# Simulation of Spherical Luneburg Lens Using Numerical Electrodynamic Methods 

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#### Abstract

This article discusses various ways of electromagnetic simulation and settings of the boundary conditions in the study of the spherical Luneburg lens using the Ansys EM software. We investigated the seven-layer spherical lens with finite element method, method of moments, hybrid boundary conditions, symmetrical boundary conditions. Numerical results are obtained and their analysis is presented.


## 1 Introduction

Development of multibeam and scanning antenna systems is an important area in the antenna technology, they are extensively used in radar and telecommunication complexes. When using the scanning in the antenna arrays the gain reduction effect appears, the width of the main lobe of the radiation pattern increases and growth of sidelobe level occur alongside the beam deviation from the normal. Antenna system with the ability to control the beam without these unwanted effects can be obtained using the Luneburg lens [1]. Lenses of inhomogeneous dielectric are known for a long time [2]. However, making of such a lens antenna is facing some technological difficulties and often the cost of materials for such structures is too high to commit design errors. In this regard, the need for correct and rapid enough electromagnetic simulation for analysis of structure under consideration and its further optimization is very strong.

To simplify the design and implementation of heterogeneous lenses in most cases, one can go from a smooth change in their electrodynamics parameters towards step permittivity profile. Thus, the challenge of rational choice of number of homogeneous layers, their thickness and dielectric permittivity in order to obtain the required field characteristics and minimize production costs becomes significant [2, 3, 4]. Simulation of such objects with an increase in the number of layers and lateral sizes of such structures is becoming substantially more complicated and requires large amounts of RAM and CPU resources, which in turn makes it difficult to design and optimize such objects. To solve the problem one can, e.g. use mathematical methods of calculation and analysis of such structures based on the use of tensor Green's functions as described in $[5,6,7,8,9]$. However, these methods require further development, e.g. when there is need to change the type of feed or insert some local inhomogeneities in the non-coordinate lens, i.e. allow solving very narrow class of radiation problems.

Now the simulation of multilayer structures can be carried out by special software for three-dimensional electromagnetic modeling, allowing the calculation of radiation pattern, gain, polarization characteristics, etc. This modern software provides a wide range of numerical methods for analysis of antenna systems, allowing to model the structure of almost arbitrary shape with varying precision of results. The choice of a particular method of calculating the characteristics of investigated antennas is the first challenge to the modern developer. This paper compares the results of the analysis of lens antennas by means of various methods, included in a software package Ansys EM [10].

## 2 Spherical Luneburg lens

The principle of operation of a lens antenna is based on slowing the phase velocity of the wave in the material of the lens. Due to the travelling of each individual piece of the wave front a certain distance with a modified speed the flattening of the phase plots and turning a cylindrical or spherical wave front, radiated by the feed in flat occurs. Effect of concentration of radiation using radial inhomogeneous dielectric lenses was first described by the mathematician Karl Rudolf Luneburg [11]. This effect appears for the radiator located on the opposite side to the direction of the main lobe of lenses, which refraction coefficient varies from $\sqrt{2}$ in the center to one on the edge by the law

$$
\begin{equation*}
n(r)=\sqrt{\varepsilon^{\prime}(r)}=\sqrt{2-(r / a)^{2}} \tag{1}
\end{equation*}
$$

where $\varepsilon^{\prime}$ - relative permittivity of the lens material, $r$ - the radial coordinate in spherical or cylindrical coordinate system, $a$ - outer lens radius.

Typically, the lens are manufactured as multi-layer structures with a step change in refraction coefficient with the sequence close to (1). The structure is shown in Fig. 1. The number of layers and their parameters are shown in table 1. This paper discusses various methods of electromagnetic modeling, optimization and analysis of this antenna. Luneburg lens is considered spherical with a diameter of 650 mm , consisting of seven layers with the dielectric loss tangent 0.001 for all layers. As a radiator half wave dipole located at a distance of 118 mm from the surface is used. The simulation was carried out at a frequency of 4 GHz .

Luneburg lens settings for the analysis were obtained from the article [12], which examines the broadband optimization of the lens structure.

Table 1: Parameters of lens layers

| Parameters | Number of the layer |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ |
| Outer layer radius, mm | 75 | 101 | 164 | 194 | 227 | 260 | 325 |
| Relative permittivity of the layer, $\varepsilon$ | 1,76 | 1,70 | 1,66 | 1,60 | 1,53 | 1,46 | 1,39 |
| Dielectric loss tangent, $\operatorname{tg} \delta$ | 0,001 | 0,001 | 0,001 | 0,001 | 0,001 | 0,001 | 0,001 |



Figure 1: Cross-section of the spherical Luneburg lens.

