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Mathematical modeling of particle stratification in jigs

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ABSTRACT

Recognizing mathematical modeling as a powerful tool for systematic process analysis and control, this paper attempts to critically review the theories and mathematical models which have been advanced to explain and simulate the behaviour of jigging process. The existing literature on mathematical modeling and quantitative analysis of jigging has been divided into six subheads : (i) classical theory, (ii) potential theory, (iii) dispersion models, (iv) energy dissipation theory, (v) stochastic analysis and (vi) empirical models. A new modeling approach based on Newtonian mechanics is used to describe the stratification behavior of particles in jig. In this approach, the motion of solid material is treated using the discrete element method (DEM) while the corresponding motion of the liquid is determined by marker and cell (MAC) technique. For illustration purpose, a jig bed consisting of 100 particles of two different densities is simulated. Preliminary results show that the model predicts the stratification of particles reasonably well.

INTRODUCTION

The jigging is a separation process in which a mineral bed is pulsated by a current of water resulting in stratification of mineral particles of different specific gravity. Thus it exploits the stratification of particulate matter under the influence of hydrodynamic and gravity forces. Jig offers economic advantages over heavy media separators owing to its simple design and operational features. In the US, jigs are used for primary cleaning of coal in 37% of the plants. It is generally believed that Indian coal is difficult to wash owing to high ash content, therefore, jigging gives way to heavy media separation. However, with the recent increase in the price of coal coupled with stringent environmental regulation, the above belief can only be vindicated. Against this background, we analyze the jigging process with a view to improve our understanding of the jigging process.

Jig operates in a cyclic manner. The jig cycle may be considered consisting of four stages, namely, inlet, expansion, exhaust, and compression. In the inlet stage the bed lifts up en masse. Near the end of the lift stroke the particles at the bottom of the bed start falling resulting in the loosening of the bed which, in turn, causes its expansion or dilation. During the third and fourth stages of jig cycle, the particles resettle through the fluid, and the bed collapses back to its original volume. The pulsation and suction is repeated to bring about stratification with respect to specific gravity across the bed height.

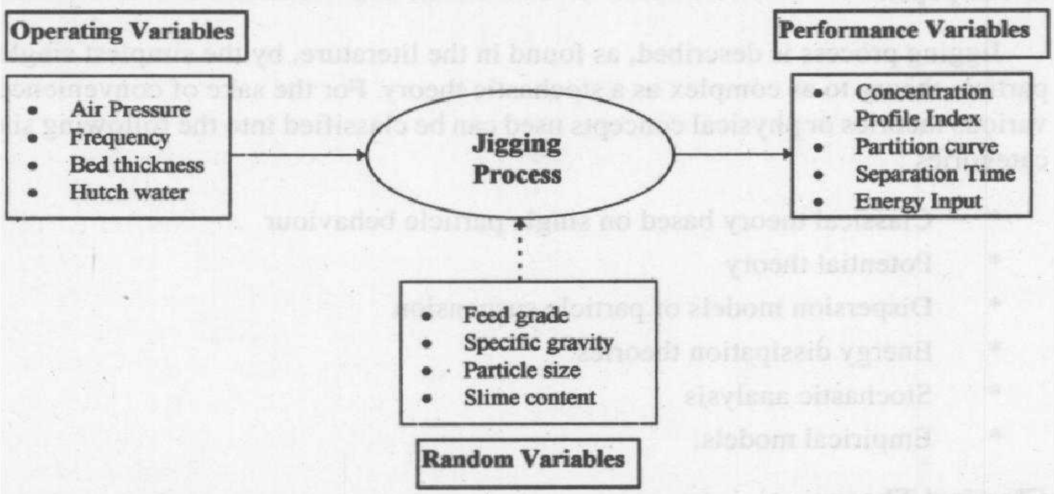

Fig. 1 : Important variables of a jigging process.

A large number of factors affect the stratification in a jig. These are schematically shown in Fig. 1. Ideally one may want to formulate a simple and practically applicable mathematical model of the jigging process correlating all the variables. However, inspite of several efforts by a large number of researchers such a model has remained elusive. This is due to the fact that although jigging is one of the oldest mineral processes, the basic principles involved are not yet well understood. With the revival of interest in gravity concentration techniques, in general, and jigging in particular, once again there is a concerted effort to improve our understanding of the jigging process. The present paper is an attempt in that direction.

