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Penrose Limit and Six-Dimensional Gauge Theories

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Abstract

We study the Penrose limit of the (p, q) fivebranes supergravity background. We consider the different phases of the worldvolume field theory and their weakly coupled descriptions. In the Penrose limit we get a solvable string theory and compute the spectrum. It corresponds to states of the six-dimensional worldvolume theory with large energy and large $U(1)$ charge. We comment on the RG behavior of the gauge theory.

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1 Introduction

The pp-wave background [1] is one of the solvable string theory backgrounds [2]. It has received much attention recently since it can be obtained by taking a Penrose limit of $\mathrm{AdS}_5 \times S^5$ [3]. Via the AdS/CFT correspondence (for a review, see [4]), it corresponds to a particular limit of $\mathcal{N} = 4$ SYM theory, where the number of colors N is taken to infinity, the Yang-Mills coupling g_{YM} is kept fixed and one considers operators with infinite R-charge J such that $J/\sqrt{N}$ is fixed. Since string theory is solvable one obtains the exact spectrum of anomalous dimensions of a particular set of operators of the gauge theory. This discussion has been generalized to a large number of other AdS/CFT examples.

It is natural to ask whether the Penrose limit is a useful tool to analyse particular sectors of non-conformal field theories [5, 6]. Non-local theories in the Penrose limit of the dual string background have been studied in [7]. In particular, the Penrose limit of the NS5-brane background is the Nappi-Witten background [8] (see also [9, 10]), which is solvable. The string spectrum and its interpretation via states of little string theory has been discussed in [7].

In this letter we will consider the (p, q) fivebranes in the Penrose limit. The worldvolume theory at low-energy is a six-dimensional $\mathcal{N} = (1, 1)$ SYM with gauge group $SU(s)$, where s is the greatest common divisor of p and q. The different (p, q) theories are characterized by different θ angles. They were considered in [11] (see also [12]). A dual string description has been studied in [13]. We will show that in the Penrose limit the string theory is solvable.

The letter is organized as follows. In section 2 we present the (p, q) fivebranes supergravity background in the decoupling limit and discuss the phase structure. In section 3 we take the Penrose limit and compute the string spectrum. It provided information on the high-energy spectrum of the field theory. Section 4 is devoted to the consideration of more general null geodesics and some discussion.

2 Field theory and the supergravity background

2.1 Field theory

We will consider the $(1, 1)$ supersymmetric six-dimensional gauge theories on the worldvolume of (p, q) fivebranes in Type IIB string theory. At low energy the theories reduce to super Yang-Mills (SYM) theories with gauge group $SU(s)$ where s is the greatest common divisor of p and

q. Consider the low-energy effective action

$$S_{eff} = \int d^6x \left(\frac{1}{g_{YM}^2} \mathrm{tr}\, F^2 + \theta_{YM} \mathrm{tr}\, F \wedge F \wedge F + \cdots \right) , \qquad (2.1)$$

where the dots correspond to higher dimension operators as well as the supersymmetric completion. Clearly, the low energy interactions dominated by the F^2 term cannot distinguish the different (p,q) theories with the same low-energy gauge group. However, it was argued in [11] that the theta term in (2.1), which is a consequence of $\pi_5(SU(s)) = \mathbf{Z}$, $s > 2$, is an observable that distinguishes the low-energy gauge theories. Our aim is to gain some information on the spectrum of these theories.

2.2 Supergravity and phase structure

Consider the supergravity (string) background

$$\begin{aligned} l_s^{-2} ds^2 &= \hat{h}^{-1/2} \left[-dx_0^2 + \sum_{i=1}^{5} dx_i^2 + \frac{q}{u^2}(du^2 + u^2 d\Omega_3^2) \right] , \\ e^{\phi} &= g \frac{\hat{h}^{-1}}{a_{\rm eff} u} , \quad \chi = \frac{1}{g} a_{\rm eff}^2 u^2 \hat{h} + \frac{p}{q} , \\ l_s^{-2} dB &= 2q\, \epsilon_3 , \quad l_s^{-2} dA = 2p\, \epsilon_3 , \end{aligned} \qquad (2.2)$$

where $\hat{h}^{-1} = 1 + a_{\rm eff}^2 u^2$, $a_{\rm eff}^2 = 2 l_{\rm eff}^2 / q$ and $g l_{eff}^2 = g_s l_s^2$. ϵ_3 is the volume 3-form of S^3. This background is obtained by taking a decoupling limit of the (p,q) fivebranes supergravity solution [13] [3].