## 3 Simulation

In the analysis options it was set to double match the result with the specified standard deviation of scattering matrix $\Delta S=0.01$ on the last iteration of the adaptive mesh of structures to improve accuracy results. As a criterion for assessing the accuracy of an analysis the directivity of model was chosen. The more precise the method is, the higher will be the calculated directivity of the antennas because the more accurate will be approximation of the surface of the lens and the closer it is to the theoretical models.

In total there were considered five different methods of numerical modeling of electromagnetic seven-layer Luneburg spherical lens. Simulation results are presented in table 2.

The first way of modeling is the analysis of spherical lens with finite element method (FEM) entirely. The half wave dipole is used as the radiator of the lens. The box surrounding the sphere with the radiator was set to be made of the air. On the edges of this box the radiation boundary condition is defined. According to the recommendations of the software developers faces of this air box around the lens with the feed must be located at a distance of not less than a quarter wavelength from the outer boundary of the lens for optimum reduction of distortion of the radiated fields. The area inside the box is fully decomposed by finite elements (tetrahedral mesh), at anchor points the calculation of the electromagnetic field is conducted. The created 3d model is shown in Fig. 2. Fig. 3 shows the simulation results for the pattern of this model.


Figure 2: Spherical Luneburg lens with radiator inside an air box


Figure 3: Radiation pattern of the model of spherical Luneburg lens inside an air box
It should be noted that the pattern in Fig. 3 represents a cross section of a 3D pattern in the YOZ plane in which the dipole is located. The angle $\varphi$ is measured from the positive direction of the $x$-axis, so the $y$-axis direction corresponds to $\varphi=90^{\circ}$. Later in the paper, the pattern is presented in the same section with the same parameters.

The second way of modeling is the analysis by the method of moments (MoM) in HFSS-IE (Integral Equations) design. Model calculations according to this technique require fewer RAM and less computing time, but usually less accurate, as
evidenced by the largest directivity. In this case, the air box is not used, because only the surface of the investigated structure components is discretized by the finite elements. This significantly reduces the amount of RAM used in calculations, and an electromagnetic field is defined only at the margins between the dielectric layers and in far field region of the antenna. Model of the structure under consideration is shown in Fig. 4. Fig. 5 shows the resulting antenna radiation pattern of this model.


Figure 4: Model of spherical Luneburg lens in HFSS-IE


Figure 5: Radiation pattern of the model of spherical Luneberg lens in HFSS-IE
The third way is to use simulation of hybrid boundary conditions of FE-BI (Finite Element Boundary Integral). In this case only part of the structure is decomposed by finite elements. On the faces of this part the hybrid boundary conditions is imposed, so the part is analyzed with the FEM and the rest of the space is analyzed with MoM. This technique is a combination of the first and second modelling approaches. It allows you to significantly reduce the amount of calculations, improving simulation time because the field outside the sphere, which surrounded by the FE-BI condition, is not calculated directly, but only if necessary it is recalculated from the equivalent currents on the surface with hybrid conditions.

Reduction of the amount of RAM is due to the lack of part of the finite elements representing free space in the air box. In this model, FE-BI hybrid condition is set on the outer surface of the outer layer of spherical Luneburg lens. The model is shown in Fig. 6. at the. In Fig. 7 the radiation pattern of the considered model of lens antenna is shown.


Figure 6: Model of Luneburg spherical lens with hybrid FE-BI boundary conditions applied


Figure 7: Radiation pattern of the model of spherical Luneburg lens of with FE-BI hybrid boundary conditions
In the fourth way, modeling of lens antenna also conducted in HFSS design. In this case, the mesh of finite elements is done only on a quarter of the first model, as the symmetry of Luneburg lens in two dimensions is used. This method is applicable to the spherical lens because it has two planes of symmetry. Application of the boundary conditions of symmetry allows one to reduce the amount of required RAM. This option proved to be the fastest in terms of computation time. The appearance of the model is shown in Fig. 8. Fig. 9 shows the radiation pattern of this model of the lens antenna.


Figure 8: Model of the quarter of spherical Luneberg lens inside an air box with the symmetry boundary conditions


Figure 9: Radiation pattern of the model of quarter of spherical Luneberg lens, with the symmetry boundary conditions
The fifth method of simulation on basic steps repeats the fourth, but with replacement of an air box with rectangular shape. When this method is used the air box is less conformal to the investigated structure so the number of finite elements increases the and, consequently, increases the amount of the RAM required for the calculation. The appearance of this model is shown in Fig. 10. Fig. 11 shows the radiation pattern of this model obtained for the lens antenna.


