ATOC 5051 INTRODUCTION TO PHYSICAL OCEANOGRAPHY

Lecture 19

Learning objectives - understand forcing & processes for Ekman currents

1. Surface Ekman layer & forces of balance (continue)
2. Wind-driven Ekman spiral, Ekman transport and coastal & equatorial upwelling
3. Open Ocean Ekman pumping
4. Bottom boundary layer

Context of this course

- Seawater properties
- Observational method & observed ocean circulation
- Equations of motion & scale analysis
- Wave dynamics
- Static & dynamical instabilities – mixing

- Wind-driven Ekman current

a) Balance of forces in the Ekman layer;
b) Obtain solutions for Ekman spiral, Ekman transport & Ekman pumping
c) Using Ekman dynamics to explain observations

1. : Previous class: The Ekman layer

Fridtjof Nansen: *Icebergs move 20-40° to the right of wind*

Walfrid Ekman (1905) :
Explained Nansen's observation –
effect of wind & Coriolis force

The ocean ***surface boundary layer -*** *sometimes referred to as Ekman layer – is subject to direct wind forcing.*

Ekman layer thickness:

$$H_{Ekman} = \sqrt{\frac{2A_z}{f}}$$

Eddy viscosity (A_z)

Coriolis parameter (f)

Large-scale: $R_o \ll 1$

$E_x \ll 1$ & $E_y \ll 1$

$E_z \sim 1$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + A_x\frac{\partial^2 u}{\partial^2 x} + A_y\frac{\partial^2 u}{\partial^2 y} + A_z\frac{\partial^2 u}{\partial^2 z}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + A_x\frac{\partial^2 v}{\partial^2 x} + A_y\frac{\partial^2 v}{\partial^2 y} + A_z\frac{\partial^2 v}{\partial^2 z}$$

Ocean surface layer: subject to direct wind forcing

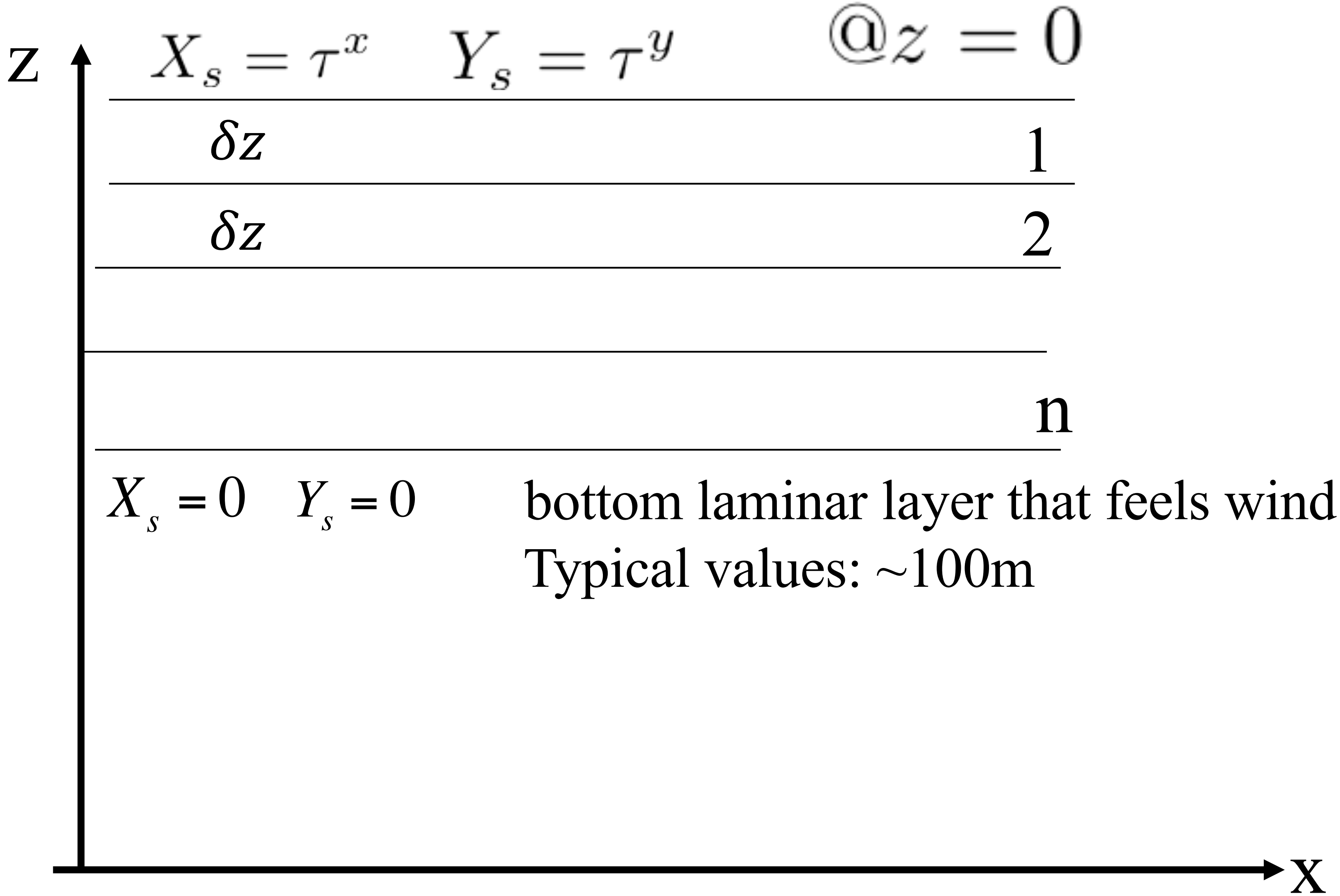
Steady state - steady wind forcing: $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0$

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + A_z\frac{\partial^2 u}{\partial^2 z} = 0$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + A_z\frac{\partial^2 v}{\partial^2 z} = 0$$

Previous class:

Surface wind exerts stress, (τ^x, τ^y) N/m^2 forces ocean



Important: the ocean is viscous; stress linearly decreasing with depth (constant viscosity, laminar flow: non-turbulent).

Previous class:

$$X_s = \tau^x \quad Y_s = \tau^y \qquad @z = 0$$

δz 1

2

…

Unit area: stress at bottom of layer 1 (top of layer 2):

$$\tau^x - \delta z \frac{\partial X}{\partial z}, \tau^y - \delta z \frac{\partial Y}{\partial z}$$

Net Force stress for unit area for layer 1:

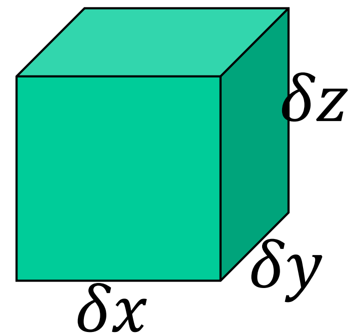
$$\tau^x - \left(\tau^x - \delta z \frac{\partial X}{\partial z}\right) = \delta z \frac{\partial X}{\partial z};$$

$$\tau^y - \left(\tau^y - \delta z \frac{\partial Y}{\partial z}\right) = \delta z \frac{\partial Y}{\partial z};$$

Also true for any Laminar layer

Net stress in x, y directions:

$$\delta z \frac{\partial X}{\partial z}, \delta z \frac{\partial Y}{\partial z}$$



Force for unit mass:

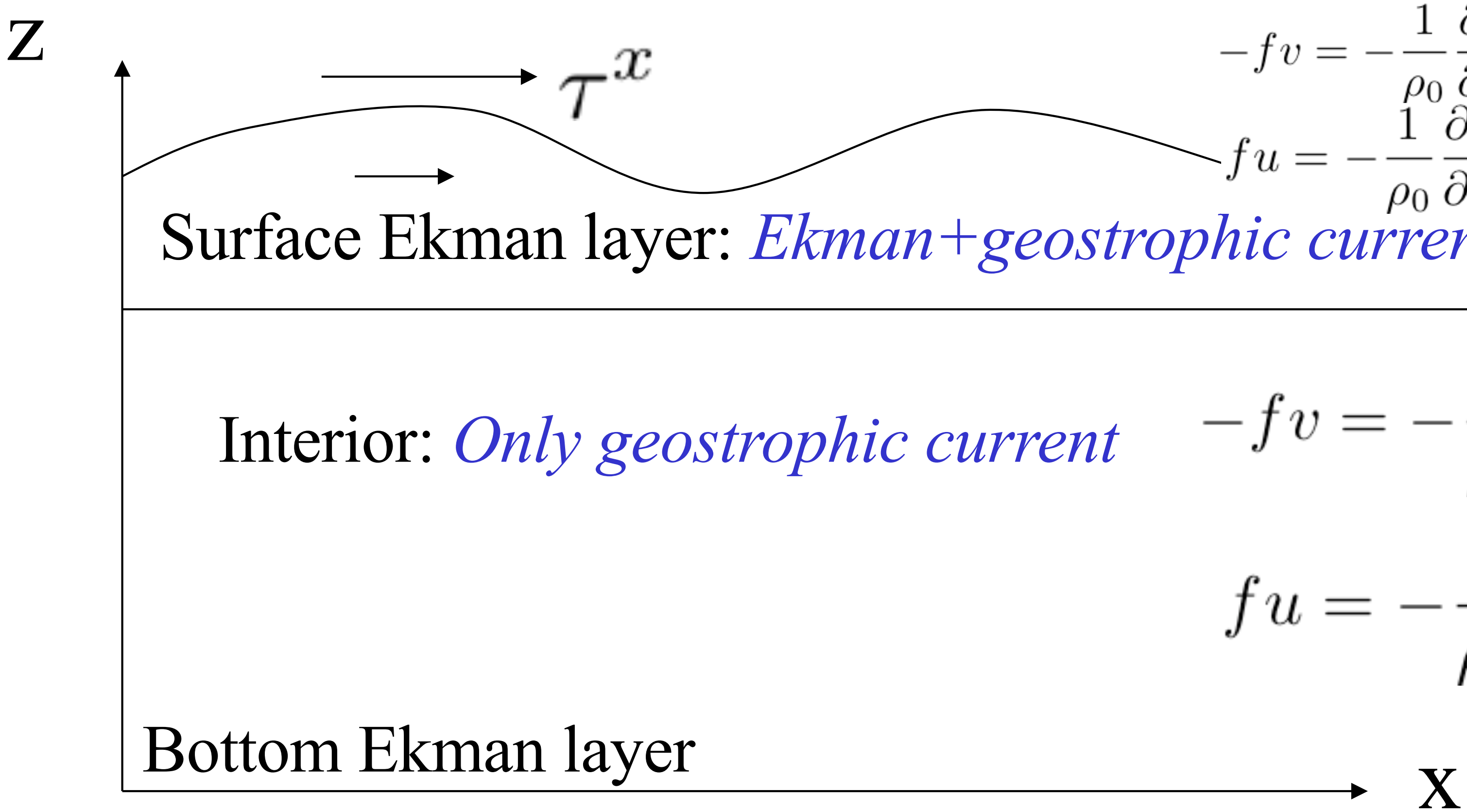
$$\delta x \delta y \delta z \left(\frac{\partial X}{\partial z}, \frac{\partial Y}{\partial z}\right) / \rho \delta x \delta y \delta z = \frac{1}{\rho}\left(\frac{\partial X}{\partial z}, \frac{\partial Y}{\partial z}\right)$$

Small Rossby number (Ro<<1), Ex~Ey<<1, **steady state**, **constant density**: the equations of motion in Ekman layer are:

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial X}{\partial z},$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{1}{\rho_0}\frac{\partial Y}{\partial z}.$$

Stress X, Y decreases quickly with depth, their direct influence is felt only in the surface boundary layer.



$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\frac{\partial X}{\partial z},$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{1}{\rho_0}\frac{\partial Y}{\partial z}.$$

$$-fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

$$fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y}$$

Because $$-fv_g = -\frac{1}{\rho_0}\frac{\partial p}{\partial x},$$

$$fu_g = -\frac{1}{\rho_0}\frac{\partial p}{\partial y},$$

$\longrightarrow$ $$-fv = -fv_g + \frac{1}{\rho_0}\frac{\partial X}{\partial z},$$

$$fu = fu_g + \frac{1}{\rho_0}\frac{\partial Y}{\partial z}.$$

Thus, $$-fv_E = -f(v - v_g) = \frac{1}{\rho_0}\frac{\partial X}{\partial z},$$

$$fu_E = f(u - u_g) = \frac{1}{\rho_0}\frac{\partial Y}{\partial z},$$

Ekman current; Ekman flow

2. Ekman Spiral: Ekman flow within the surface Ekman layer

Vertical structure of flow within the Ekman layer

Ekman 1905: a simple wind-driven ocean model

Assumptions: viscous, laminar boundary layer

$$R_{ex} = \frac{inertial}{viscous} = \frac{UL}{A_x}$$

Non-turbulent (Re is small)

Ekman assumed: internal stress is balanced by viscosity

$$\frac{1}{\rho}(X,Y) = A_z(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}),$$

Recall that in the Ekman layer:

$$-fv_E = -f(v - v_g) = \frac{1}{\rho_0}\frac{\partial X}{\partial z},$$
$$fu_E = f(u - u_g) = \frac{1}{\rho_0}\frac{\partial Y}{\partial z},$$

Applying $\frac{1}{\rho}(X, Y) = A_z(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}),$

We have:

$$-fv_E = A_z\frac{\partial^2 u_E}{\partial z^2}, \qquad (1a)$$
$$fu_E = A_z\frac{\partial^2 v_E}{\partial z^2}. \qquad (1b)$$

Using boundary conditions:

$u_E = v_E = 0$ as $z \sim -\infty$

$$A_z \frac{\partial u}{\partial z} = \frac{1}{\rho}\tau^x$$

and

$$A_z \frac{\partial v}{\partial z} = \frac{1}{\rho}\tau^y$$

at z=0,

Equations

$$-f v_E = A_z \frac{\partial^2 u_E}{\partial z^2},$$

$$f u_E = A_z \frac{\partial^2 v_E}{\partial z^2}.$$

yield:

$$u_E = \frac{e^{\sqrt{f/2A_z}z}}{\rho\sqrt{2fA_z}}[(\tau^x + \tau^y)cos(\sqrt{f/2A_z})z + (\tau^x - \tau^y)sin(\sqrt{f/2A_z})z],$$
$$v_E = \frac{e^{\sqrt{f/2A_z}z}}{\rho\sqrt{2fA_z}}[(\tau^y - \tau^x)cos(\sqrt{f/2A_z})z + (\tau^x + \tau^y)sin(\sqrt{f/2A_z})z].$$

Important features:

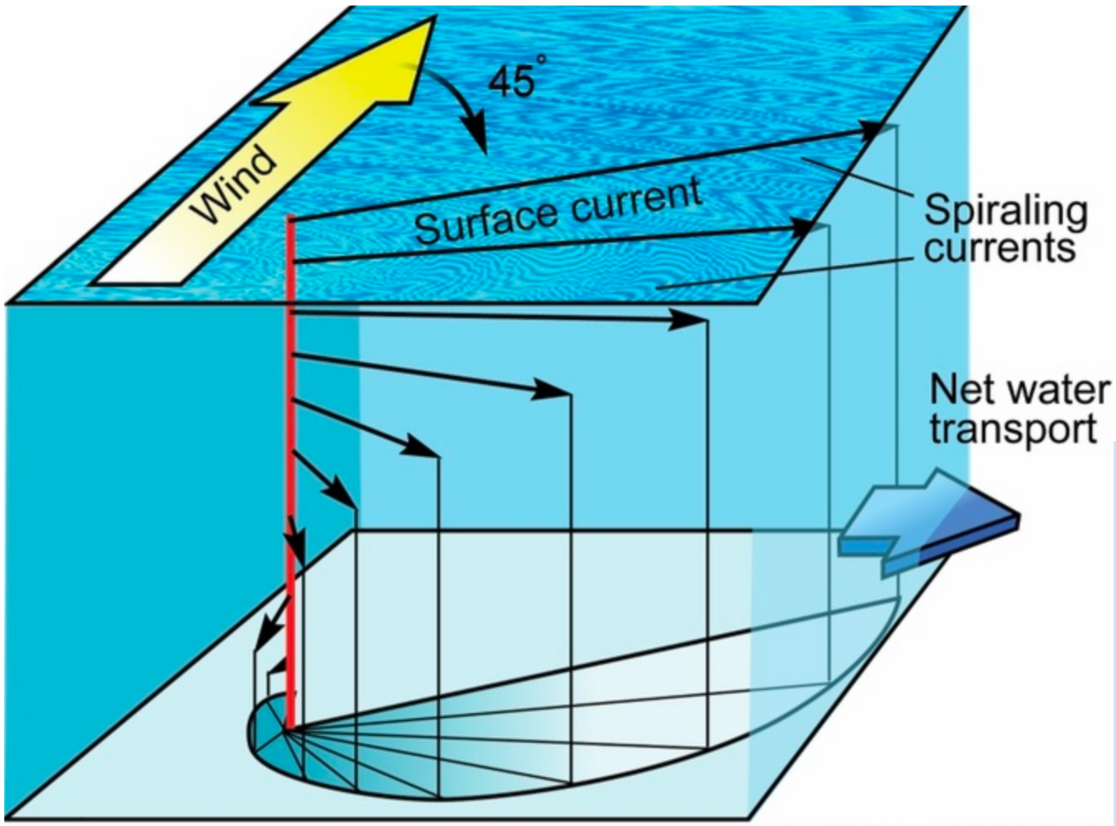
a) The Ekman layer thickness is $H_{Ekman} = \sqrt{\frac{2A_z}{f}}$

Which is the e-folding decay scale of Ekman flow.
The stronger the viscosity, the thicker the Ekman layer.

b) The flow in the Ekman layer is not in geostrophic balance because viscosity is important.

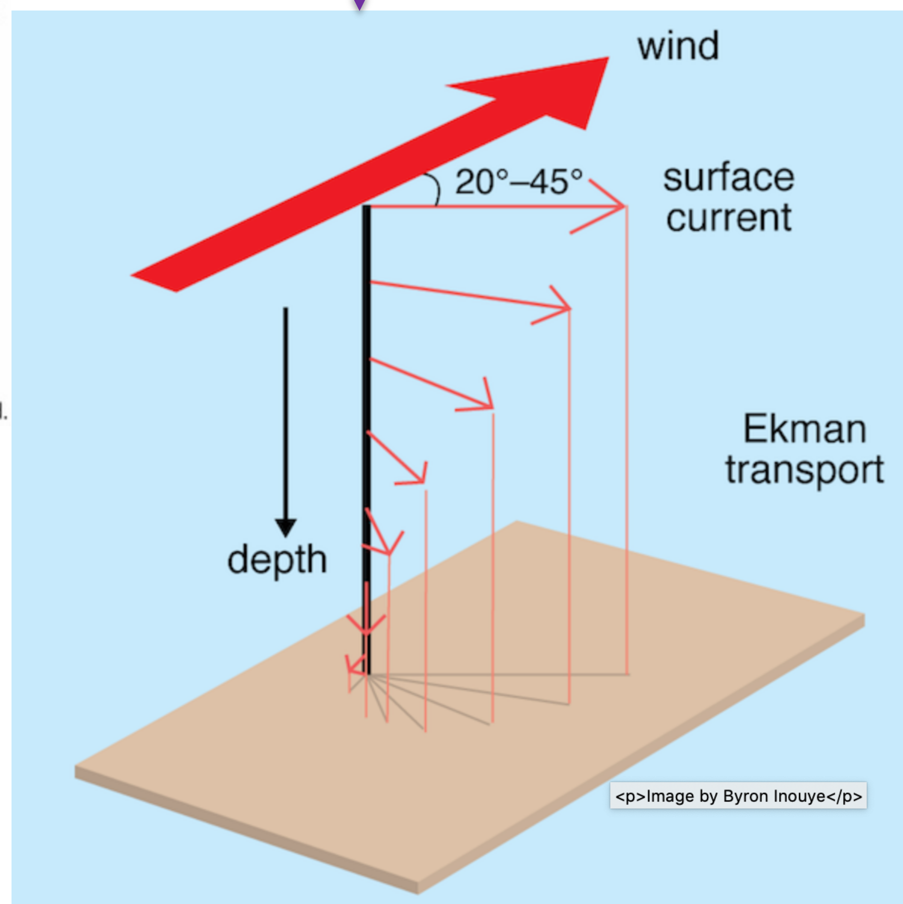
c) As we shall see below: Ekman transport is the vertical integral of Ekman spiral – 90deg to the right (left) of wind in Northern Hemisphere (Southern Hemisphere).

Ekman spiral



Steady wind forcing on infinitely deep ocean

Nansen: 20-40°: depends on how long wind lasts & how deep ocean is (smaller angle for shallow sea)



Ekman spiral is very difficult to observe in real ocean. Why?

Ekman Transport

Vertically integrate Ekman flow in the entire Ekman layer:

$$-fv_E = -f(v - v_g) = \frac{1}{\rho_0}\frac{\partial X}{\partial z},$$

$$fu_E = f(u - u_g) = \frac{1}{\rho_0}\frac{\partial Y}{\partial z},$$

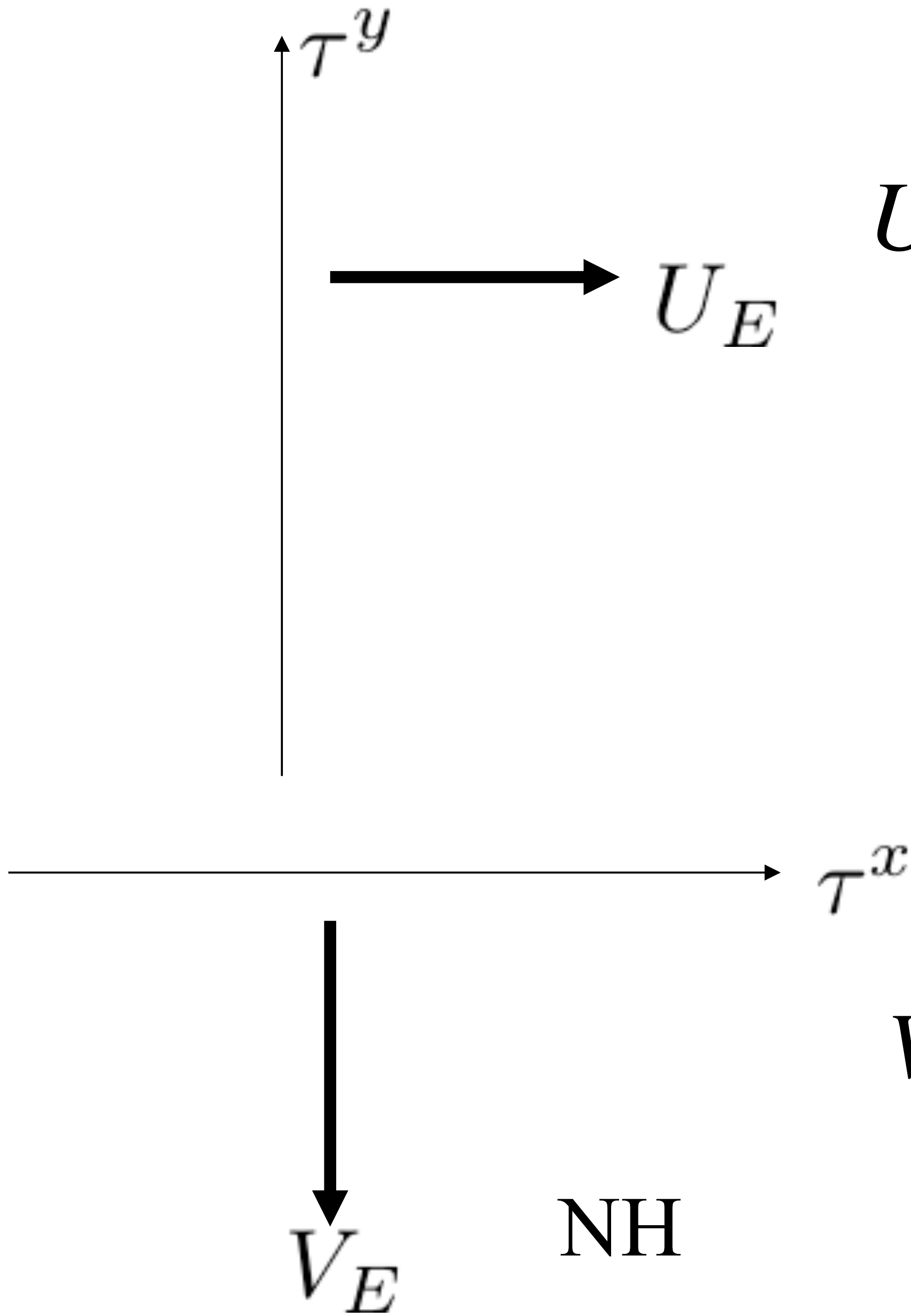
Using boundary conditions:

$$(X, Y) = (\tau^x, \tau^y) @ z = 0, \quad (X, Y) = 0 @ z = -H_E,$$

$$(u_E, v_E) = 0 @ z = -H_E.$$

$$U_E = \int_{-H_E}^{0} u_E\, dz = \frac{\tau^y}{\rho_0 f},$$

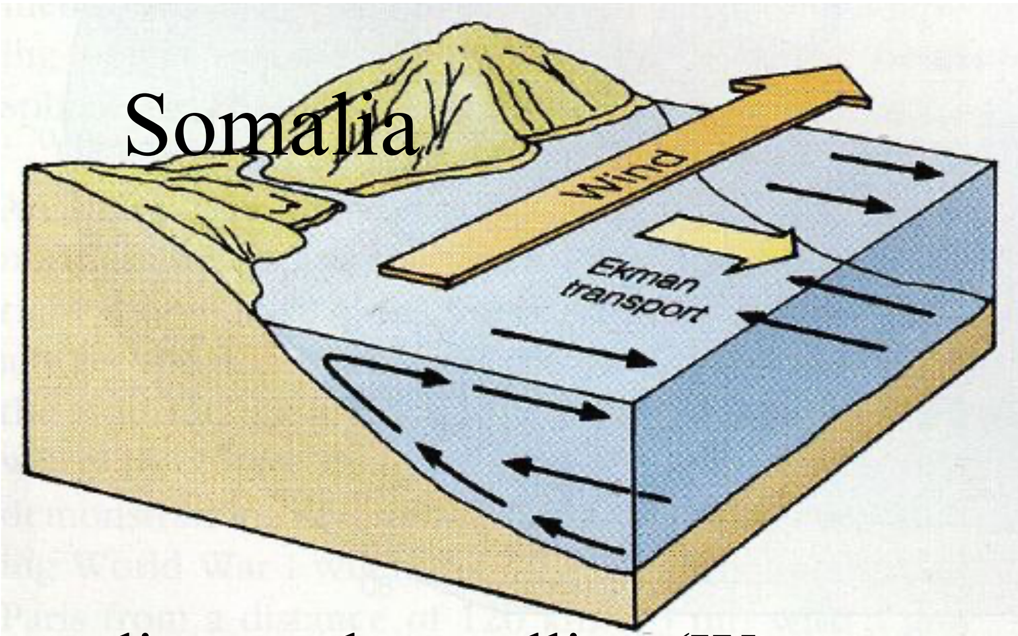
$$V_E = \int_{-H_E}^{0} v_E\, dz = -\frac{\tau^x}{\rho_0 f}$$



$$U_E = \int_{-H_E}^{0} u_E \, dz = \frac{\tau^y}{\rho_0 f},$$

$$V_E = \int_{-H_E}^{0} v_E \, dz = -\frac{\tau^x}{\rho_0 f}$$

Ekman transport & upwelling



$$U_E = \int_{-H_{mix}}^{0} u_E dz = \frac{\tau^y}{\rho_0 f},$$

$$V_E = \int_{-H_{mix}}^{0} v_E dz = -\frac{\tau^x}{\rho_0 f}.$$

Somali coastal upwelling (Western Indian Ocean summer monsoon)

Do you expect colder or warming SST along Somali coast?

Ekman transport: Coastal Ekman divergence

- Coastal upwelling; Marine life. *This does not require the wind to have curl*

Units: Ekman current & volume transport

u_E, v_E : m/s, speed

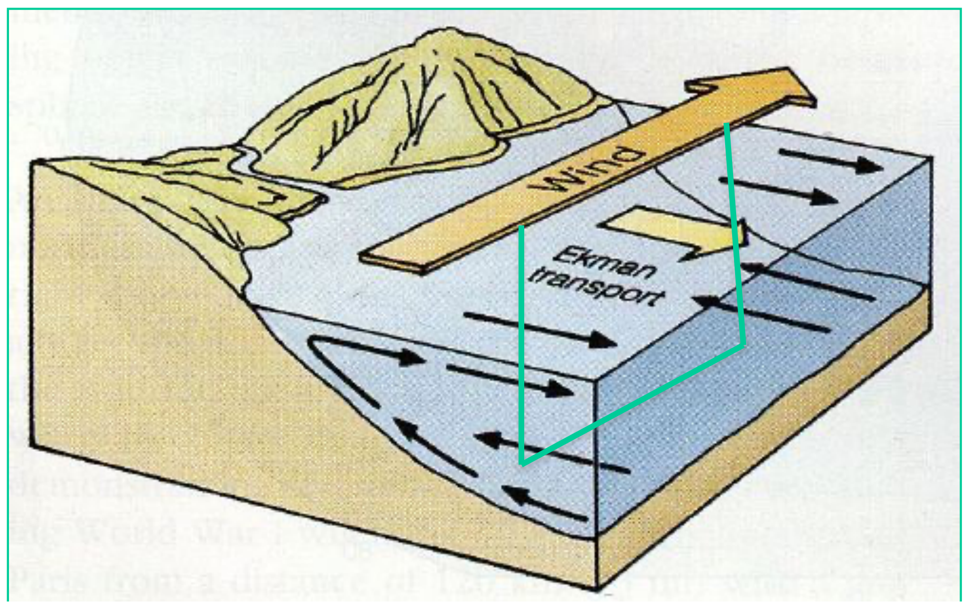
Ekman transport at a specific (lon,lat): m^2/s

$$U_E = \int_{-H_{mix}}^{0} u_E dz = \frac{\tau^y}{\rho_0 f}, \qquad V_E = \int_{-H_{mix}}^{0} v_E dz = -\frac{\tau^x}{\rho_0 f}.$$