String theory is weakly coupled if the effective string coupling is small $e^{\phi} < 1$. This implies that $g \ll 1$. We have a weakly coupled string description in the regime $g < a_{\rm eff} u < g^{-1}$.

Consider the low-energy regime $a_{\rm eff} u < g$. The effective string coupling is large and we should obtain a weakly coupled S-dual description. Assume $g \ll \frac{p}{q}$ which means $\chi = \frac{p}{q}$. Using the S-duality transformation

$$\tau \to \frac{a\tau + b}{c\tau + d}, \quad dB \to d\, dB + c\, dA, \quad dA \to b\, dB + a\, dA , \qquad (2.3)$$

with $ad - bc = 1$ and $cp + dq = 0$, we get the D5-branes background

$$ds^2 = l_s^2 \left(\frac{u}{(g_{YM}^2 s)^{1/2}} (-dx_0^2 + \sum_{i=1}^{5} dx_i^2) + \frac{(g_{YM}^2 s)^{1/2}}{u} (du^2 + u^2 d\Omega_3^2) \right) ,$$

[3]Here we correct the form of the RR 2-form potential A of [13].

$$
\begin{aligned}
e^{\phi} &= \frac{(g_{YM}^2 s u^2)^{1/2}}{s}, \quad \chi = \frac{a}{c}, \\
dB &= 0, \quad dA = 2 s l_s^2 \epsilon_3 \ .
\end{aligned} \tag{2.4}
$$

$(g_{YM}^2 s)^{1/2} = \frac{q^2 a_{\rm eff}}{sg}$ and we rescaled x by $\sqrt{\frac{q}{g_{\rm YM}^2 s}}$. s is the greatest common divisor of p and q. The S-dual string description is weakly coupled when $p \sim q \sim s$.

In the regime $(g_{\rm YM}^2 s)^{-1/2} < u < s(g_{\rm YM}^2 s)^{-1/2}$, the scalar curvature of (2.4) is small and the supergravity description is valid. When $u < (g_{\rm YM}^2 s)^{-1/2}$, the dimensionless effective gauge coupling $g_{\rm YM}^2 s u^2$ is small and the perturbative SYM theory description is valid.

In the high-energy regime $a_{\rm eff} u > g^{-1}$, the effective string coupling is large and we need to consider the S-dual description. Since $\chi = \frac{1}{g}$, g has to be rational in order to have a weakly coupled S-dual description. We rescale the coordinate

$$
x_0 \to \sqrt{q}\, x_0, \quad x_i \to \sqrt{q}\, x_i, \quad g a_{\rm eff} u = e^U . \tag{2.5}
$$

Using the S-duality transformation (2.3) with $c + gd = 0$ we get

$$
\begin{aligned}
& ds^2 = R^2 \left(-dx_0^2 + dx_i^2 + dU^2 + d\Omega_3^2 \right), \\
& e^{\phi} = c^2 e^{-U}, \quad \chi = \frac{a}{c}, \\
& dB = 2R^2 \epsilon_3, \quad dA = \chi\, dB \ ,
\end{aligned} \tag{2.6}
$$

where $R^2 = dq l_s^2$ and l_s^2 is rescaled by $\frac{g_s}{|c|}$. This is an NS5-branes metric with a constant RR scalar and RR 2-form potential.

To summarize, there are three different backgrounds which are relevant. At low energies it is the D5-branes background (2.4), at intermediate energies it is the (p, q) fivebranes background (2.2) and at high energies it is the NS5-branes background (2.6).

3 The Penrose limit

In the following we will take the Penrose limit of the three phases discussed in the previous section. The resulting backgrounds after an appropriate change of variables take the form

$$
\begin{aligned}
& ds^2 = -4dX^+ dX^- - y^2 (dX^+)^2 + d\vec{r}^2 + d\omega^2 + dy^2 + y^2 d\varphi^2, \\
& \chi = \text{const}, \qquad \phi = \text{const}, \\
& dB = a\, dX^+ \wedge dy_1 \wedge dy_2, \qquad dA = b\, dX^+ \wedge dy_1 \wedge dy_2 \ ,
\end{aligned} \tag{3.1}
$$

and

$$a = 2\cos\gamma, \quad b = 2\left(e^{-\phi}\sin\gamma + \chi\cos\gamma\right) \ . \tag{3.2}$$

a, b, γ are constants and $y_1 = y\sin\varphi, y_2 = y\cos\varphi$. It is straightforward to check that (3.1) with (3.2) is a solution to the Type IIB supergravity equations.

