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Standard Practice for Estimating Thurstonian Discriminal Distances[1]

This standard is issued under the fixed designation E2262; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice describes procedures to estimate Thurstonian discriminal distances (that is, *d*' values) from data obtained on two samples. Procedures are presented for four forced-choice methods (that is, the triangle, the Duo-Trio, the 3-alternative-forced-choice (or 3-AFC) and the 2-AFC (also called the directional difference test)), the A/Not-A method, the Same-Different method, and for data obtained from ordered category scales. Procedures for estimating the variance of *d*' are also presented. Thus, confidence intervals and statistical tests can be calculated for *d*'.

1.2 The procedures in this practice pertain only to the unidimensional, equal-variance model. Other, more complicated Thurstonian models, involving multiple dimensions and unequal variances exist but are not addressed in this practice. The procedure for forced-choice methods is limited to dichotomous responses. The procedure for the A/Not-A method assumes equal sample sizes for the two samples. The procedure for the Same-Different method assumes equal sample sizes for the matched and unmatched pairs of samples. For all methods, only unreplicated tests are considered. (Tests in which each assessor performs multiple (that is, replicated) evaluations require different analyses.)

1.3 Thurstonian scaling is a method for measuring the perceptual difference between two samples based on a probabilistic model for categorical choice decision making. The magnitude of the perceived difference, δ, can be estimated from the assessors' categorical choices using the methods described in this practice. (See Appendix X3 for a more detailed description of Thurstonian scaling.)

1.4 In theory, the Thurstonian δ does not depend on the method used to measure the difference between two samples. As such, δ provides a common scale of measure for comparing samples measured under a variety of test conditions. For example, Thurstonian scaling can be used to compare products measured under different test conditions, to compare panels (trained, consumer or both) that have evaluated the same samples (using the same or different test methods) and to compare test methods on their ability to discriminate samples that exhibit a fixed sensory difference.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.6 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E253 Terminology Relating to Sensory Evaluation of Materials and Products

E456 Terminology Relating to Quality and Statistics

2.2 *ASTM Manual:*[2]

Manual 26 Sensory Testing Methods, 2nd Edition

2.3 *ISO Standard:*[3]

ISO 5495 Sensory Analysis—Methodology—Paired Comparison

3. Terminology

3.1 *Definitions:*

3.1.1 For definitions of terms relating to sensory analysis, see Terminology E253. For terms relating to statistics, see Terminology E456.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *δ*—the Thurstonian discriminal distance is the distance between the means of the distributions of sensory magnitudes of the two samples in the test (see Appendix X3).

[1] This practice is under the jurisdiction of ASTM Committee E18 on Sensory Evaluation and is the direct responsibility of Subcommittee E18.03 on Sensory Theory and Statistics.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] Available from American National Standards Institute (ANSI), 25 W. 43rd St., 4th Floor, New York, NY 10036, http://www.ansi.org.

3.2.2 *d'*—the statistic used to estimate δ based on the data obtained from the test.

3.2.3 *choice proportion (P_c)*—the expected proportion of responses from a forced-choice method. (For example, if there is no perceptible difference between the samples in a triangle test, P_c = 1/3. If there is a perceptible difference, P_c > 1/3.)

3.2.4 *observed choice proportion (p_c)*—the statistic used to estimate choice proportion, P_c, where $p_c = x/n$, where x is the observed number of correct responses and n is the sample size.

4. Summary of Practice

4.1 Determine the type of data collected on the two samples: data from a forced-choice test, an A/Not-A test, a Same-Different test or an ordered category scale.

4.2 For forced-choice tests, reference the table that corresponds to the test method (that is, triangle test (Tables X1.1 and X1.2), Duo-Trio test (Tables X1.3 and X1.4), 3-AFC test (Tables X1.5 and X1.6), or 2-AFC test (Tables X1.7 and X1.8)). Identify the entry in the table closest to the observed choice proportion (p_c) from the test. Read the estimated value of δ (that is, *d'*) from the corresponding row and column headings of the table. Estimate the variance of *d'* by referencing the appropriate table for the test method. Find the value of *B* that corresponds to the value of *d'* obtained in the first step (see Note 1). The estimated variance of *d'* is $S^2(d') = B/n$, where n is the sample size. Use the estimates *d'* and $S^2(d')$ to construct confidence intervals and tests of hypotheses related to the objectives of the research.

NOTE 1—The variance of *d'* is a complicated function of the true value of δ and the decision rule when associated with the test method being used (see Appendix X3). However, regardless of the test method, the variance of *d'* can always be expressed as $S^2(d') = B/n$, where the parameter *B* captures all of the information concerning the test method, and n is the sample size. The values of *B* have been tabulated to make the calculation of the variance of *d'* a simple task.

4.3 For the A/Not-A method, tally the observed choice proportions of "A" responses for the A sample and the "A" responses for the Not-A sample. Read the value of *d'* from Table X1.9 in the column that corresponds to the observed choice proportion of the "A" responses for the Not-A sample (p_{na}) and the row that corresponds to the observed choice proportion of the "A" responses for the A sample (p_a). The same method is used to estimate the variance of *d'*, $S^2(d')$, using Table X1.10.

4.4 For the Same-Different method, tally the proportion of "same" responses for the matched pairs of samples (that is, A/A or B/B) and the proportion of "same" responses for the unmatched pairs of samples (that is, A/B or B/A). Read the value of *d'* from Table X1.11 in the column that corresponds to the observed proportion of "same" responses for the unmatched pairs ($p_s/_u$) and the row that corresponds to the observed proportion of the "same" responses for the matched pairs ($p_s/_m$). The same method is used to estimate the variance of *d'*, $S^2(d')$, using Table X1.12.

4.5 For ordered category scales, a rapid, table-look-up approach is used. For each sample, the category scale data are collapsed into two categories. One sample is selected to be the "A" sample and the other sample is selected to be the "Not-A" sample. Choice proportions are tallied for each sample and the values of *d'* and its variance, $S^2(d')$, are obtained from Tables X1.9 and X1.10, respectively, by the same techniques used in the A/Not-A method.

5. Significance and Use

5.1 Under the assumptions of the model, the Thurstonian model approach to measuring the perceived difference between two samples (whether overall or for a specific attribute) is independent of the sensory method used to collect the data. Converting results obtained from different test methods to *d'* values permits the assessment of relative differences among samples without requiring that the samples be compared to each other directly or that the same test methods be used for all pairs of samples.

5.2 Thurstonian scaling has been applied to:

5.2.1 Creating a historical database to track differences between production and reference samples over periods in which different test methods were used to measure the difference,

5.2.2 Comparing the relative sensitivities of different user groups and consumer segments,

5.2.3 Comparing trained panels that use different measuring techniques,

5.2.4 Comparing the relative sensitivities of consumers versus trained panels,

5.2.5 Comparing different methods of consumer testing (for example, CLT versus HUT, preference versus hedonic scales, etc.), and

5.2.6 Comparing different discrimination test methods.

6. Procedure

6.1 *Forced-choice Methods*—The relationship between δ and the expected choice proportion, P_c, is different for different forced-choice methods because the decision rule used by the assessors varies from one method to another (see Appendix X3). As a result, different tables are required to estimate δ depending on the method used. Tables for the four most commonly used methods are presented. The estimated value of δ (that is, *d'*) is obtained as follows:

6.1.1 Compute the observed choice proportion as $p_c = x/n$, where x is the observed number of correct responses and n is the sample size.

6.1.2 Obtain *d'* by entering the table in Appendix X1 that corresponds to the test method used: triangle test (Table X1.1), Duo-Trio (Table X1.3), 3-AFC (Table X1.5), or 2-AFC (Table X1.7). Find the entry in the table that is closest to the observed value of p_c. The value of *d'*, accurate to one decimal place, is the row-label of the table corresponding to the selected entry. The second decimal place of *d'* is the column-label of the table corresponding to the selected entry.

6.1.3 Obtain the estimated variance of *d'* as follows. Enter the appropriate table in Appendix X1: triangle test (Table X1.2), Duo-Trio (Table X1.4), 3-AFC (Table X1.6), or 2-AFC (Table X1.8). Find the value of *B* in the row and column that correspond to the value of *d'* obtained in 6.1.2. Compute the estimated variance of *d'* as $S^2(d') = B/n$, where n is the sample

size. Use the estimates d' and $S^2(d')$ to construct confidence intervals and tests of hypotheses related to the objectives of the research.

6.2 *A/Not-A Method*—Compute the choice proportions of the two samples, $p_a = x_a/n$ and $p_{na} = x_{na}/n$, where x_a is the number of times the "A" sample is chosen as being "A," x_{na} is the number of times the "Not-A" sample is chosen as being "A" and n is the sample size.

NOTE 2—This practice only considers the case where the number of "A" samples equals the number of "Not-A" samples, $n = n_a = n_{na}$.

6.2.1 Read the value of d' from Table X1.9 in Appendix X1 in the column that corresponds to the observed choice proportion of the "Not-A" sample (p_{na}) and the row that corresponds to the observed choice proportion of the "A" sample (p_a).

6.2.2 To obtain an estimate of the variance of d', read the value of B from Table X1.10 in Appendix X1 using the same technique as in 6.2.1. The variance estimate is $S^2(d') = B/n$, where n is the sample size.

6.3 *Same-Different Method*—Compute the choice proportions for the matched (m) and unmatched (u) pairs of samples, $p_{s/m} = x_{s/m}/n$ and $p_{s/u} = x_{s/u}/n$, where $x_{s/m}$ is the number of "same" responses for the matched pairs (A/A or B/B) evaluated, $x_{s/u}$ is the number of "same" responses for the unmatched pair and n is the number of matched or unmatched pairs evaluated.

NOTE 3—This practice only considers the case where the number of matched pairs equals the number of unmatched pairs, $n = n_m = n_u$.

6.3.1 Read the value of d' from Table X1.11 in Appendix X1 in the column that corresponds to the observed proportion of "same" responses for unmatched pair ($p_{s/u}$) and the row that corresponds to the observed proportion of "same" responses for the matched pair ($p_{s/m}$).

6.3.2 To obtain an estimate of the variance of d', read the value of B from Table X1.12 in Appendix X1 using the same technique as in 6.3.1. The variance estimate is $S^2(d') = B/n$, where n is the sample size.

6.4 *Ordered Category Scales*—A rapid, table-look-up method is described. The method collapses the category-scale data into two categories, regardless of the number of categories on the physical scale used to collect the data. It is recognized that information detail is lost by collapsing the data into two categories. However, the estimates of d' and its variance, $S^2(d')$, obtained from the technique are accurate. The computational ease offsets the small loss of accuracy incurred.