This paper consists of two parts. In the first part, a critical review of literature on mathematical modeling and quantitative analysis of physical phenomena leading to stratification in jigging is presented. The second part of the paper deals with simulation of stratification in jigs using a numerical technique known as the discrete element method.

JIGGING THEORIES

Different theories of jigging process have emerged from varying perception of jig beds by different investigators. These perceptions range from a simple heavy medium system to a substantially more complicated stochastically oscillating fluidized bed. Theories considering jig as a heavy medium[1-5] with alterable density during various conditions consider particle separation to be the same as that in a heavy medium according to the density of particles alone. Since these theories do not explain several of the main jig phenomena, these are not pursued any further in this paper.

Jigging process is described, as found in the literature, by the simplest single particle theory to as complex as a stochastic theory. For the sake of convenience various theories or physical concepts used can be classified into the following six categories :

* Classical theory based on single particle behaviour
* Potential theory
* Dispersion models of particle suspension
* Energy dissipation theories
* Stochastic analysis
* Empirical models.

Classical Theory

The physics of the stratification of particles under the action of hydrodynamic forces is quite complex. Because of this complexity that relates to the simultaneous behaviour of particles in a large ensemble, particle behavior is analyzed in isolation first and then related to the movement of masses of particles. In this approach, a hydrodynamic force balance is set-up on each particle and an attempt is made to relate the separation of particles with different physical characteristics to differences in the hydrodynamic forces acting on each particle. The theory considers the principles of equal settling of particles, interstitial currents, acceleration and suction. Gaudin has listed three major effects contributing to the stratification in jigs: (i) hindered settling, (ii) differential acceleration, and (iii) consolidated trickling [6-8].

Classical theory assumes that the solid-fluid mixture in jigging is so thick that it is essentially a loosely packed bed of solids with interstitial fluid instead of a fluid carrying a large number of suspended solid particles. In such thick slurries, the settling ratio of heavy to light minerals is greater than that for free settling conditions. Hindered settling has a marked effect on separation of coarse minerals.

The initial acceleration during settling of particles in a suspension is given by

$$\left(\frac{dv}{dt}\right)_I = \left(1 - \frac{\rho f}{\rho_S}\right)g \qquad \dots \quad (1)$$

where υ is instantaneous velocity of a settling particle, ρ_f and ρ_s are the densities of fluid and solid respectively, t is time, and g is acceleration due to gravity. It is thus evident that the initial acceleration depends only on the ratio of solid and liquid densities and is independent of particle size.

Hindered settling and differential acceleration combined together result in a bed in which small particles are on top of large particles. The large particles in some regions rest on each other in such a way that large enough passage ways exist between them allowing the trickling of small particles through them. This aspect is difficult to quantify by simple settling theory.

Recently Beck and Holtham[9] have considered a two dimensional jig bed. The equations of motion of each individual particles are formulated to include forces arising from particle-fluid and particle-particle interactions. Each particle in the bed, irrespective of its position and surroundings is assumed to experience the same fluid velocity at any given instant of time, effectively ignoring the local fluid dynamics around individual particles. Particle-particle interactions take into consideration the short term collision described by an impulsive force model, and a long term frictional contact described by a damping spring force model. The motion of individual particles within the bed is continuously tracked. This model, however, suffers from two major deficiencies:

1. The idealized water model is used. It is now well established that the water motion in a jig is considerably more complex[10,11]. The simulation model does not take into account the considerable damping of water motion that occurs due to the presence of particulate solid.
2. Simulation of a two dimensional system means that sorting mechanism such as consolidation trickling can not be modeled as there is no connection between interparticle pore spaces in the two dimensional compacted bed through which small particles could trickle.

Potential Energy Theory

The theory, first proposed by Mayer[10], assumes that the cause of stratification in a jig bed is the difference between potential energies of the system before and after stratification. An unstratified bed of particles of different densities and sizes is an unstable system under gravity potential which strives towards more stable (stratified) configuration. It does so by redistributing the particles within the bed in a manner such that there is a reduction in the overall potential energy of the bed. The change in potential energy, which is represented by the changing center of gravity of the bed, corresponds to the minimum potential energy only when complete stratification is achieved. Mayer disagreed with the assumption that the jig stroke supplies the flow energy which causes stratification. He argued that the supplied energies have only the effect of releasing the potential energy stored in the granular mixture and hence, not directly responsible for stratification.