Consider the (p, q) fivebranes background (2.2). We take the metric on S^3 to be

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta\, d\phi_1^2 + \cos^2\theta\, d\phi_2^2 \ , \tag{3.3}$$

and rewrite the background in terms of the new coordinate

$$x_0 \to \sqrt{q}\, x_0, \quad x_i \to \sqrt{q}\, x_i, \quad u a_{\rm eff} = e^U. \tag{3.4}$$

The background becomes

$$\begin{aligned} ds^2 &= R^2 \hat{h}^{-1/2}\left[-dx_0^2 + \sum_{i=1}^{5} dx_i^2 + dU^2 + d\Omega_3^2\right] \ , \\ \chi &= \frac{1}{g} e^{2U}\hat{h} + \frac{p}{q} \ , \qquad\qquad e^{\phi} = g\frac{\hat{h}^{-1}}{e^U} \ , \\ dB &= 2R^2\, \epsilon_3 \ , \qquad dA = \frac{p}{q}\, dB \ , \end{aligned} \tag{3.5}$$

where $\hat{h}^{-1} = 1 + e^{2U}$ and $R^2 = q l_s^2$.

In order to take the Penrose limit we first consider a null geodesic. As a null geodesic we can take [4] $U = U_0 = \text{const}$, $x_i = \theta = 0$, $x_0 = \phi_2 = \lambda$, with λ the affine parameter. We define a coordinate transformation such that it reduces to the null geodesic in the large R limit;

$$\begin{aligned} & U = U_0 + (1+e^{2U_0})^{-1/4}\,\frac{\omega}{R}, \quad \theta = (1+e^{2U_0})^{-1/4}\,\frac{y}{R}, \\ & x_i = (1+e^{2U_0})^{-1/4}\,\frac{r_i}{R}, \\ & x_0 = (1+e^{2U_0})^{-1/4}\left(x^+ + \frac{x^-}{R^2}\right), \quad \phi_2 = (1+e^{2U_0})^{-1/4}\left(x^+ - \frac{x^-}{R^2}\right). \end{aligned} \tag{3.6}$$

By taking the limit $q \to \infty$ or equivalently $R \to \infty$, we obtain

$$\begin{aligned} ds^2 &= -4dX^+dX^- - y^2(dX^+)^2 + d\vec{r}^2 + d\omega^2 + dy^2 + y^2 d\phi_1^2, \\ \chi &= \frac{1}{g}\,\frac{e^{2U_0}}{1+e^{2U_0}} + \frac{p}{q} \ , \qquad\qquad e^{\phi} = g\frac{1+e^{2U_0}}{e^{U_0}} \ , \end{aligned}$$

[4] A more general class of null geodesics will be discussed in the discussion section.

$$dB \;=\; 2(1+e^{2U_0})^{-1/2}\, dX^+\wedge dy_1 \wedge dy_2\ , \qquad dA=\frac{p}{q}\, dB\ , \tag{3.7}$$

where

$$x^{\pm} = (1+e^{2U_0})^{\pm 1/4}\, X^{\pm}. \tag{3.8}$$

Consider next the Penrose limit of the D5-branes background (2.4) which provides a good description at low energy. Define the new coordinate

$$u\,\frac{(g^2_{\rm YM}s)^{1/2}}{s} = e^U, \quad x_{0,\cdots,5}\rightarrow (g^2_{\rm YM}s)^{1/2}x_{0,\cdots,5}\ . \tag{3.9}$$

The background takes the form

$$ds^2 = R^2 e^U\left(-dx_0^2+\sum dx_i^2 + dU^2 + d\Omega_3^2\right), \quad e^{\phi} = e^U\ , \tag{3.10}$$

where $R^2 = s l_s^2$. The Penrose limit is defined by

$$\begin{aligned} &U = U_0 + e^{-U_0/2}\,\frac{\omega}{R}, \quad \theta = e^{-U_0/2}\,\frac{y}{R}, \quad x_i = e^{-U_0/2}\,\frac{r_i}{R}, \\ &x_0 = e^{-U_0/2}\left(x^+ + \frac{x^-}{R^2}\right), \quad \phi_2 = e^{-U_0/2}\left(x^+ - \frac{x^-}{R^2}\right)\ , \end{aligned} \tag{3.11}$$

with $R\rightarrow\infty$. Using $x^{\pm} = e^{\pm U_0/2}X^{\pm}$ we obtain

$$\begin{aligned} &ds^2 = -4dX^+dX^- - y^2\left(dX^+\right)^2 + d\vec{r}^2 + d\omega^2 + d\vec{y}^2, \\ &e^{\phi} = e^{U_0}, \\ &dB = 0, \qquad dA = 2\,e^{-U_0} dX^+\wedge dy_1\wedge dy_2\ . \end{aligned} \tag{3.12}$$

