Surface waves induced magnetic reconnection and quantification of space weather phenomena

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Abstract. The surface wave induced magnetic reconnection model (SWIMR), based on Alfvén resonance theory near the neutral point when extended to magnetosonic waves is seen to have not only the features of formation of current sheets and plasmoids, as in the incompressible case, but also gives rise to the features caused by the thermal catastrophic loss of equilibrium for the current sheet. From this point of view, the catastrophic model for substorm is reexamined and it is shown in this paper that the catastrophic model can explain not only the onset and expansion phase of the substorm, but also the recovery phase. The correspondence of the energy input due to resonance absorption to the Akasofu parameter is clearly seen. Further, the dependence of the critical velocity for the onset of K-H instability, the generation, existence and propagation of surface waves on the interplanetary parameters is discussed keeping in view that the SWIMR model depends on the dispersion of surface waves. The SWIMR model therefore can be parameterized to understand some aspects of space weather phenomena.

Index Terms. Alfvén surface waves, magnetic reconnection, magnetic substorms, space weather.

1. Introduction

There are two elements of understanding the solar wind-magnetosphere interaction, the basis for space weather phenomena, one is to derive a mathematical description of the dynamics of the magnetosphere from the basic physical principle and second is to use the derived mathematical model to forecast the disturbances of the Earth's magnetic field caused by the solar wind.

There are many theories (Lui, 1992) to model the geomagnetic storms, the extraordinary disturbances of the Earth's magnetic field. From the point of view of the fact that the reconnection by pure tearing modes cannot give the explosive phenomena such as solar flares or magnetic storms, most of these models depend on the driven magnetic reconnection, either by flow velocity or by dispersion of hydromagnetic waves. In this paper we shall consider the model of magnetic reconnection induced by Alfvén surface waves.

Considering the MHD waves in structured and inhomogeneous magnetic fields Uberoi (1994) proposed long wavelength Alfvén surface waves as a possible source for producing conditions for inducing reconnection. The surface-wave-induced-magnetic reconnection (SWIMR) model is based on the Alfvén resonance theory near the neutral point. More recently (Uberoi, 2002) it was shown that this model can be easily extended to compressional or magnetosonic waves showing the features of formation of current sheets and plasmoids as in the case of the incompressible case. In addition, the energy considerations show the triggering by the catastrophic loss of equilibrium (Smith, Goretz and Grossman, 1986). Synthesizing the SWIMR model with the concept of thermal catastrophe model, with some modifications, we show that the resultant driven model can explain not only the expansion but also the recovery phase of the substorms. Since the model depends on the existence and characteristics properties of surface wave propagation, which is closely related to interplanetary parameters we try to show that this model can be used for prediction of geomagnetic storms, an important element of space weather phenomena. The rate of the resonant absorption of Alfvén wave energy gives the coupling function for the solar-wind-magnetosphere interaction.

2. The reconnection model

The Alfvén wave equation governing the dynamics of the hydromagnetic waves in the incompressible, ideal, MHD media propagating in inhomogeneous magnetic fields with variation in the direction perpendicular to the plane of the magnetic field is given as

$$\frac{d}{dx}\left(\varepsilon\frac{d\mathrm{v}_x}{dx}\right)-k_{\parallel}^2\varepsilon\mathrm{v}_x=0, \qquad (1)$$

where $\varepsilon=[\omega^2 4\pi\rho_0(x)-k_{\parallel}^2 B_0^2(x)]$, $\mathbf{k}=(k_{\parallel},k_{\perp})$ is the wave vector in the (y,z) plane. Here

$$\mathbf{B}_0=[0,B_0(x),0] \qquad (2)$$

and v_x is the perturbed velocity component.