6.4.1 Tally the frequency distributions of category scale ratings for the two samples. Select the sample with the lower median rating to be the Not-A sample. Select the sample with the higher median rating to be the A sample.

6.4.2 Collapse the frequency data for each sample into two categories as follows. Identify the category in which the median of the Not-A sample occurs. Pool the number of responses from that category and all lower categories for each sample separately and record the totals in the two-by-two table under "Low" (that is, the y_{na} and y_a tallies, below). Pool the number of responses for the remaining, higher categories for each sample separately and record the totals in the two-by-two table under "High" (that is, the x_{na} and x_a tallies, below).

Sample	Low	High
Not-A	y_{na}	x_{na}
A	y_a	x_a

6.4.3 Compute the choice proportions of the two samples, $p_a = x_a/n$ and $p_{na} = x_{na}/n$, where x_a and x_{na} are obtained from the table above and n is the sample size, common to both samples.

6.4.4 Apply the same technique used in the A/Not-A method (see 6.2). Read the value of d' from Table X1.9 in Appendix X1 in the column that corresponds to the observed choice proportion of the Not-A sample (p_{na}) and the row that corresponds to the observed choice proportion of the A sample (p_a).

6.4.5 To obtain an estimate of the variance of d', read the value of B from Table X1.10 in Appendix X1 using the same technique as in 6.4.4. The variance estimate is $S^2(d') = B/n$, where n is the sample size.

6.5 *Statistical Tests and Confidence Intervals*—Often the objective of a sensory discrimination test is to determine if the samples in the test are perceptibly different. In other instances it is of interest to obtain an estimate of the size of the perceptible difference (and to measure the precision of the estimated difference). Because testing for a difference and estimating the size of a difference address different goals, it is not surprising that different statistical methods apply to each. For the purpose of testing if a perceptible difference exists, the binomial and chi-square tests traditionally associated with the test methods discussed in this practice are appropriate. For the purposes of estimating the size of the difference and assessing the precision of that estimate, confidence intervals are appropriate. Because δ is the difference between the means of two normal distributions and d' is an estimate of δ, it can be assumed that d' is approximately normally distributed. Based on this assumption, statistical confidence intervals concerning δ can be constructed using traditional techniques.

6.5.1 A 100(1 - α) % two-sided confidence interval on δ is calculated as: $d' \pm Z_{\alpha/2}S(d')$, where d' is the estimated value of δ, $Z_{\alpha/2}$ is the upper-α/2 percentage point of the standard normal distribution (for example, for a 90 % confidence interval $Z_{\alpha/2}$ = 1.65; for a 95 % confidence interval $Z_{\alpha/2}$ = 1.96; etc.), and $S(d')$ is the standard deviation of d', that is, the square root of, $S^2(d') = B/n$. Similarly, 100(1 - α) % one-sided confidence intervals on δ are calculated as: $d' + Z_\alpha S(d')$ for a one-sided upper confidence interval and $d' - Z_\alpha S(d')$ for a one-sided lower confidence interval, where $Z\alpha$ is the upper-α percentage point of the standard normal distribution (for example, for a 90 % confidence interval Z_α = 1.28; for a 95 % confidence interval Z_α = 1.65; etc.) and d' and $S(d')$ are as defined above.

6.5.2 To test if δ is greater than zero, that is, that the two samples in the test are perceptibly different, use the binomial or chi-square test that is traditionally associated with the discrimination method used.

6.5.3 To test if it is reasonable to believe two δ's have the same value, that is, to test the hypotheses H_0: $\delta_1 = \delta_2$ versus H_a: $\delta_1 \neq \delta_2$ form the ratio:

$$T = \frac{|\, d' - d'_2 \,|}{\sqrt{S_1^2 + S_2^2}}$$

where:

d_1' = estimated value of δ_1,
d_2' = estimated value of δ_2,
S_1^2 = variances of d_1', and
S_2^2 = variances of d_2'.

If $T > Z_{\alpha/2}$, then conclude the two δ values are unequal at the α-level of significance.

APPENDIXES

(Nonmandatory Information)

X1. STATISTICAL TABLES

TABLE X1.1 Observed Choice Proportions, p_c, (×10⁴) as Function of *d'* for Triangle Test[A]

NOTE 1—Find the entry in the table closest to the choice proportion observed in the test. Read the estimated value of *d'* from the corresponding row and column headings.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	3333	3333	3334	3334	3335	3336	3337	3338	3339	3341
0.1	3343	3344	3347	3349	3351	3354	3357	3360	3363	3366
0.2	3370	3374	3378	3382	3386	3390	3395	3400	3405	3410
0.3	3415	3421	3427	3432	3439	3445	3451	3458	3464	3471
0.4	3478	3486	3493	3501	3508	3516	3524	3533	3541	3550
0.5	3558	3567	3576	3586	3595	3604	3614	3624	3634	3644
0.6	3654	3665	3676	3686	3697	3708	3719	3731	3742	3754
0.7	3766	3778	3790	3802	3814	3827	3839	3852	3865	3878
0.8	3891	3905	3918	3932	3945	3959	3973	3987	4001	4016
0.9	4030	4045	4059	4074	4089	4104	4119	4134	4149	4165
1.0	4180	4196	4212	4228	4244	4260	4276	4292	4309	4325
1.1	4342	4358	4375	4392	4409	4426	4443	4460	4477	4494
1.2	4512	4529	4547	4564	4582	4600	4618	4636	4654	4672
1.3	4690	4708	4726	4745	4763	4782	4800	4819	4837	4856
1.4	4875	4893	4912	4931	4950	4969	4988	5007	5026	5045
1.5	5065	5084	5103	5122	5142	5161	5180	5200	5219	5239
1.6	5258	5278	5297	5317	5337	5356	5376	5396	5415	5435
1.7	5455	5474	5494	5514	5534	5554	5573	5593	5613	5633
1.8	5653	5672	5692	5712	5732	5752	5771	5791	5811	5831
1.9	5851	5870	5890	5910	5930	5950	5969	5989	6009	6028
2.0	6048	6068	6087	6107	6127	6146	6166	6185	6205	6224
2.1	6244	6263	6283	6302	6321	6341	6360	6379	6398	6418
2.2	6437	6456	6475	6494	6513	6532	6551	6570	6589	6608
2.3	6627	6645	6664	6683	6701	6720	6739	6757	6776	6794
2.4	6812	6831	6849	6867	6885	6903	6922	6940	6958	6976
2.5	6993	7011	7029	7047	7064	7082	7100	7117	7135	7152
2.6	7169	7187	7204	7221	7238	7255	7272	7289	7306	7323
2.7	7340	7356	7373	7390	7406	7423	7439	7455	7472	7488
2.8	7504	7520	7536	7552	7568	7584	7600	7616	7631	7647
2.9	7662	7678	7693	7709	7724	7739	7754	7769	7784	7799
3.0	7814	7829	7844	7859	7873	7888	7902	7917	7931	7945
3.1	7960	7974	7988	8002	8016	8030	8044	8057	8071	8085
3.2	8098	8112	8125	8139	8152	8165	8179	8192	8205	8218
3.3	8231	8243	8256	8269	8282	8294	8307	8319	8332	8344
3.4	8356	8368	8381	8393	8405	8417	8428	8440	8452	8464
3.5	8475	8487	8498	8510	8521	8532	8544	8555	8566	8577
3.6	8588	8599	8610	8620	8631	8642	8652	8663	8673	8684
3.7	8694	8704	8715	8725	8735	8745	8755	8765	8775	8784
3.8	8794	8804	8813	8823	8833	8842	8851	8861	8870	8879
3.9	8888	8897	8906	8915	8924	8933	8942	8951	8959	8968
4.0	8977	8985	8994	9002	9010	9019	9027	9035	9043	9051
4.1	9059	9067	9075	9083	9091	9099	9106	9114	9122	9129
4.2	9137	9144	9151	9159	9166	9173	9180	9188	9195	9202
4.3	9209	9216	9223	9229	9236	9243	9250	9256	9263	9269
4.4	9276	9282	9289	9295	9301	9308	9314	9320	9326	9332
4.5	9338	9344	9350	9356	9362	9368	9374	9379	9385	9391
4.6	9396	9402	9407	9413	9418	9424	9429	9434	9440	9445
4.7	9450	9455	9460	9465	9470	9475	9480	9485	9490	9495
4.8	9500	9504	9509	9514	9518	9523	9528	9532	9537	9541
4.9	9546	9550	9554	9559	9563	9567	9571	9575	9580	9584
5.0	9588	9592	9596	9600	9604	9608	9612	9615	9619	9623
5.1	9627	9630	9634	9638	9641	9645	9648	9652	9655	9659
5.2	9662	9666	9669	9673	9676	9679	9682	9686	9689	9692
5.3	9695	9698	9701	9704	9707	9710	9713	9716	9719	9722

TABLE X1.1 *Continued*

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.4	9725	9728	9731	9734	9736	9739	9742	9745	9747	9750
5.5	9753	9755	9758	9760	9763	9765	9768	9770	9773	9775
5.6	9778	9780	9782	9785	9787	9789	9792	9794	9796	9798
5.7	9800	9803	9805	9807	9809	9811	9813	9815	9817	9819
5.8	9821	9823	9825	9827	9829	9831	9833	9834	9836	9838
5.9	9840	9842	9843	9845	9847	9849	9850	9852	9854	9855
6.0	9857	9859	9860	9862	9863	9865	9866	9868	9869	9871

[A] Adapted from Ennis, D. M., "The Power of Sensory Discrimination Methods," *Journal of Sensory Studies*, 8, 1993, pp. 353–370.