Finally, consider the NS5-branes background (2.6) which is valid in the UV regime. The Penrose limit is defined by the coordinates transformation

$$\begin{aligned} &U = U_0 + \frac{\omega}{R}, \quad \theta = \frac{y}{R}, \quad x_i = \frac{r_i}{R}, \\ &x_0 = X^+ + \frac{X^-}{R^2}, \quad \phi_2 = X^+ - \frac{X^-}{R^2}\ , \end{aligned} \tag{3.13}$$

with $R\rightarrow\infty$. One gets

$$\begin{aligned} &ds^2 = -4dX^+dX^- - y^2\left(dX^+\right)^2 + d\vec{r}^2 + d\omega^2 + d\vec{y}^2, \\ &e^{\phi} = c^2 e^{-U_0}, \qquad \chi = \frac{a}{c}, \\ &dB = 2\,dX^+\wedge dy_1\wedge dy_2, \qquad dA = 2\chi dX^+\wedge dy_1\wedge dy_2\ . \end{aligned} \tag{3.14}$$

4 String spectrum

In this section, we quantize string theory on the pp-wave backgrounds obtained in the previous section. For simplicity we focus on bosonic states. The bosonic part of the Green-Schwarz action takes the form

$$2\pi\alpha' S_b = \int d\tau \int_0^{2\pi\alpha' p^+} d\sigma \left(\frac{1}{2} G_{MN}\, \partial_a X^M \partial^a X^N + B_{MN} \partial_\tau X^M \partial_\sigma X^N \right) . \tag{4.1}$$

Let us first consider the background (3.7). B is given by

$$B = 2\,(1 + e^{2U_0})^{-1/2} \mu y_2 dX^+ \wedge dy_1 \ , \tag{4.2}$$

where we rescaled $X^\pm \to \mu^{\pm 1} X^\pm$. In the light-cone gauge $X^+ = \tau$ the action reads

$$2\pi\alpha' \mathcal{L}_b = \frac{1}{2}\left[(\partial_a \vec{r})^2 + (\partial_a \omega)^2 + (\partial_a \vec{y})^2 - \mu^2 \vec{y}^2 \right] + 2\,(1 + e^{2U_0})^{-1/2} \mu\, y_2 \partial_\sigma y_1 \ . \tag{4.3}$$

$\vec{y}$ and ω are massless free bosons. The equations of motion for y_1, y_2 are given by

$$\begin{aligned} &\partial_a^2 y_1 + \mu^2\, y_1 + 2\,(1 + e^{2U_0})^{-1/2} \mu\, \partial_\sigma y_2 = 0, \\ &\partial_a^2 y_2 + \mu^2\, y_2 - 2\,(1 + e^{2U_0})^{-1/2} \mu\, \partial_\sigma y_1 = 0 \ . \end{aligned} \tag{4.4}$$

We solve these equations by Fourier expanding y_1, y_2

$$y_i(\sigma, \tau) = \Sigma_n \alpha_n^i(\tau) e^{in\sigma/\alpha' p^+} \ . \tag{4.5}$$

The coefficients read

$$\begin{aligned} \alpha_n^1(\tau) &= e^{-iw_n\tau} \tilde{\alpha}_n^+ + e^{+iw_{-n}\tau} \left(\tilde{\alpha}_{-n}^+\right)^* + e^{-iw_{-n}\tau} \tilde{\alpha}_n^- + e^{+iw_{+n}\tau} \left(\tilde{\alpha}_{-n}^-\right)^*, \\ \alpha_n^2(\tau) &= i \left[-e^{-iw_n\tau} \tilde{\alpha}_n^+ + e^{+iw_{-n}\tau} \left(\tilde{\alpha}_{-n}^+\right)^* + e^{-iw_{-n}\tau} \tilde{\alpha}_n^- - e^{+iw_{+n}\tau} \left(\tilde{\alpha}_{-n}^-\right)^* \right] \ , \end{aligned} \tag{4.6}$$

where

$$\omega_n = \sqrt{\mu^2 + \frac{n^2}{(\alpha\prime p^+)^2} + 2\,(1 + e^{2U_0})^{-1/2} \mu\, \frac{n}{\alpha' p^+}} \ . \tag{4.7}$$

