COMPUTERS AND GEOMETRY TEACHING

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ABSTRACT

At the beginning of my article I wish to emphasize the relevance of geometry learning in education and present my conception of geometry teaching. Thereby I shall point out why I don't find the orientation not only in Euklid but also in modern axiomatism sensible. A conception of geometry teaching which suits the sense and structure of general school education must be directed both to the formation of geometric concepts which are combined with each other and based on experiences and also to pure mathematical or everyday problems. This conception is similar to those of the representatives of the genetic method.

Following this explanation I want to mention the part that computers can take in my conception and how school reacts on the change in our computer society. First, in my opinion, a critical discussion with an extensive view is necessary and secondly, a qualified discussion of course needs and concrete experiences with applications of computers in selected fields. The application of computers in geometry is an example for this.

In the second part of my article I want to show with examples on school geometry how computers can support and enliven geometry teaching and what kinds of objectives should be aimed at. I will complete these reflections with examples of programs in Basic and Pascal as well as some outputs of these. Finally I want to say some words about my experiences with university students.

INTRODUCTION

Let me begin with some general remarks on geometry teaching. Unfortunately, geometry is treated in school like a stepchild though the importance of geometry teaching has been appreciated by many maths-teaching specialists during the last years. This neglect of geometry in school may have been caused for example by two things: first geometry cannot be learned in such a straight forward and simple way as algorithms in computation, and therefore geometry requires thorough knowledge. Second, the problem of developing a systematic course for school geometry remains unsolved. This second handicap can be overcome if we give up the orientation towards a system developed by mathematicians - whether Euclid, Hilbert, Bachmann or anyone else. Rejecting the orientation on mathematical systematics is no big loss, because the task of mathematics teaching at school is not to transmit the results of mathematics research, by which I mean the anticipation of areas of later mathematical studies. On the contrary, we have to look at more general aspects of working with mathematics. With respect to geometry teaching I will specify these general aspects in four points which can be found in the didactic literature (see e.g. Gemener Kongress 1978, Kirsch 1980 and Bender 1983).

1. <u>The relevance of geometry as a means of structuring our three-dimensional world and of using this structure in practice in everyday life, for engineering, the description of nature, etc.</u>

   According to this point of view we have to teach children that mathematics, in this case geometry, can be helpful in solving everyday problems. I elaborated this aspect some years ago in my conception of the so-called "practice-orientated-mathematics teaching" (see Graumann 1976-79). Furthermore, the application of mathematics in science belongs to this point of view. Also, the use of geometry for visualization and as a heuristic medium should be taught to children.

2. <u>The cultural tradition of geometry in our world.</u>
What is meant here is that all pupils should acquire a basic knowledge of standard geometric concepts and skills. Geometry has also been of great importance in architecture, especially in earlier times, and that sometimes mathematical research has influenced life in general as well as the view of life. Furthermore, the phenemenon that mathematical problems arising from real life problems after a time become indepent, belongs to this point.

3. <u>Geometry as a field for enhancing intellectual competence, as a medium for formal education such as training for creativity, problem solving, argumentation and proving theorems.</u>
From this point of view geometry represents only one field, but because of its graphic quality as well as the often complex figures and the combinations of concepts, elementary geometry is an indispensable subject for students aiming at intellectual competence at a higher level. Kirsch (1980) states with regard to this point that intellectual training must not mean stolid cramming, it can also bring pleasure and self-confidence. Last I shall mention point 4, which could also be presented in points one or two.

4. <u>Geometry for pleasure and for developing self-confidence.</u>
From this point of view we could e.g. show students the aesthetics of forms and arrangements, let them work on their own and produce, as well as let them feel their own intellectual power and the autonomy of reasoning.

Keeping these four aims in our mind, it becomes obvious that neither an orientation towards Euclid nor an orientation in modern axiomatism is desirable. Instead for geometry teaching in school, a genetically orientated course with experience in developing geometric concepts and with applications to real life problems or playful pure mathematical problems is necessary. With this kind of geometry teaching, the task of structuring the subjects will not be solved, but we have a wider scope by choosing the subjects so that the classroom situation and the above mentioned aims can be better realized. On the other hand, in this way we can include geometry lessons better in the mathematics course. Also the special problems, which can be worked out with computers, to be presented later in the article fit in this conception of geometry teaching.

THE USE OF COMPUTERS IN SCHOOL

Before going on, I will briefly deal with the use of computers in school in general. You all, I think, know enough about the role computers play with regard to change in our world. And just as in Germany, you in Finland will certainly find advocates and opponents of the present trend. But global pros and cons do not help us; for on the one hand we cannot stop history, and on the other hand the reason for a predicted negative run always lies in the details. Computers are as all technical innovations not inherently "evil" or "good".

It depends on how we use them. And for that we first have to know a lot about details. The remarkable thing about computers for me seems to be in that they intensify special trends, whether positive or negative. The administration, for example, always contains the danger of seeing individual human beings as objects of administration and produces senseless or even inhuman regulations. Because of the possibility of the multifarious processing of quantities of data and the nation-wide cable-connection this aspect becomes obvious and can become a nation-wide danger. To face such dangers, they must be detected on the basis of expert-knowledge and public discussions, and they must be restricted by courageous actions and legislative limits. Many of the dangers or negative runs could be stopped if enough many people have enough knowledge and experience. Therefore it is our task to reflect on the use of computers in school. I do not want to discuss this general problem here. I only want to say the following: A purely technologically orientated course cannot be the answer. In addition to some fundamental theoretical knowledge, we especially need knowledge and experience concerning the facilities and consequences of computers in the different fields. A task for maths-teaching specialists, therefore, is to reflect on the sensible use of computers in maths-education. By sensible I mean on the one hand the consideration of the above general objectives, and on the other hand the consideration of the adequacy of the medium computer for the given problem. To elucidate my ideas I shall point out the ways in which computers could be used in geometry teaching, and what kinds of didactic alternatives computers have to offer in geometry education.