TABLE X1.2 The *B* Values for Estimating Variance of *d'* Obtained From Triangle Test[A]

NOTE 1—Enter the table in the row and column corresponding to the value of *d'* obtained from Table X1.1. The variance of *d'* is $S^2(d') = B/n$, where *n* is the sample size.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	*******	65800.	16452.	7314.	4115.	2635.	1831.	1346.	1031.	815.
0.1	661.08	546.88	460.03	392.44	338.81	295.54	260.13	230.78	206.19	185.38
0.2	167.61	152.31	139.06	127.50	117.35	108.40	100.45	93.38	87.05	81.36
0.3	76.236	71.597	67.387	63.554	60.054	56.850	53.910	51.205	48.711	46.406
0.4	44.273	42.294	40.455	38.742	37.146	35.655	34.261	32.954	31.729	30.578
0.5	29.496	28.477	27.517	26.611	25.755	24.945	24.179	23.452	22.764	22.110
0.6	21.489	20.898	20.336	19.801	19.291	18.805	18.341	17.897	17.474	17.069
0.7	16.681	16.310	15.954	15.614	15.287	14.973	14.672	14.383	14.106	13.839
0.8	13.582	13.335	13.097	12.868	12.647	12.435	12.230	12.032	11.841	11.657
0.9	11.479	11.308	11.142	10.982	10.827	10.677	10.532	10.392	10.256	10.125
1.0	9.998	9.875	9.756	9.641	9.529	9.421	9.316	9.214	9.115	9.019
1.1	8.926	8.836	8.749	8.664	8.582	8.502	8.424	8.349	8.276	8.205
1.2	8.136	8.069	8.004	7.941	7.880	7.820	7.762	7.706	7.651	7.598
1.3	7.547	7.497	7.448	7.401	7.355	7.310	7.267	7.225	7.184	7.144
1.4	7.106	7.068	7.032	6.997	6.962	6.929	6.897	6.866	6.835	6.806
1.5	6.778	6.750	6.723	6.697	6.672	6.648	6.624	6.601	6.579	6.558
1.6	6.538	6.518	6.499	6.480	6.462	6.445	6.429	6.413	6.398	6.383
1.7	6.369	6.355	6.342	6.330	6.318	6.307	6.296	6.286	6.276	6.267
1.8	6.258	6.250	6.242	6.235	6.228	6.222	6.216	6.211	6.206	6.201
1.9	6.197	6.193	6.190	6.187	6.185	6.183	6.181	6.180	6.179	6.179
2.0	6.178	6.179	6.179	6.180	6.182	6.184	6.186	6.188	6.191	6.194
2.1	6.198	6.201	6.206	6.210	6.215	6.220	6.226	6.231	6.238	6.244
2.2	6.251	6.258	6.265	6.273	6.281	6.289	6.298	6.307	6.316	6.326
2.3	6.336	6.346	6.356	6.367	6.378	6.389	6.401	6.413	6.425	6.438
2.4	6.450	6.463	6.477	6.490	6.504	6.518	6.533	6.548	6.563	6.578
2.5	6.594	6.609	6.626	6.642	6.659	6.676	6.693	6.710	6.728	6.746
2.6	6.765	6.783	6.802	6.821	6.841	6.861	6.881	6.901	6.921	6.942
2.7	6.963	6.985	7.007	7.029	7.051	7.073	7.096	7.119	7.143	7.166
2.8	7.190	7.214	7.239	7.264	7.289	7.314	7.340	7.366	7.392	7.418
2.9	7.445	7.472	7.500	7.527	7.555	7.584	7.612	7.641	7.670	7.700
3.0	7.729	7.760	7.790	7.821	7.852	7.883	7.914	7.946	7.979	8.011
3.1	8.044	8.077	8.111	8.144	8.179	8.213	8.248	8.283	8.318	8.354
3.2	8.390	8.427	8.463	8.500	8.538	8.576	8.614	8.652	8.691	8.730
3.3	8.770	8.810	8.850	8.890	8.931	8.973	9.014	9.056	9.099	9.142
3.4	9.185	9.228	9.272	9.317	9.361	9.406	9.452	9.498	9.544	9.591
3.5	9.638	9.685	9.733	9.781	9.830	9.879	9.929	9.979	10.029	10.080
3.6	10.131	10.183	10.235	10.287	10.340	10.394	10.448	10.502	10.557	10.612
3.7	10.668	10.724	10.781	10.838	10.896	10.954	11.012	11.072	11.131	11.191
3.8	11.252	11.313	11.375	11.437	11.500	11.563	11.627	11.691	11.756	11.821
3.9	11.887	11.954	12.021	12.089	12.157	12.225	12.295	12.365	12.435	12.506
4.0	12.578	12.651	12.723	12.797	12.871	12.946	13.022	13.098	13.174	13.252
4.1	13.330	13.409	13.488	13.568	13.649	13.730	13.812	13.895	13.979	14.063
4.2	14.148	14.233	14.320	14.407	14.495	14.584	14.673	14.673	14.854	14.946
4.3	15.039	15.132	15.226	15.321	15.417	15.514	15.611	15.709	15.809	15.909
4.4	16.009	16.111	16.214	16.318	16.422	16.527	16.634	16.741	16.849	16.958
4.5	17.068	17.180	17.292	17.405	17.519	17.634	17.750	17.867	17.985	18.104
4.6	18.225	18.346	18.468	18.592	18.717	18.842	18.969	19.097	19.226	19.357
4.7	19.488	19.621	19.755	19.890	20.026	20.164	20.303	20.443	20.584	20.727
4.8	20.871	21.016	21.163	21.310	21.460	21.610	21.763	21.916	22.071	22.227
4.9	22.385	22.544	22.705	22.867	23.031	23.196	23.363	23.531	23.701	23.872
5.0	24.046	24.220	24.397	24.575	24.754	24.936	25.119	25.304	25.491	25.679
5.1	25.869	26.061	26.255	26.451	26.648	26.847	27.049	27.252	27.457	27.665

[A] Adapted from Bi, J., Ennis, D. M., and O'Mahony, M., "How to Estimate and Use the Variance of *d'* from Difference Tests," *Journal of Sensory Studies*, 12, 1997, pp. 87–104.

TABLE X1.3 Observed Choice Proportions, p_c, ($\times 10^4$) as Function of *d'* for Duo-Trio Test[A]

NOTE 1—Find the entry in the table closest to the choice proportion observed in the test. Read the estimated value of *d'* from the corresponding row and column headings.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5000	5000	5000	5001	5001	5002	5003	5005	5006	5007
0.1	5009	5011	5013	5015	5018	5021	5023	5026	5030	5033
0.2	5037	5040	5044	5048	5053	5057	5062	5066	5071	5077
0.3	5082	5087	5093	5099	5105	5111	5117	5124	5131	5137
0.4	5144	5152	5159	5166	5174	5182	5190	5198	5206	5215
0.5	5223	5232	5241	5250	5259	5269	5278	5288	5298	5308
0.6	5318	5328	5339	5349	5360	5371	5382	5393	5404	5415
0.7	5427	5438	5450	5462	5474	5486	5498	5511	5523	5536
0.8	5548	5561	5574	5587	5600	5614	5627	5641	5654	5668
0.9	5682	5695	5709	5724	5738	5752	5766	5781	5795	5810
1.0	5825	5840	5854	5869	5884	5900	5915	5930	5945	5961
1.1	5976	5992	6007	6023	6039	6055	6071	6087	6102	6119
1.2	6135	6151	6167	6183	6200	6216	6232	6249	6265	6282
1.3	6298	6315	6331	6348	6365	6381	6398	6415	6432	6448
1.4	6465	6482	6499	6516	6533	6550	6567	6584	6601	6618
1.5	6635	6652	6669	6686	6703	6720	6737	6754	6771	6788
1.6	6805	6822	6839	6856	6873	6890	6907	6923	6940	6957
1.7	6974	6991	7008	7025	7042	7058	7075	7092	7109	7125
1.8	7142	7159	7175	7192	7208	7225	7241	7258	7274	7291
1.9	7307	7323	7340	7356	7372	7388	7404	7420	7436	7452
2.0	7468	7484	7500	7516	7531	7547	7563	7578	7594	7610
2.1	7625	7640	7656	7671	7686	7701	7717	7732	7747	7762
2.2	7777	7791	7806	7821	7836	7850	7865	7879	7894	7908
2.3	7923	7937	7951	7965	7979	7993	8007	8021	8035	8049
2.4	8062	8076	8090	8103	8117	8130	8143	8156	8170	8183
2.5	8196	8209	8222	8235	8247	8260	8273	8285	8298	8310
2.6	8323	8335	8347	8360	8372	8384	8396	8408	8420	8431
2.7	8443	8455	8466	8478	8489	8501	8512	8523	8535	8546
2.8	8557	8568	8579	8590	8600	8611	8622	8632	8643	8653
2.9	8664	8674	8685	8695	8705	8715	8725	8735	8745	8755
3.0	8765	8774	8784	8794	8803	8813	8822	8831	8841	8850
3.1	8859	8868	8877	8886	8895	8904	8913	8921	8930	8939
3.2	8947	8956	8964	8973	8981	8989	8997	9006	9014	9022
3.3	9030	9038	9046	9053	9061	9069	9077	9084	9092	9099
3.4	9107	9114	9121	9129	9136	9143	9150	9157	9164	9171
3.5	9178	9185	9192	9199	9206	9212	9219	9225	9232	9239
3.6	9245	9251	9258	9264	9270	9276	9283	9289	9295	9301
3.7	9307	9313	9319	9324	9330	9336	9342	9347	9353	9359
3.8	9364	9370	9375	9381	9386	9391	9397	9402	9407	9412
3.9	9417	9423	9428	9433	9438	9443	9447	9452	9457	9462
4.0	9467	9471	9476	9481	9485	9490	9494	9499	9503	9508
4.1	9512	9517	9521	9525	9529	9534	9538	9542	9546	9550
4.2	9554	9558	9562	9566	9570	9574	9578	9582	9586	9589
4.3	9593	9597	9601	9604	9608	9612	9615	9619	9622	9626
4.4	9629	9633	9636	9639	9643	9646	9649	9653	9656	9659
4.5	9662	9665	9669	9672	9675	9678	9681	9684	9687	9690
4.6	9693	9696	9698	9701	9704	9707	9710	9713	9715	9718
4.7	9721	9723	9726	9729	9731	9734	9736	9739	9742	9744
4.8	9747	9749	9751	9754	9756	9759	9761	9763	9766	9768
4.9	9770	9772	9775	9777	9779	9781	9783	9786	9788	9790
5.0	9792	9794	9796	9798	9800	9802	9804	9806	9808	9810
5.1	9812	9814	9816	9817	9819	9821	9823	9825	9827	9828
5.2	9830	9832	9833	9835	9837	9839	9840	9842	9843	9845
5.3	9847	9848	9850	9851	9853	9854	9856	9857	9859	9860
5.6	9862	9863	9865	9866	9868	9869	9870	9872	9873	9874
5.7	9876	9877	9878	9880	9881	9882	9884	9885	9886	9887
5.8	9888	9890	9891	9892	9893	9894	9895	9897	9898	9899
5.9	9900	9901	9902	9903	9904	9905	9906	9907	9908	9909
6.0	9910	9911	9912	9913	9914	9915	9916	9917	9918	9919
6.1	9920	9921	9922	9922	9923	9924	9925	9926	9927	9928
6.2	9928	9929	9930	9931	9932	9932	9933	9934	9935	9935

[A] Adapted from Ennis, D. M., "The Power of Sensory Discrimination Methods," *Journal of Sensory Studies*, 8, 1993, pp. 353–370.