This theory has several criticisms. The major one being that it gives no fundamental description of the dynamics involved in the approach of the system to stable configuration and therefore, it cannot completely describe the rate of separation. Mayer did postulate that the stratification rate, S, could be given a simple first order equation

$$S = J\, exp(-kt) \qquad \ldots \quad (2)$$

where J is dressability (Jiggability) and k is a rate constant which is a function of several factors like the form, amplitude and frequency of the jig stroke, the displacement and frictional resistance of particles and jiggability. The above equation is, however, only empirical. The other limitation of potential theory is that it fails to account the stratification of materials with size distributions.

Dispersion Models of Particle Suspension

Potential energy theory has been extended by several investigators to allow for the dispersive effects due to particle-particle collision. Vetter *et.al.*[13] have argued against the postulate that claims the material in the jig bed is stratified once the potential energy is minimized simply because it does not describe the remixing of the material due to its natural dispersion within the bed. Drawing an analogy with diffusion mixing within the homogenized fluidized beds a mathematical formulation has been proposed to describe dispersion of particulate materials with different densities but uniform size and shape. The particle motion in the jig bed is modeled in a probabilistic manner by interpreting the conventional hydrodynamic force balance on a particle as a stochastic differential equation with particle-particle interactions included as a noise term. The force balance equation describing the motion of particle within an ensemble of similar particles located in some external force field is

$$m\frac{dv(t)}{dt} + Bv(t) + ma(t) = A(t) \qquad \ldots \quad (3)$$

where m is particle mass, B is particle drag term, $a(t)$ is an acceleration term such that $ma(t)$ represents the external force acting on the particle due to the presence of the fluid. $A(t)$ represents the collision force and is considered to be a Gaussian white noise stochastic process. Neglecting the inertial term, the above equation is reformulated in terms of particle displacement as an Ito stochastic differential equation

$$dy(t) = -\frac{ma(t)}{B}dt + dw(t) \qquad \ldots \quad (4)$$

where the function $w(t)$ is known as Wiener process and is defined as

$$w(t) = \int \frac{A(t)}{B} dt \qquad \ldots \quad (5)$$

The final dispersion equation in which concentration C_i is represented as conditional probability is formulated as

$$\frac{\partial C_i}{\partial_t} - \frac{\partial\left[k_\mu(\rho_i - \overline{\rho})C_i\right]}{\partial_y} + k_D \frac{\partial^2 C_i}{\partial_y{}^2} \qquad \ldots \quad (6)$$

where k_μ is a drag coefficient and k_D is a coefficient which is a measure of dispersive mixing of particles in the jig bed. One of the major limitations of this formulation is that it is valid only for particles of uniform size and shape. For a real jigging process the formulation fails because of this assumption.

Siwiec and Tumidajsk[14] have treated jigging process as a quasi diffusion process corrected for gravity and buoyancy forces. They assume the feed to be consisting of the light fraction having density lower than the separation density and the heavy fraction having higher density. A heavy particle makes a jump of some length in the course of each water pulsation with jump length being a random variable following a gamma distribution.

In one of the major contributions to the quantitative analysis of stratification phenomena in jigging, Tavares and King[15] recognized that the ideal stratification as visualized by Mayer[12] in his potential theory is never achieved because of dispersive forces which disturb the already stratified bed by random motion. The stratification achieved in actual practice must represent a balance between the stratification forces and dispersive forces. Extending the potential theory the separation performance of jig has been determined by calculating the density profiles of the stratified bed, which results because of jigging action, using the actual composition of the bed. Considering a dynamic state of equilibrium of the bed where the stratification flux for each particle type i is balanced by the corresponding diffusive flux they have shown that

$$\frac{dC_i(h)}{dh} = -\alpha C_i(h)\,[\rho_i - \overline{\rho}\,(h)] \quad i = 1,2...n \qquad ... \quad (7)$$

where h is a normalized height, and α is the stratification coefficient defined as

$$\alpha = \frac{ugV_pH_b}{D} \qquad ... \quad (8)$$

In the above equation V_p is the volume of the particle, u is the specific mobility due to particle fluid interactions, H_b is the bed height, D is diffusion coefficient and g is the acceleration due to gravity. Solution of n coupled differential equations represented by Eqn. 8 gives the equilibrium density distribution profile which may be used as a measure of performance of overall jigging process. One of the advantages of this procedure as compared to the traditional procedure using partition curves for performance evaluation and simulation of jigging process is that only a single parameter, α, is to be estimated from experimental data. However one of the main limitations is that it can be applied strictly to mono-size feed.

