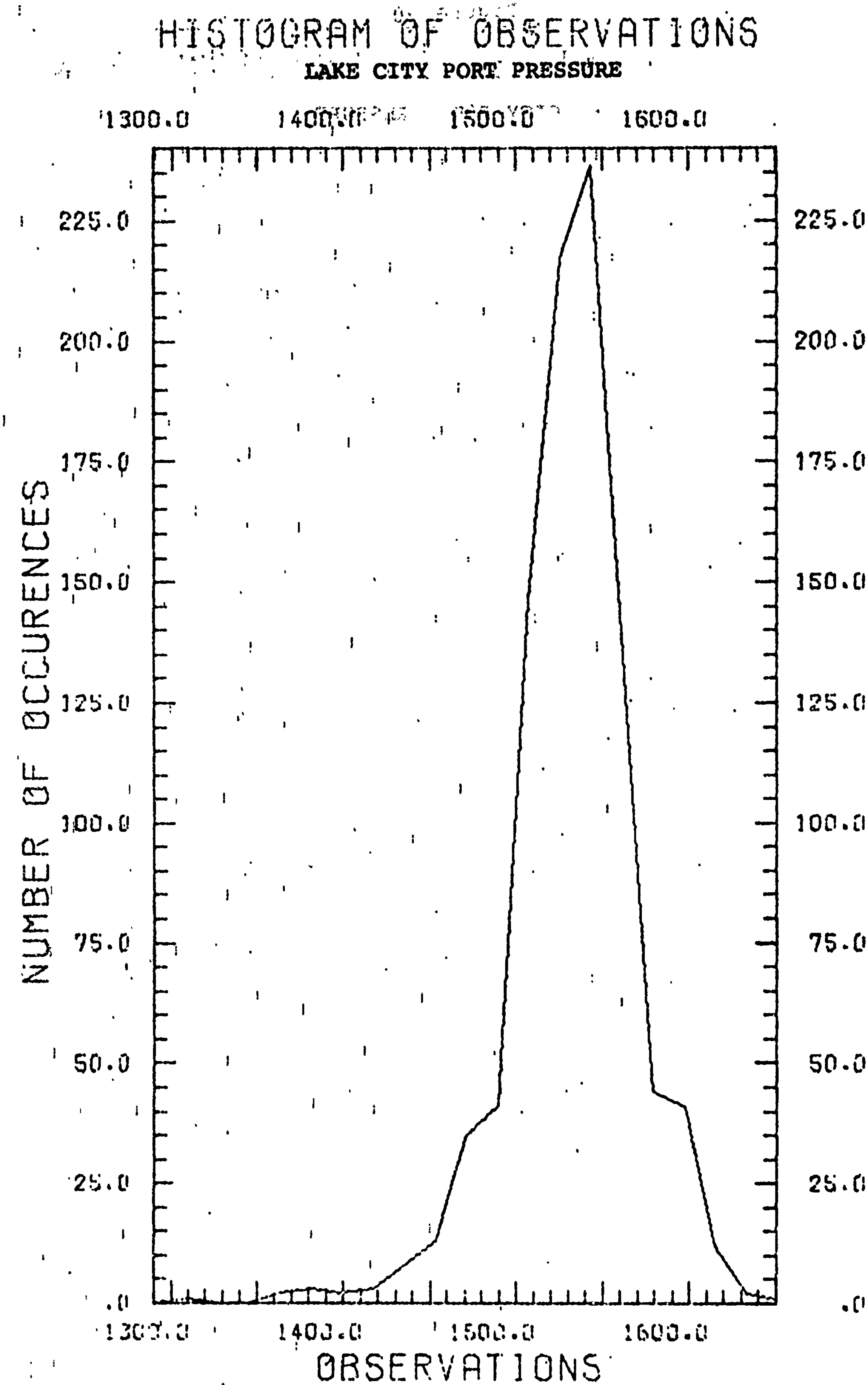

HISTOGRAM OF OBSERVATIONS
LAKE CITY PORT PRESSURE
1300.0
1400.0
1500.0
1600.0
225.0
200.0
175.0
150.0
125.0
100.0
75.0
50.0
25.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

Figure 70

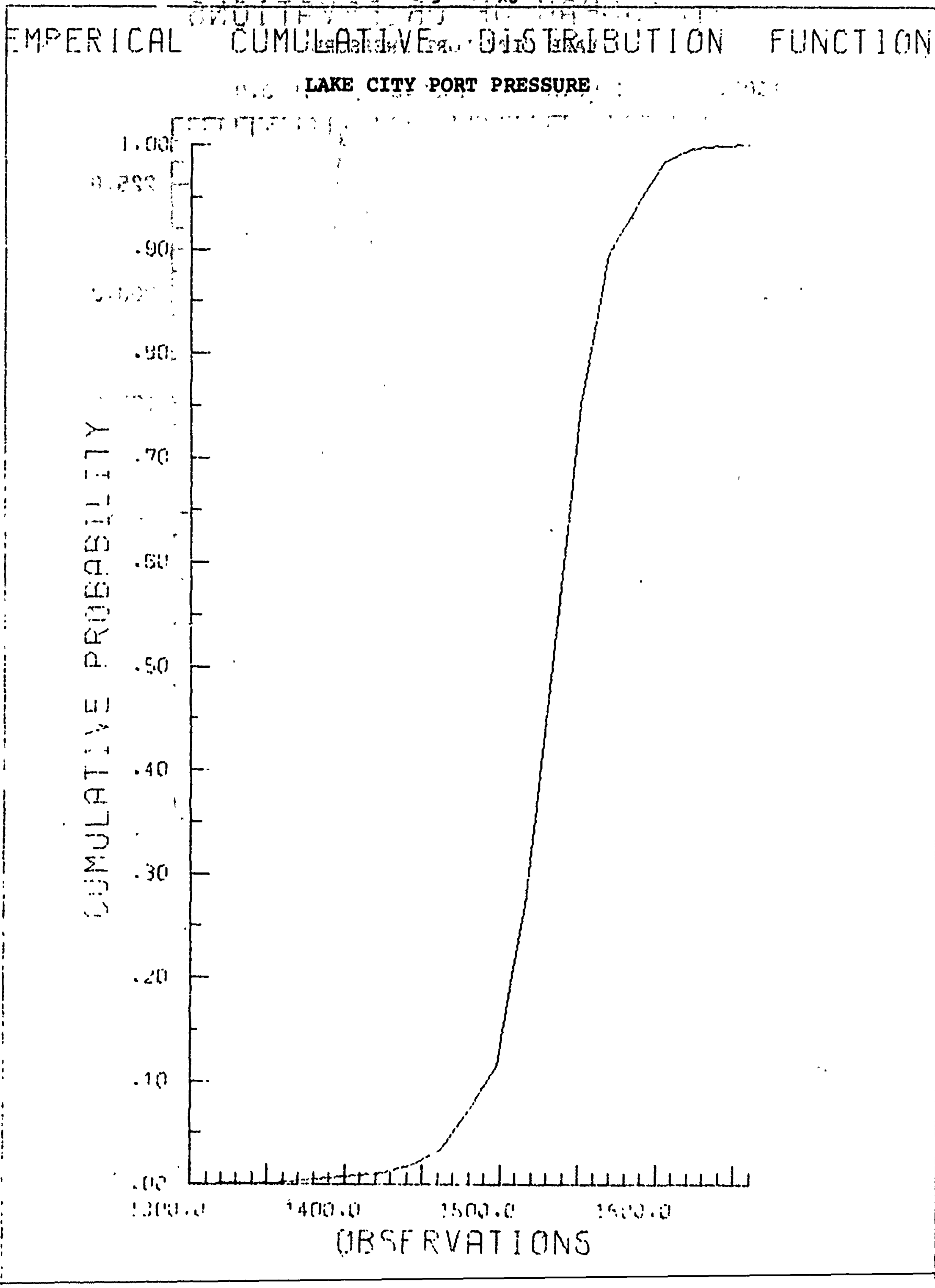

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
LAKE CITY PORT PRESSURE
CUMULATIVE PROBABILITY
OBSERVATIONS
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
1400.0
1500.0

TABLE 8

HISTOGRAM AND CUMULATIVE DISTRIBUTION FUNCTION

LAKE CITY PORT PRESSURE

Serial Number	Interval	Number of Occurences	Probability	Cumulative Probability
1	1300.00 to 1318.00	1.	.001	.001
2	1318.00 to 1336.00	0.	.000	.001
3	1336.00 to 1354.00	0.	.000	.001
4	1354.00 to 1372.00	2.	.002	.003
5	1372.00 to 1390.00	3.	.003	.006
6	1390.00 to 1408.00	2.	.002	.008
7	1408.00 to 1426.00	3.	.003	.012
8	1426.00 to 1444.00	8.	.008	.020
9	1444.00 to 1462.00	13.	.014	.034
10	1462.00 to 1480.00	35.	.037	.071
11	1480.00 to 1498.00	41.	.044	.115
12	1498.00 to 1516.00	146.	.155	.270
13	1516.00 to 1534.00	217.	.230	.500
14	1534.00 to 1552.00	236.	.250	.750
15	1552.00 to 1570.00	135.	.143	.894
16	1570.00 to 1588.00	44.	.047	.940
17	1588.00 to 1606.00	41.	.043	.984
18	1606.00 to 1624.00	12.	.013	.997
19	1624.00 to 1642.00	2.	.002	.999
20	1642.00 to 1660.00	1.	.001	1.0
		Total=942	Total=1	

TABLE 9

Lot Selection--(Based on Port Pressure Alone)

Lake City

High		Medium		Low	
Lot No.	Port Pressure psi	Lot No.	Port Pressure psi	Lot No.	Port Pressure psi
L-		L-		L-	
1-318	15870	1-359	15410	1-297	14800
1-317	15970	1-338	15450	1-287	14900
1-308	15940	1-296	15390	1-285	14720
1-298	15970	1-281	15410	1-261	14870
1-247	15930	1-274	15440	1-217	14830
		1-265	15410		
		1-250	15420		
		1-239	15430		
		1-204	15400		
		1-158	15430		

(2) Gun performance is more likely to be controlled by the parameters of piezo pressure-time curve. It is possible that for different pressure-time curves the same chamber pressure value is obtained. Same can be said regarding the port pressure. It is therefore rather unlikely that gun performance would be determined by chamber pressure alone or port pressure alone.

5.5 Selection Based Upon Chamber Pressure and Port Pressure (Lake City)

It is considered more probable that gun performance is governed jointly by the chamber pressure and the port pressure. The final selection is, therefore, based upon both the chamber pressure and the port pressure. Table 10 is the bivariate histogram of chamber pressure and port pressure excluding the outlays. This computer generated table gives the number of lots belonging to a certain chamber pressure range and a certain port pressure range. For example, there are 26 lots that have a chamber pressure between 45800 psi and 46100 psi and a port pressure between 15400 psi and 15500 psi. In this table the group width (Δ_1) along the port pressure axis is equal to half standard deviation (S.D.) and along the chamber pressure axis (Δ_2) is equal to one S.D. of chamber pressure. (The mean value of S.D. is 1330 psi. Hence, S.D. of mean chamber pressure = 1330/ 20 ≈ 300 psi).

The selected frequencies are enclosed in small rectangles in Table 10. Medium lots are the most probable, hence the central rectangle is so placed as to cover the maximum number of lots. The selection of High and Low lots is primarily governed by the chamber pressure criterion. These have the same port pressure range (2.5 S.D.) as the Medium lots but the chamber pressure range is situated around the high and low critical values determined by using chamber pressure criterion alone. For 'Normal Selection' the chamber pressure range is 2 S.D. and for 'Tight Selection' (shown by shaded rectangles) the chamber pressure range is 1 S.D.. Table 11 gives the critical values of chamber pressure and port pressure in both the cases. Table 12 gives the selected lots for Tight Selection and the selected lots for Normal Selection are given in Table 13.

5.6 Selection Based Upon Chamber Pressure and Port Pressure (Twin City)

A similar approach is now used for Twin City data. The Twin City chamber pressure and port pressure for the period of normal production are plotted in Figures 71 and 72. Fig. 73 gives the histogram of Twin City chamber pressure and Fig. 74 is the plot of empirical c.d.f.. The entire sequence of calculations is shown in Table 14. The bivariate histogram of chamber pressure and port pressure is shown in Table 15. Table 16 indicates the critical values for Normal and Tight Selection. The selected lots for Tight Selection and Normal Selection are included in Tables 17 and 18 respectively.

TABLE 11

BIVARIATE HISTOGRAM OF CHAMBER PRESSURE AND PORT PRESSURE

LAKE CITY

			14800				15400				16000				16600	Chamber Pressure (psi)	
0.	0.	0.	0.	0.	0.	0.	0.	2.	0.	0.	0.	0.	0.	0.	0.	43100	
0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	1.	2.	2.	0.	1.	0.	0.	0.	0.	0.	43700	
0.	0.	0.	1.	0.	1.	4.	0.	2.	1.	1.	0.	1.	0.	0.	0.		LOW
1.	0.	0.	1.	0.	4.	2.	2.	7.	2.	3.	2.	2.	0.	0.	0.	44300	
0.	0.	0.	0.	2.	3.	6.	5.	11.	3.	4.	1.	1.	0.	0.	0.		
0.	0.	0.	0.	1.	5.	5.	6.	11.	5.	7.	4.	2.	0.	0.	0.	44900	
0.	1.	0.	0.	3.	3.	14.	10.	21.	3.	10.	5.	1.	1.	0.	0.		
1.	0.	0.	1.	5.	9.	15.	5.	23.	11.	6.	4.	2.	0.	0.	0.	45500	
1.	0.	0.	3.	3.	10.	15.	12.	25.	7.	13.	6.	2.	3.	1.	0.		MEDIUM
0.	0.	2.	1.	4.	9.	15.	12.	25.	8.	12.	1.	0.	0.	1.	0.	46100	
0.	0.	1.	1.	7.	6.	17.	14.	19.	9.	14.	1.	0.	1.	0.	0.		
0.	0.	1.	0.	6.	8.	15.	17.	16.	8.	2.	1.	2.	0.	0.	0.	46700	
0.	0.	1.	2.	1.	8.	14.	11.	18.	6.	5.	2.	1.	0.	0.	1.		
0.	0.	2.	2.	4.	3.	12.	8.	16.	4.	5.	2.	1.	0.	0.	0.	47300	
0.	1.	1.	0.	6.	7.	12.	5.	7.	2.	4.	0.	1.	0.	0.	0.		
0.	1.	0.	0.	7.	5.	11.	5.	7.	2.	2.	0.	1.	0.	0.	0.	47900	
1.	0.	2.	2.	2.	2.	1.	2.	4.	0.	3.	1.	1.	0.	0.	0.		
0.	0.	0.	0.	2.	5.	2.	1.	2.	1.	1.	0.	0.	0.	0.	0.	48500	HIGH
0.	0.	1.	1.	1.	3.	0.	1.	2.	1.	3.	1.	0.	0.	0.	0.		
0.	0.	2.	0.	1.	0.	2.	0.	1.	0.	1.	0.	0.	0.	0.	0.	49100	
1.	1.	0.	0.	0.	0.	1.	1.	1.	0.	0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	1.	0.	2.	1.	0.	0.	0.	0.	0.	0.	49700	
0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	1.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	50300	

Port Pressure (psi)

Δ_1 = 150 psi

Δ_2 = 300 psi

Note: Values of pressures along the two axes are maximum values for that group.

TABLE 11

Critical Values for Selection Based Upon Chamber Pressure and Port Pressure

LAKE CITY

Normal Selection

Chamber Pressure Tolerance = 2 S. D.

Port Pressure Tolerance = 2.5 S.D.

	Chamber Pressure psi	Port Pressure psi
HIGH	47900 to 48500	14950 to 15850
MEDIUM	45500 to 46100	14950 to 15850
LOW	44000 to 44600	14950 to 15850

Tight Selection

Chamber Pressure Tolerance = 1 S.D.

Port Pressure Tolerance = 2.5 S.D.

	Chamber Pressure psi	Port Pressure psi
HIGH	48200 to 48500	14950 to 15850
MEDIUM	45800 to 46100	14950 to 15850
LOW	44000 to 44300	14950 to 15850

Table 12

Final Lot Selection

(Based Upon Chamber Pressure & Port Pressure)

Chamber Pressure Tolerance = 1 S.D.

Port Pressure Tolerance = 2.5 S.D.

LAKE CITY

High				Medium				Low			
Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi
LC-				LC-				LC-			
1-435	48300	1065	15060	1-292	45900	1772	15100	1-179	44200	960	15480
1-321	48500	1188	15060	1-289	46000	887	15200	1-177	44100	743	15360
1-43	48300	1552	15350	1-281	45900	1034	15410	1-158	44200	811	15430
-12879	48300	1606	15260	1-272	45800	1856	15440	1-142	44300	935	15110
-12752	48200	1484	15310	1-268	45800	732	15280	1-72	44200	1079	15510
-12620	48300	1200	15100	1-265	46000	1249	15410	1-62	44300	1397	15330
				1-229	46000	1132	15380	12764	44200	940	15390
				1-223	45800	1195	15160	12748	44100	820	15530
				1-215	46000	1847	15070	12742	44100	1147	15100
				1-191	46000	1479	15060				
				1-183	45800	1029	15290				
				1-170	46000	1365	15090				
				1-160	46000	1438	15390				
				1-120	45900	1257	15460				
				1-109	45900	761	15180				
				1- 96	45900	1406	15210				
				1- 93	46000	1134	15420				
				1- 89	45800	768	15200				

Table 13

Final Lot Selection

(Based Upon Chamber Pressure & Port Pressure)

Chamber Pressure Tolerance = 2 S.D.

Port Pressure Tolerance = 2.5 S.D.

LAKE CITY

High				Medium				Low			
Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi
LC- 1-435	48300	1065	15060	LC- 1-292	45900	1772	15100	LC- 1-279	44400	1194	15390
1-321	48500	1188	15060	1-289	46000	887	15200	1-273	44400	844	15160
1-300	48100	1720	15420	1-281	45900	1034	15410	1-254	44600	1124	15380
1-163	48000	1572	15290	1-276	46100	1572	15390	1-249	44400	1187	15350
1-43	48300	1552	15350	1-272	45800	1856	15440	1-205	44600	732	15250
1-25	47900	1313	15110	1-268	45800	732	15280	1-195	44600	870	15500
-12901	48000	1256	15470	1-265	46000	1249	15410	1-194	44600	1386	15290
-12882	48100	1457	15360	1-229	46000	1132	15380	1-182	44500	888	15379
-12879	48300	1606	15260	1-223	45800	1195	15160	1-179	44200	960	15480
-12778	47900	1339	15510	1-215	46000	1847	15070	1-178	44600	1129	15460
-12758	48100	2002	15210	1-207	46100	1139	15320	1-177	44100	743	15360
				1-191	46000	1479	15060				

<u>Table 13 (cont'd)</u>

High				Medium				Low			
Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi
				1-183	45800	1029	15290	1-175	44600	949	15560
-12752	48200	1484	15310	1-170	46000	1365	15090	1-159	44400	637	15280
-12620	48300	1200	15100	1-160	46000	1438	15390	1-158	44200	811	15430
				1-120	45900	1257	15460	1-142	44300	935	15110
				1-109	45900	761	15180	1-131	44500	927	15350
				1- 96	45900	1406	15210	1-130	44600	1162	15380
				1- 93	46000	1134	15420	1-113	44400	1162	15380
				1- 89	45800	768	15200	1- 72	44200	1-79	15510
								1- 62	44300	1397	15330
								12764	44200	940	15390
								12748	44100	820	15530
								12742	44100	1147	15100

Figure 71

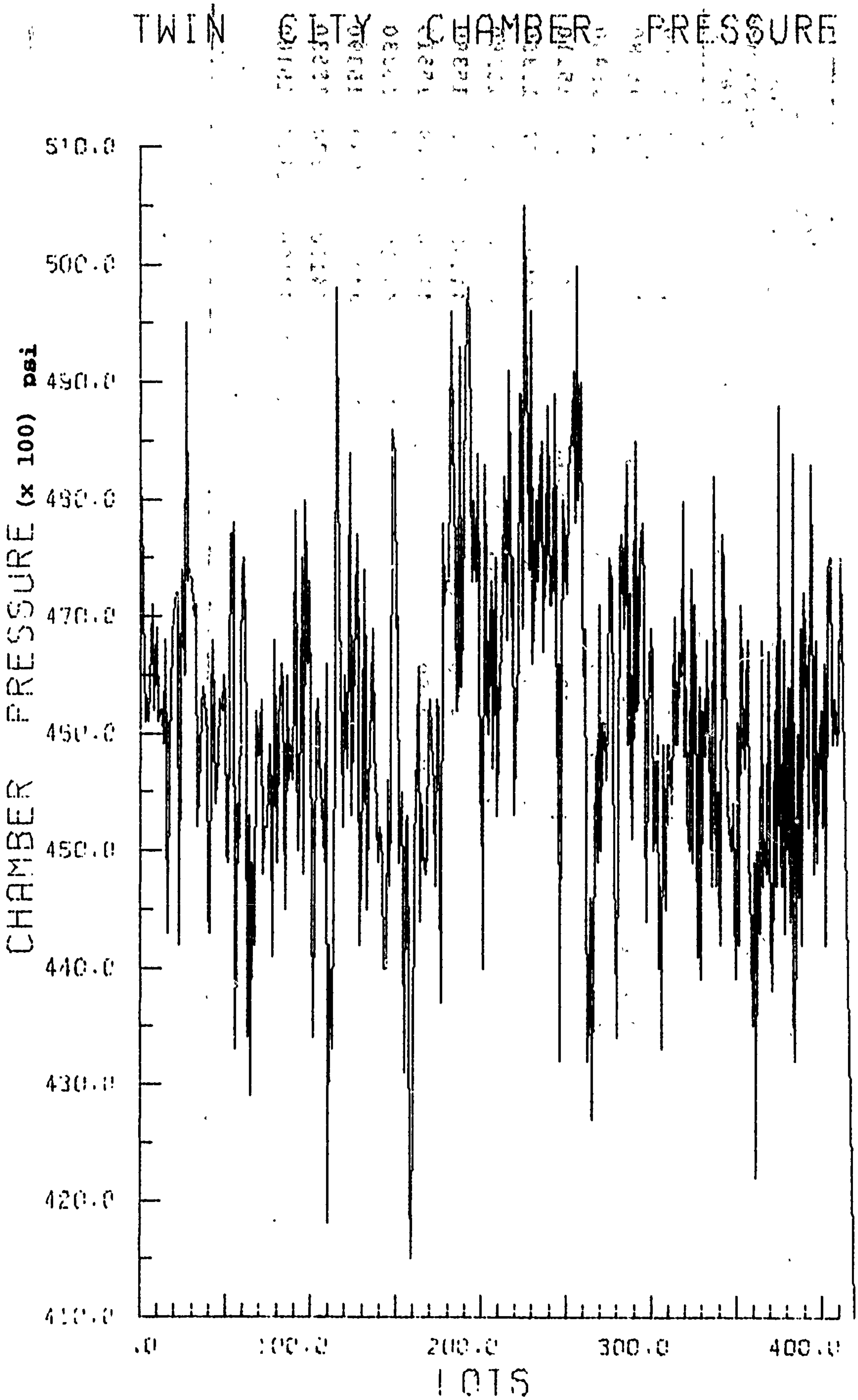

TWIN CITY CHAMBER PRESSURE
CHAMBER PRESSURE (x 100) psi
510.0
500.0
490.0
480.0
470.0
460.0
450.0
440.0
430.0
420.0
410.0
.0
100.0
200.0
300.0
400.0
LOTS

Figure 72

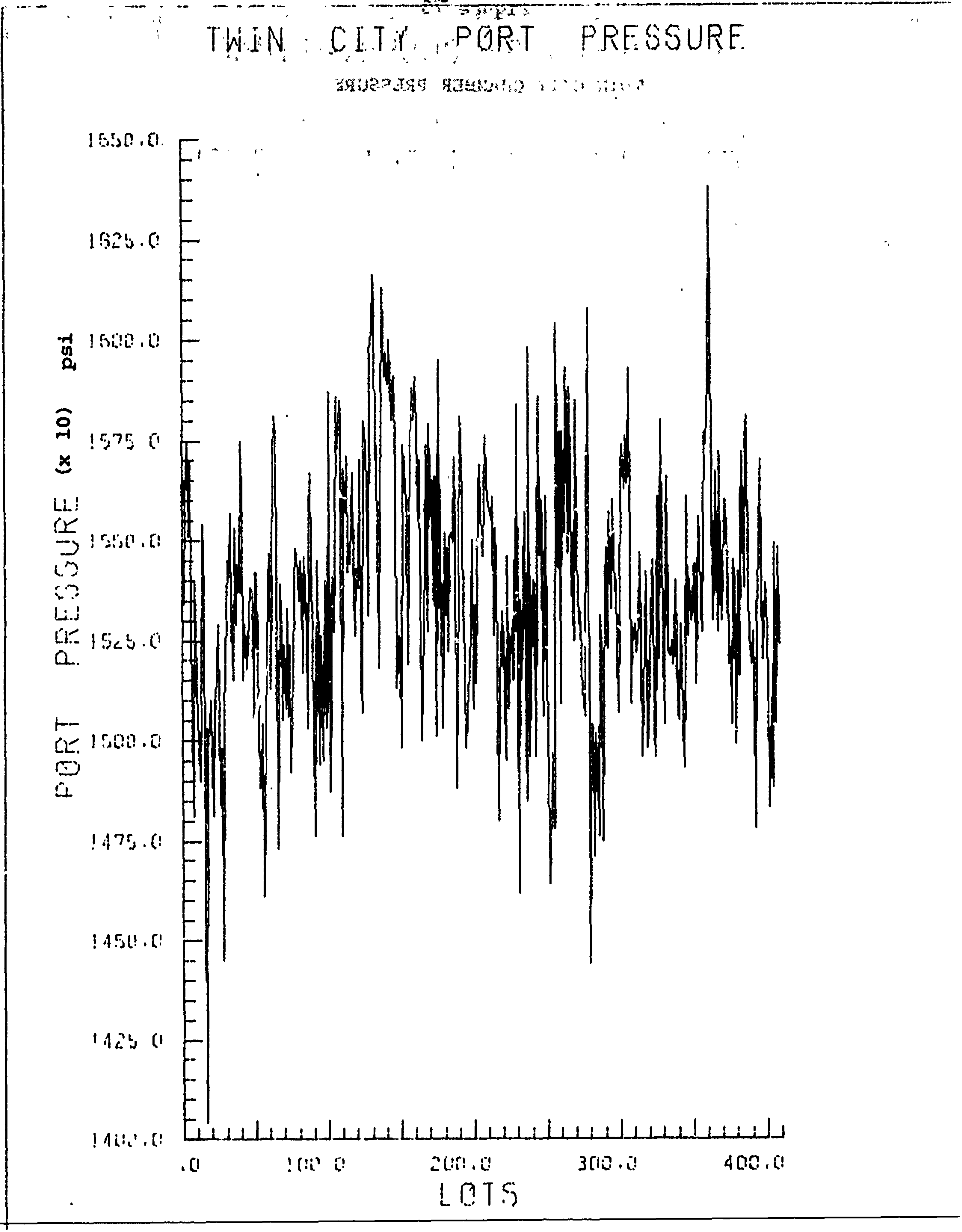

TWIN CITY PORT PRESSURE
PORT PRESSURE (x 10) psi
1650.0
1625.0
1600.0
1575.0
1550.0
1525.0
1500.0
1475.0
1450.0
1425.0
1400.0
.0
100.0
200.0
300.0
400.0
LOTS

Figure 73

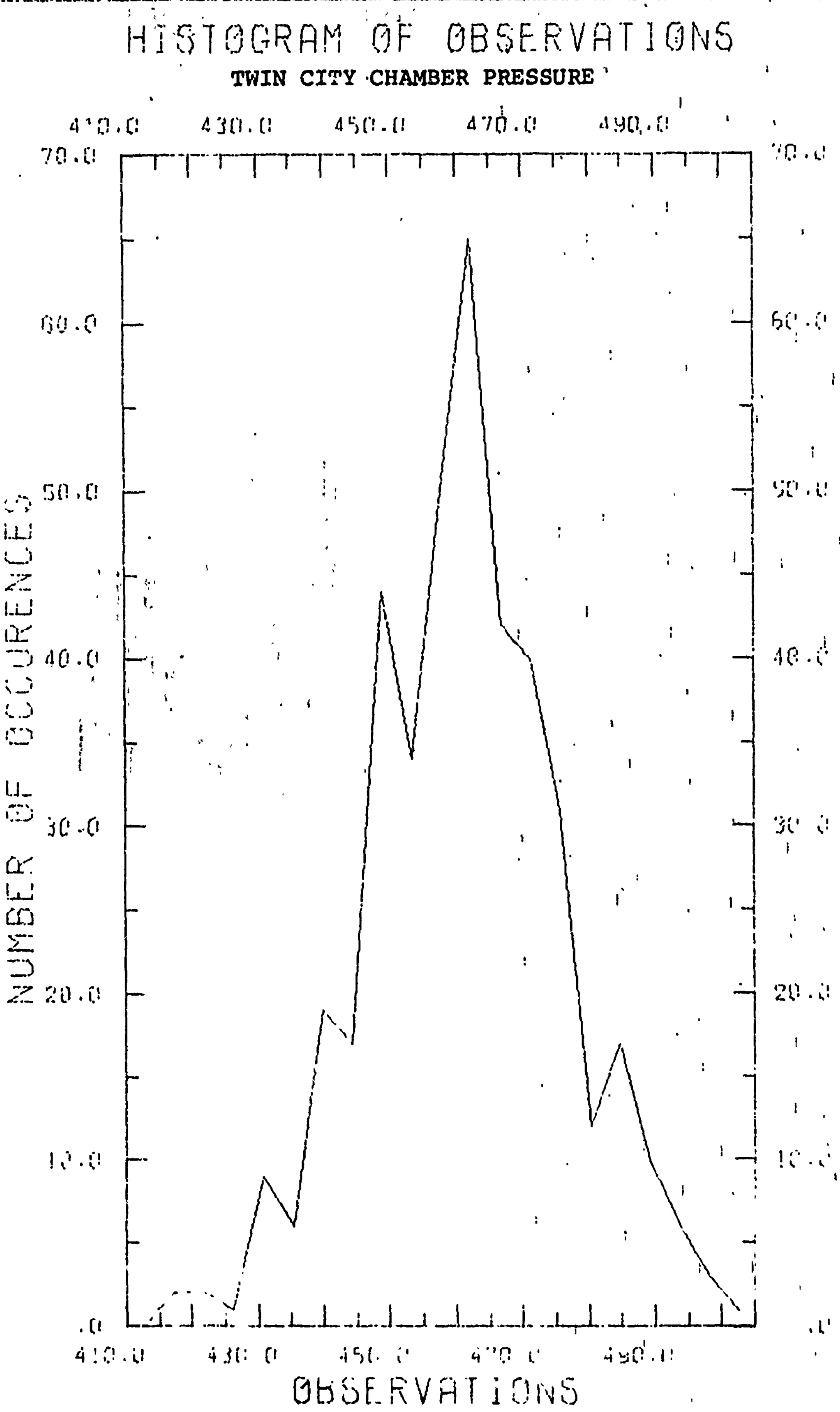

HISTOGRAM OF OBSERVATIONS
TWIN CITY CHAMBER PRESSURE
410.0
430.0
450.0
470.0
490.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
.0
NUMBER OF OCCURENCES
OBSERVATIONS