COMPUTERS AND GEOMETRY TEACHING

Geometry in this connection is only one of several fields of application where experience about the facilities and limits of computers, as well as their sensible use, could be gathered. Reversely the computer is to be seen as only one of several media that can enrich and enliven geometry teaching. You may ask why I talk about computers in geometry teaching because computers are mostly mentioned in connection with topics which have to do with algorithms. But first of all I like geometry, and secondly what is more important, the new small computers which can make graphics represent a tool, expanding the hitherto very limited possibilities of making many graphics and of carrying out long mathematical operations. Therefore computer-assisted geometry education is more able to make the best of the wealth of elementary geometry so that competencies like "creativity", "flexibility", "differentiated perception", "handling complex problems" can be trained. Especially at the present when routine -problems are increasingly taken care of by machines these general objectives are important. Another advantage of computers in geometry education is the possibility of showing the dynamics in geometric processes and working out systematic variations of geometric subjects. For example you can demonstrate how a circle changes into an ellipse or how the intersection of a plane and a cube varies. Furthermore, with the modern computers which can make graphics, the students receive visible feedback very quickly so that they can get verification of their effort as well as visible signs of their mistakes. Last but not least with the computer we can produce a lot of aesthetic pictures so that children learn about the aspect of arts and leisure in mathematics or even get motivated for advanced problems.

Before showing you examples of the use of computers in geometry teaching I want to present a survey of geometric themes, which could be supported by computers. I have grouped them the way they are used on the computer. (The following list is, of course, not complete.)

1. <u>The computer-screen-plotter-systems as a medium for the visualization of single geometric figures or sequences of figures</u>
   a) Geometric forms and patterns of polygons, circles or their parts, ornaments and symbols, coverings and special configurations
   b) Curves and families of curves, spirals, rhodoneas, roulettes or dynamical changing conics
   c) Oblique views or three-plane projections of three-dimensional figures
   d) Pairs of figure and image from geometric mappings

2. <u>The computer as a tool for calculations with geometric measures</u>
   a) Linear, square and angular measures with polygons (especially triangles and quadrangles) whereat the polygons are determined in different ways
   b) Linear, square, volume and angle computations with solids like prisms, pyramids, prismatoids and Platonic solids whereat the solids are also determined in different ways
   c) Length, curvature, surface area and volume of curves, respectively, solids of revolution

d) Qualities in analytic geometry such as slope, minimal distances, eccentricity or midpoint

3. Computer procedures as a universal tool to change figures and to test conditions

   a) Procedure for reflecting, rotating, translating, stretching or shearing any plane figure which is given by coordinates
   b) Procedure to produce an oblique view of any three-dimensional figure which is given by coordinates
   c) Procedure to determine the composition of two or more mappings
   d) Procedure to draw a triangle (or quadrangle) which is given by any three (or five) data
   e) Procedure for testing equal length, parallelity and orthogonality of two line segments
   f) Procedure which can produce from a given curve a new one through a fixed technique (such as "pedal curve" or "evolute")

4. The computer as a tool for heuristic research with respect to given problems or finding new questions

   a) Discovering Archimedean coverings
   b) Producing midpoint and diagonal figures for any plane or three-dimensional polygon
   c) Representing simple four-dimensional figures
   d) Producing polygons where a fixed number of linear and angle measures are given
   e) Discovering symmetries in any polygon
   f) Discovering special groups of mapping
   g) Perceiving patterns in ground-plans of historic buildings
   h) Examining the iteration of special mappings (Here I would like to refer to the theory of Mandelbrot and the famous computer patterns with respect to the attractor in the theory of dynamical systems).

In item 4 I have listed a lot of problems which have not been solved. I only wanted to give you an idea of the possibilities. But I think that the use of the computer as a heuristic medium (item 4) should be seen as the most important one. If we could bring at least some of the contents of item 4 into school we would have the opportunity to introduce an idea of what mathematical research really is about.

In the following I shall illustrate some themes by examples.

Example 1: Polygon

The theme "Polygon" going beyond the conventional treatment of triangles and quadrangles is in principle very well suited to the above described conception of geometry teaching, because this theme yields clarification of concepts, manysided applications and stimulations for creativity and aesthetic feeling. The patterns you see on Figure 1 for example are made with a relatively simple program (see program NORM-ECK on Figure 1). With a somewhat extended program where one also can choose the midpoint the patterns on Figure 2 are produced. The correct design reguires a lot of thinkings and computations as to length and angle. Other variations of the first program produce hexagon coverings or regular polygons with their axes of symmetry (see Figure 3) or regular polygons with their diagonals (see Figure 4). The last two programs lead us to questions about axes of symmetry and diagonal-figures of non-regular polygons. We arrive at interesting mathematical problems such as "How many axes of symmetry can a non-regular hexagon have?" or "When is the polygon built from the diagonals similar to the original polygon?".

Furthermore, I shall show you a program with some outputs, which produces a polygon with constant length at each side and an insertable constant angle (see program and outputs on Figure 5). If you want to discuss the different forms of triangles and the shape with regard to the rations of the side, you can also use a simple program (see program and outputs on Figure 6).