When compressibility is taken into account equation (1) becomes

$$\frac{d}{dx}\left[\frac{\varepsilon\alpha B_0^2}{\alpha k_\perp^2 B_0^2-\varepsilon}\frac{dv_x}{dx}\right]-\varepsilon v_x=0, \tag{3}$$

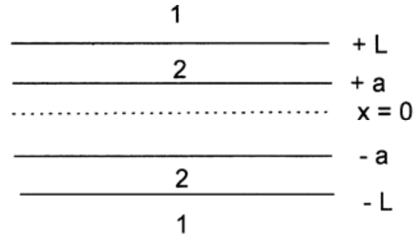
where

$$\alpha(x)=1+\frac{\omega^2 V_s^2}{V_A^2(\omega^2-k_\parallel^2 V_s^2)}.$$

With V_s being the velocity of sound and $V_A(x)$, the Alfvén velocity.

Equation (1) shows a singular behaviour at the point where the local Alfvén speed becomes equal to the phase speed of the wave and has been studied extensively by using normal mode analysis, initial value problem (Hasegawa and Uberoi, 1982) and time-dependent solution for general class of initial conditions was given in (Uberoi and Sedlacek, 1992). From all these studies it was noted (Uberoi, 1994) that the time evolution of Alfvén waves shows that the current density increases secularly with time t. Thus, showing the development of current sheets as the Alfvén wave propagates in the inhomogeneous systems. The thickness of the current sheet decreases as 1/t. These current sheets arise not due to any instability but due to the accumulation of energy around the resonance point, where Alfvén wave energy is absorbed resonantly. Therefore, propagation of MHD waves in inhomogeneous plasma with sheared magnetic field can give rise to singular current sheets. This situation continues till time $t = t_h$. For $t > t_h$, where t_h is the intrinsic timescale (Uberoi et al., 1999), the resistivity effects become important and the regularization of the singularity takes place by resonant mode conversion of surface waves along a sharp discontinuity to the resistivity modified Alfvén wave at the non-zero singular point at the centre of the resistivity layer. However, the mathematical structure of the hydromagnetic wave equation (1) remains the same at zero and non-zero singular points, but the role played by surface waves in the resonant absorption mechanism at these two points is different (Uberoi, 1994). The theories of resonant absorption of Alfvén waves consider surface waves propagating along a sharp discontinuity separating two infinity extended plasma regions. The structural discontinuities are not taken into account. Near a neutral point the structure of the discontinuity becomes important as waves are now of long wavelengths. In this case, wave propagation is to be considered for a plasma layer, which can support two types of surface modes of oscillations, symmetric and asymmetric.

Since we intend to apply results from a consideration of the shape of the magnetic tail we consider a plasma layer with thickness 2a, with the following magnetic field profile:



$$\begin{aligned}\mathbf{B}&=\left(0,B_0\frac{x}{a},0\right), \quad -a<x<a\\ \mathbf{B}=\mathbf{B}_2&=(0,B_2\cos\beta_2,B_2\sin\beta_2), \quad a\le x\le L\\ \mathbf{B}=\mathbf{B}_1&=(0,B_1\cos\beta_1,B_1\sin\beta_1), \quad x\ge L\end{aligned} \tag{4}$$

And the density being, $\rho_0(x)$ and $\rho_{1,2}$ in the inner and outer regions. The parameters on the two sides of the interfaces at $x = \pm a$ are related by the requirements of the continuity of the pressure and normal component of the velocity across the surface of discontinuity.

Equation (1) is a coupled equation for shear Alfvén and Alfvén surface waves, the coupling arising owing to the non-uniformity of the magnetic field. In case of constant $B_0(x)$ except for regions of sharp discontinuities, equation (1) decouples giving a bulk Alfvén mode and the surface modes governed by the Laplacian $\nabla^2 v_x=0$. The Laplace equation when solved for magnetic profiles with sharp discontinuities as given in equation (4) with the required mentioned boundary conditions gives the modes of surface waves propagation. In order to understand the role of surface waves and the resonance absorption near the neutral point equation (1) with magnetic profile (4) was discussed as an initial value problem and it was shown (Uberoi, 1994) that in the long wavelength limit ka << 1, the symmetric surface modes of the plasma layer resonantly couple to the low-frequency end of the Alfvén continuum.