Figure 74

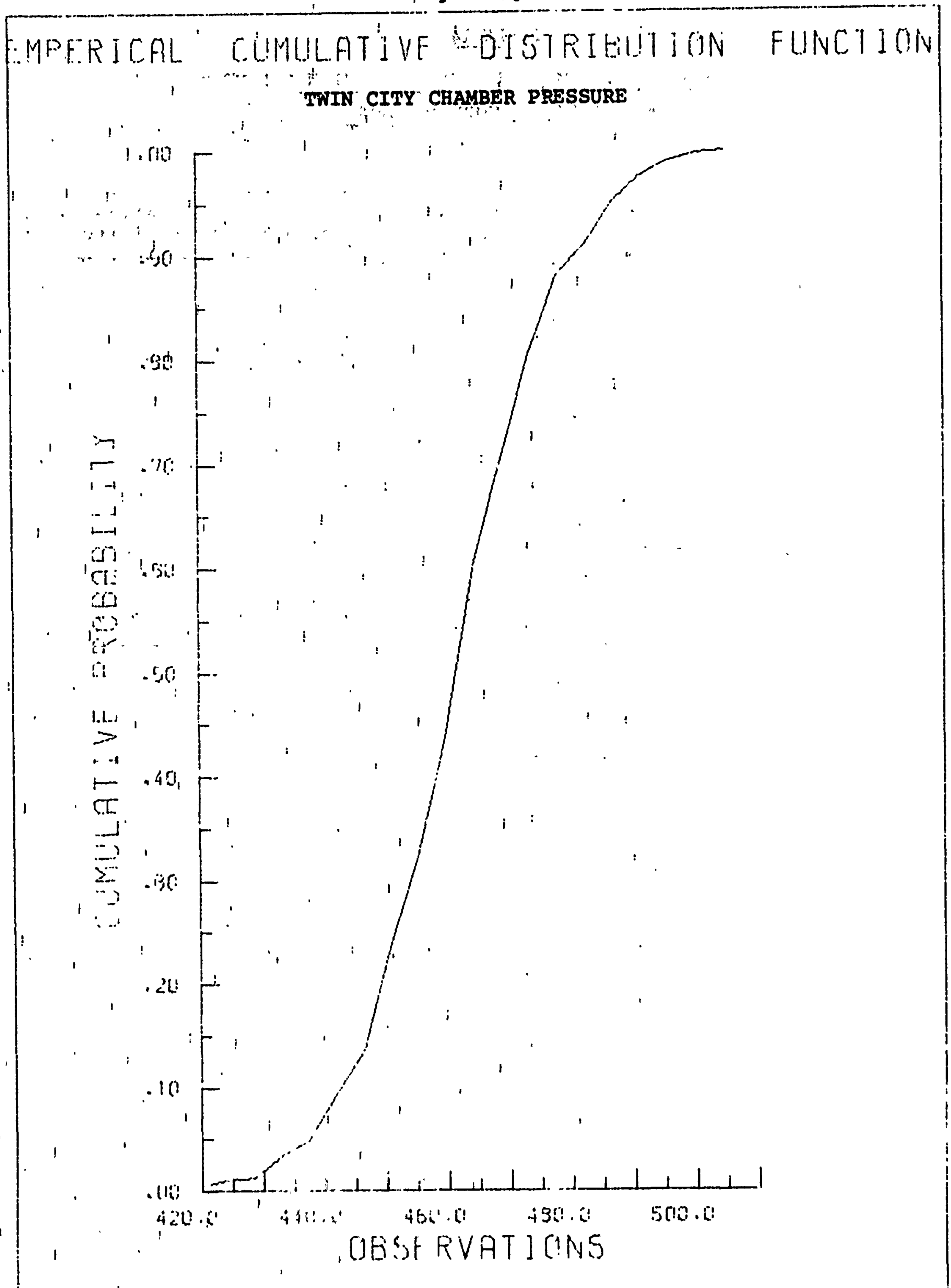

EMPERICAL CUMULATIVE DISTRIBUTION FUNCTION
TWIN CITY CHAMBER PRESSURE
CUMULATIVE PROBABILITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
.00
420.0
440.0
460.0
480.0
500.0
OBSERVATIONS

TABLE 14

HISTOGRAM AND CUMULATIVE DISTRIBUTION FUNCTION TWIN CITY CHAMBER PRESSURE

Serial Number	Interval	Number of Occurences	Probability	Cumulative Probability
1	415.00 to 419.50	2.	.005	.005
2	419.50 to 424.00	2.	.005	.010
3	424.00 to 428.50	1.	.002	.012
4	428.50 to 433.00	9.	.022	.034
5	433.00 to 437.50	6.	.015	.049
6	437.50 to 442.00	19.	.046	.095
7	442.00 to 446.50	17.	.041	.136
8	446.50 to 451.00	44.	.107	.243
9	451.00 to 455.50	34.	.083	.326
10	455.50 to 460.00	50.	.122	.448
11	460.00 to 464.50	65.	.158	.606
12	464.50 to 469.00	42.	.102	.708
13	469.00 to 473.50	40.	.097	.805
14	473.50 to 478.00	31.	.075	.881
15	478.00 to [illegible] 50	12.	.029	.910
16	482.50 to 487.00	17.	.041	.951
17	487.00 to 491.50	10.	.024	.976
18	491.50 to 496.00	6.	.015	.990
19	496.00 to 500.50	3.	.007	.998
20	500.50 to 505.00	1.	.002	1.0
		Total=411	Total=1	

TABLE 15

BIVARIATE HISTOGRAM OF CHAMBER PRESSURE AND PORT PRESSURE

TWIN CITY

												Chamber Pressure (psi)
0.	0.	0.	1.	0.	0.	0.	0.	0.	1.	0.	0.	418.82
0.	0.	0.	0.	0.	0.	0.	0.	0.	2.	0.	0.	
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	426.46
0.	0.	0.	0.	0.	0.	1.	1.	0.	1.	0.	0.	
0.	0.	2.	0.	1.	0.	1.	1.	4.	1.	0.	0.	434.10
0.	0.	0.	0.	0.	0.	2.	2.	1.	2.	0.	0.	← LOW
0.	0.	0.	0.	0.	0.	3.	3.	0.	3.	1.	0.	441.74
0.	0.	0.	0.	0.	4.	4.	5.	5.	2.	2.	0.	
0.	0.	0.	1.	0.	2.	9.	5.	4.	9.	0.	0.	449.38
0.	0.	0.	1.	1.	5.	10.	4.	7.	4.	1.	1.	
0.	0.	0.	0.	1.	3.	8.	11.	6.	2.	2.	0.	457.02
0.	0.	0.	1.	1.	6.	9.	9.	5.	3.	0.	0.	
0.	0.	0.	0.	3.	12.	13.	21.	10.	2.	1.	0.	← MEDIUM; 464.66
0.	0.	0.	0.	7.	9.	8.	8.	4.	2.	0.	0.	
0.	0.	0.	1.	5.	7.	6.	6.	6.	2.	0.	0.	472.30
0.	0.	1.	2.	3.	13.	6.	6.	2.	2.	1.	1.	
0.	0.	0.	2.	1.	3.	3.	0.	1.	0.	0.	0.	479.94
0.	0.	0.	1.	3.	3.	6.	3.	1.	0.	1.	0.	
0.	0.	0.	1.	1.	8.	1.	1.	1.	0.	0.	0.	← HIGH; 487.58
0.	0.	0.	0.	4.	0.	1.	5.	0.	0.	0.	0.	
0.	0.	0.	0.	1.	0.	0.	1.	1.	0.	0.	0.	495.22
0.	0.	0.	0.	0.	0.	1.	2.	0.	2.	0.	0.	
0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	Δ_2
0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	506.68

Δ_1

144.70 148.52 152.34 156.16 159.98 163.80

Port Pressure (psi)

Note: Values of pressure along the two axes are maximum values for that group

Δ_1 = 191 psi

Δ_2 = 382 psi

TABLE 16

Critical Values for Selection Based Upon Chamber Pressure and Port Pressure

TWIN CITY

Normal Selection

Chamber Pressure Tolerance = 2 S.D.

Port Pressure Tolerance = 2.5 S.D.

	Chamber Pressure psi	Port Pressure psi
HIGH	48376 to 49140	14852 to 15807
MEDIUM	46084 to 46848	14852 to 15807
LOW	43028 to 43792	14852 to 15807

Tight Selection

Chamber Pressure Tolerance = 1 S.D.

Port Pressure Tolerance = 2.5 S.D.

	Chamber Pressure psi	Port Pressure psi
HIGH	48376 to 48758	14852 to 15807
MEDIUM	[illegible]084 to 46466	14852 to 15807
LOW	43410 to 43792	14852 to 15807

TABLE 17

Final Lot Selection

(Based Upon Chamber Pressure and Port Pressure)

Chamber Pressure Tolerance = 1 S.D.

Port Pressure Tolerance = 2.5 S.D.

TWIN CITY

High				Medium				Low			
Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi
TW-1-55	48400	1610	15160	TW-1-182	46100	2220	15240	TW-1-132	43500	1850	15740
1-7	48500	1610	15570	1-180	46300	2020	15040	-18609	43400	1710	14870
-18734	48700	1090	15150	1-178	46400	2590	14880	-18556	43400	1930	15810
-18705	48400	2600	15140	1-172	46200	1620	15390				
-18656	48400	1470	15260	1-164	46400	1510	15270				
-18655	48600	1680	15130	1-163	46400	1540	15190				
				1-145	46200	1360	15600				
				1-128	46300	2140	15570				
				1-126	46400	1160	15140				

TABLE 18

Final Lot Selection

(Based Upon Chamber Pressure and Port Pressure)

Chamber Pressure Tolerance = 2 S.D.

Port Pressure Tolerance = 2.5 S.D.

TWIN CITY

High				Medium				Low			
Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No	Chamber Pressure psi	S.D.	Port Pressure psi
TW-				TW-							
1-155	48400	1610	15160	1-184	46800	1517	15170	1-157	43200	1210	15720
1-147	48800	1860	15440	1-182	46100	2220	15240	1-132	43500	1850	15740
1-30	49000	2140	15430	1-180	46300	2020	15040	1-78	43300	1360	15660
1-26	49100	1890	14970	1-178	46400	2590	14880	1-34	43200	1830	15520
1-15	48900	1660	14960	1-174	46600	1630	15240	1-18	43200	1470	15600
1-11	48800	1890	14960	1-172	46200	1620	15390	-18662	43100	1500	15190
1-7	48500	1610	15570	1-169	46800	1430	15700	-18619	43300	2300	15570
-18734	48700	1090	15150	1-164	46400	1510	15270	-18609	43400	1710	14870
-18733	48800	1710	15280	1-163	46400	1540	15190	-18556	43400	1930	15810
-18730	48900	1530	15460	1-150	46800	2320	15040				
-18729	48900	1750	15950	1-145	46200	1460	15600				
-18723	49100	2000	15470	1-141	46700	1880	15720				

5.7 Design of Experiments to Obtain the Effect of Ammunition Characteristics on Weapon Performance

In selecting ammunition for weapon tests, it has been assumed that the weapon performance is governed by the copper crusher chamber pressure and port pressure characteristics of the ammunition lot. This is not known to be true. The true ammunition parameters that control gun performance need to be determined. Once these parameters are known, they can be substituted for chamber pressure and port pressure and the analysis repeated to obtain proper ammunition lots for weapon tests.

Lacking a complete theoretical analysis at the present time, true ammunition parameters can only be determined experimentally. To minimize experimentation, proper experimental design techniques have to be used. One such experimental procedure is now illustrated.

Let us suppose that the effect of chamber pressure and port pressure on cyclic rate is to be determined. Figure 75 shows a replicated 2^2 factorial design with cyclic rate as response. For tests at each of the four points, two ammunition lots are selected with their chamber pressure and port pressure within the indicated ranges. Since these ranges are two standard deviations wide, the lots are considered identical. The two tests at each of the four points are, therefore, replicates. This design enables the determination of the main effects of chamber pressure and port pressure as well as the effect

TABLE 18. Cont'd

High				Medium			
Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi	Lot No.	Chamber Pressure psi	S.D.	Port Pressure psi
-18705	48400	2600	15140	1-137	46800	1320	15300
-18697	49000	1320	15400	1-129	46800	1760	15330
-18656	48400	1470	15260	1-128	46300	2140	15570
-18655	48600	1680	15130	1-126	46400	1160	15140
				1-122	46100	1280	15330
				1-115	46600	1710	15190
				1-104	46800	1270	15520
				1-102	46200	2400	15490
				1-100	46400	1160	15520

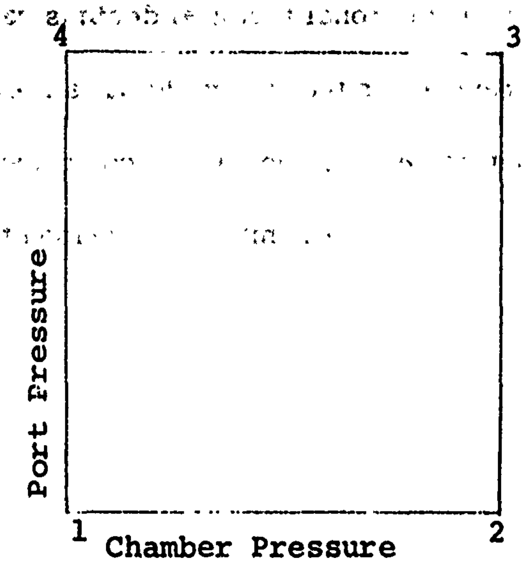
of chamber pressure-port interaction on cyclic rate in just eight experiments.

Such factorial and/or fractional factorial designs can be employed to determine ammunition parameters that show significant effects on weapon responses. These parameters can then be said to control weapon performance and would be used to select ammunition for weapon testing.

Figure 75

Replicated Factorial Design

(Lake City)



Test Points	Critical Pressure Ranges for Test Lots					
	Chamber Pressure Range (psi)			Port Pressure Range (psi)		
1	44000	to	44600	14950	to	15250
2	47900	to	48500	14950	to	15250
3	47900	to	48500	15550	to	15850
4	44000	to	44600	15550	to	15850

6. DETERMINATION OF PROPER NUMBER OF TESTS FOR ESTIMATING AMMUNITION PARAMETERS

6.1 Method of Approach

To obtain good estimates of ammunition parameters such as chamber pressure, port pressure, peak chamber pressure, velocity, etc., proper number of tests have to be conducted. If smaller than necessary number of tests are conducted, the parameters would be estimated with large variance. On the other hand, conducting more than necessary number of tests involves unnecessary expenditure. There is a need to obtain an appropriate number of tests to be performed.

This number would be determined by the desired precision with which the parameters are to be estimated and the experimental error involved. For example, in the measurement of chamber pressure by standard tests if the desired precision is $r = \frac{1}{\sigma_1^2}$ and the experimental error is known to be σ^2, then the required number of tests 'n' to get the desired precision is given by

$$n = \sigma^2 / \sigma_1^2$$

The chamber pressure is then estimated by the mean of these n readings. The mean has an error variance of σ_1^2 which is a prespecified value. The only unknown quantity is σ^2--the error variance for each test.

Hence σ^2 has to be experimentally determined. The experimental error is a function of the following:

(1) The parameter or parameters to be estimated, i.e., the error involved in the measurement of chamber pressure can be different from the error in the measurement of port pressure.

(2) Experimental setup which includes variables like barrels, measuring devices (copper crusher, piezo) cartridges from different manufacturers etc.

(3) Inherent variability of the manufacturing process in producing a lot of ammunition. Table 13 indicates that lots belong to the HIGH category have a somewhat larger chamber pressure standard deviation as compared to the lots belonging to the LOW category.

If the population of ammunition lots is considered to be homogeneous, then the average value of standard deviation of chamber pressure (e.g. 1330 psi for Lake City) is a good estimate of standard error in copper crusher testing, for the measurement of chamber pressure. However, differences exist amongst the ammunition lots. It is, therefore, necessary to conduct experiments to determine the experimental error.

6.2 Sequential Procedure for Estimation of Experimental Error

A sequential procedure is now illustrated by which an estimate of experimental error can be obtained. Figures 76, 77, and 78 are based upon

20 individual copper crusher chamber pressure readings per lot. The data were obtained from B.A.A.P. Figures 79 and 80 are based upon 10 individual piezo peak pressure readings obtained from Frankford Arsenal. The entire set of data is given in Appendix III.

The figures illustrate sequential estimation of variance of observations using 2,3,4, ... number of tests. As can be observed, the estimate of variance fluctuates considerably in the beginning but tends to stabilize as the number of observations increases. The value about which the variance stabilises is the estimate of experimental error. The figures indicate that in all cases the variance has not stabilised. The conclusion is that 20 readings are not sufficient to estimate experimental error in copper crusher testing and same is true about ten readings in piezo testing.

It should be noted that the current practice in standard acceptance testing is to conduct 20 tests to obtain an estimate of standard deviation. In view of the analyses conducted above, this estimate of standard deviation has a large variance associated with it. It is suggested that sufficiently large number of tests be conducted to obtain a stable estimate of experimental error.

Once such an estimate of experimental error is available, necessary number of tests to obtain parameter estimates with desired degree of precision can be determined.

Figure 76

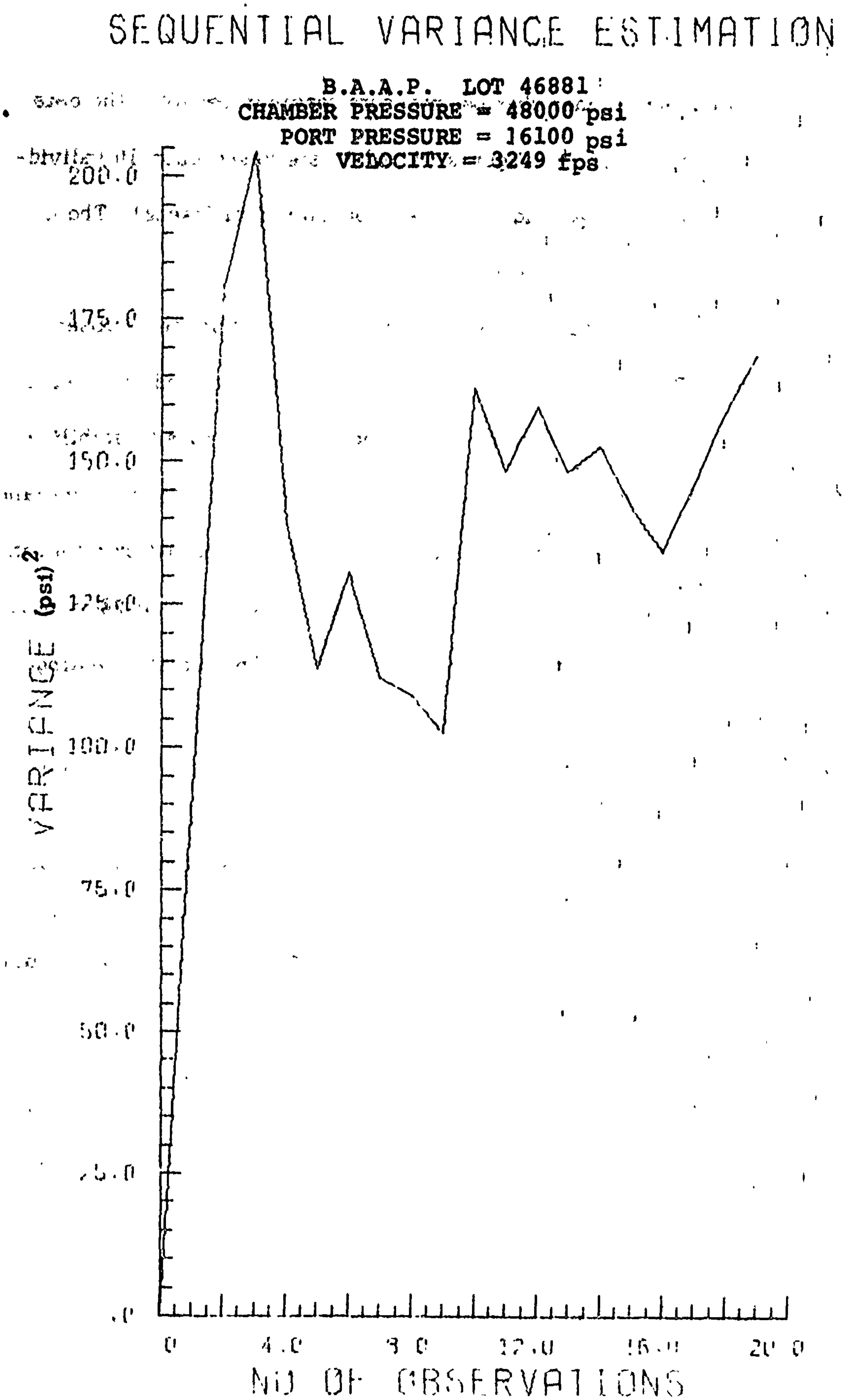

SEQUENTIAL VARIANCE ESTIMATION
B.A.A.P. LOT 46881
CHAMBER PRESSURE = 48000 psi
PORT PRESSURE = 16100 psi
VELOCITY = 3249 fps
VARIANCE (psi)²
200.0
175.0
150.0
125.0
100.0
75.0
50.0
25.0
.0
0
4.0
8.0
12.0
16.0
20.0
NO OF OBSERVATIONS

Figure 77

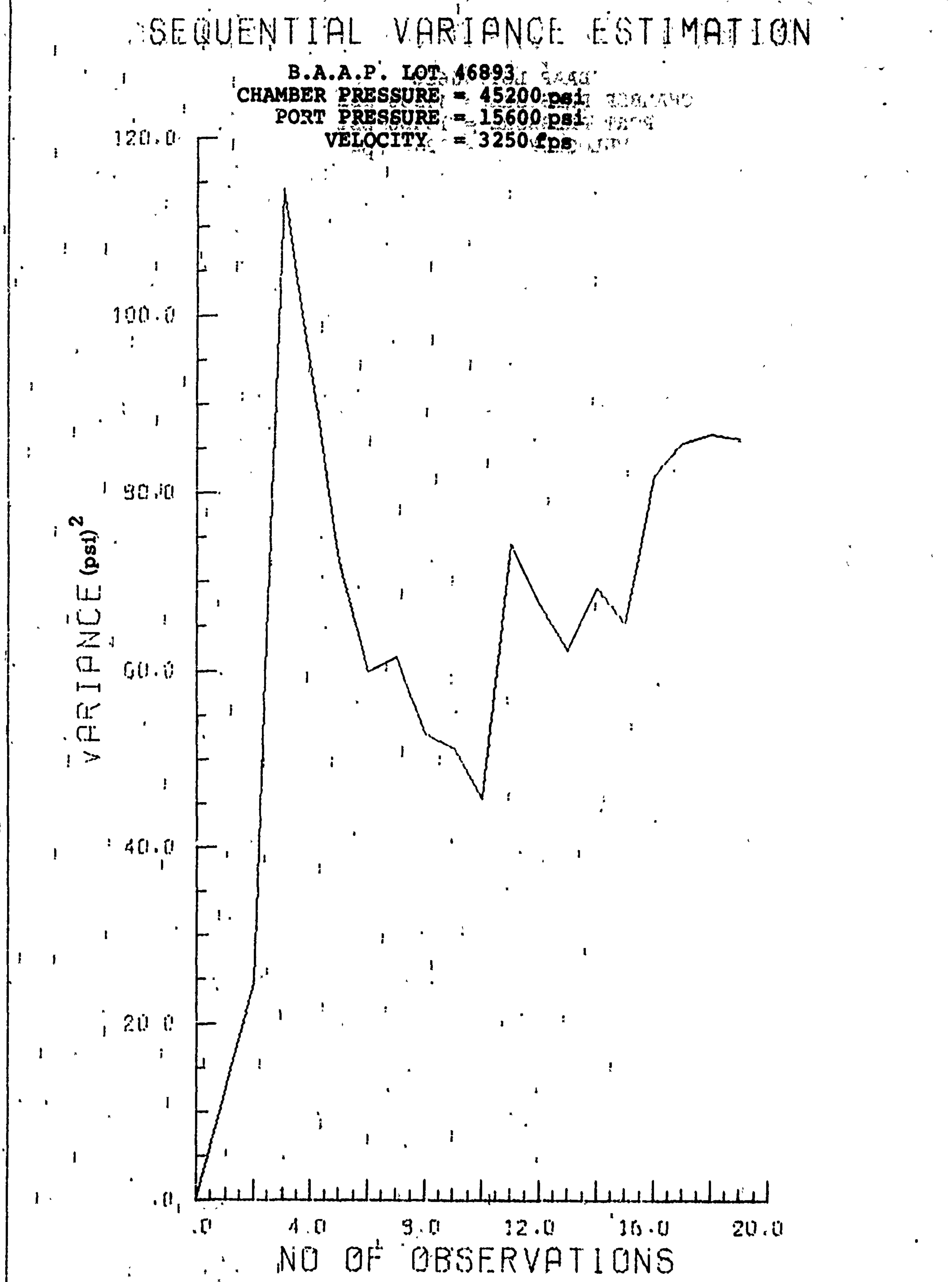

SEQUENTIAL VARIANCE ESTIMATION
B.A.A.P. LOT 46893
CHAMBER PRESSURE = 45200 psi
PORT PRESSURE = 15600 psi
VELOCITY = 3250 fps
120.0
100.0
80.0
60.0
40.0
20.0
.0
VARIANCE (psi)2
.0
4.0
8.0
12.0
16.0
20.0
NO OF OBSERVATIONS

Figure 78

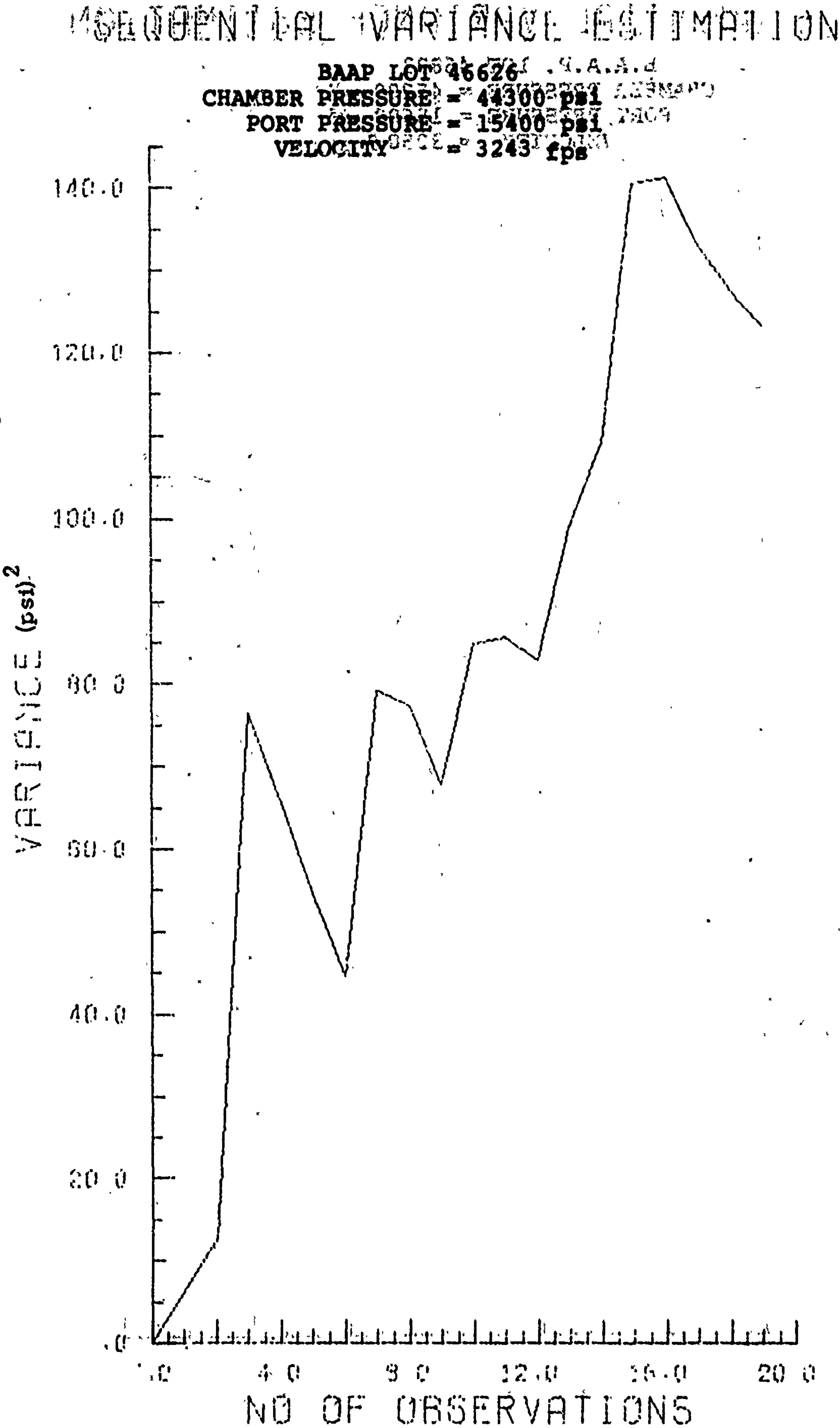

SEQUENTIAL VARIANCE ESTIMATION
BAAP LOT 46626
CHAMBER PRESSURE = 44300 psi
PORT PRESSURE = 15400 psi
VELOCITY = 3243 fps
140.0
120.0
100.0
80.0
60.0
40.0
20.0
.0
VARIANCE $(psi)^2$
4.0
8.0
12.0
16.0
20.0
NO OF OBSERVATIONS