Program for REGULAR POLYGONS
with fixed midpoint

```
  1 REM  * NORM-ECK *

 10 LPRINT CHR$(28):CHR$(37)
           (Command to start plotter)
 20 LPRINT "O";50;",";-50
           (Command to fix origin)

 30 INPUT "Number of vertices";N

 40 INPUT "Radius";R
 50 INPUT "Beginning direction";A

 60 INPUT "Number for colour";C
 70 LPRINT "J";C
           (Command to set colour)

 80 X0=R*COS(A):Y0=R*SIN(A)
 90 LPRINT "M";X0;",";Y0
           (Command to move to first point)

100 FOR I=1 TO N
110 X=R*COS(A+(360/N)*I)
120 Y=R*SIN(A+(360/N)*I)
130 LPRINT "D";",";X;",";Y
           (Command to draw to next point)
140 NEXT I

150 GOTO 40
```

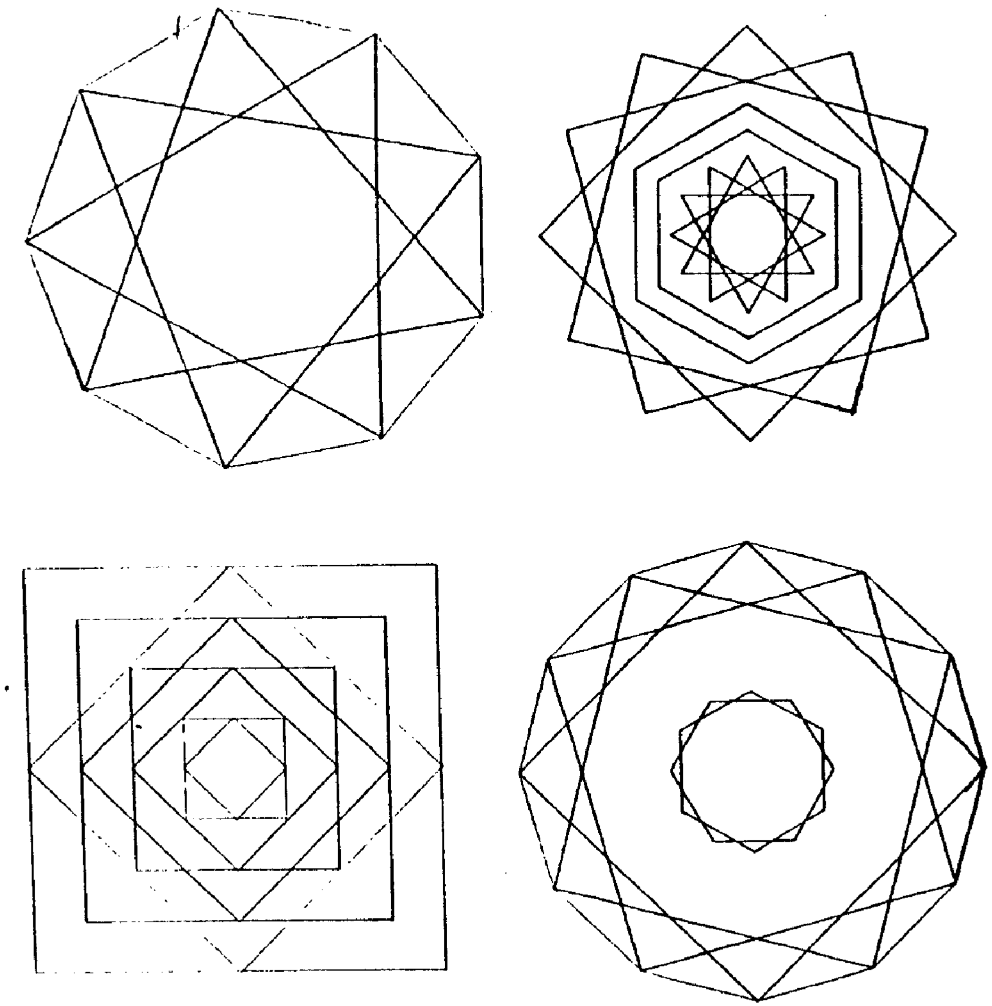

FIGURE 1.