Usually, in research: we calculate Ekman transport across an area (e.g., along 50E, 5N-10N) within the mixed layer Hmix. Then, Ekman transport is:

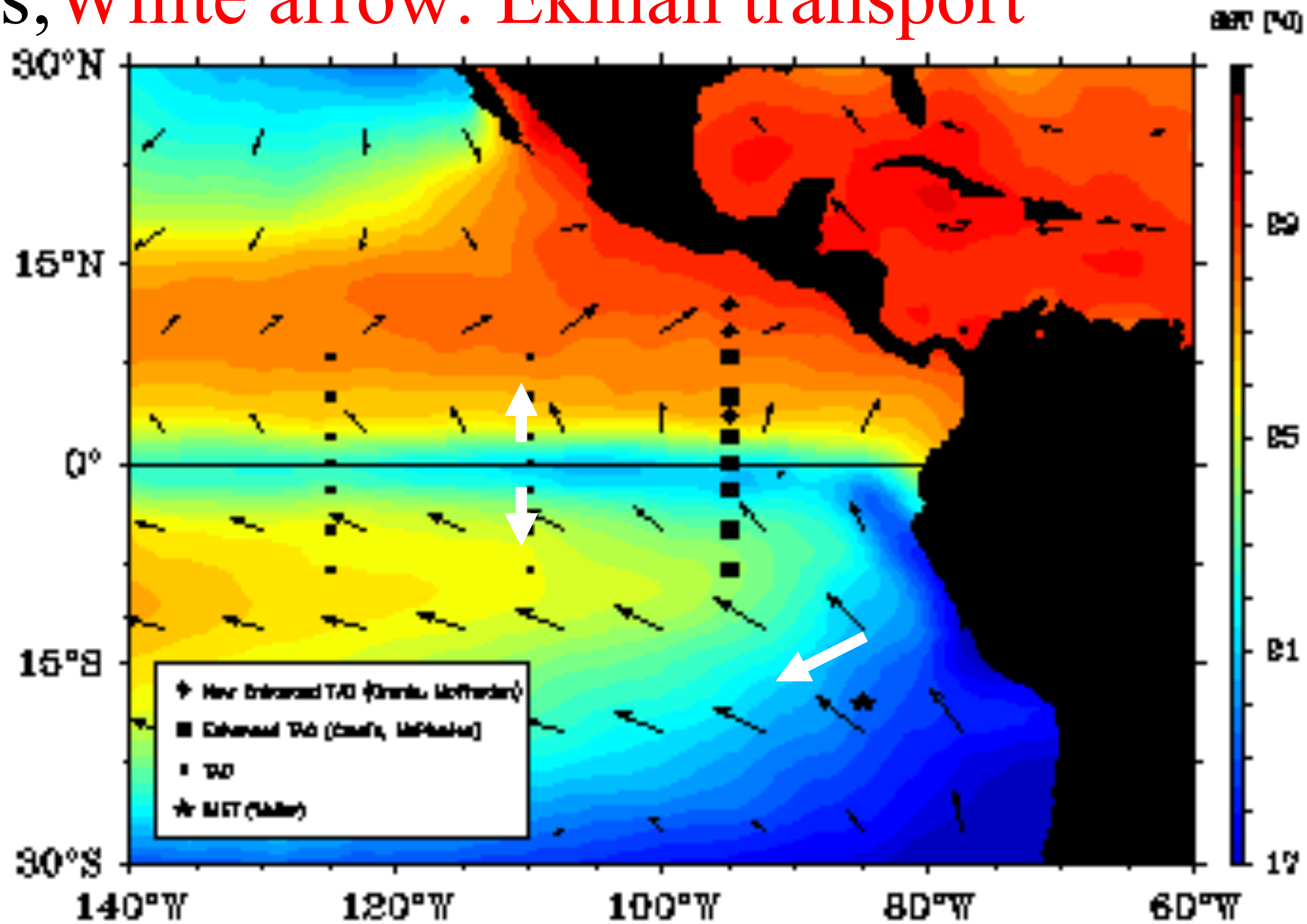
$U_E \times Ly$: m^3/s

1 Sverdrup(sv) = $10^6 m^3/s$



TAO data in the eastern Pacific: Color: SST; black arrow: winds; White arrow: Ekman transport

Coastal & Equatorial Upwelling!