When the resistivity is switched on, the long wavelength surface modes couples with the tearing mode of the layer, thus inducing magnetic reconnection on the tearing mode timescale. This mode is unstable for certain wavelengths greater than a critical value and begins to grow until magnetic islands are formed. The estimate of the linear dimensions of the islands that are formed is given by wavelength at which instability sets in as $\lambda = 2\pi a/0.64$ (Uberoi, Lanazerotti, Wolfe, 1996).

The important timescales found in this model are

$$t_h=\tau_A^{2/3}\tau_R^{1/3} \quad or \quad t_h=\tau_A S^{1/3}, \tag{5}$$

where $\tau_R=4\pi a^2/\eta$, $\tau_A=a/V_A$, with $V_A = B_0^2/4\pi\rho_2$ and $S=\tau_R/\tau_A$ is the Lundquist number. Here, η is the plasma resistivity. The resistivity effects begin to play a role for $t \ge t_h$. The other important timescale is the reconnection time:

$$t_r=\tau_A^{2/5}\tau_R^{3/5} \quad or \quad t_r=\tau_A S^{3/5}. \tag{6}$$

The width of the resistive layer scales as

$$\Delta x \alpha S^{-1/3} . \tag{7}$$

For considering the magnetosonic waves we consider the equation (3). We note that near the spatial resonance ε ≈ 0 eqn. (3) becomes,

$$\frac{d}{dx}\left(\varepsilon \frac{dv_x}{dx}\right) - k_\perp^2 \varepsilon v_x = 0 \tag{8}$$

which has the same structure as equation (1). As equation (8) is similar to equation (1) the basic features of the development of current sheets and magnetic reconnection induced by the resonant coupling of compressional surface waves leading to the formation of plasmoids will be the same as in the incompressible case. We note here that, although the dispersion relation for the surface waves along the density discontinuities are different from that of incompressible case (Uberoi, 1989), the long wavelength compressional surface waves for the layered structure have the similar dispersion equation, so without any modification the SWIMR model can be used both for the shear Alfvén and magnetosonic waves.

3. Thermal catastrophe

The equation (8) has a logarithmic singularity at $\omega = k_{\|} V_A$ i.e. at the point x = x_0, where the phase velocity of the wave $V_{ph} = \omega / k_{\|}$, becomes equal to the Alfvén wave velocity. The rigorous analysis of this equation shows that the surface waves, which are now coupled to an Alfvén bulk wave, due to inhomogeneity, as pointed out in the previous section, are resonantly absorbed near the resonant point. The surface energy, thus dissipated, irreversibly heats the plasma by coupling its energy to kinetic Alfvén waves, or if finite conductivity is a dominant factor (Hasegawa and Uberoi, 1982), to bulk Alfvén waves which, in turn, are heavily damped. The surface wave energy thus dissipated irreversibly heats the plasma.

For developing the energy equation for the problem under consideration, consider the plasma layer as given by equation (4). The surface perturbations are made at x = ± a. Following (Smith et al., 1986) we write the heating rate Q_A per unit area in the (y,z) plane due to resonant absorption of compressional waves as

$$Q_A = \frac{2\pi\omega a b_{1x}^2(a)/4\pi}{[(k_{\|}^2 B_2^2 / 4\pi\omega^2 \rho_0(0))(k/k_\perp) - 1]^2 + \pi^2 k^2 a^2}, \tag{9}$$

where $k^2 = k_\perp^2 + k_{\|}^2$ and b_{1x} is the amplitude of perturbation of a single mode of surface wave excited at x = a with frequency ω. Balancing this heating solely against convective transport of energy towards the reconnection layer around x = 0 with convection velocity V_x, the steady-state energy equation is

$$\frac{d}{dx}\left[\frac{5}{2} V_x P\right] = q_0 + \frac{1}{2} \operatorname{Re} J.E^*, \tag{10}$$

The last term on the right is the contribution from resonant absorption such that

$$\frac{dQ_A}{dx} = \frac{1}{2} \operatorname{Re} J.E^* .$$

q_0 is the combined heating rates of other heating mechanisms and *P(x)=n(x)T(x)*. While writing the energy balance equation (10) it is assumed that the plasma layer (4) reacts quickly enough to adiabatically changing inputs of Alfvén wave energy and establishes a quasi-steady state. In the steady state and with the further approximation of treating the convecting velocity as a free parameter V_x, the convective term reduces to that given in the left-had side of equation (10). A detailed discussion of such an approximation in the context of solar flares is given by Tur and Priest (1978).