TABLE X1.4 *B* Values for Estimating Variance of *d'* Obtained From Duo-Trio Test[A]

NOTE 1—Enter the table in the row and column corresponding to the value of *d'* obtained from Table X1.3. The variance of *d'* is $S^2(d') = B/n$, where *n* is the sample size.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	*****	74025	18508	8228	4630	2964	2059	1514	1160	917
0.1	743.52	615.05	517.34	441.30	380.96	332.29	292.45	259.43	231.77	208.35
0.2	188.36	171.16	156.25	143.24	131.83	121.75	112.82	104.86	97.74	91.34
0.3	85.576	80.358	75.622	71.310	67.374	63.770	60.463	57.420	54.615	52.024
0.4	49.624	47.398	45.330	43.405	41.610	39.933	38.365	36.897	35.520	34.226
0.5	33.010	31.864	30.785	29.766	28.804	27.895	27.034	26.218	25.444	24.710
0.6	24.012	23.349	22.719	22.118	21.545	21.000	20.479	19.981	19.506	19.052
0.7	18.618	18.202	17.803	17.422	17.056	16.705	16.368	16.044	15.733	15.435
0.8	15.148	14.872	14.606	14.350	14.104	13.867	13.638	13.418	13.205	13.000
0.9	12.803	12.612	12.427	12.249	12.078	11.911	11.751	11.596	11.446	11.301
1.0	11.160	11.025	10.893	10.766	10.643	10.524	10.409	10.297	10.189	10.084
1.1	9.983	9.885	9.789	9.697	9.608	9.521	9.437	9.356	9.277	9.201
1.2	9.127	9.055	8.986	8.918	8.853	8.790	8.729	8.669	8.612	8.556
1.3	8.502	8.450	9.400	8.351	8.304	8.258	8.214	8.171	8.130	8.090
1.4	8.051	8.014	7.978	7.943	7.910	7.878	7.847	7.817	7.788	7.760
1.5	7.734	7.709	7.684	7.661	7.638	7.617	7.597	7.577	7.559	7.541
1.6	7.524	7.509	7.494	7.480	7.466	7.454	7.442	7.432	7.422	7.412
1.7	7.404	7.396	7.389	7.383	7.378	7.373	7.369	7.365	7.363	7.361
1.8	7.360	7.359	7.359	7.360	7.361	7.363	7.366	7.369	7.373	7.377
1.9	7.382	7.388	7.395	7.401	7.409	7.417	7.426	7.435	7.445	7.456
2.0	7.467	7.478	7.490	7.503	7.516	7.530	7.545	7.560	7.575	7.591
2.1	7.608	7.625	7.643	7.661	7.680	7.699	7.719	7.740	7.761	7.782
2.2	7.804	7.827	7.850	7.874	7.898	7.923	7.948	7.974	8.000	8.027
2.3	8.055	8.083	8.111	8.140	8.170	8.200	8.231	8.262	8.294	8.326
2.4	8.359	8.392	8.426	8.461	8.496	8.531	8.567	8.604	8.641	8.679
2.5	8.717	8.756	8.796	8.836	8.876	8.917	8.959	9.001	9.044	9.088
2.6	9.132	9.176	9.221	9.267	9.313	9.360	9.408	9.456	9.505	9.554
2.7	9.604	9.654	9.705	9.757	9.809	9.862	9.916	9.970	10.024	10.080
2.8	10.136	10.192	10.250	10.308	10.366	10.425	10.485	10.545	10.606	10.668
2.9	10.731	10.794	10.857	10.922	10.987	11.052	11.119	11.186	11.254	11.322
3.0	11.391	11.461	11.531	11.603	11.675	11.747	11.820	11.894	11.969	12.045
3.1	12.121	12.198	12.275	12.354	12.433	12.513	12.593	12.675	12.757	12.839
3.2	12.923	13.007	13.093	13.179	13.265	13.353	13.441	13.530	13.620	13.711
3.3	13.802	13.894	13.987	14.081	14.176	14.271	14.368	14.465	14.653	14.662
3.4	14.762	14.862	14.964	15.066	15.169	15.273	15.378	15.484	15.591	15.698
3.5	15.807	15.916	16.026	16.137	16.250	16.363	16.477	16.591	16.707	16.824
3.6	16.942	17.061	17.180	17.301	17.422	17.545	17.668	17.793	17.919	18.045
3.7	18.173	18.301	18.431	18.561	18.693	18.826	18.960	19.094	19.230	19.367
3.8	19.505	19.644	19.784	19.926	20.068	20.212	20.356	20.502	20.649	20.797
3.9	20.946	21.096	21.248	21.400	21.554	21.709	21.866	22.023	22.182	22.342
4.0	22.503	22.665	22.829	22.994	23.160	23.327	23.496	23.666	23.838	24.010
4.1	24.184	24.360	24.536	24.714	24.894	25.075	25.257	25.441	25.626	25.812
4.2	26.000	26.189	26.380	26.573	26.767	26.962	27.159	27.357	27.557	27.759
4.3	27.962	28.166	28.373	28.580	28.790	29.001	29.214	29.428	29.645	29.862
4.4	30.082	30.303	30.526	30.751	30.978	31.206	31.436	31.668	31.902	32.138
4.5	32.376	32.615	32.857	33.100	33.345	33.593	33.842	34.093	34.347	34.602
4.6	34.859	35.119	35.381	35.645	35.911	36.179	36.449	36.722	36.996	37.274
4.7	37.553	37.835	38.119	38.405	38.694	38.985	39.279	39.575	39.873	40.174
4.8	40.478	40.784	41.093	41.404	41.718	42.035	42.355	42.677	43.001	43.329
4.9	43.660	43.993	44.329	44.668	45.010	45.356	45.704	46.054	46.409	46.766
5.0	47.127	47.490	47.857	48.227	48.600	48.976	49.357	49.740	50.127	50.517
5.1	50.911	51.308	51.709	52.113	52.521	52.933	53.349	53.768	54.191	54.618

[A] Adapted from Bi, J., Ennis, D. M., and O'Mahony, M., "How to Estimate and Use the Variance of *d'* from Difference Tests," *Journal of Sensory Studies*, 12, 1997, pp. 87–104.

TABLE X1.5 Observed Choice Proportions, p_c, (×10^4) as Function of *d'* for 3-AFC Test[A]

NOTE 1—Find the entry in the table closest to the choice proportion observed in the test. Read the estimated value of *d'* from the corresponding row and column headings.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	3333	3362	3390	3418	3447	3475	3504	3533	3562	3591
0.1	3620	3649	3678	3707	3737	3766	3795	3825	3855	3884
0.2	3914	3944	3974	4003	4033	4063	4093	4124	4154	4184
0.3	4214	4244	4275	4305	4336	4366	4396	4427	4458	4488
0.4	4519	4549	4580	4611	4641	4672	4703	4734	4764	4795
0.5	4826	4857	4888	4918	4949	4980	5011	5042	5072	5103
0.6	5134	5165	5195	5226	5257	5288	5318	5349	5380	5410
0.7	5441	5471	5502	5532	5563	5593	5624	5654	5684	5714
0.8	5745	5775	5805	5835	5865	5895	5925	5955	5985	6014
0.9	6044	6074	6103	6133	6162	6191	6221	6250	6279	6308
1.0	6337	6366	6395	6423	6452	6481	6509	6538	6566	6594

TABLE X1.5 *Continued*

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.1	6622	6650	6678	6706	6734	6761	6789	6816	6844	6871
1.2	6898	6925	6952	6979	7005	7032	7059	7085	7111	7137
1.3	7163	7189	7215	7241	7266	7292	7317	7342	7367	7392
1.4	7417	7442	7466	7491	7515	7539	7563	7587	7611	7635
1.5	7658	7682	7705	7728	7751	7774	7796	7819	7842	7864
1.6	7886	7908	7930	7952	7973	7995	8016	8037	8058	8079
1.7	8100	8121	8141	8162	8182	8202	8222	8242	8261	8281
1.8	8300	8319	8339	8357	8376	8395	8413	8432	8450	8468
1.9	8486	8504	8522	8539	8556	8574	8591	8608	8624	8641
2.0	8658	8674	8690	8706	8722	8738	8754	8769	8785	8800
2.1	8815	8830	8845	8860	8874	8889	8903	8917	8931	8945
2.2	8959	8973	8986	9000	9013	9026	9039	9052	9065	9077
2.3	9090	9102	9114	9127	9138	9150	9162	9174	9185	9197
2.4	9208	9219	9230	9241	9252	9262	9273	9283	9293	9304
2.5	9314	9324	9333	9343	9353	9362	9372	9381	9390	9399
2.6	9408	9417	9426	9434	9443	9451	9460	9468	9476	9484
2.7	9492	9500	9508	9515	9523	9530	9538	9545	9552	9559
2.8	9566	9573	9580	9587	9593	9600	9606	9613	9619	9625
2.9	9631	9637	9643	9649	9655	9661	9666	9672	9677	9683
3.0	9688	9693	9698	9703	9709	9713	9718	9723	9728	9733
3.1	9737	9742	9746	9751	9755	9759	9764	9768	9772	9776
3.2	9780	9784	9788	9791	9795	9799	9802	9806	9809	9813
3.3	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845
3.4	9848	9850	9853	9856	9859	9861	9864	9867	9869	9872
3.5	9874	9877	9879	9881	9884	9886	9888	9890	9892	9894
3.6	9897	9899	9901	9903	9904	9906	9908	9910	9912	9914
3.7	9915	9917	9919	9920	9922	9924	9925	9927	9928	9930
3.8	9931	9932	9934	9935	9937	9938	9939	9940	9942	9943
3.9	9944	9945	9946	9948	9949	9950	9951	9952	9953	9954
4.0	9955	9956	9957	9958	9959	9960	9961	9961	9962	9963
4.1	9964	9965	9965	9966	9967	9968	9968	9969	9970	9970
4.2	9971	9972	9972	9973	9974	9974	9975	9975	9976	9977
4.3	9977	9978	9978	9979	9979	9980	9980	9981	9981	9981
4.4	9982	9982	9983	9983	9984	9984	9984	9985	9985	9985
4.5	9986	9986	9986	9987	9987	9987	9988	9988	9988	9989
4.6	9989	9989	9989	9990	9990	9990	9990	9991	9991	9991
4.7	9991	9992	9992	9992	9992	9992	9993	9993	9993	9993
4.8	9993	9993	9994	9994	9994	9994	9994	9994	9995	9995
4.9	9995	9995	9995	9995	9995	9995	9996	9996	9996	9996
5.0	9996	9996	9996	9996	9996	9996	9997	9997	9997	9997
5.1	9997	9997	9997	9997	9997	9997	9997	9997	9998	9998
5.2	9998	9998	9998	9998	9998	9998	9998	9998	9998	9998
5.3	9998	9998	9998	9998	9998	9998	9999	9999	9999	9999

[A] Adapted from Ennis, D. M., "The Power of Sensory Discrimination Methods," *Journal of Sensory Studies*, 8, 1993, pp. 353–370.