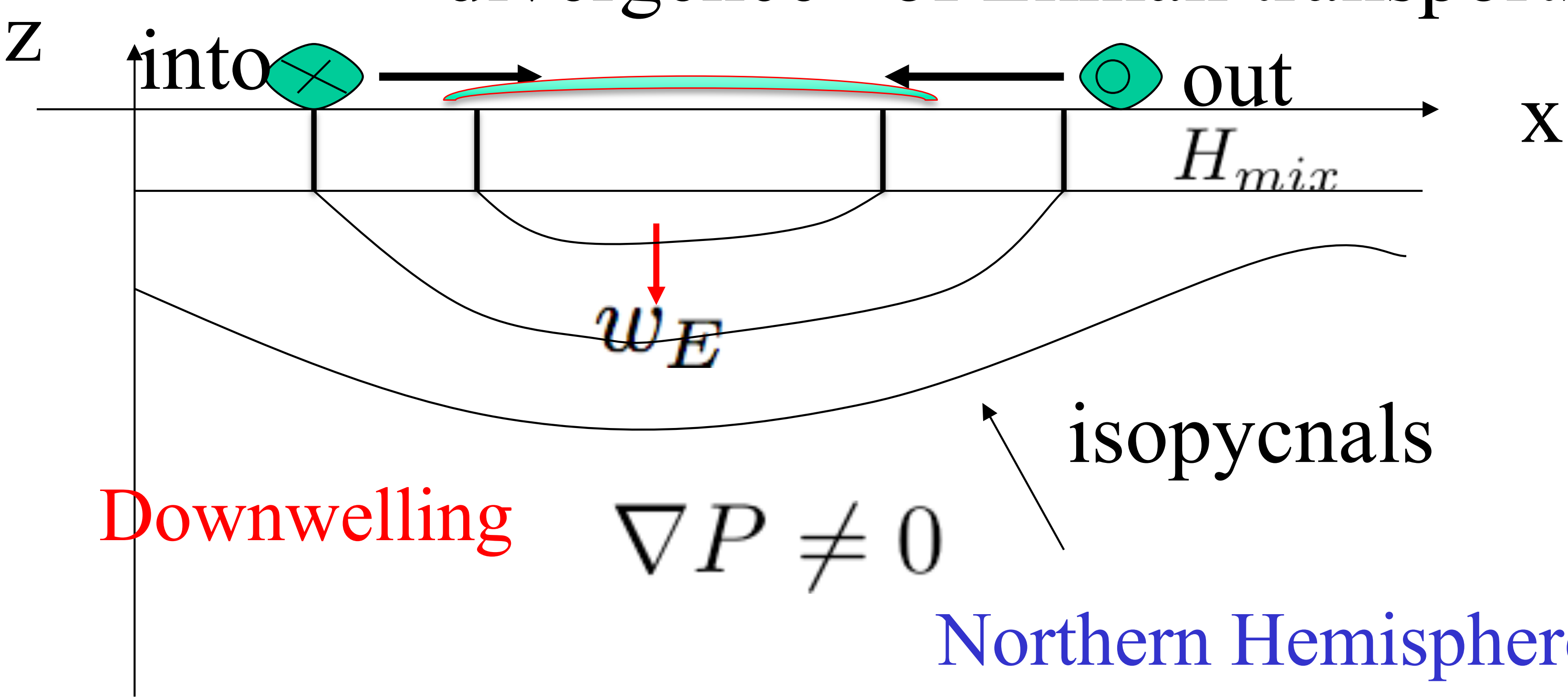


3. Open ocean Ekman pumping

Interaction between the surface Ekman layer and the interior ocean beneath

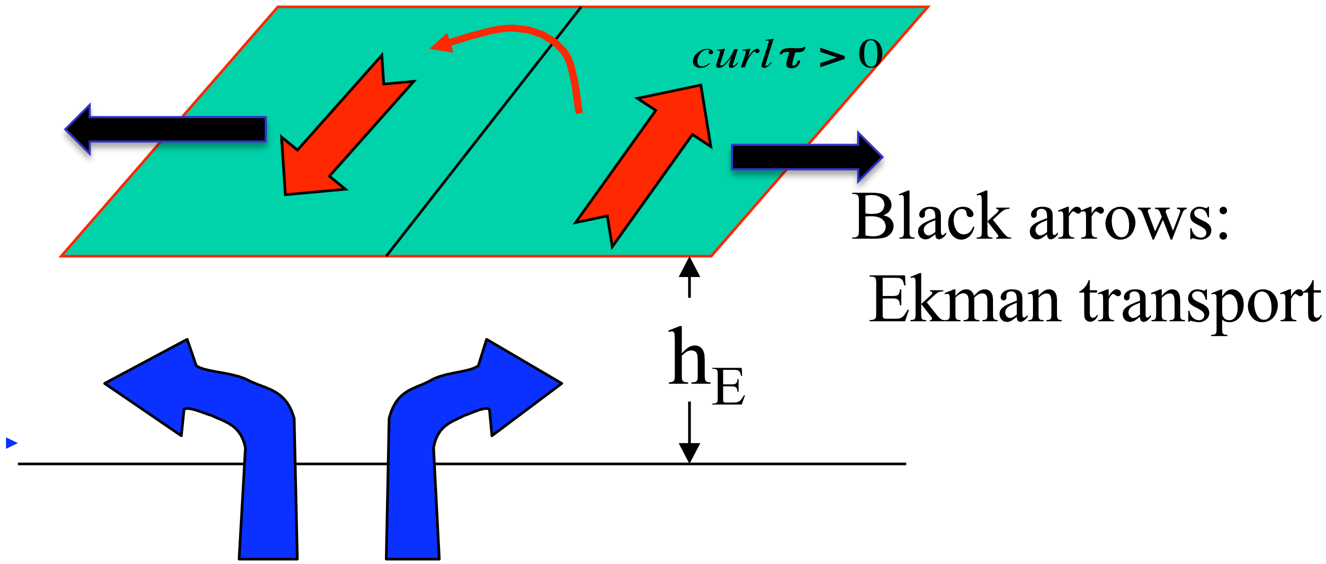
Surface windstress (τ^x, τ^y) varies spatially,

→ Producing "convergence" or "divergence" of Ekman transports



Northern Hemisphere (NH)

Red bulk arrow: Wind stress



w_E Ekman pumping velocity

Positive w_E *can induce upwelling in the open ocean!*

Ekman convergence (divergence) causes ispycnals to move down (up), generating horizontal pressure gradient force in the "ocean interior" below the Ekman layer, and thus driving the deep ocean in motion. Within the surface mixed layer, they cause both Ekman flow and geostrophic currents (the sum of the two). Note: this is in a linear system.

Ekman pumping can be clearly demonstrated by integrating the continuity equation:

$$u_x + v_y + w_z = 0$$

Or: $$u_{Ex} + u_{gx} + v_{Ey} + v_{gy} + w_z = 0$$

Because $$u_{gx} + v_{gy} = 0$$

We have $$u_{Ex} + v_{Ey} + w_{Ez} = 0$$

$$\longrightarrow \quad \int_{-H_E}^{0} u_{Ex} + v_{Ey} + w_{Ez}\, dz = 0$$

$$w_E = U_{Ex} + V_{Ey}$$ ($w_E = 0 @ z = 0$ is used in a baroclinic ocean)

At $z = -H_E$

Ekman transport convergence/divergence

$$w_E = U_{Ex} + V_{Ey}$$

$$U_E = \int_{-H_E}^{0} u_E \, dz = \frac{\tau^y}{\rho_0 f},$$

$$V_E = \int_{-H_E}^{0} v_E \, dz = -\frac{\tau^x}{\rho_0 f}$$

Constant ρ_0: Boussinesq approximation

So,

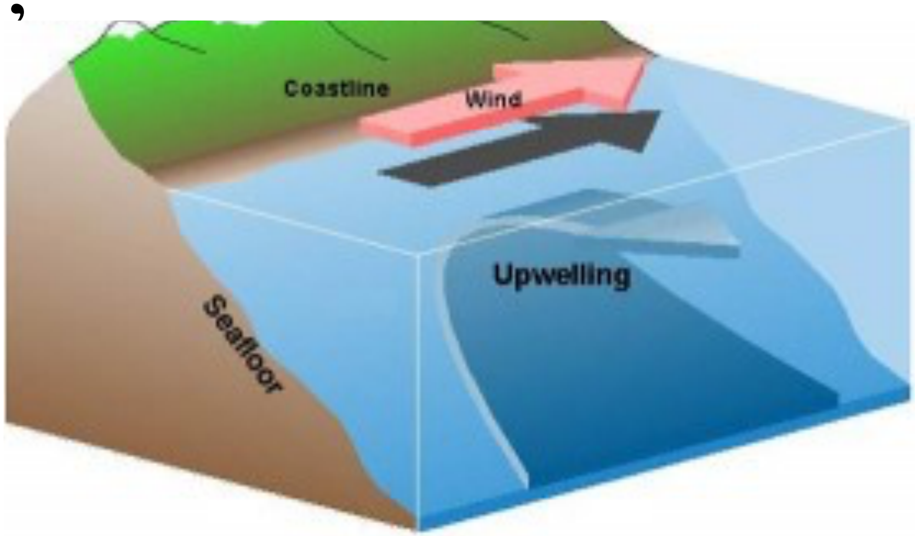
$$w_E = \frac{\partial}{\partial x}\left(\frac{\tau^y}{\rho f}\right) - \frac{\partial}{\partial y}\left(\frac{\tau^x}{\rho f}\right)$$

This expression is valid for both constant and varying density.

Open ocean: wind stress curl is NEEDED!