The equation (10) on integration from x = 0 to a gives:

$$T - T_0 = \frac{2Q_A}{5n_0 V_x}, \tag{11}$$

Here, n_0 is the particle density at x = 0 within the layer.

$$T_0 = \frac{2}{5nV_x} \int q_0 dx.$$

Defining

$$T_w = \frac{1}{2} m_i \frac{\omega^2}{k_{\|}^2}\left(\frac{k_\perp}{k}\right), x^2 = \pi^2 k^2 a^2, T' = \frac{T}{T_w}, T_0' = \frac{T_0}{T_w}$$

and noting that from equilibrium conditions $B_2^2 / 4\pi = 2n_0 T$, as the plasma pressure in medium 2 is much less than the magnetic pressure, the equation (9) on using (11) then can be written as

$$W = (T' - T_0')[(T'-1)^2 + X^2] \tag{12}$$

with

$$W = \frac{\omega(a)(b_{1x}^2)}{5n_0 V_x T_w} . \tag{13}$$

As dW/dT has two positive roots for which $d^2W/dT^2 > 0$, equation (13) has a family of solution $T'(T_0', X, W)$ in the form of mathematical catastrophe for $0 < T_0' < 1 - \sqrt{3}X$ as seen in Fig. 1. For W = W^*which is given by

$$W^* = \frac{2}{27}\left[(1 - T_0') + \sqrt{(1 - T_0')^2 - 3X^2}\right] \times \left[2(1 - T_0') - \sqrt{(1 - T_0')^2 - 3X^2}\right]^2, \tag{14}$$

the equilibrium temperature T′ jumps discontinuously or catastrophically to a higher value, see the points B and D in Fig. 1. The thermal catastrophe of the current sheet corresponds to the triggering of the substorm expansion phase.

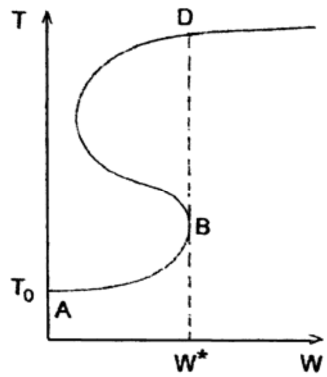


Fig. 1. Schematic of solutions T (T_0, X, W) = 0 of equation (12) showing the unphysical catastrophic jumps in temperature from B to D for the critical energy W*.

4. Model for substorms

The geometry given by equation (4), the layer of thickness 2a, with neutral sheet inside represents the plasma sheet. The perturbations of the boundary x = ± a, we assume that are due to surface waves at the magnetopause at x = ± L. We note from section 2 and 3 that there are two processes arising due to resonant absorption of MHD waves in the neutral sheet in a structured plasma medium. One is the resonant excitation of the tearing mode after time $t \geq t_h$, giving the reconnection energy, in the incompressible case, equals to the resonant absorption energy (Vekestein, 2000). In the other process the Alfvén wave heating energy when balanced with the convective transport of energy towards the current sheet gives the thermal catastrophic loss of equilibrium. The latter triggers the expansion phase of the substorm. The synthesis of these two processes gives the model of substorms.

The time t_h, after which the recovery phase sets in can be calculated as:

$$t_h = \left(\tau_A^2 \tau_R\right)^{1/3} = \left(\frac{a^2}{V_{A2}^2}\frac{4\pi a^2}{\eta}\right)^{1/3} = \left(\frac{4\pi}{\eta V_{A2}^2}\right)^{1/3} a\ .$$

For 2a=1R_E, η = 10^{-3} ohms-km, V_A = 500 km/s.