Upon quantization, $\tilde{\alpha}_n^{\pm\dagger}$ become operators that create states of light-cone energy $\omega_{\pm n}$. Defining the oscillators

$$\tilde{\alpha}_n^\pm = \frac{1}{\sqrt{4p^+ \omega_{\pm n}}}\, \alpha_n^\pm \ , \tag{4.8}$$

we see that they obey the commutation relations

$$\left[\alpha_n^+, (\alpha_m^+)^\dagger \right] = \left[\alpha_n^-, (\alpha_m^-)^\dagger \right] = i\delta_{n,m} \ . \tag{4.9}$$

Thus, the bosonic part of the light-cone hamiltonian takes the form

$$2p^- = \sum_n \left(N_n^{r,w} \frac{|n|}{\alpha' p^+} + N_n^+ \omega_n + N_n^- \omega_{-n} \right) . \tag{4.10}$$

We would like to relate the string spectrum to states in the six-dimensional field theory. Note that

$$\frac{\partial}{\partial X^+} = \frac{\partial}{\partial x_0} + \frac{\partial}{\partial \phi_2}, \quad \frac{\partial}{\partial X^-} = \frac{(1+e^{2U_0})^{-1/2}}{R^2} \left(\frac{\partial}{\partial x_0} - \frac{\partial}{\partial \phi_2} \right) . \tag{4.11}$$

It follows from this that

$$\frac{2p^-}{\mu} = q^{1/2} E - J_V, \quad 2\mu p^+ = \frac{(1+e^{2U_0})^{-1/2}}{R^2} (q^{1/2} E + J_V) \ , \tag{4.12}$$

where we used

$$q^{1/2} E = i \frac{\partial}{\partial x_0}, \quad J_V = -i \frac{\partial}{\partial \phi_2} \ . \tag{4.13}$$

Here we define the energy E in terms of the generator of the translation of x_0 in (2.2). We thus find that the string spectrum with finite $p^{\mp}$ corresponds to states in the six-dimensional field theory with

$$q^{1/2} E, J_V \sim q, \quad \text{while } E - J_V = \text{finite} \ , \tag{4.14}$$

with $q \to \infty$. Note that for large q

$$R^2 \mu p^+ = (1+e^{2U_0})^{-1/2} J_V \ . \tag{4.15}$$

Thus, the light-cone energy takes the form

$$\frac{2p^-}{\mu} = \sum_n \left[(1+e^{2U_0})^{1/2} \frac{q}{J_V} N_n^{r,w} |n| + \sqrt{1 + (1+e^{2U_0}) \frac{q^2}{J_V^2} n^2 + 2 \frac{q}{J_V} n} \ (N_n^+ + N_{-n}^-) \right] . \tag{4.16}$$

Consider now symmetry of the pp-wave background (3.7). The (p,q) fivebranes background before the Penrose limit has an $SU(2)_L \times SU(2)_R$ isometry of S^3 corresponding to the R-symmetry of the field theory. In the parametrization (3.3), the manifest $U(1)$ isometries are

$$U(1)_{1,2} : \ \phi_{1,2} \to \phi_{1,2} + \text{const} \ . \tag{4.17}$$

They are identified as

$$U(1)_1 = U(1)_A, \quad U(1)_2 = U(1)_V \ , \tag{4.18}$$

where $U(1)_{V(A)}$ is the vector (axial) subgroup of $U(1)_L \times U(1)_R$. Denote

$$Y = (y_1 + i y_2)/2 = \sum_n \left[\frac{1}{\sqrt{4p^+ \omega_n}} \alpha_n^+ e^{-i\omega_n \tau + in\sigma/\alpha' p^+} + \frac{1}{\sqrt{4p^+ \omega_{-n}}} \left(\alpha_n^-\right)^\dagger e^{i\omega_{-n}\tau - in\sigma/\alpha' p^+} \right] \ . \tag{4.19}$$

This complex scalar carries a $U(1)_A$ charge while the massless scalars are neutral.

