

# 16 PARALLAX

## OBJECTIVE

1. To become familiar with the basic technique of measuring stellar distances in units of parsec using heliocentric parallax.
2. To apply this method to simulated observations of the heliocentric parallax of stars to measure their distances from the solar system.

## PREPARATION

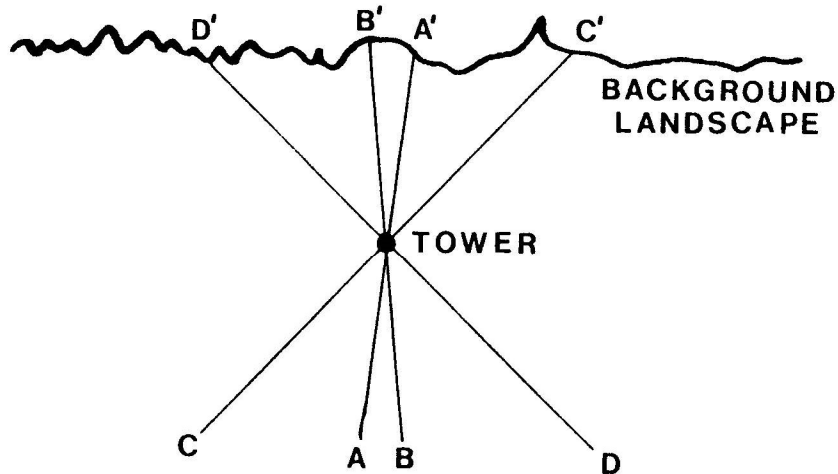
In our everyday lives we estimate distances of objects nearly automatically based on how large or small they appear compared to our knowledge of their actual size. While driving, for example, knowing that similar models of cars have about the same size, we estimate that a car which looks half as big as another car is twice as far away, since most automobiles are roughly the same size. Our minds convert our knowledge of the linear size of an object and our visual perception of its angular size into a relative distance. This method works well for familiar objects that are not very far away. It does NOT work for stars because, in spite of being extremely large objects, they are so incredibly far away from us that we can only see them as point light sources even through very powerful telescopes.

There is, however, another everyday method for determining distances of objects that does not depend on how their actual size changes with changing distance. Instead, it works by how much objects in the foreground appear to shift their position with respect to a distant background while you are moving past them. You experience this every time you drive in a car and look out your window. The objects you pass that are closer to your car, such as shrubs at the side of the road, street signs, or cars in a parking lot in front of buildings, go through a much larger shift in position with respect to distant mountains or the Moon in the sky than objects a little further away. This apparent shift of nearby objects in front of a distant background when viewed from two different observing locations is called *parallax*.

The simplest way for you to see parallax with your own eyes is to use your left eye and right eye as the two different observing locations (aka *vantage points*) by keeping one eye shut while looking through the other and then switching eyes. Use the wall in a room as the distant background and your thumb on your fully extended arm in front of you as the nearby object. Look at your thumb with one eye while the other is shut and note which location on the wall your thumb appears to be directly in front of. Switch eyes and repeat. Note how your thumb appears to be directly in front of a slightly different location on the wall! This apparent shift along the wall is due to the parallax angle of your eyes and thumb. Watch how the parallax changes with the distance of the object. Move your thumb closer to your face by bending your arm a little and repeat the observations looking at your thumb with alternating eyes. When the thumb is closer its parallax angle is larger which increases its apparent shift along the wall! Since this method involves the measurement of an angle that depends on the distance of the object and the distance between the two points of observation (aka *baseline*), it is also called trigonometric parallax.

Figure 1 below shows the general principle of parallax. When we look at a tower from points A' and B', the tower appears to be projected against points A' and B' against the background when viewed from both places. But if we look instead from points C and D, the tower appears to move from C' to D', a much larger degree of shift. Thus, to increase the parallax of an object, we want to observe it with the largest possible distance or baseline between our vantage points. In this way, it is sometimes possible to see a parallactic shift that cannot be resolved with a smaller baseline.