Figure 79

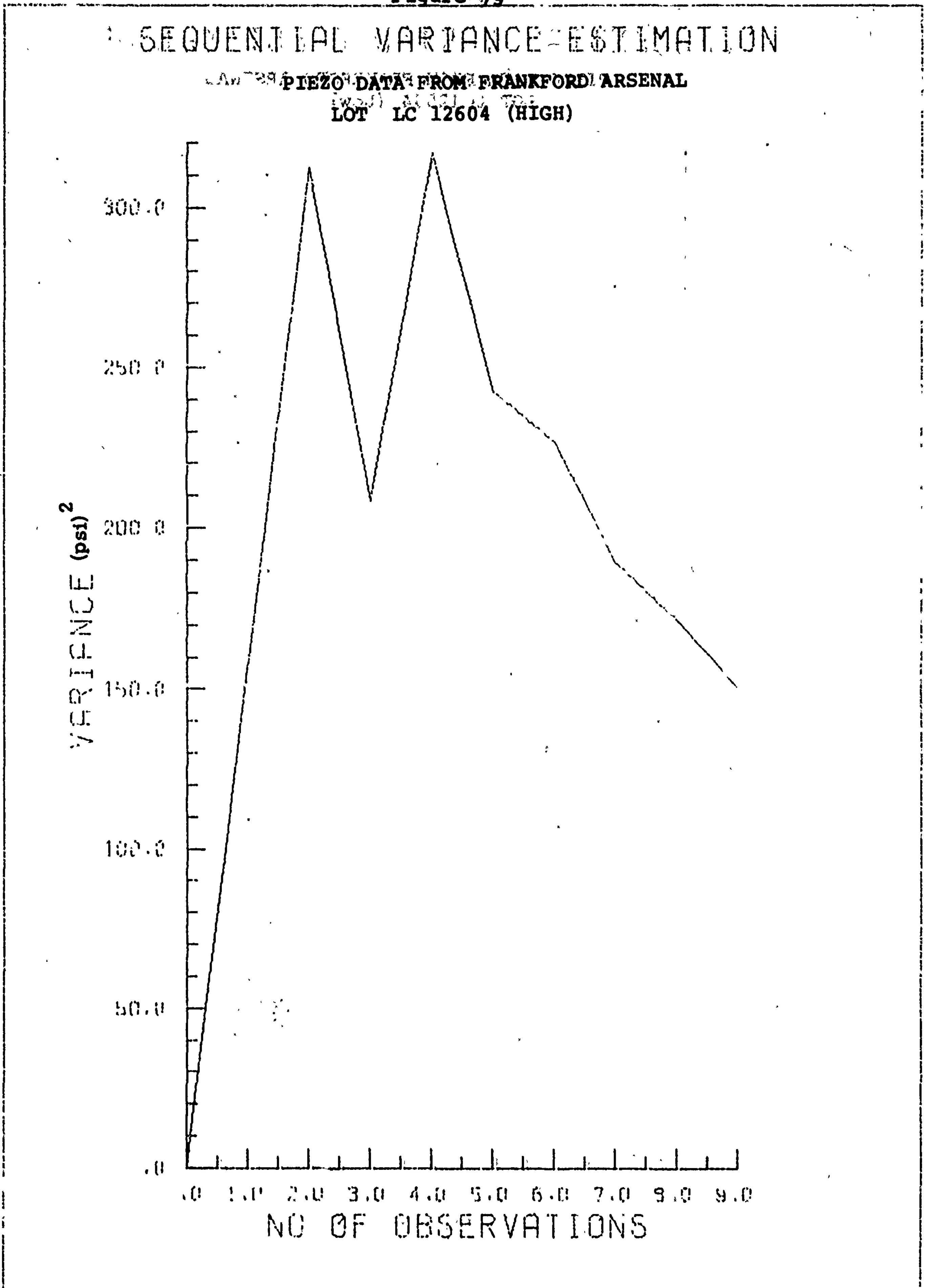
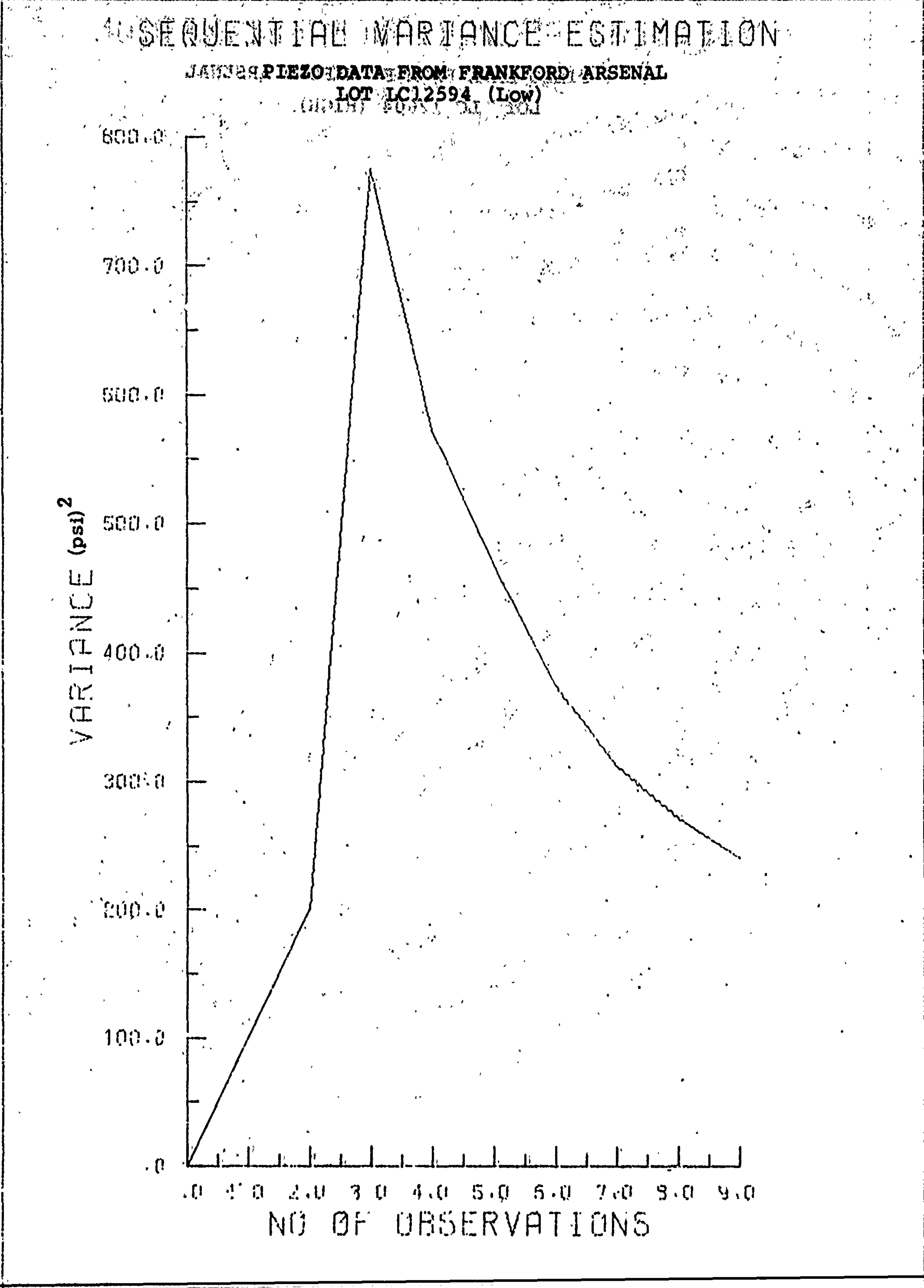

SEQUENTIAL VARIANCE ESTIMATION
PIEZO DATA FROM FRANKFORD ARSENAL
LOT LC 12604 (HIGH)
VARIANCE (psi)^2
300.0
250.0
200.0
150.0
100.0
50.0
.0
.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
NO OF OBSERVATIONS

Figure 80


SEQUENTIAL VARIANCE ESTIMATION
PIEZO DATA FROM FRANKFORD ARSENAL
LOT LC12594 (Low)
VARIANCE (psi)^2
800.0
700.0
600.0
500.0
400.0
300.0
200.0
100.0
.0
.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
NO OF OBSERVATIONS

7. ANALYSIS OF DATA AT DIFFERENT STAGES OF MANUFACTURE

Data are generated at different stages in the manufacturing process. An analysis of these data would indicate changes in ammunition characteristics during the process of manufacture. Such an analysis would help explain the final ballistic characteristics attained by a given ammunition.

7.1 Time Series Analysis of Propellant Lot Characteristics from B.A.A.P.

Since the propellant lots are produced sequentially, a relationship may be expected between successive lots. The ballistic results are obtained by adjusting the charge weight to obtain muzzle velocity close to 3250 f.p.s. Changes in lot characteristics would be reflected in the set charge weight and the resulting chamber pressure. These two responses, therefore, have been used to represent the properties of the propellant lots. The series of charge weight and chamber pressure are given in Appendix IV.

Plot of chamber pressure data is shown in Fig. 81. Fig. 82 shows the plot of the auto correlation function and the partial auto correlation function for the original series. A plot of the first difference of the original series is shown in Fig. 83 and the corresponding auto and partial correlation functions are shown in Fig. 84. The plots indicate that the first difference of the chamber pressure data is stationary and follows an MA (1) process. The model is, therefore, identified as:

$$y_t - y_{t-1} = (1 - \theta B)a_t = a_t - \theta\, a_{t-1}$$

where

y_t = chamber pressure for the t^{th} lot

a_t = NID (θ, σ^2)

θ = parameter to be estimated

The value of θ is found by nonlinear estimation and is 0.8892. Fig. 85 shows the plot of the residual a_t' s. The model is found to be adequate.

Fig. 86 through 90 show a similar analysis for the charge weight series and the time series model for this data is found to be

$$y_t - y_{t-1} = (1 - 0.5425B)a_t$$

where

y_t = charge weight for the t^{th} lot

$a_t \sim$ NID (θ, σ^2).

These models are subject to the following limitations:

(1) Testing is done using components from five different manufacturers. The differences in components are likely to influence the results.

(2) Lots do not strictly represent the time sequence of powder production process because they are obtained by blending fifteen preblends not necessarily in the order in which the preblends were produced.

Within these limitations, the successive propellant lots can be said to exhibit dependent characteristics. If it can be assumed that the successive propellant lots are used at the ammunition plants, the dependency between propellant lots would cause, to a small extent, the successive ammunition lots to show correlated characteristics.

7.2 Comparison of Acceptance Test Results, for the Same Propellant Lots, from B.A.A.P., Lake City and Twin City

Each propellant lot produced at B.A.A.P. is in effect tested for acceptance twice, once at B.A.A.P. and then at the place of cartridge manufacture. There is a time lag of a few months between the two testing dates. If it could be assumed that the testing equipments are alike, then a comparison of the two results would indicate the changes in powder properties during the elapsed time.

Table 19 gives the propellant lot acceptance test results from B.A.A.P., conducted with Lake City components. Table 20 gives the corresponding results from Lake City. Table 21 is the subtraction of Table 20 from Table 19. Tables 22, 23 and 24 give similar results with Twin City components. It can be observed that the quantities in Tables 21 and 24 have small magnitudes and random variations in signs. The conclusion is that powder properties, in terms of the responses considered, do not seem to change during the period of few months between the two testing dates.

Figure 81

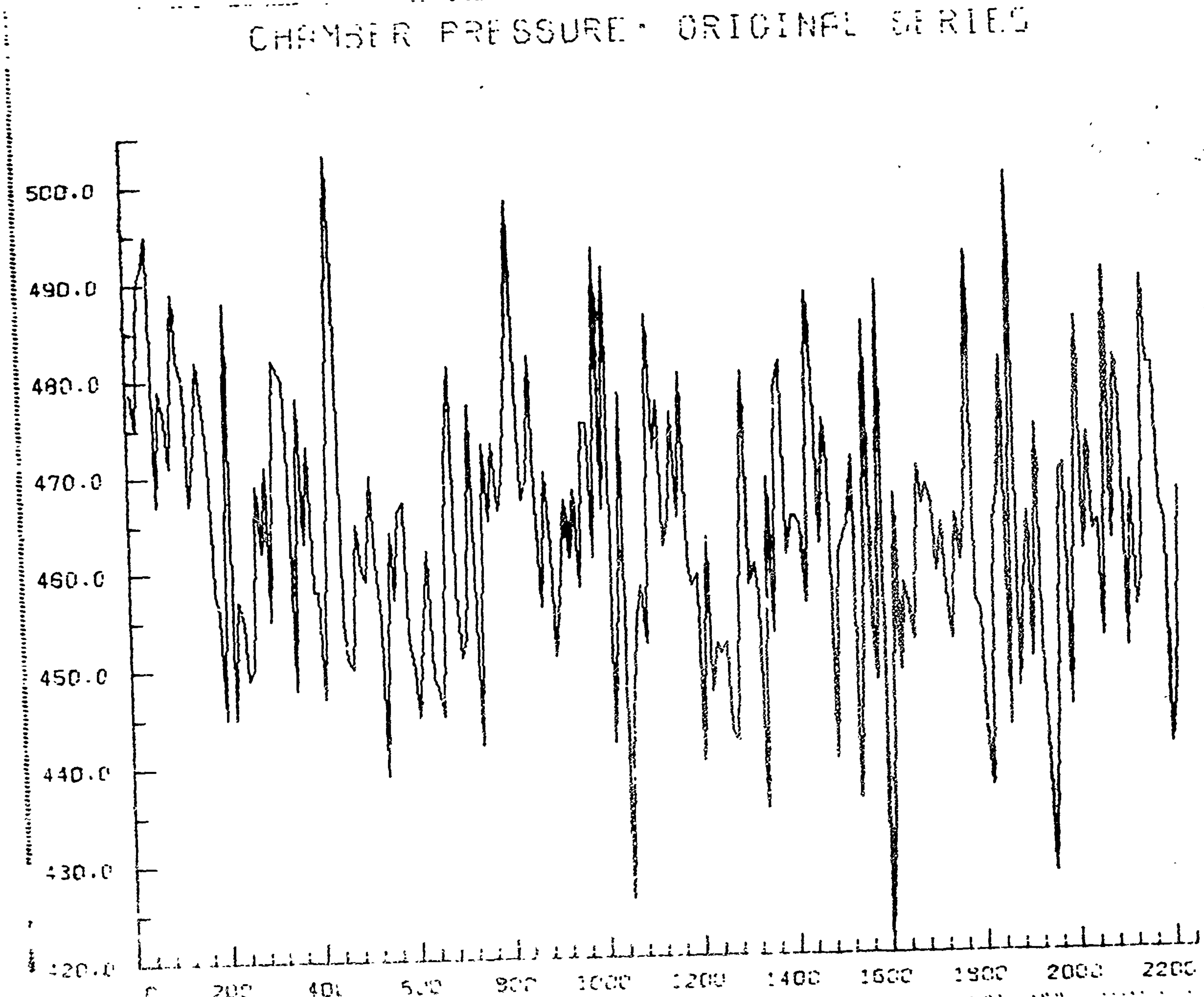

CHAMBER PRESSURE - ORIGINAL SERIES
500.0
490.0
480.0
470.0
460.0
450.0
440.0
430.0
420.0
0
200
1400
1600
1800
2000
2200

Figure 82(a)

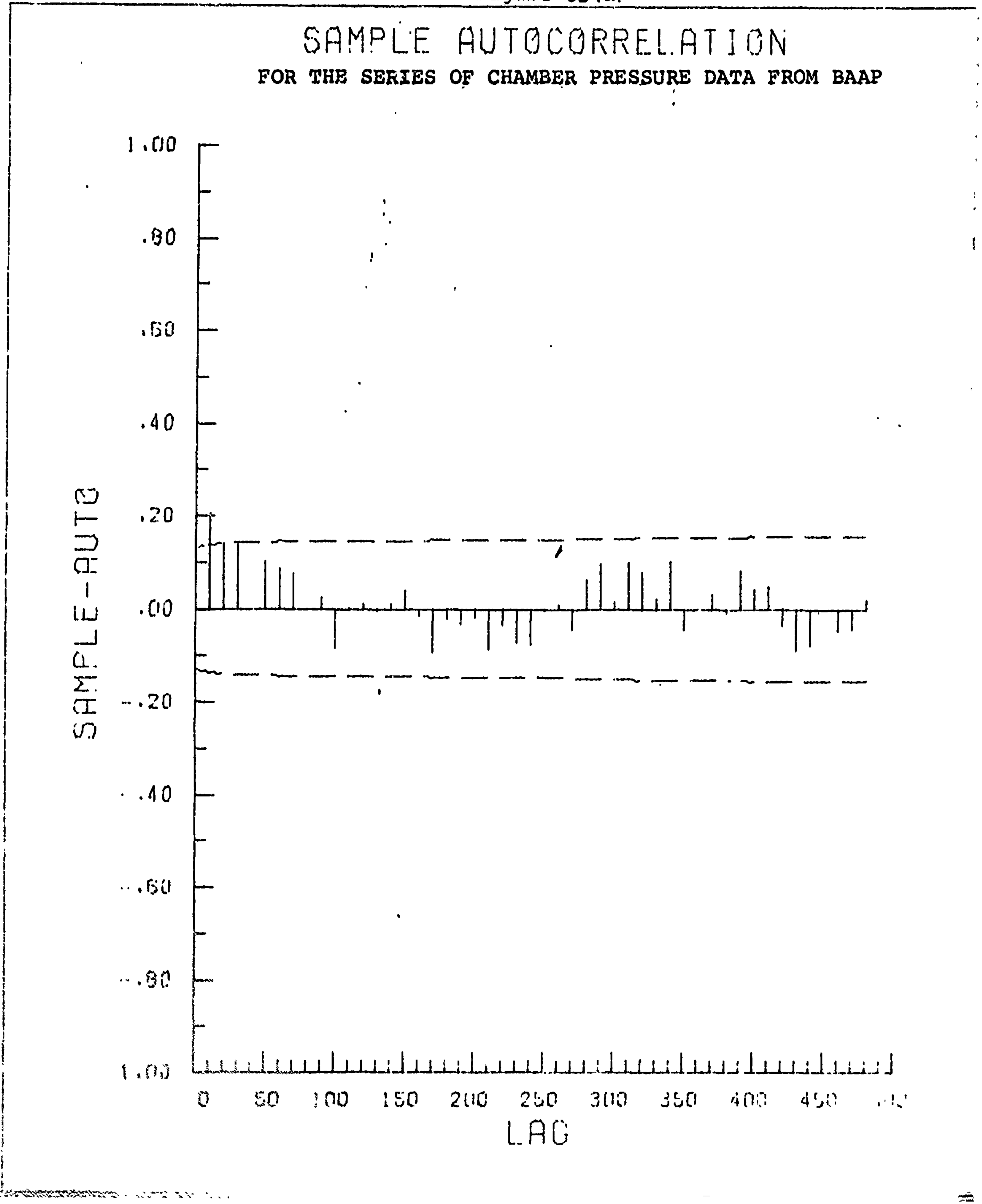

SAMPLE AUTOCORRELATION
FOR THE SERIES OF CHAMBER PRESSURE DATA FROM BAAP
1.00
.80
.60
.40
.20
.00
-.20
-.40
-.60
-.80
1.00
SAMPLE-AUTO
0
50
100
150
200
250
300
350
400
450
LAG

Figure 82 (b)

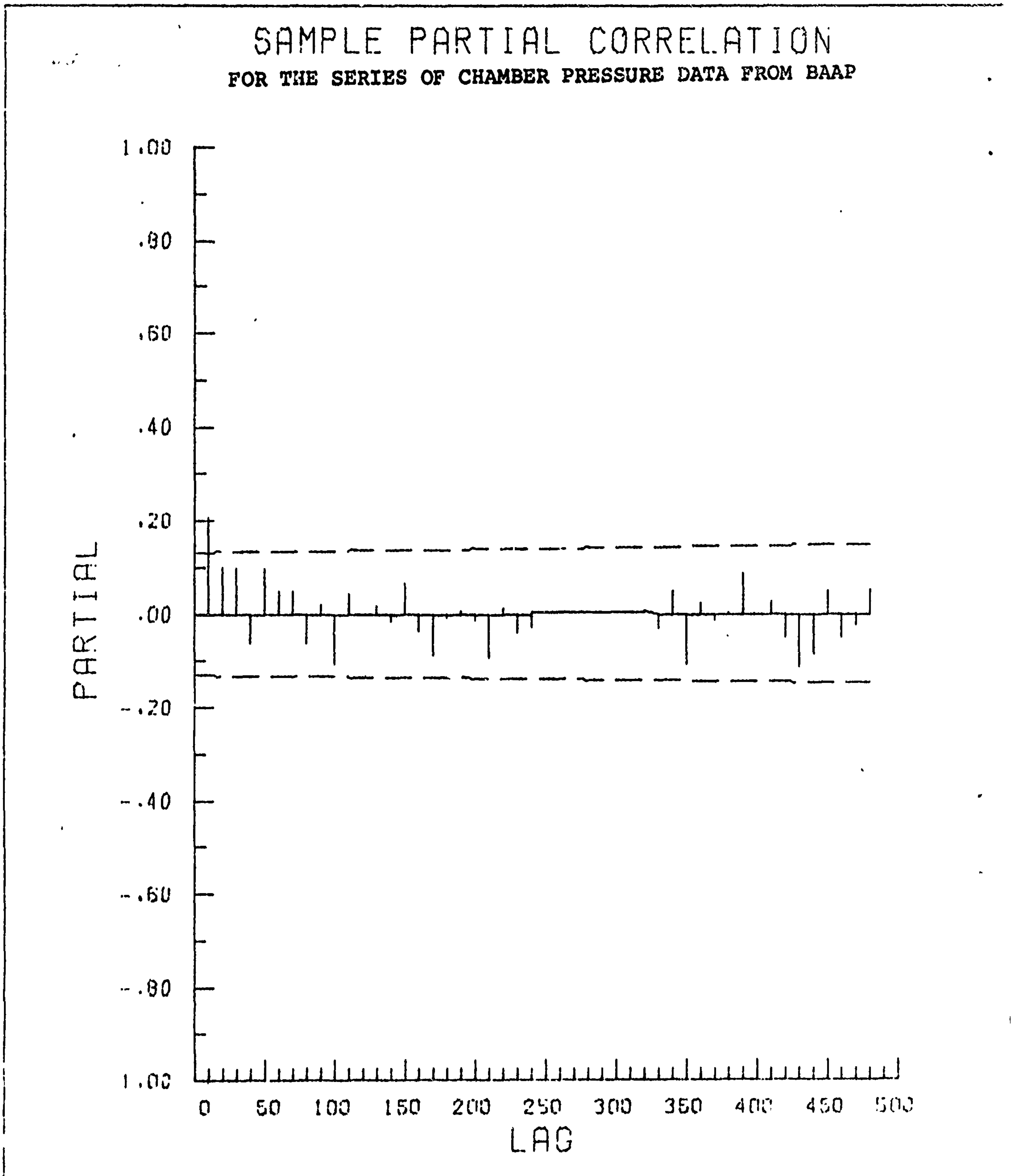

SAMPLE PARTIAL CORRELATION
FOR THE SERIES OF CHAMBER PRESSURE DATA FROM BAAP
1.00
.80
.60
.40
.20
.00
-.20
-.40
-.60
-.80
1.00
PARTIAL
0
50
100
150
200
250
300
350
400
450
500
LAG

Figure 83

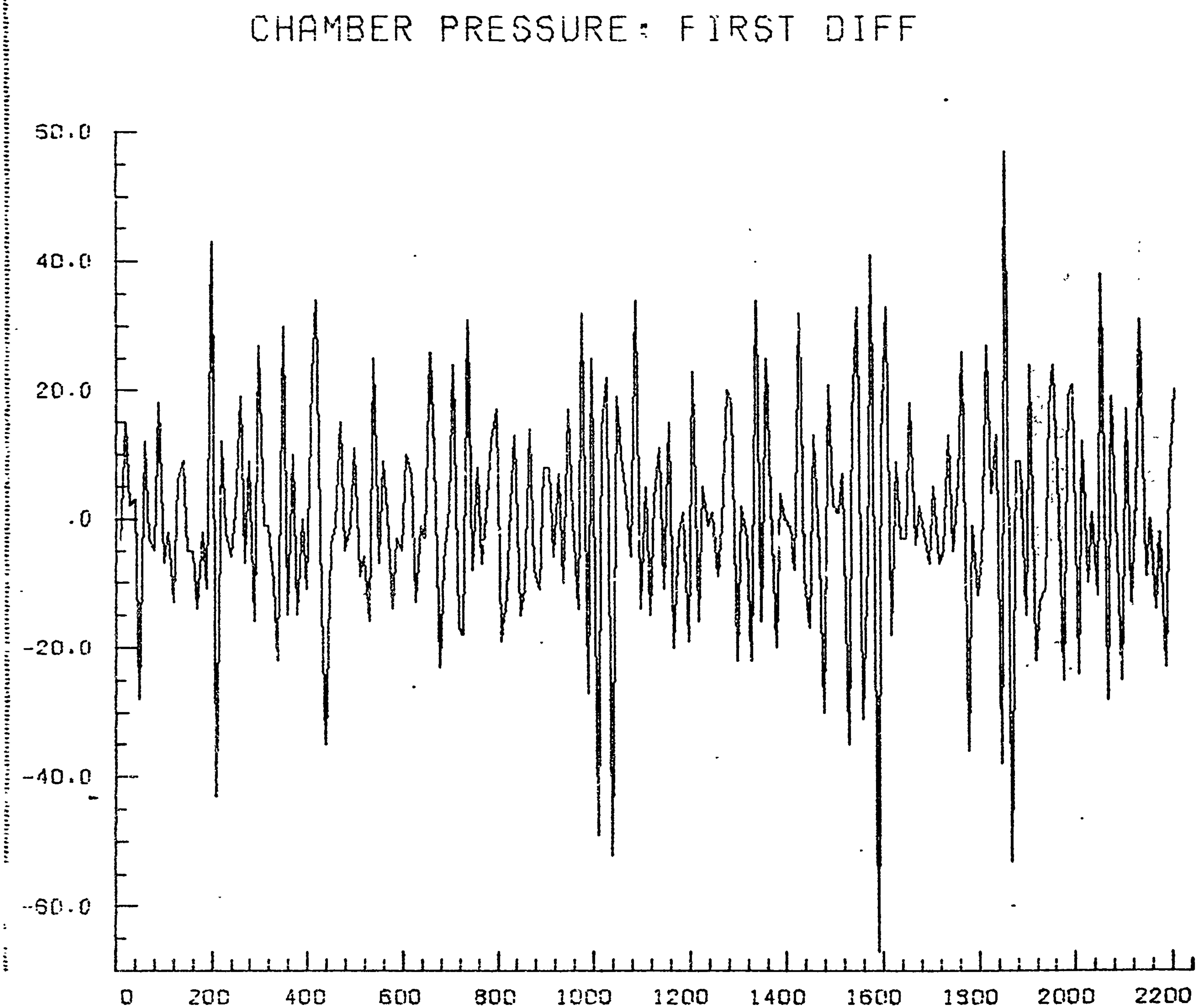

CHAMBER PRESSURE: FIRST DIFF
50.0
40.0
20.0
.0
-20.0
-40.0
-60.0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200

Figure 84 (a)