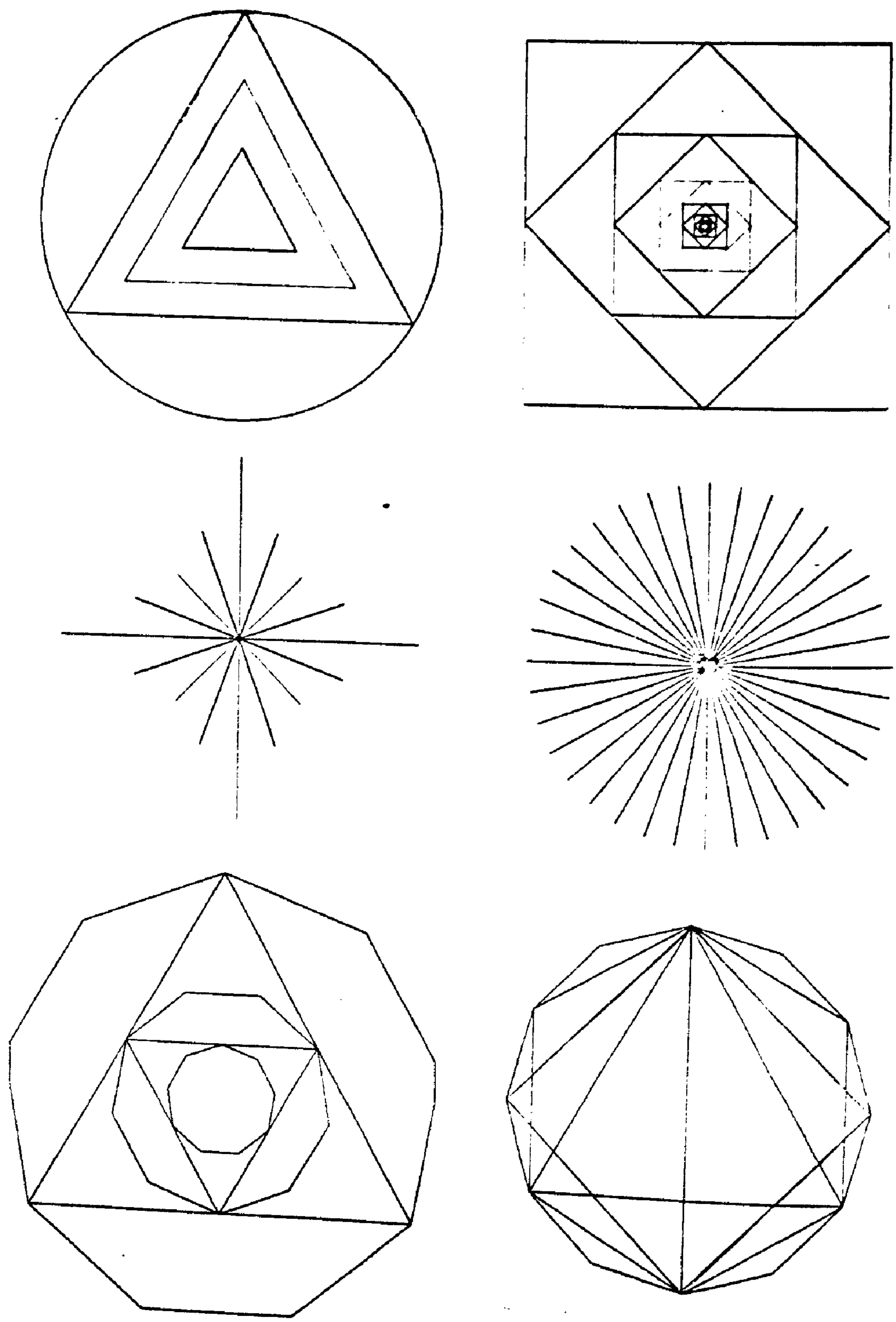

FIGURE 1. (cont.)

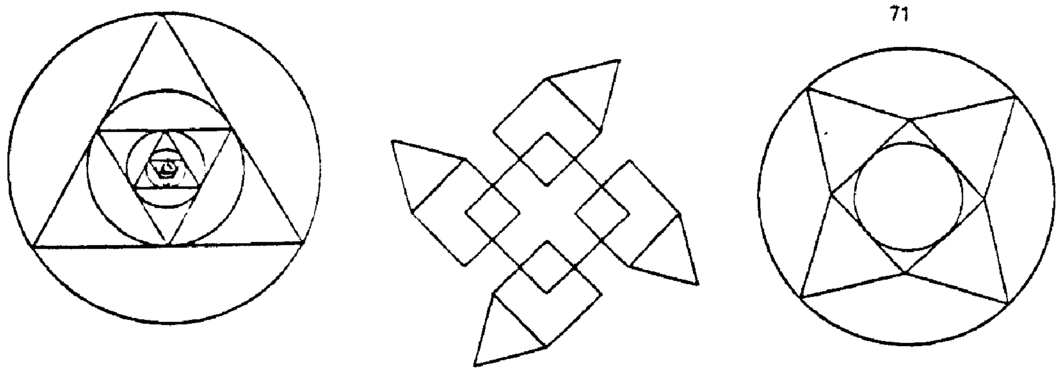

FIGURE 2.

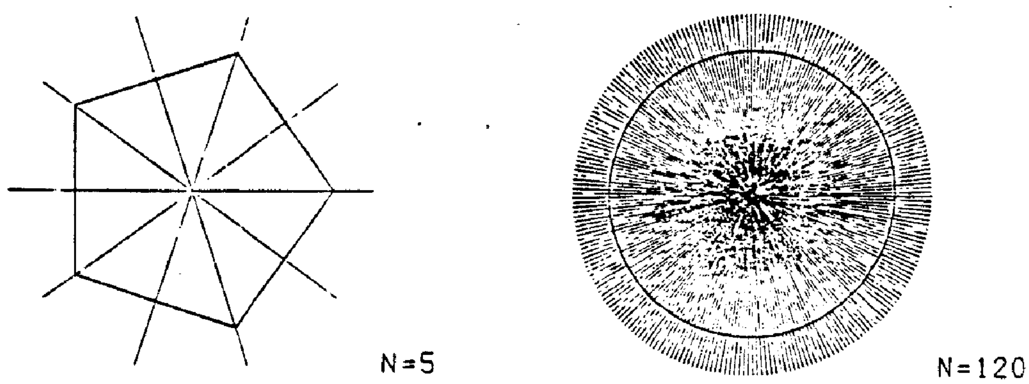

FIGURE 3.