Coastal upwelling: favorable longshore wind is needed - do not need wind stress curl

- Physical description: Alongshore winds – offshore surface Ekman divergence – coastal upwelling (colder, subsurface water upwells to the surface layer);
- Mathematics: $U_{Ex} + V_{Ey} > 0,$



Vertical velocity at mixed layer bottom:

$$w_E = U_{Ex} + V_{Ey} > 0,$$

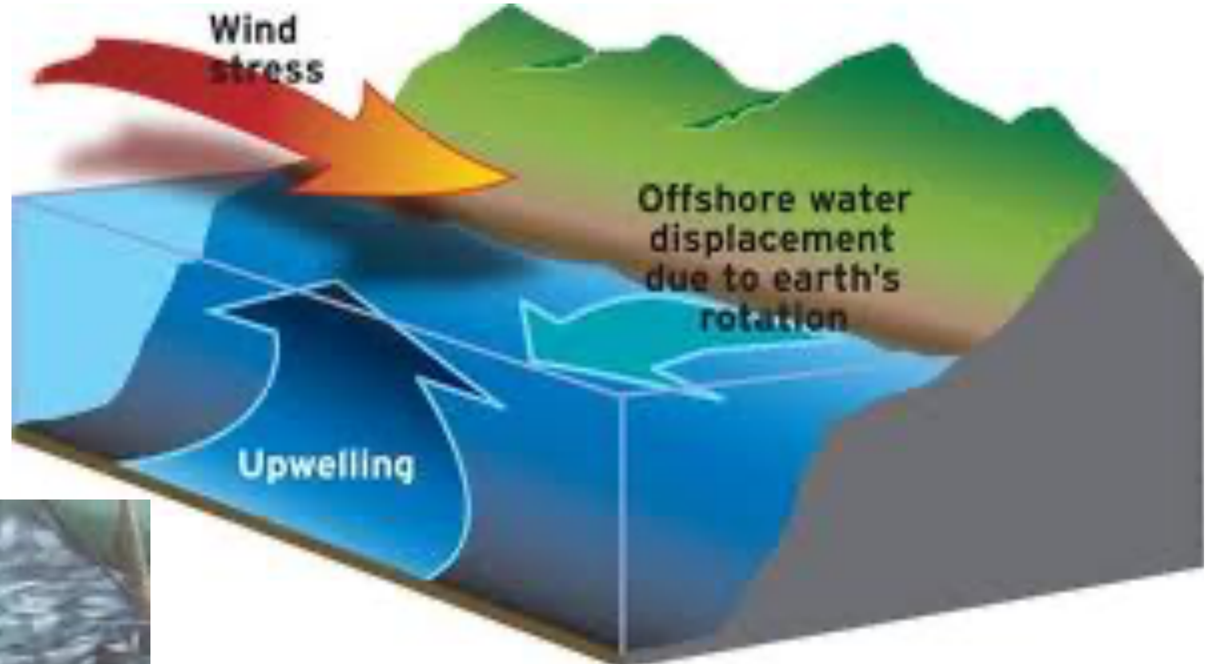
upwelling.

The above equation comes from:

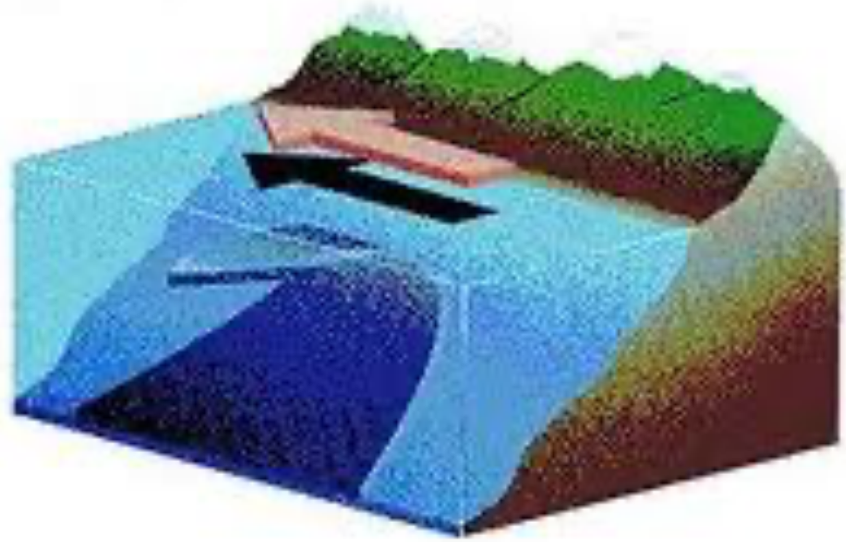
$$u_{Ex} + v_{Ey} + w_{Ez} = 0$$

$$\int_{H_{mix}}^{0} u_{Ex} + v_{Ey} + w_{Ez} dz = 0 \quad w_E = 0 @ z = 0$$

Northeast Pacific & Atlantic: ***Wind-driven Coastal upwelling, fishery***

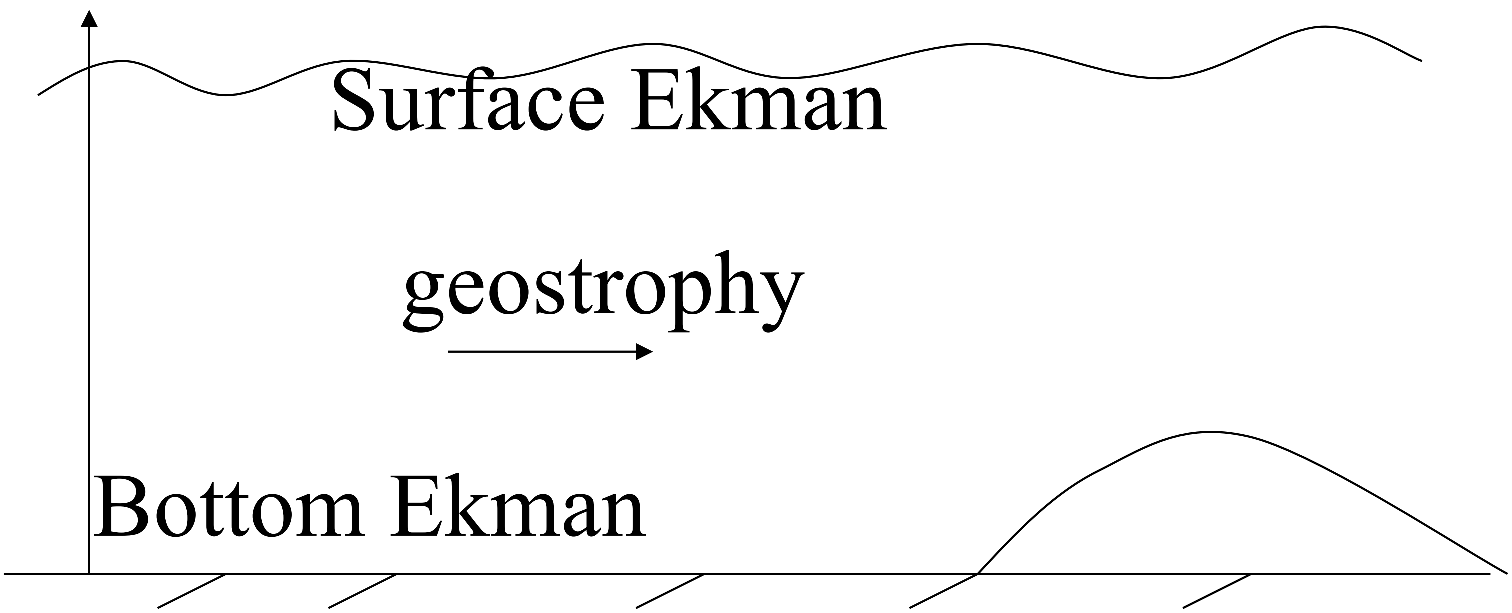


Southeast Pacific & Atlantic



*Although coastal upwelling regions account for only **1%** of the ocean surface, they contribute roughly **50%** of the world's fisheries landings.*