$t_h = 2.6 \times 10^3 = 2600$ sec.

This value can be increased with low values of η and V_A.

5. Energy coupling function

The heating rate Q_A as a function of T shows a resonance peak of width XT_w at T = T_w. Catastrophe occurs when T has a value $1-X < T/T_w < 1+X$, Q_A again decreases and the equilibrium can be stabilized. We note here that the maximum value of X is equal to $1/\sqrt{3}$.

The maximum heating rate obtained when T = T_w over an area A on both sides of the current sheet is $2AQ_A$, which from equation (10) with $\omega/k_{\|} = V_{sw}$, surface wave phase velocity, can be written as

$$E_{\max} = \frac{2AV_{sw}b_{1x}^2(a)}{2\pi X} = \varepsilon_A \tag{15}$$

The main phase of the storm in this model is therefore controlled by the parameter ε_A. Considering the plasma sheet with a = 0.5R_E and length 100 R_E, V_{sw} = 500 km/s, b_{1x} = 0.1 B_2 with $B_2 = 50\gamma = 50 \times 10^{-5}$ Gauss and X = 0.3.

$\varepsilon_A = 1.5 \times 10^{18}$ ergs/s.

Which compares well with the moderate type of storm. For the large values of V_{sw}, ε_A can be increased.

Comparing ε_A with the Akasofu parameter $\varepsilon_s = \ell_0^2 V B^2 \sin^4\theta/2$ we find that the magnetic storm is driven by the surface waves and the amplitude changes of the magnetic field $\mathbf{B}_2$ in the normal direction and not directly by the solar wind. Monitoring the surface wave velocity and $\mathbf{B}_2$, the magnetic field in the lobe region, it is possible, therefore, to estimate ε_A which can then give the prediction of Dst and AE index. We assume that the surface waves are generated at x = ± L, the magnetopause, by the K-H instability. In this case, Southwood's instability criterion, (see Uberoi, 1984) gives the critical value of the solar wind velocity for instability as

$$u > V_A \frac{\sin(\beta_2 - \beta_1)}{\sin\beta_2} = u_c\ .$$

Therefore generation of surface waves require the solar wind speed $u > u_c$.

The angle $(\beta_2 - \beta_1)$ is function of ψ, the angle between the magnetic field and solar wind velocity in the solar wind. This was seen by Lee and Olson (1986). Also, B_2 is function of the interplanetary magnetic field B_∞. Therefore

$u_c = f(\psi, B_\infty)$.

Thus measure of interplanetary parameters can predict a limit on u_c for which the surface waves can be generated, which in turn could induce a magnetic storm. For example for ψ = 60°, it was seen that (Lee and Olson, 1986) $\frac{u_c}{V_{A\infty}} = 4.5$.

Taking $V_{A\infty}$ = 70 km/s, u_c = 315 km/s. Hence, by monitoring the solar wind velocity and the interplanetary parameters, the necessary condition for the onset of a substorm can be predicted.

We had also shown (Uberoi, 1984) that for u = u_c,

$$V_{sw} = \left(\frac{\rho_1}{\rho_1 + \rho_2} \right)^{1/2} V_{A1} \frac{\sin(\beta_2 - \beta_1)}{\sin \beta_2}, \text{ for } ka << 1$$

In this case we see from equation (15) that the input coupling function can also be function of interplanetary parameters. Prediction of ε_A then can be used to get AE and Dst indices of the storm as considered by Akasofu (1981).

6. Conclusion

The two features of the Alfvén Resonance theory that the magnetic reconnection can be driven by Alfvén surface waves and that the mathematical catastrophe as shown by the energy equation are synthesized to give a model for a magnetic substorm. It is seen that using this model a fairly accurate prediction of the onset and growth of magnetic storm manifested in the AE and Dst index can be made by monitoring the generation and propagation of low frequency Alfvén surface waves at the magnetopause. The dependence of the input energy function, due to resonant absorption of Alfvén waves, on the interplanetary parameters suggests the possibility of using this model for numerical forecasting of the space weather.

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