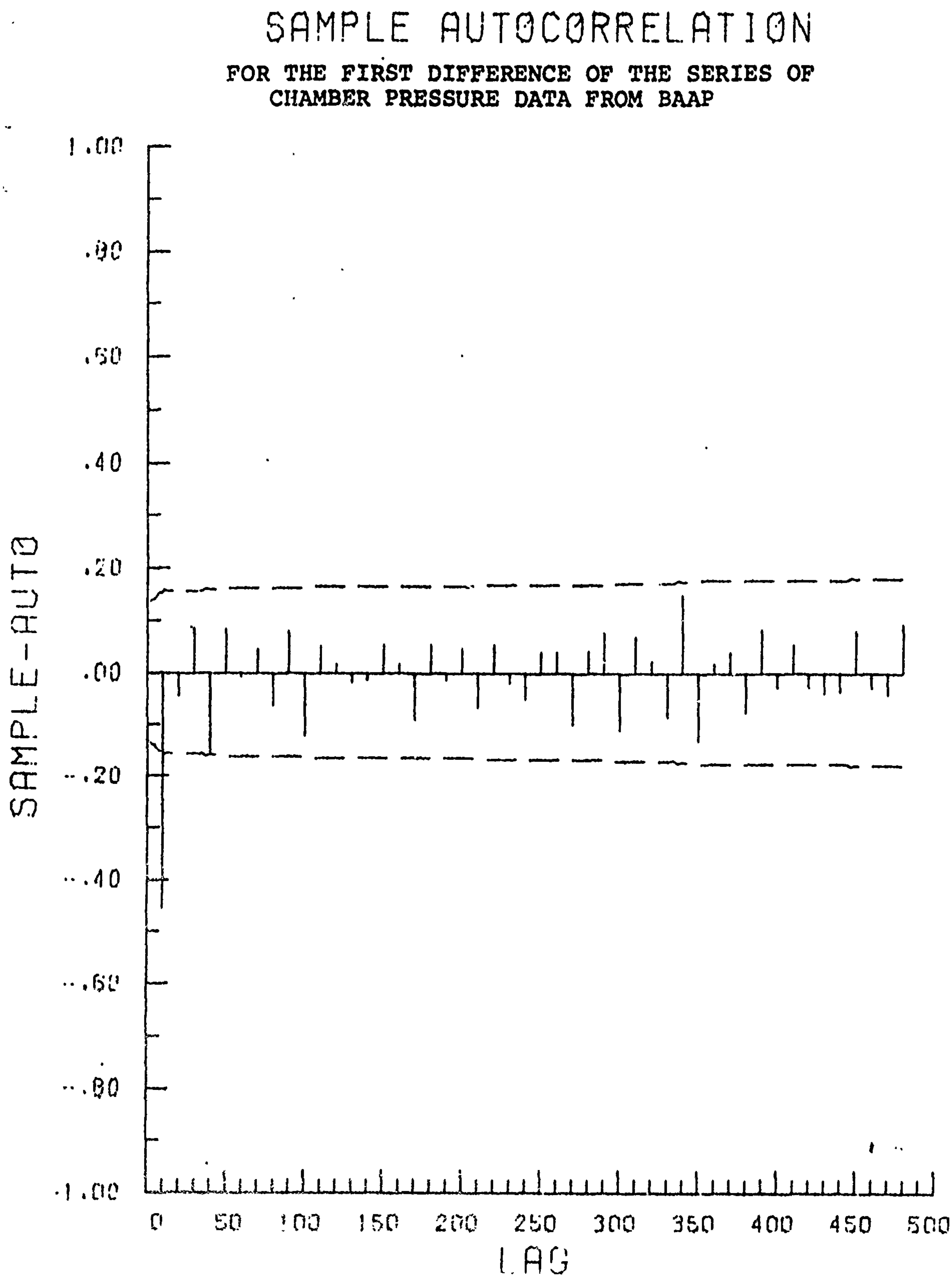
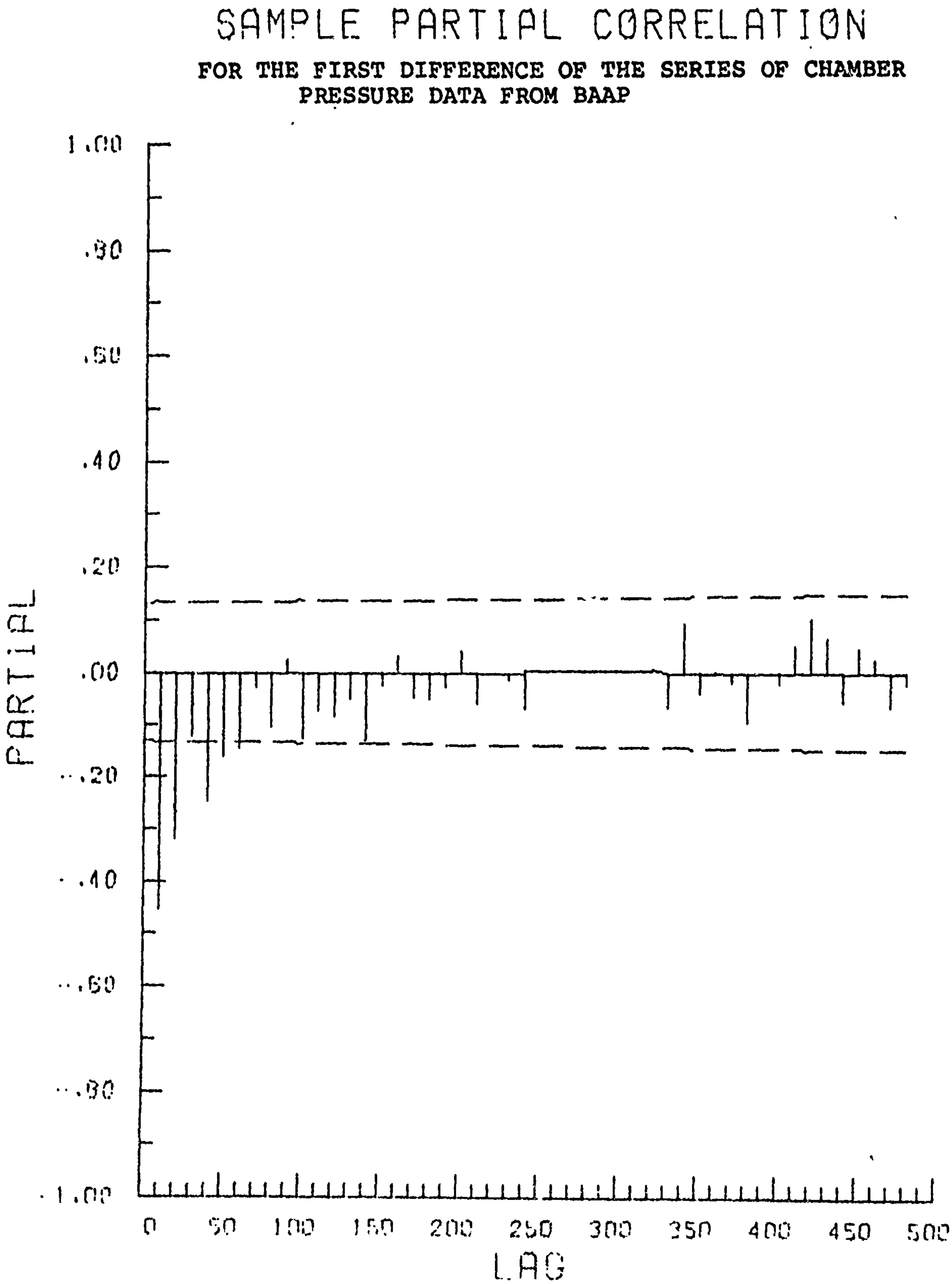

SAMPLE AUTOCORRELATION
FOR THE FIRST DIFFERENCE OF THE SERIES OF CHAMBER PRESSURE DATA FROM BAAP
1.00
.80
.60
.40
.20
.00
-.20
-.40
-.60
-.80
-1.00
SAMPLE-AUTO
0
50
100
150
200
250
300
350
400
450
500
LAG

Figure 84 (b)


SAMPLE PARTIAL CORRELATION
FOR THE FIRST DIFFERENCE OF THE SERIES OF CHAMBER PRESSURE DATA FROM BAAP
1.00
.80
.60
.40
.20
.00
-.20
-.40
-.60
-.80
-1.00
PARTIAL
0
50
100
150
200
250
300
350
400
450
500
LAG

PLOT ØF RESIDUALS

Figure 85

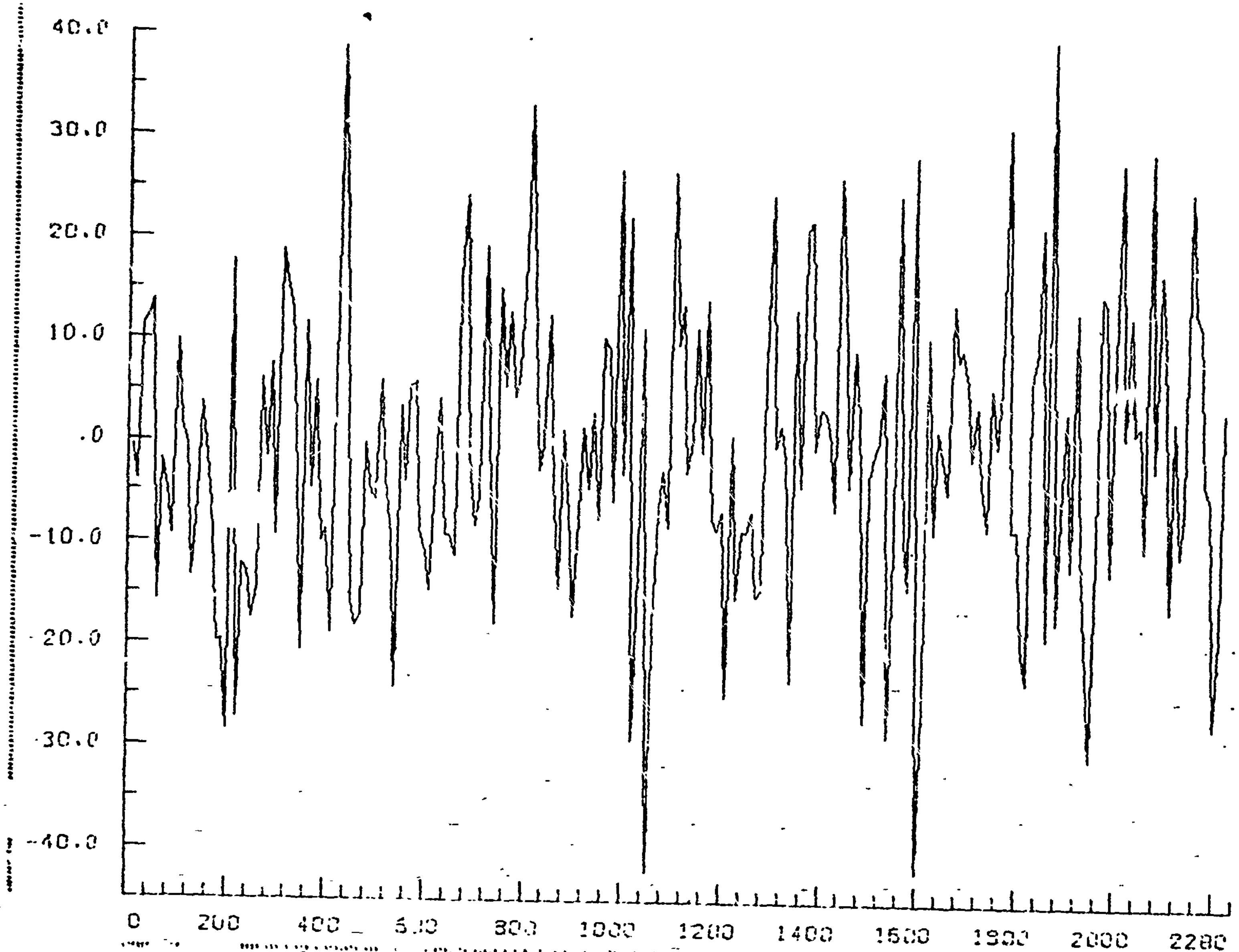


Figure 86

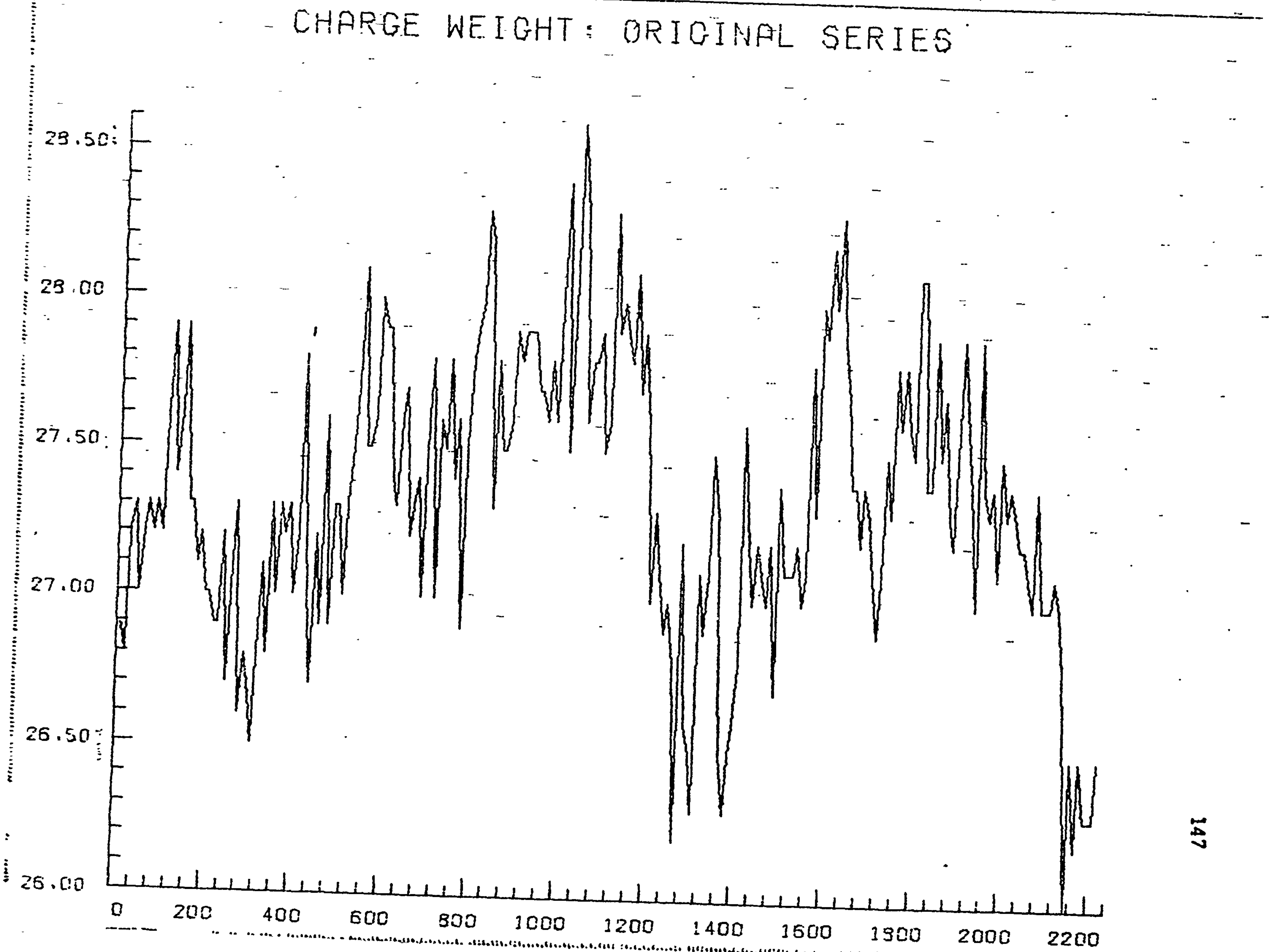

CHARGE WEIGHT: ORIGINAL SERIES
28.50
28.00
27.50
27.00
26.50
26.00
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200

Figure 87 (a)

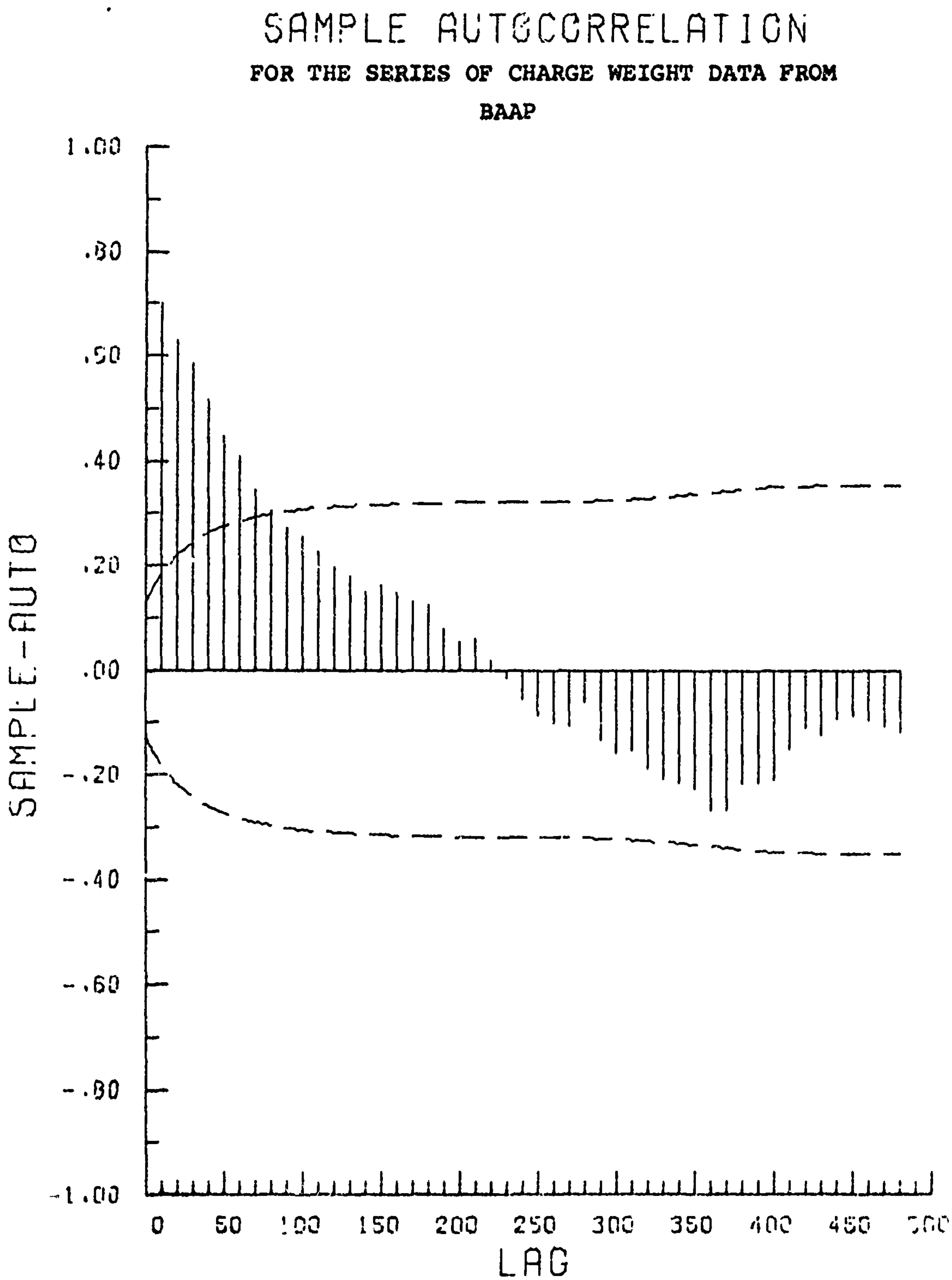

SAMPLE AUTOCORRELATION
FOR THE SERIES OF CHARGE WEIGHT DATA FROM
BAAP
1.00
.80
.60
.40
.20
.00
-.20
-.40
-.60
-.80
-1.00
SAMPLE-AUTO
0
50
100
150
200
250
300
350
400
450
500
LAG

Figure 87 (b)

SAMPLE PARTIAL CORRELATION

FOR THE SERIES OF CHARGE WEIGHT DATA FROM BAAP

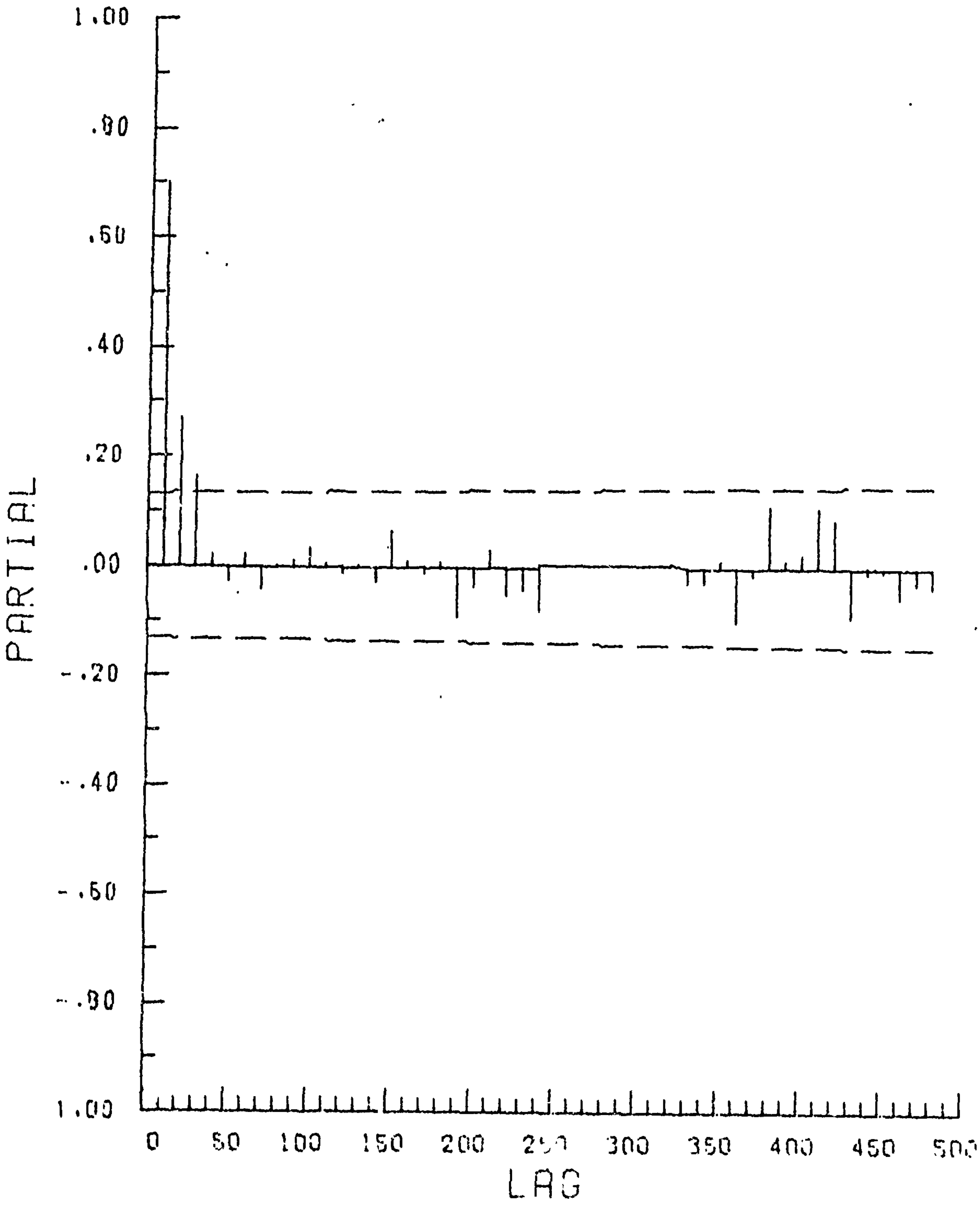


Figure 88

CHARGE WEIGHT · FIRST DIFF·

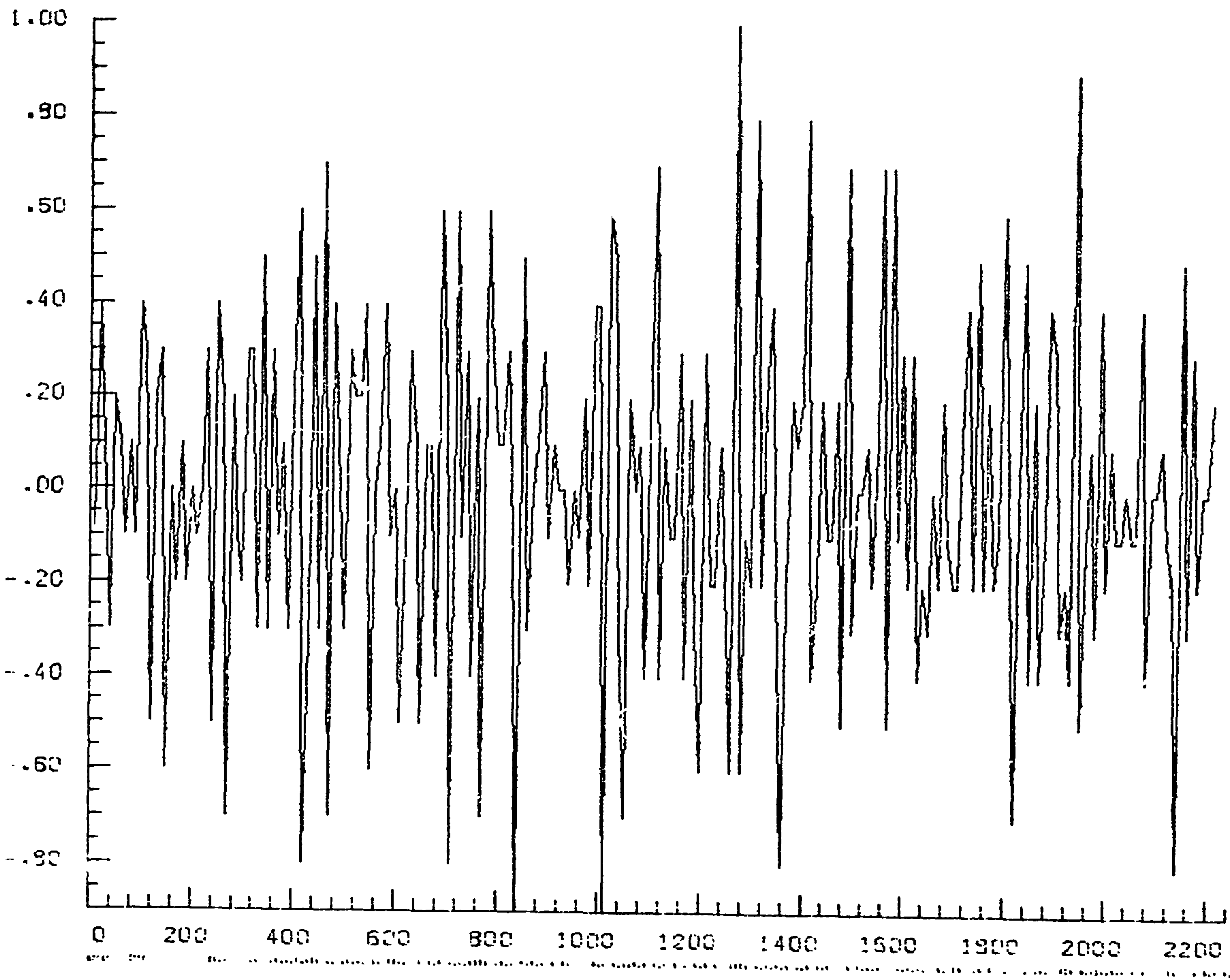


Figure 89 (a)