4. Bottom Ekman layer



Bottom drag due to roughness and torques due to bathymetry can affect fluid motion;

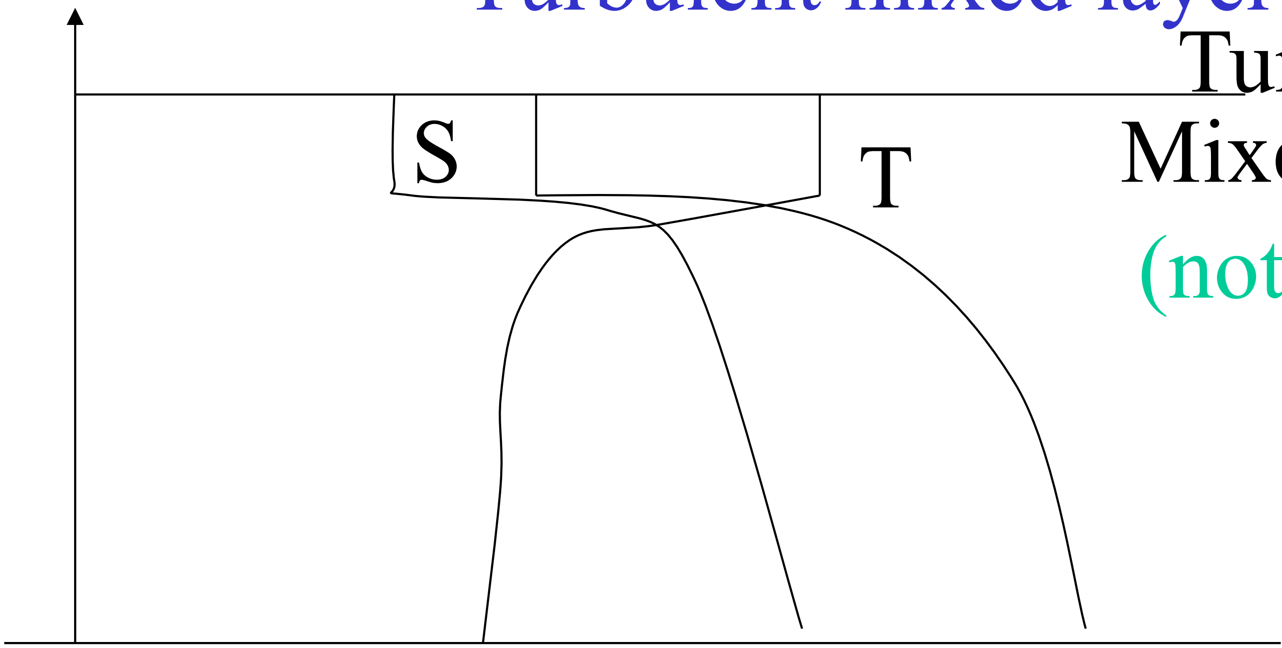
Currents slow down or use: $u = v = 0$.

No slip boundary condition

For a flat bottom ocean, when the interior geostrophic flow approaches the bottom, it is slown down by the bottom drag. Following similar procedure, we can obtain bottom Ekman layer thickness:

$$\delta_e = \sqrt{\frac{2A_z}{f}}$$

Turbulent mixed layer



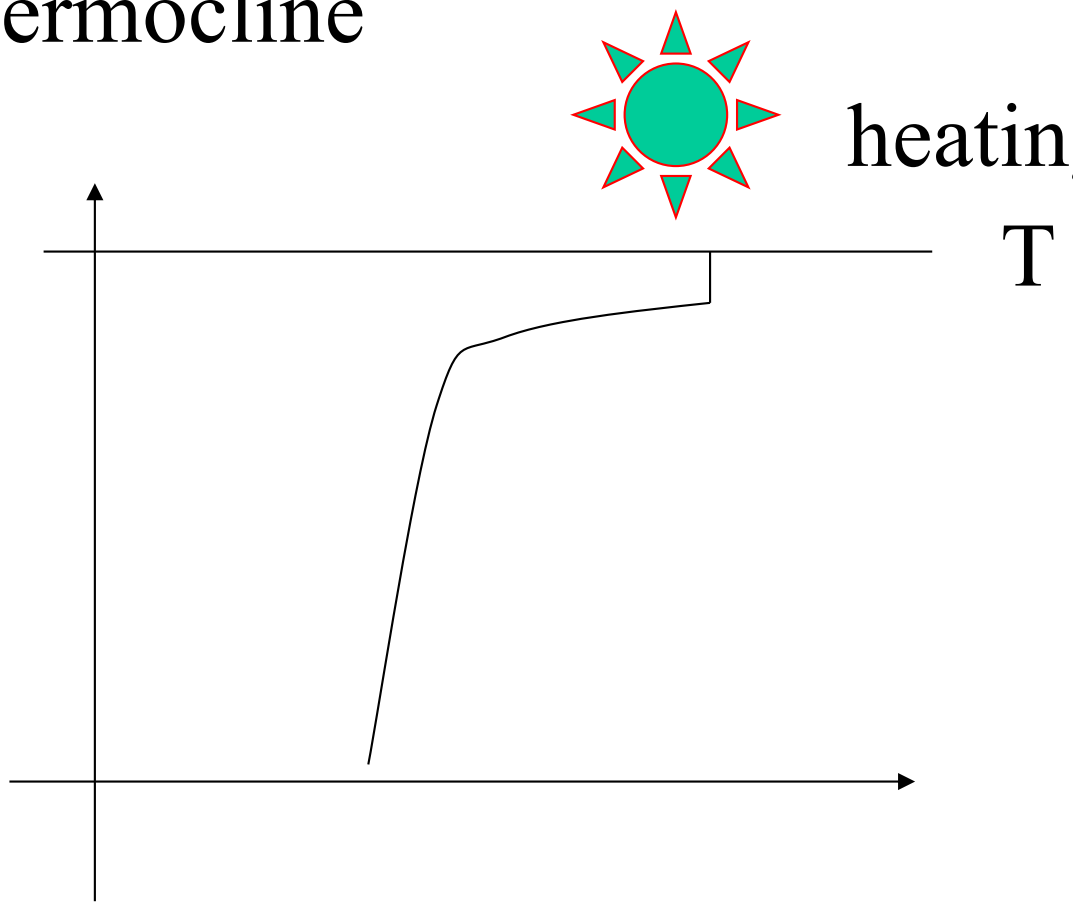
Turbulent
Mixed layer
(not laminar!)

Definition of mixed layer depth Hmix (often used by researchers): The depth at where T decreases by 0.5C from the SST; or density increases by a value that is equivalent of 0.5C decrease.

Vertical mixing processes can be affected by:

Wind mechanical stirring (Kraus-Turner
Mixed layer physics: Kraus and Turner 1967);
Surface cooling that weakens stratification;
Shear instabilities (K-H), baroclinic instabilities

a) Thermocline



Thermocline theory: Pedlosky 1987. Wind-driven ocean circulation: Won't be covered in this class.