Energy Dissipation Theories

In the jigging theories discussed hitherto, the role of water motion on stratification has been by and large ignored. Assuming that the separation and stratification of particles in a jig are directly related to bed expansion which, in turn, is governed by water pulsation and physical properties of particles, Jinnouchi *et.al.*[10,16] have modeled air pulsated jigging process viewing it as a simple mechanical system as shown in Fig. 2. The air chamber in the jig is an energy storage device (represented by a spring), the water mass that oscillates is an inertial element, the bed plate is a frictional element, and the particulate bed is another inertial element.

In this model, variations in pressure in the air chamber and motion of water in the jig are modeled using an equation of state for the air, the unsteady state Bernoulli's equation, and unsteady state material balances for air and water. The bed is visualized as a unsteady state fluidized bed whose motion is described by Richardson and Zaki[17] type correlation. The set of differential equations describing this mechanical system is solved numerically to determine the air chamber pressure, fluid velocity, and bed porosity. One of the attractive features of this model is that it is able to establish that the physical dimensions of the jig have a large bearing on how water motion will relate to the air valve settings. Based on

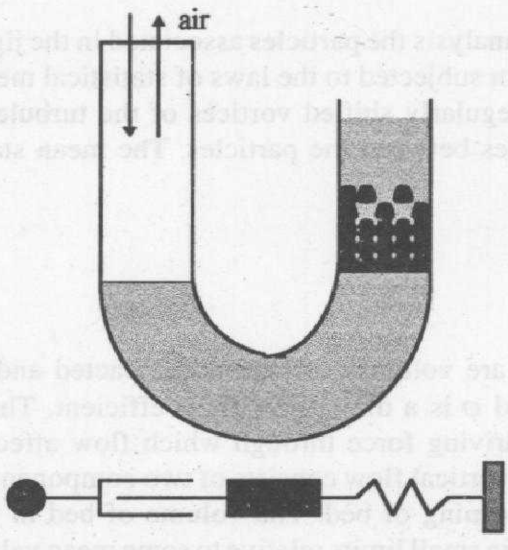

Fig. 2 : Representation of a jig by means of a simple mechanical model.

this analysis a method to control the wave pattern of water pulsation has been developed. The limitation of this model is that there is no attempt to describe the stratification of particles in the jig.

A major contribution to the understanding of air pulsated jigs has come from a series of publications by Lyman and Co-workers[11,18-22] based on well designed experimental studies and elaborate quantitative analysis of experimental data. The primary effort has been towards identifying a single para-meter that has a key influence on bed stratification which, in turn, depends on jig operating parameters, feed characteristic and the jig size. This parameter is found to be the total energy dissipated in the jig bed within a cycle. They claim that any setting of jigging parameters that produces the same energy dissipation in the bed will achieve similar stratification results. The para-meters which appear to influence the energy dissipation most strongly are the frequency of pulsation and operating air pressure. The main feature of this analysis is that the bed stratification mechanism takes into consideration the role of operating parameters, and the air water behavior in the jigging process.

Stochastic Analysis

The proponent of this Model, Vinogradov *et.al.*[23], argue that the analysis of jig behavior based on single particle analysis is not realistic. It is known that the jigging bed consists of a large number of particles which are not bound to each other. When the whole mass of the particle is displaced, as it happens in a jig, some new regularities start to act - these are inherent only in en masse processes and do not appear during the movement of an isolated particle.

In the statistical analysis the particles associated in the jigging bed are considered as a mass system subjected to the laws of statistical mechanics. During hindered conditions irregularly shifted vortices of the turbulent flow create some pressure in the spaces between the particles. The mean statistical value of the pressure is

$$P_T = \varphi \frac{V_0}{V} \rho_l u \quad \ldots \quad (9)$$

where V_0 and V are volumes of bed in compacted and loosening states, ρ_l density of liquid and φ is a dimensionless coefficient. This turbulent pressure provides the main driving force through which flow affects the particles. The movement of bed in vertical flow consists of two components : Movement of bed as a whole and Loosening of bed. The volume of bed in the loosened state is chaotically changed in small limits relative to some mean value. The state in which the volume remains constant is considered as statistically stable. If the volume, however, varies then it is statistically unstable state which is characterized through a coefficient of statistical unstability defined as

$$\varpi = \frac{V_s}{V_u} \quad \ldots \quad (10)$$

where V_s and V_u refer to bed volumes in statistically stable and unstable states. In a statistically unstable state there appears an apparent density gradient along the entire height of the bed which is given by

$$\rho_a = \rho_l + (1 + \bar{\theta})(\rho_s - \rho_l)\,\varpi \pm \frac{\rho_l \dot{u}}{g} \quad \ldots \quad (11)$$

where θ is coefficient of breaking of bed and $\dot{u}$ acceleration of the liquid. ϖ is an important parameter in the jigging process and greatly influences the efficiency of stratification.