TABLE X1.6 The *B* Values for Estimating Variance of *d'* Obtained From 3-AFC Test[A]

NOTE 1—Enter the table in the row and column corresponding to the value of *d'* obtained from Table X1.5. The variance of *d'* is $S^2(d') = B/n$, where *n* is the sample size.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	2.7925	2.7862	2.7801	2.7740	2.7681	2.7624	2.7568	2.7513	2.7460	2.7408
0.1	2.7357	2.7307	2.7259	2.7212	2.7167	2.7123	2.7080	2.7038	2.6998	2.6959
0.2	2.6921	2.6884	2.6849	2.6814	2.6782	2.6750	2.6719	2.6690	2.6662	2.6635
0.3	2.6610	2.6585	2.6562	2.6540	2.6519	2.6499	2.6481	2.6464	2.6448	2.6433
0.4	2.6419	2.6406	2.6395	2.6384	2.6375	2.6367	2.6360	2.6355	2.6350	2.6347
0.5	2.6344	2.6343	2.6343	2.6345	2.6347	2.6351	2.6355	2.6361	2.6368	2.6376
0.6	2.6385	2.6396	2.6407	2.6420	2.6434	2.6449	2.6465	2.6483	2.6501	2.6521
0.7	2.6542	2.6564	2.6587	2.6611	2.6637	2.6664	2.6692	2.6721	2.6751	2.6783
0.8	2.6815	2.6949	2.6884	2.6921	2.6958	2.6997	2.7037	2.7079	2.7121	2.7165
0.9	2.7210	2.7256	2.7304	2.7353	2.7403	2.7454	2.7507	2.7561	2.7616	2.7673
1.0	2.7731	2.7790	2.7851	2.7913	2.7976	2.8041	2.8107	2.8175	2.8244	2.8314
1.1	2.8386	2.8459	2.8534	2.8610	2.8688	2.8767	2.8847	2.8930	2.9013	2.9098
1.2	2.9185	2.9273	2.9363	2.9454	2.9547	2.9642	2.9738	2.9836	2.9936	3.0037
1.3	3.0140	3.0244	3.0351	3.0459	3.0569	3.0680	3.0794	3.0909	3.1026	3.1145
1.4	3.1265	3.1388	3.1512	3.1639	3.1767	3.1898	3.2030	3.2164	3.2301	3.2439
1.5	3.2580	3.2722	3.2867	3.3014	3.3163	3.3314	3.3468	3.3623	3.3781	3.3942
1.6	3.4104	3.4269	3.4437	3.4607	3.4779	3.4954	3.5131	3.5311	3.5493	3.5678
1.7	3.5866	3.6056	3.6249	3.6445	3.6643	3.6845	3.7049	3.7256	3.7466	3.7679
1.8	3.7895	3.8114	3.8336	3.8561	3.8790	3.9021	3.9256	3.9494	3.9735	3.9980
1.9	4.0229	4.0480	4.0736	4.0995	4.1257	4.1523	4.1793	4.2067	4.2344	4.2626
2.0	4.2911	4.3201	4.3494	4.3792	4.4093	4.4399	4.4710	4.5025	4.5344	4.5667
2.1	4.5996	4.6328	4.6666	4.7008	4.7356	4.7708	4.8065	4.8427	4.8794	4.9167
2.2	4.9545	4.9928	5.0317	5.0711	5.1111	5.1517	5.1929	5.2346	5.2769	5.3199
2.3	5.3635	5.4077	5.4525	5.4981	5.5442	5.5910	5.6386	5.6868	5.7357	5.7853

TABLE X1.6 *Continued*

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.4	5.8357	5.8868	5.9386	5.9912	6.0446	6.0988	6.1538	6.2096	6.2662	6.3236
2.5	6.3819	6.4411	6.5012	6.5622	6.6241	6.6869	6.7507	6.8154	6.8811	6.9479
2.6	7.0156	7.0843	7.1542	7.2250	7.2970	7.3701	7.4443	7.5196	7.5961	7.6737
2.7	7.7526	7.8327	7.9141	7.9967	8.0806	8.1658	8.2524	8.3403	8.4296	8.5204
2.8	8.6125	8.7061	8.8012	8.8979	8.9960	9.0957	9.1971	9.3000	9.4047	9.5110
2.9	9.619	9.729	9.840	9.954	10.069	10.186	10.305	10.426	10.549	10.674
3.0	10.801	10.930	11.062	11.195	11.331	11.469	11.609	11.752	11.897	12.045
3.1	12.195	12.347	12.503	12.660	12.821	12.984	13.150	13.319	13.491	13.666
3.2	13.844	14.025	14.209	14.396	14.587	14.781	14.978	15.179	15.383	15.591
3.3	15.802	16.018	16.237	16.460	16.687	16.918	17.154	17.393	17.637	17.886
3.4	18.138	18.396	18.658	18.925	19.196	19.473	19.755	20.042	20.334	20.632
3.5	20.936	21.245	21.559	21.880	22.206	22.539	22.878	23.224	23.575	23.934
3.6	24.299	24.672	25.051	25.438	25.832	26.234	26.643	27.061	27.486	27.920
3.7	28.362	29.813	29.273	29.741	30.218	30.706	31.203	31.710	32.225	32.752
3.8	33.290	33.838	34.397	34.968	35.549	36.142	36.748	37.365	37.996	38.638
3.9	39.295	39.964	40.647	41.343	42.055	42.781	43.522	44.278	45.051	45.837
4.0	46.642	47.463	48.301	49.157	50.031	50.925	51.835	52.765	53.716	54.686
4.1	55.676	56.687	57.722	58.778	59.856	60.959	62.083	63.232	64.406	65.609
4.2	66.835	68.085	69.363	70.679	72.011	73.378	74.776	76.203	77.663	79.153
4.3	80.676	82.231	83.826	85.456	87.122	88.824	90.568	92.347	94.168	96.029
4.4	97.94	99.89	101.87	103.91	106.00	108.13	110.30	112.54	114.82	117.15
4.5	119.54	122.00	124.49	127.06	129.67	132.36	135.11	137.91	140.79	143.72
4.6	146.75	149.83	153.00	156.23	159.55	162.92	166.39	169.94	173.60	177.32
4.7	181.14	185.06	189.07	193.16	197.35	201.65	206.08	210.59	215.24	219.93
4.8	224.81	229.85	234.91	240.13	245.52	251.01	256.65	262.39	268.29	274.38
4.9	280.57	296.97	293.44	300.11	307.05	314.10	321.37	328.68	336.35	344.18
5.0	352.08	360.42	368.70	377.37	386.31	395.30	404.63	414.26	423.99	433.91
5.1	444.34	454.81	465.84	477.06	488.24	500.04	512.35	524.87	537.15	550.34

[A] Adapted from Bi, J., Ennis, D. M., and O'Mahony, M., "How to Estimate and Use the Variance of *d'* from Difference Tests," *Journal of Sensory Studies*, 12, 1997, pp. 87–104.

TABLE X1.7 Observed Choice Proportions, p_c, (×10^4) as Function of *d'* for 2-AFC Test[A]

NOTE 1—Find the entry in the table closest to the choice proportion observed in the test. Read the estimated value of *d'* from the corresponding row and column headings.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5000	5028	5056	5085	5113	5141	5169	5197	5226	5254
0.1	5282	5310	5338	5366	5394	5422	5450	5478	5506	5534
0.2	5562	5590	5618	5646	5674	5702	5729	5757	5785	5812
0.3	5840	5868	5895	5923	5950	5977	6005	6032	6059	6086
0.4	6114	6141	6168	6195	6221	6248	6275	6302	6329	6355
0.5	6382	6408	6434	6461	6487	6513	6539	6565	6591	6617
0.6	6643	6669	6695	6720	6746	6771	6796	6822	6847	6872
0.7	6897	6922	6947	6971	6996	7021	7045	7069	7094	7118
0.8	7142	7166	7190	7214	7237	7261	7284	7308	7331	7354
0.9	7377	7400	7423	7446	7469	7491	7514	7536	7558	7580
1.0	7602	7624	7646	7668	7689	7711	7732	7754	7775	7796
1.1	7817	7837	7858	7879	7899	7919	7940	7960	7980	8000
1.2	8019	8039	8058	8078	8097	8116	8135	8154	8173	8192
1.3	8210	8229	8247	8265	8283	8301	8319	8337	8354	8372
1.4	8389	8406	8423	8440	8457	8474	8491	8507	8523	8540
1.5	8556	8572	8588	8603	8619	8635	8650	8665	8681	8696
1.6	8711	8725	8740	8755	8769	8783	8798	8812	8826	8840
1.7	8853	8867	8881	8894	8907	8920	8933	8946	8959	8972
1.8	8985	8997	9009	9022	9034	9046	9058	9070	9081	9093
1.9	9104	9116	9127	9138	9149	9160	9171	9182	9193	9203
2.0	9214	9224	9234	9244	9254	9264	9274	9284	9293	9303
2.1	9312	9321	9331	9340	9349	9358	9367	9375	9384	9393
2.2	9401	9409	9418	9426	9434	9442	9450	9458	9465	9473
2.3	9481	9488	9495	9503	9510	9517	9524	9531	9538	9545
2.4	9552	9558	9565	9571	9578	9584	9590	9596	9603	9609
2.5	9615	9620	9626	9632	9638	9643	9649	9654	9659	9665
2.6	9670	9675	9680	9685	9690	9695	9700	9705	9710	9714
2.7	9719	9723	9728	9732	9737	9741	9745	9749	9753	9757
2.8	9761	9765	9769	9773	9777	9781	9784	9788	9791	9795
2.9	9798	9802	9805	9809	9812	9815	9818	9821	9824	9828
3.0	9831	9833	9836	9839	9842	9845	9848	9850	9853	9856
3.1	9858	9861	9863	9866	9868	9870	9873	9875	9877	9880
3.2	9882	9884	9886	9888	9890	9892	9894	9896	9898	9900
3.3	9902	9904	9906	9907	9909	9911	9912	9914	9916	9917
3.4	9919	9921	9922	9924	9925	9926	9928	9929	9931	9932
3.5	9933	9935	9936	9937	9938	9940	9941	9942	9943	9944
3.6	9945	9947	9948	9949	9950	9951	9952	9953	9954	9955
3.7	9956	9956	9957	9958	9959	9960	9961	9962	9962	9963