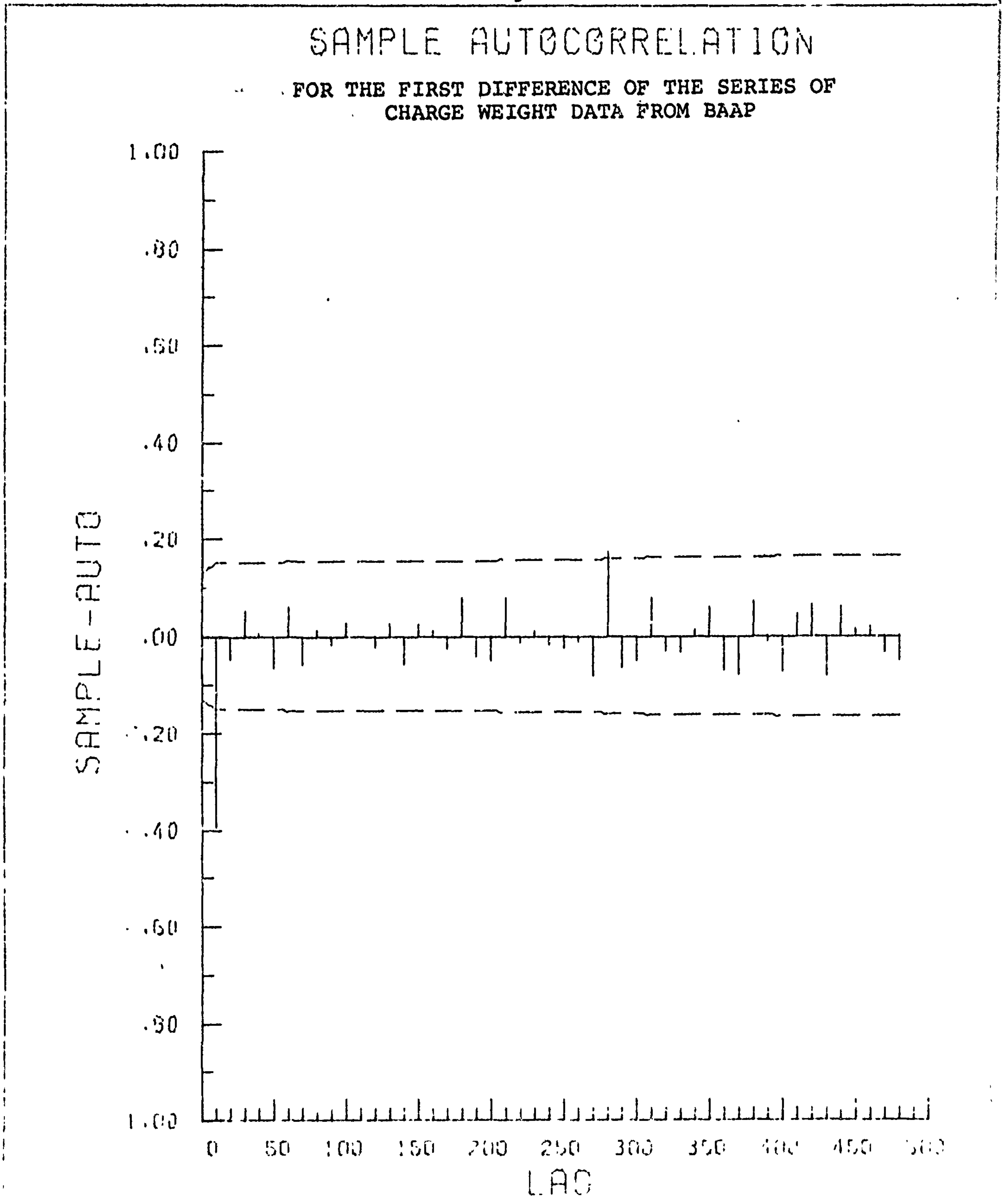
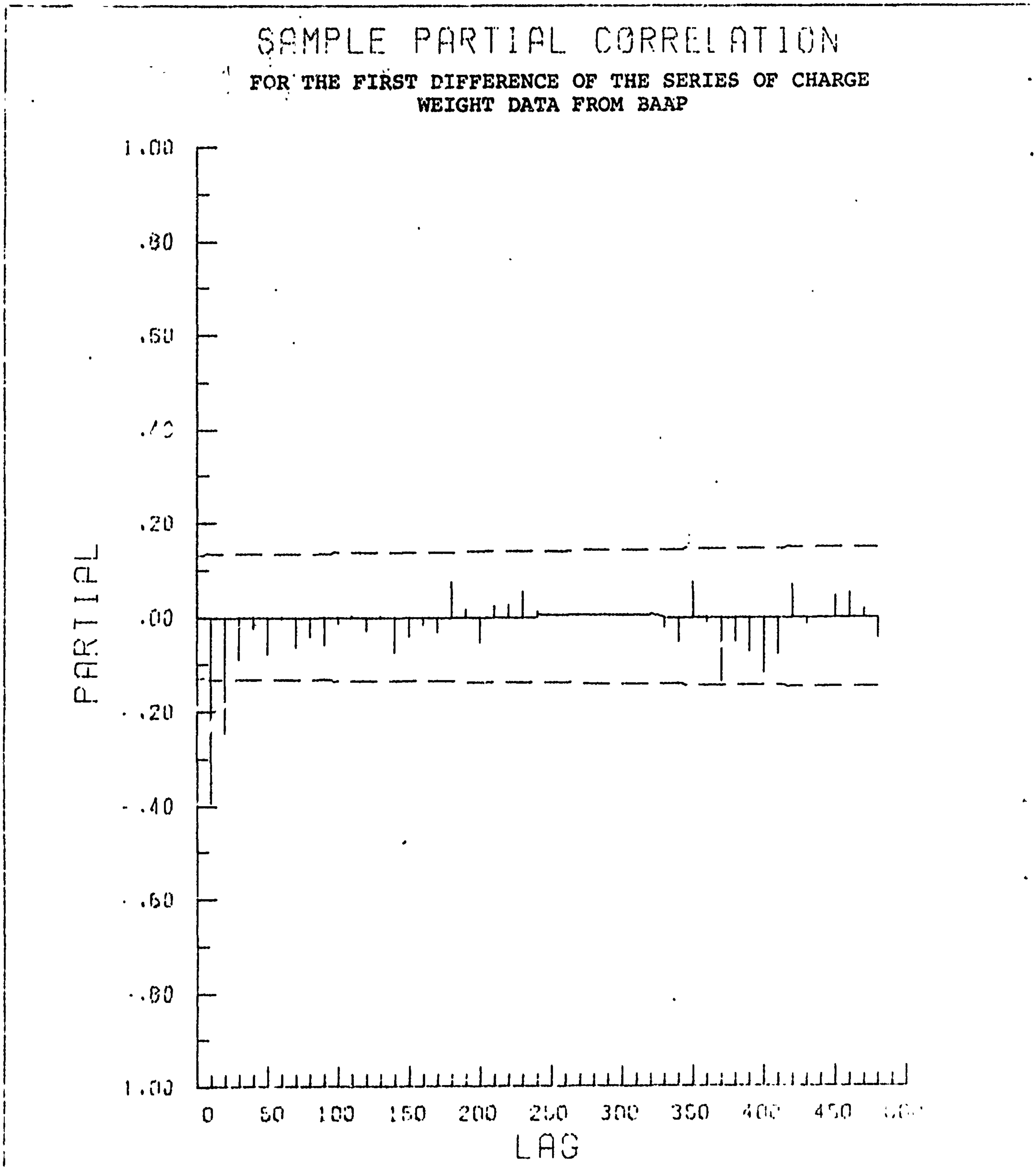

SAMPLE AUTOCORRELATION
FOR THE FIRST DIFFERENCE OF THE SERIES OF CHARGE WEIGHT DATA FROM BAAP
1.00
.80
.60
.40
.20
.00
-.20
-.40
-.60
-.80
-1.00
SAMPLE-AUTO
0
50
100
150
200
250
300
350
450
LAG

Figure 89 (b)


SAMPLE PARTIAL CORRELATION
FOR THE FIRST DIFFERENCE OF THE SERIES OF CHARGE WEIGHT DATA FROM BAAP
1.00
.80
.60
.20
.00
-.20
-.40
-.60
-.80
PARTIAL
0
50
100
150
200
300
350
450
LAG

PLØT ØF RESIDUALS

Figure 90

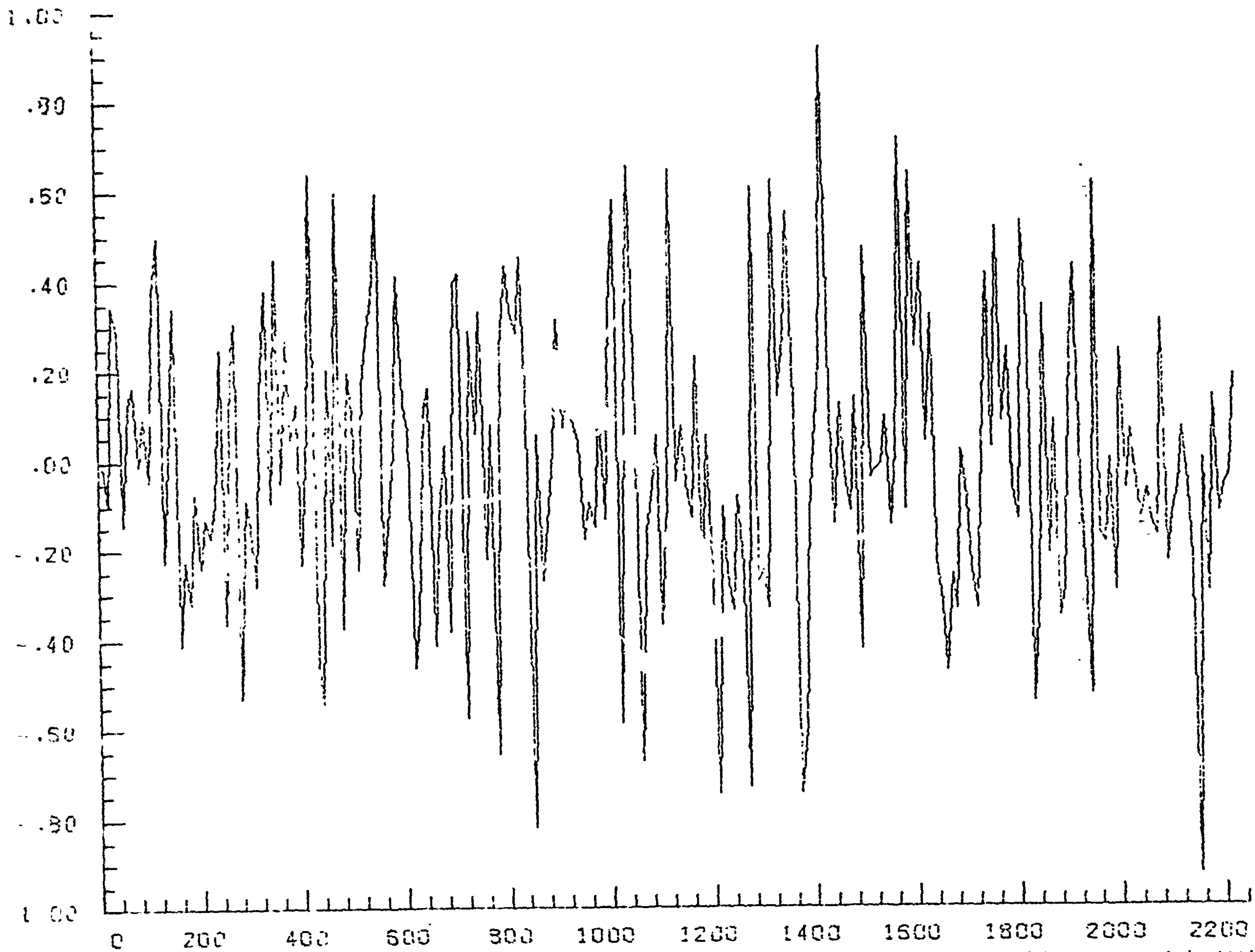


TABLE 19

PROPELLANT LOT ACCEPTANCE TEST RESULTS FROM BAAP

Date Fired	Propellant Lot No.	Charge Weight (Grains)	Corrected Velocity (fps)	Corrected Chamber Pressure (psi)	Corrected Port Pressure (psi)
8-14-69	46362	27.9	3244	45100	15500
8-18-69	46364	27.9	3243	46700	15700
8-19-69	46366	27.9	3240	46800	15400
8-21-69	46412	27.7	3255	47500	15500
9- 3-69	46414	27.8	3251	46100	15500
9-10-69	46416	28.0	3241	46600	15500
9-15-69	46418	27.5	3242	44200	15500
9-22-69	46425	27.6	3248	44500	15600
9-30-69	46427	27.9	3241	45200	15800
10-10-69	46430	28.3	3245	47700	15700
10-28-69	46881	28.1	3249	48000	16100
10-30-69	46883	27.9	3242	45800	16800
11- 7-69	46886	27.6	3242	45900	15200
11-21-69	46890	27.1	3240	44700	15700
12-16-69	46893	26.8	3250	45200	15600
1-13-70	46900	26.9	3245	45700	15200
1-29-70	46507	27.5	3252	46900	15300
2-11-70	46508	27.3	3244	45300	15600
2-24-70	46514	27.6	3248	46400	15700
2-25-70	46515	27.2	3247	45600	15700
2-27-70	46516	27.0	3247	48800	15100
3- 9-70	46517	27.2	3244	47900	15600

TABLE 19 Cont'd

Date Fired	Propellant Lot No.	Charge Weight (Grains)	Corrected Velocity (fps)	Corrected Chamber Pressure (psi)	Corrected Port Pressure (psi)
3-23-70	46520	27.2	3255	47000	15100
5-12-70	46529	27.8	3248	45400	15700
6-17-70	46607	28.0	3244	46700	15500
6-25-70	46609	27.9	3245	45800	15800
7- 8-70	46611	27.4	3255	47000	15500
		27.59	3246	46240	15588

TABLE 20

PROPELLANT LOT ACCEPTANCE TEST RESULTS FROM LAKE CITY AMMUNITION PLANT

Date Fired 1970	Propellant Lot No.	Charge Weight (Grains)	Corrected Velocity (fps)	Corrected Chamber Pressure (psi)	Corrected Port Pressure (psi)
2- 6 to 2-13	46362	27.9	3237	45440	15600
1-30-70	46364	27.95	3252	47000	15400
1-26 to 1-30	46366	28.0	3271	47200	15700
3-31 to 4-8	46412	27.8	3244	46022	15500
2-26 to 3-3	46414	27.75	3249	45677	15380
2-13 to 2-17	46416	27.9	3223	44700	15310
4-8 to 4-15	46418	27.7	3238	44955	15510
4-29 to 5-16	46425	27.8	3242	45444	15300
2-17 to 2-24	46427	27.8	3243	44477	15530
3-5 to 3-10	46430	28.1	3243	46360	15350
3-10 to 3-13	46881	27.85	3240	45833	15370
5-7 to 5-13	46883	27.8	3257	45625	15440
3-16 to 3-24	46886	27.8	4356	45622	15610
3-24 to 3-30	46890	27.0	3240	45211	15320
4-15 to 4-21	46893	26.9	3228	45325	15370
4-22 to 4-28	46900	26.9	3250	45566	15440

TABLE 20. Cont'd

Date Fired 1970	Propellant Lot No.	Charge Weight (Grains)	Corrected Velocity (fps)	Corrected Chamber Pressure (psi)	Corrected Por Pressure (psi)
5-14 to 5-19	46507	27.5	3234	44488	15446
5-21 to 5-26	46508	27.5	3246	45420	15271
5-27 to 6-14	46514	27.7	3245	45400	15387
6-5 to 6-11	46515	27.3	3253	45666	15310
6-12 to 6-26	46516	27.1	3237	46033	15116
6-18 to 6-24	46517	27.2	3234	46300	15080
6-29 to 6-30	46520	27.1	3250	46636	15150
6-30 to 7-13	46529	27.6	3258	46009	15350
7-14 to 7-22	46607	27.9	3248	46212	15300
7-22 to 7-27	46609	27.7	3242	46044	15250
7-27 to 7-31	46611	27.3	3243	45588	15480
		27.59	3244.5	45700	15374

TABLE 21

DIFFERENCE IN BAAP RESULT AND LAKE CITY AMMUNITION PLANT RESULTS (BAAP - LAKE CITY)

Time Lag (Months)	Propellant Lot No.	Difference in charge weights (grains)	Difference in Velocities (fps)	Difference in chamber Pressures (psi)	Difference in port Pressures (psi)
6	46362	0	7	340	100
5 1/2	46364	-0.05	-9	-300	300
5 1/2	46366	-0.10	-31	-400	-300
7	46412	-0.10	11	1478	0
6	46414	+0.05	2	423	120
4	46416	0.1	18	1900	190
6	46418	-0.2	4	-755	-10
7	46425	-0.2	6	-944	300
4 1/2	46427	0.1	-2	723	270
5	46430	0.2	2	1340	350
4 1/2	46881	+0.25	9	2167	730
6 1/2	46883	0.1	-15	175	1360
4 1/2	46886	-0.2	-14	453	-410
4	46890	0.1	0	-511	380
4	46893	-0.1	22	-125	230
3 1/2	46900	0	-5	134	-240
3 1/2	46507	0	18	2412	-416
3 1/2	46508	-0.2	-2	-120	329
3	46514	-0.1	3	1100	313
3 1/2	46515	-0.1	-6	-66	390
3 1/2	46516	-0.1	10	2767	-16
3 1/2	46517	0	10	1600	520
3	46520	0.1	5	364	-50
2	46524	0.2	-10	-609	350
1	46607	-0.1	-4	488	200
1	46609	0.2	3	-244	550
1/2	46611	0.1	12	1412	20

TABLE 22

PROPELLANT LOT ACCEPTANCE TEST RESULTS FROM BAAP

Date Fired	Lot Number	Charge Weight (Grains)	Velocity (fps)	Corrected Chambor Pressure (psi)	Corrected Port Pressure (psi)
4-1-70	46522	27.4	3257	46100	15400
4-22-70	46524	27.1	3242	46400	15100
5-4-70	46527	27.0	3254	45200	15400
5-7-70	46528	27.1	3242	48500	15100
5-22-70	46530	27.3	3244	44800	13500
6-26-70	46610	27.7	3245	45500	15800
7-7-70	46612	27.4	3254	45200	15700
7-17-70	46613	27.2	3245	46600	15600
7-28-70	46617	26.9	3244	46400	15200
7-2-70	46618	27.1	3253	45700	15100
8-4-70	46620	27.3	3254	46500	15600
8-4-70	46621	27.8	3249	46000	15900
8-20-70	46624	27.6	3247	45600	15600
8-24-70	46625	27.5	3247	45500	15700
9-2-70	46938	27.4	3252	46800	15600
9-1-70	46940	27.5	3248	44300	15700
9-17-70	46942	27.3	3248	44700	15200
9-23-70	46944	27.6	3257	46500	15500
9-30-70	46945	27.9	3241	45000	15800
9-30-70	46946	27.6	3251	47400	15300
1 -5-70	46948	27.4	3250	45200	15400
10-9-70	46949	27.0	3255	43900	15300
10-22-70	46953	27.1	3240	44500	14900
10-27-70	46956	27.4	3254	46100	15600
10-30-70	46958	27.2	3251	46300	15400
		27.39	3248.96	45788	15496

TABLE 21

PROPELLANT LOT ACCEPTANCE TEST RESULTS FROM TWIN CITY AMMUNITION PLANT

Date Fired 1970	Lot Number	Charge Weight (Grains)	Velocity (fps)	Corrected Chamber Pressure (psi)	Corrected Port Pressure (psi)
6-1 to 6-11	46522	27.4	3243	45920	15484
6-15 to 6-22	46524	27.2	3242	45500	15162
6-22 to 6-29	46527	27.3	3246	46180	15432
6-30 to 7-9	46528	27.0	3248	46400	15170
7-12 to 7-15	46530	27.4	3244	46100	15207
7-16 to 7-27	46610	27.3	3234	45200	15240
7-30 to 7-31	46612	27.3	3252	46150	15360
8-4 to 8-12	46613	27.3	3241	45740	15254
8-13 to 8-19	46617	27.1	3232	46475	15150
8-21 to 8-31	46618	27.0	3248	45220	15272
8-31 to 9-8	46620	27.3	3241	45325	15375
9-8 to 9-16	46621	27.4	3257	46300	15385
9-17 to 9-23	46624	27.8	3237	44425	15648
10-2 to 10-8	46625	27.5	3251	45300	15490
9-24 to 9-30	46938	27.4	3244	44575	15815
10-12 to 10-15	46940	27.5	3249	44750	15460

TABLE 23 Cont'd

Date Fired 1970	Lot Number	Charge Weight (Grains)	Velocity (rps)	Corrected Chamber' Pressure (psi)	Corrected Port Pressure (psi)
10-16 to 10-26	46942	27.3	3241	46000	15270
10-27 to 11-3	46944	27.4	3255	46225	15245
11-4 to 11-10	46945	27.7	3248	44400	15605
11-17 to 11-18	46946	27.7	3248	46220	15382
11-19 to 12-1	46948	27.5	3243	46400	15205
12-1 to 12-11	46949	27.2	3244	45900	15425
12-12 to 12-17	46953	27.0	3242	46180	15140
12-21 to 12-23	46956	27.2	3234	46125	15223
12-28 to 12-30	46958	27.4	3243	47133	15217
		27.34	3244.28	45765	15342

TABLE 24

DIFFERENCE IN BAAP RESULT AND
TWIN CITY AMMUNITION PLANT RESULTS
(BAAP - TWIN CITY)

Time Lag (Months)	Lot Number	Difference in charge weight (grains)	Difference in Velocities (fps)	Difference in corrected Chamber Pressure (psi)	Difference in corrected Port Pressure (psi)
2	46522	0	14	180	-84
2	46524	-0.1	0	900	-62
2	46527	-0.3	8	-980	-32
2	46528	0.1	-6	2100	-70
1 1/2	46530	-0.1	0	-1300	293
1	46610	0.4	11	300	560
1	46612	0.1	2	-950	340
1	46613	-0.1	4	860	346
1/2	46617	-0.2	12	-75	50
1	46618	0.1	5	480	-172
1	46620	0	13	1175	225
1	46621	0.4	-8	-300	515
1	46624	-0.2	10	1175	-48
2	46625	0	-4	200	210
1	46438	0	8	2225	-215
1	46940	0	-1	-450	240
1	46942	0	+7	-1300	-70
1	46944	0.2	2	275	255
1	46945	0.2	-7	600	195
1 1/2	46946	-0.1	+3	1180	-82
1 1/2	46948	-0.1	7	-1200	195
2	46949	-0.2	11	-2000	-125
1 1/2	46953	0.1	-2	-1680	-240
2	46956	0.2	20	-25	377
2	46958	-0.2	8	-833	183

8. CONCLUSIONS AND SUGGESTIONS

8.1 Summary and Conclusions

(1) Piezo method of testing gives better information regarding pressures inside the barrel than copper crusher method of testing. Piezo gives the maximum pressure at the gage location. It also gives ignition delay and slope which are indicators of propellant characteristics.

(2) Copper crusher deformation can be considered as a weighted integral of piezo pressure-time curve.

(3) Different responses from piezo and copper crusher are correlated. The correlative structure is given in Section 2.5. Correlation analysis is found to help present a better visual picture by effecting data reduction. Copper crusher and piezo data are found to exhibit similar pressure-velocity relationships, however, copper crusher has larger variability. Piezo data indicate a time trend in the coating process which crusher data fail to show. This time trend could be significant from process control viewpoint.

(4) Acceptance test data are found to contain time trends, in particular, there seems to exist a 'point of shift' after which the data become relatively stationary.

(5) Method of cumulative sum, with reference constant equal to the mean of the series of data, is found to give the best visual picture of the point of shift.

(6) Data after the point of shift are found to be non-stationary and therefore, nonnormal. They are also autocorrelated.

(7) Analysis of chamber pressure data from different manufacturers indicates the mean chamber pressure to be quite alike. The chamber pressure variance varies considerably, from 118×10^4 psi^2 for Remington to 629×10^4 psi^2 for Winchester. Mean chamber pressure of Ball ammunition (46600 psi) is lower than the mean chamber pressure for Tracer ammunition (49200 psi). The within lot chamber pressure variance for Twin City is 29×10^5 psi^2 and for Lake City is 17.6×10^5 psi^2. In both cases, the trend is toward a reduction in variance, indicating continued improvement in production and testing processes.

(8) The cutoff date and the corresponding control limits for the data from five manufacturers are given in Tables 4 and 5 respectively. On the average, the new control limits are found to be lower than the standard control limit of 52000 psi, by about 2000 psi.

(9) It is found that different lot selection can result if ammunition selection for weapon tests is based on chamber pressure alone or port pressure alone. Lot selection is therefore based on the bivariate histogram of chamber pressure and port pressure.

(10) The selected ammunition lots from Lake City are given in Section 5.5 and the selected lots from Twin City are given in Section 5.6.

(11) In acceptance testing, standard deviation is determined from twenty tests. It is found that twenty tests are too less for a proper estimate of experimental error.

(12) Propellant lot characteristics are found to be serially correlated.

(13) Comparison of ammunition data from B.A.A.P. and loading plants indicate no change in propellant characteristics during the elapsed period between the tests.

(14) The model building approach of Appendix I appears to be the correct way to interpret interior ballistic phenomena.

8.2 Recommendations for Future Work

(1) To compare copper crusher and piezo methods as a means of standard testing, following analysis should be conducted.

(a) Cost analysis for the two methods should be carried out. This would involve the determination of proper number of tests to be conducted by each method. Approach of

Chapter 6 can be used for this purpose.

(b) An analytical expression for copper crusher deformation should be obtained to see if it is a good estimate of impact energy.

(c) From the viewpoint of gun design, efforts should be made to obtain a suitable weighted integral of pressure-time curve as a measure of pressure inside the barrel. Such a measure could then replace the present measure of crusher deformation.

(2) Designed experiments, as suggested in Section 5.7, should be carried out to determine the ammunition factors that control gun performance. These factors can be used to refine ammunition selection criteria.

(3) Analysis in Chapter 6 indicates that 20 tests are not sufficient for proper estimation of experimental error. Further analysis should be conducted to determine the proper number of tests necessary for acceptance testing.

(4) Extensive data analysis should be undertaken to determine if identical results are obtained at B.A.A.P. and Loading Plants. For this purpose, it would also be necessary to evaluate the experimental facilities at different loading plants and B.A.A.P. If the results are identical, then there is a possibility of reduction in the extent of acceptance testing.

(5) Efforts directed at building a model for interior ballistic of guns are likely to be fruitful. Such a model would help explain the observed ballistic phenomena and suggest likely changes for improvement in the weapon system.

APPENDIX I

EMPIRICAL-MECHANISTIC MODEL FOR INTERIOR BALLISTICS OF GUNS

Introduction

Over the past several decades, considerable effort has been directed toward the theoretical analysis of interior ballistics of guns. The complexity of the phenomenon has made a complete theoretical analysis impractical. Development of interior ballistic models in the past was hampered by lack of analytical as well as experimental methods and computer facilities. As a consequence, several simplifying assumptions were made. In view of the current emphasis on high velocity weapons and the considerable improvements in the field of solid propellants (ball powder, improved deterrent coatings, etc.), the solutions obtained are no longer adequate. With today's computer facilities and the recent advances in hydrodynamics and statistical model building techniques, improved interior ballistic models appear feasible.

Models for Physical Systems

Models for physical systems can be classified into three broad categories - theoretical, mechanistic and empirical. Theoretical models are based entirely on theoretical considerations and presently do not seem feasible for interior ballistics. Mechanistic models assume a considerable knowledge about the system so that the functional form of the model can be derived. The parameters of the model are then estimated from the data. Empirical models are not based upon a physical understanding of the process. Here a functional form is devised and parameters estimated to explain the observed data. For the interior ballistics system an empirical-mechanistic approach seems necessary, since only parted information is available about the system.

Empirical-Mechanistic Model

To build a mechanistic model there is a need to know 'how the system works'. Since exact information about the system is not available, true functional form of the model cannot be determined. However, based upon partial information several models can be proposed and statistical techniques used to identify the most probable functional form of the model. From experimental results the parameters of the model can be estimated. The model is then diagnostically checked for adequacy of fit. This three stage iterative procedure of identification, estimation and diagnostic checking will lead to an adequate empirical-mechanistic model.

Once the model is available it can be used to predict the process behavior under different conditions or for optimization of the process. These aims can also be achieved entirely experimentally using empirical models and response surface techniques. For example, optimum burning rate can be determined by making powders of different composition (by varying process variables) and the experimental results analyzed using response surface methods. But the experimentation involved would be rather expensive.

Apart from optimization, the important advantage of this approach via mechanistic models is that the model is useful in the development of new processes since meaningful extrapolation is possible. Thus the ballistic performance of radically different experimental weapons or powder can be simulated using these models. The model would, therefore, be of considerable use in developing new concepts in weaponary.

Mathematical Model for the Gun

As a first step a simplified model is assumed. This is based upon the following assumptions.

A. Assumptions

(1) Products of combustion are assumed to obey ideal gas law.

(2) Gases of combustion are assumed to have negligible viscosity and thermal conductivity. Boundary layers are assumed absent, so the flow is one dimensional.

(3) Coulomb friction between projectile and barrel as well as the resistance due to compression of air ahead of the projectile is neglected.

(4) Walls of the gun are assumed perfectly insulated, so there is no energy loss by heat transfer.

(5) Effect of chambrage is neglected.

B. Statement of the Simplified Problem

In short the problem considered is as follows: There is an insulated tube sealed at one end and open at the other end. Gases and powder are trapped between the sealed end and the piston, and there are heat, mass and volume sources inside the volume behind the piston (Fig. 1). The motion of the piston and the pressure changes constitute the objects of the solution to this problem.

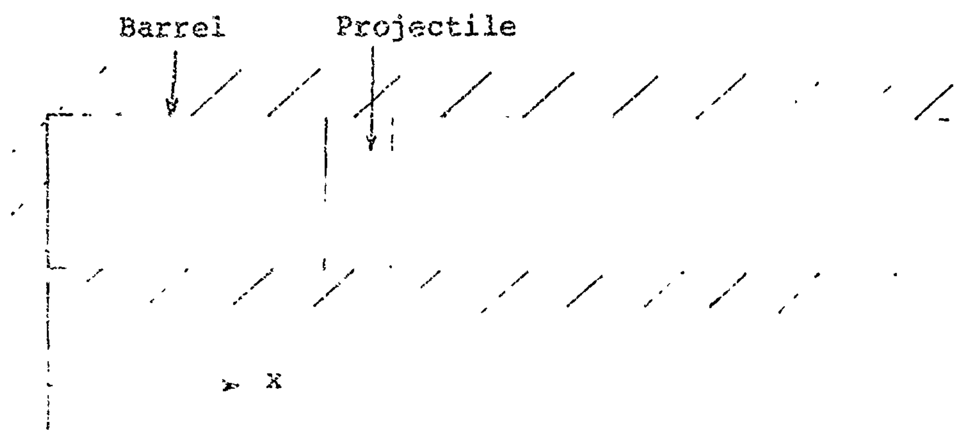


Figure 1

C. Method of Solution

The Lagrangean approach is used for the solution of this problem. That is to each fluid particle, the Lagrangean coordinate x is assigned to be its distance from the sealed end at time time $t = 0$. Then at any instant of time, t, its location is given by the function $R(x, t)$. The fundamental equations governing the motion of gases are the equations (1) through (5).

Fluid Equations:

(1) $\frac{\partial R}{\partial t} = u$ Velocity Equation

(2) $\frac{\partial u}{\partial t} = -V_o \frac{\partial p}{\partial x}$ Momentum Equation

(3) $\frac{\partial E}{\partial t} = -p \frac{\partial V}{\partial t} + \dot{Q}$ Energy Equation

(4) $V = V_o \frac{\partial R}{\partial x}$ Continuity Equation

(5) $E = f(p,V) = \frac{pV}{\gamma-1}$ Equation of state.

Here, V, u, p, E are the specific volume, the fluid velocity, the pressure and the specific energy respectively. Q is the rate of liberation of energy and γ is the ratio of specific heats. V_o is the reference specific volume. These equations can be directly used in the case of instantaneous burning but they used certain modifications for finite burning rate.

These equations of fluid motion are combined with the equation of motion of the projectile, given by

(6) $$\frac{d^2R}{dt^2} = \frac{p - pA}{m} \cdot A$$

where m is the mass of projectile and A is the cross-sectional area.

D. Numerical Integration of the Set of Differential Equations

Since an analytical solution of the system of differential equations is impossible, a numerical procedure is employed. The scheme is taken from 'Difference methods for initial value problems' by Richtmyer and Morton. Certain modifications have been made to take care of the finite burning rate.

Description of the Experimental Data Used

A piezo gauge was mounted at the mid chamber position (point A in Fig. 2(a)) and the pressure-time history was recorded for various rounds. A typical pressure-time curve is shown in Fig. 2(b). Each time the terminal velocity of the projectile was measured by clocking the time taken by the projectile to travel between two magnetic screens placed at a known distance from one another.