Based on the above analysis it has been proposed that the stratification of bed by density is effective if the bed during the whole jigging cycle is only in a statistically unstable state. Such a state can be assured by matching the frequency of oscillation of the liquid by regulating the bed thickness and consumption of under screen water. Although this theory explains the interconnections between regimes of liquid movement and degree of stratification, it fails to provide the relation between operating parameters and stratification performance.

Empirical Models

Owing to the complexity and the large number of operating variables involved in jigging process, a few investigators have attempted to develop empirical models to described the process kinetics. Assuming that the bed stratification is most sensitive to the jigging time, these models attempt to express stratification as a function of jigging time using empirical equations. Karantzavelos and Frangiscos[24] have proposed a first order model to represent the relationship between the jigging time and several parameters of jigging process. The basic equation is the two parameter Weibull distribution function given as

$$Y(t) = 1 - exp\left(-\frac{t}{\theta}\right)^{\beta} \quad \ldots \quad (12)$$

where $Y(t)$ is the yield at time t and β and θ are two empirical parameters which must be determined experimentally. The parameter β is a measure of relative delay of the separation process while θ reflects the jiggability, i.e., natural tendency of mineral particles to separate according to their specific gravities in a jig environment. In general both these parameters depend on frequency of pulsation, stroke length, water level, bed thickness, particle size, specific gravities of particles, and feed grade.

Recognizing that the jig bed stratification is significantly affected by a number of parameters affecting the behavior of water in the jigging chamber, Rong and Lyman[18] have proposed a power function equation to represent the relationship between the bed stratification indices and the jigging time. The significant water behavior parameters in relation to the bed stratification are taken to be

* the maximum and average water pressure above the bed plate within a cycle
* the average water pressure above the bed plate during the time of pulsation
* water oscillation amplitude
* duration of pulsation

A NEW APPROACH TO JIG MODELING

It is clear from the above discussion that quite a lot of mathematical modeling work has been done to describe the jigging process in its entirety. These models range from the simplest based on Stoke's settling laws to those that attempt to predict pressure and velocities in jig chamber utilizing unsteady state Bournouli's equation. Yet, as with many other mineral processing operations, there has been always a search for the best model that will become the most effective tool for process analysis and control. In the jigging context, a model that describes the flow of water as well as the solid particles would be more revealing. In the recent past similar models have been developed to describe the axial motion of the slurry in tumbling mills[25].

In the proposed model, the motion of solid materials is treated by a numerical tool known as the discrete element method and the corresponding motion of the liquid material is determined by Marker and Cell technique (MAC)[26]. Although the model is numerically involved, it makes no assumption with regard to the manner in which the particles in the jig bed stratifies. This approach is also justified particularly at a time when the power of modern day computers is growing at an exponential rate.

Keeping the above modeling philosophy in mind, an attempt is made here to simulate the stratification of particles by applying the discrete element method. At the very outset, the behavior of the fluid is represented by a drag coefficient that is computed from the properties and the physical state of the jig bed. The motion of the particles is calculated by doing a force balance on each and every particle. The model equation in its most simple form may be written as

$$m\ddot{y} = \Sigma F - F_b - F_d \qquad \ldots \quad (13)$$

where $\ddot{y}$ is the acceleration in the y-direction, F_b and F_d are the buoyancy and the drag force respectively, ΣF is all other applied forces such as the contact and other body forces. The position of the particle is determined from its acceleration and when this treatment is done to all the particles, en masse behavior of the bed results.

Discrete Element Model (DEM)

Models based on the discrete element method are primarily devised to simulate any problem dealing with discontinuum behavior of particle systems. Mineral processing systems invariably deal with particulate material systems of varying nature. Quantitative models characterizing these processing units often treat the particulate system as continuum for mathematical simplicity. DEM allows modeling of hundreds of thousands of particles at the cost of significant computational work to maintain independence of each particle; it is this computational requirement that limits its acceptance. Nevertheless, DEM results are more accurate and give better insight to the micro mechanical processes occurring at the particulate level.