TABLE X1.7 *Continued*

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
3.8	9964	9965	9965	9966	9967	9968	9968	9969	9970	9970
3.9	9971	9972	9972	9973	9973	9974	9974	9975	9976	9976
4.0	9977	9977	9978	9978	9979	9979	9980	9980	9980	9981
4.1	9981	9982	9982	9983	9983	9983	9984	9984	9984	9985
4.2	9985	9985	9986	9986	9986	9987	9987	9987	9988	9988
4.3	9988	9988	9989	9989	9989	9990	9990	9990	9990	9990
4.4	9991	9991	9991	9991	9992	9992	9992	9992	9992	9993
4.5	9993	9993	9993	9993	9993	9994	9994	9994	9994	9994
4.6	9994	9994	9995	9995	9995	9995	9995	9995	9995	9995
4.7	9996	9996	9996	9996	9996	9996	9996	9996	9996	9996
4.8	9997	9997	9997	9997	9997	9997	9997	9997	9997	9997
4.9	9997	9997	9997	9998	9998	9998	9998	9998	9998	9998
5.0	9998	9998	9998	9998	9998	9998	9998	9998	9998	9998
5.1	9998	9998	9999	9999	9999	9999	9999	9999	9999	9999

[A] Adapted from Ennis, D. M., "The Power of Sensory Discrimination Methods," *Journal of Sensory Studies*, 8, 1993, pp. 353–370.

TABLE X1.8 The *B* Values for Estimating Variance of *d'* Obtained From 2-AFC Test[A]

NOTE 1—Enter the table in the row and column corresponding to the value of *d'* obtained from Table X1.7. The variance of *d'* is $S^2(d') = B/n$, where *n* is the sample size.

d'	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	3.1416	3.1417	3.1418	3.1421	3.1425	3.1430	3.1436	3.1444	3.1452	3.1462
0.1	3.1473	3.1485	3.1498	3.1513	3.1528	3.1545	3.1562	3.1581	3.1601	3.1623
0.2	3.1645	3.1669	3.1694	3.1720	3.1747	3.1775	3.1805	3.1835	3.1867	3.1900
0.3	3.1934	3.1970	3.2007	3.2045	3.2084	3.2124	3.2166	3.2209	3.2253	3.2298
0.4	3.2345	3.2392	3.2442	3.2492	3.2544	3.2597	3.2651	3.2706	3.2763	3.2821
0.5	3.2881	3.2942	3.3004	3.3067	3.3132	3.3198	3.3266	3.3335	3.3405	3.3477
0.6	3.3550	3.3624	3.3700	3.3778	3.3857	3.3937	3.4019	3.4102	3.4187	3.4273
0.7	3.4361	3.4450	3.4541	3.4633	3.4727	3.4822	3.4920	3.5018	3.5119	3.5220
0.8	3.5324	3.5429	3.5536	3.5645	3.5755	3.5867	3.5981	3.6096	3.6213	3.6332
0.9	3.6453	3.6576	3.6700	3.6826	3.6954	3.7084	3.7216	3.7350	3.7486	3.7624
1.0	3.7763	3.7905	3.8049	3.8195	3.8343	3.8492	3.8645	3.8799	3.8955	3.9113
1.1	3.9274	3.9437	3.9602	3.9770	3.9939	4.0111	4.0286	4.0462	4.0642	4.0823
1.2	4.1007	4.1194	4.1383	4.1574	4.1768	4.1965	4.2165	4.2367	4.2571	4.2779
1.3	4.2989	4.3202	4.3418	4.3637	4.3858	4.4083	4.4310	4.4541	4.4774	4.5011
1.4	4.5250	4.5493	4.5739	4.5989	4.6241	4.6497	4.6757	4.7019	4.7285	4.7555
1.5	4.7828	4.8105	4.8386	4.8670	4.8957	4.9249	4.9544	4.9844	5.0147	5.0454
1.6	5.0766	5.1081	5.1401	5.1724	5.2052	5.2385	5.2722	5.3063	5.3408	5.3759
1.7	5.4114	5.4473	5.4838	5.5207	5.5581	5.5960	5.6344	5.6734	5.7128	5.7528
1.8	5.7933	5.8343	5.8759	5.9181	5.9608	6.0041	6.0480	6.0925	6.1375	6.1832
1.9	6.2295	6.2764	6.3240	6.3722	6.4211	6.4706	6.5208	6.5717	6.6232	6.6755
2.0	6.7285	6.7823	6.8367	6.8919	6.9479	7.0047	7.0622	7.1206	7.1797	7.2397
2.1	7.3005	7.3622	7.4247	7.4881	7.5524	7.6176	7.6837	7.7508	7.8188	7.8877
2.2	7.9577	8.0286	8.1005	8.1735	8.2475	8.3226	8.3988	8.4760	8.5543	8.6338
2.3	8.7145	8.7963	8.8793	8.9635	9.0489	9.1356	9.2235	9.3128	9.4033	9.4952
2.4	9.588	9.683	9.779	9.876	9.975	10.076	10.178	10.281	10.386	10.492
2.5	10.601	10.710	10.822	10.935	11.050	11.166	11.285	11.405	11.527	11.651
2.6	11.777	11.904	12.034	12.166	12.299	12.435	12.573	12.713	12.856	13.000
2.7	13.147	13.296	13.448	13.601	13.758	13.917	14.078	14.242	14.408	14.578
2.8	14.750	14.924	15.102	15.282	15.466	15.652	15.841	16.034	16.230	16.428
2.9	16.631	16.836	17.045	17.257	17.473	17.692	17.915	18.142	18.373	18.607
3.0	18.846	19.088	19.335	19.586	19.841	20.100	20.364	20.632	20.905	21.183
3.1	21.465	21.752	22.044	22.342	22.644	22.952	23.265	23.583	23.908	24.237
3.2	24.573	24.915	25.262	25.616	25.977	26.343	26.716	27.096	27.482	27.876
3.3	28.277	28.685	29.100	29.523	29.953	30.392	30.838	31.292	31.755	32.227
3.4	32.706	33.196	33.694	34.201	34.718	35.244	35.780	36.327	36.883	37.451
3.5	38.028	38.617	39.217	39.828	40.450	41.085	41.732	42.392	43.063	43.748
3.6	44.447	45.158	45.884	46.623	47.377	48.146	48.929	49.728	50.544	51.374
3.7	52.221	53.085	53.966	54.864	55.782	56.716	57.669	58.641	59.632	60.644
3.8	61.678	62.731	63.805	64.903	66.021	67.163	68.328	69.516	70.729	71.968
3.9	73.231	74.522	75.838	77.181	78.552	79.954	81.384	82.844	84.335	85.852
4.0	87.41	88.99	90.62	92.27	93.96	95.68	97.45	99.25	101.09	102.97
4.1	104.88	106.84	108.85	110.89	112.98	115.12	117.30	119.53	121.81	124.14
4.2	126.52	128.95	131.44	133.97	136.58	139.23	141.95	144.72	147.55	150.46
4.3	153.42	156.46	159.55	162.72	165.96	169.28	172.68	176.14	179.68	183.32
4.4	187.03	190.84	194.72	198.70	202.76	206.92	211.18	215.54	219.99	224.55
4.5	229.20	234.00	238.88	243.91	248.99	254.24	259.65	265.13	270.75	276.51
4.6	282.42	288.43	294.62	301.00	307.42	314.06	320.90	327.80	334.99	342.25
4.7	349.85	357.46	365.30	373.36	381.61	390.07	398.73	407.54	416.72	426.01
4.8	435.55	445.37	455.30	465.61	476.25	486.93	498.06	509.36	521.03	532.78
4.9	545.18	557.67	570.63	583.69	597.26	611.25	625.34	639.87	654.69	670.19
5.0	686.03	702.24	718.81	736.00	752.88	771.18	789.70	808.15	827.37	847.43
5.1	867.8	889.0	910.2	932.1	954.2	978.0	1001.6	1026.1	1051.9	1077.8

[A] Adapted from Bi, J., Ennis, D. M., and O'Mahony, M., "How to Estimate and Use the Variance of *d'* from Difference Tests," *Journal of Sensory Studies*, 12, 1997, pp. 87–104.

TABLE X1.9 *d'* Values for A/Not-A Method[A,B]

NOTE 1—Find the value of *d'* in the row corresponding to P_a = Proportion of "A" response for A sample and in the column corresponding to P_{na} = Proportion of "A" response for Not-A sample.