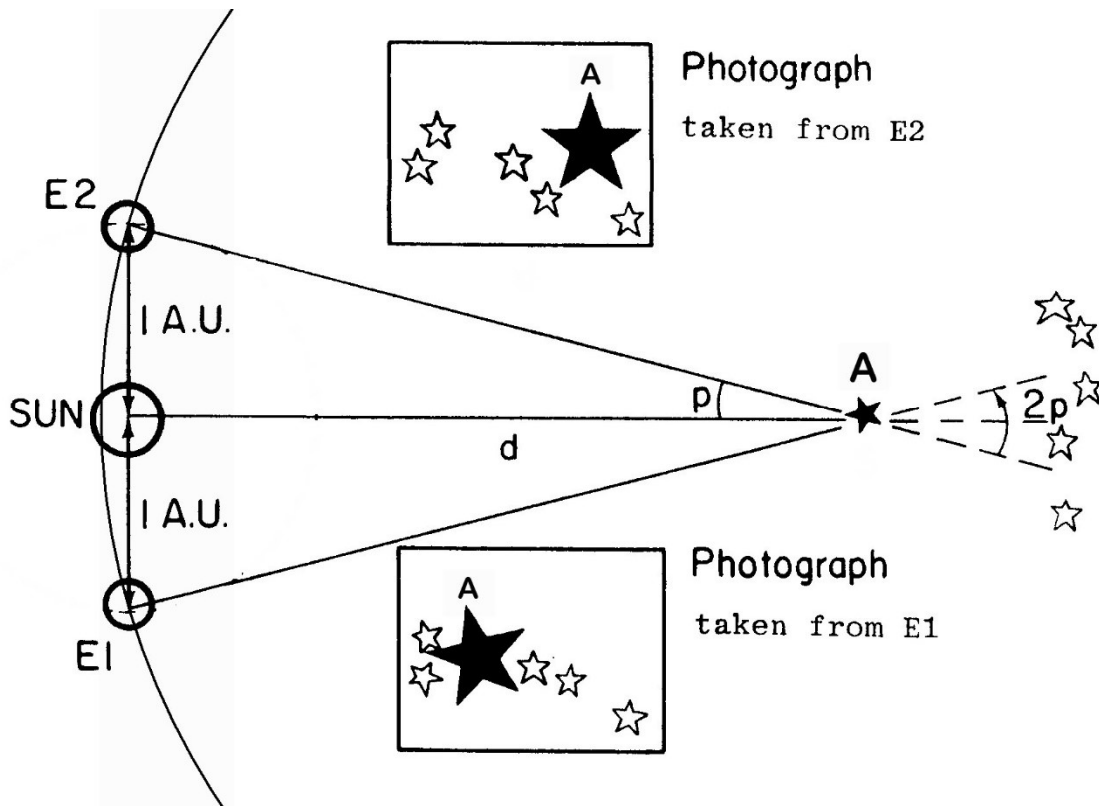
**Figure 1**



Within distances of up to 100ft, your brain registers a noticeable parallax for nearby objects. That's why keeping one eye closed makes it nearly impossible for people to judge the relative distances of nearby objects. You can experience this yourself by trying to touch the tips of their index fingers to each other with one eye closed. For object distances beyond 100ft, your eyes are simply too close together so that the parallaxes of objects become too small to notice. For stars which are all very far away, how large of a baseline would we need to get a measurable parallax? You might think that two telescopes placed on opposite sides of the Earth would suffice. But it turns out that stars are way too far away for even this planet sized baseline! One way to increase the baseline is by using the Earth's orbit where Earth reaches the two furthest possible vantage points six months apart. That is currently the longest baseline we use. Objects at a distance greater than maximum limit given by this baseline will still show parallax but too small for our instruments to detect. Parallax measured in this way is known as *heliocentric parallax* because it uses a baseline centered on the Sun. The images in this lab will simulate heliocentric parallax.