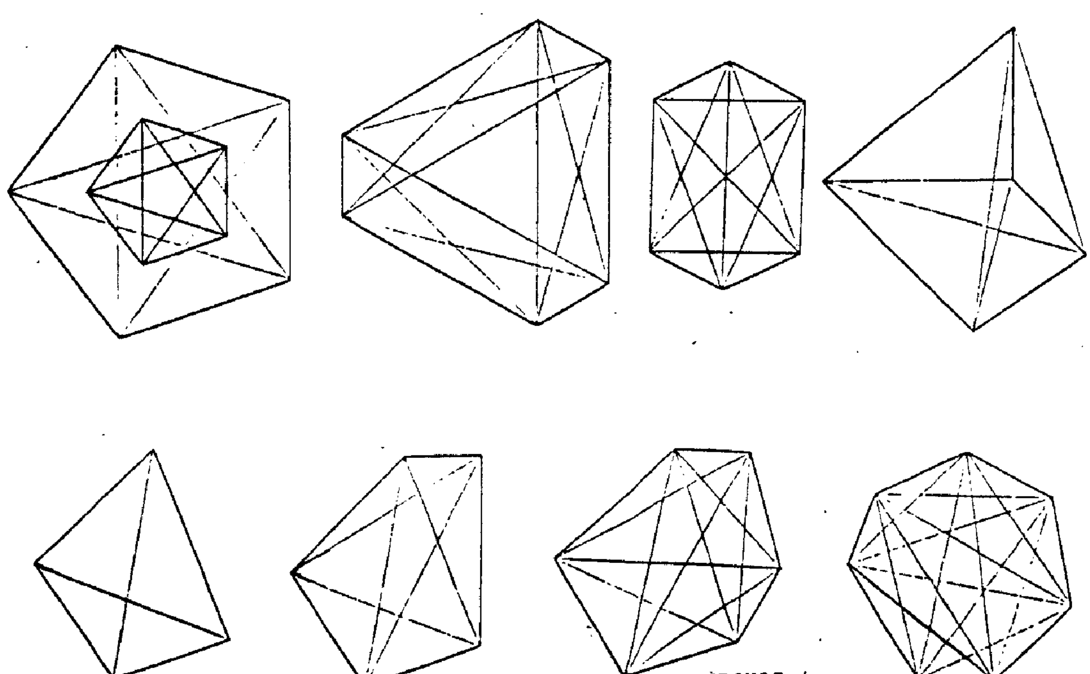

FIGURE 4.

```
10 REM **ITERATION EINES WINKELS**
20 LPRINT CHR$(28);CHR$(37)
30 INPUT "GIB DEN WINKEL EIN !";W
40 INPUT "GIB DIE ANZAHL DER
          ITERATIONEN AN !";N
45 IF N=0 THEN 130
50 INPUT "GIB DEN RADIUS AN ! ";R
60 INPUT "BEZUEGL. DER X-ACHSE
   GIB DIE ANFANGSRICHGUNG AN !";A
70 INPUT "WAEHL DIE FARBNUMMER
  #Beachte 0=SW 1=BL 2=GR 3=RT #";F
80 U=180-W:X=48+R*COSA:Y=-48+R*SINA
90 LPRINT "J";F:LPRINT "M";X;",";Y
100 FOR I=1 TO N
102 X=48+R*COS(A+I*U)
104 Y=-48+R*SIN(A+I*U)
106 LPRINT "D";",";X;",";Y
108 NEXT I
110 LPRINT "M";0;",";-110
120 GOTO 30
130 LPRINT CHR$(28);CHR$(46):END
```

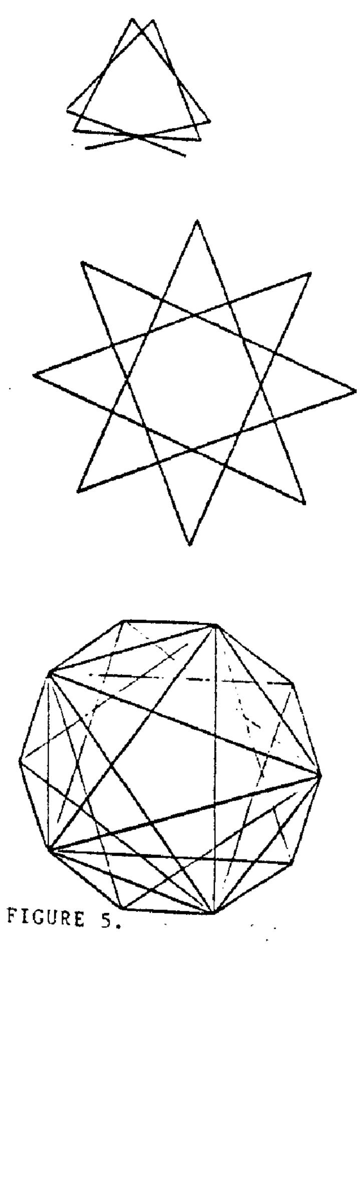

FIGURE 5.

```
10 REM **CREIECKE**
   #BEI VORGABE DER SEITENLAENGEN#
15 LPRINT CHR$(28);CHR$(37)
20 INPUT "GIB DIE LAENGEN EIN'
   #Beachte  a<10  und a<b+c #";A,B,C
30 INPUT "GIB DIE FARBNUMMER  EIN !
   #Beachte  0=SW 1=BL 2=GR 3=RT #";F
35 LPRINT "J";F
40 X=5*(A*A+B*B-C*C)/A
45 Y=-SQR(100*B*B-X*X)
50 LPRINT "D";0;",";0;",";10*A;",";0
55 LPRINT "D";",";X;",";Y;",";0;",";0
60 LPRINT "M";5*A;",";-3
65 LPRINT "P";A
70 LPRINT "M";X/2-1;",";Y/2
75 LPRINT "P";B
80 LPRINT "M";X/2+5*A-5;",";Y/2
85 LPRINT "P";C
90 LPRINT "M0,-100"
100 GOTO 20
```

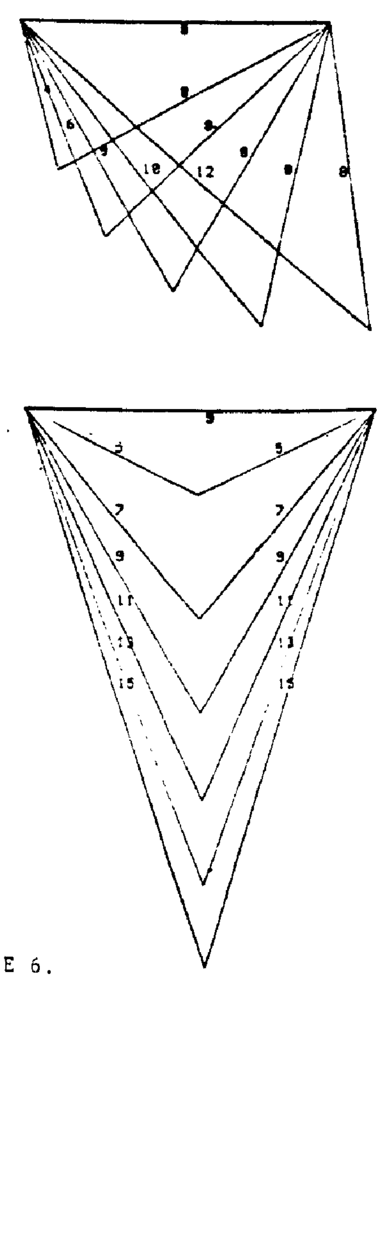

FIGURE 6.