P_a	P_{na}									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
0.01	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.02	0.273	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.03	0.446	0.173	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.04	0.576	0.303	0.13	0	. . .	. . .	. . .	. . .	. . .	. . .
0.05	0.681	0.409	0.236	0.106	0	. . .	. . .	. . .	. . .	. . .
0.06	0.772	0.499	0.326	0.196	0.09	0	. . .	. . .	. . .	. . .
0.07	0.851	0.578	0.405	0.275	0.169	0.079	0	. . .	. . .	. . .
0.08	0.921	0.649	0.476	0.346	0.24	0.15	0.071	0	. . .	. . .
0.09	0.986	0.713	0.54	0.41	0.304	0.214	0.135	0.064	0	. . .
0.1	1.045	0.772	0.599	0.469	0.363	0.273	0.194	0.124	0.059	0
0.11	1.1	0.827	0.654	0.524	0.418	0.328	0.249	0.179	0.114	0.055
0.12	1.151	0.879	0.706	0.576	0.47	0.38	0.301	0.23	0.166	0.107
0.13	1.2	0.927	0.754	0.624	0.518	0.428	0.349	0.279	0.214	0.155
0.14	1.246	0.973	0.8	0.67	0.565	0.474	0.395	0.325	0.26	0.201
0.15	1.29	1.017	0.844	0.714	0.608	0.518	0.439	0.369	0.304	0.245
0.16	1.332	1.059	0.886	0.756	0.65	0.56	0.481	0.411	0.346	0.287
0.17	1.372	1.1	0.927	0.797	0.691	0.601	0.522	0.451	0.387	0.327
0.18	1.411	1.138	0.965	0.835	0.729	0.639	0.56	0.49	0.425	0.366
0.19	1.448	1.176	1.003	0.873	0.767	0.677	0.598	0.527	0.463	0.404
0.2	1.485	1.212	1.039	0.909	0.803	0.713	0.634	0.563	0.499	0.44
0.21	1.52	1.247	1.074	0.944	0.838	0.748	0.669	0.599	0.534	0.475
0.22	1.554	1.282	1.109	0.978	0.873	0.783	0.704	0.633	0.569	0.509
0.23	1.588	1.315	1.142	1.012	0.906	0.816	0.737	0.666	0.602	0.543
0.24	1.62	1.347	1.174	1.044	0.939	0.848	0.769	0.699	0.634	0.575
0.25	1.652	1.379	1.206	1.076	0.97	0.88	0.801	0.731	0.666	0.607
0.26	1.683	1.41	1.237	1.107	1.002	0.911	0.832	0.762	0.697	0.638
0.27	1.714	1.441	1.268	1.138	1.032	0.942	0.863	0.792	0.728	0.669
0.28	1.744	1.471	1.298	1.168	1.062	0.972	0.893	0.822	0.758	0.699
0.29	1.773	1.5	1.327	1.197	1.091	1.001	0.922	0.852	0.787	0.728
0.3	1.802	1.529	1.356	1.226	1.12	1.03	0.951	0.881	0.816	0.757
0.31	1.83	1.558	1.385	1.255	1.149	1.059	0.98	0.909	0.845	0.786
0.32	1.859	1.586	1.413	1.283	1.177	1.087	1.008	0.937	0.873	0.814
0.33	1.886	1.614	1.441	1.311	1.205	1.115	1.036	0.965	0.901	0.842
0.34	1.914	1.641	1.468	1.338	1.232	1.142	1.063	0.993	0.928	0.869
0.35	1.941	1.668	1.495	1.365	1.26	1.169	1.09	1.02	0.955	0.896
0.36	1.968	1.695	1.522	1.392	1.286	1.196	1.117	1.047	0.982	0.923
0.37	1.994	1.722	1.549	1.419	1.313	1.223	1.144	1.073	1.009	0.95
0.38	2.021	1.748	1.575	1.445	1.339	1.249	1.17	1.1	1.035	0.976
0.39	2.047	1.774	1.601	1.471	1.366	1.275	1.196	1.126	1.061	1.002
0.4	2.073	1.8	1.627	1.497	1.392	1.301	1.222	1.152	1.087	1.028
0.41	2.099	1.826	1.653	1.523	1.417	1.327	1.248	1.178	1.113	1.054
0.42	2.124	1.852	1.679	1.549	1.443	1.353	1.274	1.203	1.139	1.08
0.43	2.15	1.877	1.704	1.574	1.468	1.378	1.299	1.229	1.164	1.105
0.44	2.175	1.903	1.73	1.6	1.494	1.404	1.325	1.254	1.19	1.131
0.45	2.201	1.928	1.755	1.625	1.519	1.429	1.35	1.279	1.215	1.156
0.46	2.226	1.953	1.78	1.65	1.544	1.454	1.375	1.305	1.24	1.181
0.47	2.251	1.978	1.806	1.675	1.57	1.48	1.401	1.33	1.265	1.206
0.48	2.276	2.004	1.831	1.701	1.595	1.505	1.426	1.355	1.291	1.231
0.49	2.301	2.029	1.856	1.726	1.62	1.53	1.451	1.38	1.316	1.256
0.5	2.326	2.054	1.881	1.751	1.645	1.555	1.476	1.405	1.341	1.282
0.51	2.351	2.079	1.906	1.776	1.67	1.58	1.501	1.43	1.366	1.307
0.52	2.377	2.104	1.931	1.801	1.695	1.605	1.526	1.455	1.391	1.332
0.53	2.402	2.129	1.956	1.826	1.72	1.63	1.551	1.48	1.416	1.357
0.54	2.427	2.154	1.981	1.851	1.745	1.655	1.576	1.506	1.441	1.382
0.55	2.452	2.179	2.006	1.876	1.771	1.68	1.601	1.531	1.466	1.407
0.56	2.477	2.205	2.032	1.902	1.796	1.706	1.627	1.556	1.492	1.433
0.57	2.503	2.23	2.057	1.927	1.821	1.731	1.652	1.581	1.517	1.458
0.58	2.528	2.256	2.083	1.953	1.847	1.757	1.678	1.607	1.543	1.483
0.59	2.554	2.281	2.108	1.978	1.872	1.782	1.703	1.633	1.568	1.509
0.6	2.58	2.307	2.134	2.004	1.898	1.808	1.729	1.658	1.594	1.535
0.61	2.606	2.333	2.16	2.03	1.924	1.834	1.755	1.684	1.62	1.561
0.62	2.632	2.359	2.186	2.056	1.95	1.86	1.781	1.711	1.646	1.587
0.63	2.658	2.386	2.213	2.083	1.977	1.887	1.808	1.737	1.673	1.613
0.64	2.685	2.412	2.239	2.109	2.003	1.913	1.834	1.764	1.699	1.64
0.65	2.712	2.439	2.266	2.136	2.03	1.94	1.861	1.79	1.726	1.667
0.66	2.739	2.466	2.293	2.163	2.057	1.967	1.888	1.818	1.753	1.694
0.67	2.766	2.494	2.321	2.191	2.085	1.995	1.916	1.845	1.781	1.721
0.68	2.794	2.521	2.348	2.218	2.113	2.022	1.943	1.873	1.808	1.749

TABLE X1.9 *Continued*

0.69	2.822	2.55	2.377	2.247	2.141	2.051	1.972	1.901	1.837	1.777
0.7	2.851	2.578	2.405	2.275	2.169	2.079	2	1.929	1.865	1.806
0.71	2.88	2.607	2.434	2.304	2.198	2.108	2.029	1.958	1.894	1.835
0.72	2.909	2.637	2.464	2.334	2.228	2.138	2.059	1.988	1.924	1.864
0.73	2.939	2.667	2.494	2.363	2.258	2.168	2.089	2.018	1.954	1.894
0.74	2.97	2.697	2.524	2.394	2.288	2.198	2.119	2.048	1.984	1.925
0.75	3.001	2.728	2.555	2.425	2.319	2.229	2.15	2.08	2.015	1.956
0.76	3.033	2.76	2.587	2.457	2.351	2.261	2.182	2.111	2.047	1.988
0.77	3.065	2.793	2.62	2.49	2.384	2.294	2.215	2.144	2.08	2.02
0.78	3.099	2.826	2.653	2.523	2.417	2.327	2.248	2.177	2.113	2.054
0.79	3.133	2.86	2.687	2.557	2.451	2.361	2.282	2.211	2.147	2.088
0.8	3.168	2.895	2.722	2.592	2.486	2.396	2.317	2.247	2.182	2.123
0.81	3.204	2.932	2.759	2.629	2.523	2.433	2.354	2.283	2.219	2.159
0.82	3.242	2.969	2.796	2.666	2.56	2.47	2.391	2.32	2.256	2.197
0.83	3.281	3.008	2.835	2.705	2.599	2.509	2.43	2.359	2.295	2.236
0.84	3.321	3.048	2.875	2.745	2.639	2.549	2.47	2.4	2.335	2.276
0.85	3.363	3.09	2.917	2.787	2.681	2.591	2.512	2.442	2.377	2.318
0.86	3.407	3.134	2.961	2.831	2.725	2.635	2.556	2.485	2.421	2.362
0.87	3.453	3.18	3.007	2.877	2.771	2.681	2.602	2.531	2.467	2.408
0.88	3.501	3.229	3.056	2.926	2.82	2.73	2.651	2.58	2.516	2.457
0.89	3.553	3.28	3.107	2.977	2.871	2.781	2.702	2.632	2.567	2.508
0.9	3.608	3.335	3.162	3.032	2.926	2.836	2.757	2.687	2.622	2.563
0.91	3.667	3.395	3.222	3.091	2.986	2.896	2.817	2.746	2.682	2.622
0.92	3.731	3.459	3.286	3.156	3.05	2.96	2.881	2.81	2.746	2.687
0.93	3.802	3.53	3.357	3.226	3.121	3.031	2.952	2.881	2.817	2.757
0.94	3.881	3.609	3.436	3.305	3.2	3.11	3.031	2.96	2.896	2.836
0.95	3.971	3.699	3.526	3.396	3.29	3.2	3.121	3.05	2.986	2.926
0.96	4.077	3.804	3.631	3.501	3.396	3.305	3.226	3.156	3.091	3.032
0.97	4.207	3.935	3.762	3.631	3.526	3.436	3.357	3.286	3.222	3.162
0.98	4.38	4.107	3.935	3.804	3.699	3.609	3.53	3.459	3.395	3.335
0.99	4.653	4.38	4.207	4.077	3.971	3.881	3.802	3.731	3.667	3.608