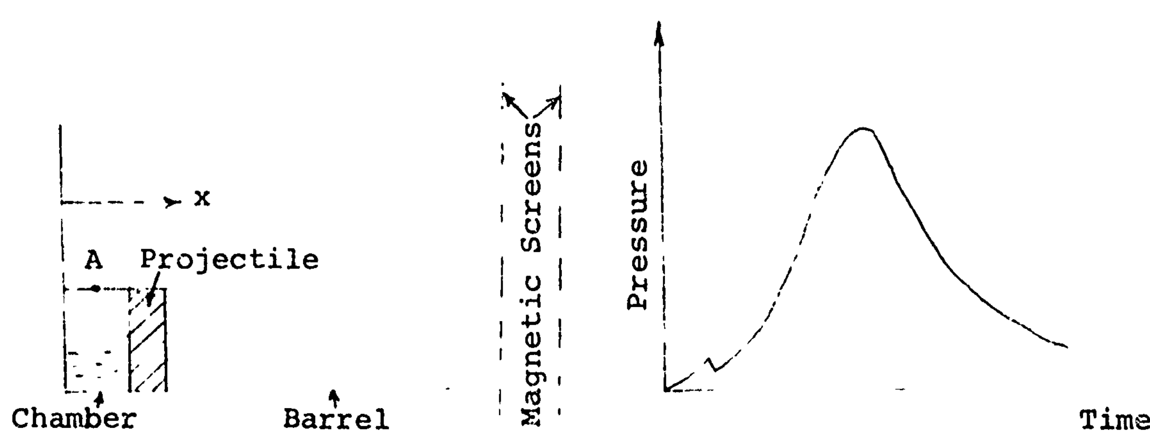


Fig. 2(a) Fig. 2(b)

In terms of the experimental data, the present model building problem is to determine an appropriate functional form of $\dot{Q}$ so that the calculated pressure-time curve and terminal velocity 'match as closely as possible' with the corresponding experimentally observed responses.

Iterative Model Building Procedure

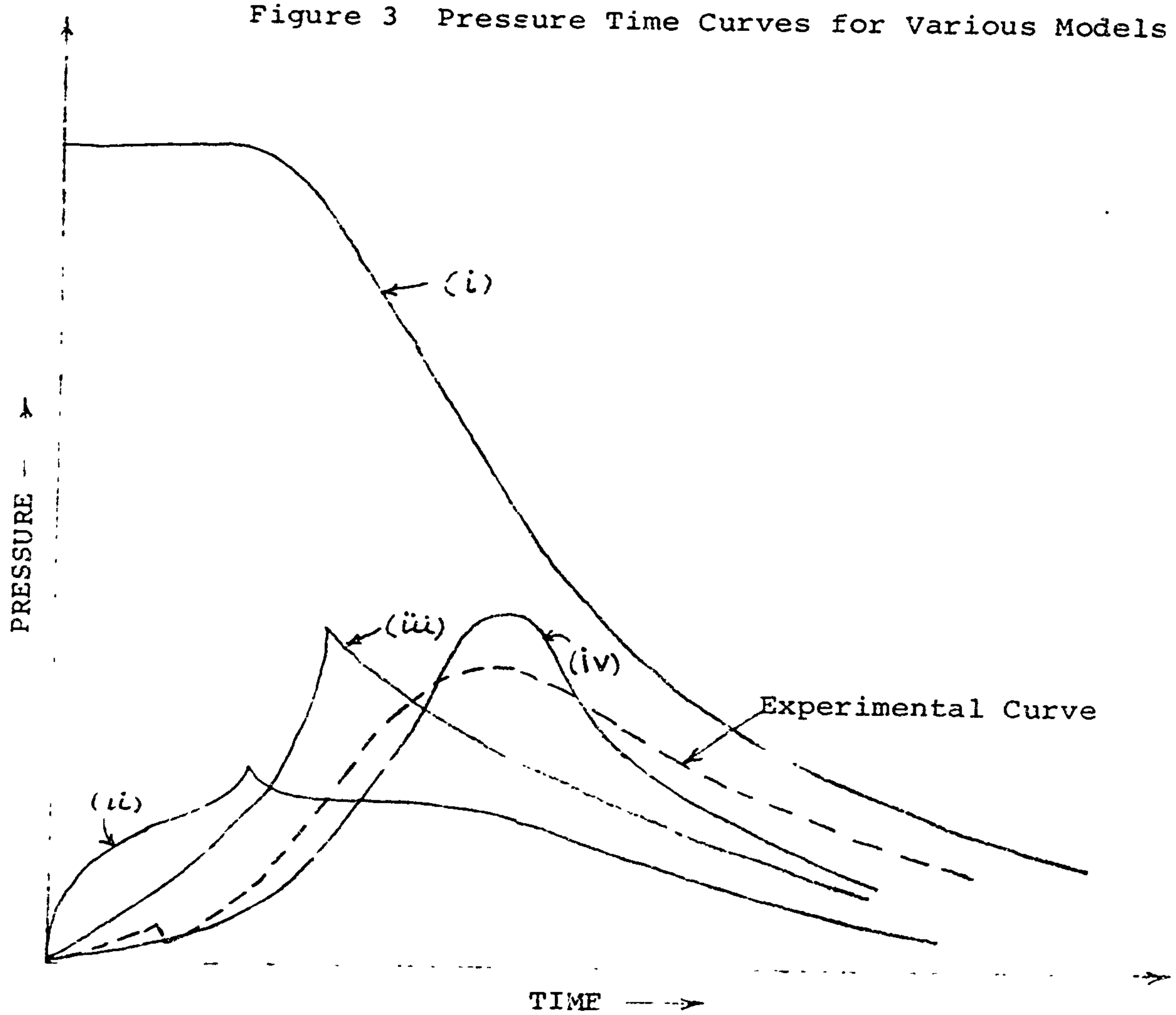
It was first assumed that all the explosive burns before the projectile starts to move (instantaneous burning). The resulting pressure-time history is shown in Fig. 3, curve (i). Obviously this model does not explain the experimental data. As a matter of fact the curve (i) suggests a model for $\dot{Q}$ such that only a portion of the explosive burns before the projectile starts to move, and that the rest of the powder continues to burn while the projectile is being accelerated. This will then avoid the excessive pressure build up in the chamber. So the next simple model was to assume that the powder burns at some constant burning rate until all the powder has burned. The resulting pressure time curve is shown in Fig. 3, curve (ii). Although it is a substantial improvement over the first model, it still does not explain the experimental curve adequately. The main differences being that the model exhibits a sharp peak wheras the experimental curve shows a smooth variation of pressure near the peak pressure and that the model predicts too high a pressure in the initial portion of the curve. Requirement of lower pressures in the beginning suggests the use of slower burning rate to start with and an increasing burning rate as time proceeds.

A study of the burning characteristics of the explosives, in the mean time, revealed that the burning rate should be proportional to the surrounding pressure; and this looked to be an ideal functional form for $\dot{Q}$ because initially the pressures are low so a smaller burning rate would be obtained and as the pressure starts building up, the burning rate would become higher. The resulting pressure-time curve is shown in Fig. 3, curve (iii). It is apparent by observing the similarities in this curve and the experimental curve that by suitably estimating the proportionality constant, the experimental curve can be explained except for the portion near the peak. The discrepancy shows that $\dot{Q}$ should be such that after a portion of the powder has burned, the burning rate should reduce. This was then taken care of in the next model. It was known that the powder is used in the form of sperical granulations. So as the burning proceeds, the surface area of the powder should go on reducing. It was also known that a deterrent coating is applied on the powder and the effect of this was assumed to nullify the effect of change in surface area. So the new model for $\dot{Q}$ is

$$\dot{Q} = HF \times RMBC \times P, \text{ until } (1-\alpha) \times 100\% \text{ of powder has burned}$$

$$= HF \times RMBC' \times P \times \text{surface area, for the remaining portion of mass.}$$

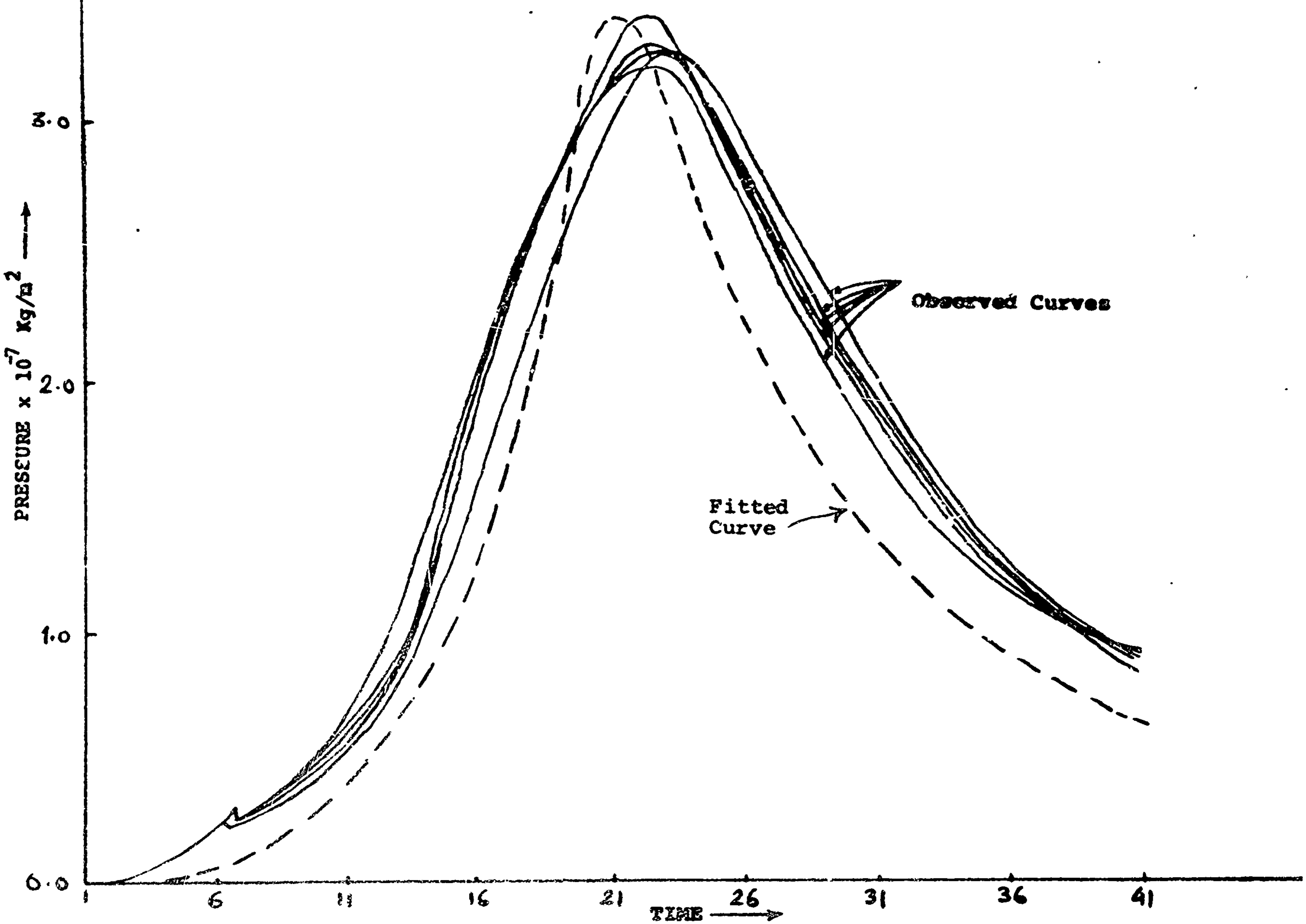
Figure 3 Pressure Time Curves for Various Models



RMBC and RMBC' are such that the two equations yield the same $\dot{Q}$ when transition takes place. HF and P are respectively the calorific value and the pressure. The resulting pressure time curve is given in Fig. 3, curve (iv). It is apparent that this curve explains the basic nature of the experimental curve. Now it was thought worthwhile to estimate the parameters α, RMBC, and the Primer energy (taken as the small amount of energy instantaneously available at the start of combustion) to get the best possible fit. In Fig. 4 the five available experimental curves are sketched. On the assumption that the errors are NID$(0, \sigma^2)$, least squares criterion is used and parameters estimated using UWHAUS. The fitted curve is also sketched on Fig. 4. (Note: the data was discretized by taking 41 points from each curve).

Using these estimated values of parameters, pressure and velocity histories along the length of the barrel were determined. Figure 5 shows the pressure variation along the length of the barrel. Figure 6 shows the breach pressure and projectile base pressure variation as a function of the distance travelled by the projectile. Breach pressure is always found to be higher than the projectile base pressure. The figure also shows the variation of projectile velocity along the length of the barrel. The predicted muzzle velocity is found to be much lower than the actual muzzle velocity. The variation of breach pressure and projectile base pressure

Figure 4 Experimental and Fitted Pressure Time Curves



as a function of time is indicated in Fig. 7. This breach pressure curve is the one that is being compared with the piezo pressure-time curve. Finally, the variation of projectile velocity and distance with time is shown in Fig. 8.

Results

In appropriate units, the parameter values and their confidence intervals are

$\alpha = 0.3491$ PRIMER = $.1504 \times 10^{-11}$ RMBC = 2.385

α: (.2917 to .4064), PRIMER $\times 10^{10}$: (-.84 to .87)

RMBC : (2.378 to 2.392)

Correlation matrix	α	PRIMER	RMBC
	1.0	-.38	-.19
		1.0	.06
			1.0

Analysis of Variance

of obs. = 41 x 5 = 202, # of parameters = 3

Residual sum sq. = $.27845 \times 10^{16}$, d. f. = 202

Pure error sum sq. = $.02649 \times 10^{16}$, d. f. = 4 x 41 = 164

Lack of fit sum sq. = 0.24856×10^{16}, d. f. = 38

Therefore,

$$\text{F statistic} = \frac{.24856 \times 10^{6}/38}{.02649 \times 10^{16}/164} = 40.6 \text{ (38,164 degrees of freedom)}$$

PRESSURE VARIATION ALONG BARREL

FIGURE 5

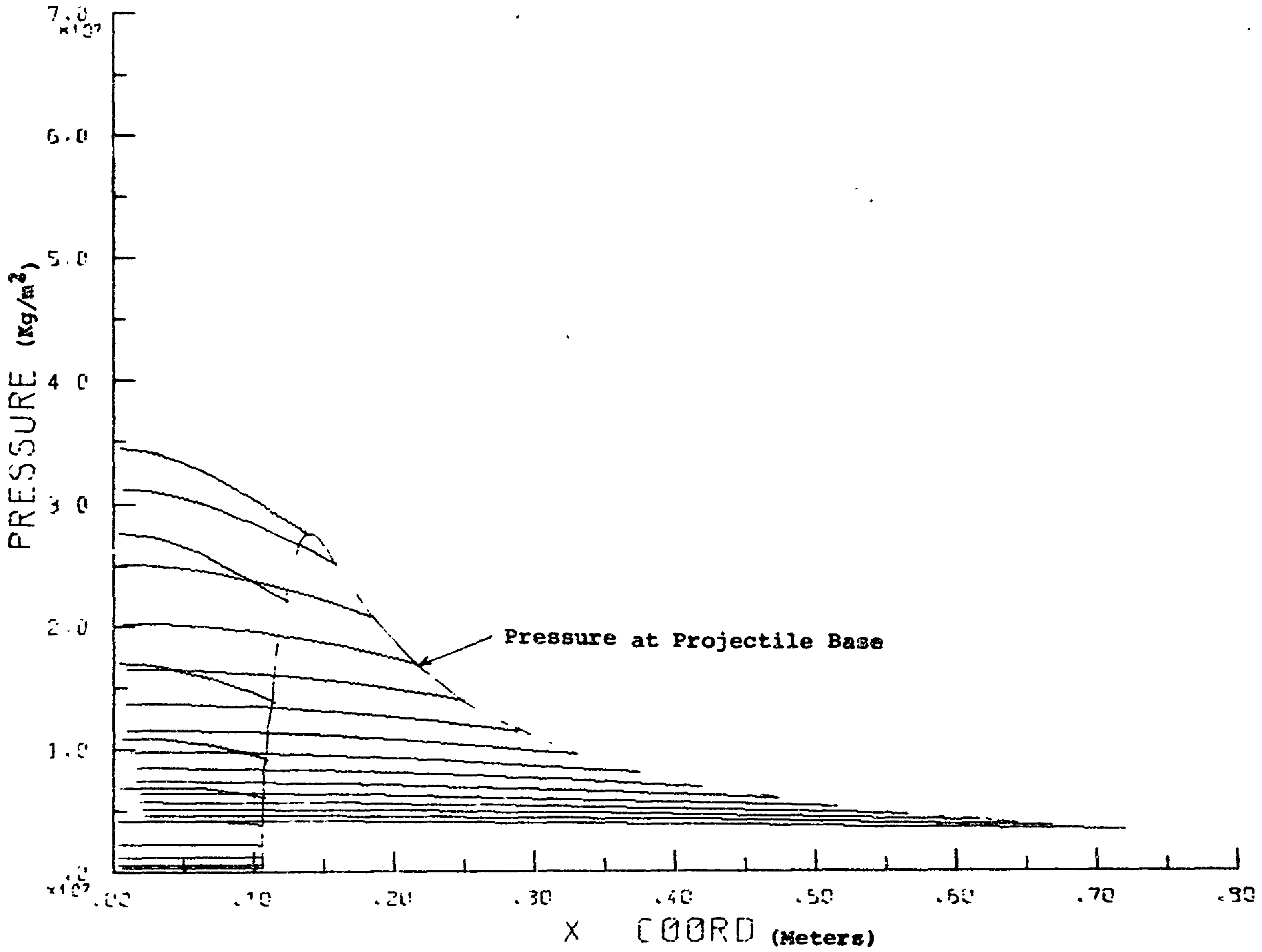
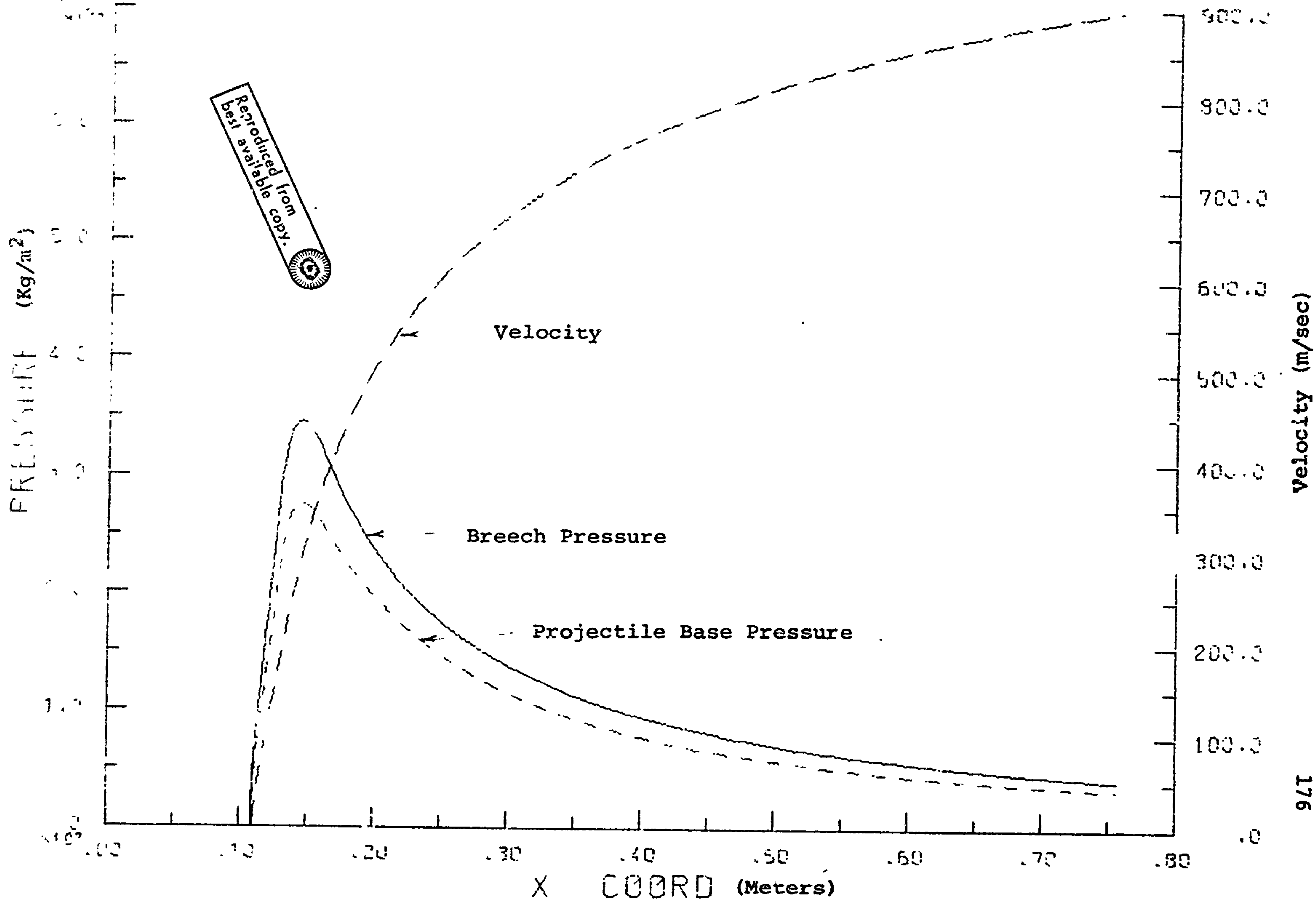
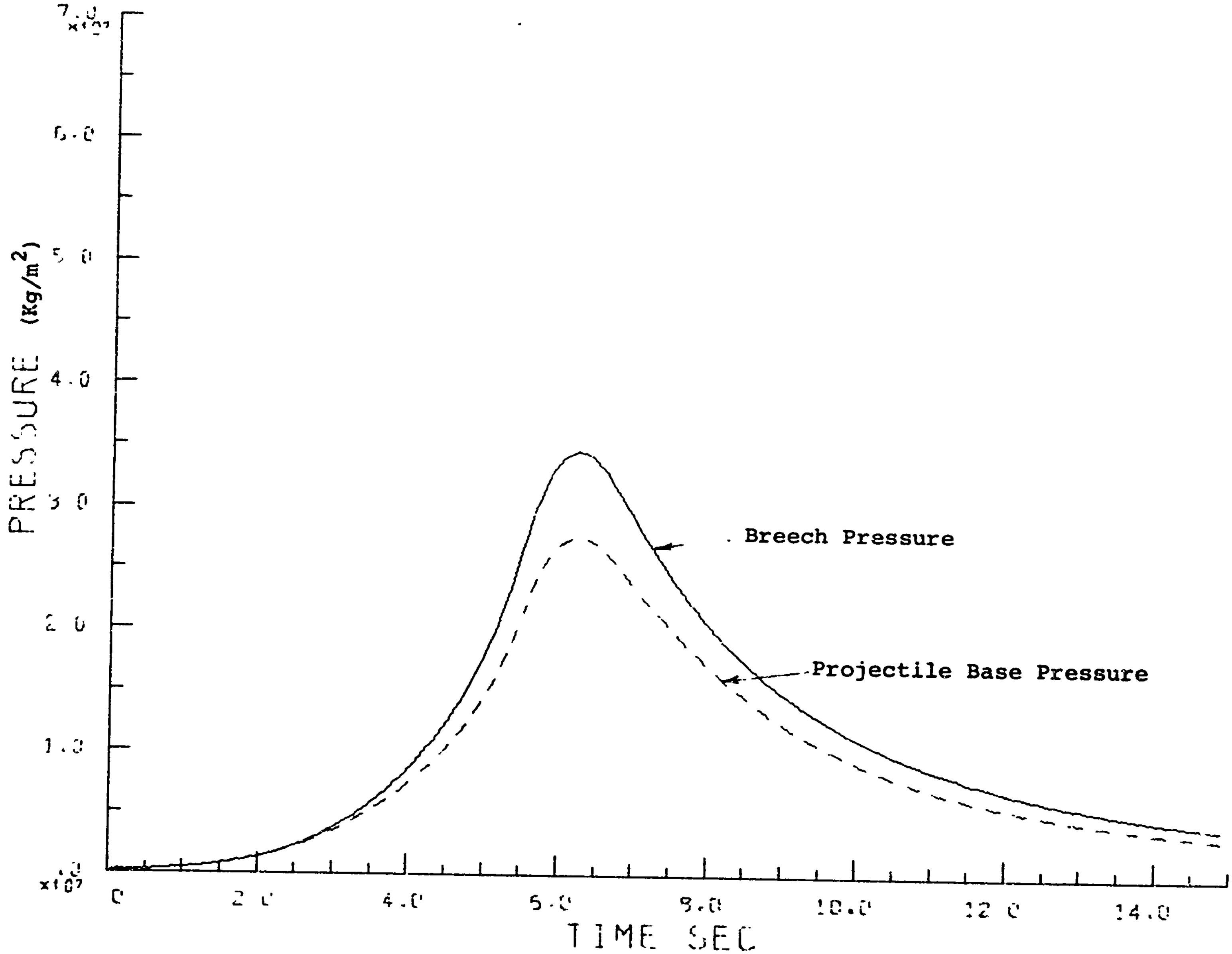


Figure 6 Pressure and Velocity Variation Along Barrel



PR VARIATION BEHIND PROJ

Figure 7



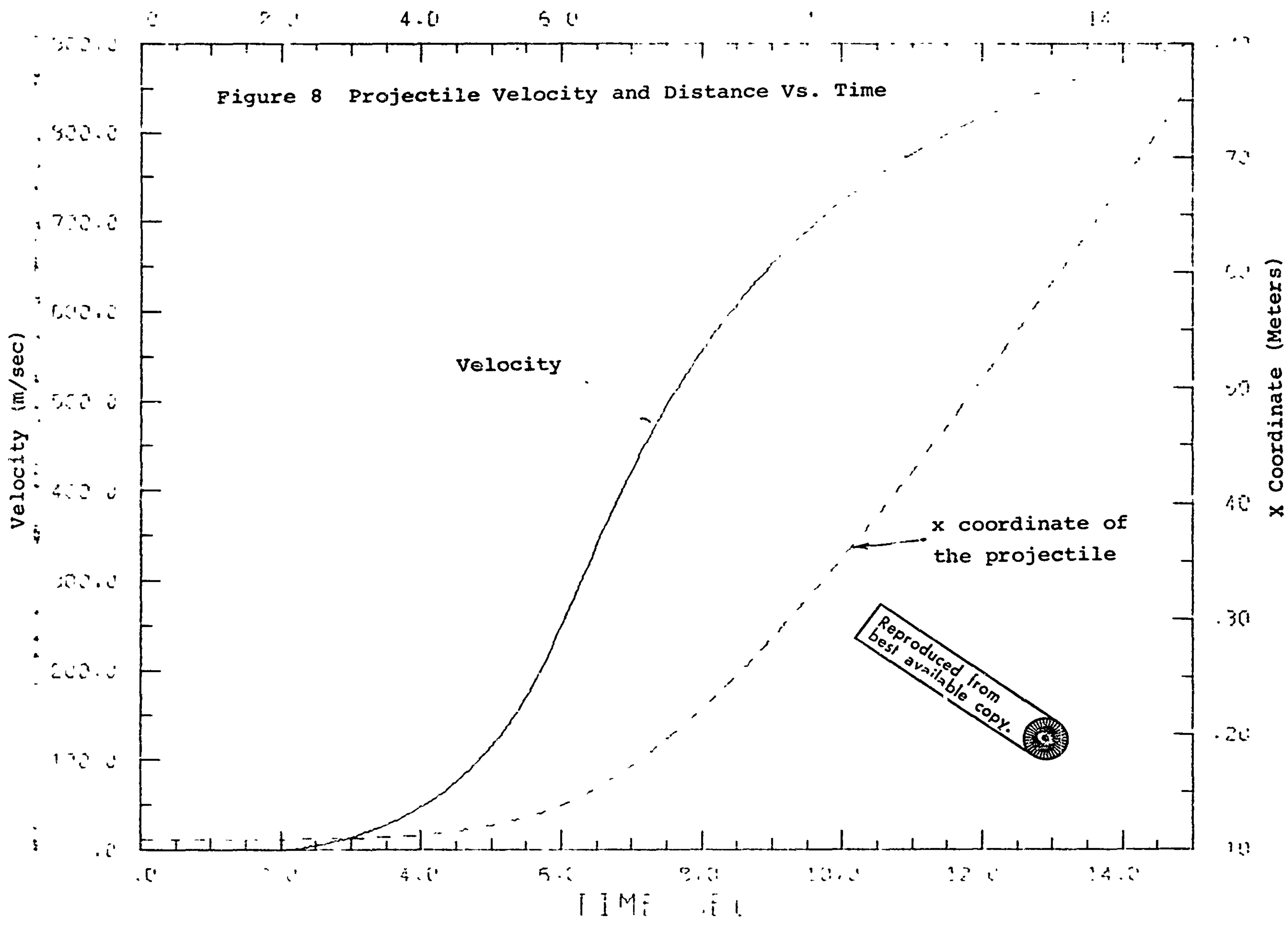


Figure 8 Projectile Velocity and Distance Vs. Time

Looking into F tables, for 10% significance level the corresponding F value is approximately 1.30. The model is found to be inadequate.

Analysis of Residuals, Diagnostic Checking

A look at the residuals tells that the model is inadequate, as also seen from the F test. Additionally, the trend in residuals shows where the inadequacy lies and how to improve upon it.

(a) The fitted values are consistently lower in the initial part of the curve. This may mean

  (1) higher initial burning rate
  (2) increased engraving force
  (3) increased primer energy

(b) By bodily translating the experimental curves to the right, a better fit for the initial part of the curve (i.e. the portion during which the pressure is increasing) is obtained. This has a relevance to the experimental conditions because of the possibility of some time lag.

(c) The fitted values have a large curvature near the peak pressure as compared with that of the experimental data. A smaller burning rate near this point would remedy this situation.

(d) The effect of the assumption of lack of heat transfer and the energy equation used should be examined further.

(e) The above modifications are expected to also take care of the one sided residuals in the position of the curve where the pressure is dropping.

Note on Estimation Procedure

It is more realistic to take the errors to be correlated when data is obtained from one experimental curve. However, from one experimental curve to the next experimental curve, the errors can be assumed uncorrelated. Then one can think of an estimation criterion similar to the one described by Box and Draper in their paper "The Bayesian Estimation of common parameters from several responses," which will be more appropriate for the present problem.