Before showing you examples of the other themes I would like to say something about the use of these programs in school. A prerequisite is that each two or three students have a mini-computer or a terminal of a bigger computer with connectable plotter. I think this will soon be possible in every school because already we can buy minicomputers with a plotter for an affordable price. Computer-languages to be used in schools I would recommend BASIC and later PASCAL; LOGO might also be possible. The students should have basic knowledge in using the special machine (computer) at school and also they should have basic knowledge of coordinates. Other knowledge might be helpful, but it could be acquired during the course. For several problems knowledge about trigonometric functions is necessary. Therefore the teacher should consider introducing these functions earlier than usual. Many programs can be worked out by the students themselves. My opinion is that some programs, especially the easy ones, should be worked out by every student, but also more advanced programs could be used during the course, because in the above sketched conception the point of main effort is the application of programs. Just as in everyday life we use technical instruments we don't know everything about, the students should above all learn to use instruments like computers. Moreover the application of programs introduce occasions for clarification and deepening of geometric concepts and relations. Computers provide opportunities for quick feed-back, and the mistakes can be seen immediately.

<u>Example 2: Circles</u>

With the above program for regular polygons it is easy to demonstrate how the regular polygons go over to a circle. If the basics of trigonometric functions are also given you can show the approach to Pi by the Archimedean method. Furthermore you can use this program to draw figures made of polygons and circles and parts of them (see Figure 7). I think you can imagine what calculations are necessary to produce this or similar patterns with the program.

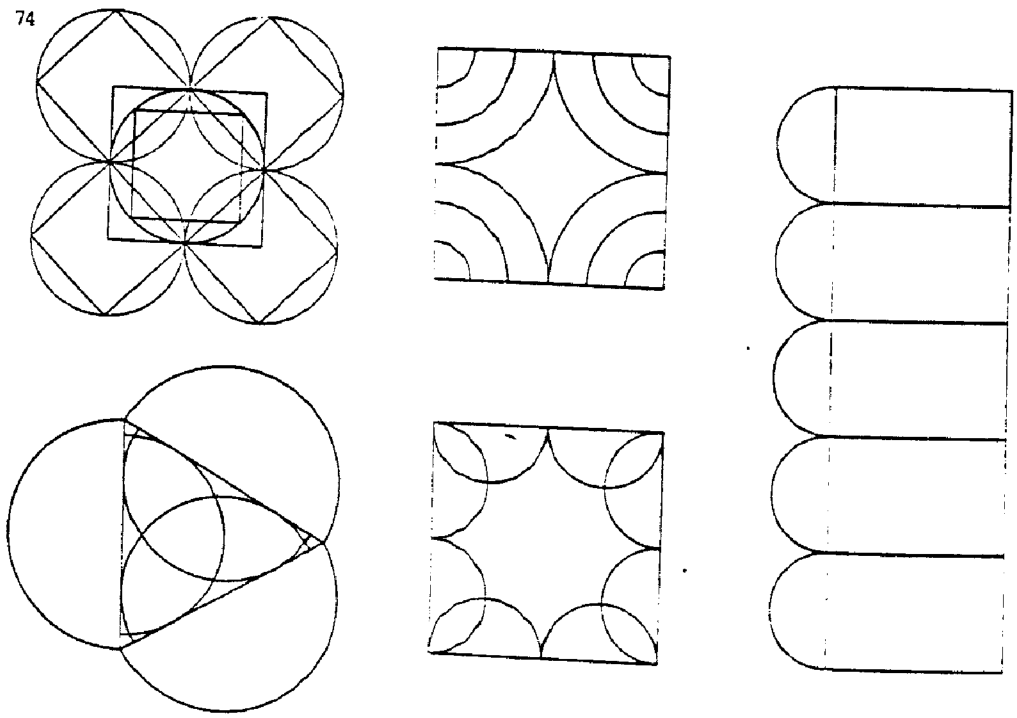

FIGURE 7.

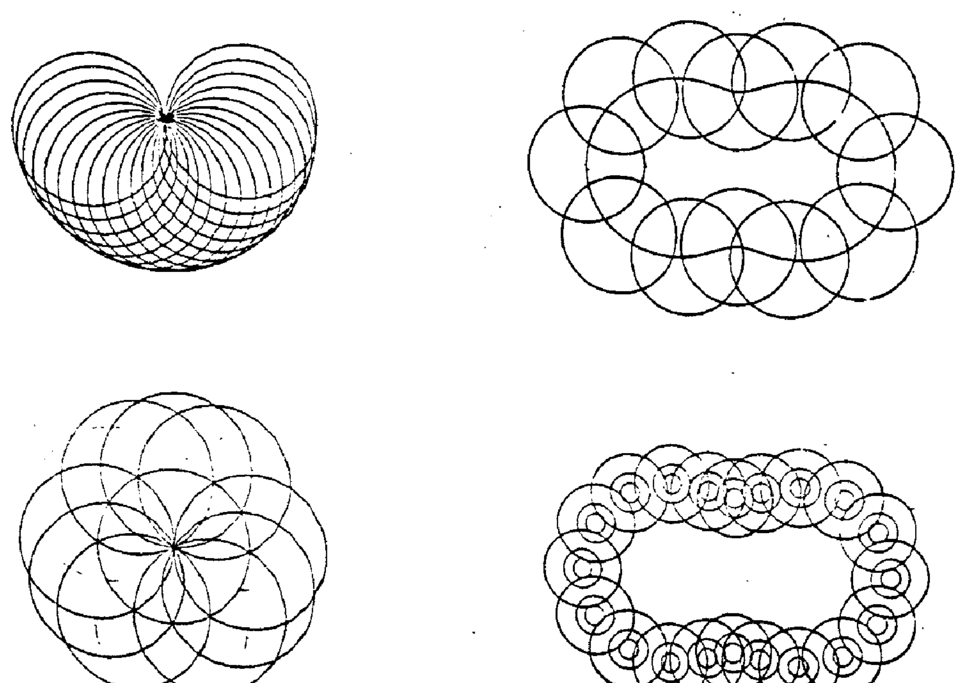

FIGURE 8.