P_a	P_{na}									
	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.2
0.11	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.12	0.052	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.13	0.1	0.049	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.14	0.146	0.095	0.046	0	. . .	. . .	. . .	. . .	. . .	. . .
0.15	0.19	0.139	0.09	0.044	0	. . .	. . .	. . .	. . .	. . .
0.16	0.232	0.181	0.132	0.086	0.042	0	. . .	. . .	. . .	. . .
0.17	0.272	0.221	0.172	0.126	0.082	0.04	0	. . .	. . .	. . .
0.18	0.311	0.26	0.211	0.165	0.121	0.079	0.039	0	. . .	. . .
0.19	0.349	0.297	0.248	0.202	0.159	0.117	0.076	0.037	0	. . .
0.2	0.385	0.333	0.285	0.239	0.195	0.153	0.113	0.074	0.036	0
0.21	0.42	0.369	0.32	0.274	0.23	0.188	0.148	0.109	0.071	0.035
0.22	0.454	0.403	0.354	0.308	0.264	0.222	0.182	0.143	0.106	0.069
0.23	0.488	0.436	0.388	0.341	0.298	0.256	0.215	0.177	0.139	0.103
0.24	0.52	0.469	0.42	0.374	0.33	0.288	0.248	0.209	0.172	0.135
0.25	0.552	0.5	0.452	0.406	0.362	0.32	0.28	0.241	0.203	0.167
0.26	0.583	0.532	0.483	0.437	0.393	0.351	0.311	0.272	0.235	0.198
0.27	0.614	0.562	0.514	0.468	0.424	0.382	0.341	0.303	0.265	0.229
0.28	0.644	0.592	0.544	0.497	0.454	0.412	0.371	0.333	0.295	0.259
0.29	0.673	0.622	0.573	0.527	0.483	0.441	0.401	0.362	0.325	0.288
0.3	0.702	0.651	0.602	0.556	0.512	0.47	0.43	0.391	0.353	0.317
0.31	0.731	0.679	0.631	0.584	0.541	0.499	0.458	0.42	0.382	0.346
0.32	0.759	0.707	0.659	0.613	0.569	0.527	0.486	0.448	0.41	0.374
0.33	0.787	0.735	0.686	0.64	0.597	0.555	0.514	0.475	0.438	0.402
0.34	0.814	0.763	0.714	0.668	0.624	0.582	0.542	0.503	0.465	0.429
0.35	0.841	0.79	0.741	0.695	0.651	0.609	0.569	0.53	0.493	0.456
0.36	0.868	0.817	0.768	0.722	0.678	0.636	0.596	0.557	0.519	0.483
0.37	0.895	0.843	0.795	0.748	0.705	0.663	0.622	0.584	0.546	0.51
0.38	0.921	0.87	0.821	0.775	0.731	0.689	0.649	0.61	0.572	0.536
0.39	0.947	0.896	0.847	0.801	0.757	0.715	0.675	0.636	0.599	0.562
0.4	0.973	0.922	0.873	0.827	0.783	0.741	0.701	0.662	0.625	0.588
0.41	0.999	0.947	0.899	0.853	0.809	0.767	0.727	0.688	0.65	0.614
0.42	1.025	0.973	0.924	0.878	0.835	0.793	0.752	0.713	0.676	0.64
0.43	1.05	0.999	0.95	0.904	0.86	0.818	0.778	0.739	0.702	0.665
0.44	1.076	1.024	0.975	0.929	0.885	0.843	0.803	0.764	0.727	0.691
0.45	1.101	1.049	1.001	0.955	0.911	0.869	0.829	0.79	0.752	0.716
0.46	1.126	1.075	1.026	0.98	0.936	0.894	0.854	0.815	0.777	0.741
0.47	1.151	1.1	1.051	1.005	0.961	0.919	0.879	0.84	0.803	0.766
0.48	1.176	1.125	1.076	1.03	0.986	0.944	0.904	0.865	0.828	0.791
0.49	1.201	1.15	1.101	1.055	1.011	0.969	0.929	0.89	0.853	0.817
0.5	1.227	1.175	1.126	1.08	1.036	0.994	0.954	0.915	0.878	0.842
0.51	1.252	1.2	1.151	1.105	1.062	1.02	0.979	0.94	0.903	0.867
0.52	1.277	1.225	1.177	1.13	1.087	1.045	1.004	0.966	0.928	0.892

TABLE X1.9 *Continued*

0.53	1.302	1.25	1.202	1.156	1.112	1.07	1.029	0.991	0.953	0.917
0.54	1.327	1.275	1.227	1.181	1.137	1.095	1.055	1.016	0.978	0.942
0.55	1.352	1.301	1.252	1.206	1.162	1.12	1.08	1.041	1.004	0.967
0.56	1.377	1.326	1.277	1.231	1.187	1.145	1.105	1.066	1.029	0.993
0.57	1.403	1.351	1.303	1.257	1.213	1.171	1.131	1.092	1.054	1.018
0.58	1.428	1.377	1.328	1.282	1.238	1.196	1.156	1.117	1.08	1.044
0.59	1.454	1.403	1.354	1.308	1.264	1.222	1.182	1.143	1.105	1.069
0.6	1.48	1.428	1.38	1.334	1.29	1.248	1.208	1.169	1.131	1.095
0.61	1.506	1.454	1.406	1.36	1.316	1.274	1.233	1.195	1.157	1.121
0.62	1.532	1.48	1.432	1.386	1.342	1.3	1.26	1.221	1.183	1.147
0.63	1.558	1.507	1.458	1.412	1.368	1.326	1.286	1.247	1.21	1.173
0.64	1.585	1.533	1.485	1.439	1.395	1.353	1.313	1.274	1.236	1.2
0.65	1.612	1.56	1.512	1.466	1.422	1.38	1.339	1.301	1.263	1.227
0.66	1.639	1.587	1.539	1.493	1.449	1.407	1.367	1.328	1.29	1.254
0.67	1.666	1.615	1.566	1.52	1.476	1.434	1.394	1.355	1.318	1.282
0.68	1.694	1.643	1.594	1.548	1.504	1.462	1.422	1.383	1.346	1.309
0.69	1.722	1.671	1.622	1.576	1.532	1.49	1.45	1.411	1.374	1.337
0.7	1.751	1.699	1.651	1.605	1.561	1.519	1.479	1.44	1.402	1.366
0.71	1.78	1.728	1.68	1.634	1.59	1.548	1.508	1.469	1.431	1.395
0.72	1.809	1.758	1.709	1.663	1.619	1.577	1.537	1.498	1.461	1.424
0.73	1.839	1.788	1.739	1.693	1.649	1.607	1.567	1.528	1.491	1.454
0.74	1.87	1.818	1.77	1.724	1.68	1.638	1.598	1.559	1.521	1.485
0.75	1.901	1.849	1.801	1.755	1.711	1.669	1.629	1.59	1.552	1.516
0.76	1.933	1.881	1.833	1.787	1.743	1.701	1.66	1.622	1.584	1.548
0.77	1.965	1.914	1.865	1.819	1.775	1.733	1.693	1.654	1.617	1.58
0.78	1.999	1.947	1.899	1.853	1.809	1.767	1.726	1.688	1.65	1.614
0.79	2.033	1.981	1.933	1.887	1.843	1.801	1.761	1.722	1.684	1.648
0.8	2.068	2.017	1.968	1.922	1.878	1.836	1.796	1.757	1.72	1.683
0.81	2.104	2.053	2.004	1.958	1.914	1.872	1.832	1.793	1.756	1.72
0.82	2.142	2.09	2.042	1.996	1.952	1.91	1.87	1.831	1.793	1.757
0.83	2.181	2.129	2.081	2.034	1.991	1.949	1.908	1.87	1.832	1.796
0.84	2.221	2.169	2.121	2.075	2.031	1.989	1.949	1.91	1.872	1.836
0.85	2.263	2.211	2.163	2.117	2.073	2.031	1.991	1.952	1.914	1.878
0.86	2.307	2.255	2.207	2.161	2.117	2.075	2.034	1.996	1.958	1.922
0.87	2.353	2.301	2.253	2.207	2.163	2.121	2.081	2.042	2.004	1.968
0.88	2.402	2.35	2.301	2.255	2.211	2.169	2.129	2.09	2.053	2.017
0.89	2.453	2.402	2.353	2.307	2.263	2.221	2.181	2.142	2.104	2.068
0.9	2.508	2.457	2.408	2.362	2.318	2.276	2.236	2.197	2.159	2.123
0.91	2.567	2.516	2.467	2.421	2.377	2.335	2.295	2.256	2.219	2.182
0.92	2.632	2.58	2.531	2.485	2.442	2.4	2.359	2.32	2.283	2.247
0.93	2.702	2.651	2.602	2.556	2.512	2.47	2.43	2.391	2.354	2.317
0.94	2.781	2.73	2.681	2.635	2.591	2.549	2.509	2.47	2.433	2.396
0.95	2.871	2.82	2.771	2.725	2.681	2.639	2.599	2.56	2.523	2.486
0.96	2.977	2.926	2.877	2.831	2.787	2.745	2.705	2.666	2.629	2.592
0.97	3.107	3.056	3.007	2.961	2.917	2.875	2.835	2.796	2.759	2.722
0.98	3.28	3.229	3.18	3.134	3.09	3.048	3.008	2.969	2.932	2.895
0.99	3.553	3.501	3.453	3.407	3.363	3.321	3.281	3.242	3.204	3.168

P_a	P_{na}									
	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.28	0.29	0.3
0.21	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.22	0.034	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.23	0.068	0.033	0	. . .	. . .	. . .	. . .	. . .	. . .	. . .
0.24	0.1	0.066	0.033	0	. . .	. . .	. . .	. . .	. . .	. . .
0.25	0.132	0.098	0.064	0.032	0	. . .	. . .	. . .	. . .	. . .
0.26	0.163	0.129	0.096	0.063	0.031	0	. . .	. . .	. . .	. . .
0.27	0.194	0.159	0.126	0.093	0.062	0.031	0	. . .	. . .	. . .
0.28	0.224	0.189	0.156	0.123	0.092	0.061	0.03	0	. . .	. . .
0.29	0.253	0.219	0.185	0.153	0.121	0.09	0.059	0.029	0	. . .
0.3	0.282	0.248	0.214	0.182	0.15	0.119	0.088	0.058	0.029	0
0.31	0.311	0.276	0.243	0.21	0.179	0.147	0.117	0.087	0.058	0.029
0.32	0.339	0.304	0.271	0.239	0.207	0.176	0.145	0.115	0.086	0.057
0.33	0.367	0.332	0.299	0.266	0.235	0.203	0.173	0.143	0.113	0.084
0.34	0.394	0.36	0.326	0.294	0.262	0.231	0.2	0.17	0.141	0.112
0.35	0.421	0.387	0.354	0.321	0.289	0.258	0.227	0.198	0.168	0.139
0.36	0.448	0.414	0.38	0.348	0.316	0.285	0.254	0.224	0.195	0.166
0.37	0.475	0.44	0.407	0.374	0.343	0.311	0.281	0.251	0.222	0.193
0.38	0.501	0.467	0.433	0.401	0.369	0.338	0.307	0.277	0.248	0.219
0.39	0.527	0.493	0.46	0.427	0.395	0.364	0.333	0.304	0.274	0.245
0.4	0.553	0.519	0.485	0.453	0.421	0.39	0.359	0.329	0.3	0.271
0.41	0.579	0.545	0.511	0.479	0.447	0.416	0.385	0.355	0.326	0.297
0.42	0.605	0.57	0.537	0.504	0.473	0.441	0.411	0.381	0.351	0.323
0.43	0.63	0.596	0.562	0.53	0.498	0.467	0.436	0.406	0.377	0.348
0.44	0.655	0.621	0.588	0.555	0.524	0.492	0.462	0.432	0.402	0.373
0.45	0.681	0.647	0.613	0.581	0.549	0.518	0.487	0.457	0.428	0.399
0.46	0.706	0.672	0.638	0.606	0.574	0.543	0.512	0.482	0.453	0.424