Figure 10: Model of quarter of spherical Luneburg lens in a rectangular air box with the symmetry boundary conditions


Figure 11: Radiation pattern of the model of quarter spherical Luneburg lens in a rectangular air box with the symmetry boundary conditions

Fourth and fifth modeling approach is possible when a simulated object has at least one symmetry plane. In addition to the availability of geometric symmetry, one must also be able to apply ideal electrical or magnetic wall to the face with the symmetry conditions, i.e. have the symmetry of the electromagnetic field structure in the modeled object.

The simulation was carried out on a personal computer with the following specifications: CPU Intel Core i7-6700 3.4 $\mathrm{GHz}, 16 \mathrm{~Gb}$ RAM, Windows 8.1. Simulation results are presented in table 2.

Table 2: Summary table

| Method \# | Directivity | Number of <br> finite elements | Elapsed time, min |  | Amount of <br> RAM, Gb |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Real | Processor (given the parallel operations <br> on different cores) |  |
| 1 | 22 | 157543 | 13,32 | 69 | 12,5 |
| 2 | 19,7 | 25728 | 8,23 | 54,05 | 3,91 |
| 3 | 21,58 | 36470 | 4,32 | 24,54 | 2,36 |
| 4 | 21,68 | 43047 | 1,08 | 3,50 | 2,18 |
| 5 | 23,28 | 81210 | 2,54 | 11,09 | 5,25 |

## 4 Conclusion

As a result of the work, it can be concluded that the use of hybrid boundary conditions gives a fairly accurate results while requiring much less computing resources than finite element method applied to the entire investigated antenna in general. In addition, one can reduce the required resources using symmetry of the object under consideration. Unfortunately, the software package Ansys EM 18.1 in the current version does not allow using symmetry and hybrid boundary conditions of FE-BI simultaneously. During the research it was found out that there are several ways to address the complex multilayered spherical Luneberg lens antennas, obtaining reasonably accurate results within a relatively short time. Such methods include the use of hybrid boundary conditions and exclusion of part of the model from analysis using its symmetry.

## References

1. Volakis J.L. Antenna engineering handbook. McGraw Hill, 2007
2. Zelkin E.G., Petrova R.A. Linzovyye antenny. [Lens antenna] Moscow, Sov. Radio Publ. 1974. 280 p. (In Russian).
3. Korotkov A.N., Shabunin S.N. Influence of the sampling cylindrical Luneberg lens on its emission characteristics. Informatsionnyye tekhnologii, telekommunikatsii i sistemy upravleniya. [Information technology, telecommunications and control systems]. Yekaterinburg, Russia. 2015. pp 20-26.
4. Panchenko B.A., Denisov D.V., Mokhova V.V., Panov R.I. Impact of stratification Luneburg lens at its antenna characteristics. Izvestiya vysshikh uchebnykh zavedeniy Rossii. Radioelektronika. 2014 No. 1 pp 3-6.
5. Panchenko B.A., Knyazev S.T. et al. Elektrodinamicheskiy raschet kharakteristik poloskovykh antenn [Electrodynamic calculation stripline antenna characteristics]. Moscow, Radio i svyaz' Publ. 2002, 256 p (In Russian).
6. Panchenko B.A. Rasseyaniye i pogloshcheniye elektromagnitnykh voln neodnorodnymi sfericheskimi telami [The scattering and absorption of electromagnetic waves by inhomogeneous spherical bodies]. Moskow, Radiotekhnika Publ. 2013, 264 p. (In Russian).
7. Panchenko B.A., Lebedeva E.V. Antenna characteristics Luneburg lens. Antenny - Antennas, 2010, № 12. pp. 5-9.
8. Shabunin S.N. Electrodynamic analysis of layered cylindrical structures by equivalent lines. Izvestiya vysshikh uchebnykh zavedeniy Rossii. Radioelektronika - News of Russian institutions of higher education. Electronics, 2007, No. 1, pp. 70-77.
9. Korotkov A., Knyazev S., Panchenko B., Shabunin S. "Investigation of spherical and cylindrical Luneburg lens antennas by Green's function method". 2015 IEEE Radio and Antenna Days of the Indian Ocean (RADIO 2015), 2015
10. Y.E. Mitelman. Designing antenna systems in Ansoft HFSS. Educational electronic edition, Ekaterinburg, 2012, 49 p.
11. Luneburg R.K. "The mathematical theory of optics", Providence, RI: Brown Univ. Press, 1944
12. M. Huang, S. Yang, J. Teng, Q. Zhu, and Z. Nie "Multiobjective optimization and design of a Luneberg lens antenna with multiband multi-polarized feed-system". Progress in Electromagnetics Research, Vol. 129, pp 251-269, 2012