Recall that the string spectrum consists of states of the form $(\omega, r)(Y, \bar{Y})|0\rangle$. Thus, the corresponding states of the six-dimensional field theory have energies varying differently with n depending on whether they carry $U(1)_A$ or not.

The computation of the string spectrum for the background (3.12) is straightforward: since $dB = 0$, the bosonic part of the GS action in the light-cone gauge takes the form

$$2\pi\alpha'\mathcal{L}_b = \frac{1}{2}\left[(\partial_a \vec{r}')^2 + (\partial_a \omega')^2 + (\partial_a \vec{y}')^2 - \mu^2 \vec{y}'^2\right]. \tag{4.20}$$

The light-cone energy is

$$2p^- = \sum_n \left(N_n^{r,w} \frac{|n|}{\alpha' p^+} + N_n^y \sqrt{\mu^2 + \frac{n^2}{(\alpha' p^+)^2}} \right). \tag{4.21}$$

Note that

$$\frac{2p^-}{\mu} = s^{1/2}E - J_V, \quad 2p^+\mu = \frac{e^{-U_0}}{R^2}\left(s^{1/2}E + J_V\right), \tag{4.22}$$

and

$$R^2 p^+ \mu = e^{-U_0} J_V \ . \tag{4.23}$$

The light-cone energy reads

$$\frac{2p^-}{\mu} = \sum_n \left[e^{U_0} \frac{s}{J_V} N_n^{r,w} |n| + \sqrt{1 + e^{2U_0} \frac{s^2}{J_V^2} n^2} \ N_n^y \right] . \tag{4.24}$$

We see that string states that carry non-vanishing $U(1)_A$ charge have a different light-cone energy compared with neutral states which are like states of strings in a flat background.

Finally consider the UV description (3.14). The bosonic light-come energy is as that of little string theory (LST). The spectrum of little string theory in the Penrose limit was discussed in [7]. Note, however, that the background has now RR fields. Therefore, we expect the interactions to differ from those of LST.

5 Discussion

In this letter, we discussed the Penrose limit of (p, q) fivebranes and the the dual six-dimensional field theory. As in the case of [3], the limit yields a solvable string background, and the string computation provides information on a subsector of states of the field theory with large energy and large $U(1)$ charge. It would be nice to confirm the string computation from the viewpoint of the six-dimensional field theory. For this purpose, it could be useful to work with the matrix theory formulation which was proposed in [11].

We have focused on a particular class of null geodesics such that the energy scale U is constant. Thus we are probing the field theory at a particular energy scale. One could be interested in a more general null geodesic with a non constant U. This is relevant to the RG flow of the field theory. Consider null geodesics of the (p,q) fivebranes background (2.2). U must satisfy

$$\left(1+e^{2U}\right)^{1/2}\dot{U}=\xi \ . \tag{5.1}$$

Here $\cdot = d/d\lambda$ with λ the affine parameter of the null geodesic, and ξ is a constant. The solution reads

$$\sqrt{1+e^{2U}}-\tanh^{-1}\sqrt{1+e^{2U}}=\xi\lambda+\eta \ , \tag{5.2}$$

with η constant. When $\xi=0$ it reduces to the case considered in the previous section. The mass of the world-sheet scalar $\vec{r}$ in light-cone gauge is [1, 6]

$$m^2=-\tilde{h}^{1/4}\frac{d^2\tilde{h}^{-1/4}}{d\lambda^2}=\frac{\xi^2}{4}\frac{e^{2U}(e^{2U}-4)}{(1+e^{2U})^3} \ . \tag{5.3}$$

The resulting string background is not solvable. Note some interesting properties: first m^2 is x^+-dependent. The light-cone time-dependent mass in the context of holographic RG in Penrose limits was noted by several authors [14, 5, 6, 7]. Second, m^2 becomes negative in the IR regime $U\to 0$. This occurs also in the Penrose limits of Dp($p<5$) backgrounds [6]. However, in the IR the relevant background is (2.4). The mass of $\vec{r}$ in the IR is

$$m^2=\frac{\xi'^2}{4}\frac{1}{(\xi' x^+ +\eta')^2} \ , \tag{5.4}$$

where ξ',η' are constants. Thus, we have m^2 positive in the IR. It would be interesting to study the Penrose limit for these null geodesics in more detail and explore the RG behavior of the six-dimensional field theory.

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