A program where the plotter draws circles whose midpoints lie on a circle or any curve was made by students at the university. Very nice patterns can be produced with it (see Figure 8). These pictures enable students e.g. to calculate with angles, to ask questions about factors of 360 or the iteration of irrational angle measures. Questions about the square measure of intersections of circles could also be dealt with in this context.

Example 3: Solids

After the regular polygon it would be good to deal with regular polyhedrons. Because drawings of these figures are not easily done, a procedure for drawing regular polyhedrons in any position can help. With questions about rotations of such a solid before plotting and about points, segments and surfaces in a special position the perception of space could be practised. In particular with the cube more operations could be made than for example intersections with planes. A computer program which delivers many pictures in a short time is therefore a good help.

The cube is already known by younger pupils. In connection with geometric puzzles you can treat them with the question for all positions of one cube. After they have made several pictures on their own to get an understanding you can let them use a program which frees them from drawing and lets them concentrate on the main point of the problem. By the way, I am not in favour of freeing the pupils from doing drawings, but in some cases it is advantageous to use an instrument such as a computer.

A computer provides helpful assistance for computations of surfaces and volumes of special solids. The computer provides an opportunity to work out problems, which are not merely dull computations. For example you can work out a program on quadratic pyramids where any two of the four data: length of the basis, height, surface, volume are given. The others will be computed. You can also introduce the equation of third degree, which gives you an opportunity to talk about the history of mathematics and the formula of Cardano.

Example 4: Mappings and Curves

A student of mine made an extensive program on geometric mappings, i.e. on rotations, reflections, central stretchings, axial stretchings and shearings. The program on the one hand comments on the mappings and on the other hand draws the original and the image of any wanted polygon.

While discussing different functions it is good if the students can get a lot of graphic representations of the various functions. Also you approach new aspects in this theme, if you give visual presentations and afterwards discuss whole families of functions. For this you may use a program which plots graphs of every function (see Figure 9).

With other curves like spirals, rhodoneas or roulettes the viewpoint of the students can be extended. Even some ready made programs give the possibility to become acquainted with such types of curves that have never been introduced in schools. Besides you can produce interesting and nice ornaments with curves, if you can draw them as easily as a computer can (see Figure 10).

The thoughts and examples presented here mark just a beginning. Many experiments must still be made. Teacher training in this area must be organized. According to the concepts introduced here one should not try to plan a systematic course. On the contrary the course should be similar to the courses in school. The task of teacher training is to accumulate experiences with the new medium, computer, on the field of elementary geometry and to reflect in a didactical way on the experiences. I, for example, did a course with students where they first had to make programs about polygons, circles and rhodoneas, similar to those I have shown. During the course the students got acquainted with the computer's graphic system and could also reflect on mathematical problems students at school will have by using such programs. We arranged five groups of two to four students and each group had to work on a bigger project. The task for each group

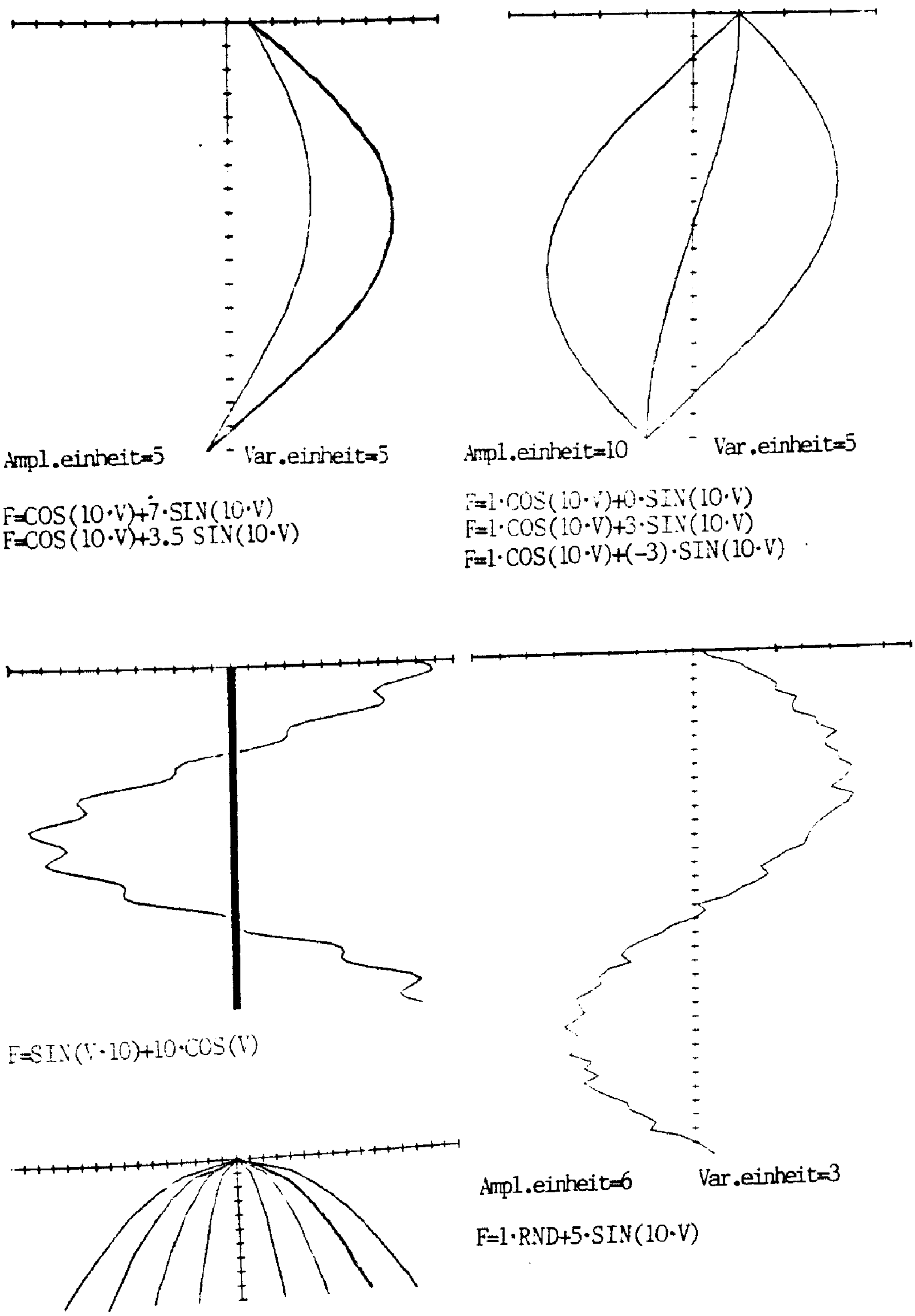
Ampl.einheit=5
Var.einheit=5
F=COS(10·V)+7·SIN(10·V)
F=COS(10·V)+3.5 SIN(10·V)
Ampl.einheit=10
Var.einheit=5
F=1·COS(10·V)+0·SIN(10·V)
F=1·COS(10·V)+3·SIN(10·V)
F=1·COS(10·V)+(-3)·SIN(10·V)
F=SIN(V·10)+10·COS(V)
Ampl.einheit=6
Var.einheit=3
F=1·RND+5·SIN(10·V)