The discrete element analysis starts with a description of elements or entities comprising the physical system. In a two dimensional representation of a simple system such as jig, these elements could be simply line-type, disc-type, or both.

More complex systems are modeled using superquadric elements to study spatial behavior of particulate systems in three dimensions. A proper contact detection scheme is established depending on the shape of these elements. Knowing the contacts for any given element and their relative displacements, equations for DEM analysis are established. These are the translational and rotational equations of motion, and a contact deformational equation. In the case of damping, additional terms defining the damping behavior of the material may be incorporated into the contact deformational equation. These equations are presented below in their most compact form.

$$[M]\ddot{x} + [C]\dot{x} + [K]x = \{f\} \quad \ldots \quad (14)$$

where x is displacement, $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrix, and $\{f\}$ is the applied load. Integration of this equation for x is done by explicit central difference time marching scheme. The computational procedure used here is in line with the pioneering work of Cundall and Stack[27].

The main calculation is done under a time-increment loop. Since DEM analyses require tens of thousands of time steps, an efficient algorithm to detect contact is required. Once one element's contacts are established the next task is to compute the relative velocity between the contacting elements. From the relative motion of neighboring elements and the interaction relationships, contact forces between these elements are computed. These contact forces are then computed for all of the neighboring contacts and are summed for each element. Applied forces resulting from boundary conditions such as buoyancy, drag and adhesive forces are computed for each element. By stepping in time, the motion of the elements are computed from the dynamic equilibrium equation. The entire solution procedure is repeated for the next time step after updating the coordinates and potential neighboring contacts for each element. This method has been successfully used for complete analysis of ball mills[28].

Mineral particles differ in size and specific gravity. In jigging the particles are separated based on their density. DEM allows particle density and specific gravity as input and based on these properties the mass and moment of inertia of the particles are computed which is used in the calculation cycle. Collision between particles is controlled by the coefficient of restitution. Similarly, coefficient of friction is used to allow for particles to slide past one another under limiting conditions. Thus, the model is able to predict the actual behavior of particles as it only considers physical and material properties.

The effect of fluid on the motion of particles is computed in a simplified manner. When a particle enters the fluid its Reynolds number R_e is computed. Then the drag coefficient is determined by using Abraham equation

$$C_d = 0.28\left(1 + \frac{9.06}{\sqrt{R_e}}\right)^2 \qquad \dots \quad (15)$$

where C_d is the drag coefficient. The above equation is generally valid for $R_e < 10^5$. For higher Reynolds number a constant value of 0.44 as drag coefficient is chosen. The force on a particle due to viscous drag is given by

$$F_d = 0.5 \times C_d \times \rho_f \times \upsilon^2 \times A \qquad \dots \quad (16)$$

where F_d is the force due to drag, υ is the velocity of the particle, ρ_f is the fluid density, and A is its area of cross section. Similarly, the buoyancy force is given by

$$F_b = m \times g \times \frac{\rho_f}{\rho_s} \qquad \dots \quad (17)$$

where F_b is the buoyancy force acting on a particle of mass m and density ρ_s. These forces are added to the right hand side of Eqn. 14 and the positions of the particles are found by numerical integration as discussed earlier. This models also allows one to compute the energy expended during jigging.

In a simpler case the model is tested to study the stratification in a bed consisting of 100 particles. The liquid pulsation rate is controlled by the movement of the piston located at the bottom of the jig. Fig 3 shows the results simulation where the progress of stratification is evident from snapshots 1 to through 6. After only 15 strokes the particles are completely separated; dense particles settle at the bottom and the lighter particles go to the top. Although simplistic in nature, the result of the simulation is consistent with experimental observation in a model jig. This model is being modified to incorporate a more rigorous treatment for the motion of fluid by using MAC technique.