The velocity has been totally ignored in the estimation procedure followed so far. This is because when the estimation is done with the pressure response alone, the expected velocity corresponding to the best fit is about 15% smaller than the observed velocity. Hence if the estimation is carried out with only the response velocity, a disjoint confidence region for the parameters would result. So it does not seem necessary to carry out a multiresponse analysis with the present model.

The confidence interval of the parameter PRIMER does include the point zero. However, because of some physical conditions, it was chosen to retain it in the estimation problem.

Uses of the Empirical-Mechanistic Model

The fitted empirical-mechanistic model can be used in several ways. Some of the uses are based upon the fact that the model explains observed data. Others stem from the mechanistic aspects of the model that permit meaningful extrapolation.

(1) The model can be used to simulate the interior ballistic performance of a weapon for any given set of initial conditions. This is especially useful in situations where actual experimentation is difficult, as in evaluating the effect of holding force.

(2) The model is useful for process optimization. For example, burning characteristics can be optimized to yield the desired muzzle velocity with small internal pressures. An anticipated problem in this regard is the development of a suitable objective function for internal pressures to be minimized.

(3) Once process optimum is defined, it can be attained by suitable changes in process variables. Because of the disturbances entering the system, the process would continually deviate from the optimum necessitating an efficient testing procedure. Such a testing procedure should detect changes quickly and also point out assignable causes.

The empirical-mechanistic model can be used to establish such a testing procedure. The effect of different process variables on (say) piezo pressure-time curve can be simulated using this model. The observed pressure-time curve would then indicate the process variables that need to be adjusted. It is possible that several process variables might influence the piezo curve in the same fashion thereby making detection difficult. This difficulty might be circumvented by considering multiple responses that distinguish between the effects of different process variables.

(4) The empirical-mechanistic model can be used to make small scale replicas of large weapons, thereby reducing cost of weapon testing.

(5) The model can be used to evaluate the performance of radically different experimental weapons or powders. For example, the effect of encapsulation or of propellants with lower molecular weights or of multiple charges can be studied with this model. The model can be used to determine process changes that would lead to increased velocity. Such inferences are possible only because the model is based upon mechanistic considerations.

APPENDIX II (A)

COPPER CRUSHER AND PIEZO DATA FROM FRANKFORD AND B.A.A.P.

FRANKFORD ARSENAL TEST DATA

Special Ammunition Tests

Cartridge - 5.56mm (Ball)

Temp. - 70 °F

	Round #	Peak Port Pressure (x100psi)	Peak Chamber Pressure (x 100psi)	Velocity (fps)	Slope	Pressure Time Integral (psi-sec)	Action Time (MS)
Lot LC12604 (High)	1	130	465	3161	3.376	27.93	1.16
	2	130	490	3207	4.165	27.40	1.237
	3	125	465	3158	3.376	28.79	1.191
	4	130	500	3192	3.606	29.43	1.148
	5	125	485	3181	3.487	29.40	1.163
	6	125	495	3203	3.732	28.98	1.261
	7	130	485	3177	3,376	29.18	1.235
	8	130	475	3180	3.487	29.04	1.29
	9	120	485	3176	3.376	29.20	1.164
	10	130	495	3193	3.606	29.23	1.265
Lot LC12594 (Low)	1	120	485	3172	3.172	29.92	1.238
	2	125	505	3193	3.271	28.23	1.225
	3	120	590	3237	3.487	28.70	1.184
	4	120	495	3172	3.271	28.98	1.227
	5	120	520	3221	3.078	29.07	1.184
	6	125	510	3222	3.867	28.57	1.155
	7	125	510	3185	3.078	29.26	1.152
	8	125	515	3237	3.867	28.96	1.165
	9	125	515	3223	3.487	28.90	1.315
	10	125	515	3225	3.376	28.68	1.244

APPENDIX II (B)

BADGER DATA (COPPER CRUSHER)
COMPOSITES
COMPONENT - TWIN
CALIBER - 5.56 MM (BALL)
TEMPERATURE = 70°F

Coating Date	Composites	Chamber Pressure (x100psi)	Velocity from Velocity Barrel (fps)	Velocity from Pressure Barrel (fps)	Charge Weight (Grains)
7- 3-70	244	429	3251	3204	27.9
7- 7-70	241	517	3245	3226	27.6
7- 9-70	243	469	3258	3230	27.7
7- 9-70	246	499	3250	3240	25.8
7- 9-70	245	450	3243	3234	27.6
7-10-70	242	424	3240	3192	26.1
7-14-70	248	469	3255	3235	27.5
7-14-70	250	524	3246	3234	27.3
7-14-70	252	479	3254	3234	27.7
7-17-70	251	472	3252	3273	26.5
7-17-70	253	446	3245	3226	27.5
7-21-70	249	447	3250	3250	27.4
7-21-70	258	416	3254	3158	26.4
7-21-70	259	459	3250	3192	28.0
7-21-70	254	432	3253	3182	27.5
7-21-70	255	479	3246	3189	27.6
7-21-70	256	420	3239	3179	27.9
7-22-70	260	438	3255	3215	28.1
7-22-70	261	455	3241	3216	28.3
7-27-70	263	453	3245	3200	28.3
7-27-70	265	435	3247	3187	27.7
7-30-70	264	483	3253	3213	26.5
7-31-70	267	482	3249	3221	28.0
8- 4-70	266	479	3254	3272	28.0
8- 4-70	268	489	3254	3216	28.5

Coating Date	Composites	Chamber Pressure (x100psi)	Velocity from Velocity Barrel (fps)	Velocity from Pressure Barrel (fps)	Charge Weight (Grains)
8- 4-70	269	449	3239	3201	26.5
8- 4-70	270	491	3250	3213	28.0
8- 4-70	271	461	3240	3230	28.3
8- 7-70	272	461	3254	3210	28.1
8- 7-70	273	447	3252	3199	27.9
8- 7-70	274	439	3242	3198	26.5
8- 7-70	275	419	3255	3192	27.9
8- 7-70	277	485	3254	3235	27.8
8-10-70	276	460	3251	3202	28.0
8-10-70	278	499	3246	3200	28.2
8-13-70	279	440	3247	3189	27.7
8-13-70	280	467	3247	3182	26.2
8-13-70	281	488	3258	3202	28.4
8-17-70	283	420	3244	3166	28.0
8-17-70	284	478	3242	3172	27.4
8-17-70	285	425	3250	3170	28.4
8-21-70	286	439	3241	3174	26.3
8-21-70	288	446	3246	3223	27.9
8-24-70	289	455	3250	3206	25.9
8-24-70	290	428	3244	3214	27.8
8-24-70	291	467	3249	3227	27.3
8-24-70	292	447	3254	3214	27.8
8-26-70	293	448	3242	3222	27.5
8-26-70	294	499	3246	3231	26.0
8-31-70	296	441	3252	3233	28.0
8-31-70	298	449	3249	3216	27.8
9- 3-70	297	467	3241	3222	27.6
9- 3-70	300	487	3246	3235	27.5
9- 4-70	301	474	3249	3230	26.2
9- 4-70	302	444	3252	3228	27.8
9- 4-70	303	459	3243	3240	27.9

Coating Date	Composites	Chamber Pressure (x100psi)	Velocity from Velocity Barrel (fps)	Velocity from Velocity Barrel (fps)	Charge Weight (Grains)
9- 4-70	304	477	3242	3242	27.8
9- 4-70	308	432	3245	3234	27.8
9- 9-70	309	477	3240	3233	27.1
9- 9-70	314	444	3245	3212	27.5
9-11-70	306	473	3256	3240	26.0
9-14-70	307	481	3240	3196	27.3
9-14-70	310	483	3253	3198	27.6
9-14-70	313	427	3249	3179	27.5
9-23-70	315	457	3240	3199	28.2
9-23-70	316	432	3256	3204	25.9
9-23-70	324	443	3245	3213	27.6
9-24-70	320	453	3251	3241	28.5
9-24-70	326	451	3250	3234	27.9
9-25-70	317	538	3252	3246	26.1
9-25-70	319	503	3252	3230	27.6
9-25-70	335	462	3238	32224	26.1

APPENDIX II (C)

BADGER DATA (Piezo Transducer)

Composites

Component - Twin

Caliber - 5.56mm (Ball)

Charge Weight Sames As In Copper Crush

Coating date	Composites	Peak Chamber Pressure	Velocity (fps) 70°	Velocity (fps) 125°	(x 100 psi) 125° Peak Chamber Pressure
7- 3-70	244	504			501
7- 7-70	241	545	3256	3295	550
7- 9-70	243	504	3243	3272	526
7- 9-70	246	518	3225	3269	542
7- 9-70	245	501	3235	3245	500
7-10-70	242	498	3209	3267	529
7-14-70	248	521	3248	3282	527
7-14-70	250	535	3253	3275	537
7-14-70	252	527	3261	3295	545
7-17-70	251	509	3229	3285	541
7-17-70	253	505	3213	3256	528
7-21-70	249	507	3246	3284	532
7-21-70	258	503	3223	3283	542
7-21-70	259	506	3226	3272	520
7-21-70	254	515	3238	3271	530
7-21-70	255	524	3252	3283	544
7-21-70	256	526	3265	3269	526
7-22-70	260	520	3239	3281	535
7-22-70	261	505	3229	3288	531
7-27-70	263	510	3227	3284	521

Coating date	Composites	(x 100 psi) 70° Peak Chamber Pressure	Velocity (fps) 70°	Velocity (fps) 125°	(x 100 psi) 125° Peak Chamber Pressure
7-27-70	265	498	3206	3274	525
7-30-70	264	494	3225	3260	513
7 31-70	267	516	3216	3274	546
8- 4-70	266	519	3252	3289	525
8- 4-70	268	494	3194	3263	531
8- 4-70	269	505	3204	3303	567
8- 4-70	270	501	3179	3244	529
8- 4-70	271	488	3206	3267	538
8- 7-70	272	469	3149	3264	533
8- 7-70	273	493	3182	3245	521
8- 7-70	274	501	3198	3270	539
8- 7-70	275	492	3198	3259	538
8- 7-70	277	520	3237	3259	538
8-10-70	276	498	3214	3255	521
8-10-70	278	508	3212	3252	528
8-13-70	279	495	3198	3241	515
8-13-70	280	491	3185	3241	521
8-13-70	281	497	3212	3242	513
8-17-70	283	488	3168	3258	522
8-17-70	284	481	3175	3217	504
8-17-70	285	492	3206	3251	510
8-21-70	286	499	3170	3271	590
8-21-70	288	543	3211	3257	571
8-24-70	290	510	3176	3217	551
8-24-70	291	513	3189	3240	558
8-24-70	292	502	3188	3245	566

Coating Date	Composites	(x 100 psi) 70° Peak Chamber Pressure	Velocity (fps) 70°	Velocity (fps) 125°	(x 100 psi) 125° Peak Chamber Pressure
8-24-70	292	515	3207	3225	535
8-26-70	293	501	3195	3244	543
8-26-70	294	523	3186	3211	549
8-31-70	296	531	3232	3215	543
8-31-70	298	533	3201	3228	551
9- 3-70	297	538	3243	3252	560
9- 3-70	300	534	3226	3275	578
9- 4-70	301	551	3242	3286	588
9- 4-70	302	558	3280	3311	569
9- 4-70	303	557	3266	3300	578
9- 4-70	304	548	3262	3288	568
9- 4-70	308	534	3238	3306	582
9- 9-70	309	529	3215	3281	584
9- 9-70	314	534	3249	3283	576
9-11-70	306	577	3272	3308	599
9-14-70	307	547	3241	3331	600
9-14-70	310	560	3243	3299	584
9-14-70	313	519	3218	3314	566
9-23-70	315	553	3225	3272	570
9-23-70	316	548	3223	3273	597
9-23-70	324	524	3239	3270	565
9-24-70	320	511	3249	3298	577
9-24-70	326	480	3209	3257	560
9-25-70	317	534	3244	3315	595
9-25-70	319	536	3229	3297	585
9-25-70	335	490	3214	3293	578

APPENDIX II (D)

BADGER DATA (Piezo Transducer)

COMPOSITE

COMPONENT - TWIN

CALIBER - 5.56 (BALL)

TEMPERATURE - 70°F

Composites	Slope	Ignition Delay (ms)
241	6.08	0.18
242	4.0	0.30
243	5.31	0.20
244	4.66	0.28
245	4.08	0.26
246	4.79	0.27
248	4.24	0.27
249	4.7	0.22
250	5.78	0.24
251	3.88	0.32
252	5.08	0.23
253	4.7	0.24
254	4.92	0.25
255	5.32	0.23
256	4.74	0.26
258	3.9	0.29

APPENDIX II (E)

BADGER DATA (COPPER CRUSHER)

Hand Blends

For Twin City

Caliber - 5.56mm(Ball)

Temp. - 70°F

Sample No.	Coating Date	Charge Weight (grains)	Velocity (Vel. Barrel) (fps)	Chamber Pressure (psi)	Velocity (Pressure Barrel) (fps)
HB-644	7- 2	27.1	3249	459	3232
HB-657	7-16	27.1	3249	452	3205
HB-662	7-24	27.0	3244	460	3187
HB-663	7-27	27.5	3241	449	3183
HB-666	7-28	27.6	3239	480	3211
HB-674	8- 4	27.4	3246	447	3199
HB-678	8- 6	27.3	3249	460	3208
HB-680	8-17	28.1	3242	462	3184
HB-692	8-25	27.3	3248	478	3239
HB-697	8-31	27.4	3251	457	3224
HB-717	9-18	27.2	3244	460	3219
HB-719	9-23	27.6	3248	488	3253
HB-720	9-23	28.0	3254	465	3230

APPENDIX II (F)

BADGER DATA (COPPER CRUSHER)

Hand Blends

For Lake City

Caliber - 5.56mm(Ball)

Temp. - 70°F

Sample No.	Coating Date	Charge Weight (grains)	Velocity (Vel. Barrel) (fps)	Chamber Pressure (psi)	Velocity (Pressure Barrel) (fps)
HB-646	7- 7	28.0	3240	440	3239
HB-647	7- 7	27.9	3251	472	3232
HB-648	7- 8	26.9	3247	456	3214
HB-652	7-13	27.5	3242	466	3245
HB-653	7-13	27.3	3251	476	3250
HB-661	7-23	27.6	3240	477	3254
HB-667	7-30	27.8	3248	464	3209
HB-679	8-10	28.2	3248	454	3222
HB-688	8-21	27.4	3249	461	3250
HB-695	8-27	27.4	3242	443	3226
HB-701	9- 3	27.9	3247	477	3232
HB-713	9-16	27.1	3238	464	3253
HB-715	9-17	27.0	3252	469	3241
HB-719	9-23	27.6	3248	488	3254
HB-726	9-28	28.0	3245	455	3256
HB-730	10-2	27.4	3245	462	3265

APPENDIX II (G)

BADGER DATA (PIEZO TRANSDUCER)

Hand Blends

For Twin City

Caliber - 5.56 (Ball)

Temp. - 70°F

Charge Weight Same as for Copper Crush

Sample No.	Coating Date	Peak Chamber Pressure (psi)	Velocity (fps)	Pressure-time Integral (psi-sec)	Slope	Ignition (ms)
HB-644	7- 2	547	3250	27-773	2.111	0.23
HB-657	7-16	540	3232	25-420	2.179	0.24
HB-662	7-24	514	3215			
HB-663	7-27	481	3168	x	x	x
HB-666	7-28	514	3224	x	x	x
HB-674	8- 4	483	3180	26-470	1.625	0.29
HB-678	8- 6	489	3172	26-456	1.516	0.30
HB-680	8-17	491	3200	26-940	1.799	0.25
HB-692	8-25	525	3220	26-039	2.050	0.30
HB-697	8-31	530	3216	26-287	1.927	0.30
HB-717	9-18	511	3237	25-420	1.958	0.29
HB-719	9-23	537	3233	26-955	2.267	0.28
HB-720	9-23	555	3268	28-697	2.117	0.25

APPENDIX II (H)

BADGER DATA (PIEZO TRANSDUCER)

Hand Blends

For Lake City

Caliber - 5.56mm (Ball)

Temp. - 70°F

Charge Weight Same as for Copper Crush

Sample No.	Coating Date	Peak Chamber Pressure (psi)	Velocity (fps)	Pressure-time Integral (psi-sec)	Slope	Ignition Delay (ms)
HB-646	7- 7	532	3224	28-167	1.67	0.19
HB-647	7- 7	536	3225	27-877	1.646	0.19
HB-648	7- 8	532	3210	29-307	1.884	0.19
HB-652	7-13	542	3243	28-813	1.828	0.20
HB-653	7-13	536	3233	28-467	1.854	0.18
HB-661	7-23	509	3245	x	x	0.19
HB-667	7-30	519	3252	x	x	0.16
HB-679	8-10	520	3264	28-806	1.754	0.19
HB-688	8-21	554	3248	28-827	2.028	0.21
HB-695	8-27	522	3216	28-136	1.896	0.23
HB-701	9- 3	532	3256	x	x	0.21
HB-713	9-16	536	3265	27-819	1.961	0.20
HB-715	9-17	539	3265	27-627	2.002	0.20
HB-719	9-23	538	3255	28-67	2.158	0.25
HB-726	9-28	465	3121	x	x	x
HB-736	10-2	447	3104	x	x	x

Appendix III

Data for Sequential Variance Estimation

Chamber Pressure Data from BAAP (psi)				Piezo Data from Frankford Arsenal (psi)	
Serial Number	Lot 46881	Lot 46893	Lot 46626	LC12604	LC12594
1	46400	45000	43700	46500	48500
2	44500	44300	43200	49000	50500
3	43600	42900	44900	46500	54000
4	45200	44900	44700	50000	49500
5	45600	44600	43600	48500	52000
6	46600	44700	43800	49500	51000
7	45800	45300	45700	48500	51000
8	46400	44500	45100	47500	51500
9	44700	45200	44300	48500	51500
10	48100	44700	45900	49500	51500
11	46100	42700	45500		
12	44000	44400	43800		
13	46100	44200	42800		
14	47100	45700	42800		
15	45400	44900	46700		
16	45200	42700	45700		
17	47500	43200	44100		
18	43800	43300	45100		
19	43800	43400	43800		
20	47000	46100	42800		

APPENDIX IV

PROPELLANT CHARGE WEIGHT AND CHAMBER PRESSURE DATA FROM BAAP

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
12-31-68	45747	LC	26.9	479
1-10-69	45748	TW	26.8	475
1-14-69	45749	TW	27.2	490
1-16-69	45750	LC	27.3	492
1-16-69	45750	FA	27.0	495
1-22-69	45751	TW	27.2	467
1-27-69	45752	TW	27.3	479
2- 7-69	45753	TW	27.2	476
2-11-69	46194	TW	27.3	471
2-13-69	46195	LC	27.2	489
2-17-69	46196	TW	27.6	482
2-18-69	46196	LC	27.9	480
2-21-69	46198	TW	27.4	467
2-22-69	46199	LC	27.6	473
2-24-69	46200	LC	27.9	482
2-26-69	46201	TW	27.3	477
2-28-69	46202	TW	27.3	472
3- 4-69	46203	LC	27.1	458
3- 5-69	46204	TW	27.2	456
3- 6-69	46205	TW	27.0	445
3-12-69	46206	LC	27.0	488
3-17-69	46207	TW	26.9	445
3-18-69	46208	TW	26.9	457
3-20-69	46209	LC	27.2	455
3-21-69	46210	TW	26.7	449
3-24-69	46211	TW	27.1	450
3-25-69	46212	LC	27.3	469
3-27-69	46213	TW	26.6	462

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
4- 7-69	46214	LC	26.8	471
4- 7-69	46215	TW	26.7	455
4- 9-69	46216	TW	26.5	482
4-10-69	46217	TW	26.8	481
4-11-69	46218	LC	27.1	480
4-16-69	46219	TW	26.8	470
4-17-69	46220	LC	27.3	448
4-22-69	46221	TW	27.0	478
4-24-69	46222	TW	27.3	463
4-25-69	46223	TW	27.2	473
4-28-69	46266	LC	27.3	458
4-29-69	46267	TW	27.0	458
5- 2-69	46268	TW	27.2	447
5- 2-69	46268	FA	27.8	469
5- 2-69	46269	TW	27.0	503
5- 6-69	46270	TW	26.7	490
5- 9-69	46271	LC	27.2	455
5-12-69	46272	TW	26.9	451
5-12-69	46273	LC	27.6	450
5-14-69	46274	TW	26.9	465
5-16-69	46275	LC	27.3	460
5-19-69	46276	TW	27.0	470
5-21-69	46278	TW	27.3	461
5-22-69	46279	TW	27.5	455
5-27-69	46280	TW	27.7	439
5-27-69	46281	LC	28.1	464
6- 3-69	46282	LC	27.5	457
6- 4-69	46283	TW	27.5	466
6- 5-69	46284	TW	27.6	467
6- 9-69	46285	LC	28.0	453

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
6-10-69	46286	TW	27.9	450
6-13-69	46287	LC	27.9	465
6-16-69	46288	LC	27.4	455
6-17-69	46289	TW	27.3	462
6-17-69	46290	LC	27.6	449
6-23-69	46337	LC	27.7	448
6-18-69	46338	TW	27.2	465
6-20-69	46339	LC	27.3	471
6-24-69	46340	TW	27.4	481
6-30-69	46341	LC	27.0	458
7- 1-69	46342	LC	27.6	451
7- 1-69	46343	LC	27.8	453
7- 2-69	46344	TW	27.0	477
7- 3-69	46346	TW	27.6	460
7- 7-69	46346	TW	27.5	442
7- 7-69	46347	LC	27.8	473
7- 8-79	46348	TW	27.4	465
7-10-69	46349	LC	27.6	473
7-14-69	46350	LC	26.9	466
7-16-69	46351	TW	27.5	469
7-17-69	46352	TW	27.8	481
7-23-69	46353	LC	27.9	498
7-24-69	46354	TW	28.0	479
7-28-69	46355	LC	28.3	467
7-28-69	46356	TW	28.2	469
8- 5-69	46357	TW	27.3	482
8- 8-69	46358	LC	27.8	467
8-11-69	46359	TW	27.5	456
8-12-69	46360	LC	27.5	470
8-13-69	46361	TW	27.6	462

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
8-14-69	46362	LC	27.9	451
8-15-69	46363	TW	27.8	459
8-18-69	46364	LC	27.9	467
8-19-69	46365	TW	27.9	461
8-19-69	46366	LC	27.9	468
8-20-69	46411	TW	27.7	458
8-21-69	46412	LC	27.9	475
9- 2-69	46413	TW	27.6	475
9- 3-69	46414	LC	27.8	461
9- 8-69	46415	TW	27.6	493
9-10-69	46416	LC	28.0	466
9-11-69	46417	TW	28.4	491
9-15-69	46418	LC	27.5	442
9-16-69	46419	LC	28.1	456
9-19-69	46420	LC	28.6	478
9-22-69	46421	LC	28.3	426
9-22-69	46425	LC	27.6	445
9-23-69	46424	TW	27.8	455
9-25-69	46426	LC	27.8	458
9-26-69	46422	LC	28.4	455
9-26-69	46423	LC	28.3	453
9-30-69	46427	LC	27.9	452
10- 2-69	46428	TW	27.5	486
10- 7-69	46429	TW	27.6	472
10-10-69	46430	LC	28.3	477
10-20-69	46422	LC	27.9	462
10-20-69	46423	LC	28.0	465
10-21-69	46884	LC	27.9	476
10-22-69	46885	LC	27.8	465
10-28-69	46881	LC	28.1	480
10-29-69	46882	TW	27.7	460

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
10-30-69	46883	LC	27.9	458
11- 7-69	46886	LC	27.6	459
11-17-69	46888	TW	27.0	440
11-21-69	46890	LC	27.1	447
12- 8-69	46891	LC	26.9	452
12-11-69	46892	TW	27.0	451
12-16-69	46893	LC	26.8	452
12-17-69	46894	TW	26.2	443
12-22-69	46895	TW	27.2	442
12-29-69	46896	TW	26.6	462
11-18-69	46889	TW	27.3	463
1- 5-70	46897	TW	26.5	480
1- 7-70	46898	TW	26.3	458
1- 8-70	46899	TW	27.1	460
1-13-70	46900	LC	26.9	457
1-15-70	46506	TW	27.1	435
1-29-70	46507	LC	27.5	469
2-11-70	46508	LC	27.3	453
2-19-70	46509	LC	26.5	481
2-19-70	46510	LC	26.3	481
2-20-70	46511	LC	26.5	461
2-20-70	46512	LC	26.6	465
2-20-70	46513	LC	26.8	465
2-24-70	46514	LC	27.6	464
2-25-70	46515	LC	27.2	456
2-27-70	46516	LC	27.0	488
3- 9-70	46517	LC	27.2	479
3-11-70	46518	LC	27.1	462
3-17-70	46519	LC	27.0	475
3-23-70	46520	LC	27.2	470
3-26-70	46521	TW	26.7	440

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
4- 1-70	46522	TW	27.4	461
4-13-70	46523	LC	27.1	463
4-22-70	46524	TW	27.1	464
5- 4-70	46526	FC	27.2	436
5- 4-70	46527	TW	27.0	452
5- 7-70	46528	TW	27.1	485
5-12-70	46529	LC	27.8	454
5-22-70	46530	TW	27.3	448
6- 2-70	46604	RA	28.0	489
6- 3-70	46605	WW	27.9	422
6- 5-70	46606	WW	28.2	434
6-17-70	46607	LC	28.0	467
6-18-70	46608	WW	28.3	449
6-25-70	46609	LC	27.9	458
4-23-70	46525	LC	27.1	471
6-26-70	46610	TW	27.7	455
7- 7-70	46612	TW	27.4	452
7- 8-70	46611	LC	27.4	470
7-17-70	46613	TW	27.2	466
7-20-70	46614	LC	27.4	468
7-20-70	46615	LC	27.3	466
7-21-70	46616	TW	27.1	459
7-28-70	46617	TW	26.9	464
7-29-70	46618	TW	27.1	457
7-30-70	46619	LC	27.5	452
8- 4-70	46620	TW	27.3	465
8- 4-70	46621	TW	27.8	460
8- 7-70	46622	TW	27.6	466
8-13-70	46623	LC	27.8	492
8-20-70	46624	TW	27.6	456

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
8-24-70	46625	TW	27.5	455
8-31-70	46626	LC	28.1	443
8-31-70	46627	WW	28.1	437
9- 2-70	46628	LC	27.4	464
9- 2-70	46938	TW	27.4	468
9-10-70	46939	LC	27.9	481
9-10-70	46940	TW	27.5	443
9-14-70	46941	LC	27.7	500
9-17-70	46942	TW	27.3	447
9-21-70	46943	LC	27.2	456
9-23-70	46944	TW	27.6	465
9-30-70	46945	TW	27.9	450
9-30-70	46946	TW	27.6	474
10- 5-70	46948	TW	27.4	452
10- 9-70	46949	TW	27.0	439
10- 9-70	46947	LC	27.9	428
10-12-70	46950	RA	27.4	445
10-14-70	46951	LC	27.3	469
10-15-70	46952	LC	27.4	470
10-22-70	46953	TW	27.1	445
10-27-70	46954	LC	27.5	464
10-27-70	46955	LC	27.3	485
10-27-70	46956	TW	27.4	461
10-29-70	46957	LC	27.3	473
10-30-70	46958	TW	27.2	463
11- 5-70	46959	WW	27.2	464
11- 9-70	46960	TW	27.1	452
11-10-70	46961	LC	27.0	490
11-10-70	46962	TW	27.4	462
11-12-70	46999	LC	27.0	481

Date Fired	Lot No.	Components	Charge Weight (Grains)	Corrected Mean Chamber Pressure x 100 psi
11-17-70	47000	TW	27.0	476
11-17-70	47001	LC	27.0	451
11-24-70	47002	TW	27.1	458
12- 1-70	47003	LC	27.0	455
12- 4-70	47004	TW	26.8	458
12- 8-70	47005	LC	26.0	489
12-10-70	47006	TW	26.5	480
12-10-70	47007	LC	26.2	480
12-15-70	47008	TW	26.5	466
12-18-70	47009	TW	26.3	464
12-22-70	47010	LC	26.3	441
12-23-70	47011	TW	26.3	447
12-23-70	47012	LC	26.5	467

APPENDIX V

COMPUTER PROGRAMS

In this appendix, different programs used for calculations in this report are included. The programs are written for UNIVAC 1108 computer.

TREND

Purpose: To estimate the underlying process from a given series of observations using Semi Average, Cumulative Sum, and Moving Average techniques.

Input:

K(I) = different values of constant K_1

AK(II) = different values of constant K_2

KK(I) = different values of constant K_3

X (I) = observed values of input variable (e.g., chamber pressure)

Output: Plots of estimated process behavior based upon the three methods of estimation.

```
C     TREND  DETERMINATION  IN  TIME  SERIES
      DIMENSION X(1000),XBAR(1000),Z(1000),SUM(1000),XXBAR(1000),
     1K(5),AK(5),KK(5)
      READ 1,N,(K(J),  J = 1,3)
    1 FORMAT(4I10)
      READ 2,(AK(I),  I = 1,3)
    2 FORMAT(3F10.0)
      READ 3, (KK(I),  I= 1,3)
    3 FORMAT(3I10)
      READ,(X(I),  I= 1,N)
      DO  4  I = 1,N
    4 X(I) = X(I)*100.
      DO  5  I = 1,N
    5 Z(I) = I
      CALL INITPL(9,10,8)
C     METHOD  OF  SEMI  AVERAGES
      DO  100  J = 1,3
      Y = K(J)
      JJ = 1
      L = N/K(J)
      DO  6  M = 1,L
      SX = 0.
      SS = 0.
      JJJ = JJ+K(J)-1
      DO  7  I = JJ,JJJ
    7 SX = SX+X(I)
      XBAR(M) = SX/Y
    6 JJ = JJ+K(J)
      CALL  GRAPH(Z,3HLIN,XBAR,3HLIN,L,4HNONE,5HSOLID,
     12H$$,2H$$,4HAXES,2H$$,4HFULL,4HNULL)
  100 CONTINUE
C     CUSUM  CHART
      DO  200  J = 1,3
      S = 0.
      DO  8  I = 1,N
      S = S+X(I)-AK(J)
    8 SUM(I) = S
      CALL  GRAPH(Z,3HLIN,SUM,3HLIN,N,4HNONE,5HSOLID,
     12H$$,2H$$,4HAXES,2H$$,4HFULL,4HNULL)
  200 CONTINUE
C     METHOD  OF  MOVING  AVERAGES
      DO  300  J = 1,3
      YY = KK(J)
      KKK = KK(J)
      SS = 0.
      DO  9  I = 1,KKK
    9 SS = SS+X(I)
      XXBAR(1) = SS/YY
      NN = N-KK(J)+1
      DO  10  I = 2,NN
      LL = I+KK(J)-1
      SS = 0.
      DO  11  M = I,LL
   11 SS = SS+X(M)
      XXBAR(I) = SS/YY
   10 CONTINUE
      CALL  GRAPH(Z,3HLIN,XXBAR,3HLIN,NN,4HNONE,5HSOLID,
     12H$$,2H$$,4HAXES,2H$$,4HFULL,4HNULL)
  300 CONTINUE
      CALL  ENDPLT
      END
```

HISTOGRAM AND CUMULATIVE DISTRIBUTION FUNCTION

Purpose: To obtain one dimension histogram and cumulative distribution function.