FIGURE 9.

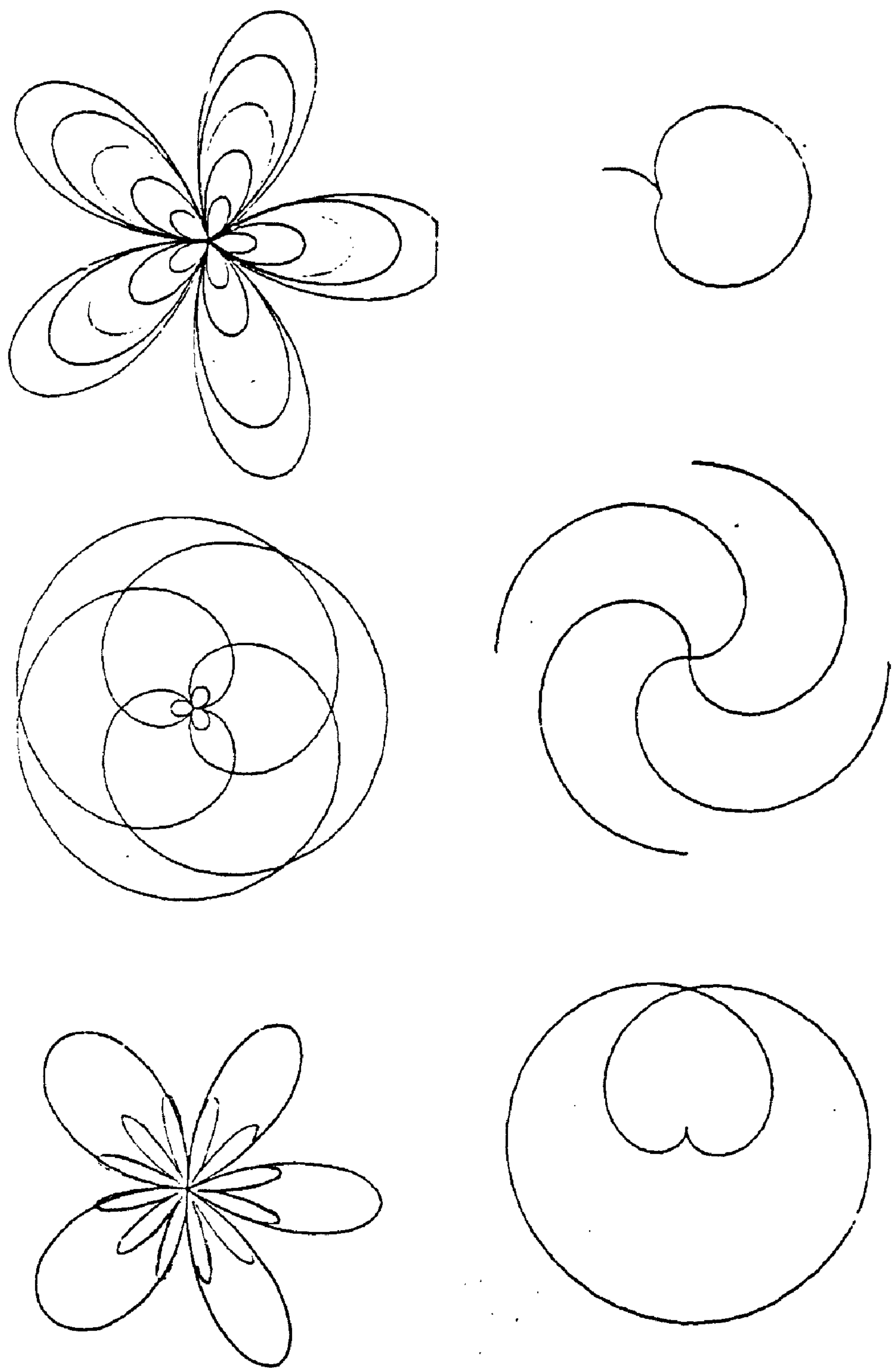

FIGURE 10.

was to make a program and consider the use of this program in school. The project themes were "mappings and symmetry of non-regular polygons", "Measures of pyramids and parallelepipeds", "Curves in Cartesian coordinates" and "Constellation of the planets at one's birthday". We learned during this course that the programming of such projects takes a lot of time and requires a lot of knowledge of computer-science. Therefore such projects can be done in school only in special study groups. But if you use good programs all students can work out interesting applications and will receive motivating experiences.

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