DISCUSSION

In this paper several mathematical models that aim at describing the jigging process are critically reviewed. Modeling of the stratification process is not a formidable task as long as a good knowledge of the physics of the stratification process is available. In particular, the understanding of the fluid flow pattern through the porous structure of the jig bed is central to the manner in which the bed stratifies. The fluid flow of this nature draws analogies to many of the main classes of fluid flow that have been extensively studied in other disciplines of engineering. In fact one can look at it as flow through a fixed bed because the fluid has to find its way through the porosity of the bed and it is this porosity that keeps changing with time that eventually leads to a stratified bed. At the same time, it does share

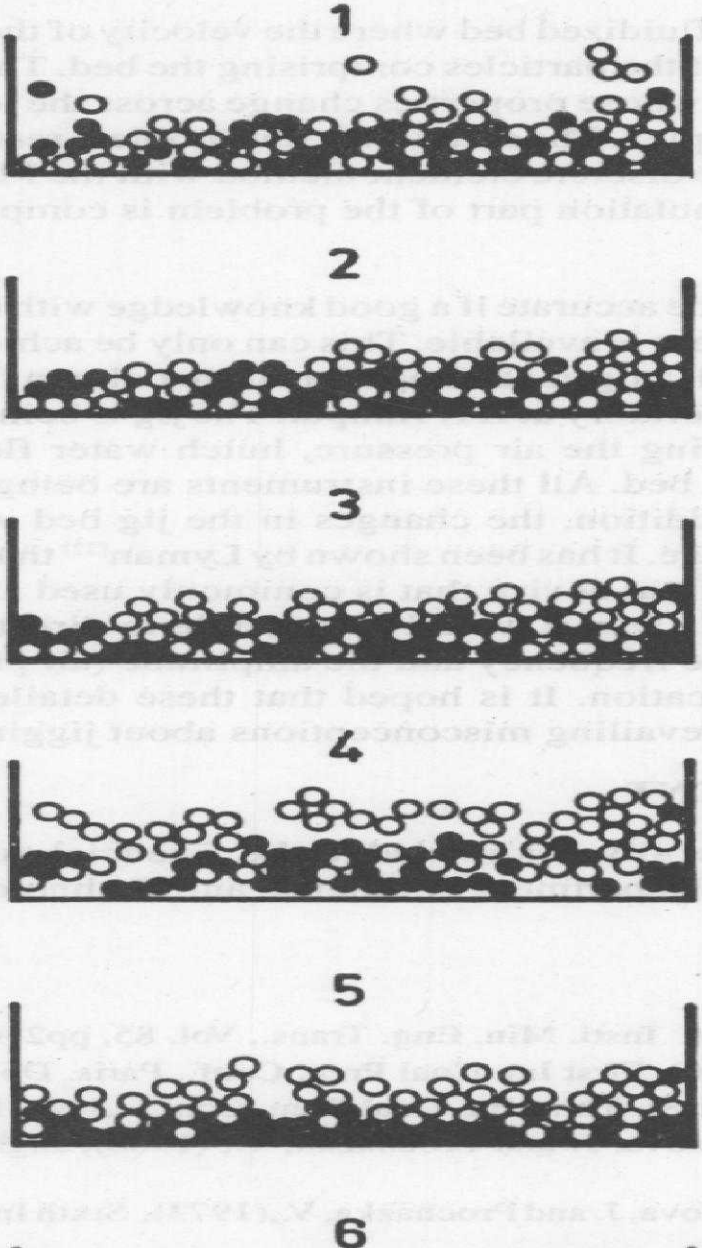

Fig. 3 : Simulation result showing progress of jigging.

some properties with fluidized bed where the velocity of the fluid has a significant effect on the motion of the particles comprising the bed. The fact that the fluid has to flow through a bed whose properties change across the length of the bed makes the problem very complex. The solution to this problem rests on a massive numerical effort coupling the discrete element method with the MAC method. While the discrete element formulation part of the problem is completed, the MAC part is still under progress.

A model can only be accurate if a good knowledge with regard to the governing principles of the process is available. This can only be achieved by careful experimentation. At this end, a jig consisting of a 30 cm column for particle bed is being constructed in our laboratory at IIT, Kanpur. The jig is being instrumented to have provision for measuring the air pressure, hutch water flow, and pressure drop across the particulate bed. All these instruments are being interfaced with a personal computer. In addition, the changes in the jig bed will be monitored by a nucleonic density gauge. It has been shown by Lyman[22] that nuclear density gauge is far superior to the float device that is commonly used as a sensor to assess the status of the bed. This way it would be possible to directly relate the operating parameters such as the frequency and the amplitude (air pressure) of pulsation to the degree of stratification. It is hoped that these detailed measurements shall dispel many of the prevailing misconceptions about jigging.

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