Input:

Z (I) = observed values of input variable (e.g. Chamber Pressure)

N = Number of values of input variable

JX = Number of groups in which the variable range is divided to obtain the histogram.

Output: Plots of histogram and cumulative distribution function

```
'FOR,SI HISTO
      SUBROUTINE HISTO(N,Z)
      DIMENSIONZ(5000),Y(50),X(50),AY(50),BY(50)
      DIMENSION FEL(16)                                                 ** FELCH
      DATA FEL / 6HHISTOG,6HRAM OF,6H OBSER,6HVATION,6HS$$    ,         ** FELCH
     16HOBSERV,6HATIONS,6H$$    ,6HNUMBER,6H OF OC,6HCURENC,6HES$$   ,  ** FELCH
     26HHISTOG,6HRAM OF,6H RESID,6HUALS$$ /                             ** FELCH
      N6=1
      JX = 20
      CALL URSRCH(0,N,Z,IBIG,B,0,DUMMY)                                 ** FELCH
      CALL URSRCH(1,N,Z,ISMA,S,0,DUMMY)                                 ** FELCH
      RANG=Z(IBIG)-Z(ISMA)
C**  4*************************************************************
      PRINT  50,Z(IBIG),Z(ISMA)
   50 FORMAT(10F10.3)
      AJX=JX
      DEL=RANG/AJX
      X(1)=Z(ISMA)
C**  5*************************************************************
      Y(1)= 0.00000000E+00
      JX = JX+1
      IF(JX.LT.1)  GO  TO  90
      DO 80 I=2,JX
      Y(I)= 0.00000000E+00
   70 X(I)=X(I-1)+DEL
   80 CONTINUE
      X(JX) = Z(IBIG)
   90 CONTINUE
C**  6*************************************************************
      PRINT  50,          RANG,DEL,X(1) ,X(JX)
      IF( N .LT. N6) GO TO 160
      DO 150  I = N6,N
      IF( JX .LT.1) GO TO 130
      DO  120  K = 2,JX
      IF(X(K)) 110, 110, 100
  100 IF(Z(I)-X(K)) 140, 140, 120
  110 IF(Z(I)-X(K)) 140, 140, 120
  120 CONTINUE
  130 CONTINUE
      GO TO  150
  140 Y(K)=Y(K)+ 1.00000000E+00
  150 CONTINUE
  160 CONTINUE
C*******HOLLERITH*CONSTANT*LONGER*THAN*6*CHARACTERS************
      PP = 0.
      IF(PP)27,27,28
   27 CONTINUE
      CALL  INITPL(9,10.8)
C*200 CALL GRAPH1(X,Y,JX,6H6X6   ,6HAUTO  ,32HHISTOGRAM OF OBSERVATIONS.
C*   1.     ,16HOBSERVATIONS..  ,24HNUMBER OF OCCURENCES..  )
  200 CALL GRAPH(X,1,Y,1,JX,4HNONE,5HSOLID,FEL(6),FEL(9),1,FEL(1),7.5,10** FELCH
     1.0)                                                               ** FELCH
   28 CONTINUE
  230 FORMAT(1H1)
  290 FORMAT(10X,F7.2,1X,3HTO ,F7.2,11X,F6.0,/)
```

```
  300 FORMAT(15X,8HINTERVAL,16X,20HNUMBER OF OCCURENCES,//)
C**   9*******************************************************
      PRINT  300
      DO  999  I = 2,JX
      J = I-1
 999  Y(J) = Y(I)
      JX=JX-1
      DO  201  I = 1,JX
  201 AY(I) = Y(I)/N
      BY(1) = AY(1)
      DO  202I = 2,JX
  202 BY(I) = BY(I-1)+AY(I)
      CALL  GRAPH(X,3HLIN,BY,3HLIN,JX,4HNONE,5HSOLID,
     1'OBSERVATIONS$$','CUMULATIVE PROBABILITY$$',
     14HAXES,'EMPERICAL  CUMULATIVE  DISTRIBUTION  FUNCTION$$',7.5,
     110.0)
      CALL  ENDPLT
      IF( JX .LT.1) GO TO 340
      DO 330 I=1,JX
C** 10*******************************************************
  310 PRINT 290,X(I),X(I+1),Y(I)
  320 CONTINUE
  330 CONTINUE
C** 11*******************************************************
      PRINT 400
  400 FORMAT(10X,11HPROBABILITY,10X,22HCUMULATIVE PROBABILITY,//)
      DO 402  I = 1,JX
      PRINT 401,AY(I),BY(I)
  401 FORMAT(8X,F10.6,16X,F10.6)
  402 CONTINUE
  340 PRINT  230
      RETURN
      END
```

BIVARIATE HISTOGRAM

Purpose: The program generates a two dimensional frequency table of chamber pressure and port pressure values. This table is a representation of the two dimensional histogram.

Input:

N	=	Number of chamber pressure values also equal to the number of port pressure values
K	=	Number of groups in which the chamber pressure range is divided
M	=	Number of groups in which the port pressure range is divided
XMIN	=	Minimum Chamber pressure value
XMAX	=	Maximum chamber pressure value
XXMIN	=	Minimum port pressure value
XXMAX	=	Maximum port pressure value
D	=	One standard deviation of chamber pressure
DD	=	Half a standard deviation of chamber pressure
Z(I)	=	Chamber pressure values
ZZ(I)	=	Port pressure values

Output: Two dimensional table, where the entire set of chamber pressure and port pressure values are grouped into groups one standard deviation wide along the chamber pressure axis and half a standard deviation wide along the port pressure axis.

```
C     BIVARIATE  HISTOGRAM
      DIMENSION Z(1000),ZZ(1000),Y(50,50),X(50),XX(50)
      READ 20,N,K,M
   20 FORMAT(3I10)
      READ  30,XMIN,XMAX,XXMIN,XXMAX,D,DD
   30 FORMAT(6F10.0)
      READ  1,(Z(I),  I=1,N)
    1 FORMAT(7F10.0)
      READ 2,(ZZ(I),  I = 1,N)
    2 FORMAT(7F10.0)
      DO  9  I = 1,N
    9 ZZ(I) = .1*ZZ(I)
      DO  50  I = 1,M
      DO  51  J = 1,K
   51 Y(I,J) = 0.
   50 CONTINUE
      X(1) = XMIN
      X(K) = XMAX
      XX(1) = XXMIN
      XX(M) = XXMAX
      KK = K-1
      DO  3  I = 2,KK
    3 X(I) = X(I-1)+D
      MM = M-1
      DO  15  I = 2,MM
   15 XX(I) = XX(I-1)+DD
      DO  4  I = 2,M
      DO  5  II = 1,N
      IF(ZZ(II).LE.XX(I).AND.ZZ(II).GT.XX(I-1))  GO  TO  6
      GO  TO  5
    6 DO  7  J = 2,K
      IF(Z(II)-X(J)) 8,8,7
    7 CONTINUE
    8 Y(I,J) = Y(I,J)+1.
    5 CONTINUE
    4 CONTINUE
      DO 10  J = 2,K
      PRINT  11,(Y(I,J),  I = 2,M)
   11 FORMAT(2X,24F4.0)
   10 CONTINUE
      END
```

AUTO AND PARTIAL CORRELATIONS (IDENTIFICATION)

Purpose: For a given series of data, the program calculates auto correlation and partial correlation functions. These functions are used to identify the form of the tentative time series model. The same program is also used in diagnostic checking of residuals.

Input:

N	=	Number of data values
Z(I)	=	One dimension array of ordered observations
KK	=	Number of auto correlations or partial correlations required + 1

Output: Plots of auto correlation function and partial correlation function with the respective significance (2 sigma) limits.

```
      SUBROUTINE AUTO(N,Z,KK,ARO1,ARO2)
      DIMENSIONZ(1000),RPLOT(1000 ),IPLOT(1000),C (99),R(99),SCLIMS(2),
     1SCPARS(10),T(50,50),VAR(99),STSR(99),STSR1(99),STSR2(99),IPLOTV(
     299),TPLOT(50),VART(50),STSP(50),STSP1(50),STSP2(50),U(50) ,S(99)
      KKFIX=KK
      KK=KK-1
      ZBAR=0.
      XN=N
      DO 102 I=1,N
  102 ZBAR=ZBAR+Z(I)
      ZBAR=ZBAR/XN
      CO=0.
      DO 103 I=1,N
  103 CO=CO+(Z(I)-ZBAR)**2
      CO=CO/XN
C     CALCULATION OF R
      RPLOT(1)=0.
      RPLOT(2)=1.
      RPLOT(3)=0.
      IPLOT(1)=0
      IPLOT(2)=0
      IPLOT(3)=0
      DO 104 K=1,KK
      C(K)=0.
      NN=N-K
      DO 105 J=1,NN
  105 C(K)=C(K)+(Z(J)-ZBAR)*(Z(J+K)-ZBAR)
      C(K)=C(K)/XN
      R(K)=C(K)/CO
      J=3*K+1
      IPLOT(J)=K
      RPLOT(J)=0.
      J=J+1
      IPLOT(J)=K
      RPLOT(J)=R(K)
      J=J+1
      IPLOT(J)=K
      RPLOT(J)=0.
  104 CONTINUE
      ARO1=R(1)
      ARO2=R(2)
      KKLAG=3*KK
      SCLIMS(1)=-1.
      SCLIMS(2)=1.
C     CALCULATION OF T
      IF(KK-50)106,106,107
  107 KKK=50-1
      GO TO 109
  106 KKK=KK-1
  109 T(1,1)=R(1)
      T(2,2)=(R(2)-R(1)**2)/(1.-R(1)**2)
      T(2,1)=T(1,1)-T(2,2)*T(1,1)
      DO 203 K=2,KKK
      B=0.
      A=0.
      DO 202 J=1,K
```

```
      A=A+T(K,J)*R(K+1-J)
 202  B=B+T(K,J)*R(J)
      A=R(K+1)-A
      B=1.-B
      T(K+1,K+1)=A/B
      DO 203 J=1,K
 203  T(K+1,J)=T(K,J)-T(K+1,K+1)*T(K,K-J+1)
C     CALCULATION OF VAR AND VART
      IPLOTV(1)=0
      VAR(1)=1./XN
      STSR(1)=SQRT(VAR(1))
      S(1)=R(1)/STSR(1)
      STSR1(1)=1.96*STSR(1)
      STSR2(1)=-1.96*STSR(1)
      A=2./XN
      DO 204 K=2,KK
      VAR(K)=VAR(K-1)+A*(R(K-1)**2)
      IPLOTV(K)=K-1
      STSR(K)=SQRT(VAR(K))
      STSR1(K)=1.96*STSR(K)
      STSR2(K)=-STSR1(K)
      S(K)=R(K)/STSR(K)
 204  CONTINUE
      KKK=KKK+1
      TPLOT(1)=0.
      TPLOT(2)=1.
      TPLOT(3)=0.
      DO 205 K=1,KKK
      A=1./(N-K)
      VART(K)=A
      STSP(K)=SQRT(A)
      STSP1(K)=1.96*STSP(K)
      STSP2(K)=-STSP1(K)
      U(K)=T(K,K)/STSP(K)
      J=3* K+1
      TPLOT(J)=0.
      J=J+1
      TPLOT(J)=T(K,K)
      J=J+1
      TPLOT(J)=0.
 205  CONTINUE
      KKKLAG=3*KKK
C     PRINT OUT
      PRINT 300
 300  FORMAT (1H1///10X,36H SAMPLE AUTOCORRELATION COEFFICIENTS//)
      PRINT 301
 301  FORMAT (20X,17H AUTO-COEFFICIENT ,5X,20H UNIFIED COEFFICIENT  /)
      PRINT 302, (I,R(I),S(I),I=1,KK)
 302  FORMAT (15X,I2,F15.4,9X,F15.4)
      PRINT 303, ZBAR,C0
 303  FORMAT( 10X,19H MEAN OF THE SERIES  ,F14.4/ 10X,
     123H VARIANCE OF THE SERIES   ,F10.4)
      PRINT 304
 304  FORMAT (1H1///10X,27H SAMPLE PARTIAL CORRELATION   //)
      PRINT 305
 305  FORMAT (20X,17HPART-CORRELATION  ,5X,20H UNIFIED COEFFICIENT/)
```

```
PRINT 302, (I, T(I,I), U(I), I=1,KKK)
KK=KKFIX
RETURN
END
```

ESTIMATION AND DIAGNOSTIC CHECKING

Purpose: The program estimates the parameters in the proposed time series model by the method of nonlinear least squares. The test for the adequacy of the model is done by considering the autocorrelation and the partial correlation functions of the series of residuals, and a chi-square statistic based on the autocorrelation function.

Input:

A general time series model can be written as

$$(1-\phi_1 B-\ldots\phi_p B^p)(1-\phi_1' B-\ldots\phi_p' B^{p'})(1-B^s)^{d_1}(1-B)^d(Z_t-\mu)$$

$$=\theta_o+(1-\theta_1 B-\ldots-\theta_q B^q)(1-\theta_1' B-\ldots-\theta_{q_1} B^{q_1})a_t$$

NREP = Number of models to be fitted

NDR = Number of observations in the data series

Z(I) = One dimensional array of data values

MAXI = $p + p' + sd_1 + d$

NP = Number of parameters to be estimated

NRD = d

NSD = s

NSEA = d_1

INC(J) = One dimensional array of size 6. It contains the information regarding the number of parameters of each type as specified by the model.

IOPA(J) = One dimensional array of size NP. It specifies the powers of B associated with each parameter in the model, in the sequence from left to right.

PA(J) = One dimensional array of size NP, specifying the initial estimates of the parameters in sequence.

Special Subroutine Used:

The subroutine UWHAUS supplied by the University of Wisconsin Computing Center is used for the nonlinear least squares estimation of the parameters.

Output:

The final estimates of the parameters, 95% confidence limits on the parameter (on linear hypothesis) and the correlation matrix of the parameters are pointed. The program also prints the autocorrelations, the partial correlations of the series of residuals and a chi-square statistic based on the autocorrelations.

```
C          MAIN PROGRAM
      DIMENSION A(999),Z(999),PA(50),IOPA(50),INC(6),NT(10),Y(100),ZP(20
     X0,10),CLL(200,10),CUL(200,10),DIFF(50),SIGNS(50),RHO(50,10),STE(50
     X,10),E(10),SCRATC(6000),F(999)
      COMMON /TWO/NDR,NRD,NSD,NSEA
      NF=0
      NTO=0
      NLOG=0
      NAD=0
      ND=NAD
      MCE=10
      MP=10
      NY=0
      MIT=50
      EPS1=.001
      EPS2=.005
      FLAM=.01
      FNU=10.
      NL=50
      READ 1, NREP,NDR
      READ 2, (Z(I), I=1,NDR)
1     FORMAT (20I4)
      DO 20 NN=1,NREP
      READ 1, MAX1,NP
      READ 1, NRD,NSD,NSEA
C     (1-B)**NRD , (1-B**NSEA)**NSD
      IF (NY .EQ. 0) GO TO 5
      READ 2, (Y(I), I=1,NY)
5     CONTINUE
      READ 1, (INC(I), I=1,6)
      READ 1, (IOPA(I), I=1,NP)
2     FORMAT(7F10.0)
      READ 2, (PA(I), I=1,NP)
      IF (NTO .EQ. 0) GO TO 10
      READ 1, (NT(I), I=1,NTO)
10    CONTINUE
      NOB=NDR-MAX1
      IF (NL .EQ. 0) GO TO 15
      DO 11 I=1,NP
      SIGNS(I)=0.
      DIFF(I)=0.005
11    CONTINUE
      NDIMS=5*NP+2*NP**2+2*NOB+NOB*NP
      NPROB=NN
      CALL ESTIM(NPROB,A,NOB,Z,PA,DIFF,SIGNS,EPS1,EPS2,MIT,FLAM,FNU,SCRA
     XTC,NDIMS,NLOG,INC,IOPA,NRD,NSD,NSEA,NL,NAD,RHO,STE,E,MCE,MP)
15    CONTINUE
      IF (NF .EQ. 0) GO TO 20
20    CONTINUE
      END

C          SUBROUTINE  1
```

```
      SUBROUTINE FSTIM(NPROB,B,NOB,YY,PA,DIFF,SIGNS,EPS1,EPS2,MIT,FLAM,F
     XNU,SCRATC,NDIMS,NLOG,IC,IP,NA,NB,NC,NL,NAD,RHO,STE,E,MCF,MP)
      DIMENSION A(999),Z(999),C(999),Y(999),PA(50),IOPA(50),INC(6),DIFF(
     X50),SIGNS(50),RHO(NL,10),STE(NL,10),E(10),SCRATC(NDIMS),B(999),YY(
     X999),IP(50),IC(6)
      COMMON A,Z,NDR,IOPA,INC,MAX11,MBO,NRD,NSD,NSEA
      EXTERNAL TSMOD
      KK=0
      NRD=NA
      NSD=NB
      NSEA=NC
      DO 3 J=1,6
      INC(J)=IC(J)
      KK=KK+INC(J)
      IF(INC(J) .LT. 0) GO TO 55
    3 CONTINUE
      IF(NSEA .LT. 0) GO TO 55
      MM=0
      IF((INC(2) .NE. 0 .OR. NSD .NE. 0) .OR. INC(6) .NE. 0)MM=1
C     IF(MM .EQ. 1 .AND. NSEA .LE. 1)GO TO 55
      IF(NRD .LT. 0 .OR. NSD .LT. 0) GO TO 55
      IF(INC(3) .GT. 1 .OR. INC(4) .GT. 1) GO TO 55
      IF(NRD .GT. 0 .AND. INC(3) .EQ. 1) GO TO 55
      IF(NSD .GT. 0 .AND. INC(3) .EQ. 1) GO TO 55
      NP=KK
      IF(NP .LE. 0 .OR. NP .GT. 50) GO TO 55
      DO 5 J=1,NP
      IOPA(J)=IP(J)
      IF(IOPA(J) .LT. 0) GO TO 55
    5 CONTINUE
      IF(NL .LE. 0 .OR. NOB .LE. 0) GO TO 55
      IF(NAD .LT. 0 .OR. NAD .GT. 10) GO TO 55
      MAX1=NRD+NSD*NSEA
      KK=INC(1)
      IF(INC(1) .NE. 0)MAX1=MAX1+IOPA(KK)
      KK=KK+INC(2)
      IF(INC(2) .NE. 0)MAX1=MAX1+IOPA(KK)
      NDR=NOB+MAX1
      IF(NDR .GT. 999) GO TO 55
      KK=KK+INC(3)+INC(4)+INC(5)
      MAX2=0
      IF(INC(5) .NE. 0)MAX2=IOPA(KK)
      IF(INC(6) .NE. 0)MAX2=MAX2+IOPA(NP)
      MBO=MAX1
      IF(MAX2 .GT. MAX1)MBO=MAX2
      IF(NDR .LE. MBO .OR. MBO .GT. 100) GO TO 55
      MAX11=MAX1+1
      DO 6 J=1,NDR
      Z(J)=YY(J)
    6 IF(NLOG .NE. 0) Z(J)=LOG(Z(J))
      DO 10 J=MAX11,NDR
      K=J-MAX1
   10 Y(K)=Z(J)
      DO 12 J=1,MAX1
   12 A(J)=0
```

```
      CALL UWHAUS(NPROB,TSMOD,NOB,Y,NP,PA,DIFF,SIGNS,EPS1,EPS2,MIT,FLAM,
     XFNU,SCRATC)
      DO 14 I=MAX11,NDR
      J=I-MAX1
   14 C(J)=A(I)
      PRINT 17
   17 FORMAT(1H1,43X,25HRESIDUAL AUTOCORRELATIONS//)
      CALL ACOR(C,NL,NOB,NAD,RHO,STF,F,MCE,MP)
      SUM=0
      DO 20 I=MAX11,NDR
   20 SUM=SUM+(A(I)-F(1))**2
      DIV=NOB*(NOB-NP)
      VABAR=SUM/DIV
      TNSTD=ABS(F(1))/SQRT(VABAR)
      PRINT 23,TNSTD
   23 FORMAT(///10X,40HMEAN OF ORIGINAL SERIES OF RESIDUALS IS ,F6.3,31H
     X STANDARD DEVIATIONS FROM ZERO.)
      DO 26 J=1,NDP
   26 B(J)=A(J)
      GO TO 60
   55 PRINT 58
   58 FORMAT(1H1,10X,24HPARAMETER ERROR IN ESTIM)
   60 RETURN
      END
C           SUBROUTINE  2
      SUBROUTINE TSMOD(NPROB,PA,F,NOB,NP)
      DIMENSION PA(50),F(999),A(999),Z(999),IOPA(50),INC(6),T(100),C(100
     X),CF(100),D(10,10),DS(10,100),W(999)
      COMMON A,Z,NDR,IOPA,INC,MAX11,MBO,NRD,NSD,NSFA
      COMMON /ONE/C,CF
      DO 3 J=1,MBO
      T(J)=0
      C(J)=0
    3 CF(J)=0
      L=0
      IF(NRD .EQ. 0) GO TO 17
      L=NRD
      D(1,1)=-1.0
      IF(NRD .EQ. 1) GO TO 8
      DO 4 J=2,NRD
    4 D(J,1)=D(J-1,1)-1.0
    8 DO 13 J=1,NRD
      DO 13 K=2,NRD
      IF(K .GT. J) GO TO 10
      D(J,K)=D(J-1,K)-D(J-1,K-1)
      GO TO 13
   10 D(J,K)=0
   13 CONTINUE
      DO 15 K=1,NRD
   15 C(K)=D(NRD,K)
   17 IF(NSD .EQ. 0) GO TO 43
      MAX=NSD*NSFA
      DO 20 J=1,NSD
      DO 20 K=1,MAX
```

```
20   DS(J,K)=0
     DS(1,NSEA)=-1.0
     MIN=2*NSEA
     IF(NSD .EQ. 1) GO TO 28
     DO 24 J=2,NSD
24   DS(J,NSEA)=DS(J-1,NSEA)-1.0
28   DO 33 J=1,NSD
     DO 33 K=MIN,MAX,NSEA
     IF((K/NSEA) .GT. J) GO TO 30
     DS(J,K)=DS(J-1,K)-DS(J-1,K-NSEA)
     GO TO 33
30   DS(J,K)=0
33   CONTINUE
     L=NSEA*NSD
     DO 37 M=1,NRD
     T(M)=T(M)+C(M)
     DO 37 J=NSEA,L,NSEA
     IF(M .EQ. 1)T(J)=T(J)+DS(NSD,J)
     N=J+M
37   T(N)=T(N)+C(M)*DS(NSD,J)
     L=L+NRD
     DO 40 J=1,L
     C(J)=T(J)
40   T(J)=0
43   MIN=1
     MAX=0
     DO 60 I=1,6
     IF(INC(I) .EQ. 0) GO TO 60
     MAX=MAX+INC(I)
     IF(I .EQ. 3 .OR. I .EQ. 4) GO TO 60
     DO 48 M=MIN,MAX
     K=IOPA(M)
     T(K)=T(K)-PA(M)
     IF(L .EQ. 0) GO TO 48
     DO 45 J=1,L
     IF(M .EQ. MIN .AND. I .LE. 2)T(J)=T(J)+C(J)
     IF(M .EQ. MIN .AND. I .EQ. 6)T(J)=T(J)+CF(J)
     N=J+IOPA(M)
     IF(I .EQ. 6)T(N)=T(N)-PA(M)*CF(J)
45   IF(I .LE. 2)T(N)=T(N)-PA(M)*C(J)
48   CONTINUE
     L=L+IOPA(MAX)
     DO 51 J=1,L
     IF(I .GE. 5)CF(J)=T(J)
     IF(I .LE. 2)C(J)=T(J)
51   T(J)=0
     IF(I .EQ. 2)L=0
60   MIN=MIN+INC(I)
     DO 66 J=1,MRO
     CF(J)=-CF(J)
66   C(J)=-C(J)
     KK=INC(1)+INC(2)+INC(3)
     DO 70 J=1,NDR
     W(J)=Z(J)
70   IF(INC(3) .EQ. 1)Z(J)=Z(J)-PA(KK)
     KK=KK+INC(4)
```

```
      IF(INC(4) .EQ. 1)CONST=PA(KK)
      IF(INC(4) .EQ. 0)CONST=0
      DO 79 K=MAX11,NDR
      A(K)=Z(K)-CONST
      IF(K .GT. MBO) GO TO 76
      KK=K-1
      DO 74 J=1,KK
   74 A(K)=A(K)-C(J)*Z(K-J)+CF(J)*A(K-J)
      GO TO 79
   76 DO 78 J=1,MBO
   78 A(K)=A(K)-C(J)*Z(K-J)+CF(J)*A(K-J)
   79 CONTINUE
      DO 81 J=1,NDR
   81 Z(J)=W(J)
      MAX1=MAX11-1
      DO 83 J=1,NOB
      LL=J+MAX1
   83 F(J)=Z(LL)-A(LL)
      RETURN
      END

C        SUBROUTINE  3

      SUBROUTINE ACOR (Z,KK,N,ND,R,STSR,E,MCE,MP)
      DIMENSION Z(999),C(101),R(101),T(50,50),VAR(101),STSR(101),
     1VART(50),STSP(50),U(50),S(101),F(5)
      COMMON /TWO/NDR,NRD,NSD,NSFA
      ZBAR=0.
      XN=N
      DO 102 I=1,N
  102 ZBAR=ZBAR+Z(I)
      ZBAR=ZBAR/XN
      F(1)=ZBAR
      CO=0.
      DO 103 I=1,N
  103 CO=CO+(Z(I)-ZBAR)**2
C     CALCULATION OF R
      DO 104 K=1,KK
      C(K)=0.
      NN=N-K
      DO 105 J=1,NN
  105 C(K)=C(K)+(Z(J)-ZBAR)*(Z(J+K)-ZBAR)
      R(K)=C(K)/CO
  104 CONTINUE
C     CALCULATION OF T
      IF(KK-50)106,106,107
  107 KKK=50-1
      GO TO 109
  106 KKK=KK-1
  109 T(1,1)=R(1)
      T(2,2)=(R(2)-R(1)**2)/(1.-R(1)**2)
      T(2,1)=T(1,1)-T(2,2)*T(1,1)
      DO 203 K=2,KKK
      Q=0.
      A=0.
      DO 202 J=1,K
```

```
      A=A+T(K,J)*R(K+1-J)
 202  B=B+T(K,J)*R(J)
      A=R(K+1)-A
      B=1.-B
      T(K+1,K+1)=A/B
      DO 203 J=1,K
 203  T(K+1,J)=T(K,J)-T(K+1,K+1)*T(K,K-J+1)
C     CALCULATION OF VAR AND VART
      VAR(1)=1./XN
      STSR(1)=SORT(VAR(1))
      S(1)=R(1)/STSR(1)
      A=2./XN
      DO 204 K=2,KK
      VAR(K)=VAR(K-1)+A*(R(K-1)**2)
      STSR(K)=SORT(VAR(K))
      S(K)=R(K)/STSR(K)
 204  CONTINUF
      KKK=KKK+1
      DO 205 K=1,KKK
      A=1./(N-K)
      VART(K)=A
      STSP(K)=SORT(A)
      U(K)=T(K,K)/STSP(K)
 205  CONTINUF
C     PRINT OUT
      PRINT 300
 300  FORMAT (1H1///10X,36H SAMPLE AUTOCORRELATION COEFFICIENTS//)
      PRINT 301
 301  FORMAT (20X,17H AUTO-COEFFICIENT ,5X,20H UNIFIED COEFFICIENT  /)
      PRINT 302, (I,R(I),S(I),I=1,KK)
 302  FORMAT (15X,I2,F15.4,9X,F15.4)
      CHISQ=0.
      DO  400  I=1,30
 400  CHISQ=CHISQ+R(I)*R(I)
      XXN=NDR-NRD-NSD*NSFA
      CHISQ=CHISQ*XXN
      PRINT  401,  CHISQ
 401  FORMAT(//,5X,'CHI SQUARE STATISTIC BASED ON 30 AUTOCORRELATIONS ='
     X,F12.5,//)
      CO=CO/XN
      PRINT 303, ZBAR,CO
 303  FORMAT( 10X,19H MEAN OF THE SERIES  ,F14.4/ 10X,
     123H VARIANCE OF THE SERIES  ,F10.4)
      PRINT 304
 304  FORMAT (1H1///10X,27H SAMPLE PARTIAL CORRELATION   //)
      PRINT 305
 305  FORMAT (20X,17HPART-CORRELATION  ,5X,20H UNIFIED COEFFICIENT/)
      PRINT 302, (I, T(I,I), U(I), I=1,KKK)
      RETURN
      END
```