Figure 2 below shows the general principle of heliocentric parallax. Using Earth's orbit as baseline, if we take one photograph of a star field, say, on January 1 and another photograph of the same star field, with the same telescope, on July 1, the Earth will have moved halfway around its orbit of the Sun. The baseline is then 2AU (twice the radius of the Earth's orbit), which is 23,500 times larger than the diameter of the Earth. The detailed geometry of heliocentric parallax is shown below.

Figure 2

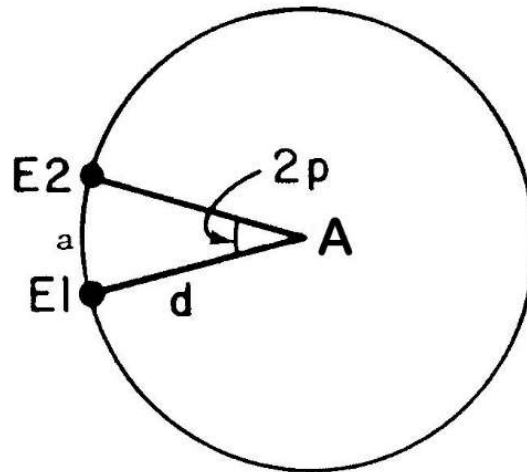


The first photograph of a field of stars is taken from Earth in position E1. The second photograph of the same field of stars is taken six months later when Earth is in position E2. When both photographs are laid on top of each other (aka *superimposed*), the distant background stars will lie on top of each other but any near star, such as A, will have shifted by an angle  $2p$  which is the full parallactic angle. The optional derivation of the parallax-distance formula below will show that the simplest form only uses half of the full parallactic angle:  $p$ .

## DERIVATION – Going from Here to There (optional)

To convert the parallax of a star into a distance, use Figure 3 which is basically a zoomed out and simplified version of Figure 2.

Figure 3



Imagine that we draw a circle centered on star A which goes through E1 and E2 (large dashed circular section in Figure 2, solid circle in Figure 3). Since the angle  $2p$  is very small, the distance between E1 and E2 (the diameter of Earth's orbit) is nearly equal to the length of the arc (labeled "a") along the circle between E1 and E2. The distance to the star  $d$  is just the radius of the circle, and the circumference is equal to  $2\pi \times 206,265$ :

$$\frac{\text{distance between E1,E2}}{\text{circumference}} = \frac{2p}{2\pi \times 206,265}$$

But the distance between E1 and E2 =  $a = 2\text{AU}$ , and the circumference =  $2\pi d$ . Therefore,

$$\frac{a}{2\pi d} = \frac{2[\text{AU}]}{2\pi d} = \frac{2p[\text{arcsec}]}{2\pi \times 206,265[\text{arcsec}]}$$

Therefore,

$$d = \frac{206,265[\text{arcsec}] \times 1[\text{AU}]}{p[\text{arcsec}]} = \frac{206,265}{p}[\text{AU}]$$

The distance equal to 206,265 AU is called a parsec. If  $d$  is measured in parsecs and  $p$  is measured in arcseconds, then

$$d = \frac{1}{p}$$

The parallax of a star is thus defined to be half the angular shift in order to bring the parallax-distance equation (equation 1 below) into its simplest form. This equation is used to determine distances of stars in the unit of parsec where 1 parsec equals the distance of a star with a parallactic shift of 1 arc second away from us. The distance of 1 parsec equals 3.3 light years.

$$d = \frac{1}{p} \quad \text{(Eq. 1)}$$

From its mission (1989 -1993), the European Space Agency (ESA) satellite Hipparcos has increased the accuracy of the parallax measurements from previously 0.01 to 0.001 arc seconds which increased accurate distance measurements to stars out to several hundred light years from previously about 100 light years. The Hipparcos catalogue based on its measurements of over 100,000 stars was released in 1997. The most advanced astrometry mission to date, ESA's Gaia satellite was launched in December 2013 and had two major data releases of the positions of nearly 1.7 billion stars in our Milky Way Galaxy to date. For about 10% of these stars (about 170 million) the parallax measurements are accurate enough to determine individual distances of stars. Gaia's mission has been extended into 2022.